Metastable multiparticle resonances: from Q-balls to helium-3 droplets

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Congratulations, Valery!

References

- Dam Thanh Son, Misha Stephanov, Ho-Ung Yee, to appear

Plan

- Metastable Q-balls
- Decay rate of metastable Q-balls
- Many-particle resonances: general theory
- Helium-3 droplets

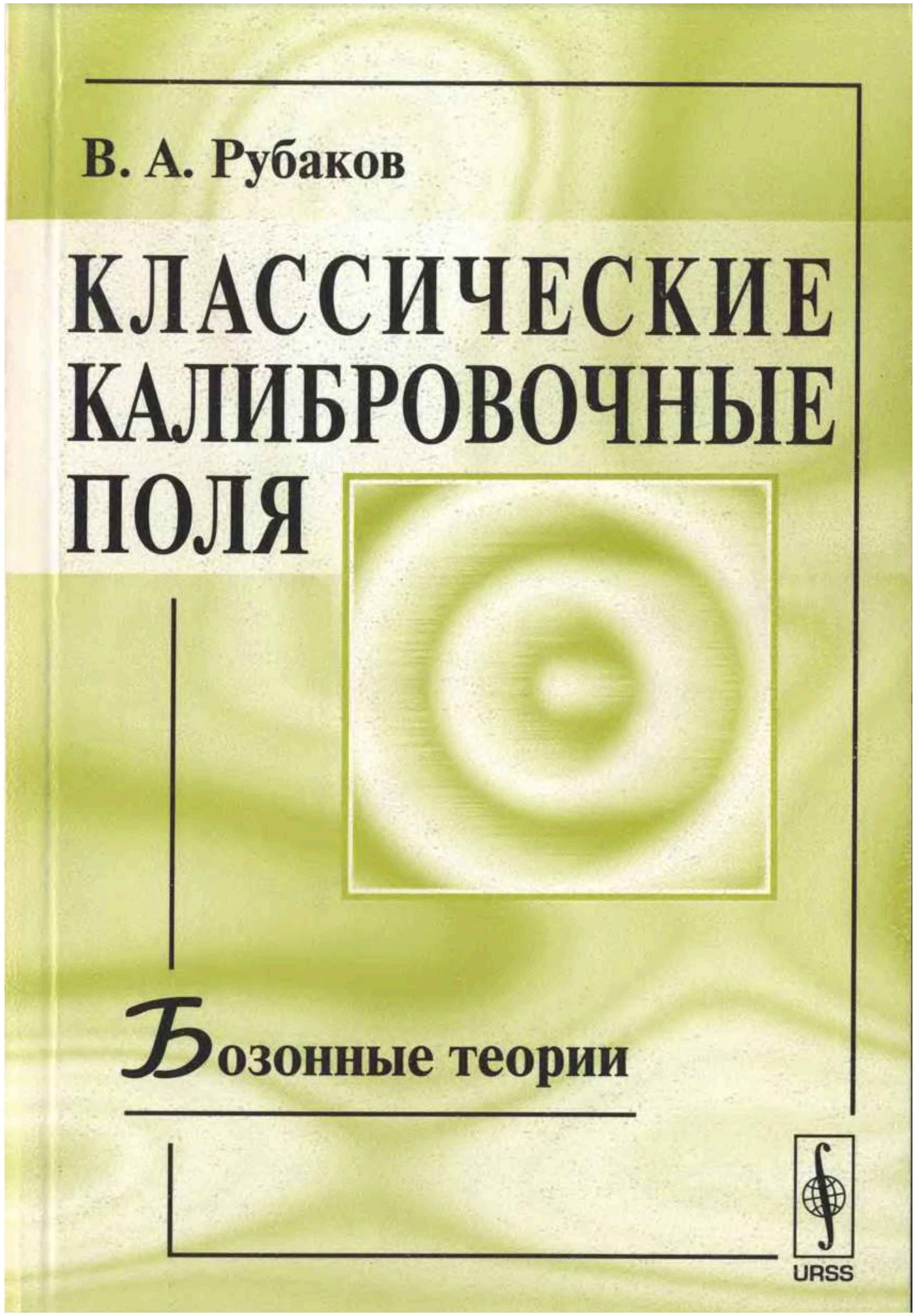
Model

- Consider a complex scalar field theory

$$\mathscr{L} = |\partial_\mu \phi|^2 - V(\phi),$$

$$V(\phi) = m^2 |\phi|^2 - \lambda |\phi|^4 + \lambda_6 |\phi|^6$$

- attractive two-body interaction, repulsive three-body
- This model has Q-balls (nontopological solitons)



V. Rubakov, *Classical Gauge Fields. Bosonic Theories*

Existence of Q-balls

10.2. Q-шары в теориях с плоскими направлениями

Если в модели имеется всего одно комплексное скалярное поле, то существование или отсутствие нетопологических солитонов (Q-шаров) существенно зависит от формы скалярного потенциала. В этом разделе мы рассмотрим модель с лагранжианом

$$\mathcal{L} = \partial_\mu \Phi^* \partial_\mu \Phi - V(|\Phi|), \tag{10.23}$$

где Φ — комплексное скалярное поле, а скалярный потенциал $V(|\Phi|)$ имеет абсолютный минимум при $\Phi = 0$, так что глобальная $U(1)$-симметрия

$$\Phi \to e^{i\alpha}\Phi$$

не нарушена в основном состоянии. Мы увидим, что Q-шары в этой модели существуют, если функция

$$\frac{V(|\Phi|)}{|\Phi|^2}$$

имеет минимум при $|\Phi| \neq 0$ (Коулмен, 1985). Этот минимум может иметь место при $|\Phi| = \infty$; для этого требуется, чтобы скалярный потенциал рос при больших Φ медленнее, чем $|\Phi|^2$, т. е. был достаточно плоским. Плоские направления скалярного потенциала (модули) характерны для суперсимметричных теорий, так что в таких теориях вполне возможно существование Q-шаров. Исследование свойств Q-шаров в суперсимметричных расширениях Стандартной модели физики частиц и возможных проявлений этих объектов в астрофизике и космологии представляет значительный интерес, при этом в качестве глобального заряда может (хотя и не обязательно) выступать барионное число, лептонное число или их комбинация (Кусенко 1997, Двали, Кусенко, Шапошников 1997).

"We will see that in this model Q-balls exist when the function

$$\frac{V(|\Phi|)}{|\Phi|^2}$$

has a minimum at $|\Phi| \neq 0$"

$$\frac{V(\phi)}{|\phi|^2} = m^2 - \lambda|\phi|^2 + \lambda_6|\phi|^4$$

- Finite chemical potential:

$$\mathscr{L} = |(\partial_0 - i\mu)\phi|^2 - |\vec{\nabla}\phi|^2 - V(\phi)$$

Effective potential

$$V_{\text{eff}}(\phi) = (m^2 - \mu^2)|\phi|^2 - \lambda|\phi|^4 + \lambda_6|\phi|^6$$

Q-balls

- 1st order phase transition between vacuum and and a finite-density liquid state
- Large number of boson: form a finite density "bag"
- $$E = -\frac{4\pi}{3}R^3\varepsilon + 4\pi R^2\sigma$$
  - for a large enough bag (large enough N) gain in volume energy overwhelms loss in surface tension
- But what happens if N is not large enough?

Nonrelativistic limit

- For simplicity, consider the nonrelativistic limit

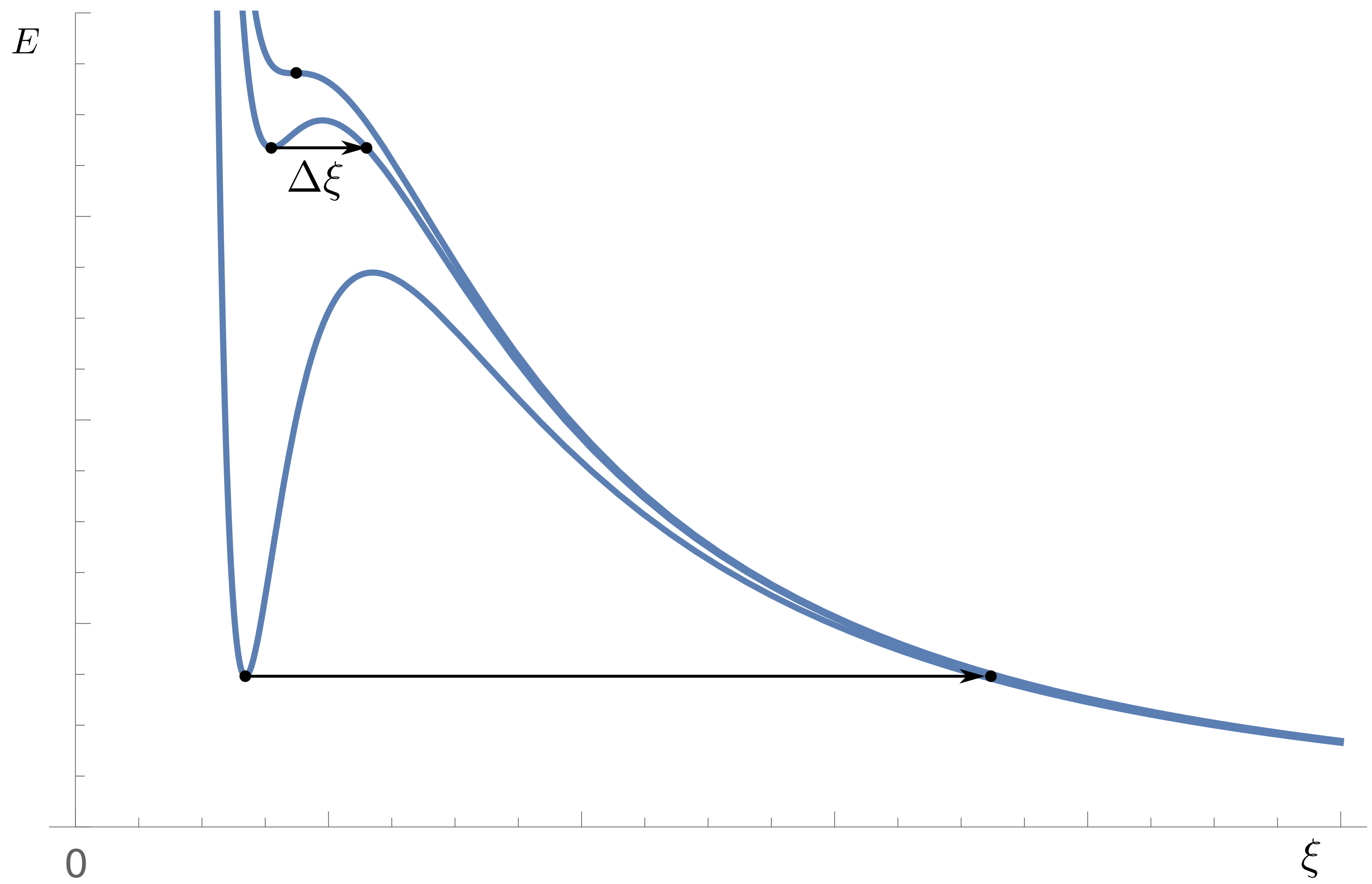
$$\mathscr{L} = i\psi^\dagger \partial_t \psi - \frac{|\vec{\nabla}\psi|^2}{2m} + \frac{g}{4}|\psi|^4 - \frac{g_6}{6}|\psi|^6$$

- Minimize the energy

$$H = \int d\mathbf{x} \left(\frac{|\vec{\nabla}\psi|^2}{2m} - \frac{g}{4}|\psi|^4 + \frac{g_6}{6}|\psi|^6 \right)$$

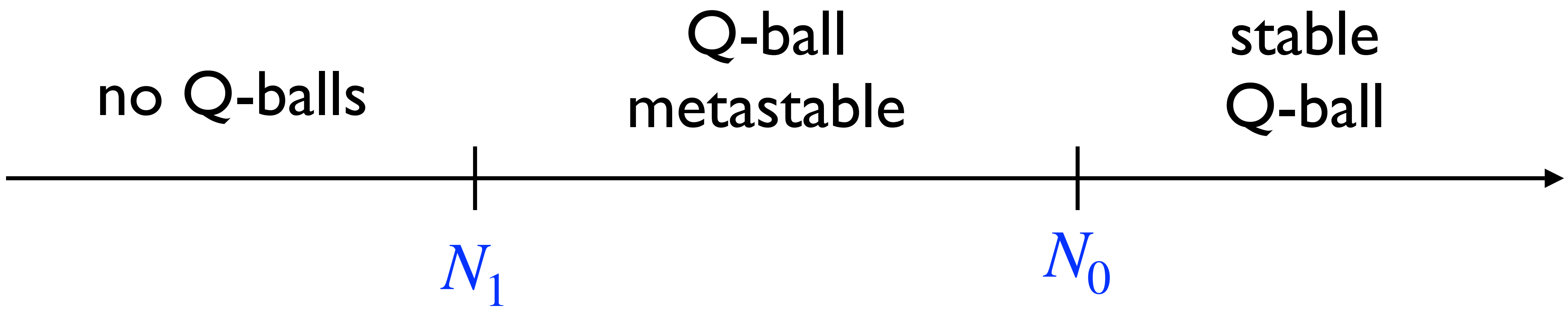
at fixed number of particles $N = \int d\mathbf{x} |\psi|^2$

- Variational ansatz: $\psi = \sqrt{\frac{N}{(2\pi\xi^2)^{3/2}}} \exp\left(-\frac{r^2}{4\xi^2} \right)$


E
$\Delta\xi$
0
ξ

Q-ball “phase diagram”

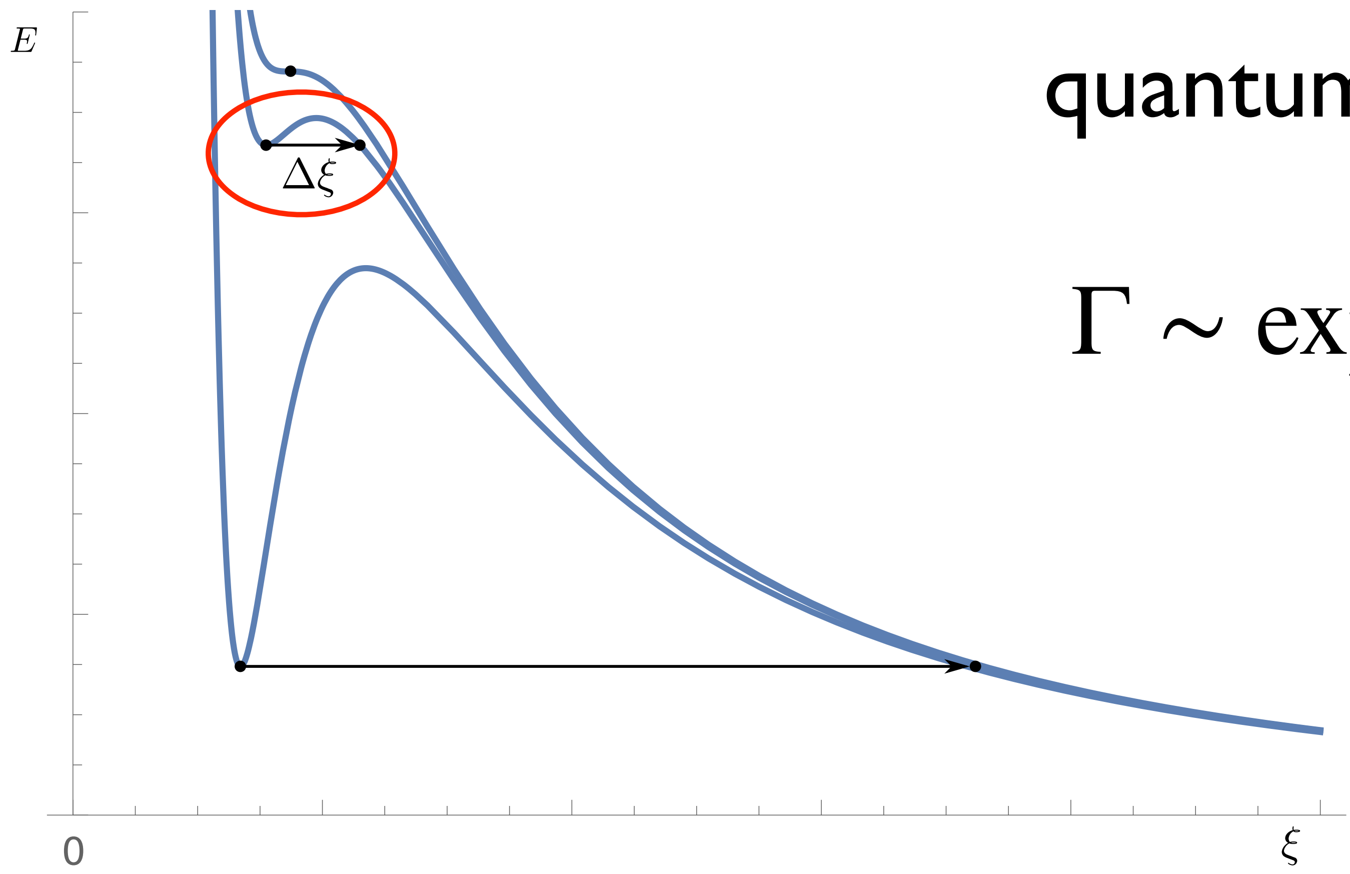
see also Levkov, Nugaev, Popescu 2017



$$N_1 = 189.4\frac{\sqrt{g_6}}{g^2} \qquad N_0 = 240.4\frac{\sqrt{g_6}}{g^2}$$

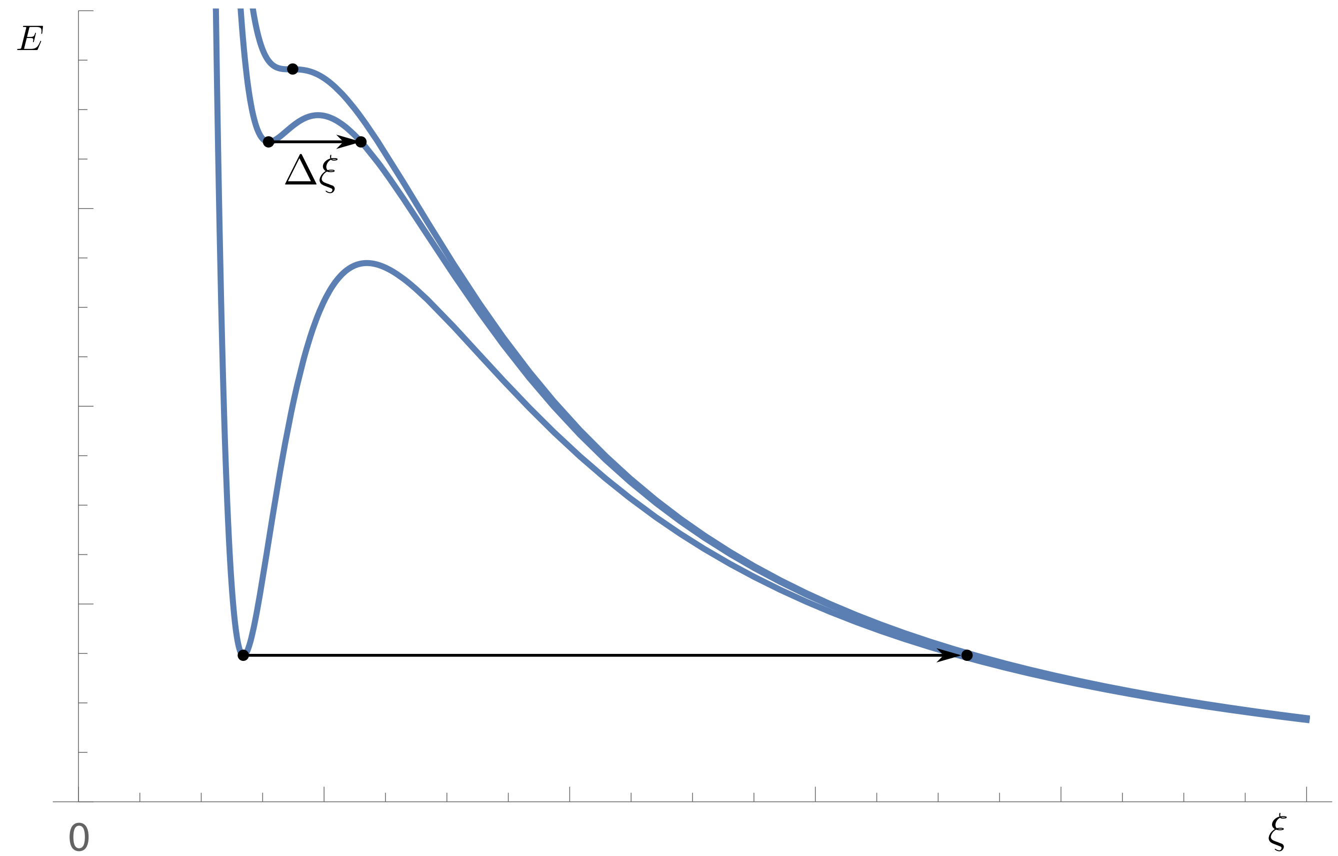
- What is the lifetime of a metastable Q-ball?
  - we need to find the instanton (bounce) solution

Almost unstable Q-ball



quantum mechanics of 1 d.o.f.

$$\Gamma \sim \exp\left[-cN\left(\frac{N_1 - N}{N}\right)^{5/4}\right]$$



Qualitatively: tunneling under a $1/R^2$ potential

$$S_E = \int dR \sqrt{2M(V(R) - E)} \sim N \ln \frac{R_{\text{max}}}{R_{\text{min}}}$$

$$\Gamma \sim e^{-2S_E} \sim \left(\frac{E}{E_0}\right)^{cN} \qquad c = ?$$

Euclidean equation of motion

- $\psi = e^{i\theta} f$
- Go to imaginary time $t = -i\tau, \theta = i\varphi$

$$\partial_\tau (f^2) + \nabla \cdot (f^2 \nabla \varphi) = 0$$

$$\partial_\tau \varphi + \frac{(\nabla \varphi)^2}{2} + \frac{\nabla^2 f}{2f} = 0$$

$(V = 0)$

$$f = \frac{\sqrt{N}}{(2\pi\tau)^{3/4}} \exp\left(-\frac{r^2}{4\tau}\right)$$

$$\varphi = \frac{r^2}{4\tau} + \frac{3}{4} \ln \tau$$

Action: logarithmic integral: $\frac{3N}{2} \int \frac{dR}{R} = \frac{3N}{2} \ln \frac{R_{\text{max}}}{R_{\text{min}}}$

width: $\Gamma \sim e^{-2S} \sim \left(\frac{E}{E_0}\right)^{3N/2}$

Lessons learned so far

- Bound state of N bosons does not disappear right away if its energy becomes positive
  instead it remains a resonance
- Resonance is narrow $\Gamma \ll E$ for small E if N is large
- Questions:
  - What happens at smaller N? say N=3 or N=4?
  - is resonance narrow at small energy?
    - if yes, what is $\Gamma(E)$?
  - (N=2: bound state simply disappears, no resonance)

Width of resonance at small N

- For small N, classical instanton calculation is not available
- But we can still find the width of the resonance at low energy
- Imagine that the interaction can be tuned, N-body bound state disappears: energy crosses 0

  What is the width as function of energy $\Gamma(E)$?

Effective field theory

- When the energy of the resonance is small: low-energy effective field theory
  - vertex $g\Psi^\dagger\phi^N$
  - decay rate determined by phase space

$$\Gamma \sim \int \prod_{a=1}^{N} d\mathbf{p}_a\, g^2 \delta\Big(\sum \mathbf{p}_a\Big)\delta\Big(\sum \frac{p_a^2}{2m} - E\Big) \sim E^{\frac{3N}{2}-\frac{5}{2}}$$

  - reproduces instanton calculation at large N, but valid at small N as well (only for small E)
  - N=3: $\Gamma \sim E^2 \ll E$, narrow resonance

Fermion droplets

- For a fermonic droplet: no semiclassical instanton calculation
- Behavior of width can still be found by EFT
- Vertex: $g\Psi^\dagger\,\psi\,\partial_x\psi\,\partial_y\psi\,\partial_z\psi\,\partial_x^2\psi\,\partial_x\partial_y\psi\,\partial_x\partial_z\psi\ldots$
  $= g\Psi^\dagger O_N[\psi]$
- Result: $\Gamma(E) \sim E^{\Delta-\frac{5}{2}}, \quad \Delta = \mathrm{dim}[O_N]$
  - Δ = ground state energy of N fermions in harmonic potential of unit frequency

Helium droplets

- Helium has 2 isotopes: He-4 and He-3
- both are self-bound liquids at zero temperature in the thermodynamic limit
- Helium-4 droplets are bound for any number of atoms
  - but He-3 droplets are stable only for $N \geq N_0$
  - numerical estimate: $20 < N_0 < 40$

Helium-3 droplets

TABLE II. Binding energy (in K) determined at the JCI3 approximation for several ^3He_N drops as a function of the number of spin-up ($N_\uparrow$) and spin-down ($N_\downarrow$) atoms. Results are given for the two Aziz potentials HFDHE-2 [10] and HFD-B(HE) [11].

N	$N_\uparrow$	$N_\downarrow$	S_z	HFDHE-2	HFD-B(HE)
40	20	20	0	-2.55 ± 0.07	-3.90 ± 0.07
39	20	19	1/2	-1.87 ± 0.09	-3.17 ± 0.10
38	19	19	0	-1.05 ± 0.11	-2.29 ± 0.11
37	20	17	3/2	-0.42 ± 0.08	-1.62 ± 0.09
36	20	16	2	0.06 ± 0.09	-1.09 ± 0.09
36	19	17	1	0.30 ± 0.10	-0.86 ± 0.10
35	19	16	3/2	0.76 ± 0.08	-0.33 ± 0.09
34	20	14	3	1.13 ± 0.06	0.09 ± 0.06
34	17	17	0	1.71 ± 0.06	0.67 ± 0.06
33	20	13	7/2	1.49 ± 0.09	0.56 ± 0.09
33	19	14	5/2	1.58 ± 0.08	0.66 ± 0.09
33	17	16	1/2	2.07 ± 0.09	1.15 ± 0.10
32	19	13	3	1.92 ± 0.09	1.04 ± 0.09
32	16	16	0	2.68 ± 0.07	1.81 ± 0.08
31	20	11	9/2	2.24 ± 0.07	1.42 ± 0.07
31	17	14	5/2	2.46 ± 0.09	1.62 ± 0.09
30	20	10	5	2.15 ± 0.09	1.35 ± 0.09
30	19	11	4	2.53 ± 0.07	1.73 ± 0.07
30	17	13	2	2.82 ± 0.06	2.02 ± 0.06
30	16	14	1	2.89 ± 0.06	2.09 ± 0.07
20	10	10	0	3.44 ± 0.05	3.01 ± 0.05

Guardiola, Navarro, Phys.Rev.Lett. 84 (2000) 1144

Metastable He-3 droplets

- Let N_0 be the minimal number of He-3 atoms that can form a bound droplet
  $E(N_0) < 0,\ E(N_0 - 1) > 0$
- Then droplet of $N_0 - 1$ atoms is metastable
- but cannot decay by emitting one atom:
  $E(N_0 - 2) > E(N_0 - 1)$
- The droplet can only decay by "explosion" into free particles
- How long the droplet lives?

Energy dependence

- Let's assume $N_0 = 29$ (smallest in literature), consider a droplet with 28 atoms
- Ground state of 28 atoms in harmonic potential

$$\Delta = 2 \times \frac{3}{2} + 6 \times \frac{5}{2} + 12 \times \frac{7}{2} + 8 \times \frac{9}{2} = 96$$

- $\Gamma \sim \left(\frac{E}{E_0}\right)^{\Delta - 5/2} = \left(\frac{E}{E_0}\right)^{93.5}$
- $E \sim 1\,\mathrm{K}, E_0 \sim 40\,\mathrm{K}$: life time $\gg$ age of Universe
- but will become shorter for 27-, 26-atom droplets

- There exist metastable He-3 droplets of $O(10)$ atoms with lifetimes ranging between fraction of nanosecond and the age of the Universe
- A more precise statement is difficult to make

Conclusion

- Metastable multiparticle resonances exist in various contexts in physics
- Behavior of the width at small energy is power-law with a known power
- Helium-3 metastable droplets exist, decay by explosion into free atoms