QFT without infinities and hierarchy problem

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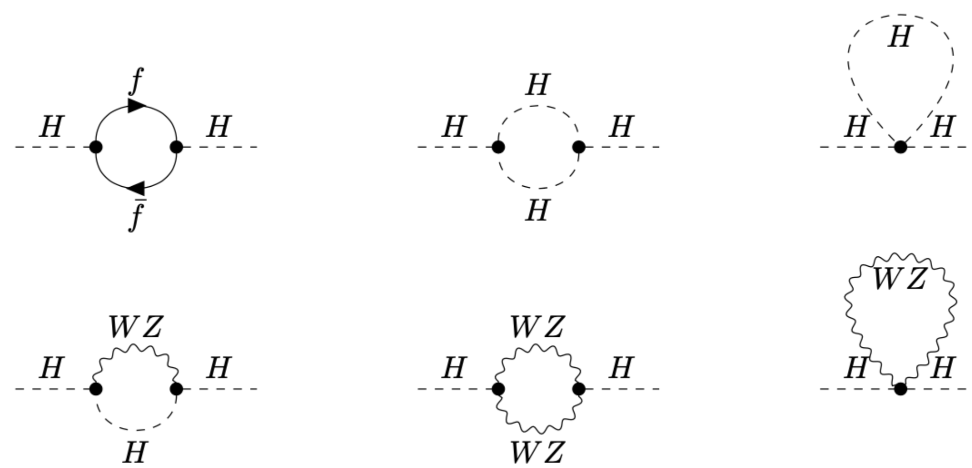
Outline

- Introduction: Naturalness
- Finite approaches to QFT
- Callan-Symanzik method as a finite description of QFT
    - No divergences
    - No fine tunings
- Conclusions

Based on: Sander Mooij and MS, 2110.05175, 2110.15925

Higgs mass fine-tuning

- The puzzle: take the Standard Model and consider radiative corrections to the Higgs mass. Quadratically divergent diagrams



lead to the term $\delta m_H^2 \propto f_t^2 \Lambda^2$, f_t - top quark Yukawa coupling, Λ - the ultraviolet cutoff of the theory, i.e. the place where the Standard Model is substituted by the more fundamental theory of Nature. Since $m_H \ll \Lambda$, one has to fine-tune the tree Higgs mass M_{tree} to cancel the radiative correction(s). The amount of fine-tuning:

$$\epsilon_H = \frac{M_{\text{tree}}^2 - \delta m_H^2}{\Lambda^2} \sim \left(\frac{100 \text{ GeV}}{4\pi\Lambda} \right)^2 \ll 1$$

Cosmological constant fine-tuning

The similar logic can be applied to vacuum energy ϵ_{vac}:

The radiative corrections are proportional to the fourth power of the cutoff scale, $\delta\epsilon_{\text{vac}} \propto f_t^4 \Lambda^4$ leading to even higher degree of fine-tuning

$$\epsilon_{\text{cc}} = \frac{\epsilon_{\text{vac}}^{\text{tree}} - \delta\epsilon_{\text{vac}}}{\Lambda^4} \sim \left(\frac{10^{-3}\ \text{eV}}{\Lambda}\right)^4 \ll \ll 1$$

Wilsonian approach

Similar picture. Low energy description of Nature provided by the SM: take all sorts of gauge-invariant operators $\mathcal{O}_n$ of mass dimension n, constructed from the SM fields. Power counting: two operators in the expansion of the action with respect to possible operators come with positive powers of the cutoff, namely

$$\mathcal{O}_2 \propto \Lambda^2 h^\dagger h,$$

giving the mass of the Higgs boson (h is the scalar field of the SM), and

$$\mathcal{O}_0 = \epsilon_{\text{vac}} \propto \Lambda^4,$$

representing the vacuum energy. The so-called fine-tuning puzzle is why the high energy contributions to these quantities are nearly cancelled by the low energy radiative corrections.

Two problems

1. Why the physical values of the Higgs mass and of the cosmological constant are much smaller than the scale of new physics (cutoff Λ) ?

2. Why the tree values of these parameters are so fine-tuned to the radiative corrections?

Naturalness:

- These fine tunings must be avoided at any price!

- The cutoff Λ must be of the order of the Fermi scale to screen the influence of high energy domain from low energy domain (SUSY, composite Higgs boson, large extra dimensions) ?

- Cosmological evolution leading to $m_H \ll \Lambda$?

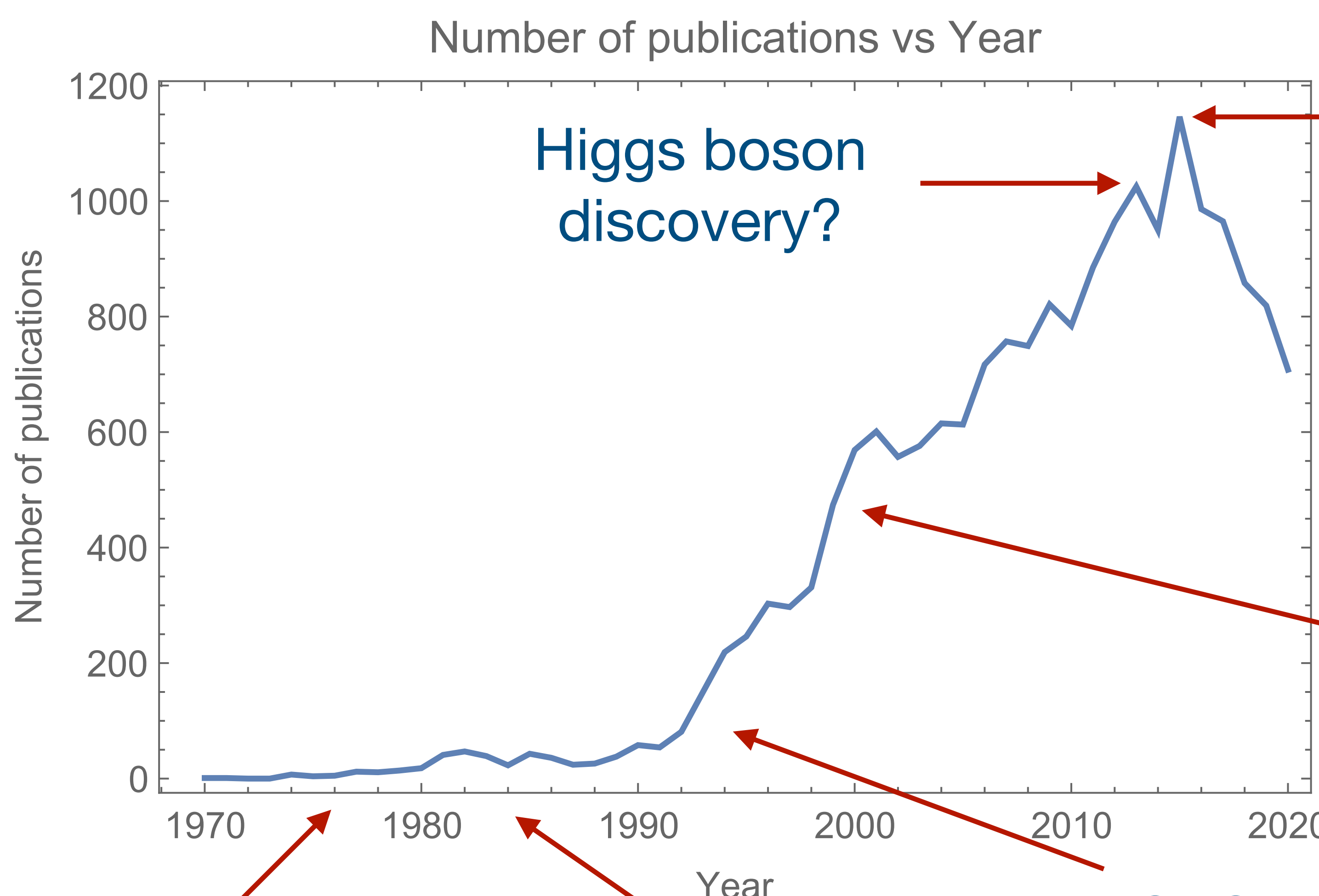
- Environmental selection leading to $m_H \ll \Lambda$?

Generically, these proposals lead to some kind of new physics right above the Fermi scale.

This problem attracted a lot of attention

The number of articles which mention
“hierarchy problem” or
“fine-tuning” or
“naturalness”
is very large:

Credit: Oleg Ruchayskiy
https://www.prophy.science



750 GeV resonance?

Large extra dimensions?

SUSY ?

Susskind ’84:
The Gauge Hierarchy Problem

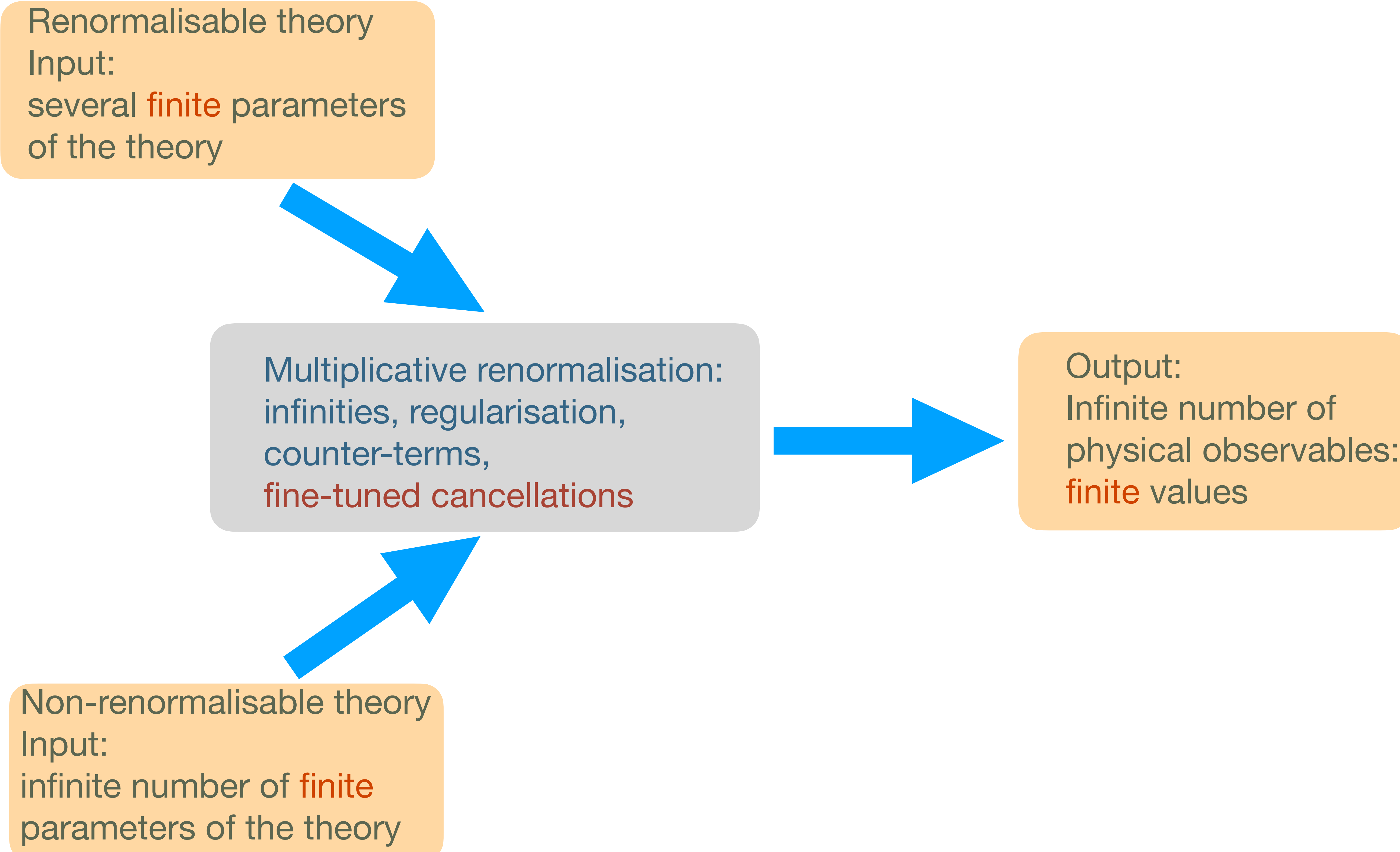
Gildener ’76:
Gauge symmetry hierarchies

Origin of the fine-tunings

The core of the problem: quadratic (or quartic, if we talk about the cosmological constant) divergences, inevitably appearing in Feynman diagrams with loops in theories with fundamental scalar fields

Renormalisation:

- Regularise UV divergent expressions
- Subtract divergences (this is exactly where fine-tunings show up)
- Get finite values for physical observables


Renormalisable theory
Input:
several finite parameters
of the theory
Multiplicative renormalisation:
infinities, regularisation,
counter-terms,
fine-tuned cancellations
Output:
Infinite number of
physical observables:
finite values
Non-renormalisable theory
Input:
infinite number of finite
parameters of the theory

Renormalisable theory
Input:
several finite parameters of the theory

Finite formulation of QFT

Output:
Infinite number of physical observables:
finite values

Non-renormalisable theory
Input:
infinite number of finite parameters of the theory

Hierarchy problem in finite formulations of QFT?

No infinities (quartic, quadratic, log) in finite QFT - perhaps, no fine-tunings? Indeed, if all expressions are finite, the computation of low energy observables should not require the knowledge of the UV domain of the theory.

The existence of such a formalism without large cancellations would challenge the “naturalness” paradigm.

If just one particular formalism of computations in QFT without necessity of fine-tunings is found, it will provide a strong argument that the problem of quantum stability of the electroweak scale against radiative corrections is formalism dependent and thus unphysical.

Finite formulations of QFT

Bogolubov-Parasuk-Hepp-Zimmermann
(BPHZ)

A certain procedure, called “R-operation” is applied to any Feynman graph before performing integrations over internal momenta) changing the integrand prescribed by the Feynman rules to another one. The resulting expression is then integrated, with no infinities encountered. The R-operation can be used in both renormalisable and non-renormalisable field theories.

INTRODUCTION TO THE THEORY OF QUANTIZED FIELDS

THIRD EDITION

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Steklov Mathematical Institute, Academy of Sciences Moscow, USSR
Joint Institute for Nuclear Research, Dubna, USSR

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The Bogolyubov $\mathcal{R}$-Operation in Nonrenormalizable Theories

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Finite formulations of QFT

Callan-Symanzik - inspired finite renormalisation equations

Usually, CS equations are represented as a tool for the renormalisation group investigation of the high energy behaviour of the renormalised amplitudes. However, the same equations can be used for the construction of the divergence-free and thus completely finite perturbation theory.

Broken Scale Invariance in Scalar Field Theory*

Curtis G. Callan, Jr.†
California Institute of Technology, Pasadena, California 91109
and
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 4 June 1970)

We use scalar-field perturbation theory as a laboratory to study broken scale invariance. We pay particular attention to scaling laws (Ward identities for the scale current) and find that they have unusual anomalies whose presence might have been guessed from renormalization-group arguments. The scaling laws also appear to provide a relatively simple way of computing the renormalized amplitudes of the theory, which sidesteps the overlapping-divergence problem.

Commun. math. Phys. 18, 227—246 (1970)

Small Distance Behaviour in Field Theory and Power Counting

K. Symanzik
Deutsches Elektronen-Synchrotron DESY, Hamburg

Received May 12, 1970

FIELD THEORY RENORMALIZATION USING THE CALLAN-SYMANZIK EQUATION

A.S. BLAER
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K. YOUNG
Physics Department, The Chinese University of Hong Kong, Hong Kong

Received 18 June 1974

INTRODUCTION TO RENORMALIZATION THEORY

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Department of Physics, Joseph Henry Laboratories, Princeton University, Jadwin Hall, PO Box 708, Princeton, NJ 08540

Finite formulations of QFT

t'Hooft: Exact equations for irreducible two-, three -, and four-point vertices which do not contain any ultraviolet infinities. The idea is that any divergent n-point function can be rendered finite by subtracting the same n-point function evaluated at different values for the external momenta. This difference can be interpreted as a new irreducible Feynman diagram with n+1 external lines. Integrating these "difference diagrams" with respect to the external momenta yields renormalisation group equations. Potentially, these equations may result in a completely non-perturbative and divergence-free definition of the theory.

Lehmann, Symanzik and Zimmermann

Nishijima

ITP-UU-04/13
SPIN-04/07
hep-th/0405032

RENORMALIZATION WITHOUT INFINITIES *

Gerard 't Hooft

IL NUOVO CIMENTO VOL. I, N. 1 1° Gennaio 1955

Zur Formulierung quantisierter Feldtheorien.

H. LEHMANN, K. SYMANZIK und W. ZIMMERMANN
Max-Planck-Institut für Physik - Göttingen (Deutschland)

(ricevuto il 22 Novembre 1954)

PHYSICAL REVIEW VOLUME 119, NUMBER 1 JULY 1, 1960

Asymptotic Conditions and Perturbation Theory

K. NISHIJIMA*
Department of Physics, Osaka City University, Osaka, Japan and Department of Physics, University of Illinois, Urbana, Illinois
(Received December 21, 1959)

Callan-Symanzik method as a finite approach to QFT

Ingredients for the simplest scalar theory:

- Lagrangian:

Everything is finite!

$$L = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

- Postulated (but can be derived) equations for vertices with n legs $\bar{\Gamma}^{(n)}$ and new, θ-type vertices:

$$2im^2\,(1+\gamma)\cdot\bar{\Gamma}^{(n)}_{\theta} = \left[\left(m\,\frac{\partial}{\partial m} + \beta\,\frac{\partial}{\partial\lambda}\right) + n\cdot\gamma\right]\bar{\Gamma}^{(n)}$$

$$2im^2\,(1+\gamma)\cdot\bar{\Gamma}^{(n)}_{\theta\theta} = \left[\left(m\,\frac{\partial}{\partial m} + \beta\,\frac{\partial}{\partial\lambda}\right) + n\cdot\gamma + \gamma_\theta\right]\bar{\Gamma}^{(n)}_{\theta}$$

Callan-Symanzik method as a finite approach to QFT

Postulated (but can be derived) boundary conditions, valid in all orders of λ:

$$\left[\frac{d}{dk^2}\,\bar{\Gamma}^{(2)}(k^2)\right]_{k^2=0} = i, \qquad \bar{\Gamma}^{(2)}\left(k^2=0\right) = im^2, \qquad \bar{\Gamma}^{(4)}\left(k^2=0\right) = -\,i\lambda\,.$$

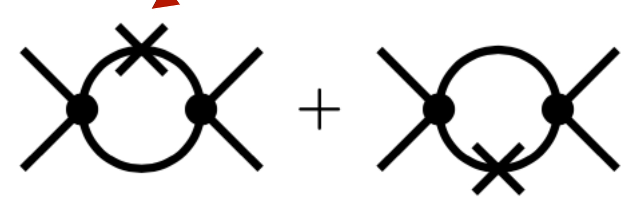
First order, tree approximations for 2 and 4 point functions, computed:

$$\left[\bar{\Gamma}^{(2)}\right]_{\lambda^0} = i\left(k^2+m^2\right) \quad , \quad \left[\bar{\Gamma}^{(4)}\right]_{\lambda} = -\,i\lambda$$

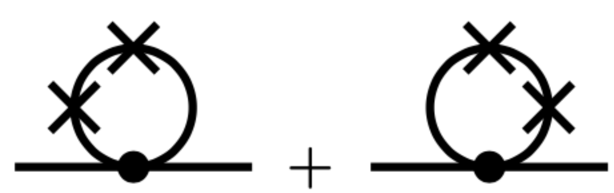
θ-operation: cuts the propagator in two

One-loop finite expressions, computed:

$$\left[\bar{\Gamma}^{(4)}_{\theta}\right]_{\lambda^2} = -\frac{\lambda^2}{32\pi^2}\sum_{3\ \mathrm{opt}}\int_0^1 dx\ \frac{1}{x(1-x)\kappa_1^2+m^2}$$



$$\left[\bar{\Gamma}^{(2)}_{\theta\theta}\right]_{\lambda} = -\frac{i\lambda}{32\pi^2}\,\frac{1}{m^2}$$



The unknown quantities β, γ, γ_θ , and the vertices are to be out by iterative procedure from these equations and boundary conditions. No infinities appear at any step of computation at any loop order.

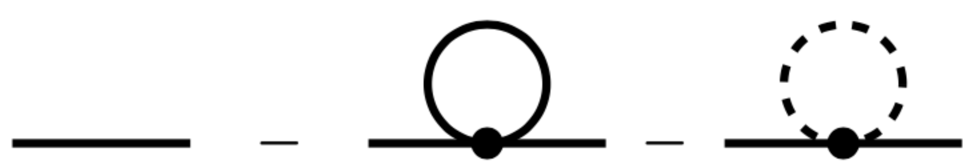
Fine-tunings with two mass scales in multiplicative renormalisation

Theory with two well separated physical mass scales, $M_{\text{phys}} \gg m_{\text{phys}}$

$$L = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\Phi\partial_\mu\Phi - \frac{m^2}{2}\phi^2 - \frac{M^2}{2}\Phi^2 - \frac{\lambda_\phi}{4!}\phi^4 - \frac{\lambda_{\phi\Phi}}{4}\phi^2\Phi^2 - \frac{\lambda_\Phi}{4!}\Phi^4$$

Standard approach, multiplicative renormalisation MS bar scheme, need to highly fine-tune the Lagrangian parameters m and M:

$\Gamma^{(2\phi)} =$



$+\ \mathcal{O}\left(\lambda^2\right)$

$$\bar{\Gamma}^{(2\phi)} = i(k^2 + m^2) - \frac{i\lambda_\phi m^2}{32\pi^2}\left(1 + \ln\frac{\mu^2}{m^2}\right) - \frac{i\lambda_{\phi\Phi}M^2}{32\pi^2}\left(1 + \ln\frac{\mu^2}{M^2}\right)$$

Absence of fine-tunings in finite QFT

- The same Lagrangian

- Postulated (but can be derived) equations for vertices $\bar{\Gamma}^{(n)}$ and new, θ-type vertices: $\bar{\Gamma}^{(n)}_{\theta,m}$, $\bar{\Gamma}^{(n)}_{\theta,M}$, $\bar{\Gamma}^{(n)}_{\theta\theta,mm}$, $\bar{\Gamma}^{(n)}_{\theta\theta,MM}$, and $\bar{\Gamma}^{(n)}_{\theta\theta,mM}$

First CS equation: 2x1 matrix equation

$$i \cdot \mathcal{G} \cdot \begin{pmatrix} \bar{\Gamma}^{(n,N)}_{\theta,m} \\ \bar{\Gamma}^{(n,N)}_{\theta,M} \end{pmatrix} = \left[\begin{pmatrix} \partial/\partial m^2 \\ \partial/\partial M^2 \end{pmatrix} + \sum_i \begin{pmatrix} \frac{1}{2m^2}\beta_{\lambda_{i,m}} \\ \frac{1}{2M^2}\beta_{\lambda_{i,M}} \end{pmatrix} \frac{\partial}{\partial \lambda_i} \right.$$

$$\left. + \frac{n}{2} \begin{pmatrix} \frac{1}{m^2}\gamma_{\phi,m} \\ \frac{1}{M^2}\gamma_{\phi,M} \end{pmatrix} + \frac{N}{2} \begin{pmatrix} \frac{1}{m^2}\gamma_{\Phi,m} \\ \frac{1}{M^2}\gamma_{\Phi,M} \end{pmatrix} \right] \bar{\Gamma}^{(n,N)}$$

$$\mathcal{G} = \begin{pmatrix} 1 + \gamma_{\phi,m} & \frac{M^2}{m^2}\gamma_{\Phi,m} \\ \frac{m^2}{M^2}\gamma_{\phi,M} & 1 + \gamma_{\Phi,M} \end{pmatrix}$$

Second CS equation: 2x2 matrix equation

$$i \cdot \begin{pmatrix} 1+\gamma_{\phi,m} & \frac{M^2}{m^2}\gamma_{\Phi,m} \\ \frac{m^2}{M^2}\gamma_{\phi,M} & 1+\gamma_{\Phi,M} \end{pmatrix} \times \begin{pmatrix} \bar{\Gamma}_{\theta\theta,mm} & \bar{\Gamma}_{\theta\theta,mM} \\ \bar{\Gamma}_{\theta\theta,Mm} & \bar{\Gamma}_{\theta\theta,MM} \end{pmatrix}$$
$$= \left[\begin{pmatrix} \partial/\partial m^2 \\ \partial/\partial M^2 \end{pmatrix} + \sum_i \begin{pmatrix} \beta_{\lambda_i,m}/2m^2 \\ \beta_{\lambda_i,M}/2M^2 \end{pmatrix} \frac{\partial}{\partial \lambda_i} + \frac{n}{2} \begin{pmatrix} \gamma_{\phi,m}/m^2 \\ \gamma_{\phi,M}/M^2 \end{pmatrix} + \frac{N}{2} \begin{pmatrix} \gamma_{\Phi,m}/m^2 \\ \gamma_{\Phi,M}/M^2 \end{pmatrix} \right] \begin{pmatrix} \bar{\Gamma}_{\theta,m} & \bar{\Gamma}_{\theta,M} \end{pmatrix}$$
$$+ \left[\frac{1}{2m^2} \times \begin{pmatrix} \gamma_{\theta,mmm} & \gamma_{\theta,Mmm} \\ \gamma_{\theta,mmM} & \gamma_{\theta,MmM} \end{pmatrix} \bar{\Gamma}_{\theta,m} + \frac{1}{2M^2} \times \begin{pmatrix} \gamma_{\theta,mMm} & \gamma_{\theta,MMm} \\ \gamma_{\theta,mMM} & \gamma_{\theta,MMM} \end{pmatrix} \bar{\Gamma}_{\theta,M} \right],$$

Absence of fine-tunings in finite QFT

- Postulated (but can be derived) boundary conditions, valid in all orders of λ:

$$\left[\frac{d}{dk^2}\,\bar{\Gamma}^{(2\phi)}(k^2)\right]_{k^2=0} = i\,, \qquad \bar{\Gamma}^{(2\phi)}\left(k^2=0\right) = im^2$$

$$\left[\frac{d}{dk^2}\,\bar{\Gamma}^{(2\Phi)}(k^2)\right]_{k^2=0} = i\,, \qquad \bar{\Gamma}^{(2\Phi)}\left(k^2=0\right) = iM^2$$

Absence of fine-tunings in finite QFT

- First order, tree approximations for 2 point functions, computed:

- $\left[\bar{\Gamma}^{(2\phi)}\right]_{\lambda^0} = i\left(k^2 + m^2\right)$, $\left[\bar{\Gamma}^{(2\Phi)}\right]_{\lambda^0} = i\left(k^2 + M^2\right)$.

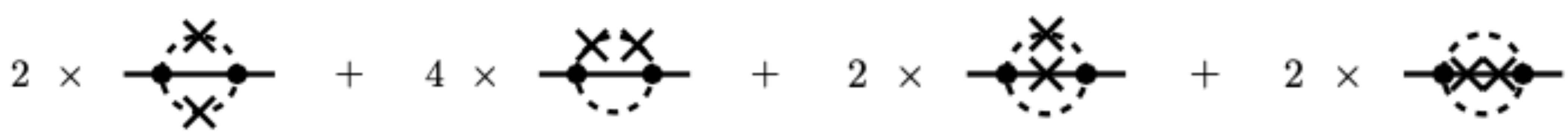
- One-loop finite expressions, computed:

$$\left[\bar{\Gamma}^{(2\phi)}_{\theta\theta,mm}\right]_{\lambda} = -\frac{i\lambda_\phi}{32\pi^2}\frac{1}{m^2}\,,\left[\bar{\Gamma}^{(2\phi)}_{\theta\theta,mM}\right]_{\lambda} = 0\,,\left[\bar{\Gamma}^{(2\phi)}_{\theta\theta,MM}\right]_{\lambda} = -\frac{i\lambda_{\phi\Phi}}{32\pi^2}\frac{1}{M^2}$$

Quantum corrections to $\Gamma^{(2,0)}$

- Begins at order λ^2
- Standard zero momentum renormalisation scheme: order M^2 corrections from

- Finite CS approach: begin from convergent (proportional to m^{-2} or M^{-2}) diagrams



Quantum corrections to $\Gamma^{(2,0)}$

- Result from integrating CS equations:

$$\left[\bar{\Gamma}^{(2\phi)}\right]_{\lambda^2} \supset \lambda^2 \times M^2 \times c_1 \times \ln\left[1 + c_2 \frac{k^2}{M^2}\right]$$

- Expand in region $k^2 << M^2$:

$$\left[\bar{\Gamma}^{(2\phi)}\right]_{\lambda^2} \supset \lambda^2 \times c_1 \times c_2 \times \left[k^2 + \mathcal{O}\left(k^4/M^2\right)\right]$$

- Result, valid in all orders: no fine-tunings are needed, m and m_{phys} are small, M and M_{phys} are large.

Cosmological constant

The same consideration applies to the cosmological constant $\epsilon_{vac} \equiv \Lambda$, related to the zero point function $\bar{\Gamma}^{(0)}$ (Casimir effect, effective potential, etc). New object:

$$\bar{\Gamma}^{(0)}_{\theta\theta\theta} = 2 \times \frac{1}{2} \times (-1)^3 \int \frac{d^4 l}{(2\pi)^4} \left(\frac{-i}{l^2 + m^2} \right)^3 = \frac{1}{32\pi^2} \frac{1}{m^2}$$

Equations for $\bar{\Gamma}^{(0)}$:

$$i \cdot \bar{\Gamma}^{(0)}_{\theta} = \left(\frac{\partial}{\partial m^2} + \beta \frac{\partial}{\partial \lambda} + \frac{\gamma_\Lambda}{2m^2} \frac{\partial}{\partial \Lambda} \right) \bar{\Gamma}^{(0)}$$

$$i \cdot \bar{\Gamma}^{(0)}_{\theta\theta} = \left(\frac{\partial}{\partial m^2} + \beta \frac{\partial}{\partial \lambda} + \frac{\gamma_\Lambda}{2m^2} \frac{\partial}{\partial \Lambda} \right) \bar{\Gamma}^{(0)}_{\theta}$$

$$i \cdot \bar{\Gamma}^{(0)}_{\theta\theta\theta} = \left(\frac{\partial}{\partial m^2} + \beta \frac{\partial}{\partial \lambda} + \frac{\gamma_\Lambda}{2m^2} \frac{\partial}{\partial \Lambda} \right) \bar{\Gamma}^{(0)}_{\theta\theta}$$

lead to finite and tuning free computation of physical observables, such as the Casimir energy or effective potential.

How to reconcile the different conclusions ?

In both approaches, m, M and λ are just Lagrangian parameters, devoid of physical meaning. Only in a tree level analysis they are directly related to physical observables. As soon as loop corrections are included, they become mere tools. Their job is to convert a finite number of initial measurements into predictions for new measurements of physical observables, like particle lifetimes and cross sections. Since the Lagrangian parameters do not carry any physical meaning, neither does an alleged fine-tuning between them. The "unphysicalness" of such a fine-tuning is precisely proven by the existence of the CS method: it does not require any fine-tuning but still arrives at the same predictions for physical experiments.

Conclusions

- Finite QFT formulation based on Callan-Symanzik equations does not require any fine-tunings in the theories with well separated mass scales.

- The so-called hierarchy problem (the sensitivity of low energy physics to high energy physics) depends on the formulation of quantum field theory, and, therefore, is devoid of physical meaning, at least for renormalisable theories.

- The conclusions drawn about new physics in finite QFT approach are very different from those of the standard one: “naturalness” leads to the conjecture about the existence of new physics right above the Fermi scale, whereas the use of a finite formulation of QFT says that no such a conclusion can be made on physical grounds.

- Though the problem of the quantum stability of the Higgs mass and of cosmological constant can be resolved by finite QFT, the question about the origin of widely separated scales in Nature (such as vacuum energy, Fermi, GUT or Planck scale) remains.

Remarks

Our discussion can be extended in several directions:

- "Naturalness" in other approached to finite QFTs: the BPHZ (or its modifications) and 't Hooft formulations of finite QFT ?

- Non-renormalisable case: repeating the "θ-operation" as many times as needed?

- At the technical side, the CS method as it stands cannot work for massless particles, such as gauge bosons. However, this problem occurs in the infrared rather than in the UV. Therefore, we expect that a "gauge symmetry preserving generalisation" of the CS method does not change the hierarchy discussion. The 't Hooft method does not seem to have this problem.