



A Study of Top Anomalous Couplings at a Future e^+e^- Collider

Julie Munch Torndal, DESY

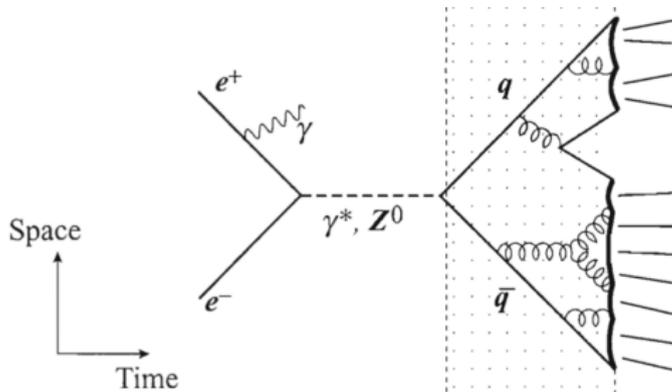
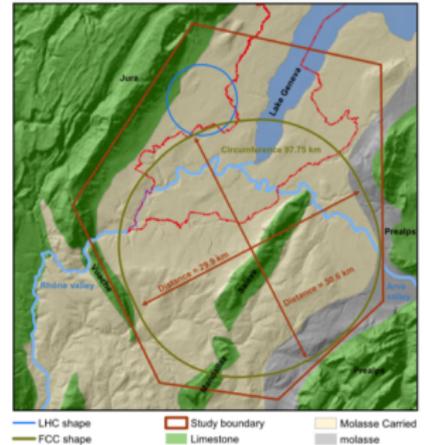
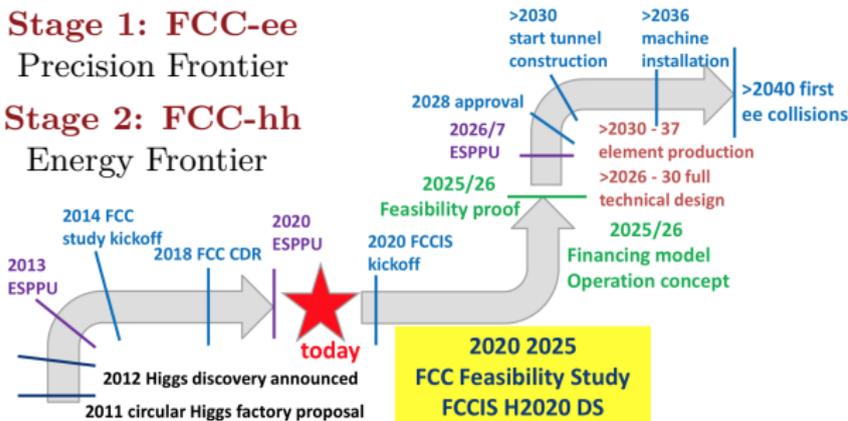
Supervisor: Jørgen Beck Hansen, NBI
November 23th, 2021

Stage 1: FCC-ee

Precision Frontier

Stage 2: FCC-hh

Energy Frontier



- No substructure of electrons
- No underlying event
- No PDFs i.e. initial state is known is all directions

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_j \frac{C_j^{(8)}}{\Lambda^4} O_j^{(8)} + \dots$$

- 7 independent top dim-6 operators \rightarrow 10 anomalous couplings

$$\begin{aligned} \mathcal{L}_{Wtb} = & -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L) P_L + (V_R) P_R t W_\mu^- \\ & -\frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q_\nu}{M_W} (g_L) P_L + (g_R) P_R t W_\mu^- + H.c. \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{Ztt} = & -\frac{g}{2c_W} \bar{t} \gamma^\mu (X_{tt}^L) P_L + (X_{tt}^R) P_R - 2s_W^2 Q_t t Z_\mu \\ & -\frac{g}{2c_W} \bar{t} \frac{i\sigma^{\mu\nu} q_\nu}{M_Z} (d_V^Z) + i(d_A^Z) \gamma_5 t Z_\mu \end{aligned}$$

$$\mathcal{L}_{\gamma tt} = -e Q_t \bar{t} \gamma^\mu A_\mu - e \bar{t} \frac{i\sigma^{\mu\nu} q_\nu}{m_t} (d_V^\gamma) + i(d_A^\gamma) \gamma_5 t A_\mu$$

δd_A^γ 

δd_V^γ 

δd_A^Z 

δd_V^Z 

δX_{tt}^L 

δX_{tt}^R 

δg_L 

δg_R 

δV_L 

δV_R 

Angular distributions:

Total cross section:

$t\bar{t}$ production	$t \rightarrow Wb$	$W \rightarrow \ell\nu$
θ_{et}	$\theta_{tb}^*, \phi_{tb}^*$	$\theta_{W\ell}^*, \phi_{W\ell}^*$

$t\bar{t}$ production
σ_{tot}

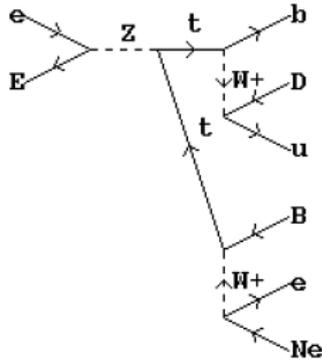
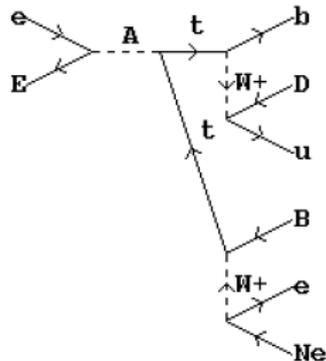


Signal:

Semileptonic channel

$$e^+ e^- \rightarrow t\bar{t} \rightarrow bW^+ \bar{b}W^- \rightarrow b\bar{b}q\bar{q}l\nu_e$$

Planned phase of FCC-ee @ $\sqrt{s} = 365$ GeV



Backgrounds:

Process	σ [pb]
$t\bar{t} \rightarrow b\bar{b}q\bar{q}l\nu_e$	0.1933
$\mu\mu$	0.7942
$\tau\tau$	0.7937
$\sum q\bar{q}$	4.143
$b\bar{b}$	0.7448
γZ	3.386
WW	10.72
ZZ	0.6428
ZH	0.1173
ZWW	$15.91 \cdot 10^{-3}$
ZZZ	$0.7633 \cdot 10^{-3}$
Single top	$3.337 \cdot 10^{-3}$

- All MC files are generated in the FCCSW framework with DelphesPythia8_EDM4HEP and IDEA Delphes Card
- Anomalous couplings available with Whizard

Simplified Detector Transverse View

Muon Spectrometer

HadCAL

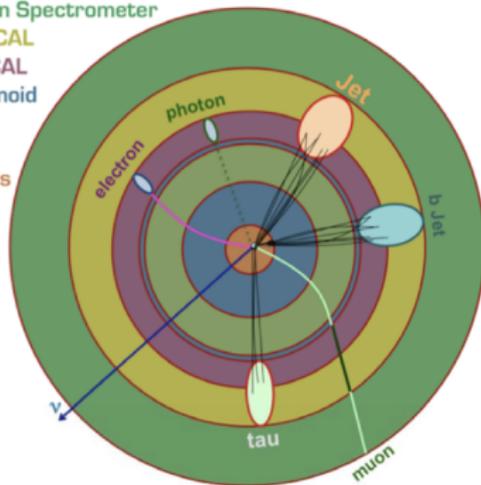
EMCAL

Solenoid

TRT

SCT

Pixels



Lepton Identification:

$$\ell = \{e, \mu\}$$

- Assumes perfect PID

Main tau decay modes:

- $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$ (17.8%),
- $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ (17.4%),
- $\tau^- \rightarrow \pi^- (n\pi^0) \nu_\tau$ (48%)
- $\tau^- \rightarrow \pi^- \pi^+ \pi^- (n\pi^0) \nu_\tau$ (15%)

Highest energy lepton:

Selecting highest energy lepton as signature lepton has an acceptance of

$$96.6 \pm 0.7 \%$$

- Find RP highest energy lepton
- Match to MC particle
- Parent history (EDM4Hep gives parent and daughter history for MCParticleData)
- Stopping criteria with PDG and status code



Jet Clustering Interface:

- flexible "after burner" using FastJet
- run multiple jet reconstructions at once
- select input particles
- access to jet constituents

Jet Tagging Interface:

- Inspired by Delphes

Jet Reconstruction:

Jet Algorithms

k_{\perp}
 Durham ($e^+e^- k_{\perp}$)
 e^+e^- Anti- k_{\perp}
 e^+e^- Cambridge/Aachen
 Valencia
 Jade



Recombination Schemes

E -scheme
 $E0$ -scheme
 p -scheme

FCCAnalyses / JetClustering

> [analyzers/dataframe/JetClustering.h](#)

[analyzers/dataframe/JetClustering.cc](#)

[analyzers/dataframe/JetClusteringUtils.h](#)

[analyzers/dataframe/JetClusteringUtils.cc](#)

FCCAnalyses / JetTagging

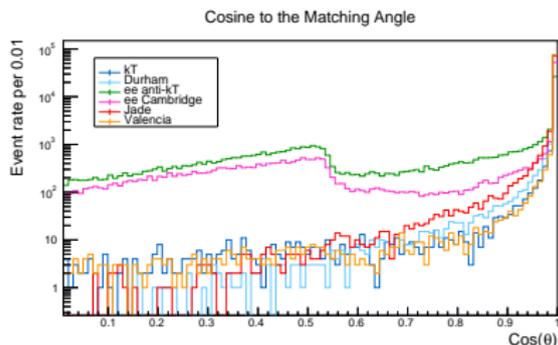
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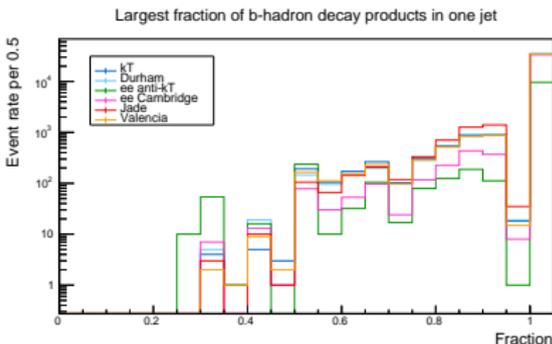
- sequential recombination
- exclusive clustering with *exactly* 4 jets
- highest energy lepton excluded from the clustering



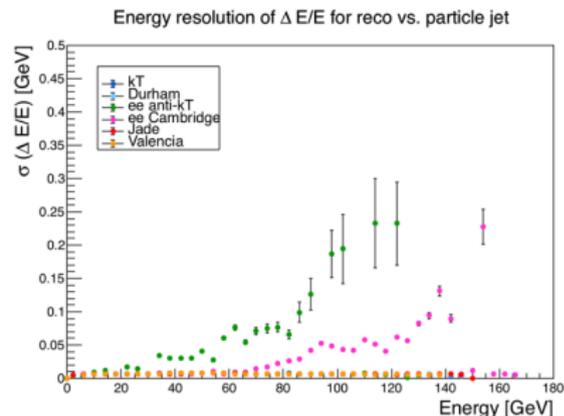
Matching angle between reco and particle jets



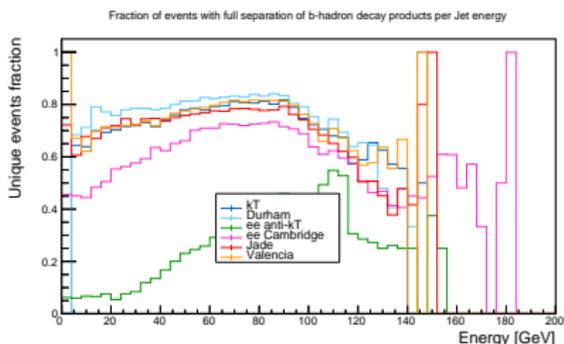
Largest fraction of b-hadron decay products in one jet



Energy resolution of jets

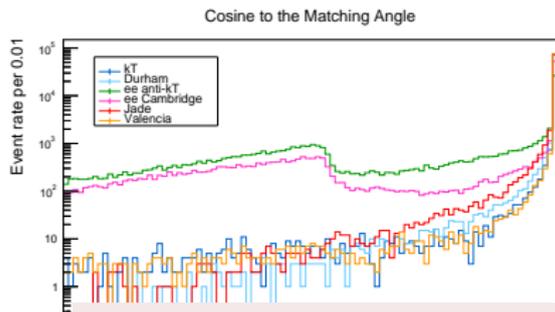


Fraction of events with full separation of b-hadron decay products

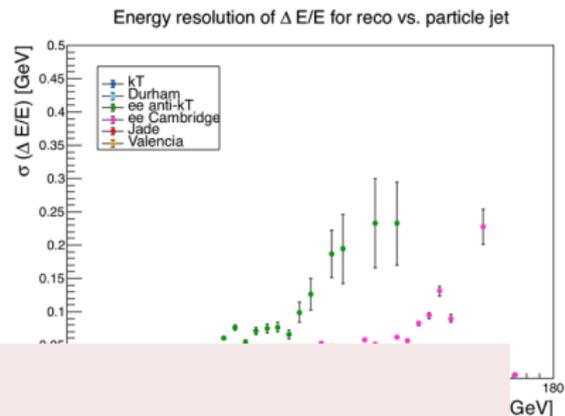




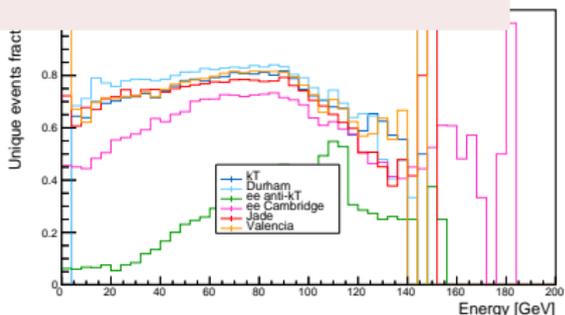
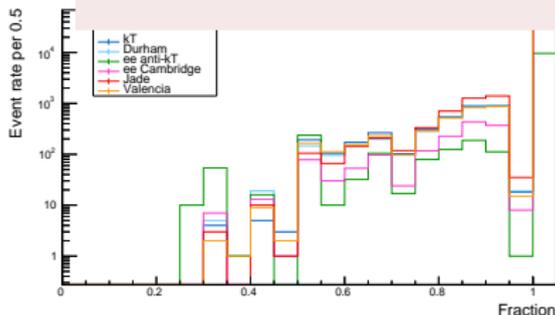
Matching angle between reco and particle jets



Energy resolution of jets

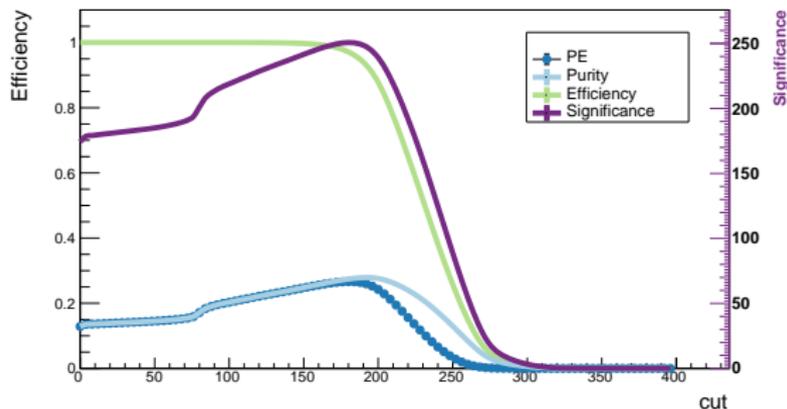


Larg prod → Jet Definition = Durham + E-scheme

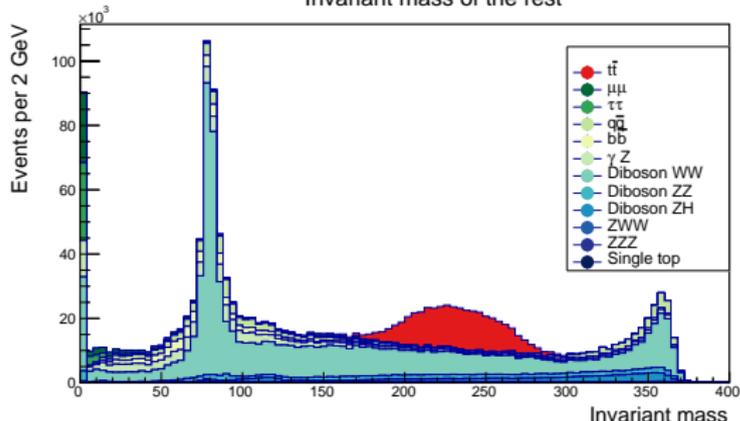




Lower cut on Invariant mass of the rest



Invariant mass of the rest



$$\text{Significance} = \frac{\text{sig}}{\sqrt{\text{sig} + \text{bkg}}}$$

$$\text{Efficiency} = \frac{\text{sig}}{\text{sig}_{\text{tot}}}$$

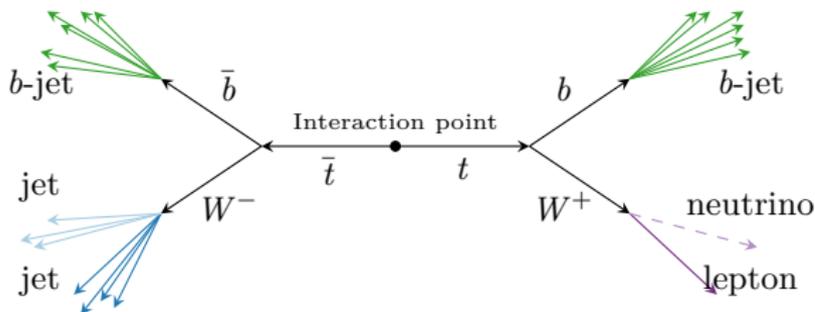
$$\text{Purity} = \frac{\text{sig}}{\text{sig} + \text{bkg}}$$

Signal region cuts

- At least 1 lepton
 - Thrust < 0.85
 - $M(\text{rest}) > 160 \text{ GeV}$
 - $M(\ell_{HE}, \bar{E}) > 50 \text{ GeV}$
 - $p_{\ell_{HE}} < 100 \text{ GeV}$
 - $p_{\ell_{HE}} > 15 \text{ GeV}$
 - $p_{\ell_{2^{nd}HE}} < 40 \text{ GeV}$
 - Exactly 4 jets
 - At least 1 b-tag
-

Event Reconstruction:

- Combinatorics
- Measurement corrections



χ^2 -function
to minimise:

$$L(y) = S(y) + g(x) + 2 \sum_{k=1}^m \lambda_k f_k(a, y, x)$$

- **Least Squares Principle:** $S(y) = \Delta y^T \mathbf{V}(y)^{-1} \Delta y = \min$
- **Lagrange multipliers:** $2 \sum_{k=1}^m \lambda_k f_k(a, y)$
- **Model expressed as m constraints:** $f_k(\bar{a}, \bar{y}) = 0$, $k = 1, \dots, m$
- **Penalty function:** $g(x) = -2 \ln(\text{pdf}(x))$



χ^2 -function
to minimise:

$$L(y) = S(y) + g(x) + 2 \sum_{k=1}^m \lambda_k f_k(a, y, x)$$

Solution:

$$\begin{pmatrix} y^{n+1} \\ x^{n+1} \end{pmatrix} = \begin{pmatrix} y_0 \\ x^n \end{pmatrix} - \tilde{\mathbf{V}} \begin{pmatrix} 0 \\ \frac{1}{2} \frac{d^2 g}{dx^2} \Big|_{x=x^n}^{-1} \end{pmatrix} + \tilde{\mathbf{V}} B^T (B \tilde{\mathbf{V}} B^T)^{-1} \\ \times \left[A(a^n - a_0) + B \begin{pmatrix} y^n - y_0 \\ \frac{dg}{dx} \Big|_{x=x^n}^{-1} / \frac{d^2 g}{dx^2} \Big|_{x=x^n} \end{pmatrix} - f(a^n, y^n, x^n) \right]$$

$$a^{n+1} = a_0 + W_A^{-1} A^T W_B \\ \times \left[A(a^n - a_0) + B \begin{pmatrix} y^n - y_0 \\ \frac{dg}{dx} \Big|_{x=x^n}^{-1} / \frac{d^2 g}{dx^2} \Big|_{x=x^n} \end{pmatrix} - f(a^n, y^n, x^n) \right]$$

with

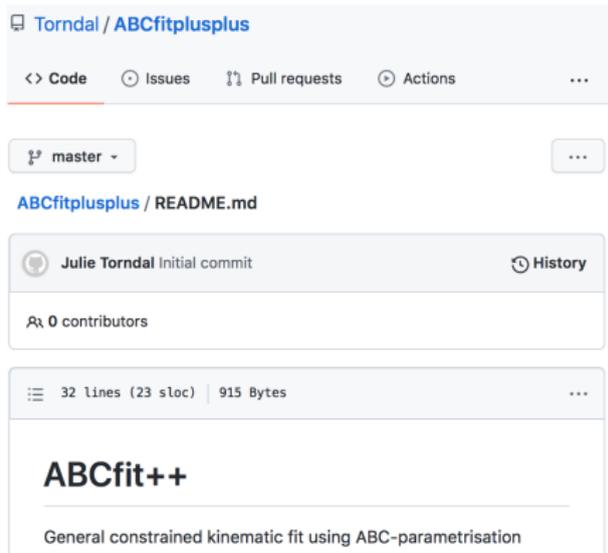
$$\tilde{\mathbf{V}} = \begin{pmatrix} \mathbf{V} & 0 \\ 0 & \left(\frac{1}{2} \frac{d^2 g}{dx^2} \Big|_{x=x^n} \right)^{-1} \end{pmatrix}, \quad B = \frac{\partial f(a, y, x)}{\partial (y, x)}, \quad A = \frac{\partial f(a, y, x)}{\partial a}, \\ W_B = (B \tilde{\mathbf{V}} B^T)^{-1} \quad \text{and} \quad W_A^{-1} = (A^T W_B A)^{-1}$$

General software package for constrained fitting

Base Classes:

- Coordinate Representation
- Particle Object
- Constraint
- Composite constraint
- Probability distribution functions
- Matrix Algebra
- ABC Fit

[Link to GitHub](#)

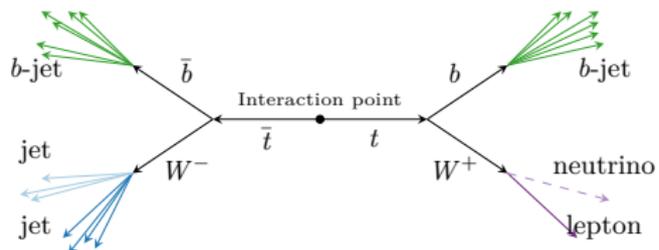


The screenshot shows the GitHub repository page for Torndal / ABCfitplusplus. At the top, there are navigation tabs for Code, Issues, Pull requests, and Actions. Below this, the current branch is set to 'master'. The repository name 'ABCfitplusplus / README.md' is displayed. A commit by Julie Torndal is shown as the initial commit, with 0 contributors listed. The file details show 32 lines (23 sloc) and 915 Bytes. The main content of the README is visible, featuring the title 'ABCfit++' and the description 'General constrained kinematic fit using ABC-parametrisation'.

Constraints:

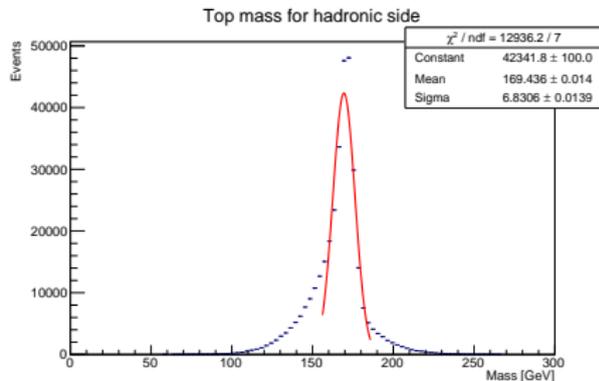
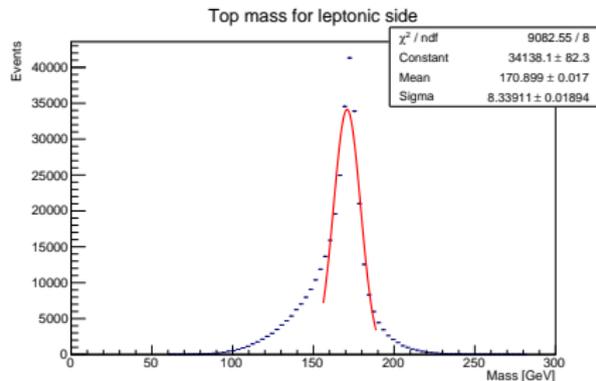
- Energy and momentum conservation

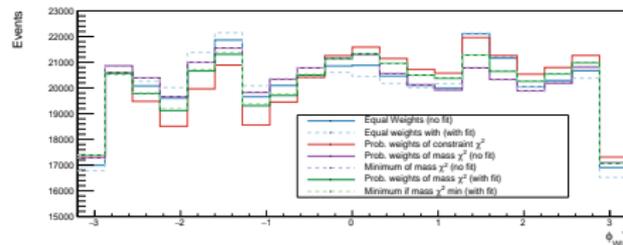
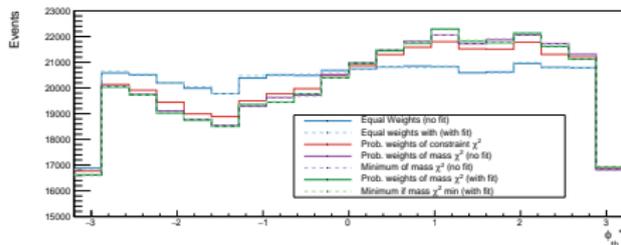
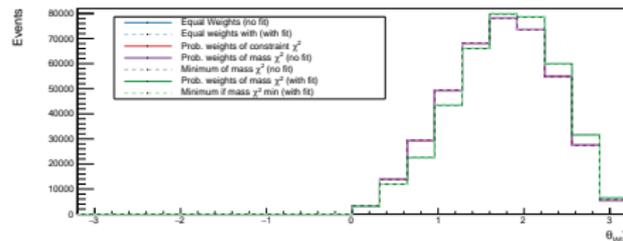
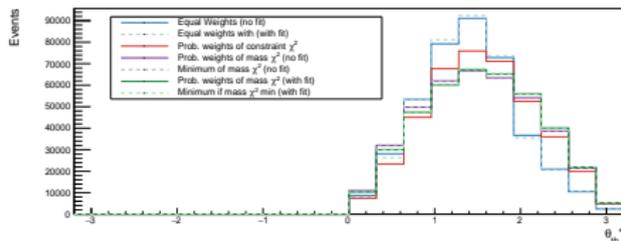
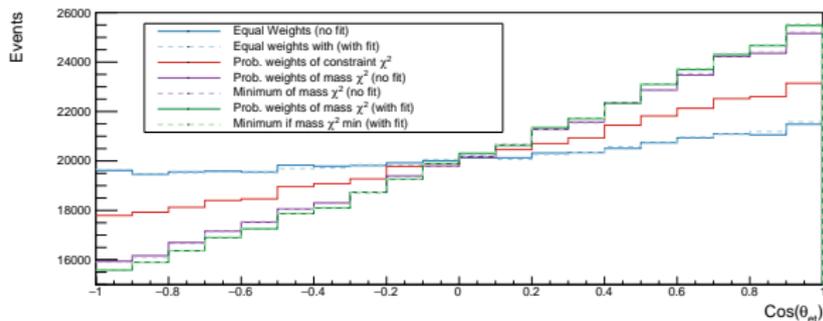
$$\Sigma \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} 365 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ GeV}$$



- Mass constraints for the two top systems

$$\mu = 173 \text{ GeV}, \quad \sigma_{\text{lep}} = 10 \text{ GeV} \quad \text{and} \quad \sigma_{\text{had}} = 9 \text{ GeV}$$







Parabolic dependence on a single coupling:

$$\begin{aligned}
 |\mathcal{M}|^2 &= |\mathcal{M}_{SM} + \alpha \mathcal{M}'_{D=6}|^2 \\
 &= |\mathcal{M}_{SM}|^2 + \alpha (\mathcal{M}_{SM}^* \mathcal{M}'_{D=6} + \mathcal{M}_{SM} \mathcal{M}'_{D=6}^*) \\
 &\quad + \alpha^2 |\mathcal{M}'_{D=6}|^2
 \end{aligned}$$

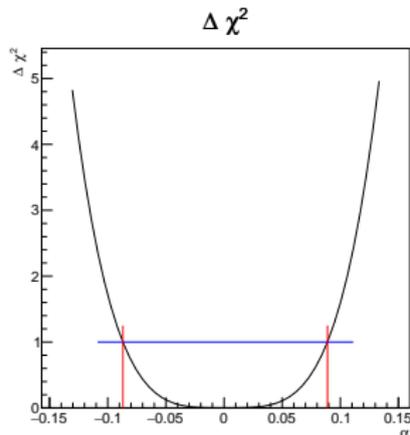
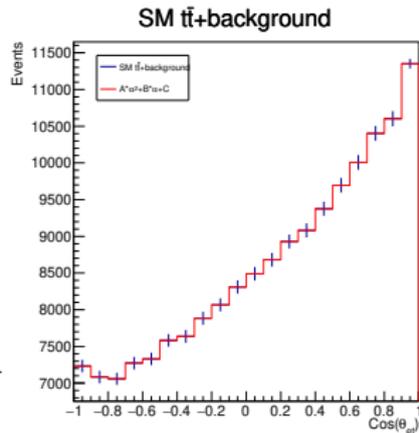
Constructing the parabola:

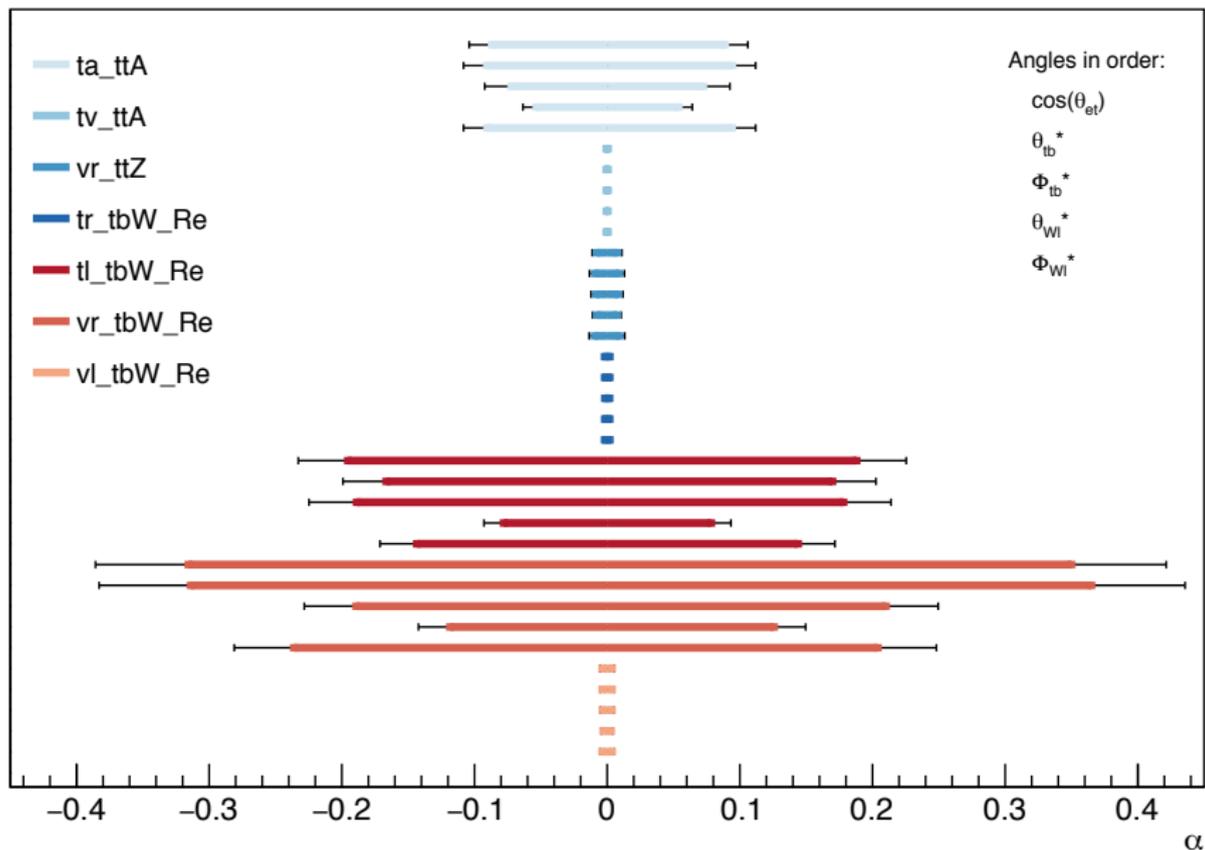
$$\begin{aligned}
 f(\alpha^0) &= C \\
 f(\alpha^+) &= A\alpha^2 + B\alpha + C \quad \Rightarrow \quad A = \frac{f(\alpha^+) + f(\alpha^-)}{2\alpha^2} - \frac{f(SM)}{\alpha^2} \\
 f(\alpha^-) &= A\alpha^2 - B\alpha + C \quad \Rightarrow \quad B = \frac{f(\alpha^+) - f(\alpha^-)}{2\alpha} \\
 &\quad C = f(SM)
 \end{aligned}$$

Fit model:

$$f(x) = A \cdot x^2 + B \cdot x + f(SM) + f(Bkg)$$

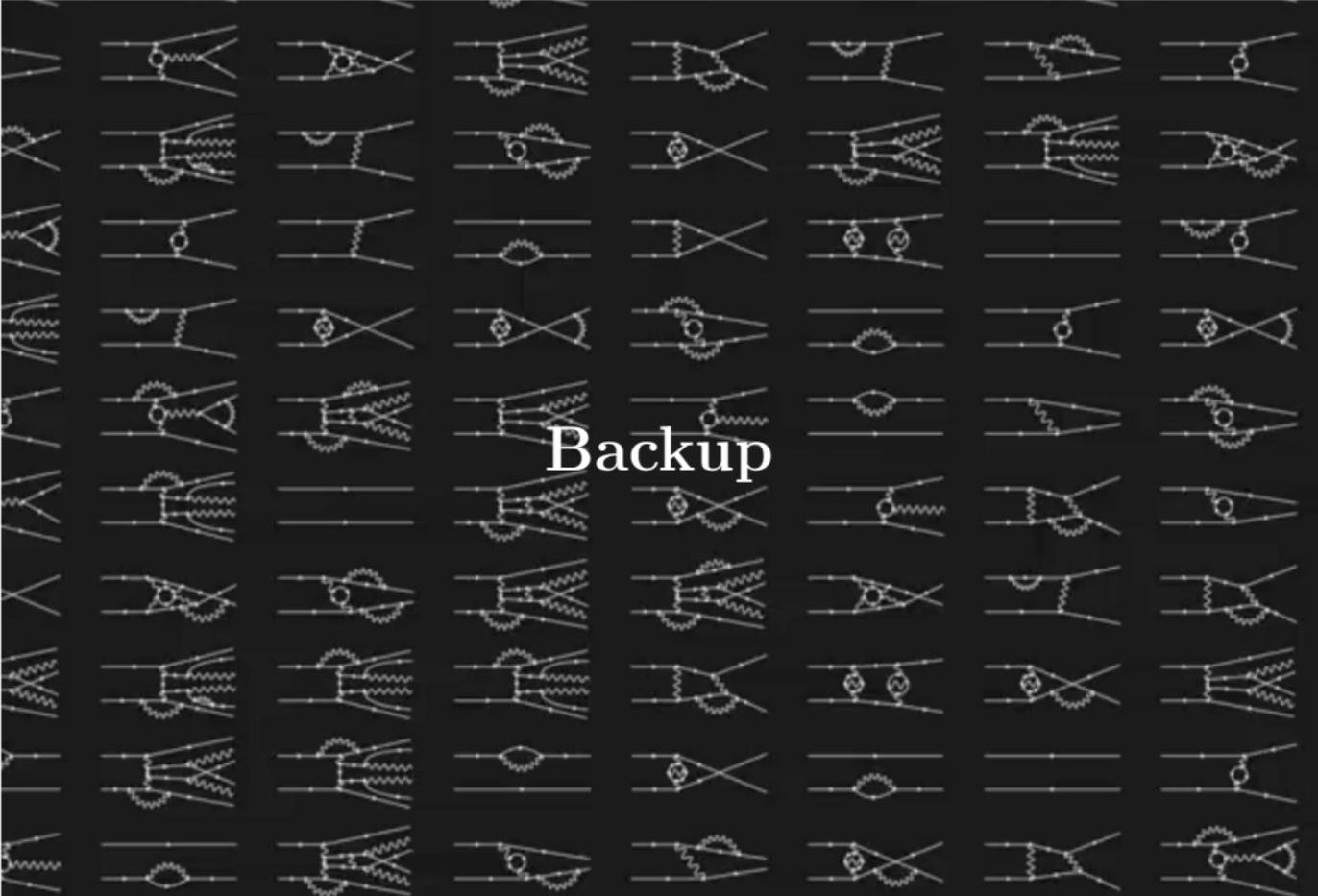
$$\chi^2 = \sum_{i=1}^n \frac{(y_i - f(x_i; \alpha))^2}{\sigma_{y_i}^2 + \sigma_{f(x_i; \alpha)}^2}$$







- **Sensitivity to top anomalous couplings** determined from 1σ confidence intervals in semileptonic channel for $t\bar{t}$ @ FCC-ee
- **Event selection** and **event reconstruction** performed
- **Jet studies** shed light on reevaluating jet definitions at future circular colliders
- **Jet Clustering and Jet Tagging tools** included in FCCAnalyses
- **ABCfit++** written as a general package for kinematic fitting



Backup



Jet Algorithms		
k_t	clustering_kt	} suitable for pp collisions
Anti- k_t	clustering_antikt	
Cambridge/Aachen	clustering_cambridge	
Generalised- k_t	clustering_genkt	
Durham	clustering_ee_kt	} suitable for e^+e^- collisions
Generalised- k_t for e^+e^-	clustering_ee_genkt	
Valencia	clustering_Valencia	
Jade	clustering_Jade	

Plugins {

Arguments for jet definition:

- ① Jet cone radius
- ② Clustering
 - 0=inclusive clustering,
 - 1=exclusive clustering with dcut,
 - 2=exclusive clustering to exactly njets,
 - 3=exclusive clustering up to exactly njets,
 - 4=exclusive clustering with ycut.
- ③ Cut-value depending on clustering
- ④ Ordering of returned jets
 - 0=sorted by p_t ,
 - 1=sorted by E .
- ⑤ Recombination scheme
- ⊕ Additional input parameters specific to jet algorithm
 - see JetClustering.h



Recombination Schemes
E -scheme
p_t -scheme
p_t^2 -scheme
E_t -scheme
E_t^2 -scheme
Boost-invariant p_t -scheme
Boost-invariant p_t^2 -scheme
$E0$ -scheme
p -scheme

- FastJet is focused towards hadron colliders and besides the E -scheme it does not have dedicated schemes for e^+e^- collisions.
- $E0$ - and p -scheme are external recombination schemes

E-scheme: Parton i and j are replaced by a pseudojet k with four-momentum

$$\mathbf{p}_k = \mathbf{p}_i + \mathbf{p}_j$$

- Lorentz invariant, energy and momentum conserved, non-zero mass for pseudojet k .

E0-scheme: The four-momentum of pseudojet k is rescaled to have zero invariant mass

$$E_k = E_i + E_j \quad , \quad \vec{p}_k = \frac{E_k}{|\vec{p}_i + \vec{p}_j|} \cdot (\vec{p}_i + \vec{p}_j)$$

- Not Lorentz invariant, only conserves energy.

p-scheme: The four-momentum is constructed to have zero invariant mass

$$\vec{p}_k = \vec{p}_i + \vec{p}_j \quad , \quad E_k = |\vec{p}_k|$$

- Not Lorentz invariant, only conserves momentum.

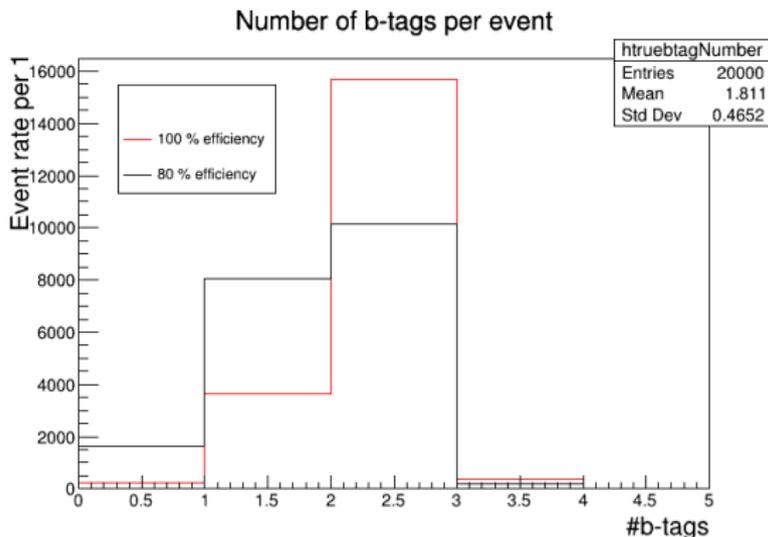
Jet Tagging Interface

- Jet is matched to parton by
 $Angle(jet, parton) < 0.3 \text{ rad}$
- Flavour priority: $b > c > \text{light flavour}$
- Flat efficiency for individual flavours

FCCAnalyses / JetTagging

> `analyzers/dataframe/JetTaggingUtils.h`

`analyzers/dataframe/JetTaggingUtils.cc`





Selection	Signal/ 10^3	Background/ 10^3	ϵ_{sig}	ϵ_{bkg}	Signif
Initial	262.89 ± 0.24	$29\,052 \pm 22$	1.0	1.0	48.6
At least 1 lepton	246.01 ± 0.23	$13\,565 \pm 14$	0.94	0.47	66.2
Thrust < 0.85	236.07 ± 0.23	1599.1 ± 4.7	0.90	0.055	174.3
$M(\text{rest}) > 160$ GeV	234.61 ± 0.23	676.4 ± 2.9	0.89	0.023	245.8
$M(\ell_{HE}, \cancel{E}) > 50$ GeV	181.21 ± 0.20	178.3 ± 1.5	0.69	0.0061	302.2
$p_{\ell_{HE}} < 100$ GeV	179.72 ± 0.20	133.6 ± 1.3	0.68	0.0046	321.0
$p_{\ell_{HE}} > 15$ GeV	176.01 ± 0.20	117.8 ± 1.2	0.67	0.0041	324.7
$p_{\ell_{2^{nd}HE}} < 40$ GeV	175.07 ± 0.20	103.5 ± 1.1	0.67	0.0036	331.7
Exactly 4 jets	175.07 ± 0.20	93.4 ± 1.1	0.67	0.0032	337.9
At least 1 b-tag	160.62 ± 0.19	11.14 ± 0.25	0.61	0.00038	387.6

} 15 %
impr.

- $\epsilon_{\text{sig}}(e, \mu) = 84$ % and $\epsilon_{\text{sig}}(\tau) = 15$ %
- Largest background contributions: ZH, ZZ and single top

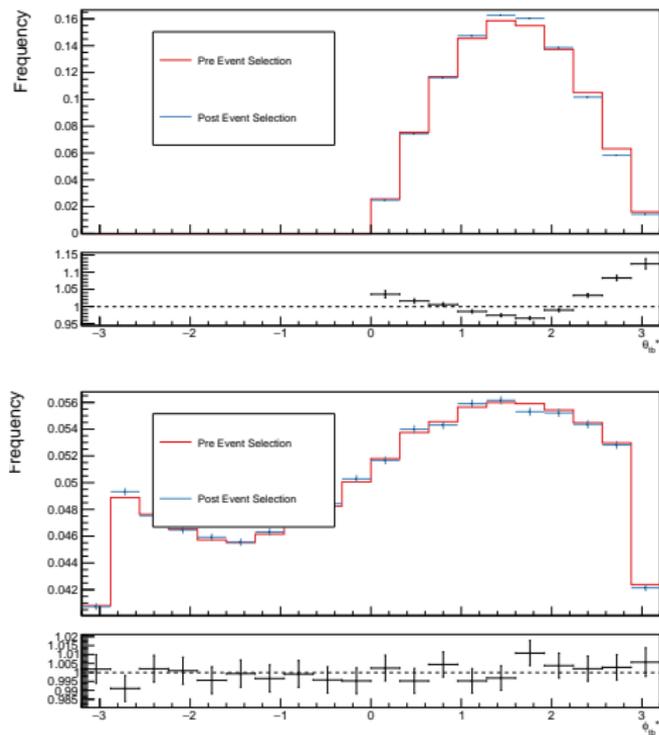


List of observables

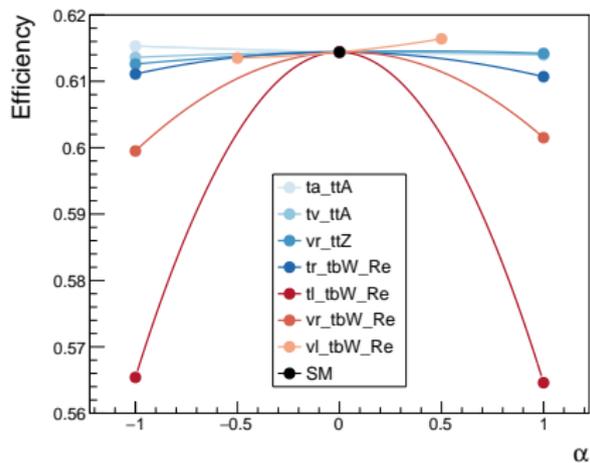
- Highest energy lepton
 - 2nd highest energy lepton
 - Lepton momentum
 - Lepton momentum excluding highest energy lepton
 - Momentum difference between highest and second highest energy lepton
 - Missing momentum
 - Invariant mass of lepton-neutrino pair
 - Invariant mass of 1st and 2nd highest energy leptons
 - Invariant mass of event excluding highest energy lepton
 - Invariant mass of entire event
 - Thrust of event excluding highest energy lepton
 - Thrust of entire event
 - Mass of jet
 - Energy of jet
 - Number of b-tagged jets
 - Significance distribution
 - Minimum of distance measure, d_{\min}
 - Invariant mass of one jet, $\Delta(m_i - m_W/2)$
 - Invariant mass of di-jet system, $\Delta(m_{ij} - m_W)$
 - Invariant mass of tri-jet system, $\Delta(m_{ijk} - m_t)$
 - Invariant mass of lepton-neutrino-jet system, $\Delta(m_{l,\nu\ell,i} - m_t)$
-

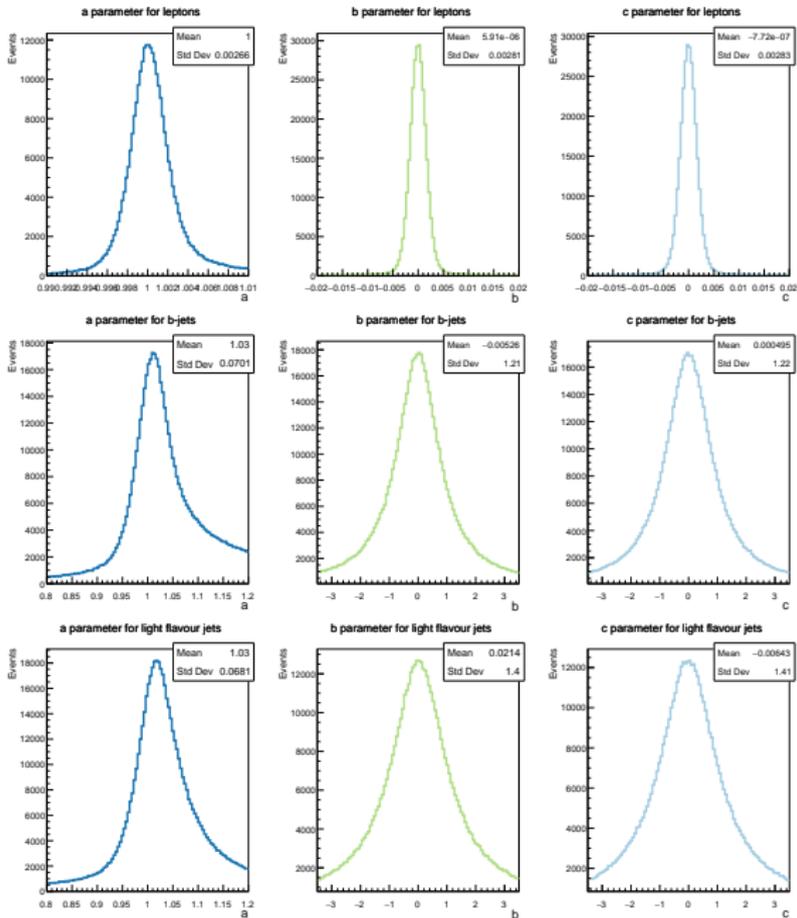


Angular distributions:



Total cross section:





**Gaussian
ABC-parameters:**

$$\vec{p}_j^r = a_j |\vec{p}_j^m| \vec{p}_j^a + b_j \vec{p}_j^b + c_j \vec{p}_j^c$$

**Cartesian unit
vectors:**

$$\vec{p}_j^a = \frac{\vec{p}_j^m}{|\vec{p}_j^m|},$$

$$\vec{p}_j^a \cdot \vec{p}_j^b = 0,$$

$$\vec{p}_j^c = \vec{p}_j^a \times \vec{p}_j^b.$$

$$(a, b, c)_{\text{lepton}} = (1, 0, 0)$$

$$(a, b, c)_{\text{b-jet}} = (1.03, 0, 0)$$

$$(a, b, c)_{\text{lf-jet}} = (1.03, 0, 0)$$



$$\vec{p}_j^r = a_j |\vec{p}_j^m| \vec{p}_j^a + b_j \vec{p}_j^b + c_j \vec{p}_j^c \quad (1)$$

Parameters: a_j , b_j and c_j

Unit vectors forming a Cartesian system: \vec{p}_j^a , \vec{p}_j^b and \vec{p}_j^c

Measured jet momentum: \vec{p}_j^m ,

$$\vec{p}_j^a = \frac{\vec{p}_j^m}{|\vec{p}_j^m|},$$

$$\vec{p}_j^b = \frac{1}{\sqrt{p_{x,m}^2 + p_{y,m}^2}} (p_y^m, -p_x^m, 0),$$

$$\vec{p}_j^c = \frac{1}{\sqrt{|\vec{p}_j^m|^2 (p_{x,m}^2 + p_{y,m}^2)}} (-p_x^m p_z^m, -p_y^m p_z^m, p_{x,m}^2 + p_{y,m}^2).$$