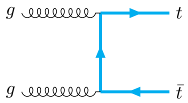
Non-factorisable two-loop contribution to t-channel single-top production

Based on arXiv:2108.09222 with Christian Brønnum-Hansen, Kirill Melnikov & Chen-Yu Wang

Jérémie Quarroz | 23 Nov 2021 | Terascale AM

Motivation

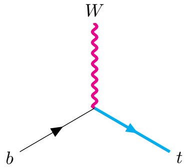
- Top quark is the heaviest particle of the Standard Model.
  - ➥ Better understanding of electroweak symmetry breaking and hopefully, physics beyond the Standard Model.



- Primarily produced in pair. However, single-top production also occurs frequently
  - ➥ tWb interaction

$$\sigma_{t,\text{single}} \approx \frac{1}{4}\sigma_{t\bar{t}}$$

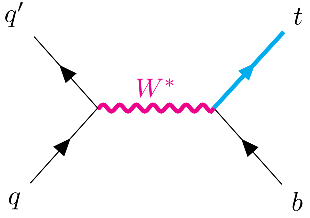
- tWb interaction is interesting due to:
  - ➥ determination of the CKM matrix element V_{bt}
  - ➥ an indirect determination of Γ_t and the top-quark mass m_t
  - ➥ a constrain of bottom-quark PDF $f_b(x_1)$



Single-top production

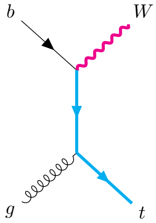
There are three single-top production modes

s channel: $q\bar{q}' \to W^* \to t\bar{b}$



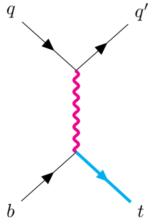
$\mathcal{O}$ (5%)

associated production: $gb \to Wt$



$\mathcal{O}$ (25%)

t channel: $qb \to q' t$



$\mathcal{O}$ (70%)

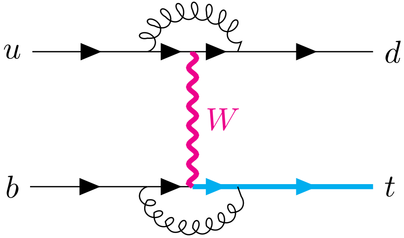
➽ The main production mode is the t-channel.

NNLO QCD corrections to t-channel single-top production

Pertubative QCD corrections are known up to an advanced stage.

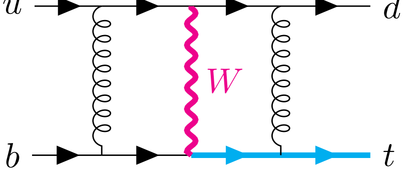
- NLO corrections are known since a while. *Harris et al. 2002; Campbell et al. 2004; Sullivan 2004; Cao and Yuan 2005; Sullivan 2005; Schwienhorst et al. 2011*
- NNLO corrections are known except for *non-factorisable corrections*. *Brucherseifer, Caola and Melnikov 2014; Berger, Edmond, Gao, Yuan, Zhu 2016; Campbell, Neumann and Sullivan 2021*

Factorisable



$$\mathrm{tr}\left(t^a t^a\right)\,\mathrm{tr}\left(t^b t^b\right) = \frac{1}{4}(N_c^2 - 1)^2$$

Non-factorisable



$$\mathrm{tr}\left(t^a t^b\right)\,\mathrm{tr}\left(t^a t^b\right) = \frac{1}{4}(N_c^2 - 1)$$

Non-factorisable contributions

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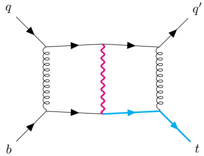
- These *non-factorisable* corrections are expected to be negligible because they are colour-suppressed.
- *Non-factorisable* corrections vanish at NLO because of color$\Rightarrow$ $\mathcal{A}_{\text{LO}} \otimes \mathcal{A}_{\text{NLO}} = 0$
- It is not obvious *non-factorisable* corrections are negligible.
  - ➥ Factorisable NNLO QCD corrections are small (few %).
  - ➥ Possible π^2 enhancement in non-factorisable contribution. *Liu, Melnikov, et al. 2019*
- A better understand of *non-factorisable* corrections to single-top production at LHC is necessary.

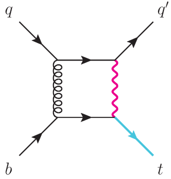
Purposes of this work

Our goal is to calculate the non-factorisable two-loop virtual amplitude.

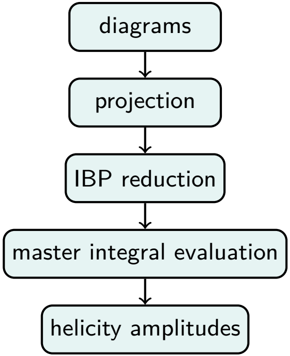
We need $\mathcal{A}_{\text{LO}} \otimes \mathcal{A}^{(2)}_{n.f}$ and $\mathcal{A}^{(1)}_{n.f} \otimes \mathcal{A}^{(1)}_{n.f}$

$\mathcal{A}^{(2)}_{n.f} =$ 

- every integral is expressed as a linear combination of master integrals keeping the **exact dependence** on m_t and m_W.
- Master integrals are computed with **auxiliary mass flow method**.
- Non-factorisable amplitude is evaluated.

$\mathcal{A}^{(1)}_{n.f} =$ 

Overview

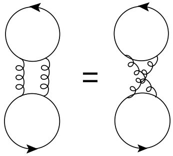


The process we study is

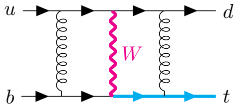
$$u(p_1) + b(p_2) \to d(p_3) + t(p_4)$$

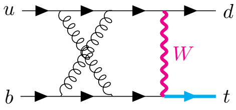
where

$$\begin{cases} p_i^2 = 0 & , i = 1,2,3 \\ p_4^2 = m_t^2 & \end{cases}$$



- Colour factor is $\mathrm{tr}\left(t^a t^b\right)\mathrm{tr}\left(t^a t^b\right) = \frac{1}{4}(N_c^2 - 1)$
- Diagrams with non-abelian gluon vertices do not contribute.
- Only **18 non-vanishing diagrams** contributes. Eg:





Spinor structures and γ_5

- Projection on to 11 spinor stuctures *Assadsolimani et al. 2014*

$$
\begin{aligned}
S_1 &= \bar{t}(p_4)\; b(p_2) \times \bar{q}'(p_3)\, \not{p}_4\, b(p_1) \\
S_2 &= \bar{t}(p_4)\, \not{p}_1\, b(p_2) \times \bar{q}'(p_3)\, \not{p}_4\, b(p_1) \\
S_3 &= \bar{t}(p_4)\, \gamma^{\mu_1}\, b(p_2) \times \bar{q}'(p_3)\, \gamma_{\mu_1}\, b(p_1) \\
S_4 &= \bar{t}(p_4)\, \gamma^{\mu_1} \not{p}_1\, b(p_2) \times \bar{q}'(p_3)\, \gamma_{\mu_1}\, b(p_1) \\
&\vdots \\
S_{11} &= \bar{t}(p_4)\, \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5}\, b(p_2) \times \bar{q}'(p_3)\, \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\, b(p_1)
\end{aligned}
$$

- Exploit anti-commutativity of γ_5 to move left-handed projectors to external *massless* fermions.
- Non-factorisable amplitude is expressed in terms of 11 form factors $\mathcal{A}_{nf}^{(2)} = \vec{f} \cdot \vec{S}$
- Form factors does not depend on helicities of external states.
  $\rightarrow$ **one can compute them with vector currents**.

Helicity amplitudes

- *'t Hooft-Veltman scheme*: external momenta in $d = 4$ and internal in $d = 4 - 2\epsilon$
- *At least two* matrices in $d = 4 - 2\epsilon$ are needed between two $d = 4$ spinors to have a support in -2ϵ space.

$$\bar{u}_d(p_3)\gamma_\mu\gamma_\nu \not{p}_4 u_u(p_1) \to \begin{pmatrix} \bar{u}_d(p_3) \\ 0 \end{pmatrix} \begin{pmatrix} \underbrace{\gamma_\mu}_{d=4} & 0 \\ 0 & \underbrace{\gamma_\mu}_{d=-2\epsilon} \end{pmatrix} \begin{pmatrix} \underbrace{\gamma_\nu}_{d=4} & 0 \\ 0 & \underbrace{\gamma_\nu}_{d=-2\epsilon} \end{pmatrix} \begin{pmatrix} \underbrace{\not{p}_4}_{d=4} & 0 \\ 0 & \underbrace{0}_{d=-2\epsilon} \end{pmatrix} \begin{pmatrix} \bar{u}_u(p_1) \\ 0 \end{pmatrix}$$

- ϵ dependence can be explicitly and unambigously extracted and γ_5 restored

$$\begin{cases} \mathcal{S}_{1,..,4} = \mathcal{S}^{(4)}_{1,..,4}\,, \\ \mathcal{S}_{5,6} = \mathcal{S}^{(4)}_{5,6} - 2\epsilon\mathcal{S}^{(4)}_{1,2}\,, \\ \mathcal{S}_{7,8} = \mathcal{S}^{(4)}_{7,8} - 6\epsilon\mathcal{S}^{(4)}_{3,4}\,, \\ \mathcal{S}_{9,10} = \mathcal{S}^{(4)}_{9,10} - 12\epsilon\mathcal{S}^{(4)}_{5,6} + \left(12\epsilon^2 + 4\epsilon\right)\mathcal{S}^{(4)}_{1,2}\,, \\ \mathcal{S}_{11} = \mathcal{S}^{(4)}_{11} - 20\epsilon\mathcal{S}^{(4)}_{7} + \left(60\epsilon^2 + 20\epsilon\right)\mathcal{S}^{(4)}_{3} \end{cases}$$

IBP reduction

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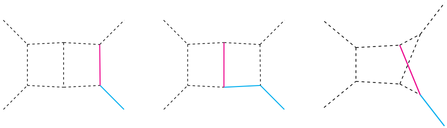
- Find symmetry relations with `REDUZE` 2 *Manteuffel and Studerus 2012.*
- Reduction performed **analytically** with `KIRA` 2.0: *Klappert, Lange, et al. 2020* and `FireFly` *Klappert and Lange 2020; Klappert, Klein, et al. 2021:*

$$\langle A^{(0)} | A_{\rm nf}^{(2)} \rangle = \sum_{i=1}^{428} c_i(d, s, t, m_t, m_W) I_i$$

- Analytic reduction is possible with four scales (s, t, m_t, m_W): $\mathcal{O}(1)$ day
- 428 master integrals I_i in 18 families
- file size of the simplified coefficients c_i: $\mathcal{O}(1)$ MB

Master integrals evaluation



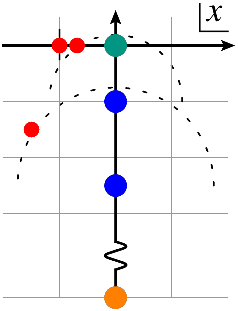
- Based on the **auxiliary mass flow method** *Liu, Ma, and Wang 2018; Liu, Ma, Tao, et al. 2020; Liu and Ma 2021*

$$m_W^2 \to m_W^2 - i\eta.$$

- Solve differential equations at each kinematic point

$$\partial_x \boldsymbol{I} = \boldsymbol{M}\boldsymbol{I}, \quad x \propto -i\eta.$$

with boundary condition $x \to -i\infty$.



Stepping from the boundary at $x \to -i\infty$, via regular points, to the physical mass. Step size is limited by singularities of the equation.

Master integral evaluation

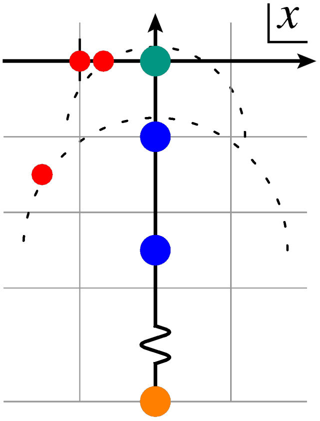
- Expand $\boldsymbol{I}$ around **boundary** in variable $y = x^{-1} = 0$:

$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_k^N \sum_l \boldsymbol{c}_{jkl} y^k \ln^l y + \dots$$

- Evaluate and expand around **regular points**:

$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_{k=0}^N \boldsymbol{c}_{jk} x'^k + \dots$$

- Evaluate at the **physical point.** $x = 0 \leftarrow$ **regular point**
- **Path** is fixed by **singularities** and desired precision.

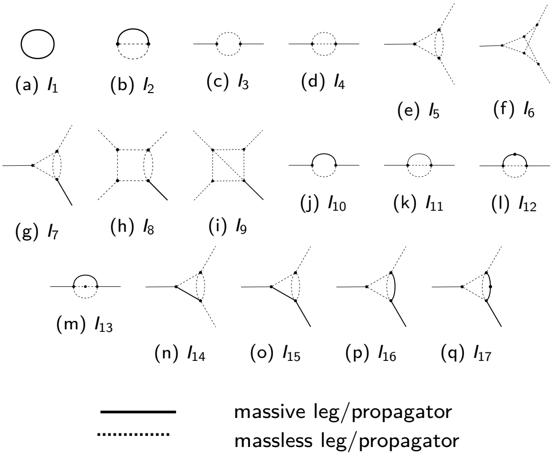


$m_W^2 \to m_W^2(1+x)$

Boundary conditions

- Most of the integrals needed can be found in the literature.
- Some of them are not available or are not known to sufficiently high ϵ order.
- All **428** master integrals evaluated numerically to 20 digits in ~ 30 minutes on a single core.

Master integrals for the boundary conditions



Results

- Comparison of poles at a typical phase space point $s \approx 104.337$ GeV^2 and $t \approx -5179.68$ GeV^2.

	ϵ^{-2}	ϵ^{-1}
$\langle \mathcal{A}^{(0)} \vert \mathcal{A}^{(2)}_{\text{nf}} \rangle$	$-229.0940408654660 - 8.978163333241640i$	$-301.180298894476 4 - 264.1773596529505i$
IR poles	$-229.0940408654665 - 8.978163333241973i$	$-301.1802988944791 - 264.1773596529535i$

- Double-virtual cross-section calculation from fixed grid of 100k points

$$\sigma^{ub}_{pp\to dt} = \left(90.3 + 0.3 \left(\frac{\alpha_s(\mu_{\text{nf}})}{0.108}\right)^2\right) \text{pb}$$

- Correction of about 0.3% for $\mu_{\text{nf}} = 173$ GeV

- **Typical transverse momentum**: $\mu_{nf} = 40 - 60$ GeV. The Magnitude of the non-factorisable corrections will **increase** by a factor $\mathcal{O}(1.5)$ and become close to **half a percent**.

Kinematic distributions

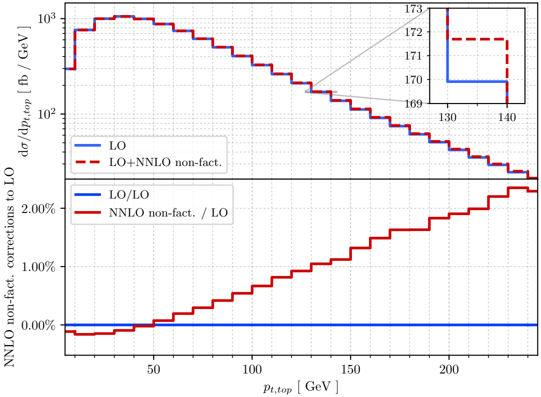
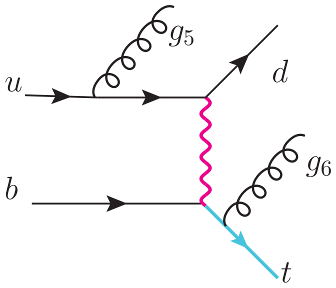


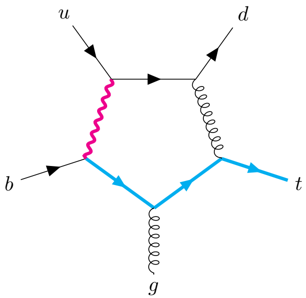
Figure: The top quark transverse momentum distribution.

- There is a significant $p_{\perp}$-dependence of the non-factorisable corrections.
- Non-factorisable virtual corrections vanishes around 50 GeV. The factorisable corrections vanish around 30 GeV.
- In some part of the phase space at low $p_{t,\text{top}}$, non-factorisable corrections are **dominant**.

Caveat

- Real corrections are to be included. → Work in progress !





Conclusion

- We computed **the missing part** of NNLO QCD corrections to the t-channel single-top production amplitude: **the two-loop non-factorisable virtual corrections**.

- **The auxiliary mass flow method** has been used for integrals evaluation. It is sufficiently **robust** to produce results relevant for phenomenology.

- Non-factorisable are smaller than, but **quite comparable** to, the factorisable ones.

- The non-factorisable real-emission contributions **still need to be included**. We are currently working on this.

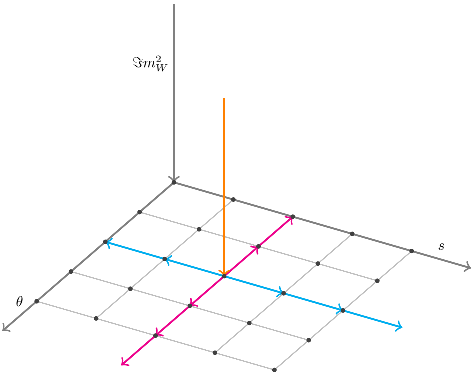
Thank you for your attention !

Master integral evaluation

- We can use the differential equation w.r.t s and t to generate phase space points.
- Solving differential equation in each direction:

$$(s_1, t_1) \xrightarrow{s} (s_2, t_1) \xrightarrow{t} (s_2, t_2)$$

- This also serves as a consistency check.



Evaluation of the cross-section

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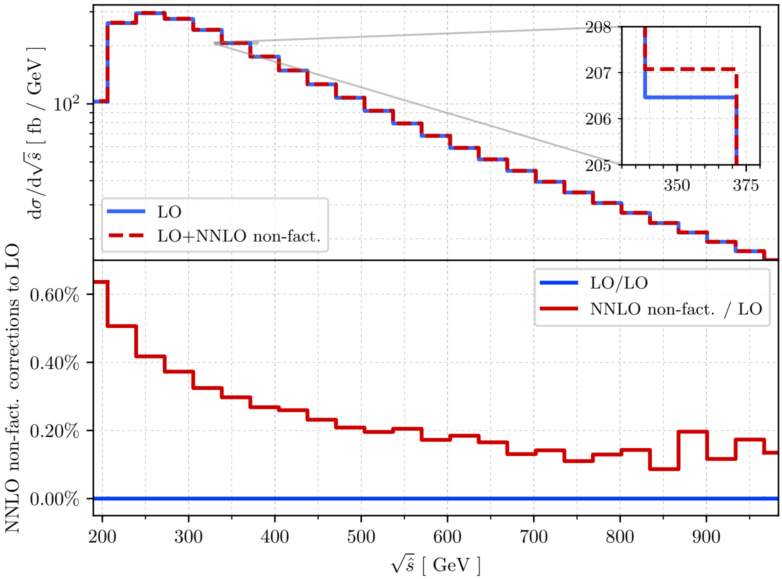
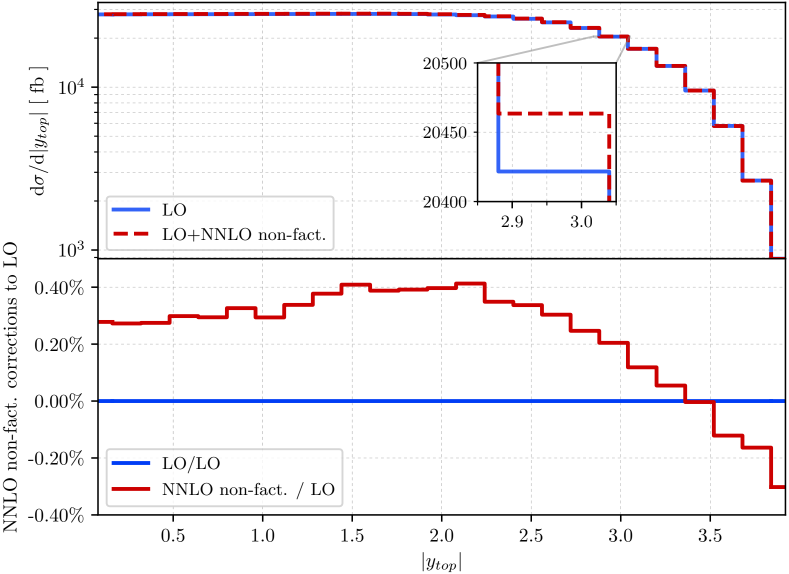
- The cross-section is evaluated with the help of a **Vegas integrator**.
- 10 grids of 10^4 points are prepared **on the Born squared amplitude**.
- $\mathcal{A}^{(1)}_{nf} \otimes \mathcal{A}^{(1)}_{nf}$ and $\mathcal{A}^{(0)} \otimes \mathcal{A}^{(2)}_{nf}$ are evaluated for each of the 10^5 points. ($\approx \mathcal{O}(1 \text{ day})$)
- The 10 different set of points give an estimation of the error of the total cross-section. (2%)



$d\sigma/d\sqrt{\hat{s}}$ [fb / GeV]
10^2
208
207
206
205
350
375
LO
LO+NNLO non-fact.
NNLO non-fact. corrections to LO
0.60%
0.40%
0.20%
0.00%
LO/LO
NNLO non-fact. / LO
200
300
400
500
600
700
800
900
$\sqrt{\hat{s}}$ [GeV]



10^4
10^3
$\mathrm{d}\sigma/\mathrm{d}|y_{top}|$ [fb]
20500
20450
20400
2.9
3.0
LO
LO+NNLO non-fact.
0.40%
0.20%
0.00%
-0.20%
-0.40%
NNLO non-fact. corrections to LO
LO/LO
NNLO non-fact. / LO
0.5
1.0
1.5
2.0
2.5
3.0
3.5
$|y_{top}|$

UV and IR singularities

- **No UV divergences** if we consider only non-factorisable contributions at NNLO.
- IR divergences are predicted using colour-space operators. *Catani 1998; Becher and Neubert 2009; Czakon and Heymes 2014*

$$|\mathcal{A}_{\rm nf}\rangle = \boldsymbol{Z}_{\rm nf}|\mathcal{F}_{\rm nf}\rangle, \qquad \mu\frac{d}{d\mu}\boldsymbol{Z}_{\rm nf} = -\boldsymbol{\Gamma}_{\rm nf}\boldsymbol{Z}_{\rm nf}$$

where the anomalous dimension operator, $\boldsymbol{\Gamma}_{\rm nf}$, is limited to non-factorisable relevant contributions

$$\boldsymbol{\Gamma}_{\rm nf} = \left(\frac{\alpha_s}{4\pi}\right)\boldsymbol{\Gamma}_{0,\rm nf} = \left(\frac{\alpha_s}{4\pi}\right)4\left[\boldsymbol{T}_u\cdot\boldsymbol{T}_b\ln\left(\frac{\mu^2}{-s-i\varepsilon}\right) + \boldsymbol{T}_b\cdot\boldsymbol{T}_d\ln\left(\frac{\mu^2}{-u-i\varepsilon}\right)\right.$$
$$\left. + \boldsymbol{T}_u\cdot\boldsymbol{T}_t\ln\left(\frac{\mu\, m_t}{m_t^2-u-i\varepsilon}\right) + \boldsymbol{T}_d\cdot\boldsymbol{T}_t\ln\left(\frac{\mu\, m_t}{m_t^2-s-i\varepsilon}\right)\right]$$

- Divergences of non-factorisable amplitude starts at $1/\epsilon^2$ due to **absence of collinear contributions**.

$$\langle\mathcal{A}^{(0)}|\mathcal{A}^{(2)}_{\rm nf}\rangle = -\frac{1}{8\epsilon^2}\langle\mathcal{A}^{(0)}|\boldsymbol{\Gamma}^2_{0,\rm nf}|\mathcal{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathcal{A}^{(0)}|\boldsymbol{\Gamma}_{0,\rm nf}|\mathcal{A}^{(1)}_{\rm nf}\rangle + \langle\mathcal{A}^{(0)}|\mathcal{F}^{(2)}_{\rm nf}\rangle,$$
$$\langle\mathcal{A}^{(1)}_{\rm nf}|\mathcal{A}^{(1)}_{\rm nf}\rangle = \frac{1}{4\epsilon^2}\langle\mathcal{A}^{(0)}||\boldsymbol{\Gamma}_{0,\rm nf}|^2|\mathcal{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathcal{A}^{(1)}_{\rm nf}|\boldsymbol{\Gamma}_{0,\rm nf}|\mathcal{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathcal{A}^{(0)}|\boldsymbol{\Gamma}^{\dagger}_{0,\rm nf}|\mathcal{A}^{(1)}_{\rm nf}\rangle + \langle\mathcal{F}^{(1)}_{\rm nf}|\mathcal{F}^{(1)}_{\rm nf}\rangle.$$

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