Nonlinear Compton – γ beam profiles

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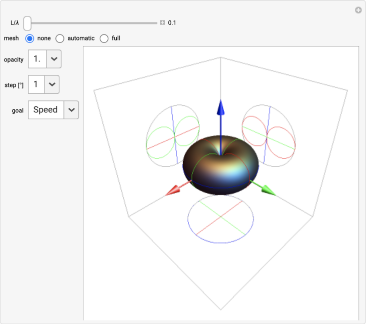
August 31st, 2021

Weak Field $\xi \ll 1$: (Boosted) Dipole



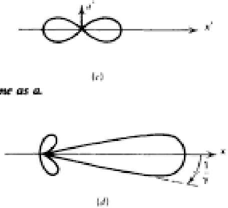
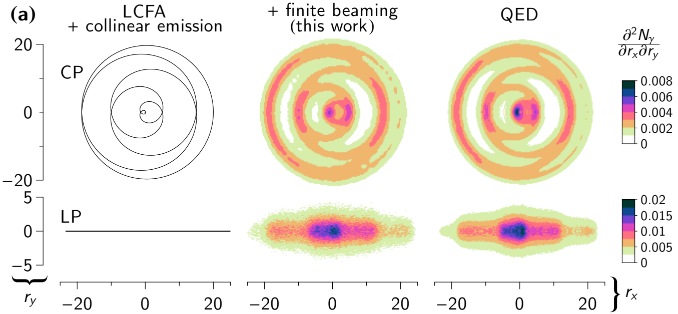


Figure 4.11c Same as a.

Figure 4.11d Angular distribution of radiation emitted by a particle with perpendicular acceleration and velocity.

- $\xi \ll 1$ pure harmonic electron oscillation $\boldsymbol{u} = \hat{\boldsymbol{x}}\,\xi \cos \omega t$ in rest frame
- relativistic beaming in LAB frame, typical emission angle $\theta \sim 1/\gamma$
- LUXE: 16.5 GeV $\Rightarrow \theta \sim 0.031$ mrad: **Size of gamma beam is 0.31 mm after 10 m propagation**

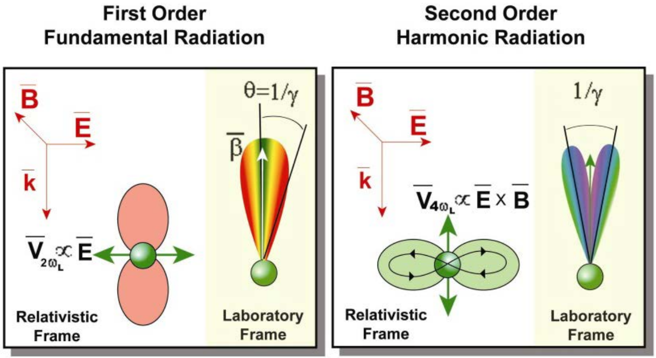
https://demonstrations.wolfram.com/DipoleAntennaRadiationPattern/

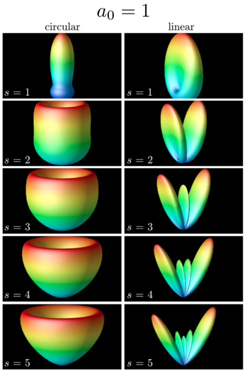


- Radiation is emitted in the direction of the instantaneous electron direction $\boldsymbol{\vartheta}$ with inherent angular spread $\theta \sim 1/\gamma$
- LP: $\tan \vartheta_x = \frac{u_x}{u_z} = \frac{\xi f(\phi)}{\gamma} \sin \phi$
- $\Rightarrow \theta \sim \xi/\gamma$ in polarization direction, $\theta \sim 1/\gamma$ perpendicular

T. Blackburn et. al. PRA **101**, 012505 (2020).

Intermediate Regime $\xi \sim 1$, LUXE

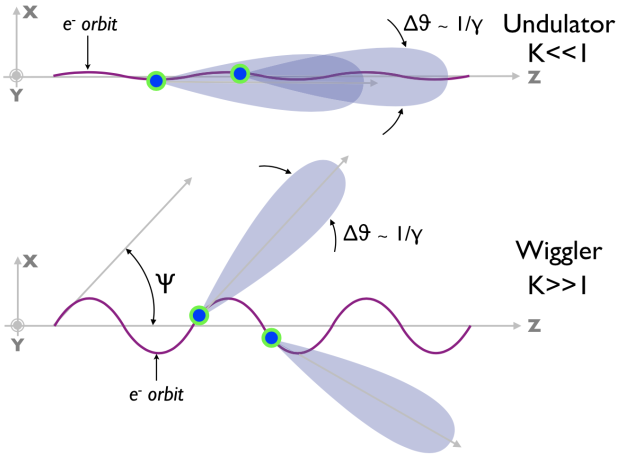




- LCFA not applicable
- An-harmonic oscillations $\Rightarrow$ harmonics have higher multipoles
- Higher harmonics have larger emission angles, $\theta \sim \xi/\gamma$

left viewgraph courtesy of BNL

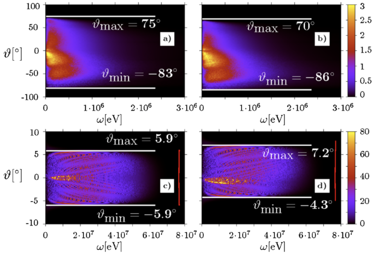
Small ξ vs Large ξ :: $K \sim \xi$



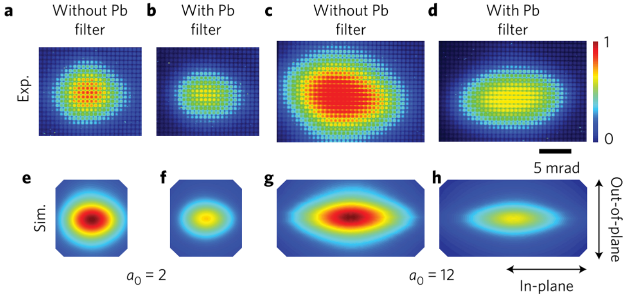
from S. Corde et al, Rev. Mod. Phys. 85, (2013).

Diagnostics using gamma-profiles

CEP[1]



ξ in nonlinear Thomson scattering[2,3]



Model-independent ξ-inferrence[4]: $\xi^2 = 4\sqrt{2}\langle\gamma_i\rangle\langle\gamma_f\rangle(\sigma_\parallel^2 - \sigma_\perp^2)$

LCFA, $\xi \gg 1$, $\chi \lesssim 1$

[1] F. Mackenroth et al PRL (2013), [2] Har-Shemesh et al Opt Lett (2012), [3] Yan et al Nature Phot (2017), [4] T. Blackburn et al PRAB (2020)

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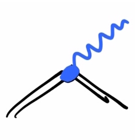
$$\frac{d^3N}{ds\,dr_x\,dr_y} = \left| \quad \right|^2$$



$$= \frac{\alpha}{16\pi^2 m^2 \eta^2}\frac{s}{1-s}\left|\bar{u}_{p'}\mathcal{M}u_p\right|^2$$

$$\mathcal{M} = \int d\phi\; e^{i\int d\phi\, \frac{k.\pi_p(\phi)}{\kappa.p(1-s)}} \left[1 + \frac{\not{a}(\phi)\not{\kappa}}{2\kappa.p(1-s)}\right] \not{\epsilon}' \left[1 + \frac{\not{\kappa}\not{a}(\phi)}{2\kappa.p}\right]$$

$$\pi_p = p - a(\phi) + \kappa\frac{a(\phi).p}{\kappa.p} - \kappa\frac{a(\phi).a(\phi)}{2\kappa.p}$$

$$s = \frac{k^+}{p^+} \approx \frac{\omega}{E_i}\,, \qquad r_{x,y} = \frac{k_{x,y}}{ms} \approx \gamma\theta_{x,y}$$

Difficulties for calculation:

- Only plane wave
- Phase integrals have highly oscillating integrands
- Need good sampling of final state phase space $s \to s_j$, $\boldsymbol{r} \to \boldsymbol{r}_{kl}$
- Interference structures in phase space $\propto 1/$pulse duration

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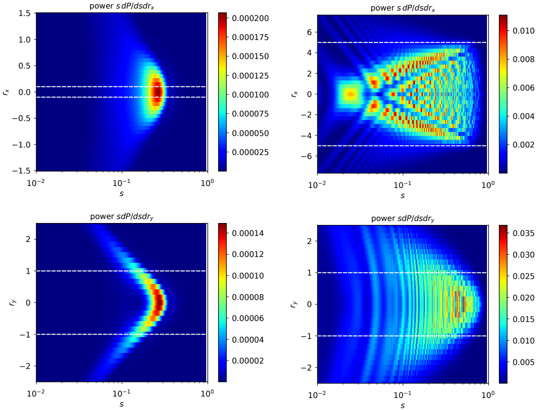
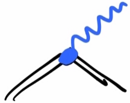
Calculation of triple-differential NLC rate (LP)

Numerical calculation: 1 or 2-cycle flat-top plane-wave @ $\xi = const.$



weak ($\xi = 0.1$) || strong ($\xi = 5$)



$$\left| \quad \right|^2 \sim \frac{d^3N}{ds\,dr_x\,dr_y}$$

$$s = \frac{k^+}{p^+} \approx \frac{\omega}{E_i}$$

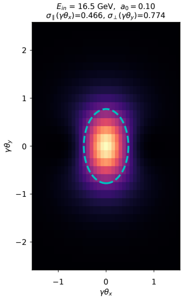
$$r_{x,y} = \frac{k_{x,y}}{ms} \approx \gamma\theta_{x,y}$$

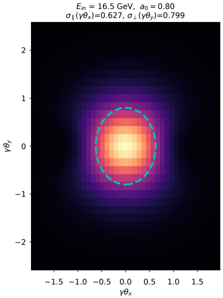
Extract angular moments of intensity distribution for fixed ξ

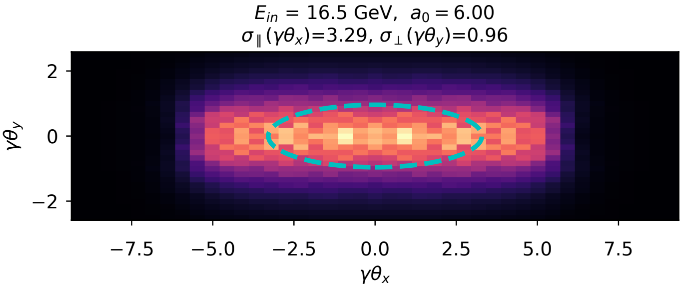
$$\text{intensity distribution: } \frac{d^2\mathcal{P}}{dr_x dr_y} = \int s \frac{d^3 N}{dr_x dr_y ds} ds$$

$$\text{total emitted power: } \mathcal{P}(\xi) = \int \frac{d^2\mathcal{P}}{dr_x dr_y} dr_x dr_y$$

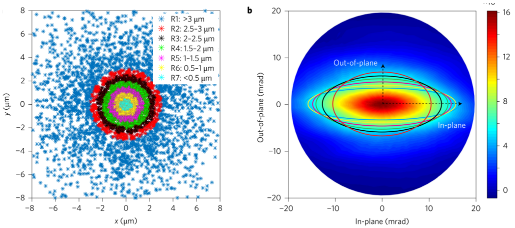
$$\text{angular variance: } \sigma_\parallel^2(\xi) = \mathcal{P}^{-1} \int (r_x - \bar{r}_x)^2 \frac{d^2\mathcal{P}}{dr_x dr_y} dr_x dr_y$$







Gaussian Focus Average (Infinite Rayleigh Range Approximation)

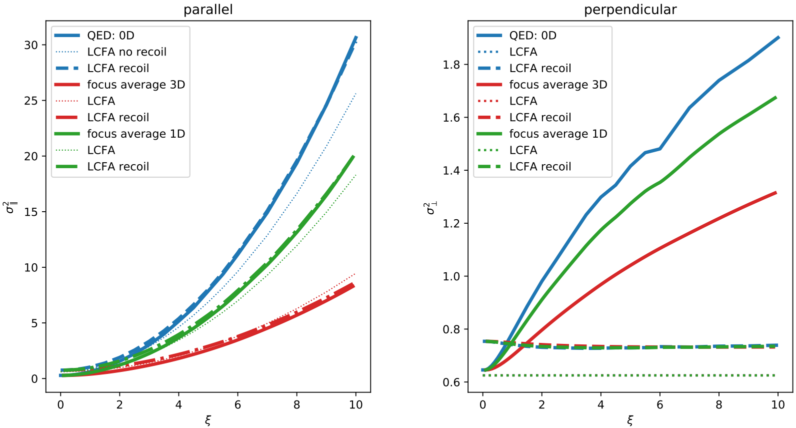


$$\text{1D:} \quad \xi = \xi(\phi) = \xi_0\, e^{-\phi^2/2\Delta^2} \quad \Rightarrow \quad \langle \sigma^2_{\perp/\parallel} \rangle = \frac{\int \sigma^2_{\perp/\parallel}(\xi)\mathcal{P}(\xi)d\phi}{\int \mathcal{P}(\xi)d\phi}$$

$$\text{3D:} \quad \xi = \xi(\boldsymbol{x}_\perp, \phi) = \xi_0 e^{-\boldsymbol{x}_\perp^2/w_0^2} e^{-\phi^2/2\Delta^2} \quad \Rightarrow \quad \langle \sigma^2_{\perp/\parallel} \rangle = \frac{\int \sigma^2_{\perp/\parallel}(\xi)\mathcal{P}(\xi)d^2x_\perp d\phi}{\int \mathcal{P}(\xi)d^2x_\perp d\phi}$$

W. Yan et al, Nature Phot. **11**, 514 (2017).

Gaussian Focus Average (Infinite Rayleigh Range Approximation)



LCFA: $$\sigma_\perp^2 = \frac{5\gamma_i}{8\gamma_f}\,, \qquad \sigma_\parallel^2 = \frac{\xi^2}{4}\frac{\gamma_i}{\gamma_f} + \sigma_\perp^2\,,$$

Blackburn et al PRAB 2020

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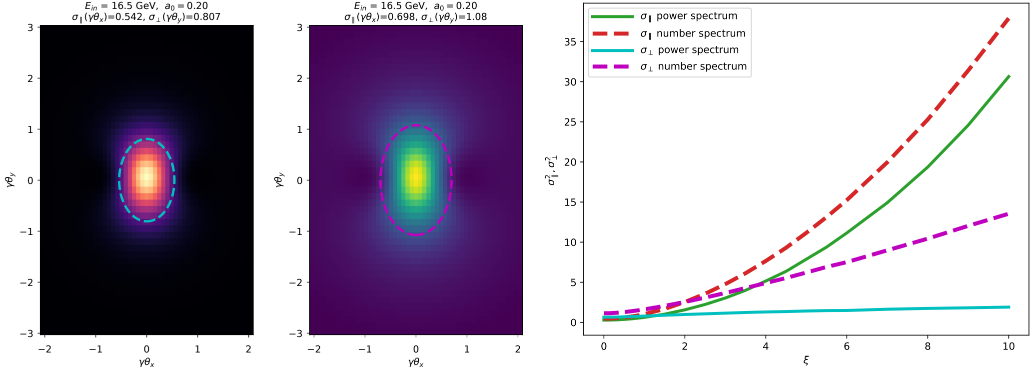
No, because (1):

- Simulations done here are restricted to *thin targets*: strictly only one emission per electron
- How to include multi-emission events (RR energy loss) in this averaging approach?
- **Conjecture:** Only first emission contributes to gamma profile since radiated power $\sim \gamma^2$ thus radiated power decreasing as electron loses energy (but smaller γ means larger emission angle)

But (2): Power or Photon Number?

- What does the γ-profiler actually measure?
- Whether to use in sims photon number spectrum or power spectrum *depends on detector*
- Studies in the literature so far based on power spectrum ($\sim$ keV X-rays)
- Differences seem not too large



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Suggestion/Discussion

- Gianluca/Marco: It's most likely the number spectrum!
- Matt: take into account the detector response $R(s)$

$$\frac{d^2\mathcal{X}_R}{dr_x dr_y} = \int R(s) \frac{d^3N}{dr_x dr_y ds} ds$$