Matching Parton Showers to NLO cross sections in the Deductor framework

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14th Annual Meeting of the Helmholtz Alliance "Physics at the Terascale"

Hamburg, 24.11.2021

Motivation

Fixed order (FO) calculations in QCD

> Systematic expansion in strong coupling :

$$\sigma[O_J] = \sigma^0[O_J] + \frac{\alpha_s}{2\pi}\sigma^1[O_J] + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^2[O_J] + \dots$$

> O_J inclusive / single scale $(\mu \approx Q) \;\Rightarrow\; \alpha_s \ll 1,$ FO descriptive ✔

> O_J exclusive / multiple scales $(\mu_J \ll \mu \approx Q) \;\Rightarrow\; \frac{\alpha_s}{2\pi} \to \frac{\alpha_s}{2\pi}$ $\log^2 \frac{\mu^2}{\mu_J^2}$, perturbation series spoiled ✖

Analytic resummation

> Find universal structure in coefficients of leading (next-to-leading, …) logs at all orders, exponentiate ✔

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Is there another solution?

Parton showers can give an accurate (to some logairthmic order) and observable independent description.

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Parton Showers (PS)

- Take the partonic state at a high scale $\mu_s = \mu_h \sim Q$, where FO is accurate
- Evolve the partonic state by dressing it with radiative corrections characterized by decreasing μ_s
- Finish the evolution at a low scale $\mu_s = \mu_f \sim \mathcal{O}(1\,\text{GeV})$
- Apply hadronization model at μ_f
- Measurement O_J at this low scale, on the generated exclusive final state

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Requirements

- Unitarity: shouldn't induce change in inclusive cross section
- The evolution should take color and spin correlations into account, at least at the formal level
- Otherwise the practical approximation in color, should be able to be improved systematically
- Level of logarithmic accuracy can be checked against analytic resummation:
  - analytically for some specific observables
  - numerically

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What's left

One still has to match the parton shower to fixed order results.

Formalism

Statistical space

> Renormalized amplitudes in color⊗spin space:

$$\big|M(\{p,f\}_n,\mu^2)\big\rangle = \sum_{\{s,c\}_n} \overbrace{m(\{p,f,s,c\}_n,\mu^2)}^{\text{color-helicity sub-amplitudes}} \; \big|\{s,c\}_n\big\rangle$$

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- > Density matrix of partonic states ⇔ states in statistical space

$$|\rho(\mu^2)) = \sum_n \frac{1}{n!}\int [d\{p\}_n] \sum_{\{f\}_n} \sum_{\{s,c,s',c'\}_n} \rho(\{p,f,s,c,s',c'\}_n,\mu^2)|\{p,f,s,c,s',c'\}_n)$$

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- The cross section this way: $\sigma[O_J] = (1|\underbrace{O_J\big[\overbrace{\mathcal{F}(\mu^2)\circ\mathcal{Z}_{\mathcal{F}}(\mu^2)}^{\text{bare PDFs}}\big]}_{\text{operators on density states}}|\rho(\mu^2))$

Formalism

Example, $gg \to gg$

$$\left|M^{(0)}(\{p\}_2)\right\rangle = \sum_{\{s\}_2} \sum_{S_C\{2,3,4\}} |\{s\}_2\rangle \otimes \Big(\big|(1,2,3,4)\big\rangle + \big|(4,3,2,1)\big\rangle\Big) m(1^{s_1}, 2^{s_2}, 3^{s_3}, 4^{s_4})$$

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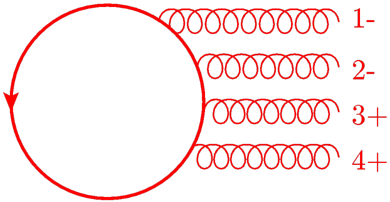

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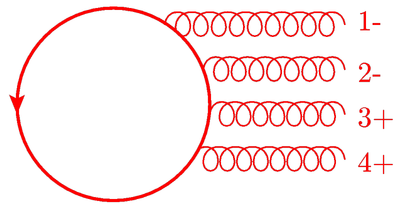

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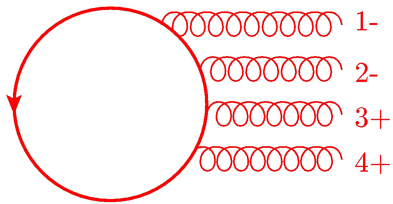
> The spin averaged density operator ($x = \frac{s_{a1}}{s_{ab}}$):

$$\mathrm{Tr_s}\left(\left|M^{(0)}\right\rangle\left\langle M^{(0)}\right|\right) = 32(4\pi\alpha_s)^2 C_F^4 \frac{1 + x^4 + (1-x)^4}{x^2(1-x)^2} \times \sum_{I,J} \rho^{(0)}_{IJ}(\{p\}_2) \, |I_c\rangle \langle J_c|$$

$$\rho^{(0)}_{IJ}(\{p\}_2) = \begin{bmatrix} x^2 & x(1-x) & -x \\ x(1-x) & (1-x)^2 & x-1 \\ -x & x-1 & 1 \end{bmatrix}$$



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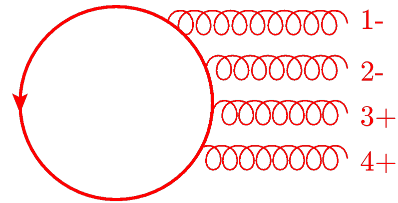

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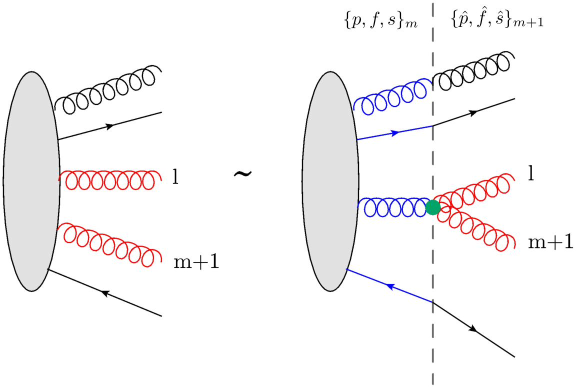
- nr of color bases is 6 $\Rightarrow$ 36 color density states
- +1 $g \to$ 576 states
- all evolve individually in a shower
- in general, nr of bases in color density $\propto (n!)^2$

Matrix elements factorize on IR poles

- Consider $\hat{p}_{m+1} \to \lambda \hat{p}_l$:

$$\left| M(\{\hat{p}, \hat{f}\}_{m+1}) \right\rangle$$

$$\sim \underbrace{t_l^\dagger(f_l \to \hat{f}_l + \hat{f}_{m+1})}_{\text{color factor}} \overbrace{V_l^\dagger(\{\hat{p}, \hat{f}\}_{m+1})}^{\text{splitting function}} \underbrace{|M(\{p, f\}_m)\rangle}_{\text{hard state}}$$



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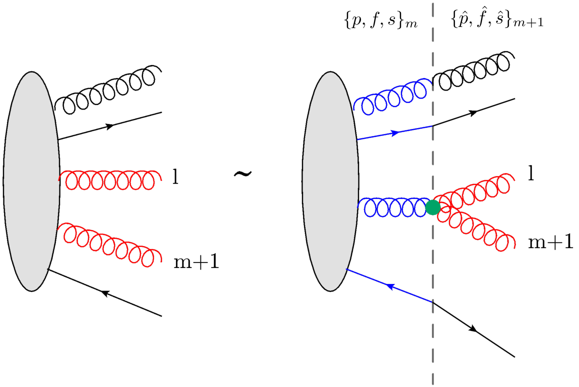

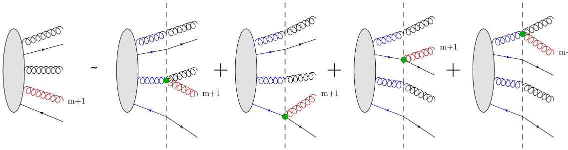

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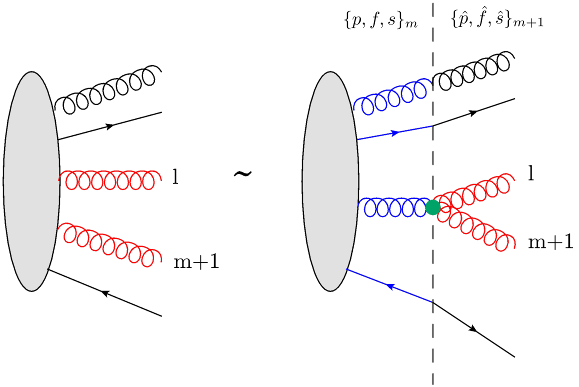
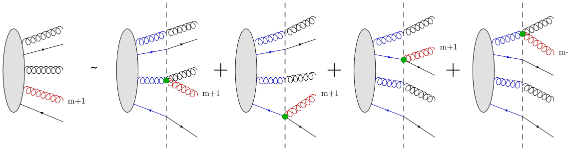
- Exact in the limit
- Splitting functions from Feynman diagrams
- Momentum mapping either global or local





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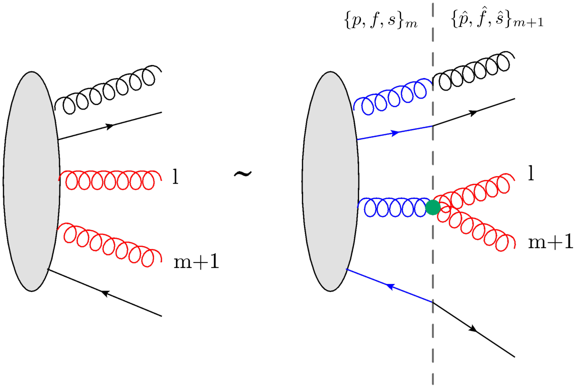

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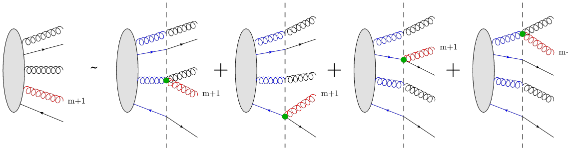


In statistical space?

This sort of factorisation directly translates to stastical space states as well!

IR-sensitive operator

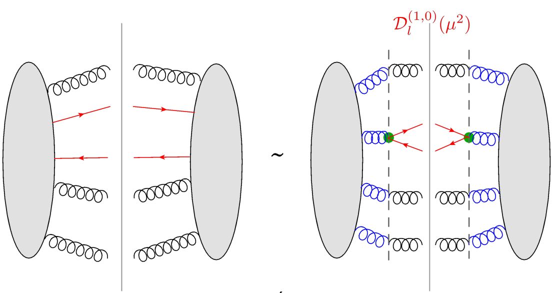
- Consider a state $|\rho(\mu^2))$ with (n_R, n_V) real + loop momenta close to IR poles
- We would expect a factorisation of the form $|\rho(\mu^2)) \sim \mathcal{D}^{(n_R,n_V)}(\mu^2)|\rho_{\text{hard}}(\mu^2))$
- $\mathcal{D}(\mu^2)$ is the "IR-sensitive operator", that contains the process independent singularities

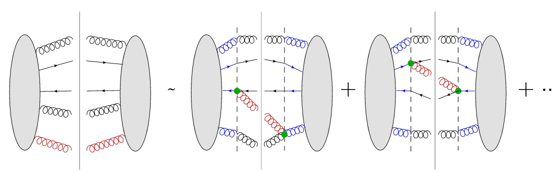
$$\begin{aligned}\mathcal{D}(\mu^2) &= 1 + \frac{\alpha_s}{2\pi}\mathcal{D}^{(1)}(\mu^2) + \left(\frac{\alpha_s}{2\pi}\right)^2\mathcal{D}^{(2)}(\mu^2) + \dots \\ &= 1 + \frac{\alpha_s}{2\pi}\Big(\mathcal{D}^{(1,0)}(\mu^2) + \mathcal{D}^{(0,1)}(\mu^2)\Big) \\ &\quad + \left(\frac{\alpha_s}{2\pi}\right)^2\Big(\mathcal{D}^{(2,0)}(\mu^2) + \mathcal{D}^{(1,1)}(\mu^2) + \mathcal{D}^{(0,2)}(\mu^2)\Big) + \dots\end{aligned}$$

- Example:

$\hat{p}_{m+1} \to \lambda\hat{p}_l$: $|\rho(\mu^2)) \sim \mathcal{D}_l^{(1,0)}(\mu^2)|\rho_h(\mu^2))$

$\hat{p}_{m+1} \to 0$: $\sum_{l,k}\mathcal{D}_{lk}^{(1,0)}(\mu^2)|\rho_h(\mu^2))$





IR-sensitive operator: construction

- We can construct $\mathcal{D}$ at each order based on the factorisation of matrix elements

$$
\begin{aligned}
&(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n_R}|\mathcal{D}^{(n_R,n_V)}(\mu^2,)|\{p,f,s,s',c,c'\}_m) \\
&= \sum_G \int d^d\{l\}_{n_V}\ {}_D\langle\{\hat{s},\hat{c}\}_{m+n_R}|\,\mathbf{V}_L(G;\{\hat{p},\hat{f}\}_{m+n_R},\{l\}_{n_V},\mu^2)\,|\{s,c\}_m\rangle \\
&\quad\times \langle\{s,c\}_m|\,\mathbf{V}_R^\dagger(G;\{\hat{p},\hat{f}\}_{m+n_R},\{l\}_{n_V},\mu^2)\,|\{\hat{s},\hat{c}\}_{m+n_R}\rangle_D \\
&\times \sum_{I\in\text{Regions}(G)} (\{\hat{p},\hat{f}\}_{m+n_R}|\mathcal{P}_G(I)|\{p,f\}_m)
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&\times \sum_{I\in\text{Regions}(G)} (\{\hat{p},\hat{f}\}_{m+n_R}|\mathcal{P}_G(I)|\{p,f\}_m)\Theta_G(I;\{\hat{p},\hat{f}\}_{m+n_R},\{l\}_{n_V};\mu_s^2)
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- Introduce a UV cutoff to capture only the IR parts
- Cuts off is in the IR-sensitive splitting variable (e.g.: virtuality or $k_\perp$) at the splitting scale, μ_s

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$$

- Introduce a UV cutoff to capture only the IR parts
- Cuts off is in the IR-sensitive splitting variable (e.g.: virtuality or $k_\perp$) at the splitting scale, μ_s

Up to first order

$$\mathcal{D}\mathcal{D}^{-1} = 1 \quad \Rightarrow \quad \mathcal{D}^{-1}(\mu^2,\mu_s^2) = 1 - \frac{\alpha_s(\mu^2)}{2\pi}\mathcal{D}^{(1)}(\mu^2,\mu_s^2)$$

Parton shower cross section

Fixed order cross section:

$$\sigma[O_J] = (1|O_J(\mu_J^2)\big[\mathcal{F}(\mu_H^2) \circ \mathcal{Z}_{\mathcal{F}}(\mu_H^2)\big]|\rho(\mu_H^2))$$

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- $|\rho_H(\mu_H^2)) = \mathcal{D}^{-1}(\mu_H^2)|\rho(\mu_H^2))$ is the hard state, $\mathcal{D}^{-1}$ supplies subtraction (see in a minute)
- $\mathcal{X}(\mu_H^2)$ is an operator derived from $\mathcal{D}$ and PDFs; it changes the particle number, flavor, spin and color
- As long as $\mu_J \gg \mu_H$ the effect of $\mathcal{X}$ is unresolved, and

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However...

Usually $\mu_J \ll \mu_H \Rightarrow$ terms of $\log^2 \frac{\mu_H^2}{\mu_J^2}$ **appear**

Parton shower cross section

We introduce a low scale, $\mu_f \ll \mu_J$, $\mu_f \sim \mathcal{O}(1\text{ GeV})$, and through a series of derivation we would reach:

$$\sigma[O_J] = (1|O_J(\mu_J^2)\underbrace{X^{-1}(\mu_f^2)\mathcal{X}(\mu_H^2)}_{\mathcal{U}(\mu_f^2,\mu_H^2)}\mathcal{V}(\mu_H^2)\mathcal{F}(\mu_H^2)|\rho_H(\mu_H^2))$$

- The operator $\mathcal{U}(\mu_f^2, \mu_H^2)$ is the parton shower evolution:

$$\mathcal{U}(\mu_f^2, \mu_H^2) = \mathbb{T}\,\text{exp}\left\{ \int_{\mu_f^2}^{\mu_H^2} \frac{d\mu^2}{\mu^2}\mathcal{S}(\mu^2) \right\}$$

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Matching

Use of subtracted cross sections at the same order as the evolution kernel!

Matching

Matching to LO-shower ⇔ NLO subtraction

$$
\begin{aligned}
|\rho_H(\mu^2)) &= \mathcal{D}^{-1}(\mu^2, \mu_s^2 = \mu^2)|\rho(\mu^2)) \\
&= |\rho^{(0)}) + \frac{\alpha_s\left(\mu^2\right)}{2\pi}\left[|\rho^{(1,0)}(\mu^2)) - \mathcal{D}^{(1,0)}(\mu^2)|\rho^{(0)})\right] \\
&\quad + \frac{\alpha_s\left(\mu^2\right)}{2\pi}\left[|\rho^{(0,1)}(\mu^2)) - \mathcal{D}^{(0,1)}(\mu^2)|\rho^{(0)})\right]
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$$

- $|\rho^{(0)})$ Born process
- $|\rho^{(1,0)}(\mu^2))$ Real radiation corrections
- $|\rho^{(0,1)}(\mu^2))$ Virtual corrections

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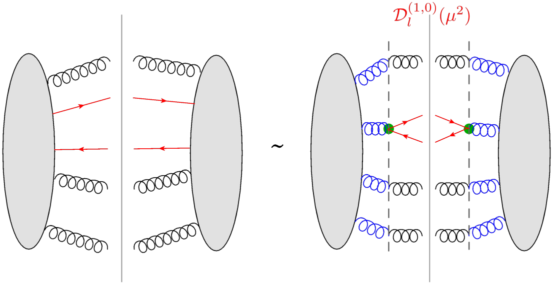
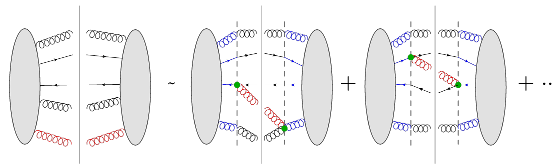
This form of subtractions

Instead of $\sigma_H[O_J]$ calculation, we have " $\sigma_H[O_J] \times (\text{nr color bases})^2$ " calculations!

- Acts on the density state level, instead of ME-squared
- Defines subtractions for all possible colour states to cancel all occuring singularities
- Serves as input for a parton shower evolution that can treat color
- The criterion for matching is using the same $\mathcal{D}$ operator as the shower does

Matching





Tracing over spins (exact in the easiest case of $2 \to 2$ as LO)

> The real radiation part directly from amplitudes:

$$\mathcal{D}^{(1,0)}(\mu^2)|\{p,f,c,c'\}_m) = \sum_{l,k} \int d\xi \, |\{\hat{p},\hat{f}\}_{m+1}) \, \Theta(f(\xi) < \mu^2) \, \lambda_{lk}(p,f_m,\xi)$$
$$\times \sum_{\{\hat{c},\hat{c}'\}_{m+1}} |\{\hat{c},\hat{c}'\}_{m+1}) \, G(k,l,\xi_f,\{\hat{c},\hat{c}'\}_{m+1},\{c,c'\}_m)$$

> Real part of virtuals defined from this:

$$\mathrm{Re}(\mathcal{D}^{(0,1)}(\mu^2)) = -\overline{\mathcal{D}^{(1,0)}}(\mu^2)$$

Matching - status

Jet production at the LHC

- LO: all $2 \to 2$ partonic processes, e.g.: $gg \to gg$, $gg \to q\bar{q}$, $qg \to qg$, $\ldots$
  - already at this level color is not trivial (vs. e^+e^- or DY) $\Rightarrow$ first non trivial matching in this sence
- NLO (real): all $2 \to 3$ partonic processes, e.g.: $gg \to ggg$, $q\bar{q} \to r\bar{r}$, $gg \to q\bar{q}g$
  - nr of colour bases blow up
  - tracing over spins (this is the last level where this is exact)
- NLO (virtual): loop corrections to LO
  - Real part of subtraction from the real subtraction term (KLN)

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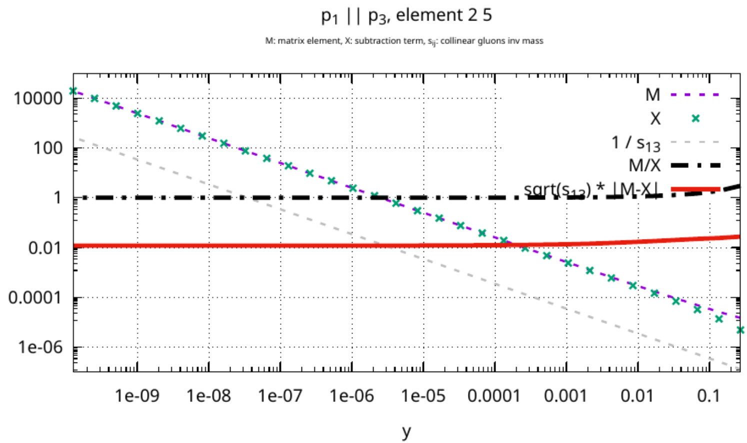
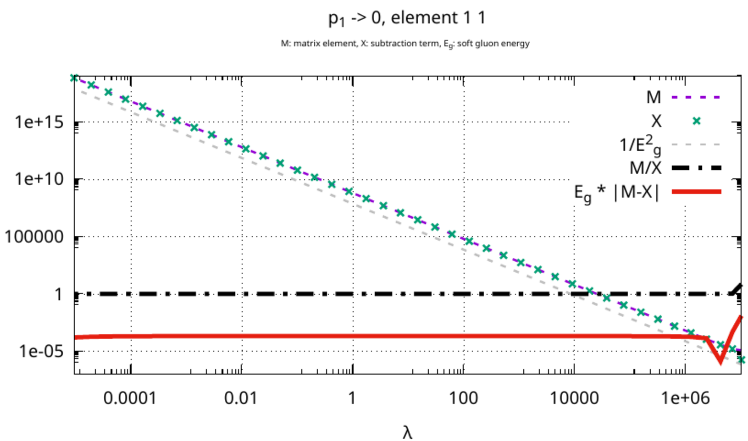
- Born, subtracted real corretion are implemented and checked ✔
- Virtual corrections are under implementation now


p1 || p3, element 2 5
M: matrix element, X: subtraction term, sij: collinear gluons inv mass
10000
100
1
0.01
0.0001
1e-06
1e-09
1e-08
1e-07
1e-06
1e-05
0.0001
0.001
0.01
0.1
y
M
X
1 / s13
M/X
sqrt(s13) * |M-X|


p1 -> 0, element 1 1
M: matrix element, X: subtraction term, Eg: soft gluon energy
1e+15
1e+10
100000
1
1e-05
0.0001
0.01
1
100
10000
1e+06
λ
M
X
1/E2g
M/X
Eg * |M-X|

Summary

- Parton showers are process independent tools that work in conditions when fixd order perturbation theory breaks down
- These parton shower and fixed order results should matched
- We saw, that formally a parton shower can handle full color
- So the matching should happen at this level too

Thank you!

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