

Constructing Effective Field Theories to Higher Mass Dimensions

in collaboration with R.V. Harlander & T. Kempkens

Magnus C. Schaaf

Institute for Theoretical Particle Physics and Cosmology
RWTH Aachen University

24.11.2021 – 14th Annual Meeting of the Helmholtz Alliance “Physics at the Terascale”

Outline

Search for new physics with EFTs

Constructing EFT operator bases

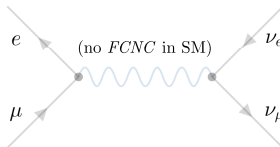
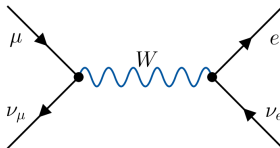
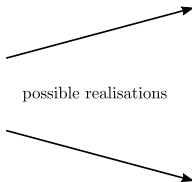
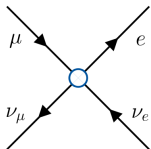
Some details on the explicit construction

Implementation of the algorithm

New Physics in Effective Field Theories

Fermi Theory (EFT)

Standard Model



$\ll m_W$

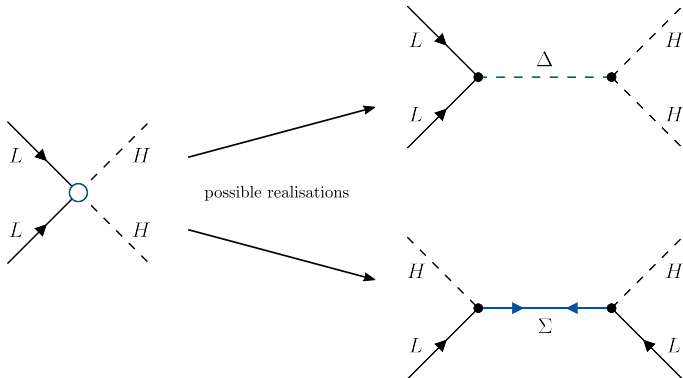
$\sim m_W$

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New Physics in Effective Field Theories

SMEFT

High Energy Theory ?



—————→ Λ

The Standard Model as EFT (SMEFT)

Lagrangian

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + \dots$$

Operators

$$\mathcal{O}^{(5)} = (L_\alpha H)(L^\alpha H)$$

$$\mathcal{O}^{(6)} = (H^\dagger H) G_{\mu\nu}^A G^{A\mu\nu}, (H^\dagger \tau^I H) W_{\mu\nu}^I B^{\mu\nu}, \dots$$

$$\vdots$$

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Brief History of SMEFT

- 1980 • Weinberg: $d = 5$ & $n_f = 1$
→ 1 Operator ($LLHH$)
- 1986 • Buchmüller-Wyler: $d = 6$ & $n_f = 1$
→ 80 Operators (22 redundant, 1 missing)
- ⋮ •
- 2010 • Grzadkowski *et al.*: $d = 6$ & $n_f = 1$
→ 59 Operators
- 2020 • H.-L. Li *et al.*: $d = 8, 9$ & $n_f = 1, 3$

Redundancies

1. Fierz identities: $g_{\mu\nu}\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\beta\dot{\beta}}^{\nu} = 2\epsilon_{\alpha\beta}\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}$
2. Equations of motion: $i\not{D}\psi + J_{\psi} = 0$
3. Integration by parts: $X D_{\mu} Y \sim -D_{\mu} X Y$
4. Flavour relations: $(L^{f_1} H)(L^{f_2} H) = (L^{f_2} H)(L^{f_1} H)$

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Standard Model fields

Felder	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
G	8	1	0
W	1	3	0
B	1	1	0
L	1	2	$-1/2$
e_C	1	1	1
Q	3	2	$1/6$
u_C	$\bar{\mathbf{3}}$	1	$-2/3$
d_C	$\bar{\mathbf{3}}$	1	$1/3$
H	1	2	$1/2$

Explicit index notation

$$\begin{aligned}\epsilon^{abc} (L^i Q_{j,a}) (Q_{i,b} Q_c^j) &= \epsilon^{abc} L_\alpha^i Q_{j,a}^\alpha Q_{i,b}^\beta Q_{c,\beta}^j \\ &= \underbrace{\epsilon^{abc}}_{T_{\text{SU}(3)}} \underbrace{\epsilon^{ik} \epsilon^{jl}}_{T_{\text{SU}(2)}} \underbrace{\epsilon_{\alpha\beta} \epsilon_{\gamma\delta}}_{T_{\text{Lorentz}}} \underbrace{L_i^\alpha Q_{j,a}^\beta Q_{k,b}^\gamma Q_{l,c}^\delta}_{\Pi_n \Phi_n}\end{aligned}$$

operators of same *type* only differ in choice of T

e.g.: $T_{\text{SU}(2)} = \epsilon^{ij} \epsilon^{kl}$

$\Rightarrow T_{\text{SU}(3)}, T_{\text{SU}(2)} \text{ \& } T_{\text{Lorentz}}$ govern the symmetry

General operator

$$\mathcal{O} = T_{\text{SU}(3)}^{\{g\}} T_{\text{SU}(2)}^{\{h\}} T_{\text{Lorentz}}^{\{l\}} \times \prod_{i=1}^N (D^{n_i} \Phi_i)_{\{g\}, \{h\}, \{l\}}$$

Invariant tensors

$$T_{\text{SU}(3)} \in \langle f^{ABC}, d^{ABC}, \delta^{AB}, (\lambda^A)^b_a, \epsilon_{abc}, \epsilon^{abc} \rangle$$

$$T_{\text{SU}(2)} \in \langle \epsilon^{IJK}, \delta^{IJ}, (\tau^I)^j_i, \epsilon_{ij}, \epsilon^{ij} \rangle$$

$$T_{\text{Lorentz}} \in \langle \sigma^{\mu\nu}_{\alpha\beta}, \bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}, \sigma^{\mu}_{\alpha\dot{\alpha}}, \sigma^{\mu\dot{\alpha}\alpha}, \sigma^{\mu\nu}_{\alpha\beta}, \epsilon^{\alpha\beta}, \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \rangle$$

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Fundamental Representation

Lorentz group: $SL(2, \mathbb{C}) = SU(2)_l \times SU(2)_r$

$$\phi \in (0, 0) \quad \psi_{\alpha} \in (1/2, 0) \quad F_{L\alpha\beta} = \frac{i}{2} F_{\mu\nu} \sigma_{\alpha\beta}^{\mu\nu} \in (1, 0)$$

$$D_{\alpha\dot{\alpha}} = D_{\mu} \sigma_{\alpha\dot{\alpha}}^{\mu} \in (1/2, 1/2)$$

Gauge group: $SU(3)_C \times SU(2)_W \times U(1)_Y$

$$G_{abc} = \epsilon_{acd} (\lambda^A)^d_b G^A, \quad W_{ij} = \epsilon_{jk} (\tau^I)^k_i W^I, \quad \dots$$

Explicit Contraction of Lorentz Indices

Integration by parts

$$-D_\mu \phi_1 \phi_2 \phi_3 D^\mu \phi_4 = \phi_1 D_\mu \phi_2 \phi_3 D^\mu \phi_4 + \phi_1 \phi_2 D_\mu \phi_3 D^\mu \phi_4$$

Tableaux $\Leftrightarrow$ Operators

$$- \begin{array}{|c|c|} \hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} = - \underbrace{\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}}_{\text{semi-standard Young tableaux}}$$

$\Rightarrow$ 2 independent Lorentz invariants

Explicit Contraction of Gauge Group Indices

Littlewood–Richardson rule

$$Q: \boxed{j} \quad Q: \boxed{k} \quad Q: \boxed{l} \quad L: \boxed{i} \quad W: \boxed{m_1 m_2}$$

$$\boxed{j} \rightarrow \boxed{j \ k} \rightarrow \boxed{j \ k \ l} \rightarrow \begin{array}{|c|c|c|} \hline j & k & l \\ \hline i & & \\ \hline \end{array} \rightarrow \begin{array}{|c|c|c|} \hline j & k & l \\ \hline i & m_1 & m_2 \\ \hline \end{array} \sim \epsilon^{ji} \epsilon^{km_1} \epsilon^{lm_2} Q_j Q_k Q_l L_i W_{m_1 m_2}$$

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$\Rightarrow$ 3 independent contractions of SU(2) indices

Multiple Generations

Relations between generations

$$\mathcal{O}_{LLHH}^{f_1 f_2} = (L^{f_1} H)(L^{f_2} H) = \mathcal{O}_{LLHH}^{f_2 f_1}$$

$\Rightarrow$ 3 independent operators for $n_f = 2$:

$$\mathcal{O}_{LLHH}^{ee}, \quad \mathcal{O}_{LLHH}^{e\mu}, \quad \mathcal{O}_{LLHH}^{\mu\mu}$$

Higher rank tensors

$$\mathcal{O}^{f_1 f_2 f_3} = \mathcal{O}_{[3]}^{f_1 f_2 f_3} + \mathcal{O}_{[2,1]}^{f_1 f_2 f_3} + \mathcal{O}_{[1^3]}^{f_1 f_2 f_3} \qquad \mathcal{O}_{\lambda}^{f_1 f_2 f_3} = b_{\lambda}^{\lambda} \mathcal{O}^{f_1 f_2 f_3}$$

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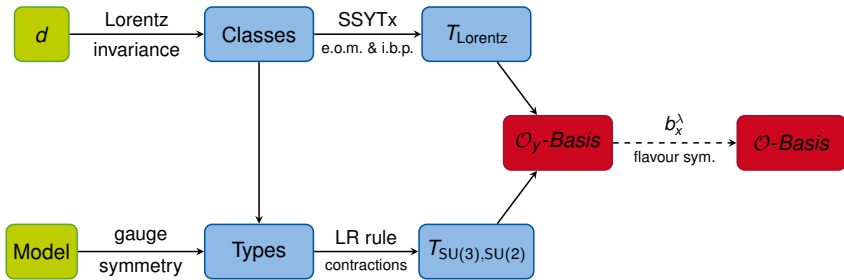
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Some details on the explicit construction

Implementation of the algorithm

Automated Construction

[Li,Ren,Shu,Xiao,Yu,Zheng, 2020]



SageMath + FORM

Custom Models

```
name: SMEFT
```

```
# arbitrary SU(N) and  
# U(1) groups
```

```
symmetries:
```

```
  SUN:
```

```
    SU3:
```

```
      n: 3
```

```
    SU2:
```

```
      n: 2
```

```
  U1: [U1]
```

```
fields:
```

```
# arbitrary fields in  
# arbitrary representations
```

```
  G:
```

```
    reps:
```

```
      Lorentz: (1,0)
```

```
      SU3: [2,1]
```

```
  uC:
```

```
    reps:
```

```
      Lorentz: (1/2,0)
```

```
      SU3: [1,1]
```

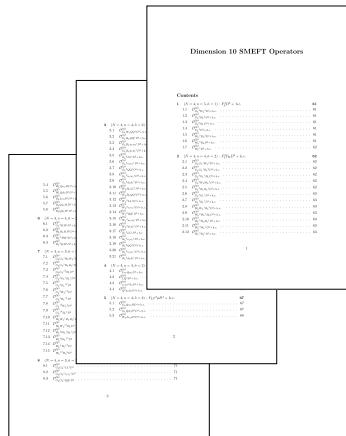
```
      U1: -2/3
```

```
  ac: True
```

```
  nf: 3
```

Operator Basis

- Complete and minimal
- Automated construction
- Custom low energy models
- Arbitrary mass dimensions d
- $d = 9$ SMEFT in ~ 2 min



Backup

Redundancies

Fierz identities

$$\begin{aligned}g_{\mu\nu}\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\beta\dot{\beta}}^{\nu} &= 2\epsilon_{\alpha\beta}\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \\ \epsilon^{\alpha\beta}\delta_{\kappa}^{\gamma} + \epsilon^{\beta\gamma}\delta_{\kappa}^{\alpha} + \epsilon^{\gamma\alpha}\delta_{\kappa}^{\beta} &= 0 \\ \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}\delta_{\dot{\gamma}}^{\dot{\kappa}} + \tilde{\epsilon}_{\dot{\beta}\dot{\gamma}}\delta_{\dot{\alpha}}^{\dot{\kappa}} + \tilde{\epsilon}_{\dot{\gamma}\dot{\alpha}}\delta_{\dot{\beta}}^{\dot{\kappa}} &= 0\end{aligned}$$

Field strength tensor

$$[D_{\mu}, D_{\nu}] = -iF_{\mu\nu}$$

Redundancies

Equations of motion

$$D^2\phi + J_\phi = 0$$

$$i\not{D}\psi + J_\psi = 0$$

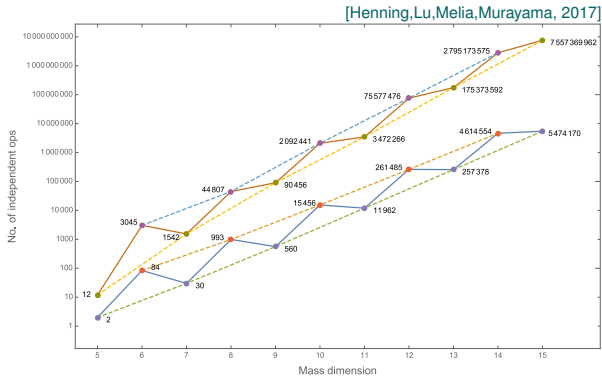
$$D_\mu F^{\mu\nu} + J_A^\nu = 0$$

Integration by parts

$$D_\mu(XY) \sim 0 \implies X D_\mu Y \sim -D_\mu X Y$$

Hilbert Series

$$H(p, \phi_1, \dots, \phi_n) = \int d\mu_{\text{conf.}} d\mu_{\text{gauge}} \sum_{i=1}^{\infty} p^i \chi_{[i;0]}^* \prod_j \text{PE}[\phi_j \chi_j(q, \alpha, \beta)]$$



$$(D^{r-|h|}\Phi)_{\alpha^{(1)}\dots\alpha^{(r-h)}\dot{\alpha}^{(1)}\dots\dot{\alpha}^{(r+h)}} \equiv \begin{cases} D_{\alpha^{(1)}\dot{\alpha}^{(1)}} \dots D_{\alpha^{(r+h)}\dot{\alpha}^{(r+h)}} \Phi_{\alpha^{(r+h+1)}\dots\alpha^{(r-h)}} , & h < 0 \\ D_{\alpha^{(1)}\dot{\alpha}^{(1)}} \dots D_{\alpha^{(r-h)}\dot{\alpha}^{(r-h)}} \Phi_{\dot{\alpha}^{(r-h+1)}\dots\dot{\alpha}^{(r+h)}} , & h > 0 \end{cases}$$

$$D_{[\alpha\dot{\alpha}} D_{\beta]\dot{\beta}} \phi = -\epsilon_{\alpha\beta} \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} D^2 \phi + \frac{i}{2} \epsilon_{\alpha\beta} \bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}} [D_\mu, D_\nu] \phi$$

$$D_{[\alpha\dot{\alpha}} \psi_{\beta]} = -\epsilon_{\alpha\beta} (\not{D}\psi)_{\dot{\alpha}}$$

$$D_{[\alpha\dot{\alpha}} F_{L\beta]\gamma} = 2\epsilon_{\alpha\beta} \sigma_{\gamma\dot{\alpha}}^\nu D^\mu F_{\mu\nu}$$

$$(D^{r-|h|}\Phi)_{\alpha^{(1)}\dots\alpha^{(r-h)}\dot{\alpha}^{(1)}\dots\dot{\alpha}^{(r+h)}} \equiv (D^{r-|h|}\Phi)_{\alpha^{r-h}}^{\dot{\alpha}^{r+h}} \in \left(\frac{r-h}{2}, \frac{r+h}{2} \right)$$

Lorentz Structure as $SU(N)$ State

Young diagrams for $N = 4$ fields

$$\epsilon \sim \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \quad \tilde{\epsilon} \sim \overline{\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

$\Downarrow$

$$T_{\text{Lorentz}} = (\tilde{\epsilon}_{\dot{\alpha}k\dot{\alpha}l})^{\otimes \tilde{n}} (\epsilon^{\alpha_i\alpha_j})^{\otimes n} \quad n \underset{\sim}{=} \tilde{n} = 1$$



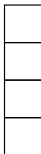
primary

+



total derivative

+



Field Definitions

$$q_L = \begin{pmatrix} Q \\ 0 \end{pmatrix}, \quad u_R = \begin{pmatrix} 0 \\ u_{\mathbb{C}}^{\dagger} \end{pmatrix}, \quad d_R = \begin{pmatrix} 0 \\ d_{\mathbb{C}}^{\dagger} \end{pmatrix}, \quad l_L = \begin{pmatrix} L \\ 0 \end{pmatrix}, \quad e_R = \begin{pmatrix} 0 \\ e_{\mathbb{C}}^{\dagger} \end{pmatrix}$$

$$F_{L/R} = \frac{1}{2} (F \pm i\tilde{F})$$

Magnus C. Schaaf

Institute for Theoretical Particle Physics and Cosmology
RWTH Aachen University
Sommerfeldstr. 16
52074 Aachen, Germany

www.particle-theory.rwth-aachen.de