

Quantum geometry effects on superconductivity, light-matter interactions, and Bose-Einstein condensation

Päivi Törmä Aalto University

Hamburg Theoretical Physics Symposium 2021









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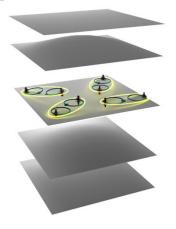
Quantum geometry and superconductivity

- Can we reach room temperature superconductivity?
- What does quantum geometry have to do with this?

Quantum geometry and BEC

Quantum geometry and light-matter interactions

Briefly: Bose-Einstein condensation and magnetic switching in a plasmonic lattice (experiment)



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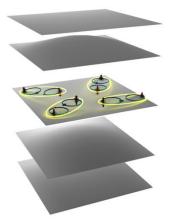
Quantum geometry and superconductivity

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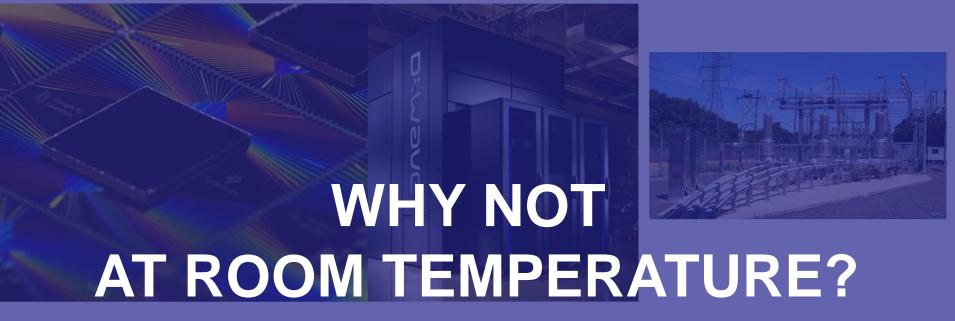
Quantum geometry and BEC

Quantum geometry and light-matter interactions

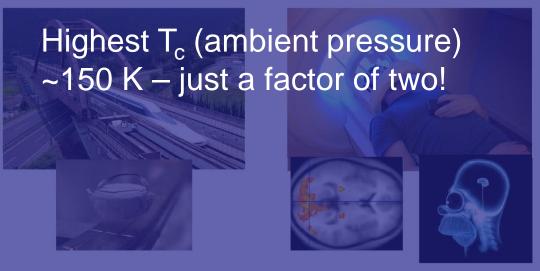
Briefly: Bose-Einstein condensation and magnetic switching in a plasmonic lattice (experiment)



SUPERCONDUCTIVITY







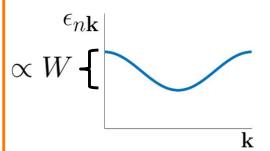
Superconductivity: BEC of Cooper pairs

Weak interaction U Large kinetic energy (Fermi level) $T_c \propto e^{-1/(U n_0(E_f))}$ Low critical temperature

Remove the kinetic energy to maximize the effect of interactions!

Flat bands: interactions dominate

Dispersive band U<<W:



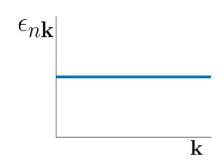
$$\psi_n(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{n\mathbf{k}}(\mathbf{r})$$

(periodic part of) the Bloch function

 $T_{\it c}$ for Cooper pairing

$$T_c \propto e^{-1/(Un_0(E_f))}$$

Flat band U>>W:



$$\epsilon_{n{f k}}={
m constant}$$

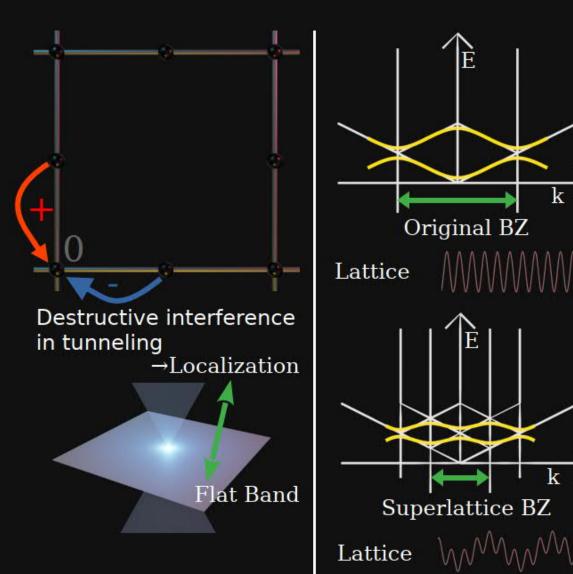
Group velocity:
$$\frac{\partial \epsilon_{n\mathbf{k}}}{\partial k} = 0$$

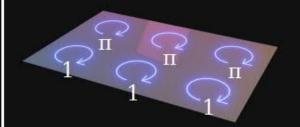
No interactions: insulator at any filling

$$T_c \propto UV_{\rm flat\ band}$$

High T_c for pairing (Khodel, Shaginyan, Volovik, Kopnin, Heikkilä)

Formation of flat bands





Landau levels

But is supercurrent stable at a flat band?

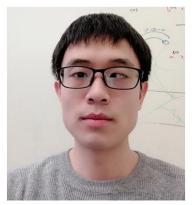
Supercurrent density: given by superfluid weight and Cooper pair momentum

$$\mathbf{J} = \frac{1}{4} D_s \hbar \mathbf{q}$$

Conventional BCS:
$$D_s = \frac{n_{\rm p}}{m_{\rm eff}} \left(1 - \left(\frac{2\pi\Delta}{k_{\rm B}T}\right)^{1/2} e^{-\Delta/(k_{\rm B}T)}\right)$$
 Zero at a flat band!!!
$$\frac{1}{m_{\rm eff}} \propto J \propto \partial_{k_i} \partial_{k_j} \epsilon_{\bf k}$$
 Bandwidth $i,j=x,y,z$

Superfluidity and quantum geometry













Sebastiano Peotta

Long Liang

Sebastian Huber

Murad Tovmasyan

Aleksi Julku

Tuomas Vanhala

Peotta, PT, Nat Comm 2015
Julku, Peotta, Vanhala, Kim, PT, PRL 2016
Tovmasyan, Peotta, PT, Huber, PRB 2016
Liang, Vanhala, Peotta, Siro, Harju, PT, PRB 2017
Liang, Peotta, Harju, PT, PRB 2017
Tovmasyan, Peotta, Liang, PT, Huber, PRB 2018
PT, Liang, Peotta, PRB(R) 2018







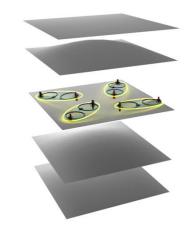
Topi Siro



Dong-Hee Kim

Our multiband approach

MULTIBAND BCS MEAN-FIELD THEORY multiband two-component attractive Fermi-Hubbard model -U < 0



$$H = -\sum_{ij\alpha\beta\sigma} t^{\sigma}_{i\alpha j\beta} c^{\dagger}_{i\alpha\sigma} c_{j\beta\sigma} - U \sum_{i\alpha} n_{i\alpha\uparrow} n_{i\alpha\downarrow} - \mu \sum_{i\alpha\sigma} n_{i\alpha\sigma}$$

Introduce a modulation of the order parameter phase to generate supercurrent

$$\Delta({f r}) o \Delta({f r}) e^{2i{f q}\cdot{f r}} \qquad 2{f q}$$
 : Cooper pair momentum

$$[D_s]_{ij} \propto \frac{\partial^2 \Omega}{\partial q_i \partial q_j} \Big|_{\mathbf{q}=0} \qquad \mathbf{j}(\mathbf{q}, \omega) = K(\mathbf{q}, \omega) \mathbf{A}(\mathbf{q}, \omega)$$
$$D_s = \lim_{\mathbf{q} \to 0} K(\mathbf{q}, \omega = 0)$$

Superfluid weight in a multiband system

$$D_s = D_{s, ext{conventional}} + D_{s, ext{geometric}}$$
 i, $j = x, y, z$ $\propto \partial_{k_i} \partial_{k_j} \epsilon_{\mathbf{k}}$ Can be nonzero also in a flat band Present only in a multiband case Proportional to the quantum metric $[D_s, ext{geometric}]_{ij} \propto Ug_{ij}$

Quantum geometric tensor

Metric for the distance between quantum states

$$d\ell^2 = ||u(\mathbf{k} + d\mathbf{k}) - u(\mathbf{k})||^2 = \langle u(\mathbf{k} + d\mathbf{k}) - u(\mathbf{k})|u(\mathbf{k} + d\mathbf{k}) - u(\mathbf{k})\rangle$$

$$\approx \sum_{i,j} \langle \partial_{k_i} u | \partial_{k_j} u \rangle dk_i dk_j$$
Introduce gauge invariant version
$$(u(\mathbf{k}) \leftrightarrow u(\mathbf{k})e^{i\phi(\mathbf{k})})$$

Quantum geometric tensor

$$\mathcal{B}_{ij}(\mathbf{k}) = 2\langle \partial_{k_i} u | (1 - |u\rangle\langle u|) | \partial_{k_j} u \rangle$$
 $\operatorname{Re} \mathcal{B}_{ij} = g_{ij}$ quantum metric $d\ell^2 = \sum_{ij} g_{ij} dk_i dk_j$
 $\operatorname{Im} \mathcal{B}_{ij} = [\mathbf{\Omega}_{\mathrm{Berry}}]_{ij}$ Berry curvature

Provost, Vallee, Comm. Math. Phys. **76**, 289 (1980)

Quantum metric is the same as Fubini-Study metric, and related to Fisher information

Lower bound for flat band superfluidity

The quantum geometric tensor \mathcal{B}_{ij} is complex positive semidefinite

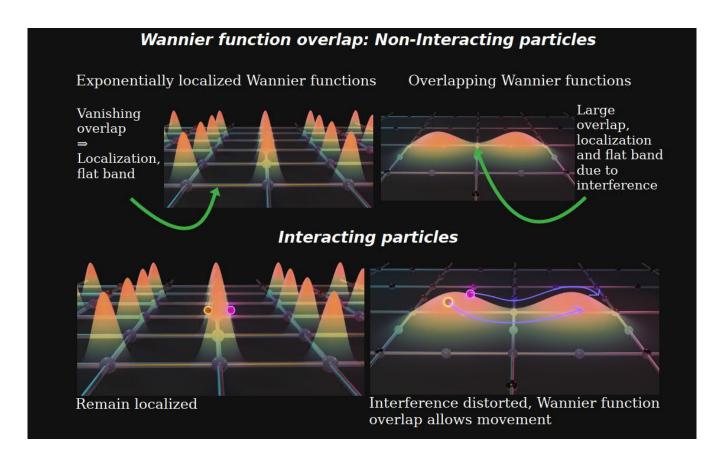
$$ightharpoonup D_s \geqslant \int_{B.Z.} d^d \mathbf{k} |\mathbf{\Omega}_{Berry}(\mathbf{k})| \geqslant C$$

Berry curvature: $\Omega(\mathbf{k}) = i\hat{z} \cdot \nabla \times \langle u_{n\mathbf{k}} | \partial_{\mathbf{k}} u_{n\mathbf{k}} \rangle$

Chern number: $C = \frac{1}{2\pi} \int_{B.Z.} d^2 \mathbf{k} \ \Omega(\mathbf{k})$

Mean-field results confirmed by: exact diagonalization, DMFT, DMRG, perturbation theory

Why can there be transport in a flat band?



$$C \neq 0 \Leftrightarrow$$
 non-localized $w(\mathbf{r}) = \mathcal{F}[u(\mathbf{k})]$

Brouder, Panati, Calandra, Marzari, PRL 2007

$$D_s \propto g_{ij} \geqslant C$$

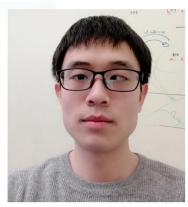
Twisted bilayer graphene (TBG) superconductivity and quantum metric



Aleksi Julku



Teemu Peltonen



Long Liang



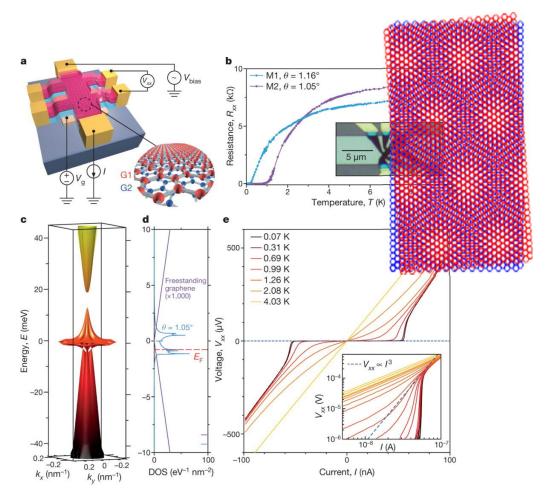
Tero Heikkilä

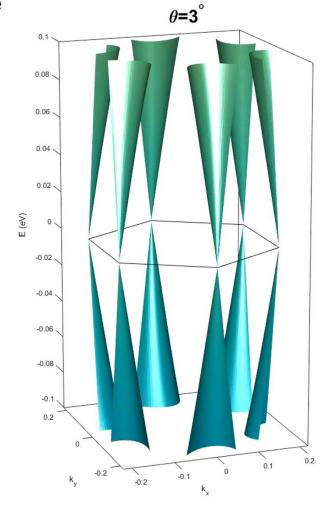
Julku, Peltonen, Liang, Heikkilä, PT, PRB(R) (2020); Editors' Suggestion For APS Physics news, google Geometry resques superconductivity

MA-TBG: Magic Angle-Twisted Bilayer Graphene

Twisting graphene layers produces flat bands

(unconventional) superconductivity





Y Cao et al. Nature **556**, 43–50 (2018)

Also Nature 556, 80 (2018) Science 363, 1059 (2019) Nature 574, 653-657 (2019)

PDF Version







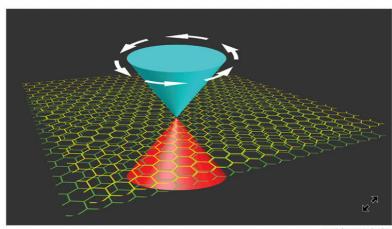
Geometry Rescues Superconductivity in Twisted Graphene

Laura Classen

School of Physics and Astronomy, University of Minnesota, Minneapolis, MN, USA

February 24, 2020 • Physics 13, 23

Three papers connect the superconducting transition temperature of a graphene-based material to the geometry of its electronic wave functions.



APS/Alan Stonebrake

Figure 1: Electrons moving through the sheets of twisted bilayer graphene (TBG) have special points in their band structure where two cone-shaped bands meet. The inherent "curvature" of the states in these bands turns out to contribute to the magnitude of TBG'... Show more

On its own, a sheet of graphene is a semimetal—its electrons interact only weakly with each other. But as experimentalists discovered in 2018 [1, 2], the situation changes when two sheets of graphene are stacked together, with a slight (\sim 1°) rotation between them (Fig. 1). At this so-called magic twist angle [3] and at low temperatures [1], the electrons become correlated, forming insulating or superconducting phases depending on the carrier density [2–7]. These phases appear to come from a twist-induced flattening of the electronic energy bands, which

Geometric and Conventional Contribution to the Superfluid Weight in Twisted Bilayer Graphene

Xiang Hu, Timo Hyart, Dmitry I. Pikulin, and Enrico Rossi

Phys. Rev. Lett. 123, 237002 (2019)

Published December 5, 2019

Read PDF

Superfluid weight and Berezinskii-Kosterlitz-Thouless transition temperature of twisted bilayer graphene

A. Julku, T. J. Peltonen, L. Liang, T. T. Heikkilä, and P. Törmä

Phys. Rev. B 101, 060505 (2020)

Published February 24, 2020

Read PDF

Topology-Bounded Superfluid Weight in Twisted Bilayer Graphene

Fang Xie, Zhida Song, Biao Lian, and B. Andrei Bernevig

Phys. Rev. Lett. 124, 167002 (2020)

Published April 24, 2020

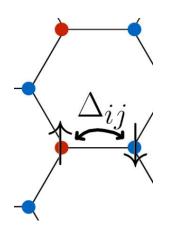
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Fermi-Hubbard lattice model with TBG geometry:

$$H = \sum_{ij\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + H_{\text{int}}$$

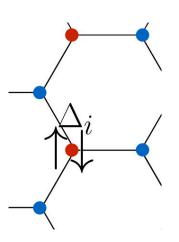
Two distinct pairing schemes:





$$H_{\rm int} = \frac{J}{2} \sum_{\langle ij \rangle} h_{ij}^{\dagger} h_{ij}$$

$$h_{ij} = c_{i\downarrow}c_{j\uparrow} - c_{i\uparrow}c_{j\downarrow}$$



J< 0 is attractive interaction strength

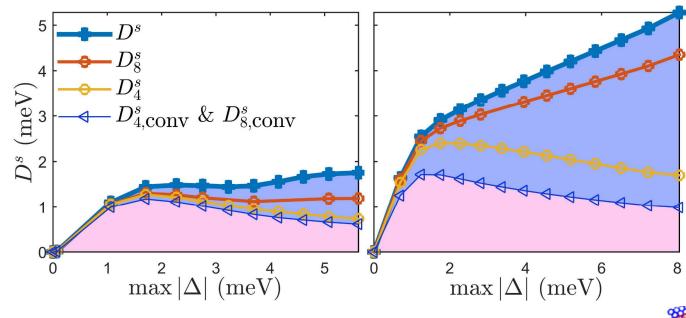
Geometric contribution in TBG

$$D^s = D^s_{\text{conv}} + D^s_{\text{geom}}$$

$$T_{\mathrm{BKT}} = \frac{\pi}{8} \sqrt{\det D^s(T_{\mathrm{BKT}})}$$

Single-particle energy

0 10 20 30 40 50 k in MBZ (arb. units, path from $K \to \Gamma \to M$



Non-local (RVB) interaction

Local (s-wave) interaction

Julku, Peltonen, Liang, Heikkilä, PT, PRB(R) (2020); Editors' Suggestion Confirmed by (only s-wave): Hu, Hyart, Pikulin, Rossi, PRL (2019) For APS Physics news, google Geometry resques superconductivity

Review

arXiv.org > cond-mat > arXiv:2111.00807

Search...

Help | Advance

Condensed Matter > Superconductivity

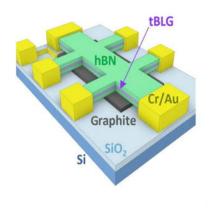
[Submitted on 1 Nov 2021]

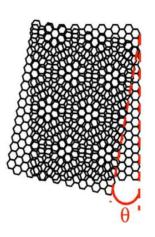
Superfluidity and Quantum Geometry in Twisted Multilayer Systems

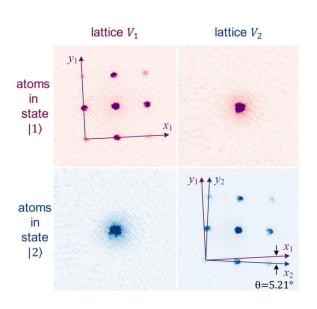
P. Törmä, S. Peotta, B.A. Bernevig

$$\det \mathcal{M}^{\mathrm{R}} \geq \mathcal{C}^2$$
, with $\mathcal{M}_{ij}^{\mathrm{R}} = \frac{1}{2\pi} \int d^2 \mathbf{k} \, g_{ij}(\mathbf{k})$

$$\frac{1}{4\pi} \int_{BZ} d^2k \operatorname{tr} g(\mathbf{k}) \ge \frac{1}{2\pi} \int_{BZ'} d^2k |f_{xy}|$$
$$\ge \left| \frac{1}{2\pi} \int_{BZ'} d^2k f_{xy} \right| = |e_2|.$$







J. Zhang group

Flat band transport and Josephson effect through a saw-tooth lattice









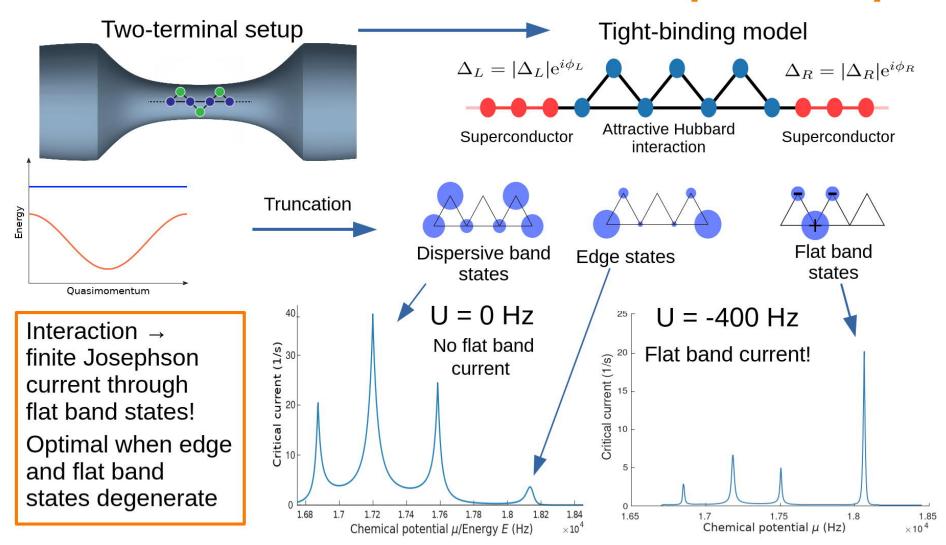


Ville Pyykkönen Sebastiano Peotta

Philipp Fabritius Jeffrey Mohan Tilman Esslinger

Pyykkönen, Peotta, Fabritius, Mohan, Esslinger, PT, PRB (2021)

Ultracold sawtooth lattice transport setup





Pyykkönen, Peotta, Fabritius, Mohan, Esslinger and Törmä: Flat band transport and Josephson effect through a finite-size sawtooth lattice, PRB 103, 144519, 2021

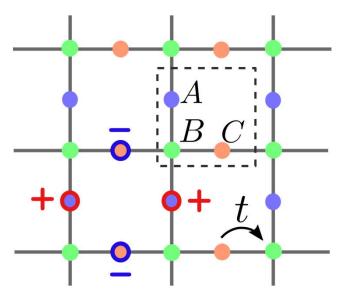
Insulator – pseudogap crossover in the Lieb lattice normal state



Kukka-Emilia Huhtinen

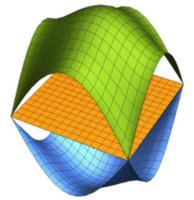
KE Huhtinen, PT, PRB(L) (2021)

Hubbard model on the Lieb lattice



Attractive Hubbard model

$$H = \sum_{\sigma} \sum_{i\alpha,j\beta} t_{ij} c_{\sigma,i\alpha}^{\dagger} c_{\sigma,j\beta} - \sum_{\sigma} \sum_{i\alpha} \mu_{\sigma} n_{\sigma,i\alpha}$$
$$+U \sum_{i\alpha} (n_{\uparrow,i\alpha} - 1/2)(n_{\downarrow,i\alpha} - 1/2)$$

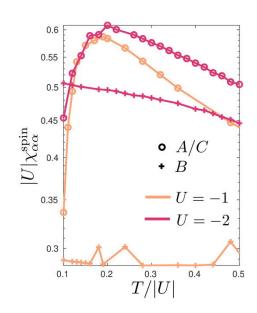


Flat band states reside at $\,A\,$ and $\,C\,$ sites

DMFT cluster: A, B and C

FOCUS ON THE NORMAL STATE ABOVE SUPERCONDUCTIVITY

Large (U>t) interactions: pseudogap

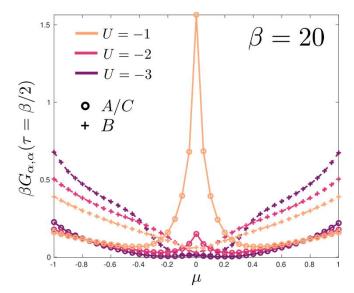


Generalized spin susceptibility:

$$\chi_{\alpha\alpha}^{\text{spin}} = \frac{2}{\beta^2} \sum_{\omega,\omega'} \left(\chi_{\uparrow\alpha,\uparrow\alpha,\uparrow\alpha,\uparrow\alpha}^{\text{ph},\omega,\omega',\nu=0} - \chi_{\uparrow\alpha,\uparrow\alpha,\downarrow\alpha,\downarrow\alpha}^{\text{ph},\omega,\omega',\nu=0} \right)$$

$$\chi_{ijkl}(\tau_1,\tau_2,\tau_3) = G_{ijkl}^{(4)}(\tau_1,\tau_2,\tau_3) - G_{ij}(\tau_1,\tau_2)G_{kl}(\tau_3,0)$$

$$G_{ijkl}^{(4)}(\tau_1, \tau_2, \tau_3) = G_{ijkl}(\tau_1, \tau_2, \tau_3) - G_{ij}(\tau_1, \tau_2)G_{kl}(\tau_3, \tau_3) - G_{ijkl}(\tau_1, \tau_2, \tau_3) - G_{ijkl}(\tau_1, \tau_2, \tau_3) - G_{ij}(\tau_1, \tau_2)G_{kl}(\tau_3, \tau_3) - G_{ijkl}(\tau_1, \tau_2, \tau_3) - G_{ijkl}(\tau_1, \tau_2, \tau_3) - G_{ij}(\tau_1, \tau_2, \tau_3) - G_{ijkl}(\tau_1, \tau_2, \tau_3) - G_{ij}(\tau_1, \tau_2, \tau_3) - G_{ijkl}(\tau_1, \tau_2, \tau_3, \tau_3) - G_{ijkl}(\tau_1, \tau_2, \tau_3, \tau_3) - G_{ijkl}(\tau_1, \tau_2, \tau_3) - G_{ijkl}(\tau_1$$

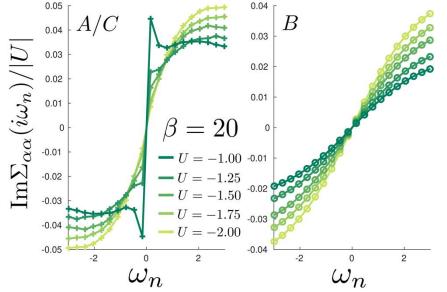


Local contribution to spin susceptibility decreases sharply with temperature at A/C sites.

At low temperatures, $\beta G_{\alpha\alpha}(\beta/2) \approx \mathcal{A}_{\alpha}(\omega=0)$, where \mathcal{A}_{α} is the orbital-resolved spectral function.

As interaction is increased, the spectral function becomes depleted around half-filling.





$$U = -0.5$$

$$0.04$$

$$0.03$$

$$-A/C$$

$$-B$$

$$-7.9 \cdot 10^{-3} \cdot T^{-0.52}$$

$$0.1$$

$$0.2$$

$$0.1$$

$$0.2$$

$$0.3$$

$$-7.9 \cdot 10^{-3} \cdot T^{-0.52}$$

$$0.1$$

$$0.2$$

$$0.1$$

$$0.2$$

$$0.3$$

$$0.1$$

$$0.2$$

$$0.3$$

$$0.1$$

$$0.2$$

$$0.3$$

$$0.4$$

$$0.5$$

$$T/|U|$$

$$T/|U|$$

$$Z = \left(1 - \frac{\operatorname{Im}\Sigma(i\omega_n)}{\omega_n}\Big|_{\omega_n \to 0}\right)^{-1}$$

In DMFT, $Z=m/m^{*}$, where m is the bare mass and m^{*} is the effective mass.

The self-energy diverges at low frequencies when the interaction strength is decreased.

The temperature dependence is $T^{-1/2}$ rather than T^{-1} found for Mott insulator.

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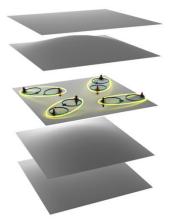
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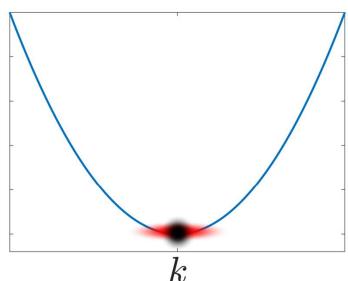
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Briefly: Bose-Einstein condensation and magnetic switching in a plasmonic lattice (experiment)



BEC in continuum

$$\epsilon(\mathbf{k}) = \frac{k^2}{2m_{eff}}$$



$$n_{\rm ex}(\mathbf{k}) = \frac{\epsilon(\mathbf{k}) + Un_0}{\sqrt{\epsilon(\mathbf{k})[\epsilon(\mathbf{k}) + 2Un_0]}} - 1$$

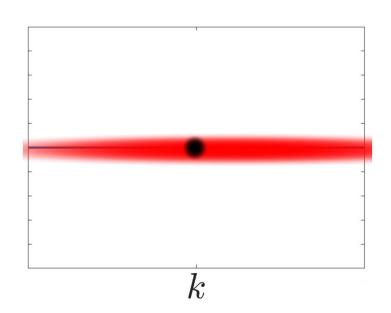
condensate density U repulsive contact interaction strength

= density of non-condensed bosons (quantum depletion)

BEC in continuum

$$m_{eff} \to \infty$$

$$\epsilon(\mathbf{k}) = \frac{k^2}{2m_{eff}} \to 0$$



For any finite interaction:

$$n_{\rm ex} o \frac{U n_0}{\sqrt{\epsilon(\mathbf{k})2U n_0}} - 1 o \infty$$

Obviously, flat band BEC in a single band model not possible.

BEC in a flat band?

- Clearly, BEC in a flat band should not be possible ???
- True for a single band system, not for multiband model (as shown in earlier works)

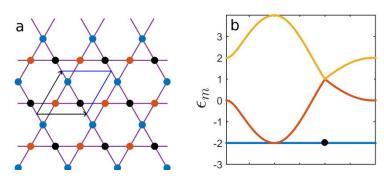
Huber & Altman, PRB 82, 184502 (2010) You et al., PRL 109, 265302 (2012) (H. Zhai group)

. What determines the stability?

Flat band BEC & quantum geometry

DISPERSIVE BAND FLAT BAND k=q k=0 $k=k_c$ $k=k_c+q$

Kagome lattice:







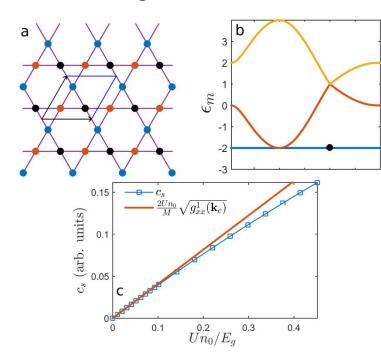
Aleksi Julku Georg Bruun

Julku, Bruun, PT, PRL 2021

Flat band BEC & quantum geometry

DISPERSIVE BAND FLAT BAND k=q k=0 $k=k_c$ $k=k_c+q$ n_0 Condensate density n_e Excitation density U Interaction u(k) Bloch function

Kagome lattice:



SPEED OF SOUND





Aleksi Julku Georg Bruun

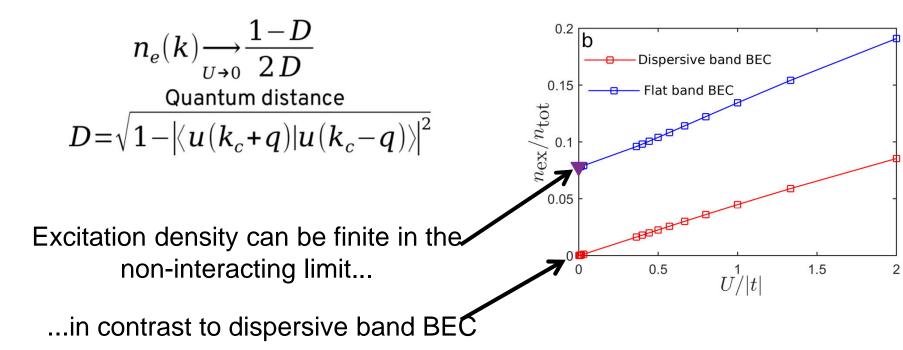
Quantum metric dictates the speed of sound

Julku, Bruun, PT, PRL 2021

Flat band BEC & quantum geometry

Excitations do not cost energy? Can BEC stable?

Answer: Yes it can, finite **quantum distance** between Bloch states sets the limit for excitation density -> stable BEC



Interaction effects prominent even in the limit of vanishing interactions

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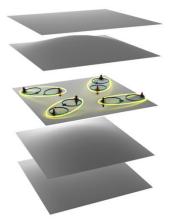
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Light-matter coupling (LMC) in multi-band

systems





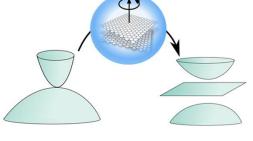


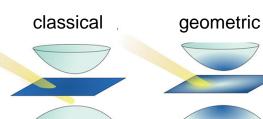


G. E. Topp, C. J. Eckhardt, D. M. Kennes, M. A. Sentef, and PT, PRB 2021



$$H_{\rm LMC}^{\rm single} = \sum_{\mu} \partial_{k\mu} \epsilon(k) \cdot A_{\mu} + \frac{1}{2} \sum_{\mu\nu} \partial_{k\mu} \partial_{k\nu} \epsilon(k) \cdot A_{\mu} A_{\nu}$$
 paramagnetic diamagnetic





Linear (A_{μ})

Quadratic $(A_{\mu}A_{\nu})$

Intra-band (n)

$$\partial_{\mu} \varepsilon_n$$

$$\partial_{\mu}\partial_{\nu}\varepsilon_{n}$$

$$\partial_{\mu}\partial_{\nu}\varepsilon_{n} - \sum_{n\neq n'} (\varepsilon_{n} - \varepsilon_{n'}) \left(\langle \partial_{\mu}n | n' \rangle \langle n' | \partial_{\nu}n \rangle + \text{h.c.} \right)$$

$$(\varepsilon_n - \varepsilon_m) \langle m | \partial_\mu n \rangle$$

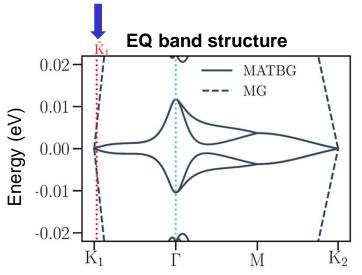
Inter-band
$$(n, m)$$

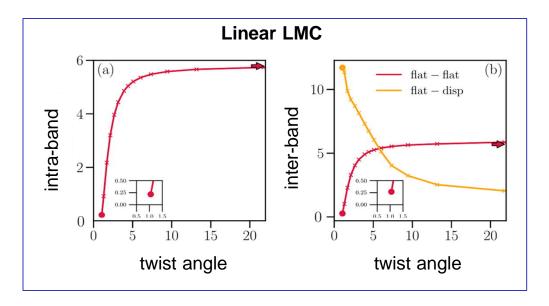
$$\left[(\partial_{\mu} \varepsilon_{n} - \partial_{\mu} \varepsilon_{m}) \langle m | \partial_{\nu} n \rangle + \frac{1}{2} \varepsilon_{m} \langle \partial_{\mu} \partial_{\nu} m | n \rangle + \frac{1}{2} \varepsilon_{n} \langle m | \partial_{\mu} \partial_{\nu} n \rangle + \sum_{n'} \varepsilon_{n'} \left(\langle \partial_{\mu} m | n' \rangle \langle n' | \partial_{\nu} n \rangle \right) \right] + (\mu \leftrightarrow \nu)$$

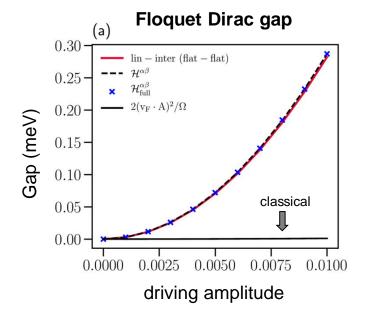
- 'classical' = determined by band dispersion
- 'geometric' = determined by Bloch states

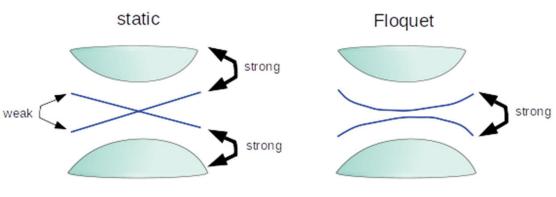
Application: Light-induced Dirac gap in TBG

G. E. Topp, C. J. Eckhardt, D. M. Kennes, M. A. Sentef, and PT, PRB 2021









$$\langle m|\,H_{\rm FLOQ}^A\,|n\rangle = \frac{\mathrm{i}A_0^2}{2\Omega} \left[\sum_l \langle m|\,\frac{\partial H_0}{\partial k_x}\,|l\rangle\,\langle l|\,\frac{\partial H_0}{\partial k_y}\,|n\rangle - \langle m|\,\frac{\partial H_0}{\partial k_y}\,|l\rangle\,\langle l|\,\frac{\partial H_0}{\partial k_x}\,|n\rangle \right]$$

Contents

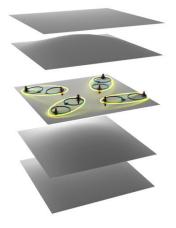
Quantum geometry and superconductivity

- Can we reach room temperature superconductivity?
- What does quantum geometry have to do with this?

Quantum geometry and BEC

Quantum geometry and light-matter interactions

Briefly: Bose-Einstein condensation and magnetic switching in a plasmonic lattice (experiment)



Bose-Einstein condensation in a plasmonic lattice

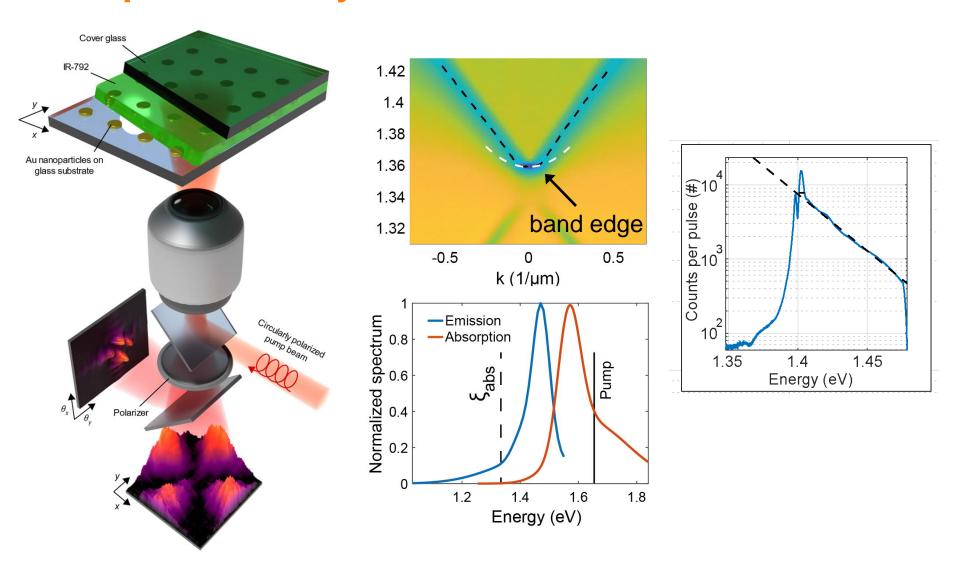
Hakala, Moilanen, Väkeväinen, Guo, Martikainen, Daskalakis, Rekola, Julku, PT, Nature Physics 14, 739 (2018)

Väkeväinen, Moilanen, Necada, Hakala, Daskalakis, PT, Nature Communications 11, 3139 (2020)

Taskinen, Kliuiev, Moilanen, PT, Nano Letters 14, 1721 (2021)

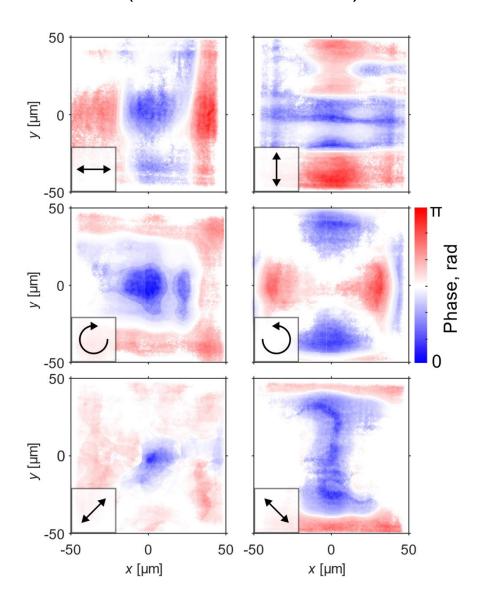
Moilanen, Daskalakis, Taskinen, PT, arXiv:2103.10397

Nanoparticle array + molecules



Nature Physics 2018, Nature Communications 2020, arXiv2021

Condensate phase determined for the first time by phase retrieval (Nano Letters 2021)



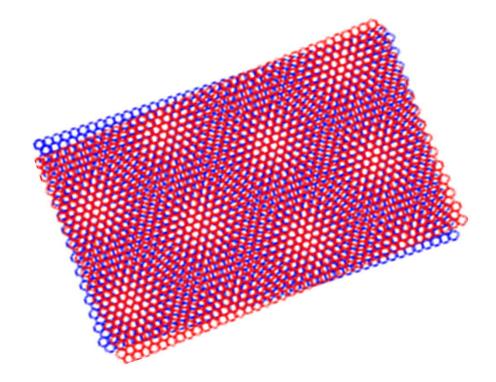
Magnetic switching of plasmonic lasing

Freire-Fernandez, Cuerda, Daskalakis, Perumbilavil, Martikainen, Arjas, PT, van Dijken, Nature Photonics in press (2021), arXiv:2104.14321

Summary

Quantum geometry governs

- flat band superfluidity
- BEC excitations
- light-matter interactions



Outlook

Towards room temperature superconductivity

Role of quantum geometry and interactions in photonic systems











