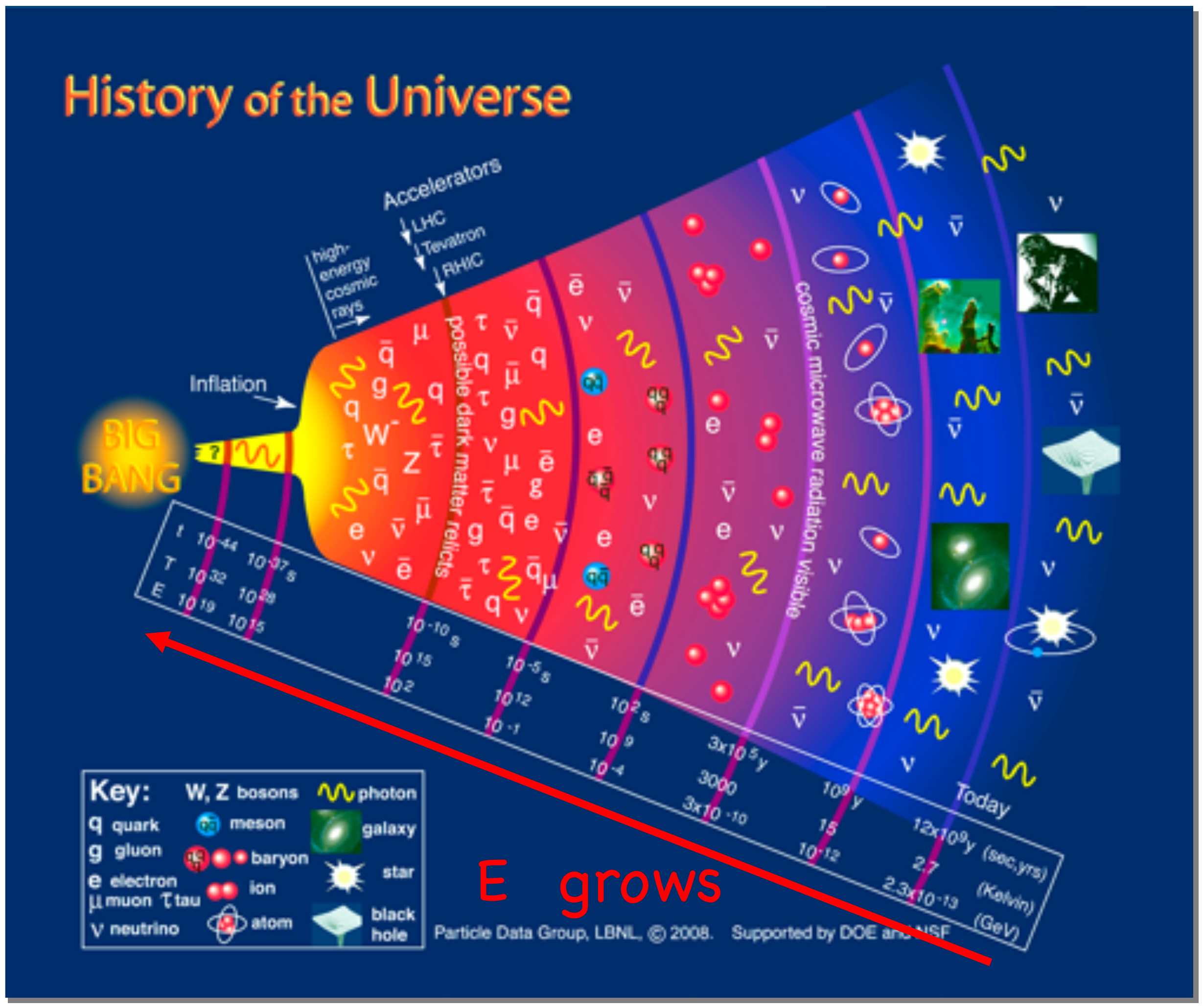
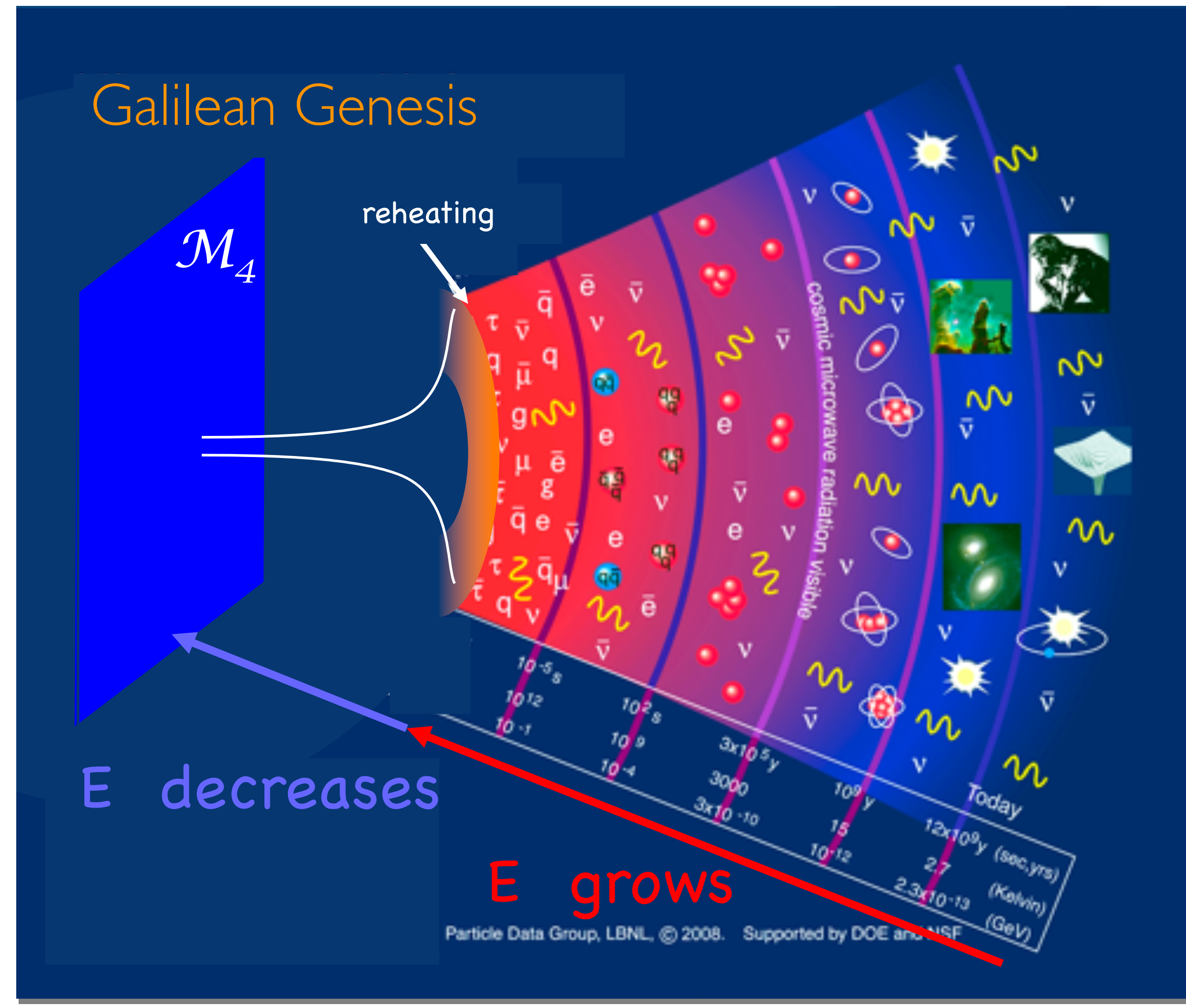
BLACK HOLES BEYOND GR

ENRICO TRINCHERINI
(SCUOLA NORMALE SUPERIORE)

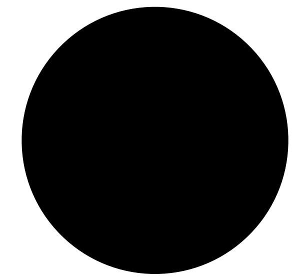




Creminelli, Nicolis, ET 2010

(Another) Study in Impossibility

What is a black hole?



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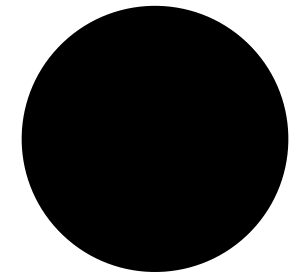

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Mass, Spin

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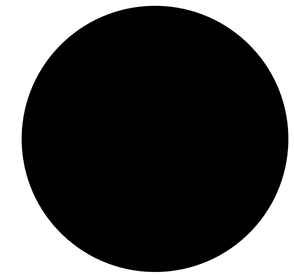

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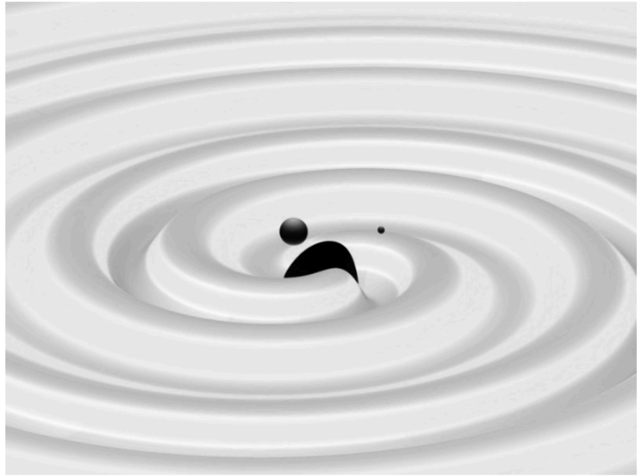
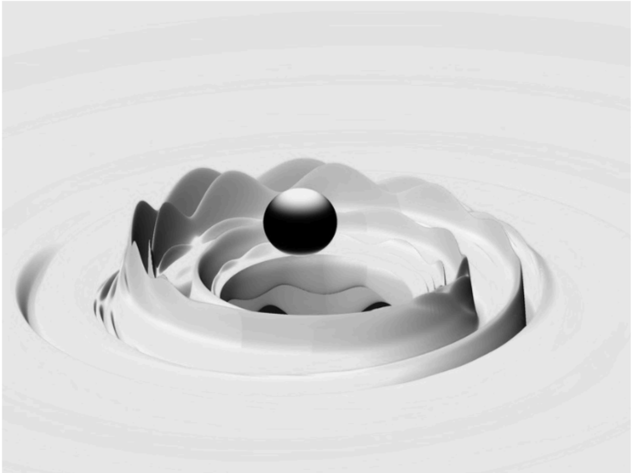
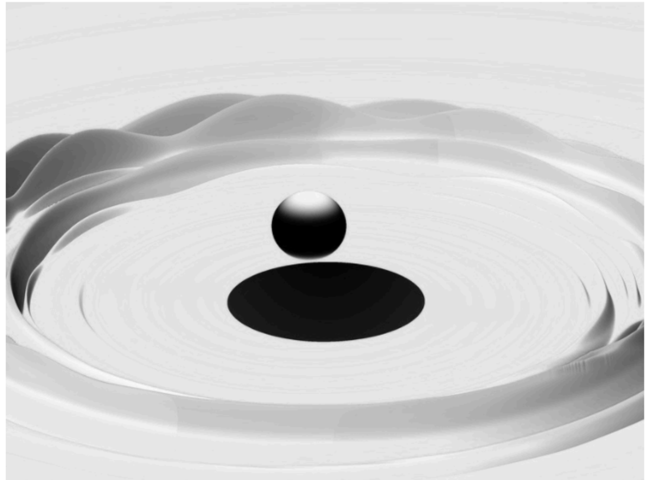


Inspiral



Merger



Ringdown

Black Hole Perturbations

$$g_{\mu\nu} = g^{\rm BH}_{\mu\nu}(r) + h_{\mu\nu}$$

Schwarzschild: static, spherically symmetric background

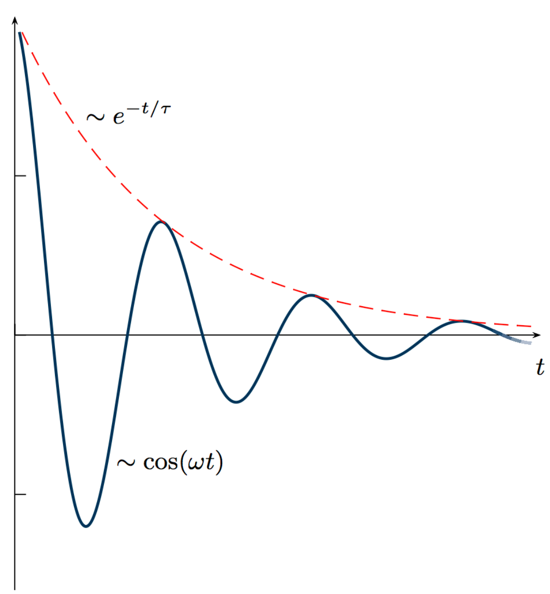
$$h(t,r,\theta,\phi) = \sum_{lm} h_{lm}(r) Y_{lm}(\theta,\phi) e^{i\omega t}$$

Classified accordingly to the behavior under parity

$$h_{\mu\nu} = \begin{cases} \text{7 even} \Longrightarrow \text{1 d.o.f.} \\ \text{3 odd} \Longrightarrow \text{1 d.o.f.} \end{cases}$$

Exponentially damped sinusoidal waves: Quasi Normal Modes (QNM)

Spectrum of characteristic (complex) frequencies ω_{nlm}



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0	0.747 343 + 0.177 925i	1.198 887 + 0.185 406i	1.618 36 + 0.188 32i
1	0.693 422 + 0.547 830i	1.165 288 + 0.562 596i	1.593 26 + 0.568 86i
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Isospectrality

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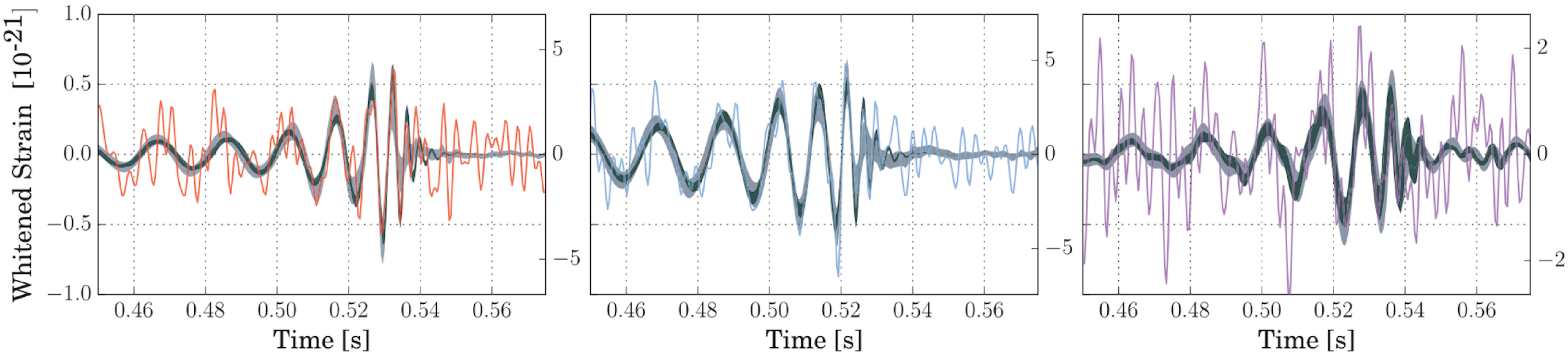

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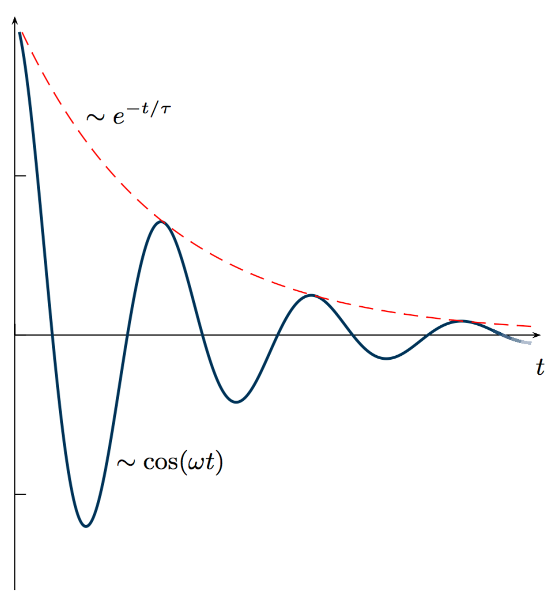

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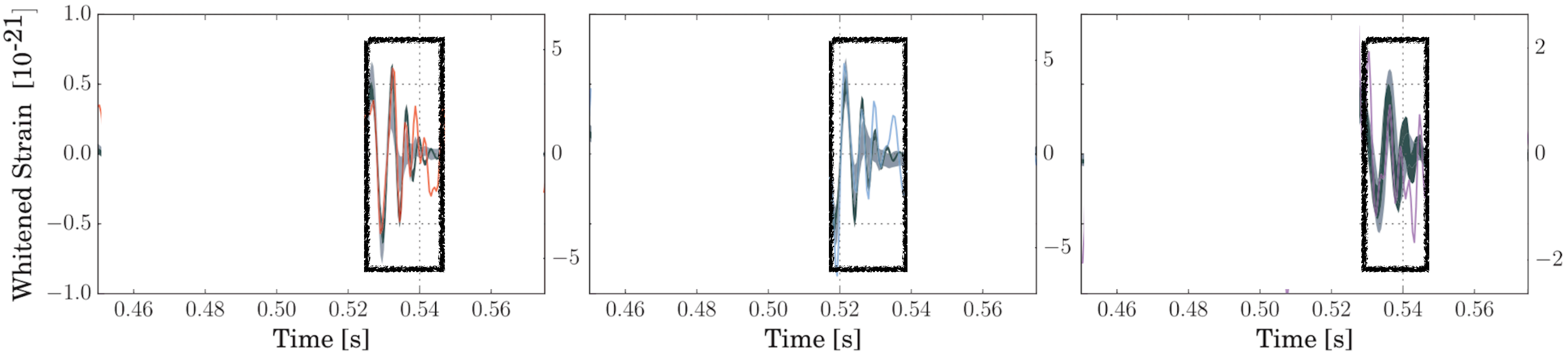

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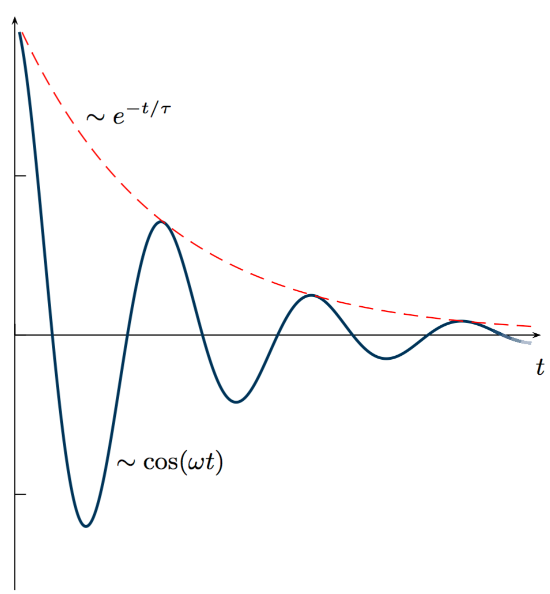

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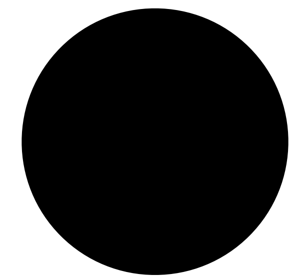

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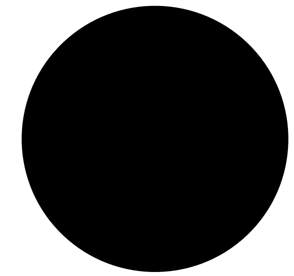

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BH response to an external field — Love numbers

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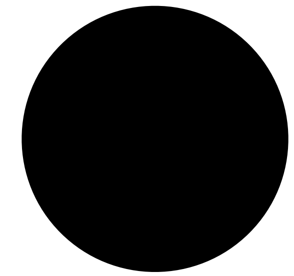

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BH response to an external field — Love numbers

Hypothesis: deviations observable by LIGO/Virgo, …

Two ways

(1) New states heavier than the BH curvature

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$$S = M_{\rm Pl}^2 \int d^4x \, \sqrt{-g}\left(R + c_3 \frac{R^3_{\mu\nu\rho\sigma}}{\Lambda^4} + c_4 \frac{R^4_{\mu\nu\rho\sigma}}{\Lambda^6} + \ldots \right)$$

Endlich, Gorbenko, Huang, Senatore 2017

Observable effects ⇒ $\Lambda \sim \mathrm{km}^{-1}$

Two ways

(2) Additional light DOF

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GR + photon

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Geometry

Mass, Spin, Charge

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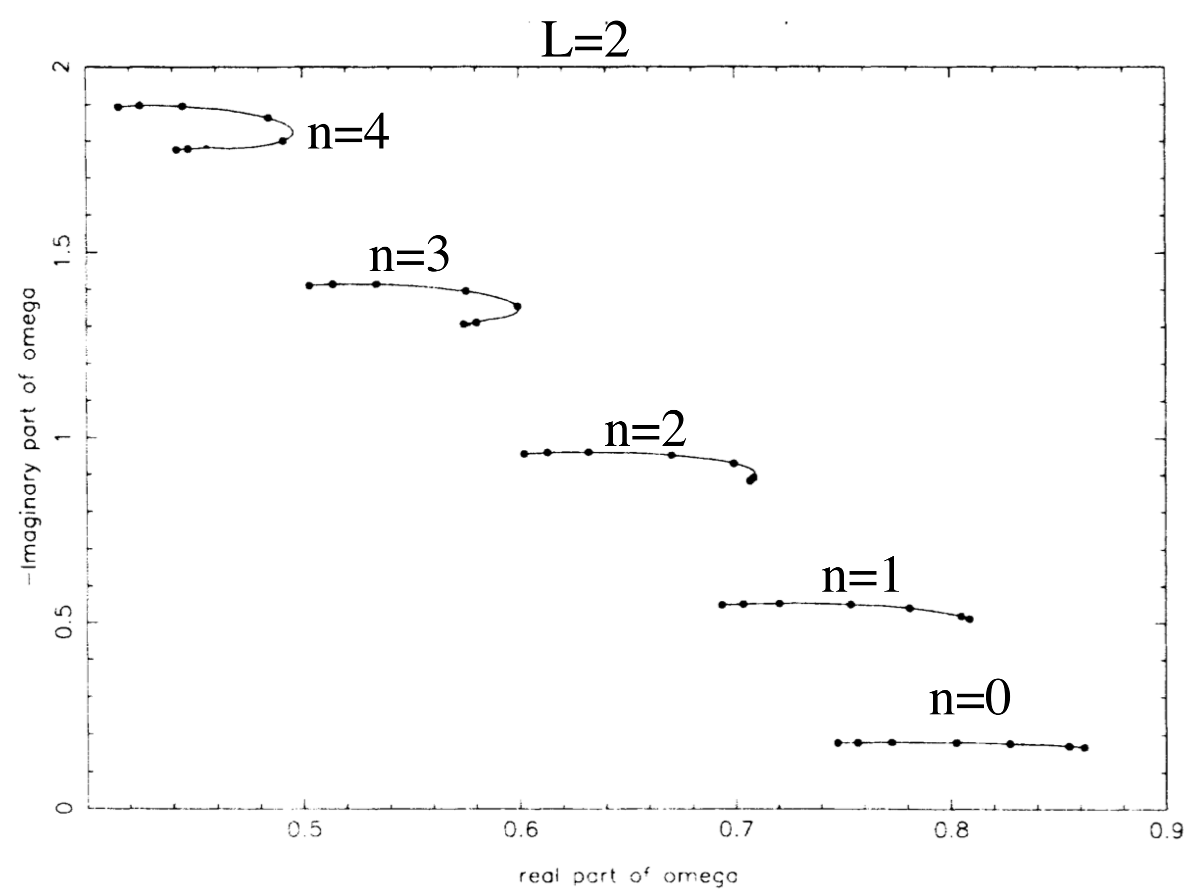

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GR + (shift-symmetric) scalar field

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If the field has zero background $\Rightarrow$ QNM are the same as GR + extra spectrum

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(2) Additional light DOF

GR + (shift-symmetric) scalar field
not coupled directly to matter

If the field has zero background ⇒ QNM are the same as GR + extra spectrum

If the field has a background ⇒
- GR frequencies are shifted
- Isospectrality can be broken
- even/odd mixing

No-Hair Theorem

Hui, Nicolis 2012

EOM $\nabla_\mu J^\mu = 0$

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + \rho^2(r)d\Omega^2$$

Assuming spherically symmetric, time-independent solutions only $J^r \neq 0$

$J^\mu J_\mu = (J^r)^2/f$ should be regular at the horizon $\Longrightarrow$ $J^r = 0$ at the horizon

using the conservation of the current $\Longrightarrow$ $J^r(r) = 0$

One last step to conclude that a vanishing current implies a constant scalar

If the dependence on the scalar in the Lagrangian starts quadratically then

$J^r = \phi' F[\phi', g, g']$ with a regular function F

F asymptotes to a constant at infinity

Then $\phi'(r) = 0$

An interesting exception

Sotiriou, Zhou 2013

$$S = \int d^4x\sqrt{-g}\Big(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \alpha M_{\rm Pl}\phi\mathcal{R}^2_{\rm GB} + \dots\Big)$$

The Gauss-Bonnet invariant is a total derivative

The linear coupling gives a ϕ-independent contribution to the scalar EOM

$\phi'(r) = 0$ is no longer a solution

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$J^2_{\rm GB}$ diverges at the horizon

Babichev, Charmousis, Lehébel 2017

$J^2_{\rm GB}$ is not a scalar operator, $J^\mu_{\rm GB}$ not covariant under diffs
Its value depends on the coordinates, the divergence is immaterial
All scalar quantities built with the metric and ϕ are bounded on the BH

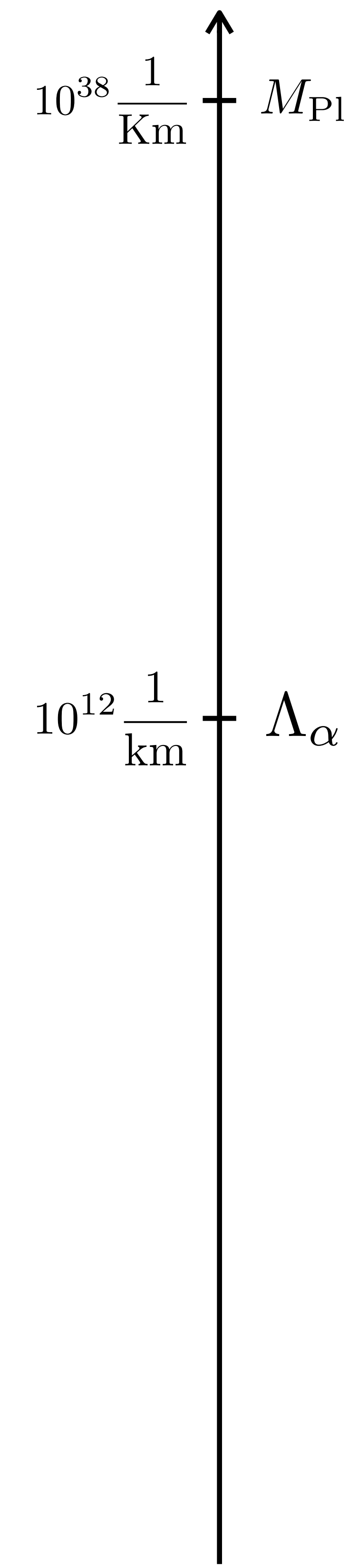
Creminelli, Loayza, Serra, ET, Trombetta 2020

The EFT of scalar Gauss-Bonnet

$$S = \int d^4x \sqrt{-g}\Big(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \alpha M_{\rm Pl}\phi\mathcal{R}_{\rm GB}^2 + \ldots\Big)$$

$\alpha \sim r_s^2$ observable effects

$$\frac{\phi\,\partial^2 h\,\partial^2 h}{\Lambda_\alpha^3} \qquad \Lambda_\alpha \equiv \Big(\frac{M_{\rm Pl}}{\alpha}\Big)^{1/3}$$



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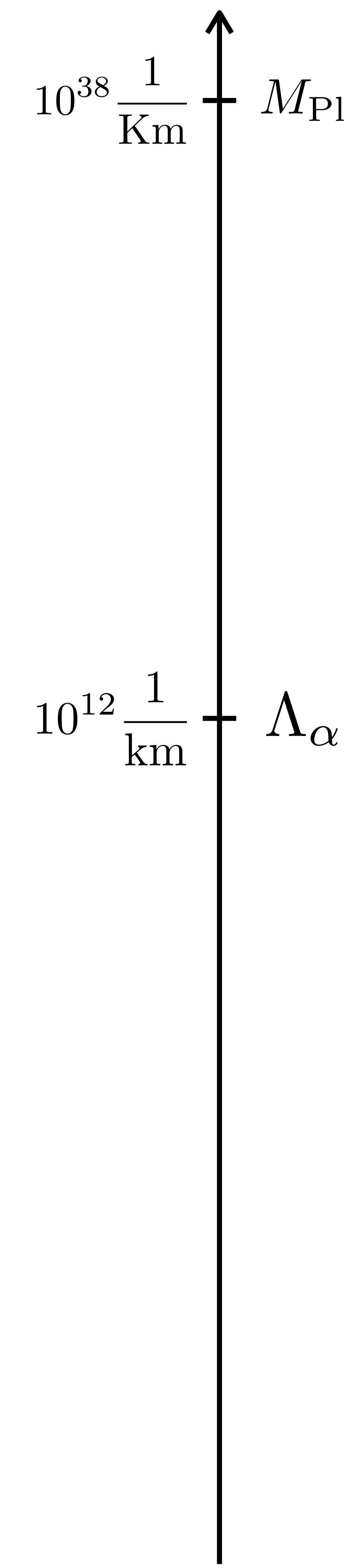
Such an EFT is **constrained by causality**

F. Serra, J. Serra, ET, L. Trombetta, *in progress*



Causality Constraints from 3-point interactions

Consider higher derivative corrections to the graviton 3-point coupling

Camanho, Edelstein, Maldacena, Zhiboedov, 2014

Huber, Brandhuber, De Angelis, Travaglini, 2020

$$S = M_{\rm Pl}^2 \int d^4x \sqrt{-g}\Big(R + \alpha_4 R^3_{\mu\nu\rho\sigma}\Big) \qquad \alpha_4 = [L]^4$$

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Small angle scattering of a graviton off a massive particle

$m \gg \omega \gg |\vec{q}|$ Eikonal limit

It experiences a time delay

Both helicities contribute: eikonal phase $\longrightarrow$ eikonal phase matrix

$$\delta t \propto 4\frac{m}{M_{\rm Pl}^2}\left[\log\frac{b_0}{b} \pm \frac{\alpha_4}{b^4}\right]$$ observable while the computation is under control

Time delay becomes a time advance when $b \sim (\alpha_4)^{1/4}$

The EFT of scalar Gauss-Bonnet

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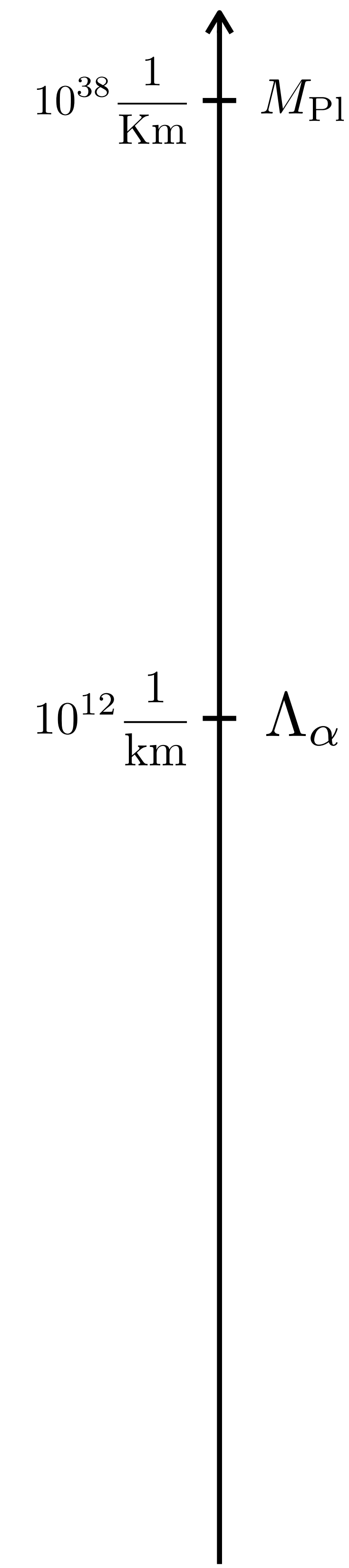
Time advance when $b \sim \sqrt{\alpha}$



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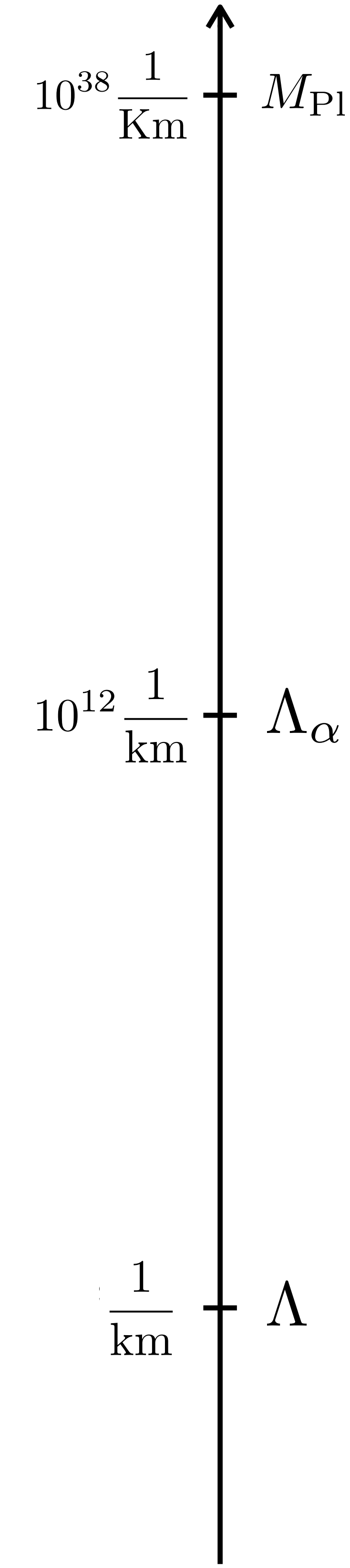

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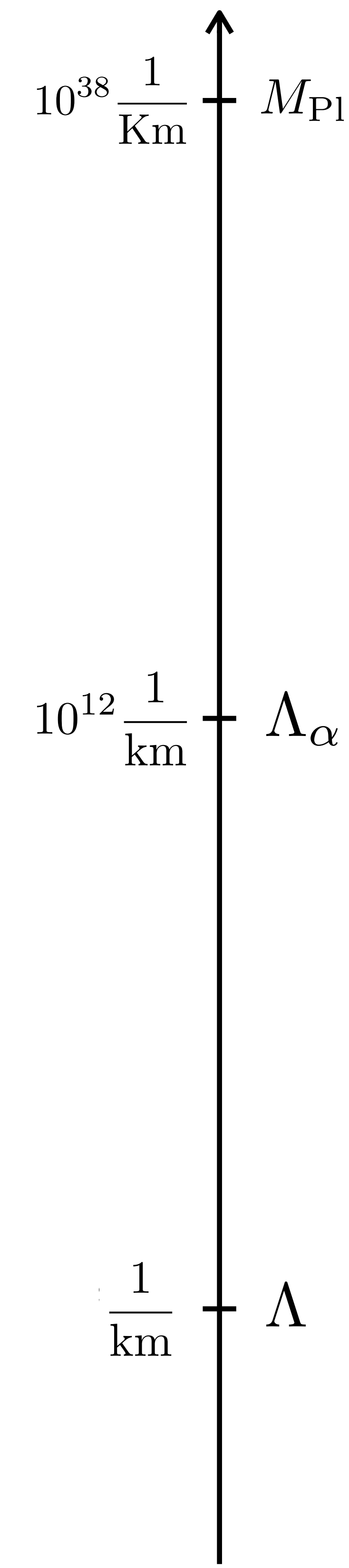
$$M_{\rm Pl}^2 R + \frac{\Lambda^4}{g^2}\Big[\hat{\mathcal{L}}^{(0)}\Big(\frac{\partial}{\Lambda},\frac{R}{\Lambda^2},\frac{g\phi}{\Lambda}\Big) + \frac{g^2}{16\pi^2}\hat{\mathcal{L}}^{(1)}\Big(\frac{\partial}{\Lambda},\frac{R}{\Lambda^2},\frac{g\phi}{\Lambda}\Big) + \dots\Big]$$

$$g \sim \frac{\Lambda}{M_{\rm Pl}}$$



A different point of view

So far in flat space but there can be different boundary conditions (ex: ULDM)

Usually studied on a model by model basis: a more systematic approach can be useful

New generation & space-based detectors will enable "black hole spectroscopy"

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New generation & space-based detectors will enable "black hole spectroscopy"

Not explain where the background comes from, take it as given

write an action that governs the dynamics of perturbations, guided by symmetries

EFT around space-time dependent backgrounds

Inflation

Late time acceleration

Gubitosi, Vernizzi, Piazza 2012

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

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Black Holes

Franciolini, Hui, Penco, Santoni, ET 2018
Hui, Podo, Santoni, ET 2021

The EFT of BH perturbations

Franciolini, Hui, Penco, Santoni, ET 2018

Start from a background solution $\mathrm{d}s^2 = -a^2(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{b^2(r)} + c^2(r)\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$

Choose a foliation of spacetime (unitary gauge) such that $\delta\Phi(r) = 0$

Write down in a derivative expansion all the operators that are invariant under the residual symmetries: (t, θ, ϕ) - diffs.

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Write down in a derivative expansion all the operators that are invariant under the residual symmetries: (t, θ, ϕ) - diffs.

The EFT can contain:

- generic functions of the radial coordinates
- free r indices, like g^{rr}
- geometric objects of the 3d spatial slices such as $K^{\mu\nu},\ R^{(3)}$

Expand in perturbations: e.g. $g^{rr} = \bar{g}^{rr} + \delta g^{rr} \qquad K_{\mu\nu} = \bar{K}_{\mu\nu} + \delta K_{\mu\nu}$

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big[\frac{1}{2}M_1^2(r)R - \Lambda(r) - f(r)g^{rr} - \alpha(r)\bar{K}_{\mu\nu}K^{\mu\nu} + M_2^4(r)(\delta g^{rr})^2 + M_3^3(r)\delta g^{rr}\delta K + M_4^2(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta K^{\mu\nu} + \text{11 other terms} + \ldots\Big]$$

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Set to a constant with a conformal transformation

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The EFT of BH perturbations

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Start quadratic in the perturbations (do not contribute to the background)

The EFT of BH perturbations

$$S = \int \mathrm{d}^4x \sqrt{-g}\Big[\frac{1}{2}M_1^2(r)R - \Lambda(r) - f(r)g^{rr} - \alpha(r)\bar{K}_{\mu\nu}K^{\mu\nu}$$
$$+ M_2^4(r)(\delta g^{rr})^2 + M_3^3(r)\delta g^{rr}\delta K + M_4^2(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta K^{\mu\nu}$$
$$+ \text{11 other terms} + \dots\Big]$$

Set to a constant with a conformal transformation

Fixed by the background solution

Start quadratic in the perturbations (do not contribute to the background)

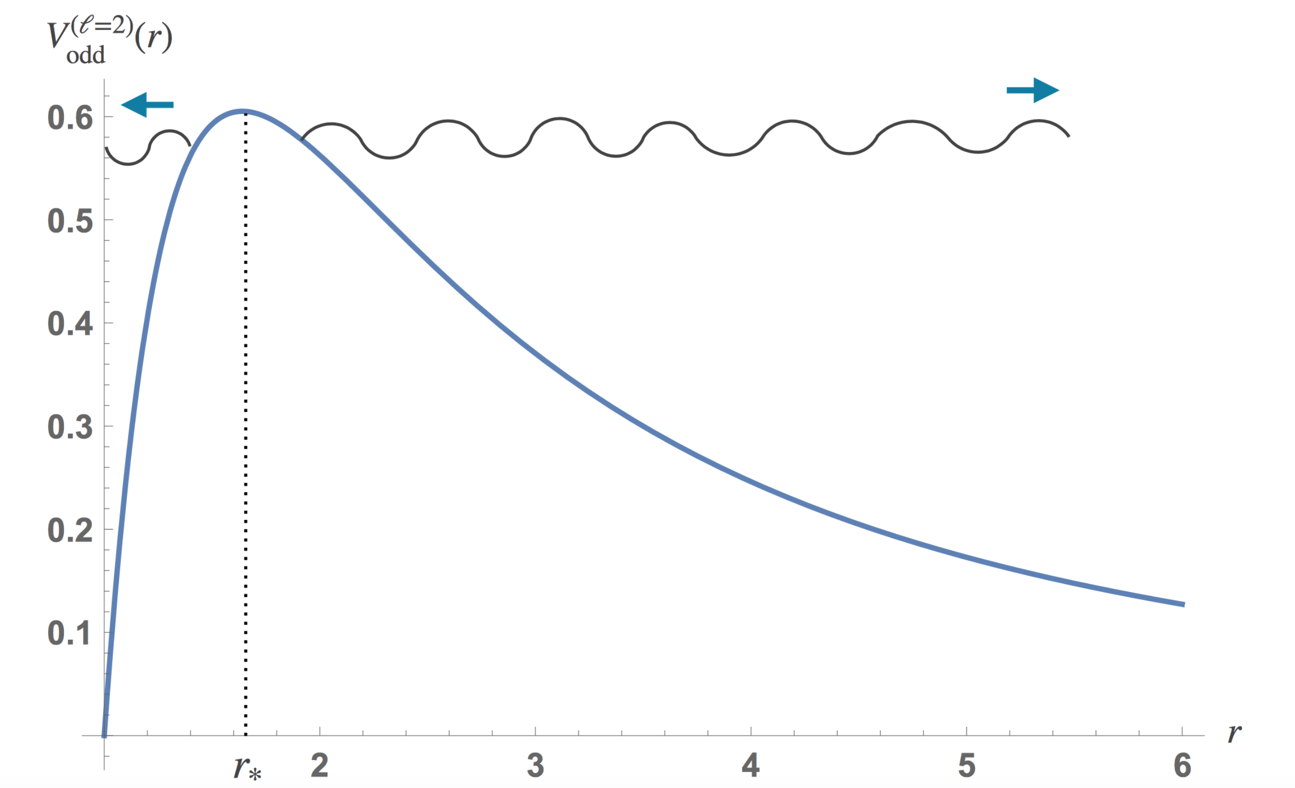
This EFT gives linear equations with 2 derivatives for the propagating d.o.f.
It contains the full information about the QNM spectra

BUT

EFT coefficients are free functions of the radius

WKB approximation

Schutz, Will 1985



$$\frac{\omega^2 - V_l}{(-2\partial_{\tilde{r}}^2 V_l)^{1/2}}\Big|_{\tilde{r}=\tilde{r}_*} = -i\left(n+\frac{1}{2}\right)$$

Accuracy of the leading order approximation for Schwarzschild BH: 3% $\mathrm{Re}[\omega_{3,0}]$, 0.5% $\mathrm{Im}[\omega_{3,0}]$

Improves with larger l

Higher orders can be included

Iyer, Will 1987

Only the values of the EFT coefficients and their derivatives at the light ring are needed

Light ring expansion

2 problems:

The light ring position r_* depends on the potential

The potential depends on l

Light ring expansion

2 problems:

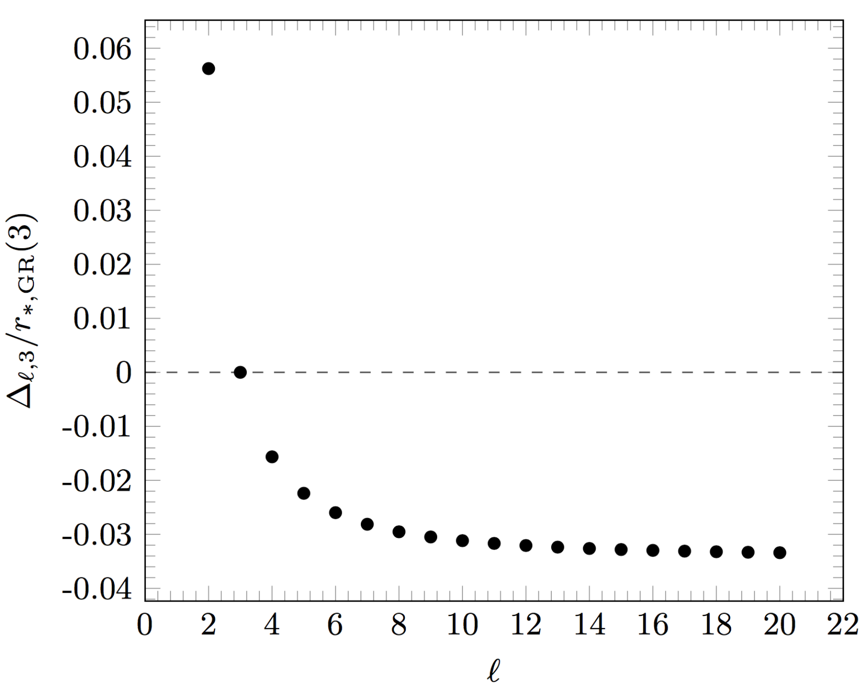
The light ring position r_* depends on the potential

The potential depends on l

i) Assume the background is "quasi-Schwarzschild": r_* is close to the GR value

$$r_*(\ell) = r_{*,GR}(\ell) + \delta r_*(\ell)$$

ii) Depends on l mildly: choose a fiducial one (l=3) and expand in $\Delta_{\ell,3} \equiv r_{*,\mathrm{GR}}(\ell) - r_{*,\mathrm{GR}}(3)$



The final WKB formula in the "light ring" approx. depends only on EFT coeff. + background and their derivatives evaluated at the *same point for every* l

The EFT of BH perturbations

Inflation	QNMs of Hairy BHs
scalar="clock"	scalar="hair"
quasi-deSitter	quasi-Schwarzschild
Slow-roll expansion	Light-ring expansion
Derivatives of inflation potential	Derivatives of QNM potential
horizon-crossing: $k/a(t_*) = H(t_*)$	maximum of potential: $r_*(\ell)$
EFT of inflation	Our EFT

Credit: Riccardo Penco