Hidden Symmetry of Vanishing Love

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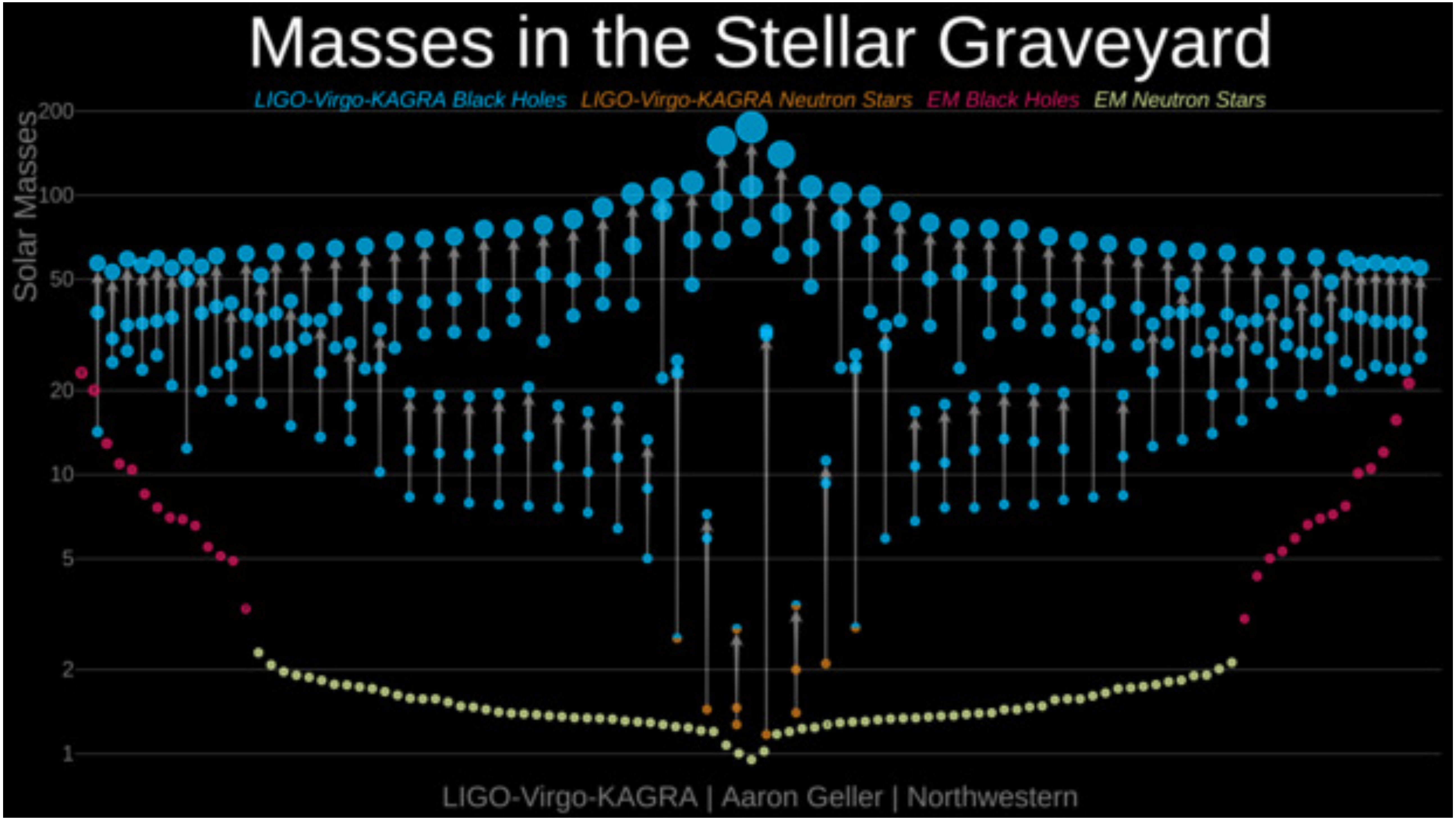
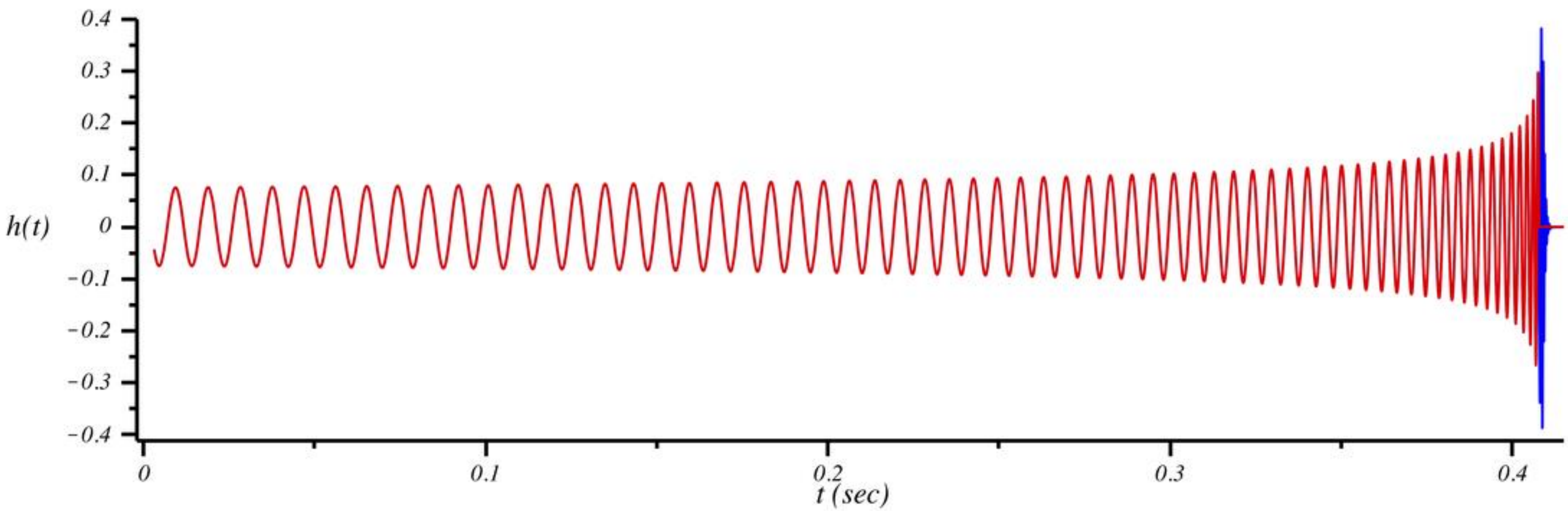
a fishing expedition in progress
with Panos Charalambous and Misha Ivanov 2102.08917, 2103.0123, + to appear



We are witnessing the dawn of precision black hole physics

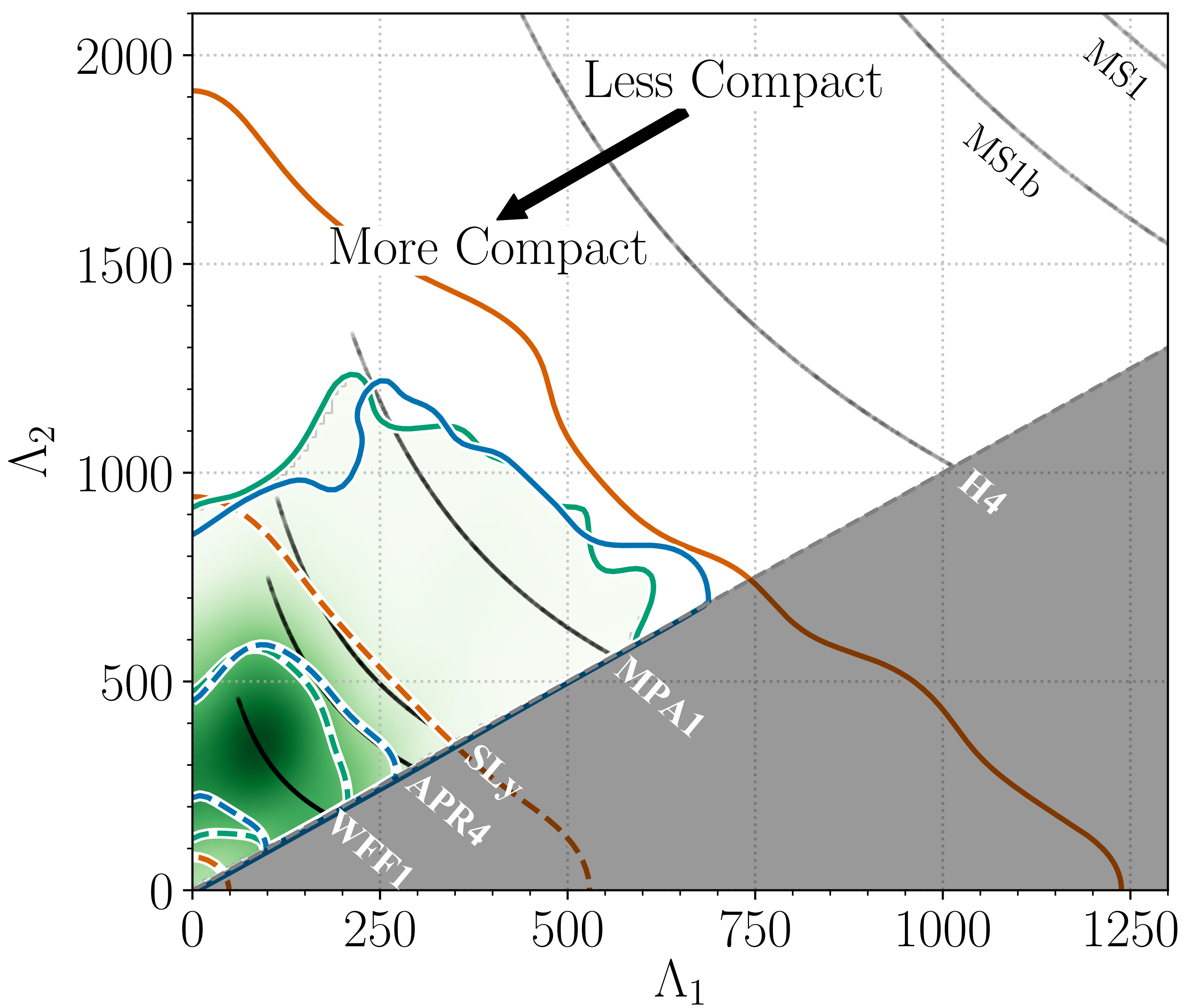
A typical binary merger



is sensitive to tidal responses of the mergers

Tidal response of neutron stars from GW170817

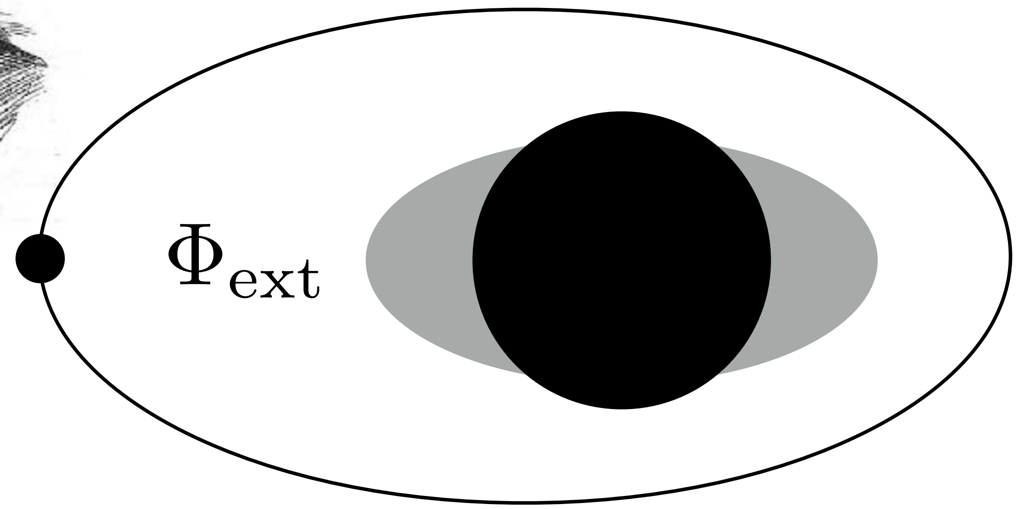
LIGO/Virgo 1805.11581



Tidal Love numbers in Newtonian gravity

$$\Phi = \Phi_0 + \Phi_2 + \Phi_3 + \ldots$$

$$\Phi_0 = -\frac{M}{r}$$



$$\Phi_2 = r^2 E_{ij} n^i n^j + \frac{Q_{ij} n^i n^j}{r^3}$$

tidal source tidal response

$$\Phi_l = r^l E_{i_1 \ldots i_l} x^{i_1} \ldots x^{i_l} + Q_{i_1 \ldots i_l} \frac{n^{i_1} \ldots n^{i_l}}{r^{l+1}}$$

Love numbers

linear response

$$\boxed{Q_l = \Lambda_l E_l}$$

Tidal Love numbers in Einstein gravity

Fang, Lovelace gr-qc/0505156
Binnington, Poisson 0906.1366
Damour, Nagar 0906.0096

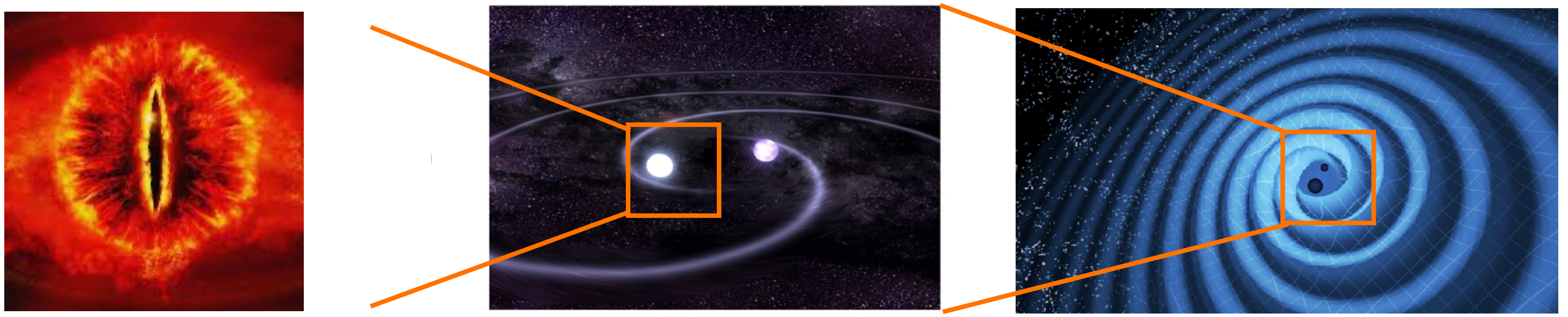
Morally, one wants to say that far away from an object gravity is Newtonian, and use the same definition.

However,

- ✶One may worry whether this definition is reparametrization invariant
- ✶Einstein gravity is non-linear, one may worry that the source/resource separation is not well-defined

A conceptually clean definition is provided by the worldline effective theory

Goldberger, Rothstein hep-th/0409156, 0511133
Kol, Smolkin 1110.3764
Porto 1601.04914



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EFT is the tool to disentangle physics from different length scales

at $r \gg r_s$ a black hole is described by the worldline effective action

$$S = -M \int d\tau + \dots$$

To keep formulas short in what follows gravity -> massless scalar

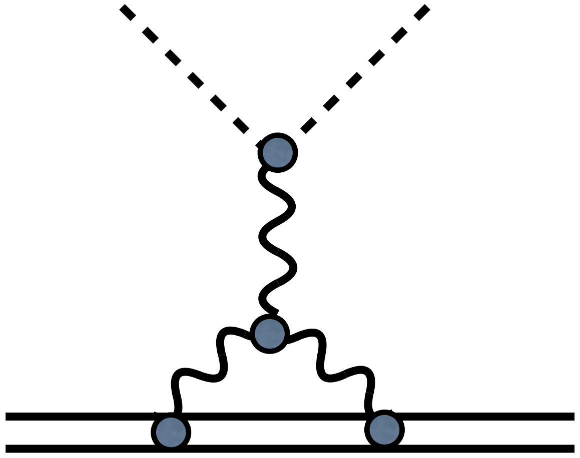
$$S_{worldline} = -M\int d\tau + \lambda_1 \int (\partial_i\phi)^2 + \lambda_2 \int (\partial_i\partial_j\phi)^2 + \ldots$$

Love numbers -> Wilson coefficients

$$\lambda_l \sim \Lambda_l r_s^{2l-1}$$

Provides unambiguous gauge-invariant definition of Love numbers

- One expects to find $\Lambda_l \gtrsim \mathcal{O}(1)$
- One expects to find (classical) RG running



$$+ \quad \cdots$$

Schwarzschild Love numbers

Fang, Lovelace gr-qc/0505156
Binnington, Poisson 0906.1366
Damour, Nagar 0906.0096
Kol, Smolkin 1110.3764
Hui, Joyce, Penco, Santoni, Solomon 2010.00593

- ✴No Love in 4d: $\Lambda_l = 0$

- ✴d>4, generically Love is a non-vanishing constant

$$\Lambda_l = \frac{2\hat{l}+1}{2\pi}\frac{\Gamma(\hat{l}+1)^4}{\Gamma(2\hat{l}+2)^2}\tan\pi\hat{l} \qquad \hat{l} = \frac{l}{d-3}$$

Love vanishes at integer $\hat{l}$

Love runs logarithmically at half-integer $\hat{l}$

Kerr Love numbers all vanish (in 4d)

Chia 2010.07300
Charalambous, SD, Ivanov 2102.08917
also Le Tiec, Casals, Franzin 2010.15795

a bit subtle, the actual worldsheet action (also for Schwarzschild)

$$S_{worldline} = -M\int d\tau + \lambda_1 \int (\partial_i\phi)^2 + \lambda_2 \int (\partial_i\partial_j\phi)^2 + \dots$$
$$+ \int d\tau \partial_i \phi \mathcal{O}_i + \dots$$

$$\phi \sim (\lambda + \langle \mathcal{O}\mathcal{O}\rangle)\phi_{ext} \sim \nu \partial_t \phi_{ext} + \dots$$ Schwarzschild

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Charalambous, SD, Ivanov 2102.08917
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$$\phi \sim (\lambda + \langle \mathcal{O}\mathcal{O} \rangle)\phi_{ext} \sim \nu(\partial_t + \Omega \partial_\phi)\phi_{ext} + \dots$$ Kerr

static dissipative response due to frame dragging

Holographic interpretation

$$S_{worldline} = -M \int d\tau + \lambda_1 \int (\partial_i \phi)^2 + \lambda_2 \int (\partial_i \partial_j \phi)^2 + \dots$$
$$+ \int d\tau \partial_i \phi \mathcal{O}_i + \dots$$

Remarkably, in some cases we know what $\mathcal{O}$'s are. For instance, for black 3-branes in type IIA string theory these are operators of N=4 supersymmetric Yang—Mills.

Love numbers characterize how black hole is "glued" to the rest of the space-time.

All Love properties follow from the Teukolsky equation

$$\varphi = \Phi(t,r,\phi)S(\theta) = R(r)S(\theta)e^{-i\omega t+im\phi}$$

$$\partial_r(\Delta(r)\partial_r R) + V(\omega,m,r)R = AR$$

in 4d

$$\Delta = (r-r_+)(r-r_-)$$

and

$$A = l(l+1) + \mathcal{O}(\omega\Omega)$$

is the eigenvalue of the angular equation

To identify Love numbers one extracts a decaying tail of the solution which is regular at the horizon

Looks like a good fishing place for some interesting hidden structure (symmetry?)

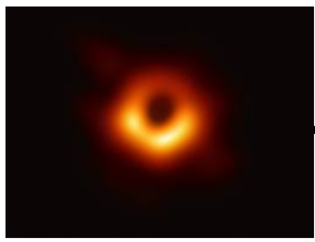
NB: static Love numbers only care about $\omega = 0$ limit. Let's look at the whole thing nevertheless...

Near zone expansion

Starobinsky'73, Maldacena, Strominger hep-th/9702015, Castro, Maloney, Strominger 1004.0996, Bertini, Cacciatori, Klemm 1106.0999, Chia 2010.0730

near zone

$\omega r \ll 1$



r

near horizon region

$r \sim r_+$

far zone

$r \gg r_+$

At low frequencies, $\omega r_+ \ll 1$, near zone covers horizon and overlaps with the asymptotically flat region

$$\boxed{V = V_0 + \epsilon V_1}$$

$\epsilon = 1$ is a formal expansion parameter. In what follows $\epsilon = 0$.

- $V_0 \gg V_1$ in the near zone
- $V_1 = 0$ at zero frequency

Let’s choose the following near zone split

$$V_0 = \frac{(2Mr_+)^2}{\Delta}\left((\omega - \Omega m)^2 - 4\omega\Omega m\frac{r - r_+}{r_+ - r_-}\right)$$

$$V_1 = \frac{2M(\omega a m\beta + 4M^2\omega^2 r_+)}{r_+(r - r_-)} + \omega^2(r^2 + 2Mr + 4M^2)$$

$$\beta = \frac{4Mr_+}{r_+ - r_-}$$

and define

$$L_0 = -\beta\partial_t \; ,$$

$$L_{\pm 1} = e^{\pm\beta^{-1}t}\left(\mp\Delta^{1/2}\partial_r + \beta\partial_r(\Delta^{1/2})\partial_t + \frac{a}{\Delta^{1/2}}\partial_\phi\right)$$

Love Symmetry

- ✶ *L*'s are regular at the horizon
- ✶ Satisfy *SL(2,R)* algebra

$$\boxed{[L_n, L_m] = (n-m)L_{n+m}\,, \quad n,m = -1,0,1}$$

- ✶ Near zone Teukolsky equation turns into

$$\boxed{\mathcal{C}_2 \Phi = \hat{l}(\hat{l}+1)\Phi}$$

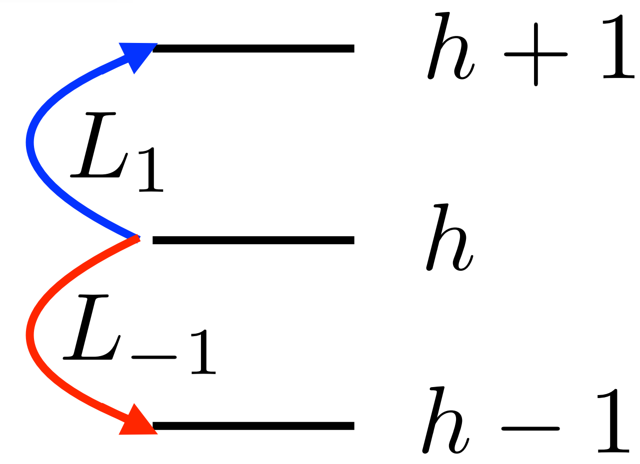
where $\mathcal{C}_2$ is the quadratic Casimir

$$\mathcal{C}_2 \equiv L_0^2 - \frac{1}{2}(L_{-1}L_1 + L_1 L_{-1})$$

All properties of Love numbers can be nicely explained in terms of SL(2,R) representation theory

Highest Weight Banishes Love

a vector of weight h $\qquad L_0 v_h = -hv$



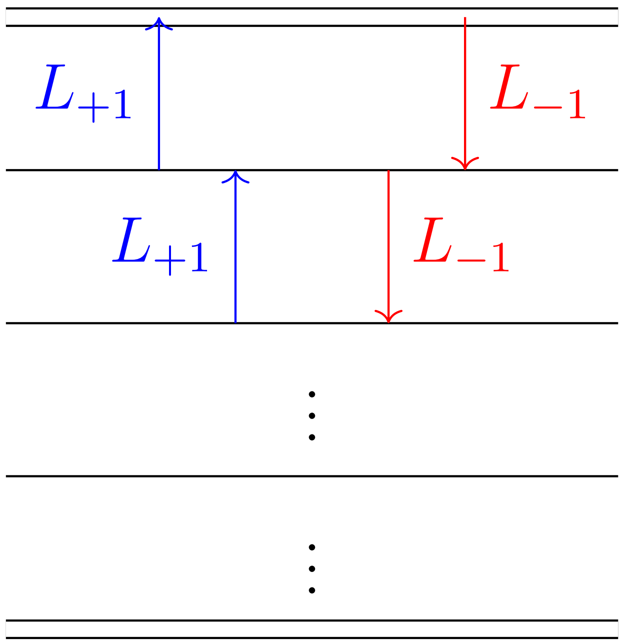
$L_0 = -\beta \partial_t \quad \Longrightarrow \quad$ static= weight 0

integer $\hat{l}$ $\quad \Longleftrightarrow \quad$ regular static solution belongs to a highest weight representation

indeed, solve $\begin{cases} L_1 v_{\hat{l}} = 0 \\ L_0 v_{\hat{l}} = -\hat{l} v_{\hat{l}} \end{cases} \quad \Longrightarrow \quad v_0 = L_{-1}^{\hat{l}} v_{\hat{l}}$

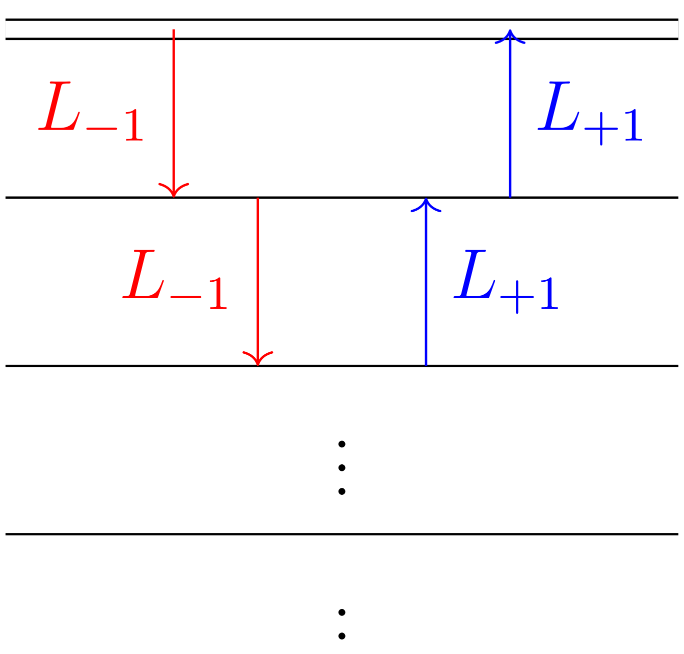
highest weight $\quad \Longrightarrow \quad$ zero Love $\quad L_1^{\hat{l}+1} v_0 = 0$

Schwarzschild



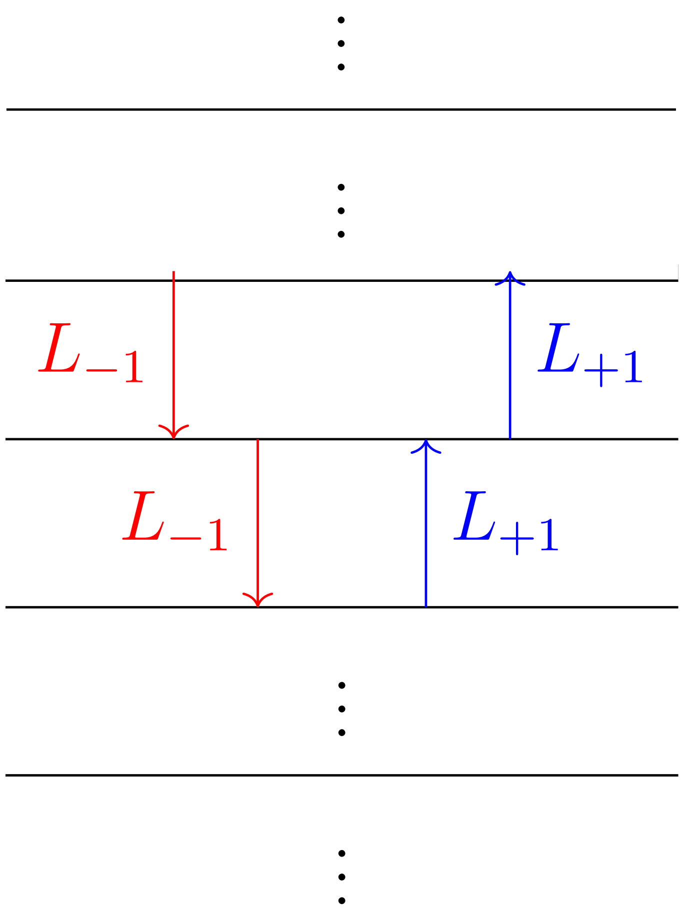
finite dim rep as a consequence
of regularity at the white hole horizon

Kerr



semi-infinite Verma module
related to a singularity at the white hole horizon

✳generic $\hat{l}$



infinite rep, non-zero Love numbers,
no running because regular and singular
solutions belong to algebraically different reps

✳half-integer $\hat{l}$

infinite rep, regular and singular solutions
belong to isomorphic reps, log running, cf.
resonance condition in conformal perturbation
theory

Further comments

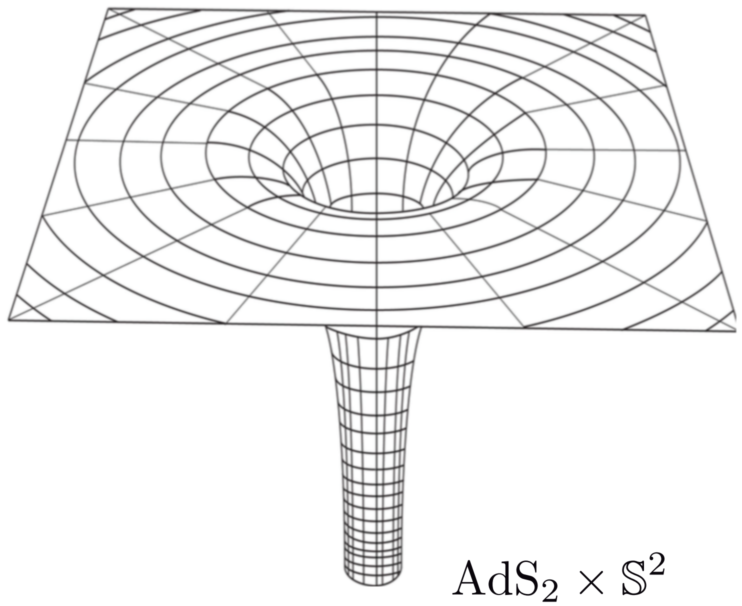
∗a story for vectors and tensors is similar.

$$L_0^{(s)} = L_0 + s\,,$$
$$L_{\pm 1}^{(s)} = L_{\pm 1} - s\,e^{\pm \beta t}(1 \pm 1)\partial_r(\Delta^{1/2})$$

c.f. GHP Lie derivatives

Geroch, Held, Penrose '73

∗a story for charged black holes is similar. In 4d Love symmetry turns into a near-horizon isometry in the extremal $Q = M$ limit.



∗R^3 gravity in 4d: no symmetry, log's are generic.

Is this a triumph of naturalness a la 't Hooft?

In the current version looks more like an example of a "UV miracle"

$$L_0 = -\beta \partial_t \, ,$$
$$L_{\pm 1} = e^{\pm \beta^{-1} t} \left(\mp \Delta^{1/2} \partial_r + \beta \partial_r (\Delta^{1/2}) \partial_t + \frac{a}{\Delta^{1/2}} \partial_\phi \right)$$

Love symmetry mixes UV and IR modes!

The story is likely to be incomplete:

- ✳Starobinsky near zone has another SL(2,R) leading to an identically vanishing response at the locking frequency $\omega = m\Omega$. Combining the two leads to an infinite extension of the Love algebra.

- ✳There is a glimpse of a symmetry argument operating strictly in the static limit, at least for the 4d Schwarzschild.

Hui, Joyce, Penco, Santoni, Solomon 2105.01069

The verdict in this case is not out yet. Independently of the result, it is fascinating to entertain a possibility that some of the fine-tunings in Nature can be explained by symmetries which mix UV and IR .