Two-loop splitting in double parton distributions.

the colour non-singlet case

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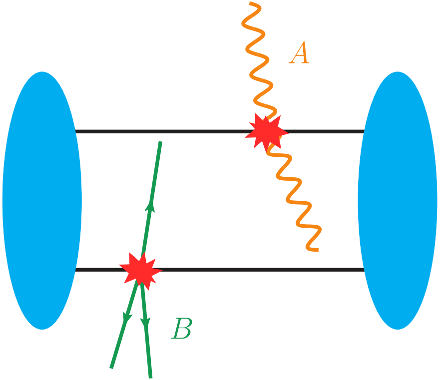
Introduction.

What is double parton scattering?

Double parton scattering (DPS) describes two individual hard interactions in a single hadron-hadron collision:

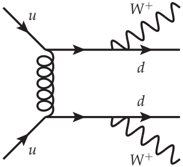


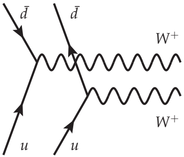
DPS is naturally associated with the situation where the final state can be separated into two subsets with individual hard scales.

Why is DPS interesting?

- Whilst generally suppressed compared to SPS, DPS may be enhanced for final states with small transverse momenta or large separation in rapidity.

- When production of final states via SPS involves small coupling constants or higher orders, DPS may give leading contributions (like-sign W production):





$\longrightarrow$ background to the search for new physics with like-sign lepton pairs.

- Relative importance of DPS increases with collision energy ($\sigma_{\text{DPS}} \sim \text{PDF}^4$ vs. $\sigma_{\text{SPS}} \sim \text{PDF}^2$).

- DPS gives access information about hadron structure not accessible in other processes: spatial, spin, and colour correlations between two partons.

Factorization for DPS.

Pioneering work already in the 80's:

LO factorisation formula based on a parton model picture [Politzer, 1980; Paver and Treleani, 1982; Mekhfi, 1985]

$$\sigma_{pp\to A,B} = \hat{\sigma}_{ik\to A}(x_1\bar{x}_1 s)\,\hat{\sigma}_{jl\to B}(x_2\bar{x}_2 s) \times \int \mathrm{d}^2\boldsymbol{y}\, F_{ij}(x_1, x_2, \boldsymbol{y}; Q_1^2, Q_2^2)\, F_{kl}(\bar{x}_1, \bar{x}_2, \boldsymbol{y}; Q_1^2, Q_2^2)$$

Increasing interest in DPS in the LHC era:

- First experimental data already from previous colliders at CERN and Tevatron, new measurements from LHC with more to come.
- Progress also from theory:
  - Systematic QCD description. [Blok et al., 2011; Diehl et al., 2011; Manohar and Waalewijn, 2012; Ryskin and Snigirev, 2012]
  - Factorization proof for double DY. [Diehl, Gaunt, Plößl, and Schäfer, 2015; Diehl and Nagar, 2019]
  - Disentangling SPS and DPS. [Gaunt and Stirling, 2011; Diehl, Gaunt, and Schönwald, 2017]

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Theory: DPD basics.

Definition of DPDs.

Bare position space DPDs:

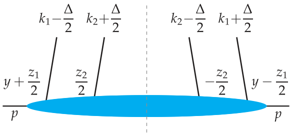
$$F^{r_1 r_1' \, r_2 r_2'}_{Bus,a_1 a_2}(x_1, x_2, \boldsymbol{y}) = (x_1 p^+)^{-n_1} (x_2 p^+)^{-n_2} \, 2p^+ \int dy^- \, \frac{dz_1^-}{2\pi} \, \frac{dz_2^-}{2\pi} \, e^{i(x_1 z_1^- + x_2 z_2^-) p^+}$$

$$\times \langle p | \, \mathcal{O}^{r_1 r_1'}_{a_1}(y, z_1) \, \mathcal{O}^{r_2 r_2'}_{a_2}(0, z_2) \, | p \rangle \big|_{y^+=0} \, ,$$

$$\mathcal{O}^{ii'}_q(y,z) = \bar{q}_{j'}(\xi_-) \, [W^\dagger(\xi_-, v_L)]_{j'i'} \, \frac{\gamma^+}{2} \, [W(\xi_+, v_L)]_{ij} \, q_j(\xi_+) \, ,$$

$$\mathcal{O}^{aa'}_g(y,z) = \left[G^{+k}(\xi_-)\right]^{b'} \left[W^\dagger(\xi_-, v_L)\right]^{b'a'} \left[W(\xi_+, v_L)\right]^{ab} \left[G^{+k}(\xi_+)\right]^{b} ,$$

with $\xi_\pm = y \pm z/2$, $z^+ = 0$, $\boldsymbol{z} = \boldsymbol{0}$.



Bare momentum space DPDs:

$$F^{r_1 r_1' \, r_2 r_2'}_{Bus,a_1 a_2}(x_1, x_2, \boldsymbol{\Delta}) = \int d^{2-2\epsilon}\boldsymbol{y} \; e^{i\boldsymbol{y}\boldsymbol{\Delta}} \, F^{r_1 r_1' \, r_2 r_2'}_{Bus,a_1 a_2}(x_1, x_2, \boldsymbol{y}) \, .$$

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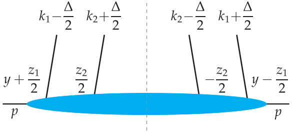

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Decomposing the colour structure of DPDs.

The colour indices in the definition of the DPDs can be coupled to an overall colour singlet in a variety of ways. [Mekhfi, 1985] In order to make this more systematic we:

- Couple the fields pairwise (r_i and r_i') to irreducible representations R_i of $SU(N)$ such that $R_1 R_2$ is a colour singlet.
- Decompose the full colour structure in terms of these combinations:

$$F^{r_1 r_1' \, r_2 r_2'}_{B\text{us},a_1 a_2}(x_1, x_2, \boldsymbol{y}) \sim \sum_{R_1, R_2} P^{r_1 r_1' \, r_2 r_2'}_{R_1 R_2} \; {}^{R_1 R_2}F_{B\text{us},a_1 a_2}(x_1, x_2, \boldsymbol{y})$$

In addition to $R_1 R_2 = 1\,1$ one finds the following colour non-singlet channels:

- $R_1 R_2 = 8\,8$ for $a_1 a_2 = q q'$.
- $R_1 R_2 = 8\,A$ and $8\,S$ for $a_1 a_2 = q g$.
- $R_1 R_2 = A\,A$, $S\,S$, $A\,S$, $S\,A$, $10\,\overline{10}$, $\overline{10}\,10$ and $27\,27$ for $a_1 a_2 = g g$.

Rapidity divergences in colour non-singlet DPDs.

DPDs in colour non-singlet channels exhibit rapidity divergences, which cancel only when combined with the DPS soft factor [Buffing, Diehl, and Kasemets, 2017]:

$$ {}^{R_1R_2}F_B(x_1, x_2, y, \zeta_p) = \lim_{\rho\to\infty} \frac{{}^{R_1R_2}F_{B\,\mathrm{us}}(x_1, x_2, y, \rho)}{\sqrt{{}^{R_1}S_B(y, 2\ell_L(\rho, \zeta_p))}} \,, $$

DPS analog for TMD subtraction [Collins, 2011].

where the limit $\rho \to \infty$ corresponds to removing the rapidity regulator.

$\longrightarrow$ DPDs pick up a rapidity dependence, which is governed by a Collins-Soper type equation:

$$ \frac{\partial}{\partial \log \zeta_p} \log {}^{R_1R_2}F(x_1, x_2, y; \mu, \zeta_p) = {}^{R_1}J(y, \mu)/2 \,, \qquad \text{with} \quad \frac{\partial}{\partial \log \mu^2} {}^{R}J(y; \mu) = -{}^{R}\gamma_J(\mu) \,. $$

Renormalization of UV divergences.

Renormalized position space DPDs:

$$ {}^{R_1 R_2}F(x_1, x_2, y, \mu, \zeta_p) = \sum_{R_1' R_2'} {}^{R_1 \overline{R}_1'}Z(\mu, x_1^2 \zeta_p) \underset{1}{\otimes} {}^{R_2 \overline{R}_2'}Z(\mu, x_2^2 \zeta_p) \underset{2}{\otimes} {}^{R_1' R_2'}F_B(y, \mu, \zeta_p) \,. $$

with individual renormalization factors Z for each of the twist-2 operators in the definition of bare DPDs.

Double DGLAP equation for position space DPDs:

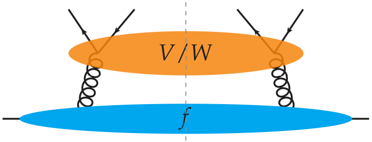
$$ \frac{\partial}{\partial \log \mu^2} \, {}^{R_1 R_2}F_{a_1 a_2}(x_1, x_2, y, \mu, \zeta_p) = \sum_{b_1, R_1'} {}^{R_1 \overline{R}_1'}P_{a_1 b_1}(\mu, x_1^2 \zeta_p) \underset{1}{\otimes} {}^{R_1' R_2}F_{b_1 a_2}(y, \mu, \zeta_p) + \sum_{b_2, R_2'} {}^{R_2 \overline{R}_2'}P_{a_2 b_2}(\mu, x_2^2 \zeta_p) \underset{2}{\otimes} {}^{R_1 R_2'}F_{a_1 b_2}(y, \mu, \zeta_p) \,, $$

Perturbative splitting in DPDs.

In the limit of small distance $\boldsymbol{y}$ (and correspondingly large $\boldsymbol{\Delta}$) the leading contribution to a DPD is due to the perturbative splitting of one parton into two:

$$^{R_1R_2}F(x_1,x_2,\Delta;\mu,\zeta_p) = {}^{R_1R_2}W(\Delta;\mu,x_1x_2\zeta_p) \underset{12}{\otimes} f(\mu)\,,$$

$$^{R_1R_2}F(x_1,x_2,y;\mu,\zeta_p) = \frac{\Gamma(1-\epsilon)}{(\pi y^2)^{1-\epsilon}}\, {}^{R_1R_2}V(y;\mu,x_1x_2\zeta_p) \underset{12}{\otimes} f(\mu)\,,$$



where

$$\Big[V \underset{12}{\otimes} f\Big](x_1,x_2) = \int_x^1 \frac{dz}{z^2}\, V\Big(\frac{x_1}{z},\frac{x_2}{z}\Big)\, f(z) = \frac{1}{x}\int_x^1 dz\; V(uz,\bar{u}z)\, f\Big(\frac{x}{z}\Big)$$

with

$$x = x_1 + x_2\,, \qquad u = \frac{x_1}{x_1+x_2}\,, \qquad \bar{u} = 1 - u\,.$$

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formally OPE of $\mathcal{O}(y,z_1)\mathcal{O}(0,z_2)$ for $y \to 0$

$$^{R_1R_2}F(x_1,x_2,y;\mu,\zeta_p) = \frac{\Gamma(1-\epsilon)}{(\pi y^2)^{1-\epsilon}}\, {}^{R_1R_2}V(y;\mu,x_1x_2\zeta_p) \underset{12}{\otimes} f(\mu)\,,$$

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Calculation: Goals.

What we calculate and how we do this.

One of the last missing piece for colour non-singlet NLO DPS calculations in the framework of [Diehl, Gaunt, and Schönwald, 2017] are the NLO coefficients of the V splitting kernels.

⟶ Already calculated these for the colour singlet case. [Diehl, Gaunt, Plößl, and Schäfer, 2019]

⟶ Extend this now to the colour non-singlet sector. This will also allow us to study colour correlations in DPS.

For the actual calculation we first calculate ${}^{R_1R_2}W^{(2)}_{B\,\mathrm{us}}(\Delta, \rho)$ and then extract the renormalized ${}^{R_1R_2}V^{(2)}$ by performing a RGE analysis.

We perform the calculation for two different rapidity regulators:

- Collins regulator using space-like Wilson lines. [Collins, 2011]
- δ regulator. [Echevarria, Scimemi, and Vladimirov, 2016]

⟶ First application (to our knowledge) of the Collins regulator to a two loop calculation!

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- Collins regulator using space-like Wilson lines. [Collins, 2011]
- δ regulator. [Echevarria, Scimemi, and Vladimirov, 2016]

⟶ Obtain identical results in both schemes!

Calculation: $W^{(2)}_{B\mathrm{us}}$.

From Feynman diagrams to $W^{(2)}_{B\text{us}}$.

The NLO $a_0 \to a_1 a_2$ kernel $W^{(2)}_{B\text{us},a_1 a_2,a_0}$ can be obtained by calculating the DPD for partons a_1, a_2 in parton a_0:

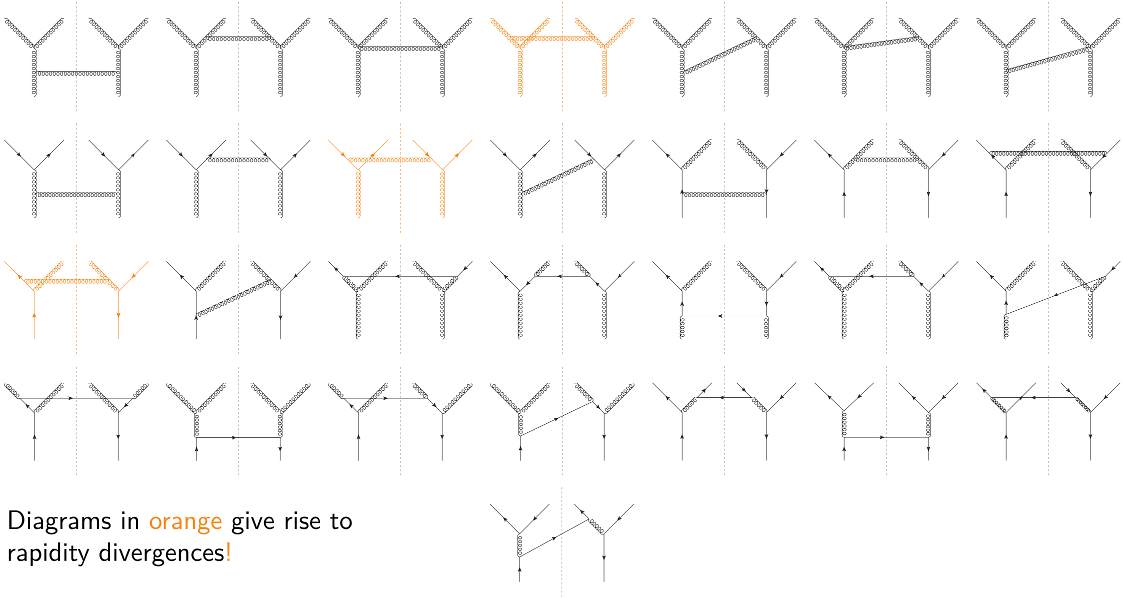
$$F^{(2)}_{B\text{us},a_1 a_2/a_0}(\Delta,\rho) = \sum_b \Big[W^{(2)}_{B\text{us},a_1 a_2,b}(\Delta,\rho) \underset{12}{\otimes} f^{(0)}_{B,b/a_0} + W^{(1)}_{B\text{us},a_1 a_2,b}(\Delta,\rho) \underset{12}{\otimes} f^{(1)}_{B,b/a_0} \Big] = W^{(2)}_{B\text{us},a_1 a_2,a_0}(\Delta,\rho)$$

At $\mathcal{O}(\alpha_s^2)$ we find the following splitting kernels:

- *LO* channels: $g \to gg$, $g \to q\bar{q}$, and $q \to qg$
- *NLO* channels: $g \to qg$, $q \to gg$, $q_j \to q_j q_k$, $q_j \to q_j \bar{q}_k$, $q_j \to q_k \bar{q}_k$

Note: Only *LO* channels exhibit rapidity divergences.

Calculating $W_{Bus}^{(2)}$.



Diagrams in orange give rise to rapidity divergences!

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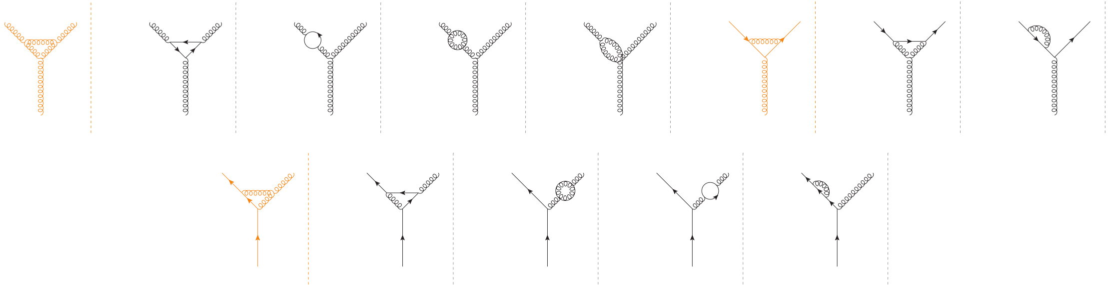
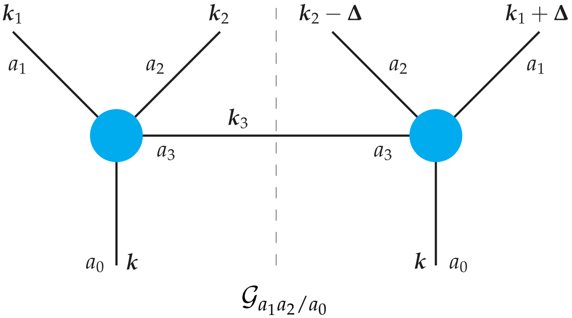


Evaluating real diagrams.



- $k_3 = k - k_1 - k_2$,
- $k_1^+ = z_1 k^+$, $k_2^+ = z_2 k^+$, $\Delta^+ = 0$
- $k_3^+ = z_3 k^+ = (1 - z_1 - z_2) k^+$

$F^{(2)}_{B\text{us}}$ and thus $W^{(2)}_{B\text{us}}$ is obtained from these diagrams by integrating over k_1^-, k_2^-, Δ^-, $\boldsymbol{k}_1$, and $\boldsymbol{k}_2$:

$$F^{(2),\text{real}}_{B\text{us},a_1 a_2/a_0}(z_1, z_2, \boldsymbol{\Delta}) = \sum_{\mathcal{G}} \left[\prod_{i=1}^{2} (x_i p^+)^{-n_i} \int \frac{\mathrm{d}k_i^- \, \mathrm{d}^{D-2}\boldsymbol{k}_i}{(2\pi)^D} \right] 2p^+ \int \frac{\mathrm{d}\Delta^-}{2\pi} \mathcal{G}_{a_1 a_2/a_0}(k_1, k_2, \Delta)$$

Performing momentum integrations.

First perform the integrations over minus components:

- The on-shell condition for parton a_3 can be used to perform one of the minus integrations, yielding:

$$k_3^- = \frac{\boldsymbol{k}_3^2}{2z_3 k^+}$$

- For the remaining minus integrations we use Cauchy's theorem.

After this perform the transverse momentum integrations:

- Use IBP relations to reduce the Feynman integrals to a finite set of master integrals.
- Use the method of differential equations to compute the master integrals:
  - Transform to Henn's canonical basis.
  - Use method of regions to obtain boundary conditions for the differential equations.

How do we implement the rapidity regulators?

Wilson line propagator in the Collins regulator scheme:

$$\lim_{\varepsilon\to 0}\frac{1}{v_L^- k_3^+ + v_L^+ k_3^- + i\varepsilon} + \text{c.c.} = \frac{2}{v_L^- k^+}\,\text{PV}\,\frac{z_3}{z_3^2 - \boldsymbol{k}_3^2 z_1 z_2/\rho} \qquad \text{with} \quad \rho = 2k_1^+ k_2^+\, v_L^-/|v_L^+|\,,$$

for space-like Wilson lines along direction v_L with $v_L^- > 0$ and $v_L^+ < 0$.

Wilson line propagator in the δ regulator scheme:

$$\frac{1}{k_3^+ + i\delta^+} + \text{c.c.} = \frac{2}{k^+}\,\frac{z_3}{z_3^2 + z_1 z_2/\rho} \qquad \text{with} \quad \rho = k_1^+ k_2^+/(\delta^+)^2\,.$$

⟶ Keep only real parts of WL propagators (sum over complex conjugate diagrams)!

⟶ Taking $\rho \to \infty$ removes the rapidity regulator and restores the light-like Wilson lines.

How do we implement the rapidity regulators?

Using the following distributional expansions makes the rapidity divergences explicit as logarithms of the parameter ρ:

Collins regulator:

$$\lim_{\rho\to\infty} \mathrm{PV} \frac{z_3}{z_3^2 - \boldsymbol{k}_3^2 z_1 z_2/\rho} = \frac{1}{[z_3]_+} + \frac{1}{2}\delta(z_3)\left[\log\frac{\rho}{\boldsymbol{\Delta}^2} - \log(z_1 z_2) - \log\frac{\boldsymbol{k}_3^2}{\boldsymbol{\Delta}^2}\right],$$

δ regulator:

$$\lim_{\rho\to\infty} \frac{z_3}{z_3^2 + z_1 z_2/\rho} = \frac{1}{[z_3]_+} + \frac{1}{2}\delta(z_3)\left[\log\rho - \log(z_1 z_2)\right].$$

Note: For the Collins regulator it is crucial to use this identity before performing the IBP reduction!

Results: analytical results.

General structure of results.

Colour non-singlet kernels:

$$
{}^{R_1R_2}V^{(2)}_{a_1a_2,a_0}(z,u,y,\mu,\zeta) = {}^{R_1R_2}V^{[2,0]}_{a_1a_2,a_0}(z,u) + L\ {}^{R_1R_2}V^{[2,1]}_{a_1a_2,a_0}(z,u) + \left(L\log\frac{\mu^2}{\zeta} - \frac{L^2}{2} + c_{\overline{\mathrm{MS}}}\right)\frac{{}^{R_1}\gamma_J^{(0)}}{2}\ {}^{R_1R_2}V^{(1)}_{a_1a_2,a_0}(z,u)
$$

where $L = \log\frac{y^2\mu^2}{b_0^2}$ and $b_0 = 2e^{-\gamma}$ and

$$
V^{[2,0]}(z,u) = V^{[2,0]}_{\text{regular}}(z,u) + \delta(1-z)\,V^{[2,0]}_{\delta}(u)\,,
$$

$$
V^{[2,1]}(z,u) = V^{[2,1]}_{\text{regular}}(z,u) + \frac{1}{[1-z]_+}V^{[2,1]}_{+}(u) + \delta(1-z)\,V^{[2,1]}_{\delta}(u)
$$

Results: numerical investigations.

Impact of NLO corrections on small y DPDs.

We study how including the NLO corrections effects the small y gg DPD for the following set of parameters:

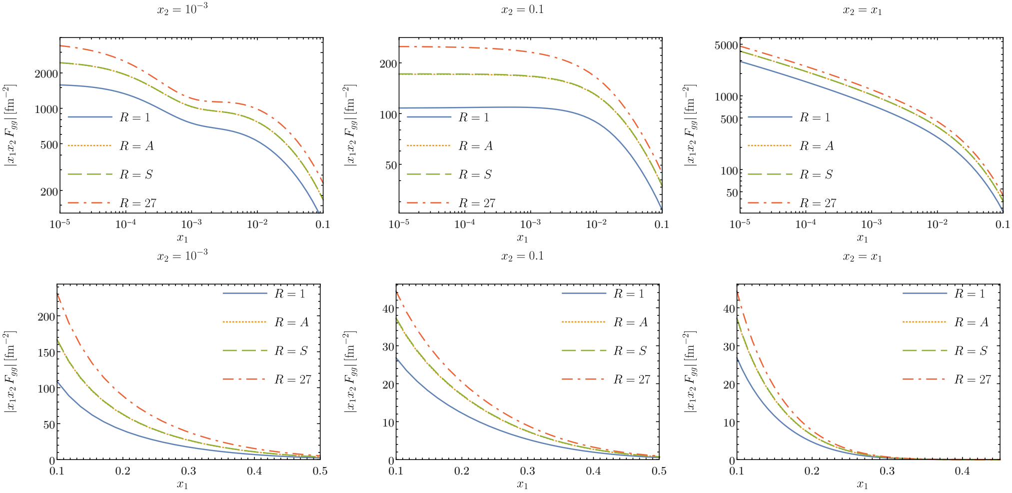
- $y = 0.022\,\mathrm{fm}$
- $\mu = \frac{b_0}{y} = 10\,\mathrm{GeV}$
- $x_1 x_2 \zeta_p = \mu^2 = 100\,\mathrm{GeV}^2$

For this choice of parameters only the $V^{[2,0]}$ part of the kernels contributes to the final DPD.

In order to get a feeling for the relative importance of the logarithmic $V^{[2,1]}$ and double logarithmic $V^{(1)}$ parts we vary μ and $\sqrt{x_1 x_2 \zeta_p}$ by a factor of two around their central values.

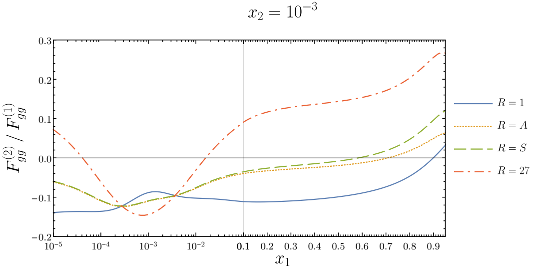
Numerics.

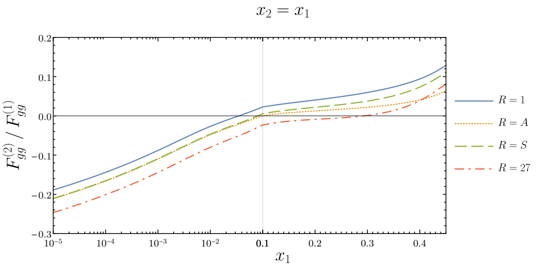
$\left|x_1 x_2 \, {}^{RR}F_{gg}\right|$.

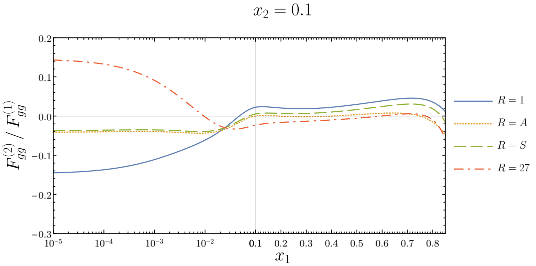


Numerics.

${}^{RR}F_{gg}^{(2)} \,/\, {}^{RR}F_{gg}^{(1)}$.





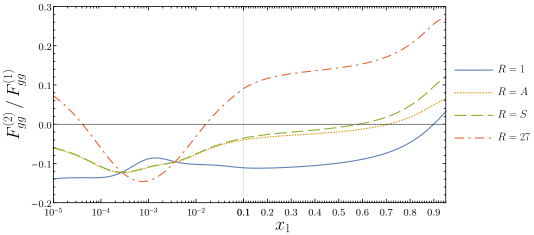


- moderate ($\mathcal{O}(10\%)$) NLO corrections.
- varied structure as a function of x_1 and x_2.
- results rather independent of PDF sets used.

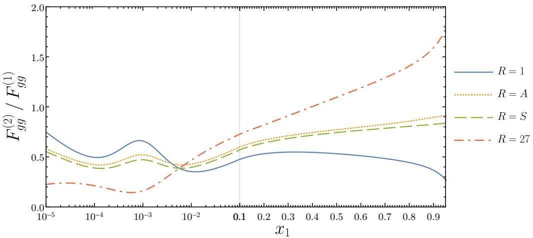
Numerics.

${}^{RR}F_{gg}^{(2)} \, / \, {}^{RR}F_{gg}^{(1)}$.

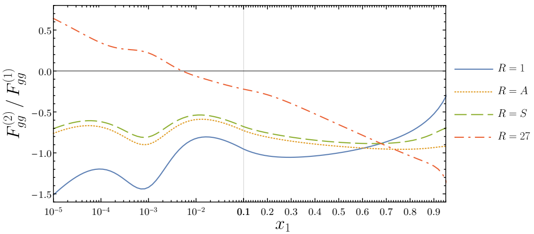
$x_2 = 10^{-3}, \mu = \mu_y$



$x_2 = 10^{-3}, \mu = 2\mu_y$



$x_2 = 10^{-3}, \mu = \mu_y/2$



- large ($\mathcal{O}(100\%)$) NLO corrections for $\mu \neq \mu_y$.
- splitting form should be evaluated at $\mu \sim \mu_y$ to avoid large higher order corrections.

Conclusion and outlook.

Conclusion and outlook.

What's been done so far?

- Calculated the unpolarised NLO small y splitting kernels ${}^{R_1R_2}V^{(2)}_{a_1a_2,a_0}$ for all parton and colour channels.
- Used different rapidity regulator schemes, providing a strong cross check.
- First application of the Collins regulator in a two loop calculation.
- Studied numerically the impact of the NLO corrections to small y DPDs.

What's left to do?

- Calculate the polarised NLO small y splitting kernels.
- Calculate the NLO colour non-singlet evolution kernels (work in progress [Diehl, Fabry, and Vladimirov]).
- Calculate NLO colour non-singlet hard scattering cross sections.

Backup.

From $W^{(2)}_{B\text{us}}$ to $V^{(2)}$.

Performing the rapidity subtraction.

A Fourier transform gives the bare unsubtracted NLO position space kernel as:

$$\frac{\Gamma(1-\epsilon)}{(\pi y^2)^{1-\epsilon}}\ {}^{R_1R_2}V^{(2)}_{B\text{us}}(y,\rho) = \int \frac{d^{2-2\epsilon}\boldsymbol{\Delta}}{(2\pi)^{2-2\epsilon}}\, e^{-i\boldsymbol{\Delta y}}\ {}^{R_1R_2}W^{(2)}_{B\text{us}}(\Delta,\rho)\,.$$

With this and the definition of the rapidity subtracted DPDs one then gets:

$${}^{R_1R_2}V^{(2)}_{B} = \lim_{\rho\to\infty}\left\{{}^{R_1R_2}V^{(2)}_{B\text{us}}(\rho) - \frac{1}{2}\,{}^{R_1}S^{(1)}_{B}(2\ell_L(\rho,\zeta))\ {}^{R_1R_2}V^{(1)}_{B}\right\},$$

where the involved quantities on the right-hand side generally differ in the two regulator schemes, while the left-hand side is already independent of this choice!

From $V_B^{(2)}$ to $V^{(2)}$.

Performing the UV renormalization.

From the renormalization prescription for the DPDs one easily obtains that the renormalized position space splitting kernel is given by:

$$
{}^{R_1R_2}V(y,\mu,\zeta) = {}^{R_1\overline{R}'_1}Z(\mu,\zeta) \underset{1}{\otimes} {}^{R_2\overline{R}'_2}Z(\mu,\zeta) \underset{2}{\otimes} {}^{R'_1R'_2}V_B(y,\mu,\zeta) \underset{12}{\otimes} \left({}^{11}Z\right)^{-1}(\mu)
$$

The NLO position space splitting kernel ${}^{R_1R_2}V^{(2)}$ is then obtained by this relation in α_s to $\mathcal{O}(\alpha_s^2)$ as:

$$
\begin{aligned}
V^{(2)} = V_{\text{fin}}^{(2)} &- \left(\hat{P}^{(0)} \underset{1}{\otimes} \left[V_B^{(1)}\right]_1 + \hat{P}^{(0)} \underset{2}{\otimes} \left[V_B^{(1)}\right]_1 - \left[V_B^{(1)}\right]_1 \underset{12}{\otimes} P^{(0)} + \frac{\beta_0}{2}\left[V_B^{(1)}\right]_1\right) \\
&+ \left(L\log\frac{\mu^2}{\zeta} - \frac{L^2}{2} + c_{\overline{\text{MS}}}\right)\frac{\gamma_J^{(0)}}{2}V^{(1)} + L\left(\hat{P}^{(0)} \underset{1}{\otimes} V^{(1)} + \hat{P}^{(0)} \underset{2}{\otimes} V^{(1)} - V^{(1)} \underset{12}{\otimes} P^{(0)} + \frac{\beta_0}{2}V^{(1)}\right)
\end{aligned}
$$

with $L = \log\frac{\mu^2 \boldsymbol{y}^2}{b_0^2}$ and $b_0 = 2e^{-\gamma}$.

Rescaling of the rapidity parameter.

The rapidity parameters ζ_p and $\zeta_{\bar{p}}$ in this work are normalised as:

$$\zeta_p \zeta_{\bar{p}} = (2p^+ \bar{p}^-)^2 = s^2 \,,$$

which differs from the convention in the TMD case

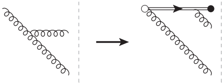
$$\zeta \bar{\zeta} = x^2 \bar{x}^2 (2p^+ \bar{p}^-)^2 = Q^4 \,,$$

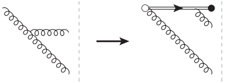
where the rapidity parameters are normalized w.r.t. the extracted parton, which would be awkward in the DPD case where parton momenta often appear in convolution integrals.

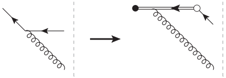
⟶ need to rescale the rapidity parameter in renormalisation factors and evolution kernels!

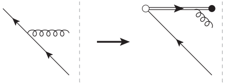
⟶ reason: can only depend on the plus-momentum $x_i p^+$ of the parton to which they refer!

From light-cone gauge diagrams to Wilson line diagrams in Feynman gauge.









Kinematic limits of the small y DPDs: Large $x = x_1 + x_2$.

$$
{}^{R_1R_2}V^{(2)} \underset{12}{\otimes} f \underset{x\to 1}{=} L\, {}^{R_1R_2}V_{+}^{[2,1]}(u)\, \log(1-x)\, f(x)
$$

$$
+ \left[{}^{R_1R_2}V_{\delta}^{[2,0]}(u) + L\, {}^{R_1R_2}V_{\delta}^{[2,1]}(u) + \left(L \log \frac{\mu^2}{x_1 x_2 \zeta_p} - \frac{L^2}{2} + c_{\overline{\text{MS}}} \right) \frac{{}^{R_1}\gamma_J^{(0)}}{2}\, {}^{R_1R_2}V^{(1)}(u) \right] f(x)
$$

$\longrightarrow$ logarithmic enhancement for plus distribution terms in the large x limit.

As a consequence one finds dominant contributions from the following kernels:

- ${}^{R_1R_2}V_{gg,g}^{(2)}$ for $R_1R_2 \neq AA, SS$.
- ${}^{R_1R_2}V_{q\bar{q},g}^{(2)}$ for all R_1R_2.
- ${}^{R_1R_2}V_{qg,q}^{(2)}$ for $R_1R_2 = 11$.

Kinematic limits of the small y DPDs: Small $x = x_1 + x_2$.

For $V(uz, \bar{u}z) = w(u)/z^2$ one finds

$$\left[V \underset{12}{\otimes} f\right](x) = \frac{w(u)}{x}\left[\frac{1}{z} \otimes f\right](x) = w(u)\, E(x)\, \frac{f(x)}{x}$$

with an enhancement factor

$$E(x) = \begin{cases} k^{-1} \log(1/x) & \text{for } x f(x) = c \log^{k-1}(1/x)\,, \\ (1 - x^{\alpha})/\alpha & \text{for } x f(x) = c\, x^{-\alpha}\,, \end{cases}$$

$\longrightarrow$ small x enhancement for z^{-2} terms in the kernels (in analogy to z^{-1} terms in DGLAP kernels).

As a consequence one finds leading small x contributions from the following kernels:

- ${}^{R_1R_2}V^{(2)}_{gg,g}$ for all R_1R_2.
- ${}^{R_1R_2}V^{(2)}_{q\bar{q},g}$ for all R_1R_2.
- ${}^{R_1R_2}V^{(2)}_{gg,q}$ for $R_1R_2 \neq AS, SA$ ($(q - \bar{q})(z)$ is not steep enough for small z).

Kinematic limits of the small y DPDs: Small x_1 or x_2.

Corresponds to the small $u = \frac{x_1}{x_1+x_2}$ and small $\bar{u} = \frac{x_2}{x_1+x_2}$ limit, with leading contributions going as u^{-1} and $\bar{u}^{-1}$ associated with slow gluons.

- u^{-1} & $\bar{u}^{-1}$: ${}^{R_1R_2}V^{(2)}_{gg,g}$, ${}^{R_1R_2}V^{(2)}_{gg,q}$, and ${}^{R_1R_2}V^{(2)}_{qg,g}$ for all R_1R_2,
- $\bar{u}^{-1}$: ${}^{R_1R_2}V^{(2)}_{qg,q}$, and ${}^{R_1R_2}V^{(2)}_{qq',q}$ for all R_1R_2.

Find two sources for this behaviour in small y DPDs:

- Explicit u^{-1} and $\bar{u}^{-1}$ terms in the kernels.
- Terms in the kernels producing u^{-1} and $\bar{u}^{-1}$ behaviour in the convolution with a PDF: $(1-z\bar{u})^{-1} \sim (k^+ - k_2^+)^{-1}$ (u^{-1}), $(1-zu)^{-1} \sim (k^+ - k_1^+)^{-1}$ ($\bar{u}^{-1}$) and similar terms.

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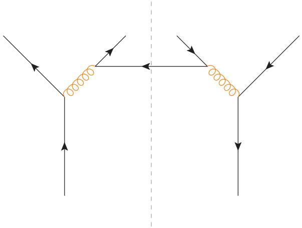

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