THE UNIVERSITY OF TOKYO

National Institute of Advanced Industrial Science and Technology AIST

WPC Theoretical Physics Symposium,
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Designing superconducting and topological systems in and out of equilibrium

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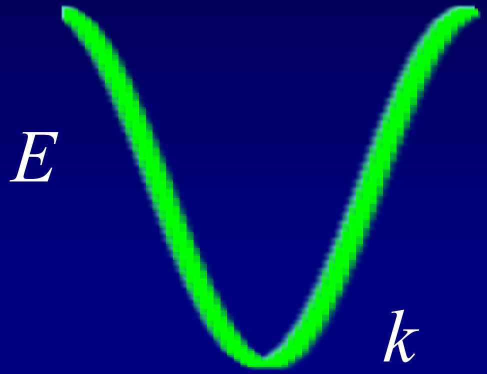
Plan of the talk

	Superconductivity (from repulsion)	Topological
Equilibrium	✓ 1-band vs multiband * Incipient-band SC * Flat-band SC * Nematic SC (new)	✓ Topological states * Dispersive vs flat bands
Non-equil	✓ d-wave SC → Floquet topological d + id SC (new)	✓ Floquet topological insulator * Dispersive vs flat bands

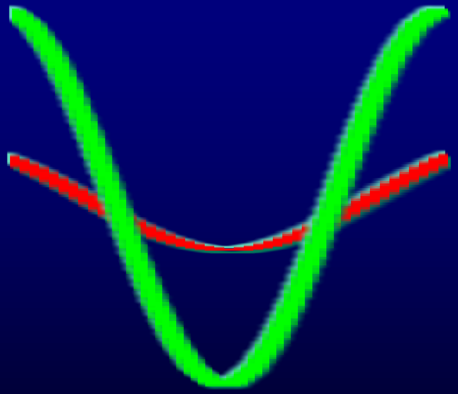
Which is most favourable for SC?

One-band

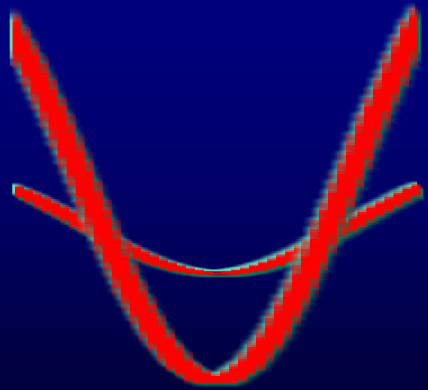


Multi-band

multi-orbital

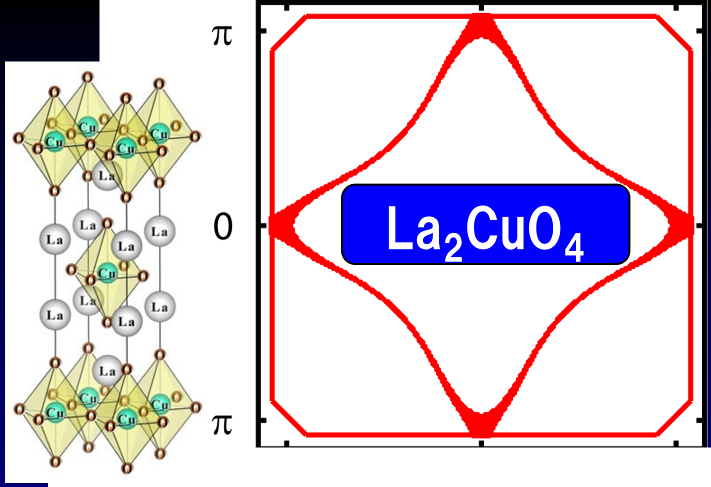


single-orbital

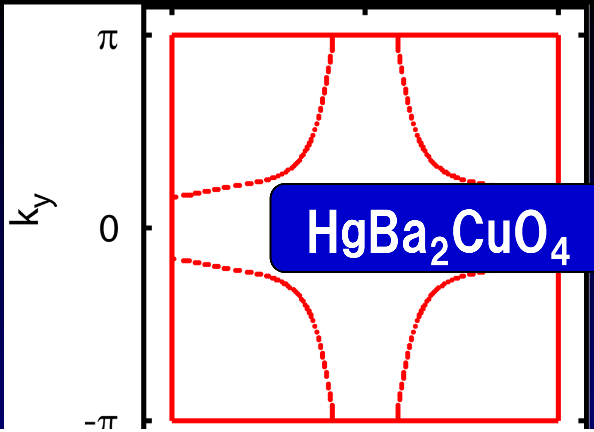


colour: orbital character

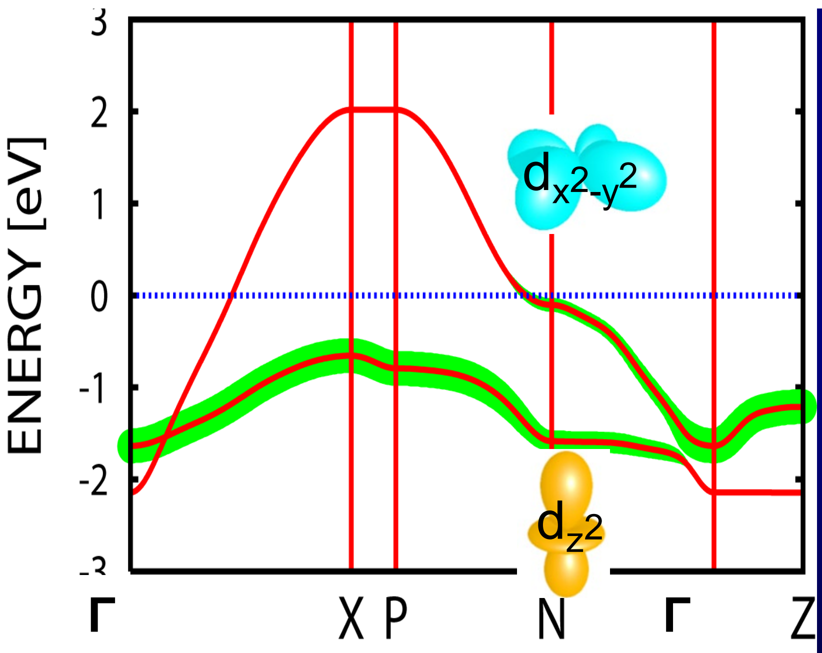
HTC cuprates

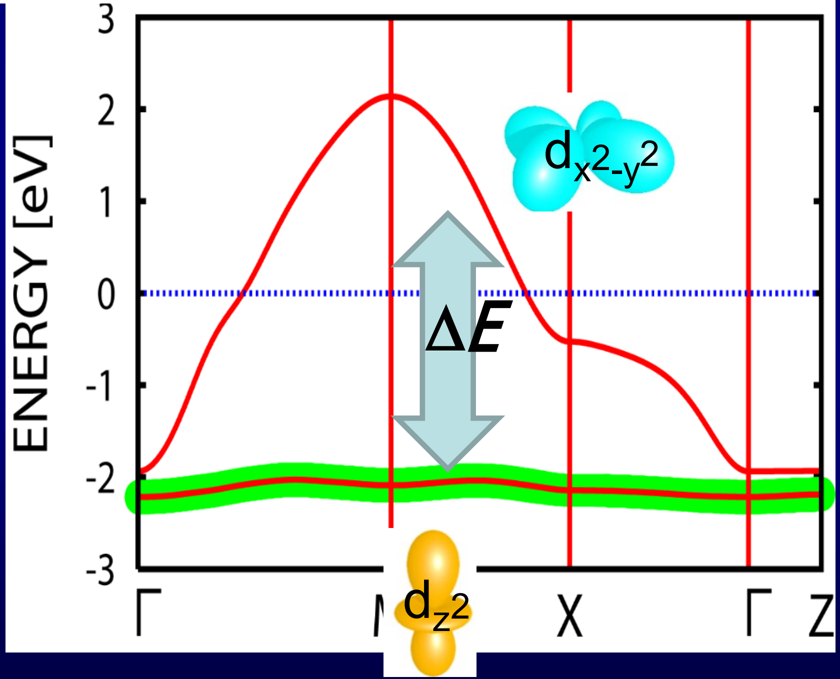


Tc = 40 K < 90 K

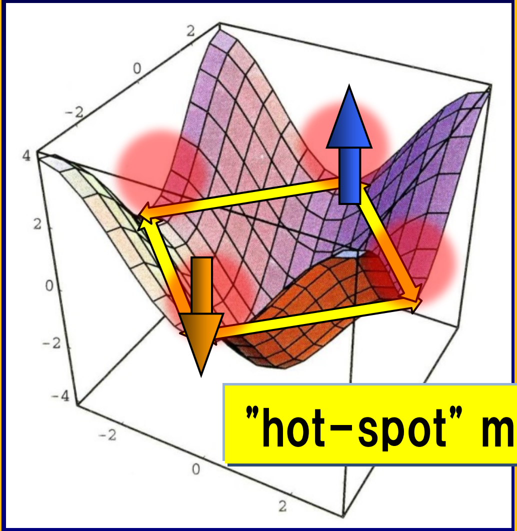
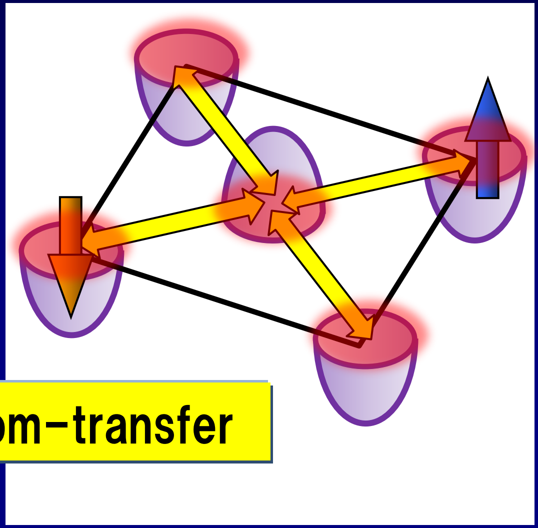
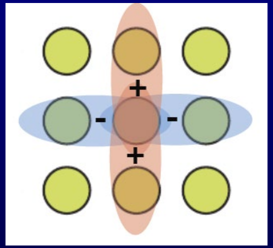
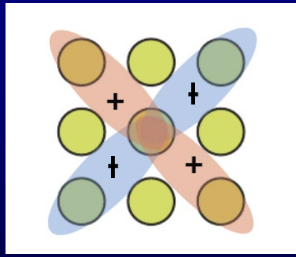


(Sakakibara et al, PRL 2010)





Pairing from repulsion in k- and real-spaces

	single-orbital, one-band	multi-o, multi-b
k - space		
real space		
	d (as in cuprates)	s+- (as in iron-based)

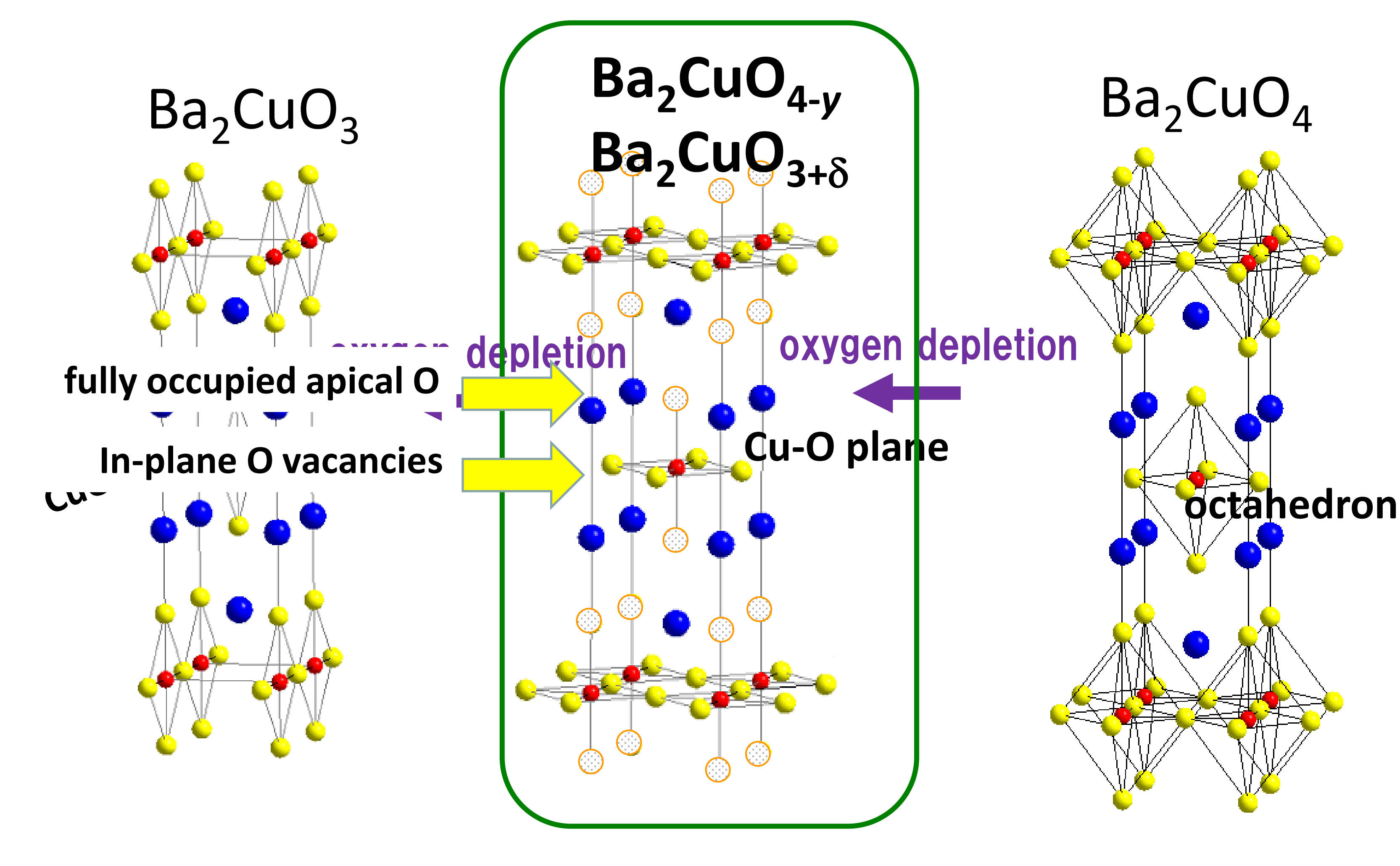
"hot-spot" mom-transfer

(Hosono & Kuroki, Physica C 2015)

However, even for a cuprate,
multi-orbital, multi-band can become crucial

← interband pairing

High Tc in single-layer Ba_2CuO_{4-y} (= $Ba_2CuO_{3+\delta}$)

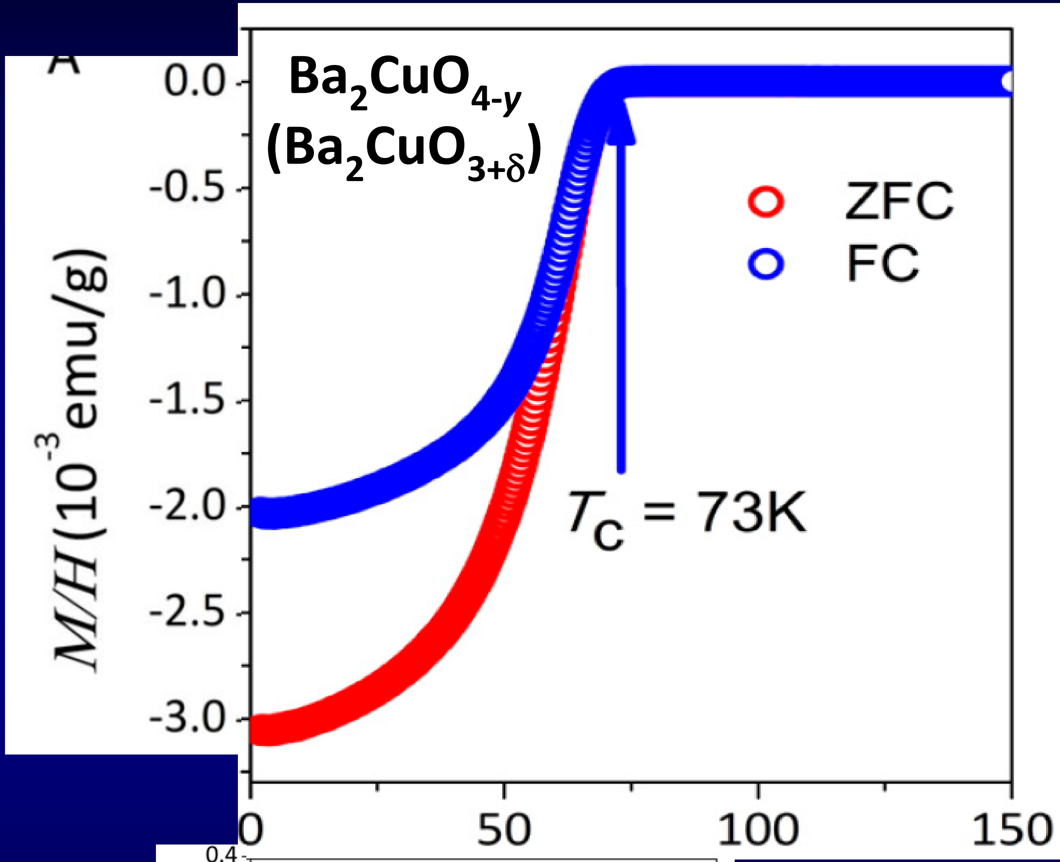


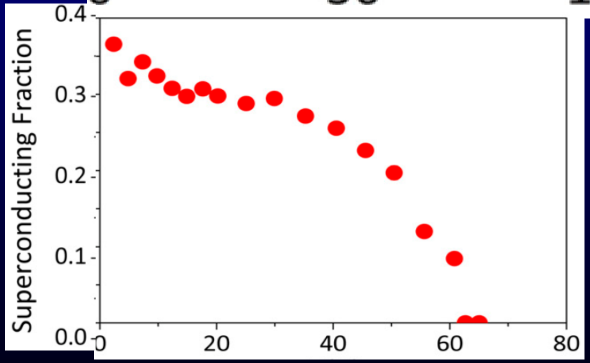
a = 3.4954 Å
b = 3.9057 Å
c = 12.68 Å

a = 3.76 Å
c = 12.57 Å

La_2CuO_4
a = 3.78 Å
c = 13.2 Å

(Li et al, PNAS 2019)





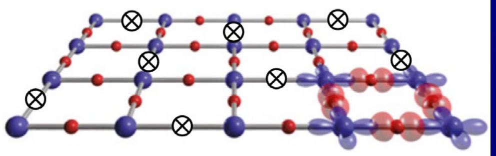
Extraordinary features of Ba_2CuO_{4-y}

- **Heavily hole-doped**;

  nominal hole density, $p = 2\delta \approx 0.4$/Cu

- **Heavily O-deficient Cu-O plane**;

  about 40% in-plane O vacancies



Mother (undoped) compound: Ba_2CuO_3

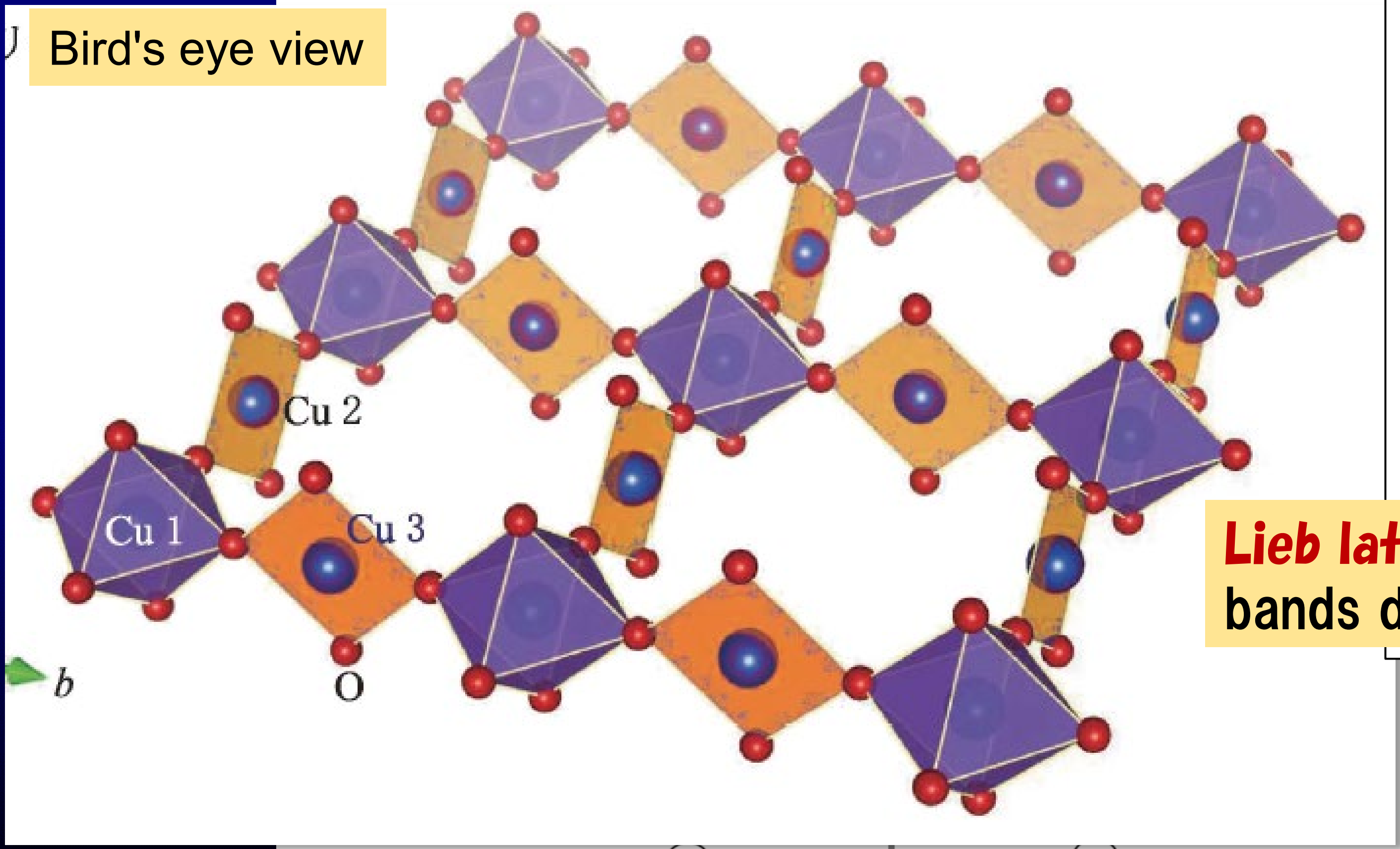
(Yamazaki et al, PRR 2020)

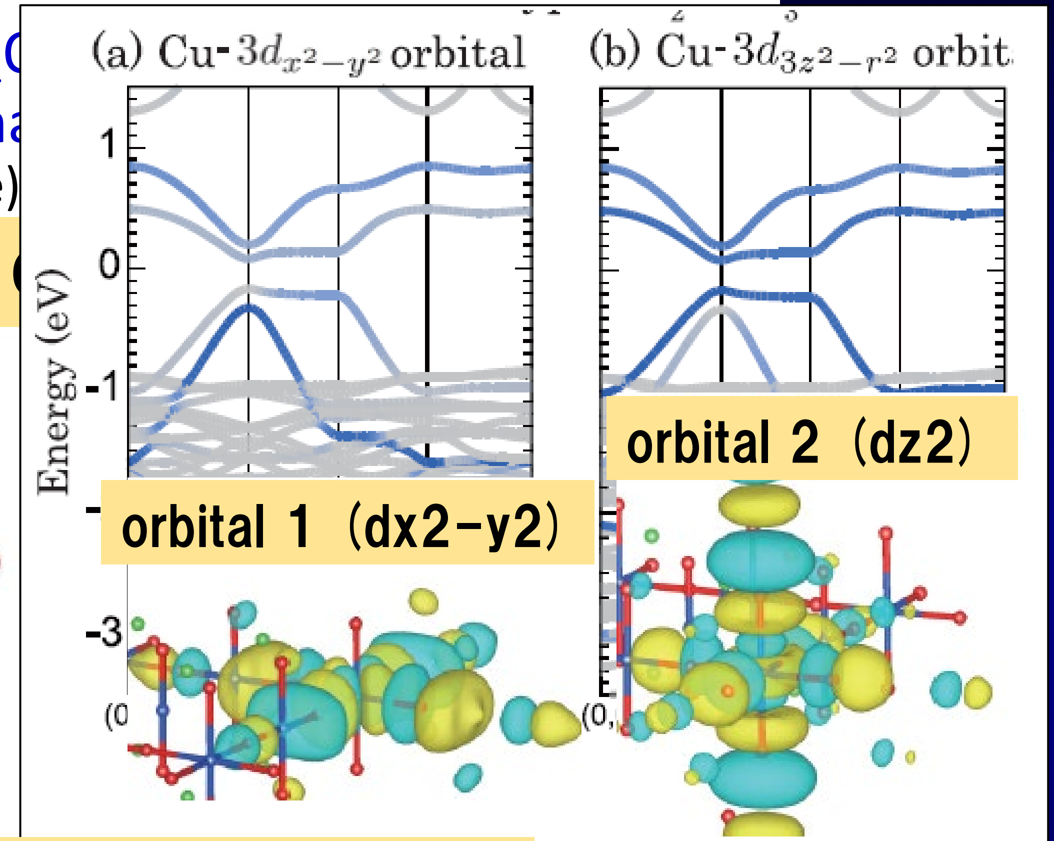
$(Ba_2CuO_3)_4 = Ba_8Cu_4O_{12}$ = $(Cu_4O$

Plana

◯ O (in-plane)

Cu_4O_4



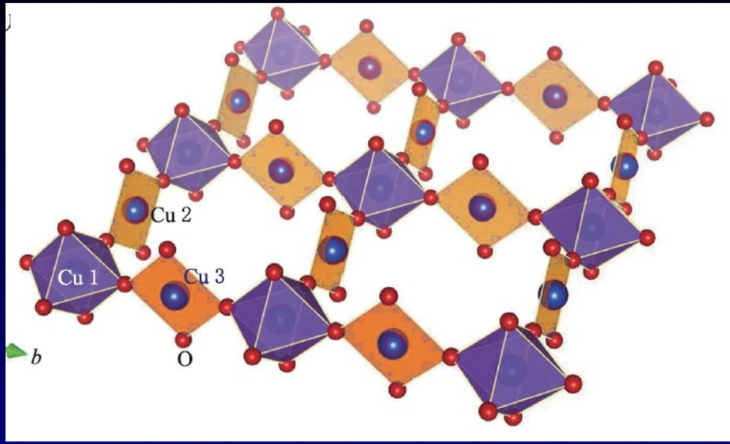


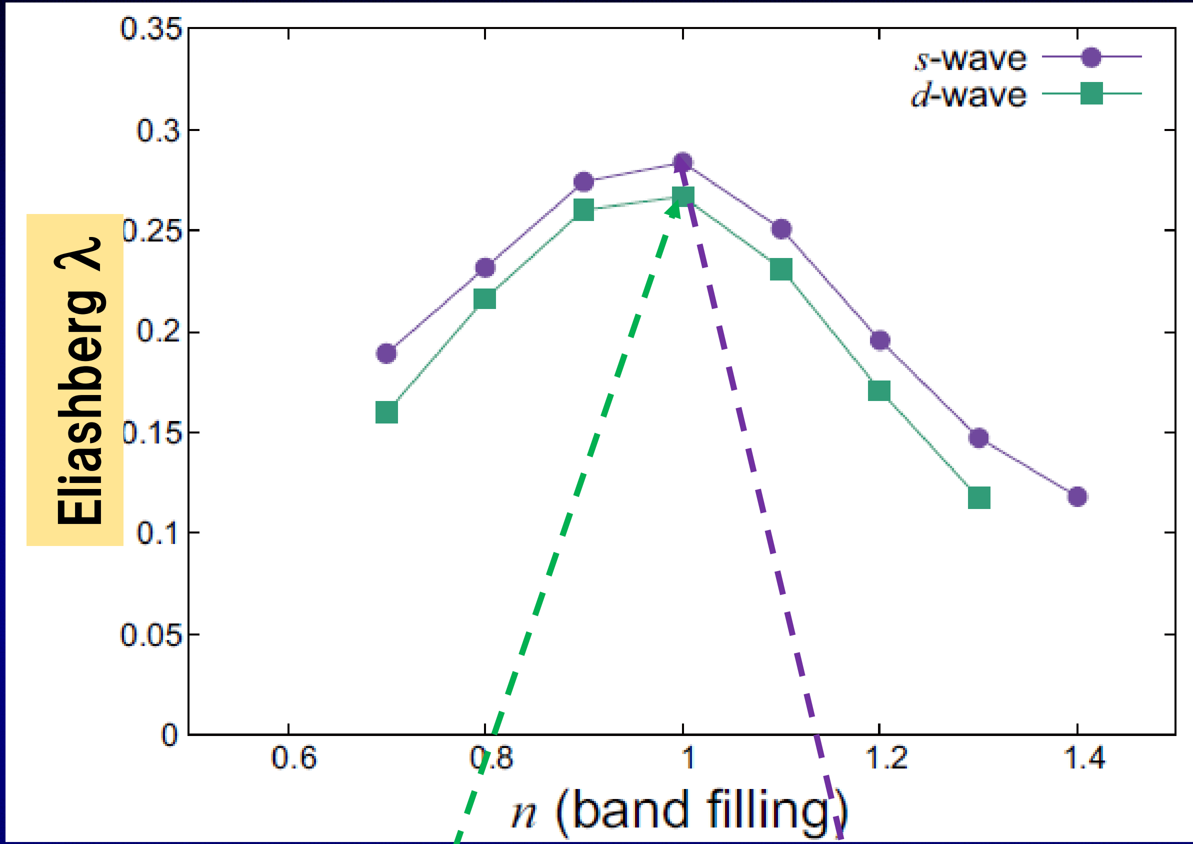
Lieb lattice, but no flat bands due to multi-orbitals

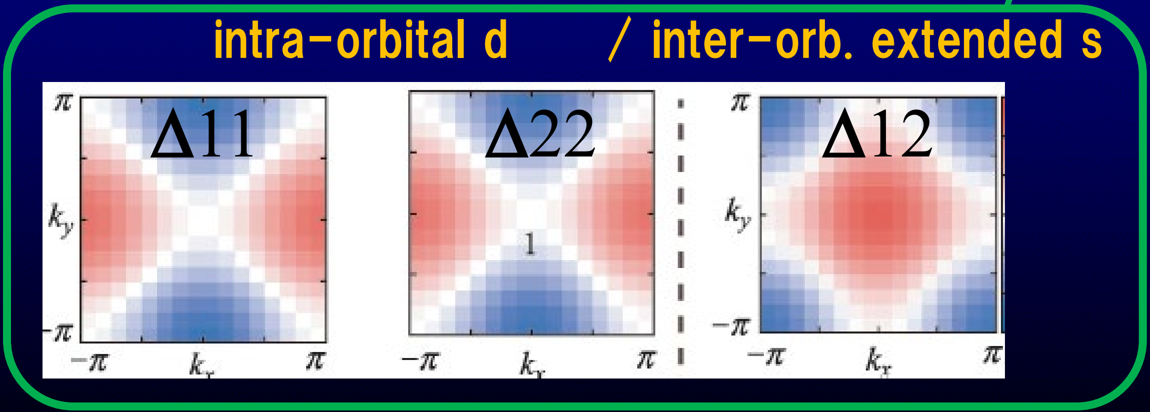
Superconductivity

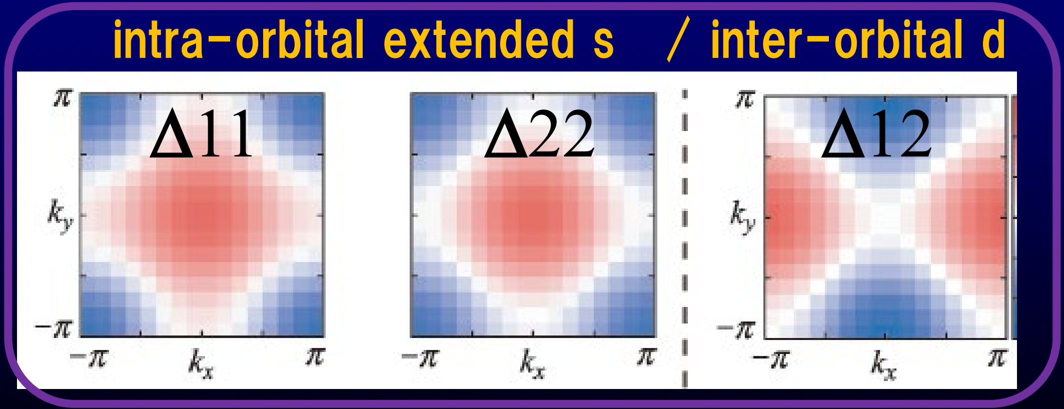
(Yamazaki et al, PRR 2020)

Method: 6-orbital FLEX



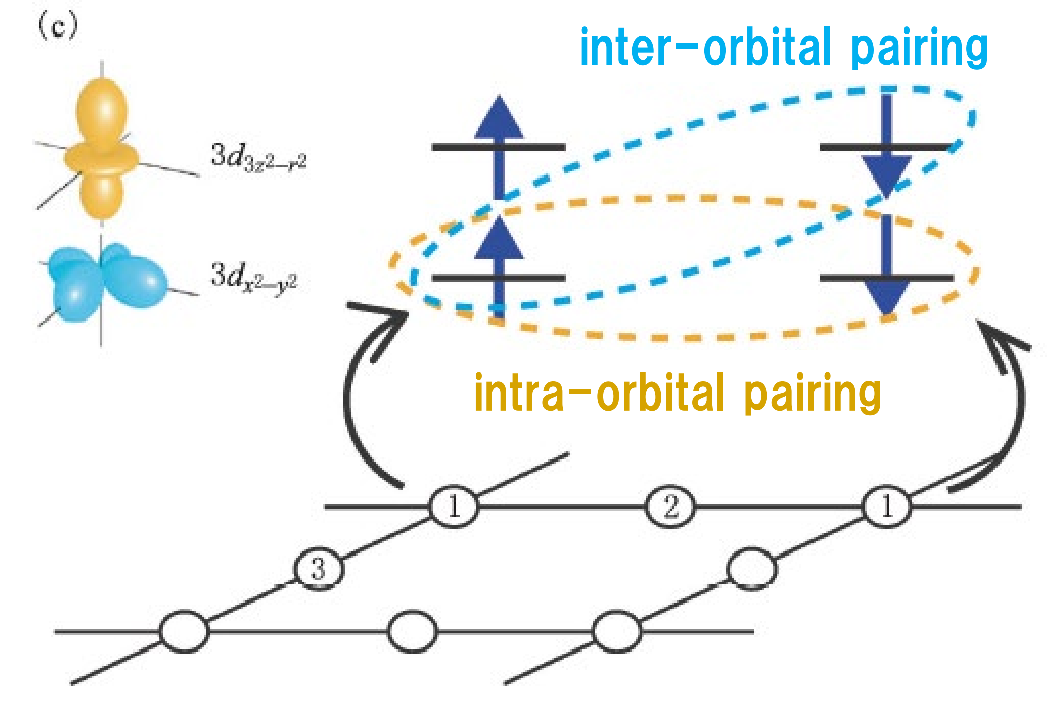


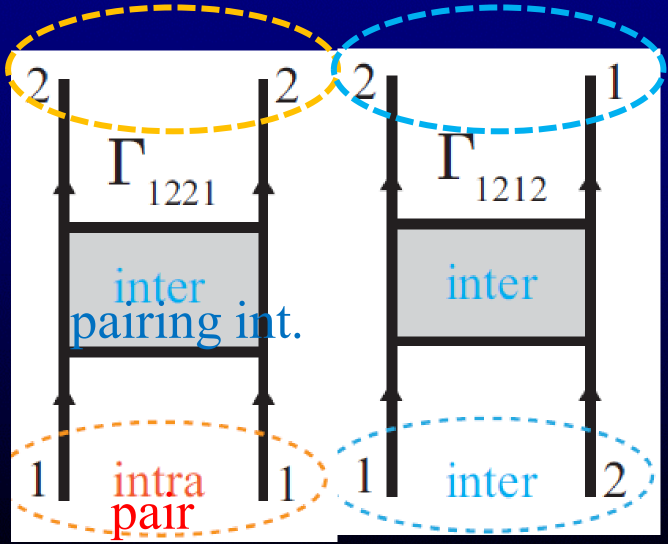


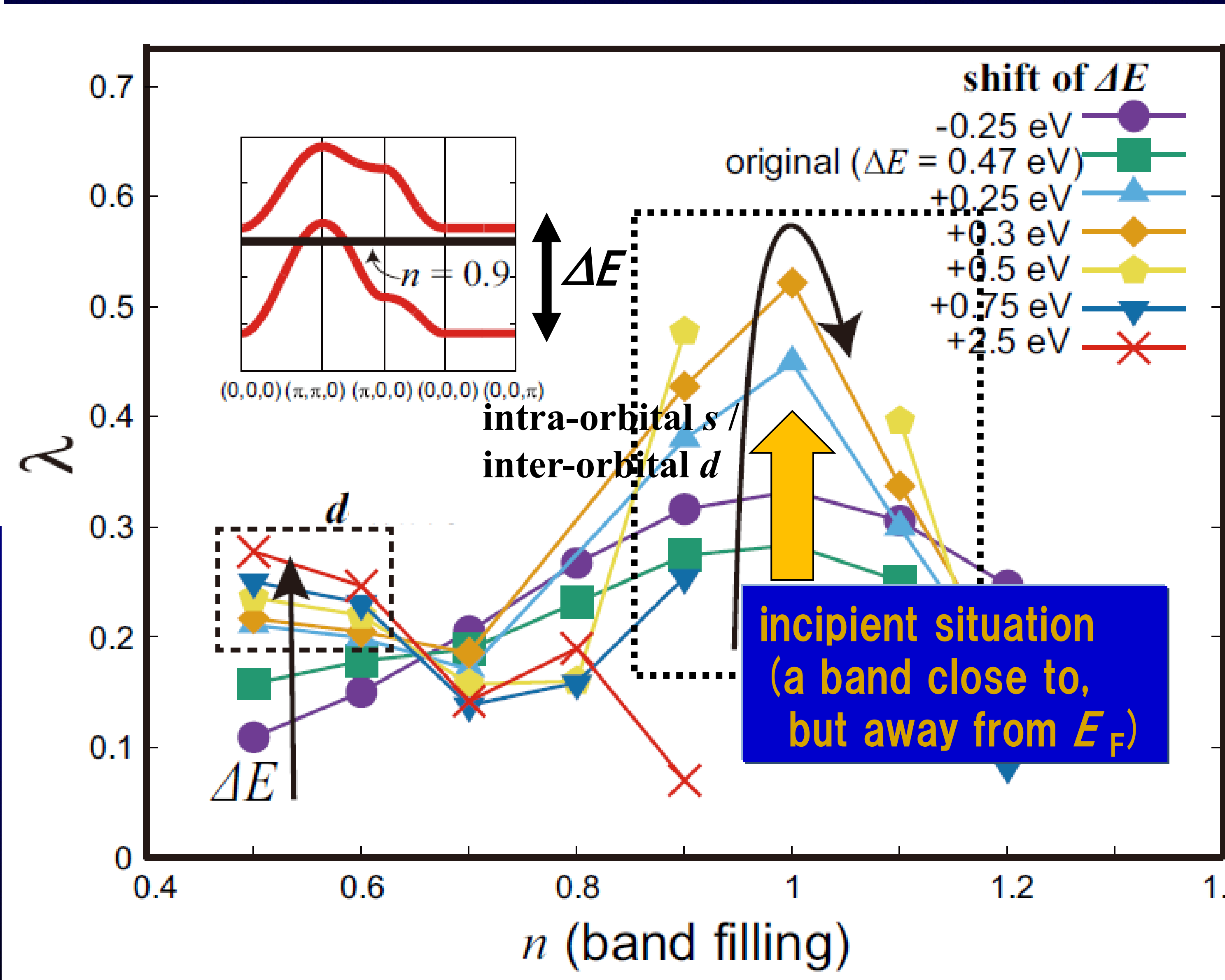


Intra- vs inter-orbital pairing

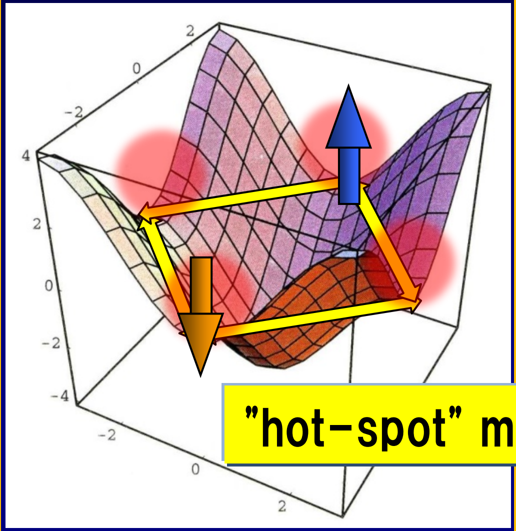
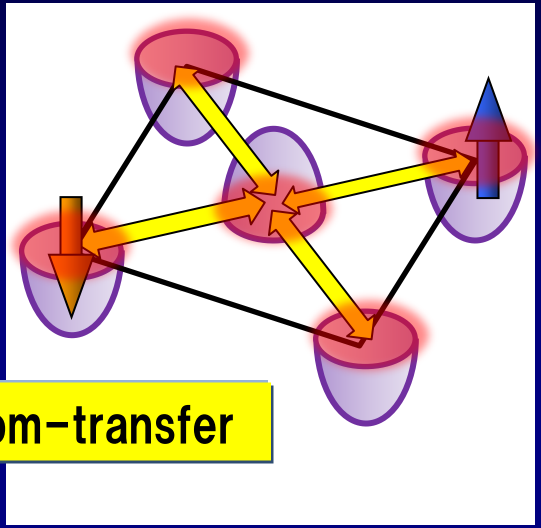
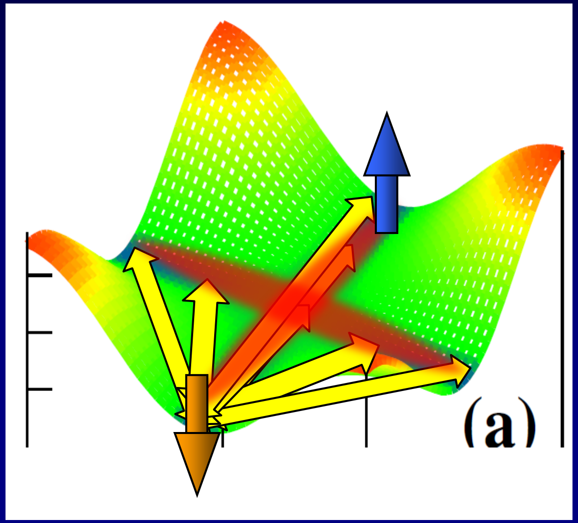
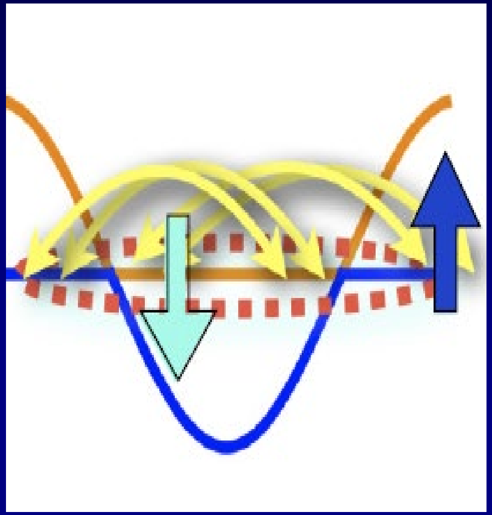
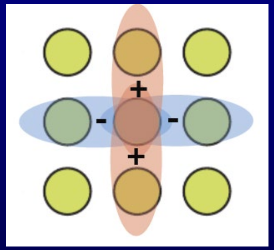
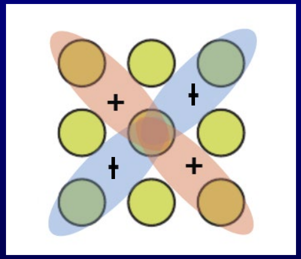
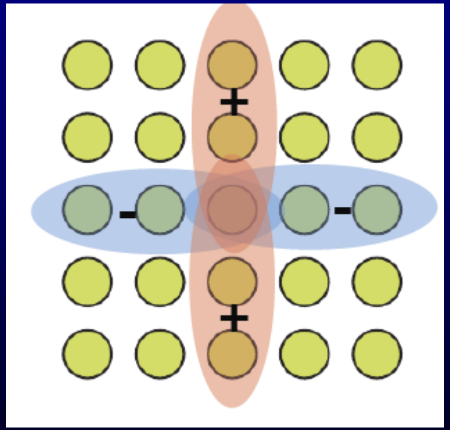
(Yamazaki et al, PRR 2020)







Pairing from repulsion in k- and real-spaces

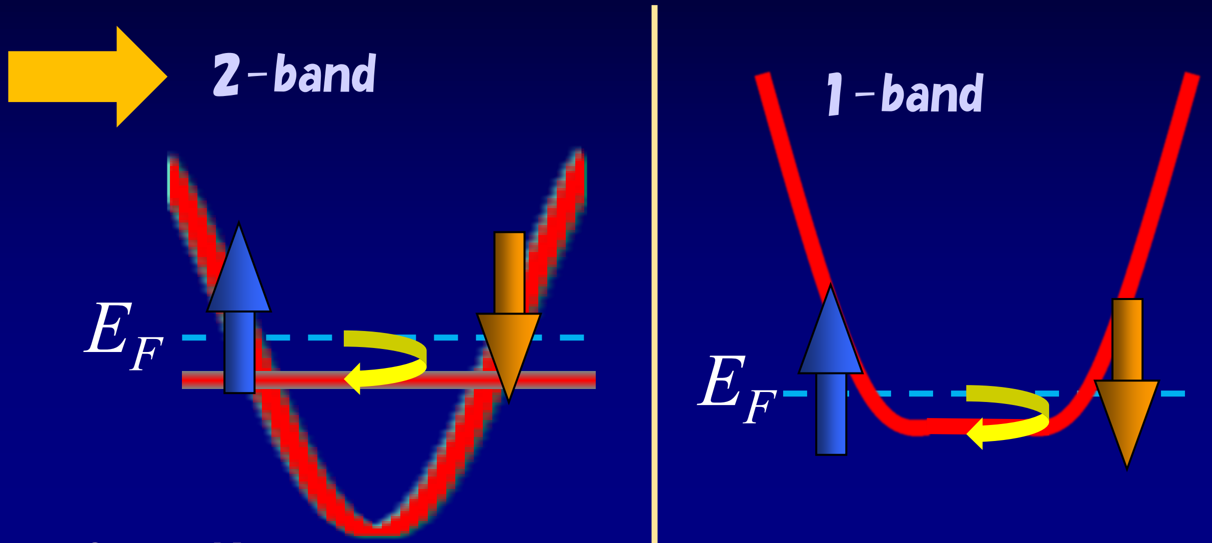
	single-orbital, one-band	multi-o, multi-b	flat-band	
			single-o, one-b	single-o, multi-b
k - space				
real space				
	d (as in cuprates)	s+- (as in iron-based)		

"hot-spot" mom-transfer

(Hosono & Kuroki, Physica C 2015)

(Sayyad et al, PRB 2020)

Flat-band SC

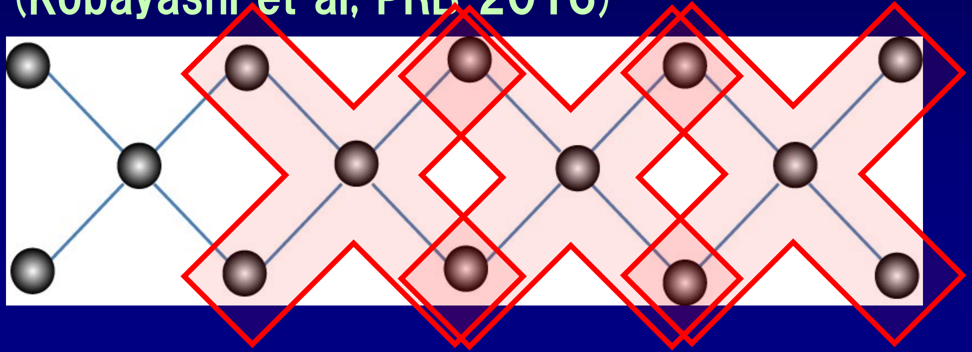


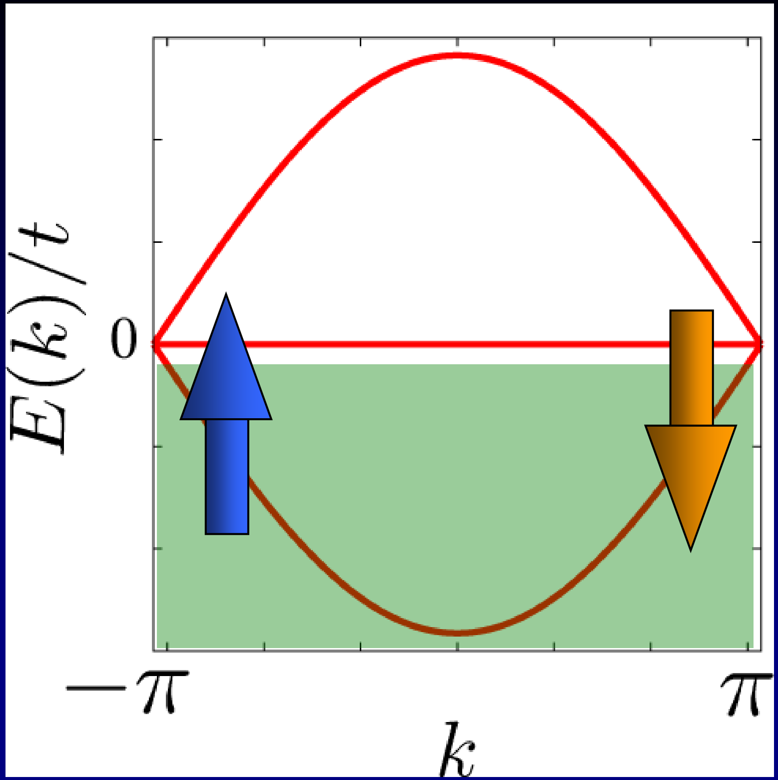
* Attractive model ← Suhl-Kondo mechanism for dispersive bands,
  but here we are talking about repulsion (spin-fl mediated pairing)

* Higher Tc when flat band is incipient
  (ie, close to, but away from, E_F)

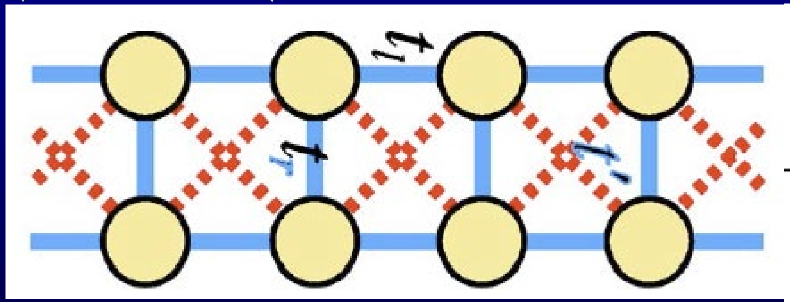
Flat-band SC in

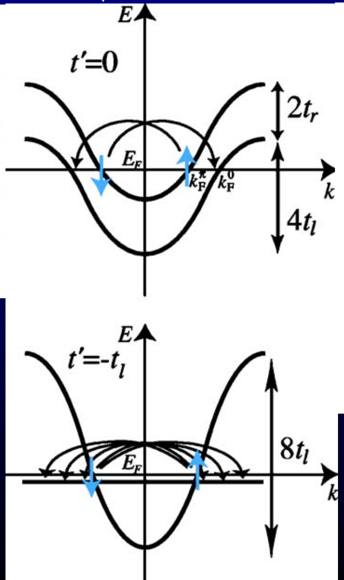
repulsive Hubbard model
on diamond chain
(a simplest possible 1D flat-b)
(Kobayashi et al, PRB 2016)

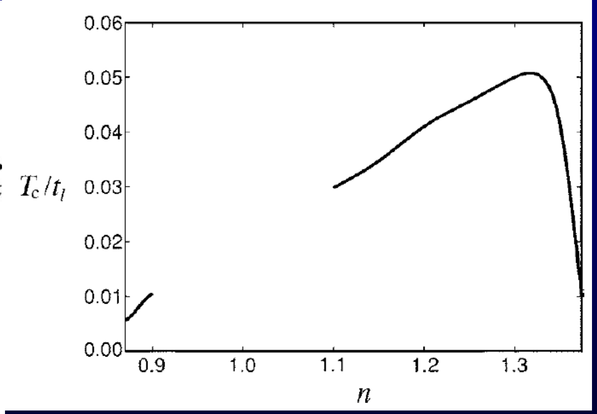




"Narrow-wide band system"
(Kuroki et al, PRB 2005; Matsumoto et al, PRB 2018)

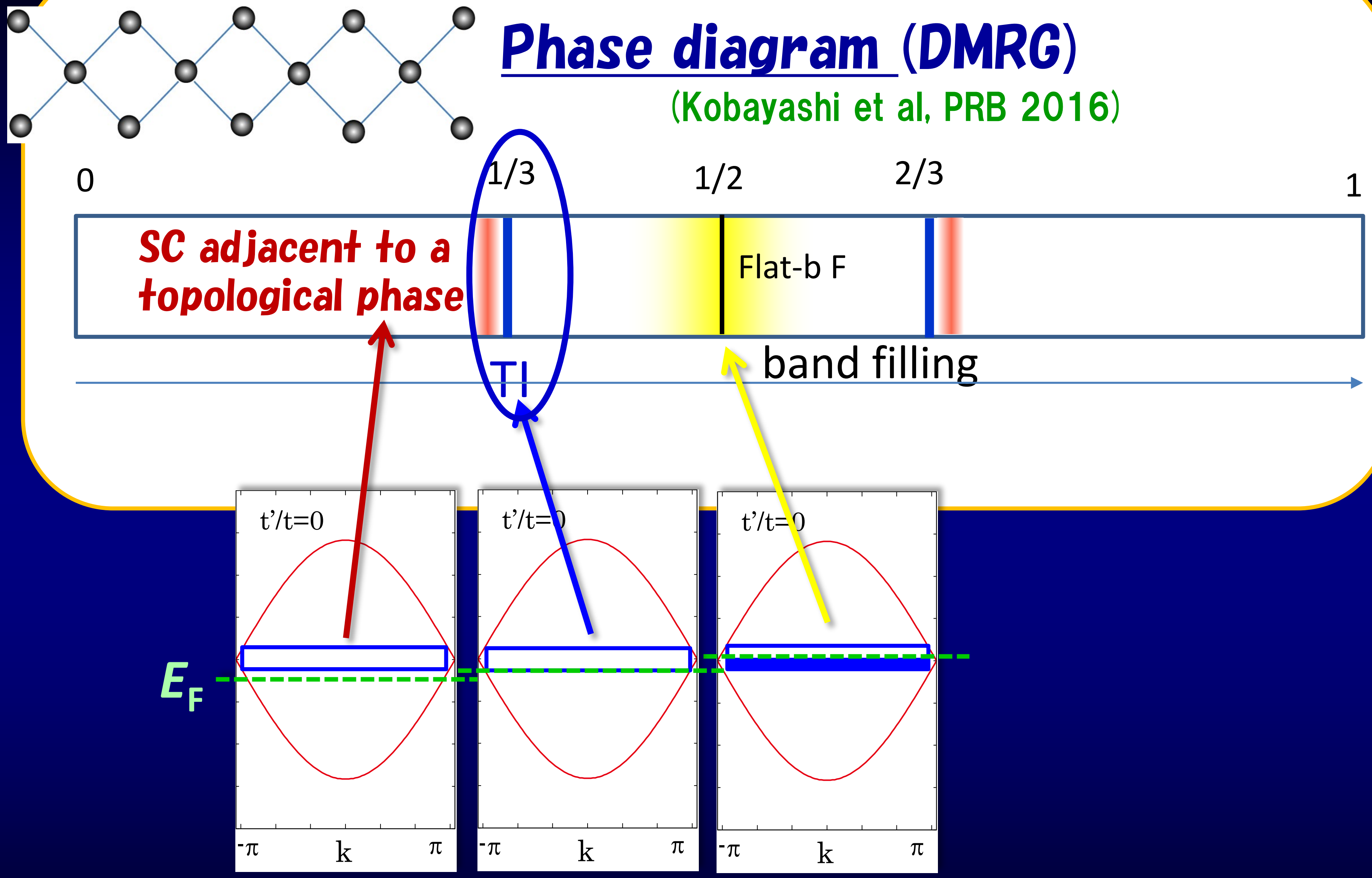






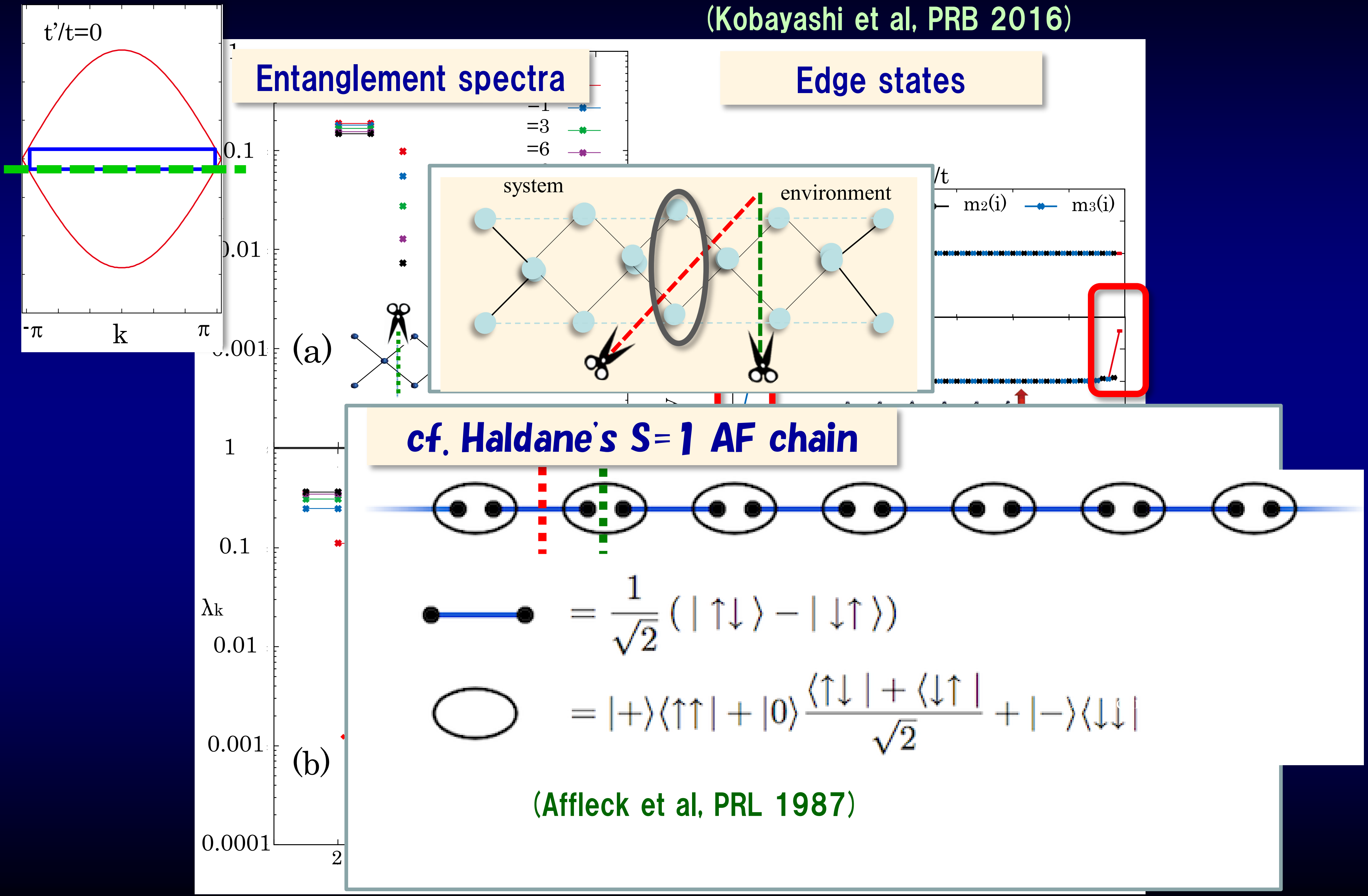
Phase diagram (DMRG)

(Kobayashi et al, PRB 2016)



Topological when exactly 1/3 filled (ED result)

(Kobayashi et al, PRB 2016)



Topological flat band → can favour SC (Törmä on Wed)

nature COMMUNICATIONS

Received 17 Sep 2015 | Accepted 19 Oct 2015 | Published 20 Nov 2015

DOI: 10.1038/ncomms9944 OPEN

Superfluidity in topologically nontrivial flat bands

Sebastiano Peotta[1] & Päivi Törmä[1,2]

$$D_s \geq (U/h^2)|C|$$

↑ Attractive Hubbard

Superfluid weight "topologically lower-bounded" in the mean field

D_s : superfluid weight ($\sigma_1(\omega) = D_s\,\delta(\omega) + \ldots$)

C : Chern # of the flat band

(see also Kopnin et al, PRB 2011; Tovmasyan et al, RPB 2016; Julku et al, PRL 2016; Liang et al, PRB 2017)

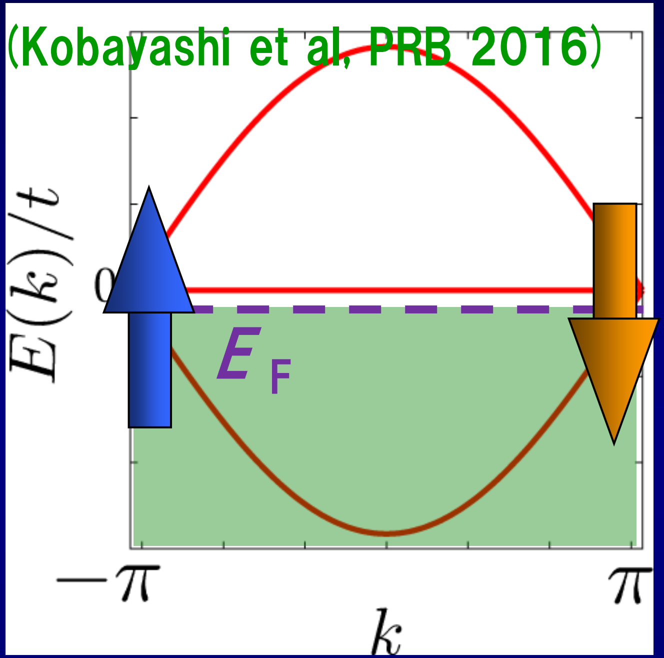
*Result beyond mean-field:

Attractive Creutz lattice with DMRG + ED (Mondaini et al, PRB 2018);

Attractive Lieb lattice (Julk et al, PRL 2016; Huhtinen et al, PRB 2021)

→ Repulsive Hubbard model ? --- an interesting question

Repulsion-induced SC in incipient flat band

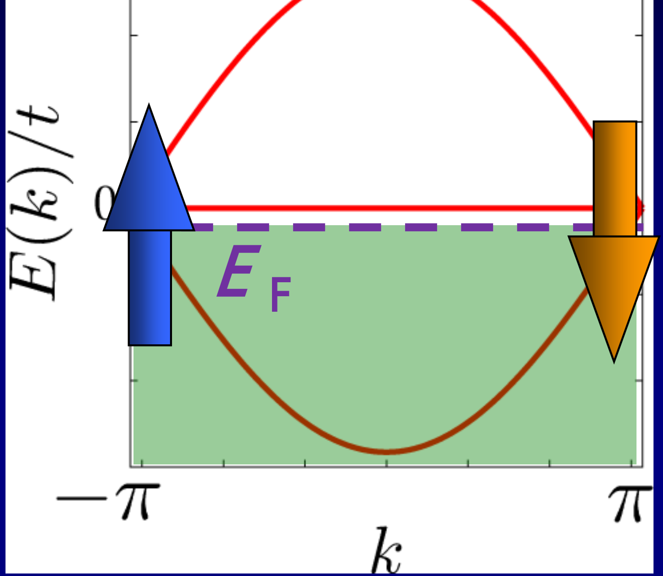


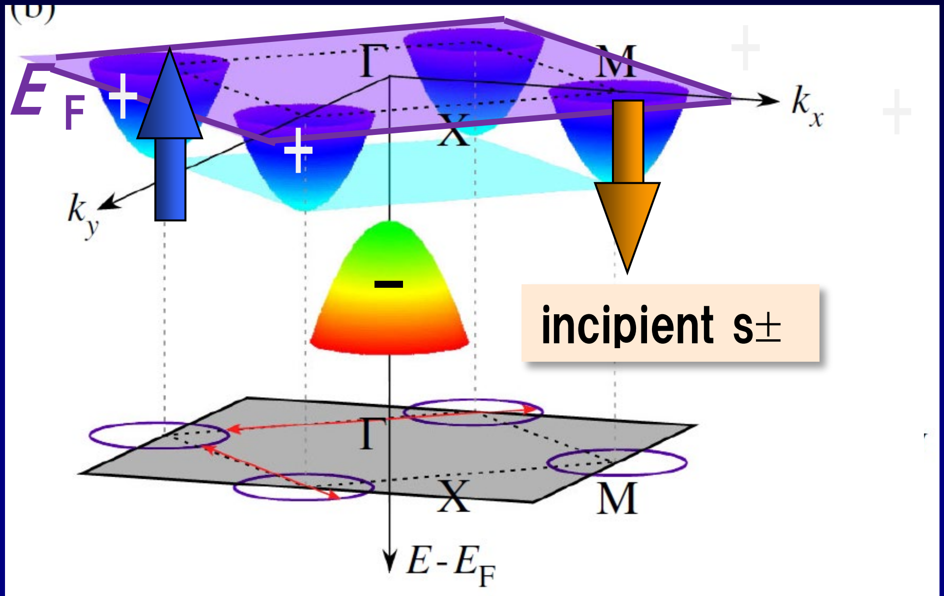
Incipient flat band

Incipient dispersive band

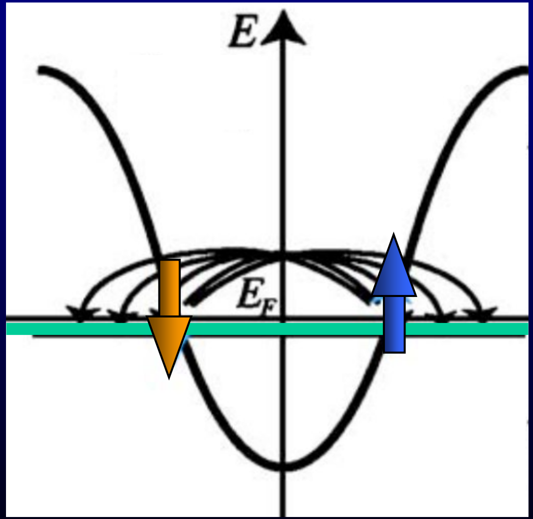
as in FeSe (Qian et al, PRL 2011; …)

(Kobayashi et al, PRB 2016)



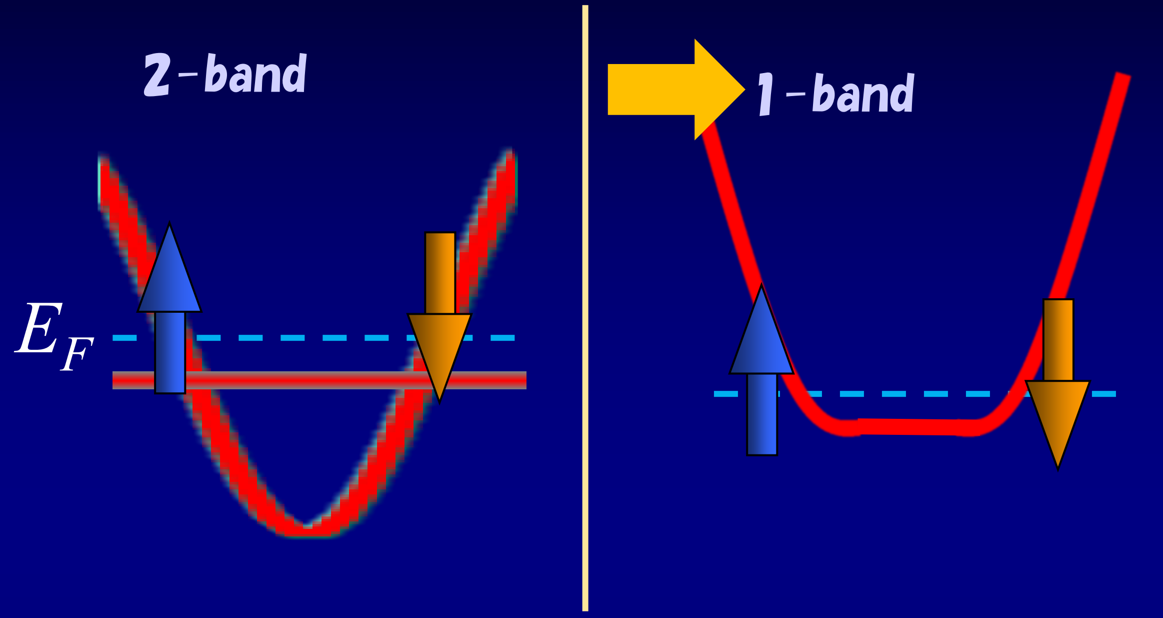


Originally, the concept of "incipient bands" introduced by Kuroki et al's narrow/wide, PRB 2005



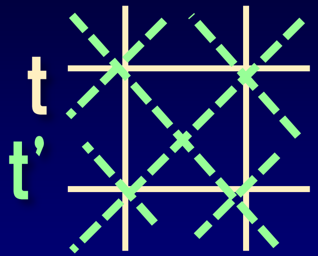
Effective pairing interaction in an incipient band
(DCA: Maier, …, Scalapino, PRB 2019)

2-band vs 1-band flat-band SC

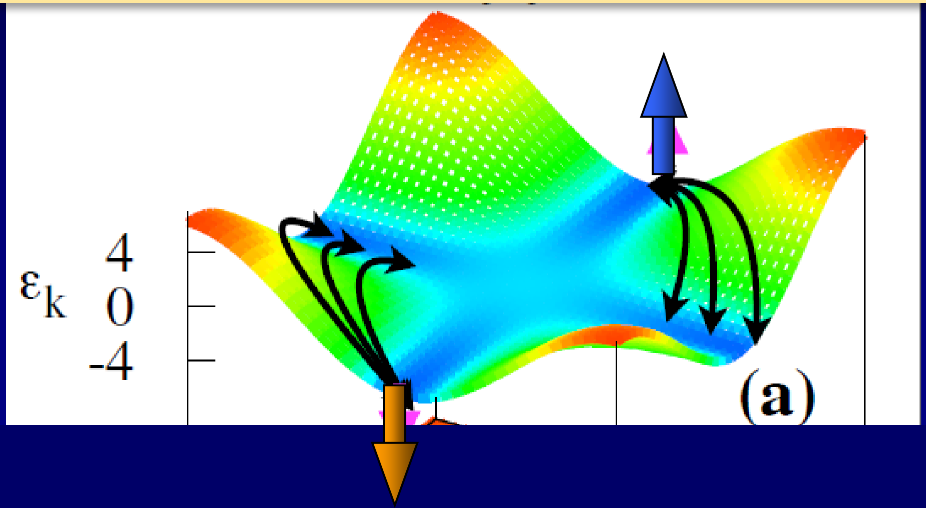


SC for repulsive U in a partially-flat band

(Sayyad et al, PRB 2020)

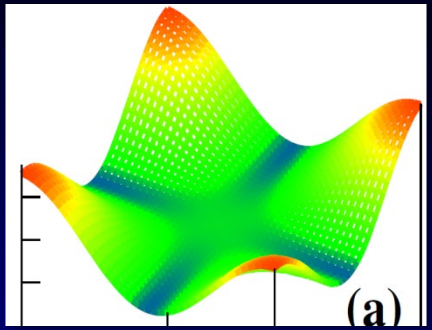


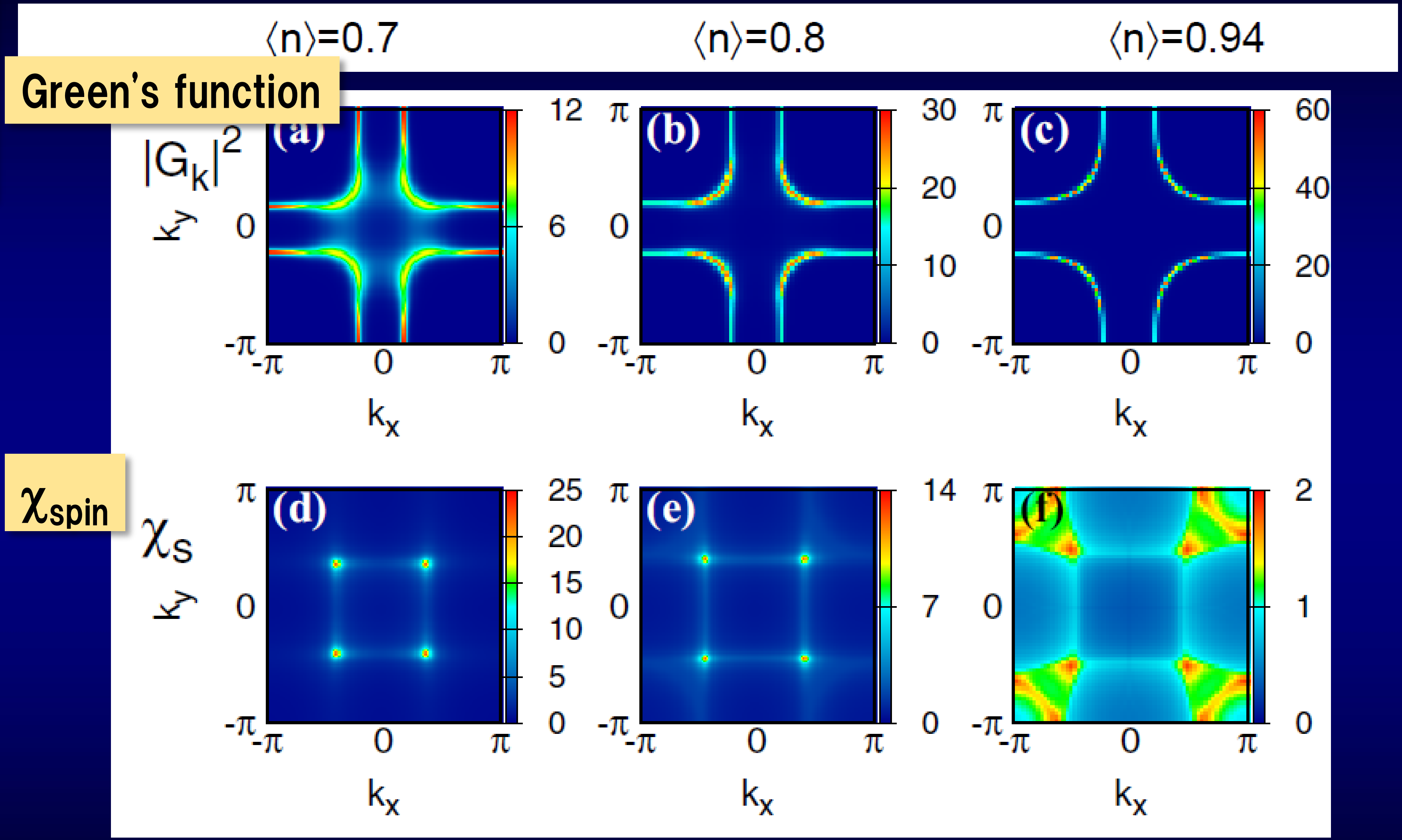
t-t' square lattice with t'~ −0.5t (a kind of frustration)



Method:
FLEX+DMFT, DQMC

Spin susceptibility χ_s

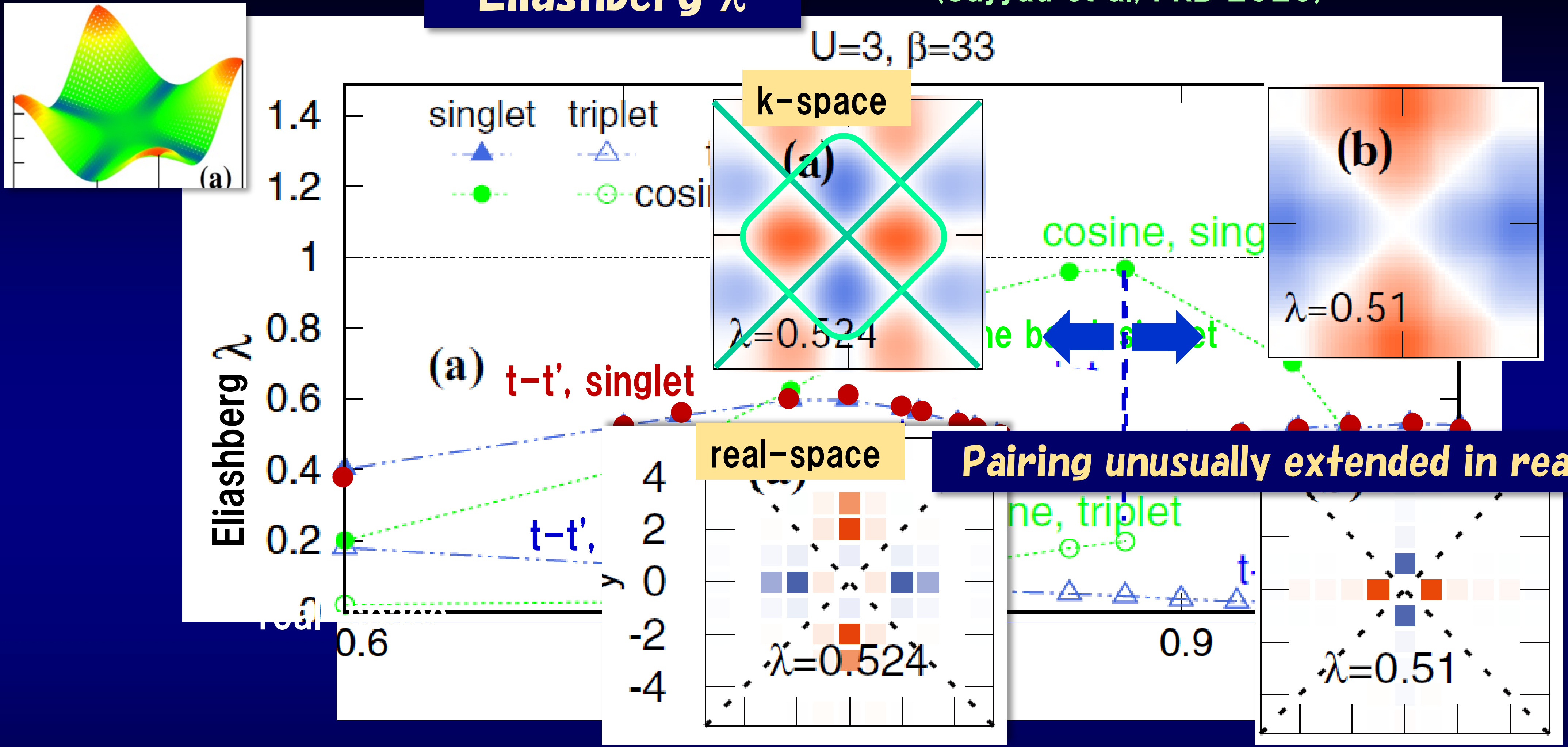




(Sayyad et al, PRB 2020)

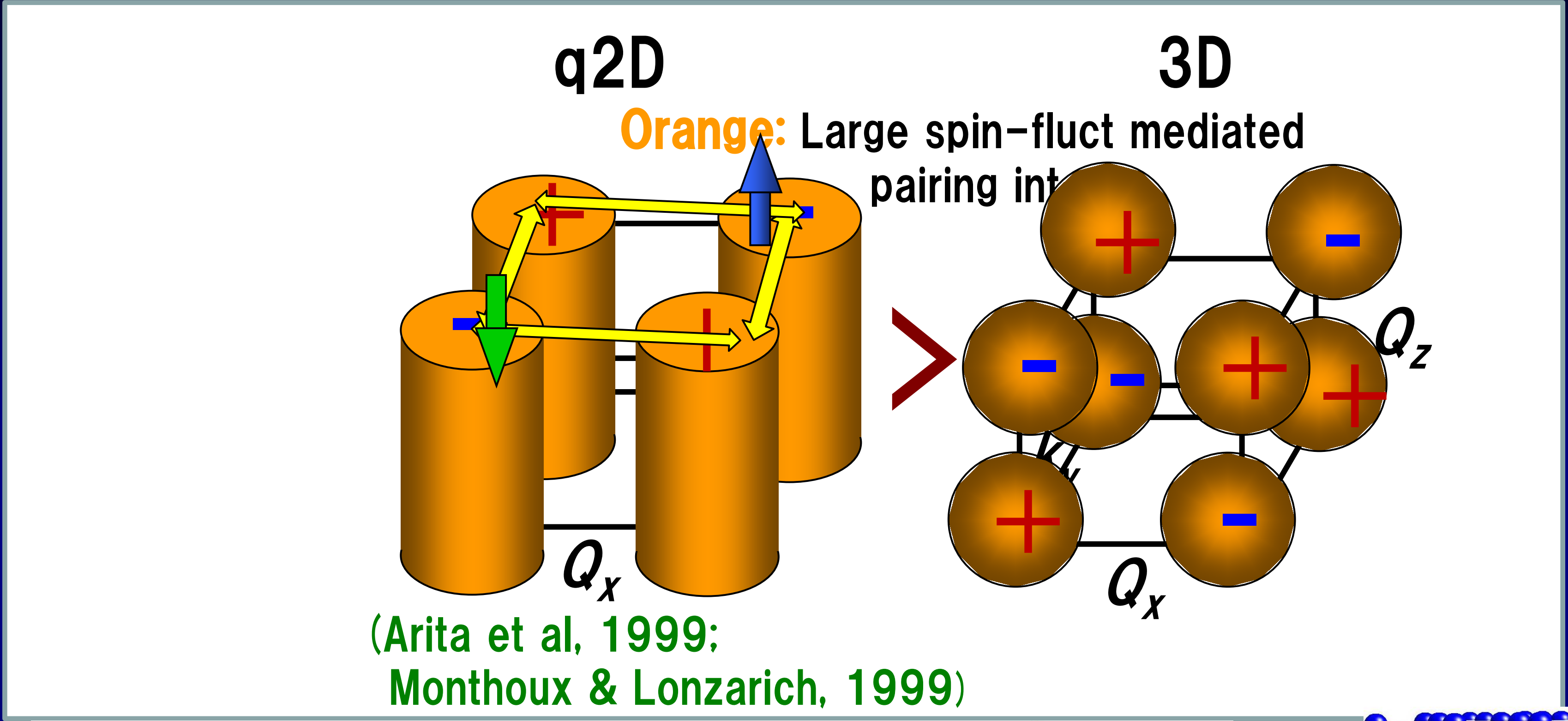
Eliashberg λ

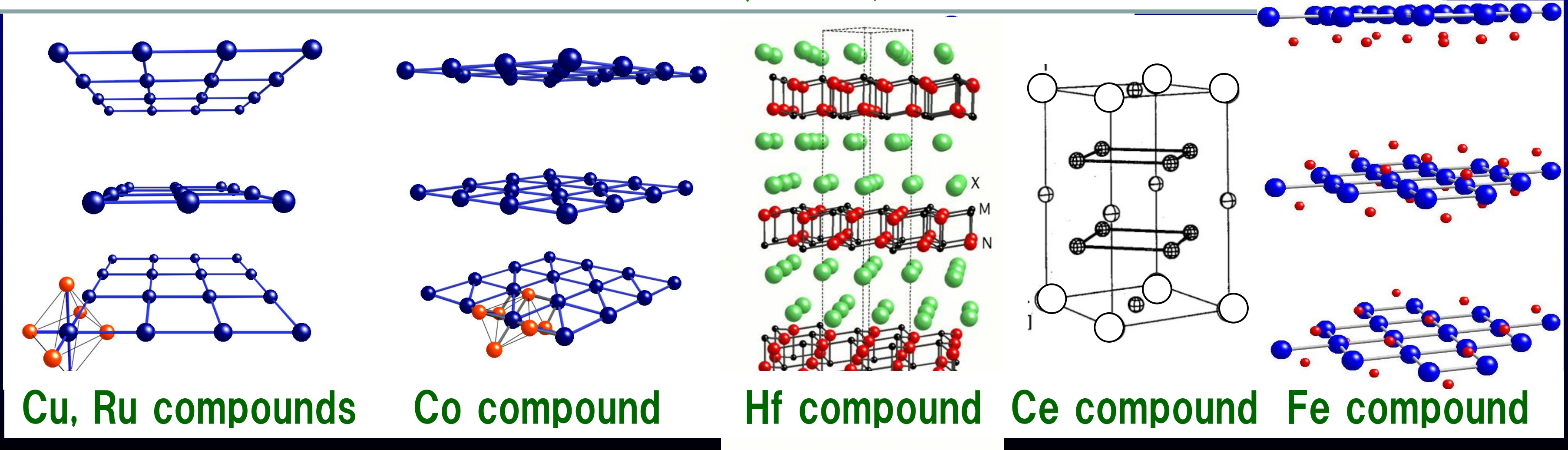
(Sayyad et al, PRB 2020)



Pairing unusually extended in real space

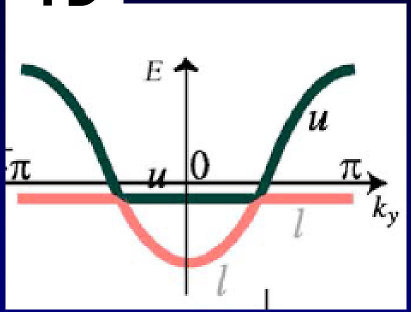
Usually, SC from el-el repulsion works much better in 2D than in 3D

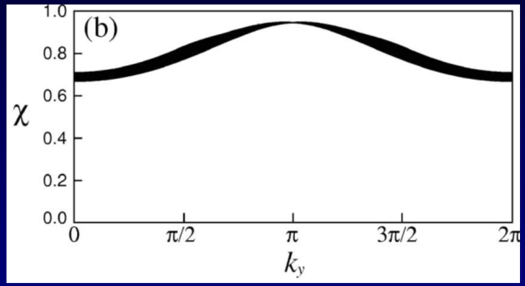




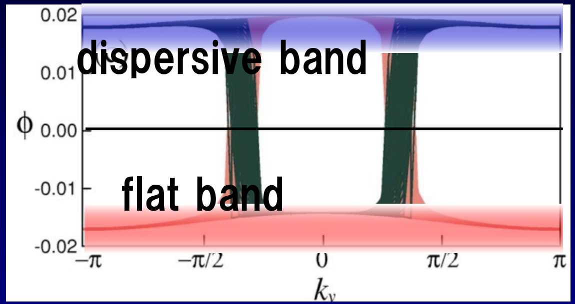
Flat-band SC has totally different dim-dep

1D Narrow-wide band system (Kuroki et al, PRB 2005)



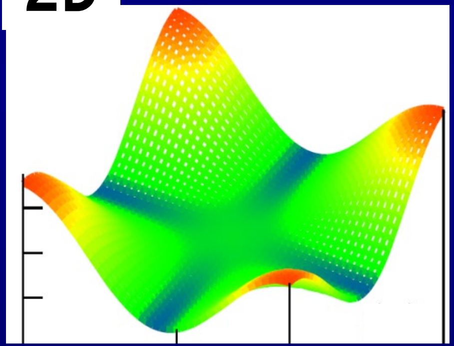


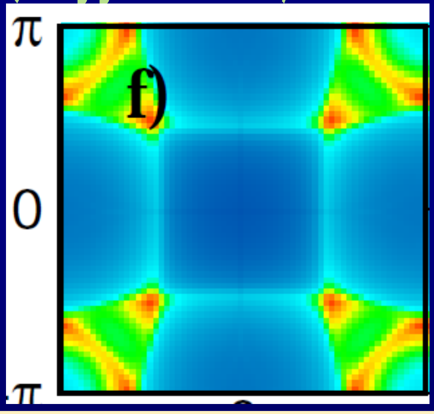
spin susc featureless

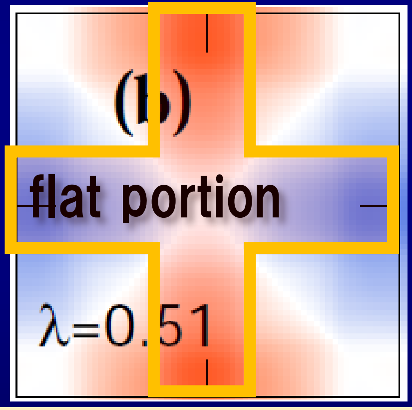


s± pairing

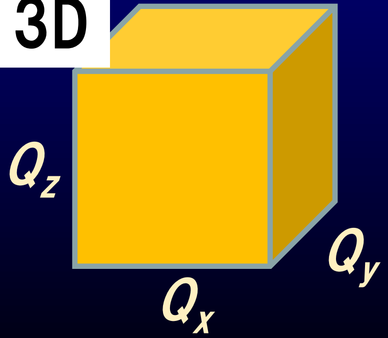
2D Partially-flat band (Sayyad et al, PRB 2020)







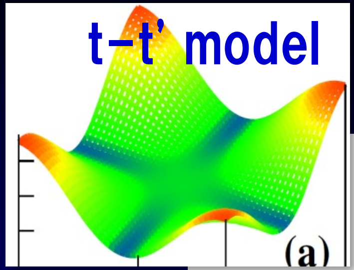
3D

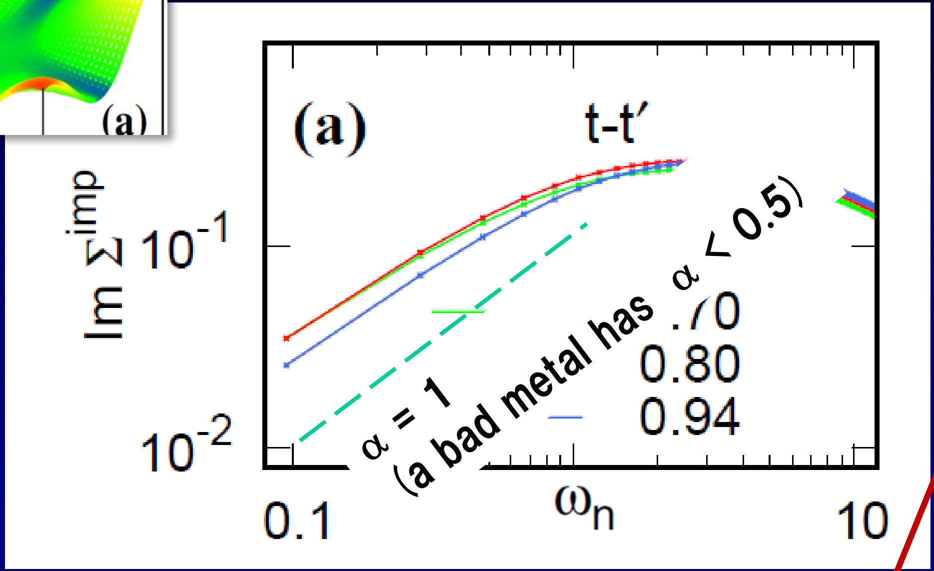


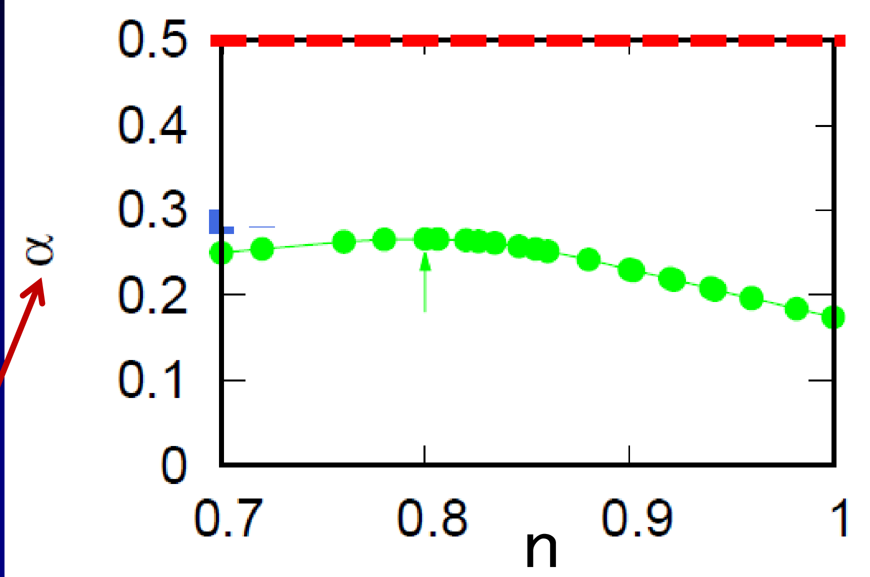
Specialty of the flat-band SC (beyond the nesting physics):
→ 3D as good as 2D
(outside Arita & Lonzarich's theorem)

Flat-band Hubbard → Very non-Fermi liquid !

(Sayyad et al, PRB 2020)







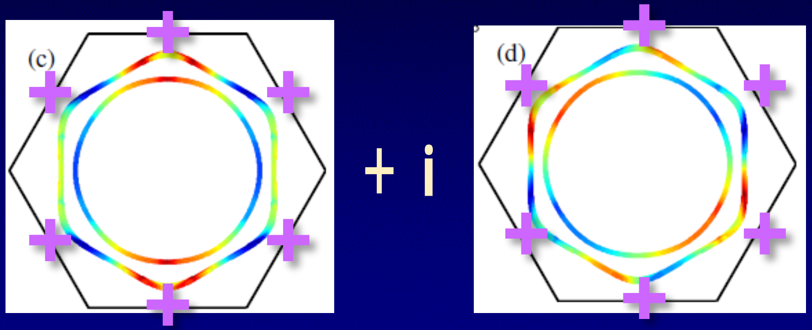
$$|\mathrm{Im}\Sigma_{\mathrm{loc}}(i\omega_n)| \propto \omega_n^{\alpha}$$

Fermi liquids:
Im Σ (ω) ~ ω^2 (real axis)
→ Im Σ (iω) ~ iω (Matsubara axis)
(Werner et al , PRL 2008;
PRB 2016 on a cuprate model)

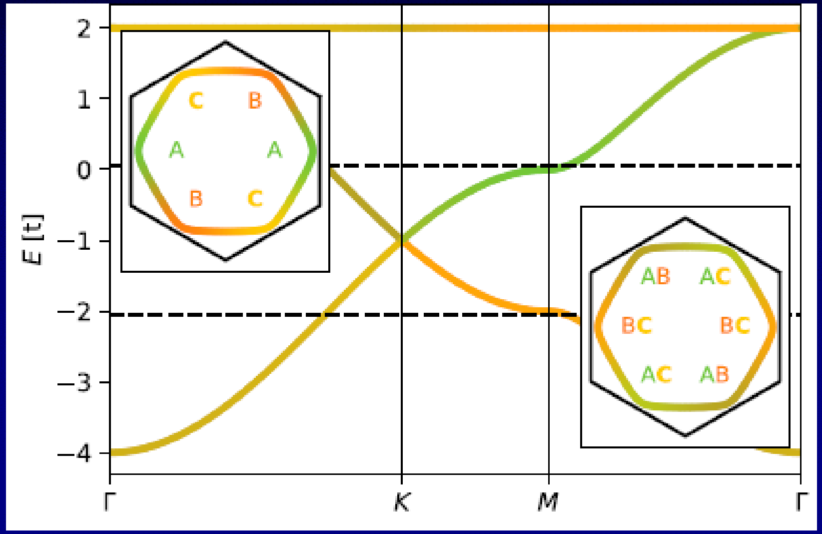
→ *the flat-band SC resides in a non-Fermi liquid regime ("non-Fermi SC")*

V_{group} = 0 at a point: van Hove sing. → unconventional SC (d+id, etc)

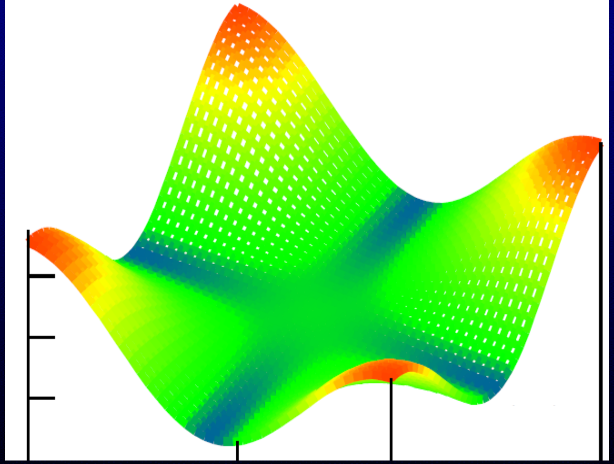
(Liu et al, PRL 2018)



(Wu et al, PRL 2021)



V_{group} = 0 in finite areas in partially-flat band



(Sayyad et al, PRB 2020)

Plan of the talk

	Superconductivity (from repulsion)	Topological
Equilibrium	✔ 1-band vs multiband* * Incipient SC * Flat-band SC * Nematic	✔ Topological states *Dispersive vs flat bands
Non-equil	✔ Non-equil induced SC ?	✔ Floquet topological insulator *Dispersive vs flat bands

My specific question: Can electronic nematicity enhance SC?

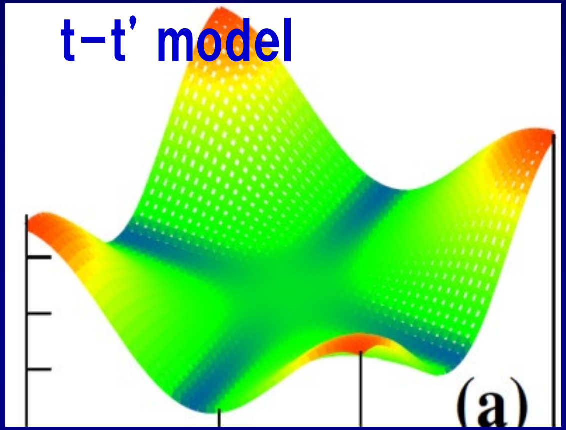
- ✔ Usually, no, or only small (2nd-order) effects
  (e.g., Kitatani, Tsuji & Aoki, PRB 2017)

- ✔ Here we deliberately consider the triangular lattice
  * frustration → enhances nematicity
  → concomitantly enhances SC
  (almost doubles Tc, a 1st-order effect)
  (Sayyad et al, arXiv:2110.14268)

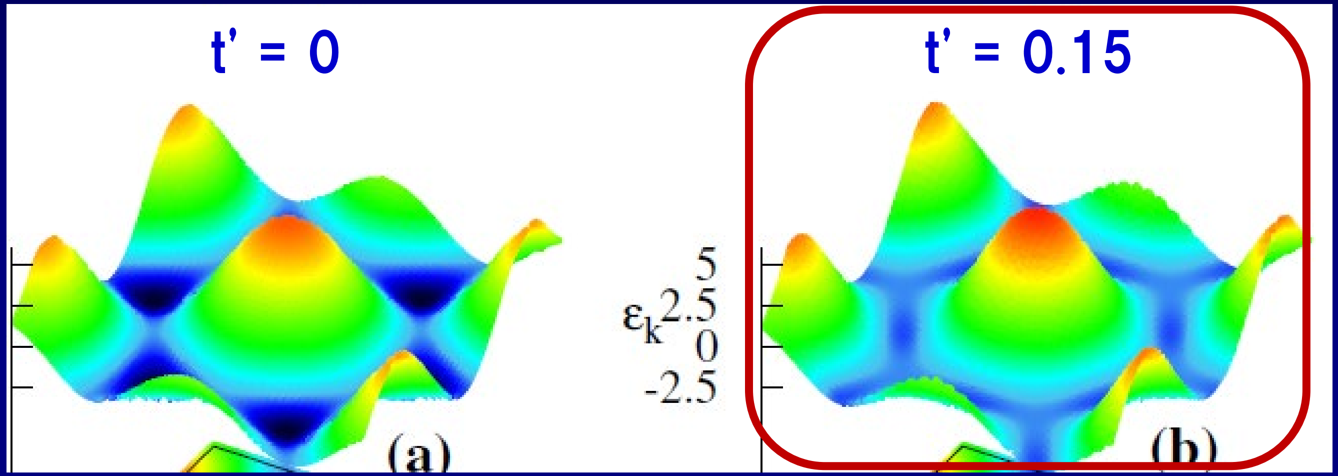
Partially-flat bands: Square → Triangular

(Sayyad et al, arXiv:2110.14268)

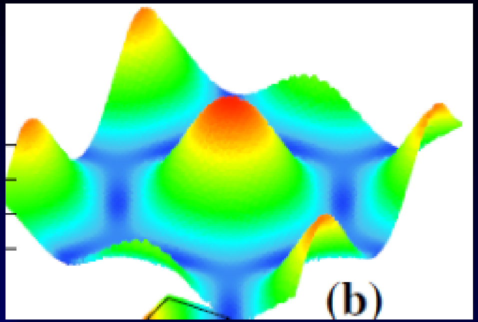
Square



Triangular



(Sayyad et al, arXiv:2110.14268)



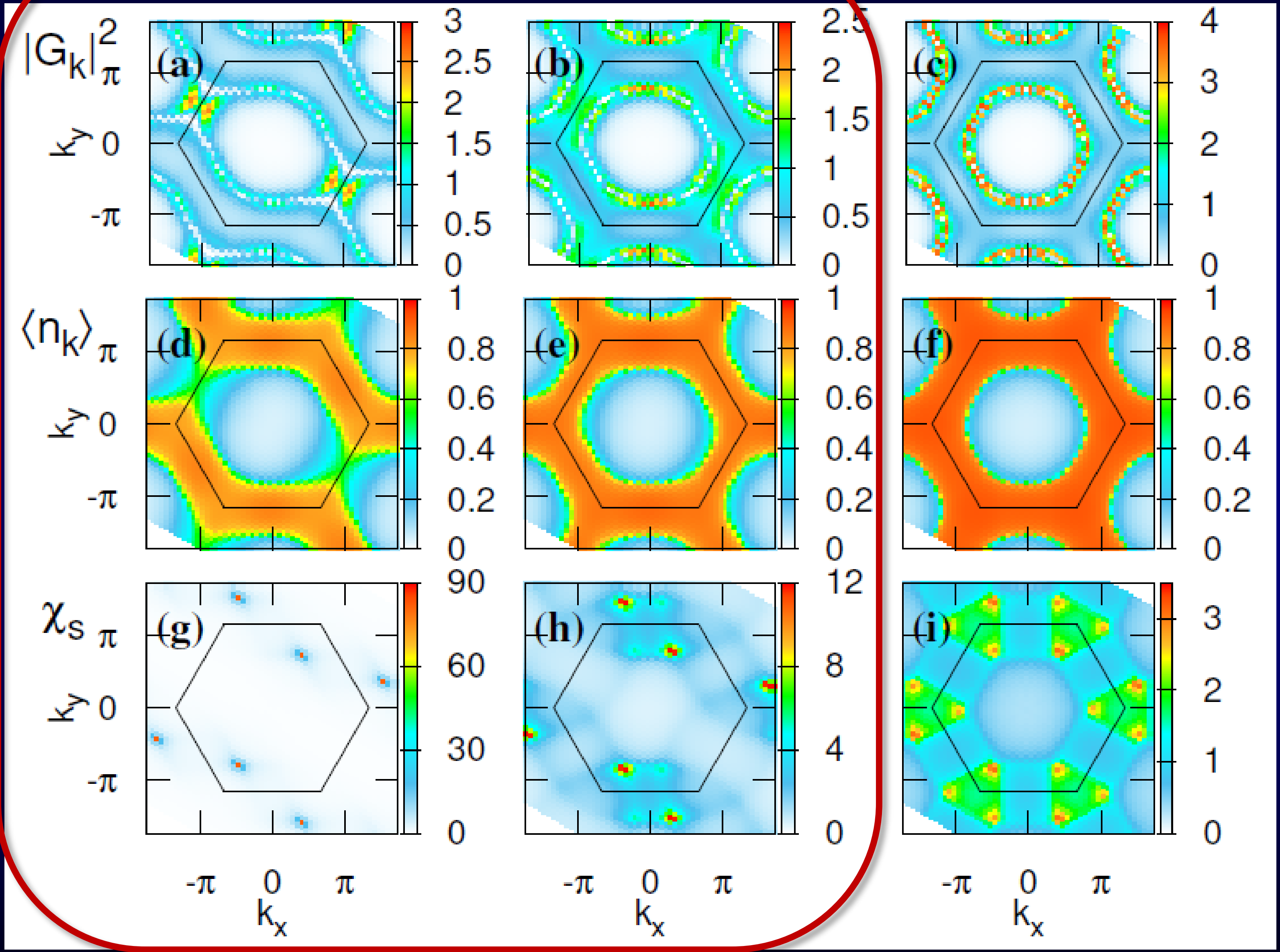
Nematic (C6 symmetry → C2)

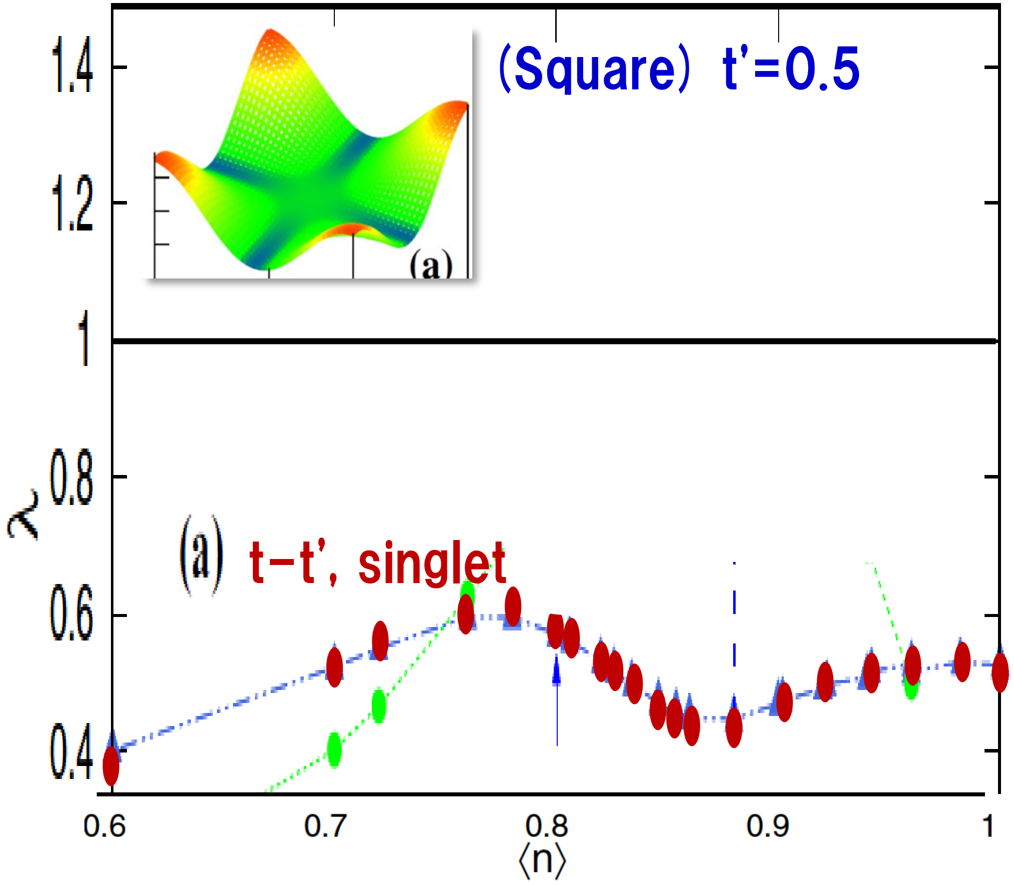
n = 0.9 n = 1.0 n = 1.1

Green's function

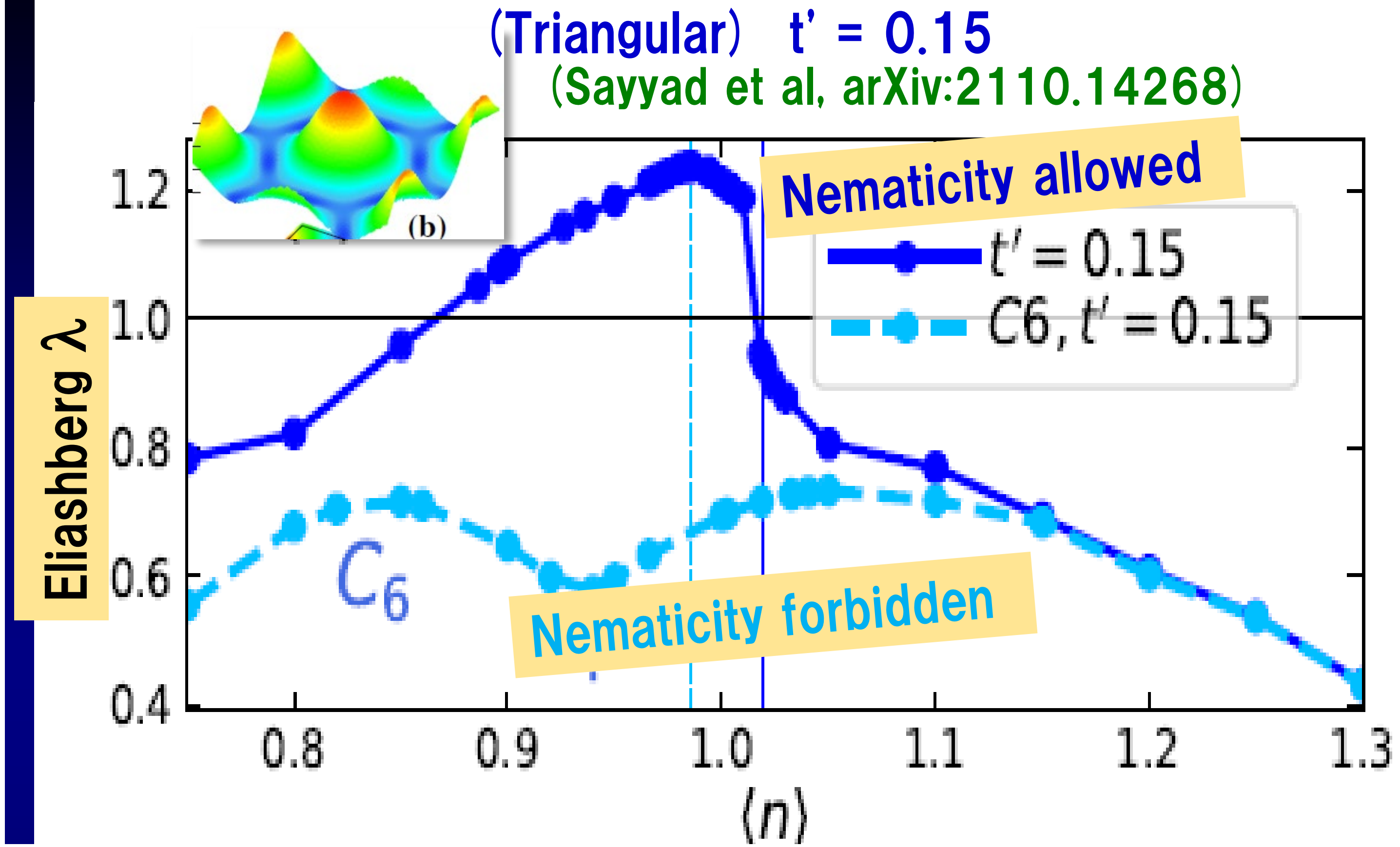
Momentum distr

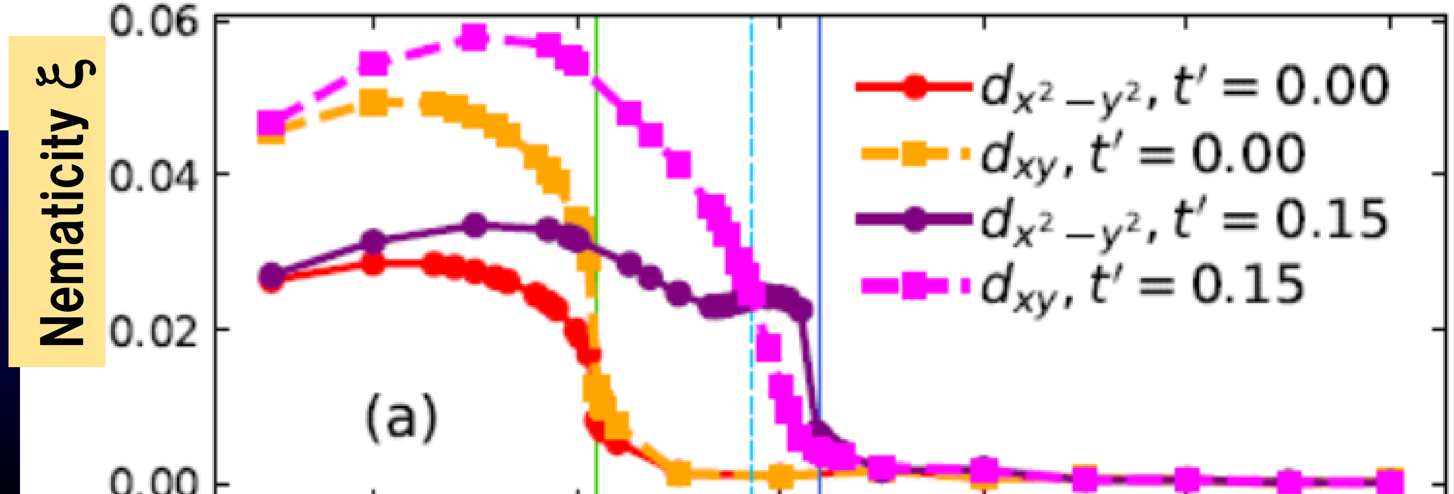
χ_{spin}

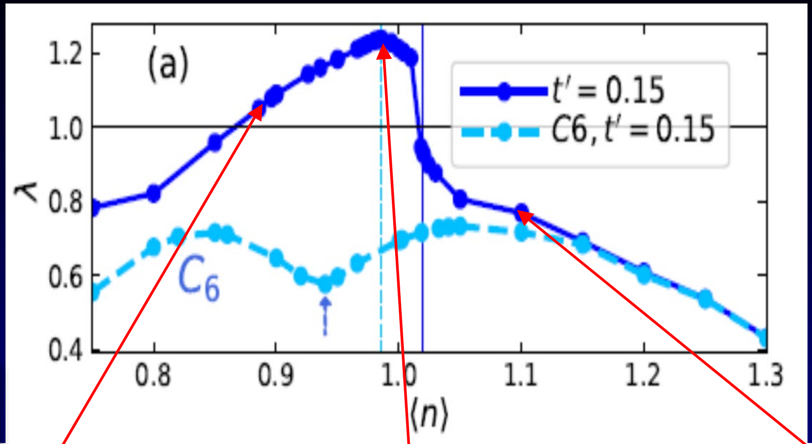




(Sayyad et al, PRB 2020)



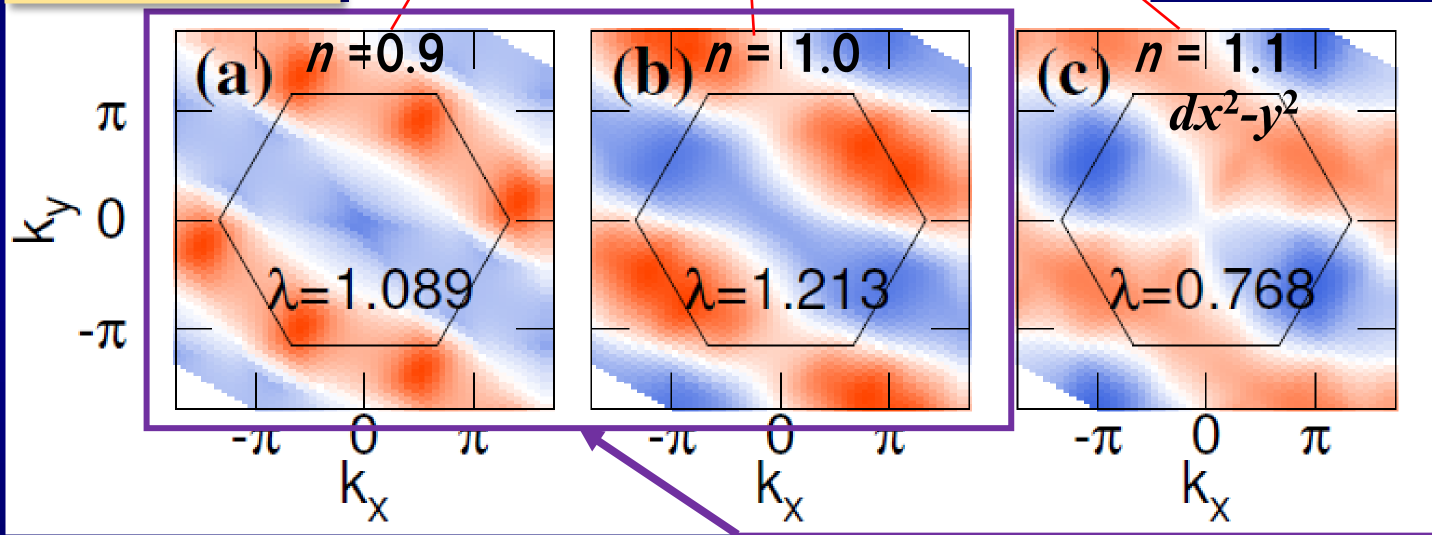


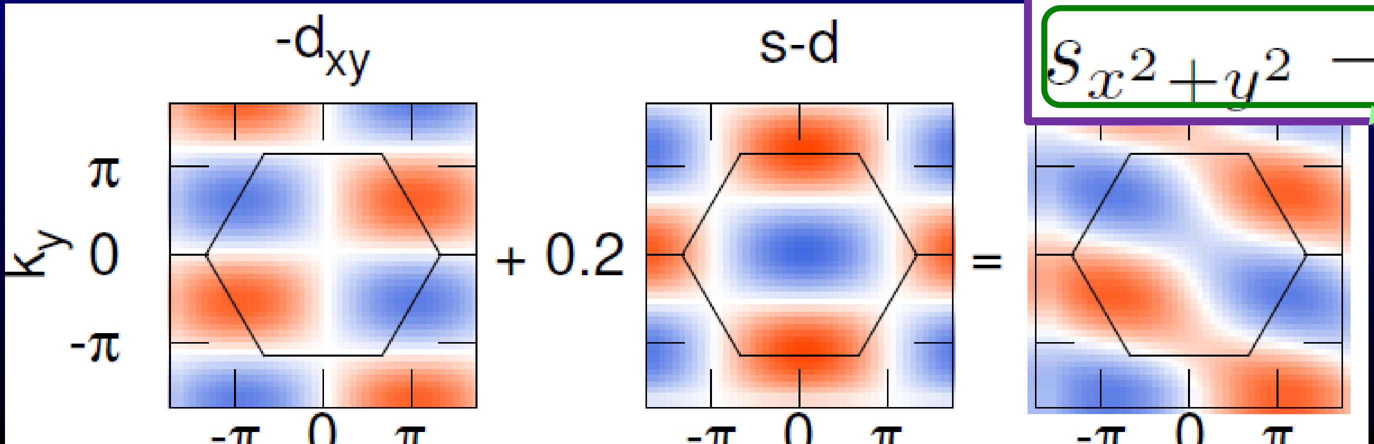


(Sayyad et al, arXiv:2110.14268)

SC systematically depends on repulsion U (intermediate), and other flat-band models also exhibit the SC

Gap function





$s_{x^2+y^2} - d_{x^2-y^2} - d_{xy}$-wave

in C_6 point-group irred rep

shorthand for $3\cos(\sqrt{3}k_y/2)\cos(k_x/2)$

(y^2 in C_2 (nematic phase) point-gr)

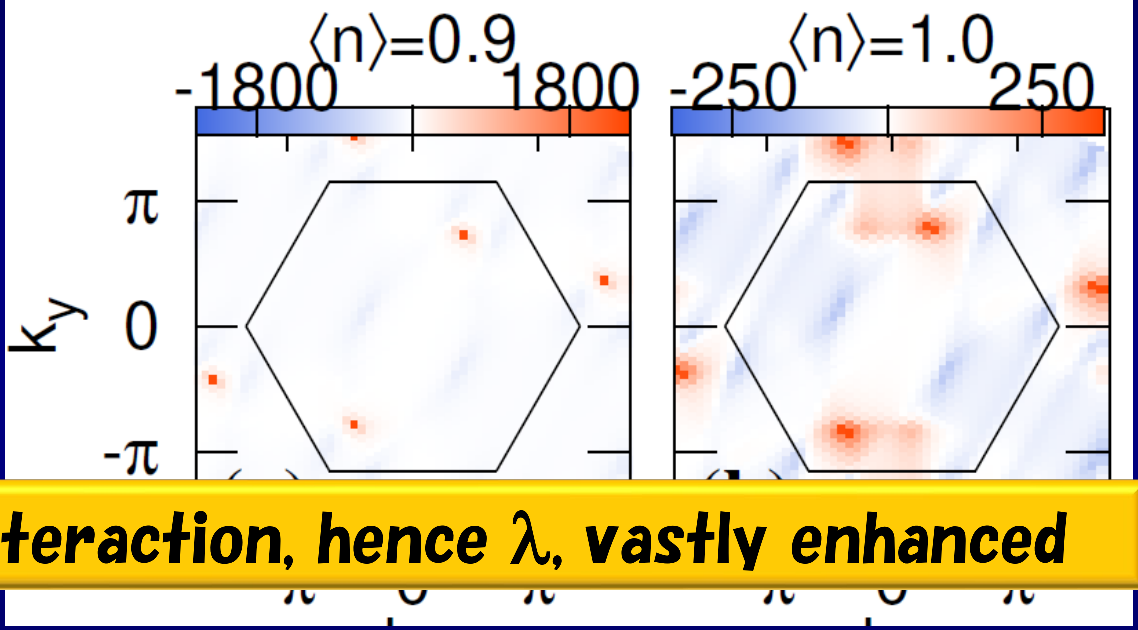
Paring interaction in the nematic phase

(Sayyad et al, arXiv:2110.14268)

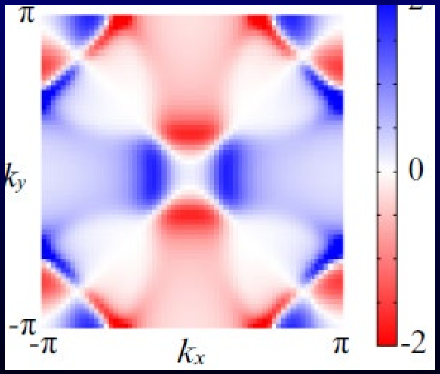
Nematicity allowed

Nematicity forbidden

$$\Delta V_{\rm eff} = V_{\rm eff} - V_{\rm eff}^{C_6}$$

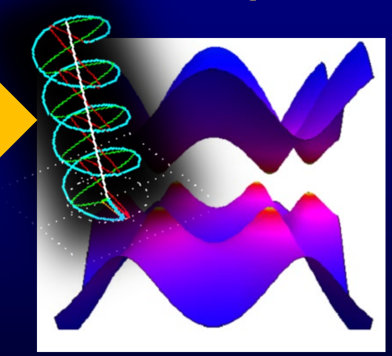


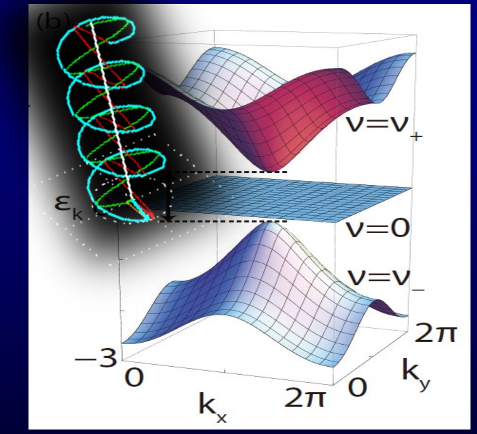
Pairing interaction, hence λ, vastly enhanced



contrasts with tetragonal lattices, where $\Delta V_{\rm eff} \sim$ d-wave symmetry →
1st-order correction to Eliashberg λ identically vanishes
(Kitatani et al, PRB 2017)

	Superconductivity	Topological
Equilibrium	✔ Flat-band SC ✔ Non-Fermi liquid	Flat-band topological states
Non-equil		✔ Floquet topological insulator

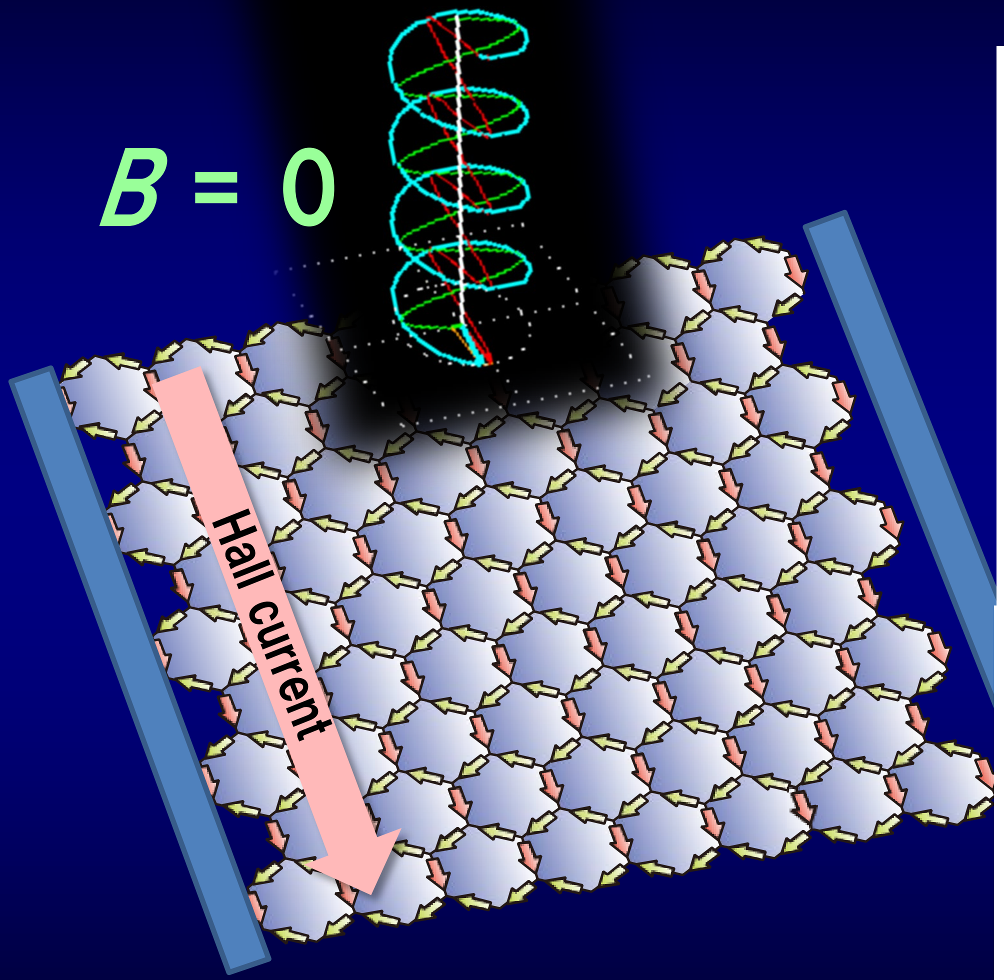


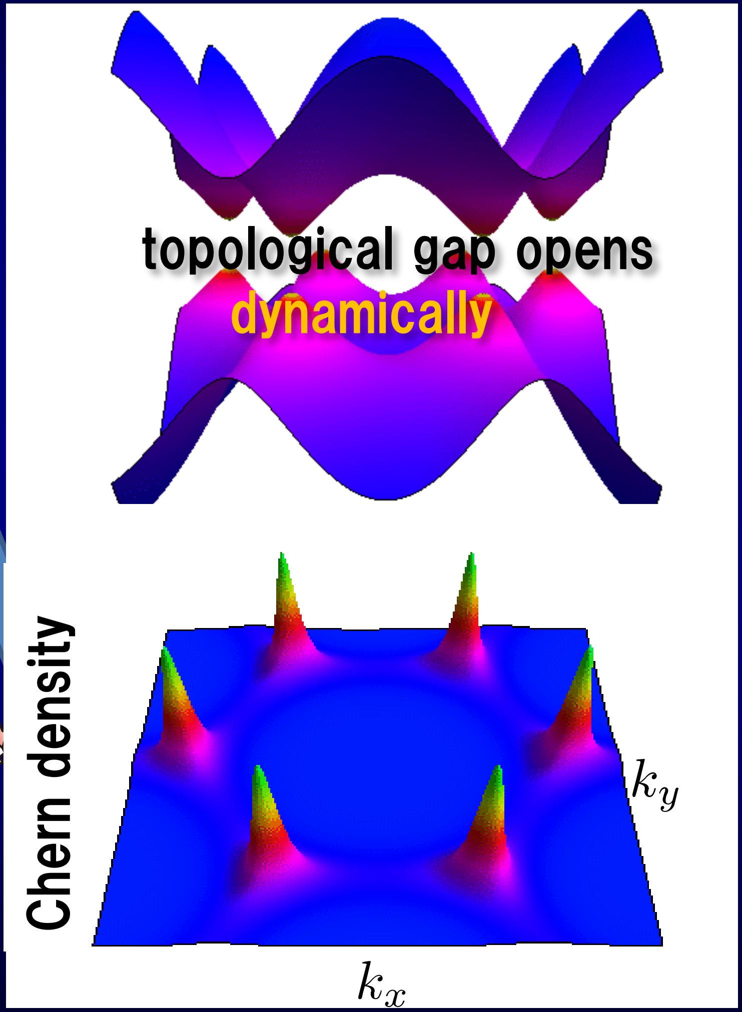


ordinary bands (e.g. graphene) → Flat bands

Floquet topological insulator

(Oka & Aoki, PRB 2009)

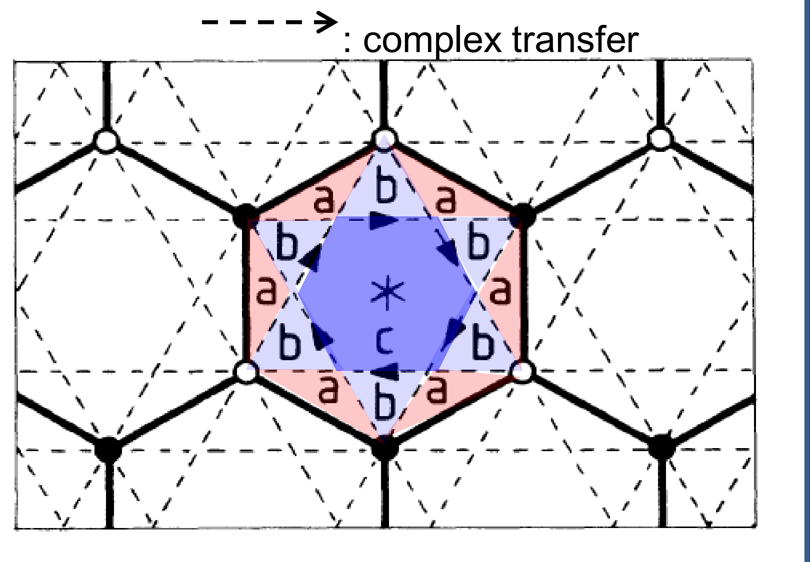




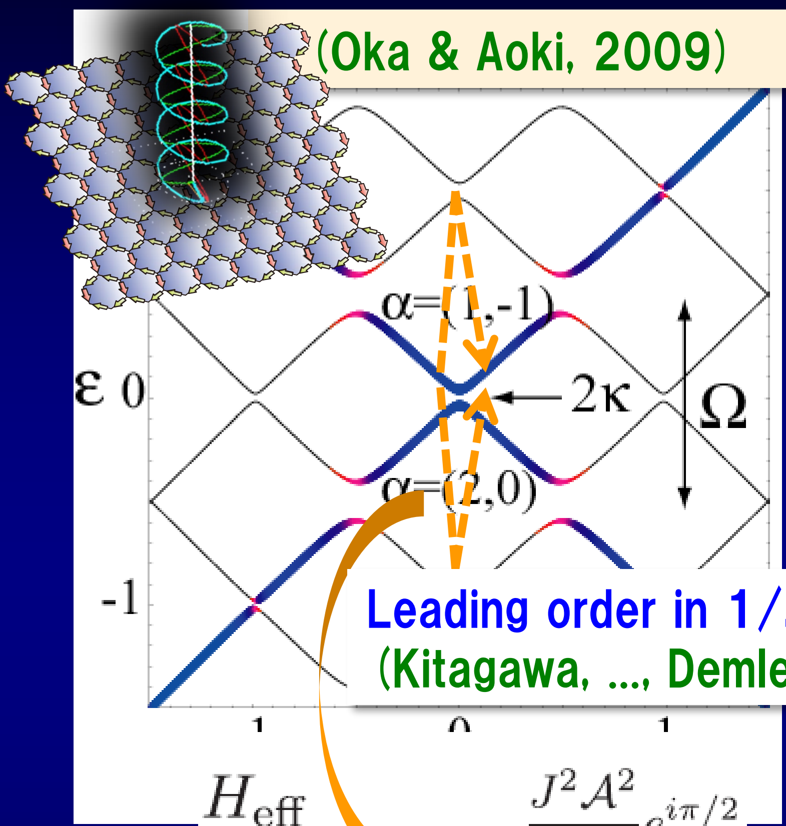
(McIver et al, 2020, Hamburg)

Quantum anomalous Hall effect (QHE in B=0)

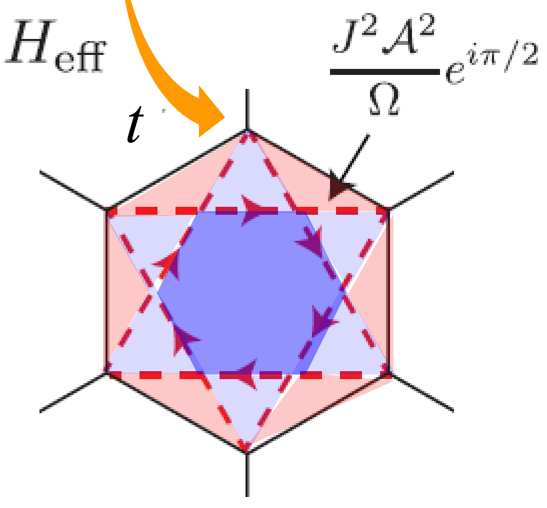
First conceived by Haldane (PRL 1988)



(Oka & Aoki, 2009)



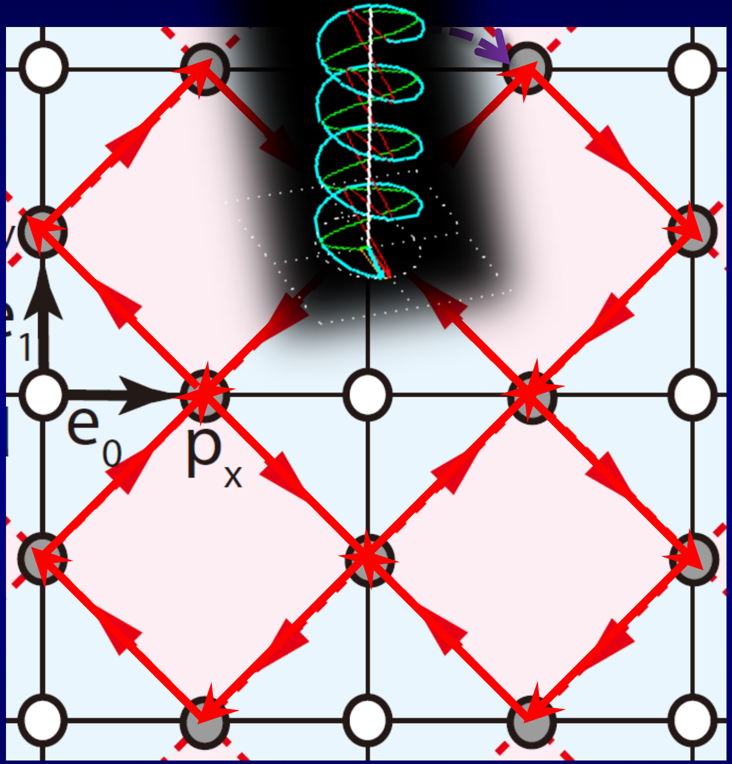
Leading order in $1/\Omega$
(Kitagawa, ..., Demler, PRB 2011)

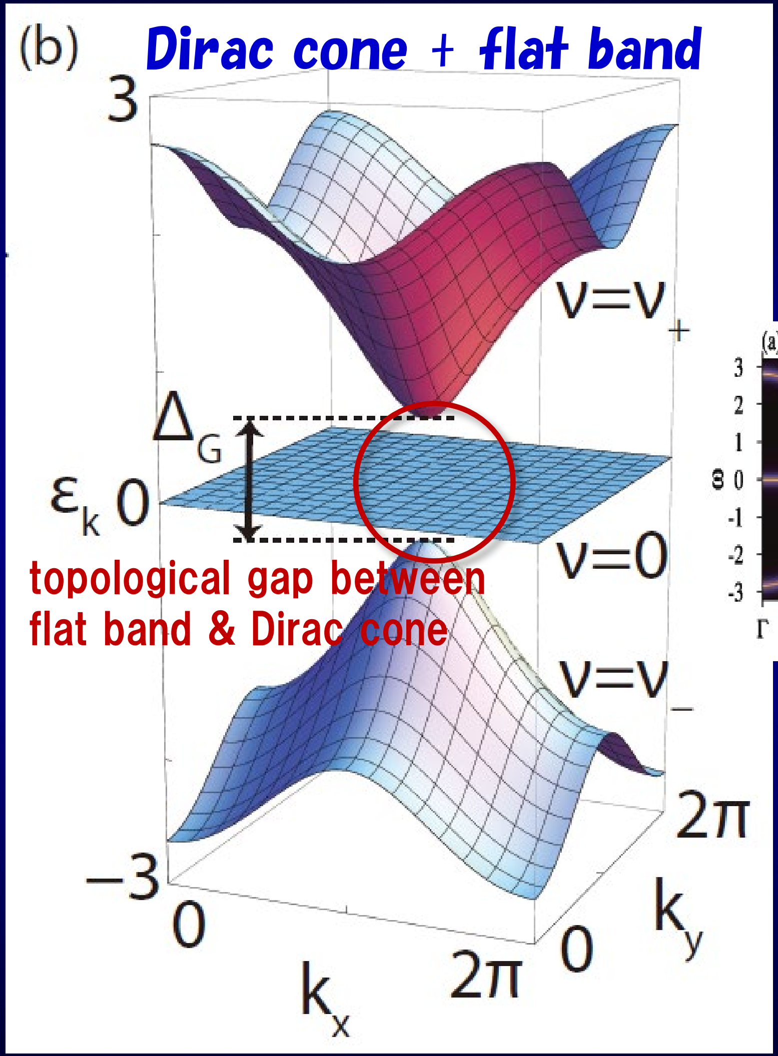


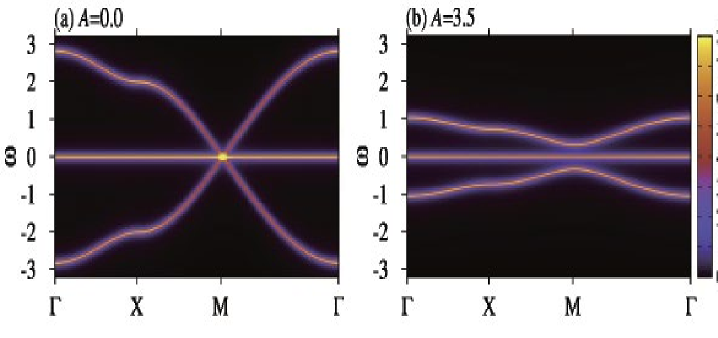
Flat-band Floquet topological insulators

(Mikami, ..., Aoki, PRB 2016)

Lieb lattice

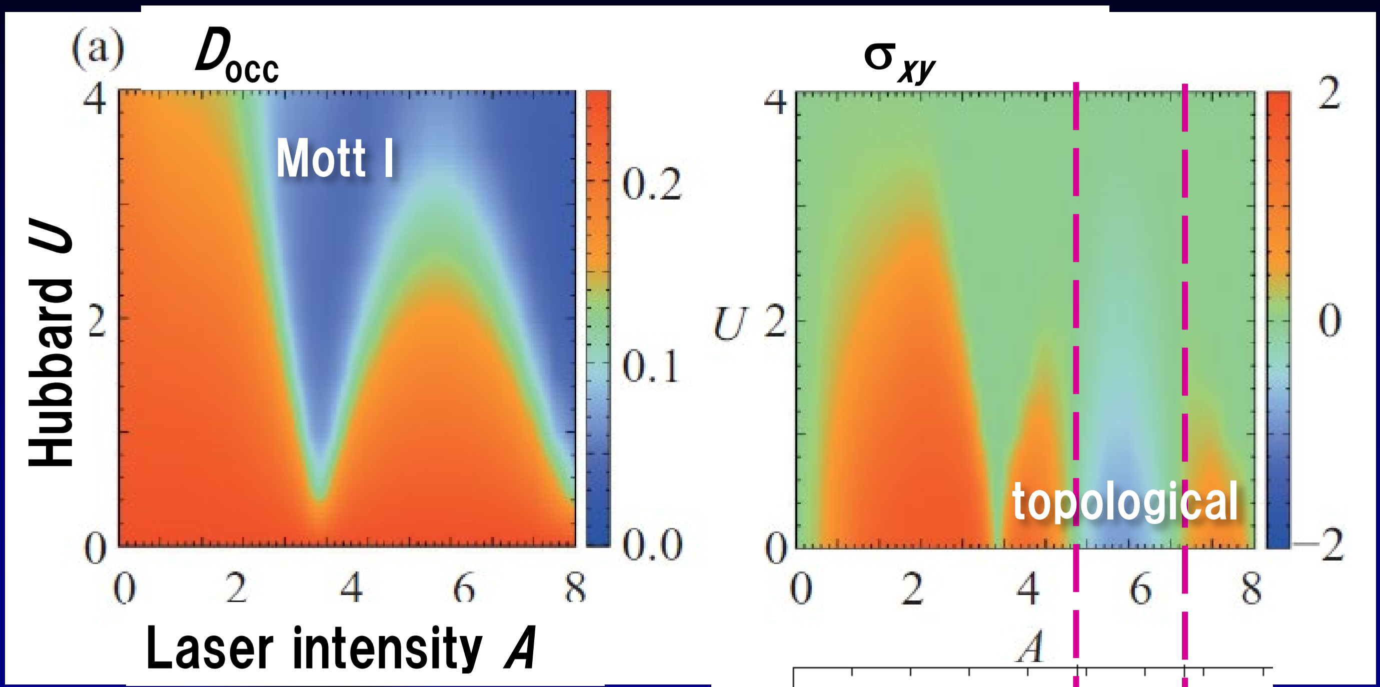


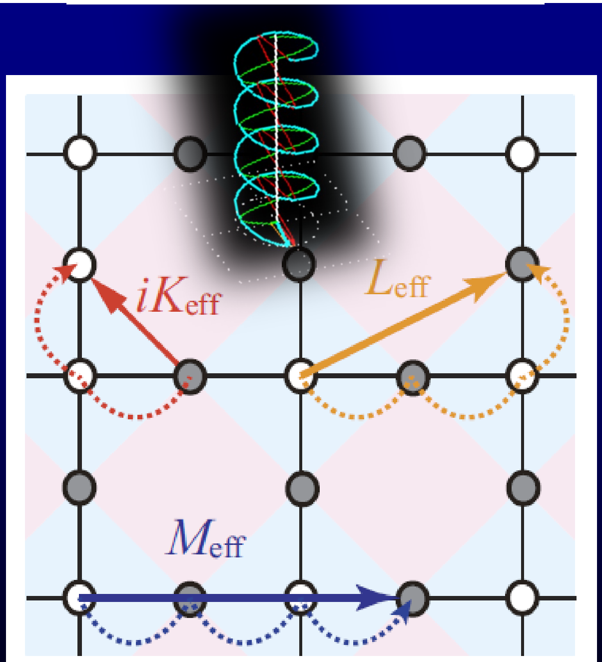


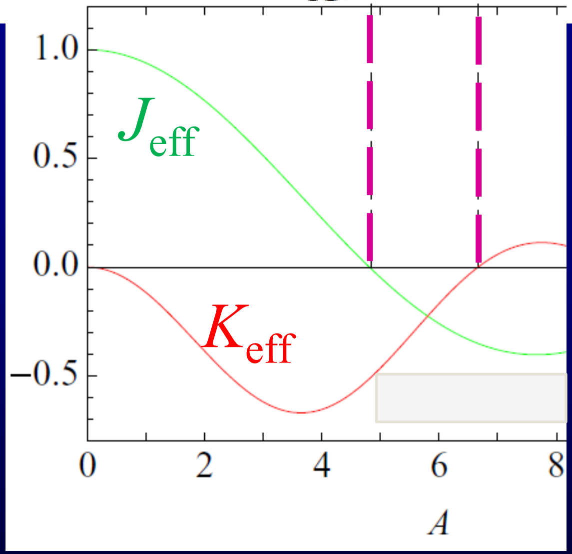


Non-equilibrium phase diagram: Lieb Hubbard model

(Mikami et al, PRB 2016)

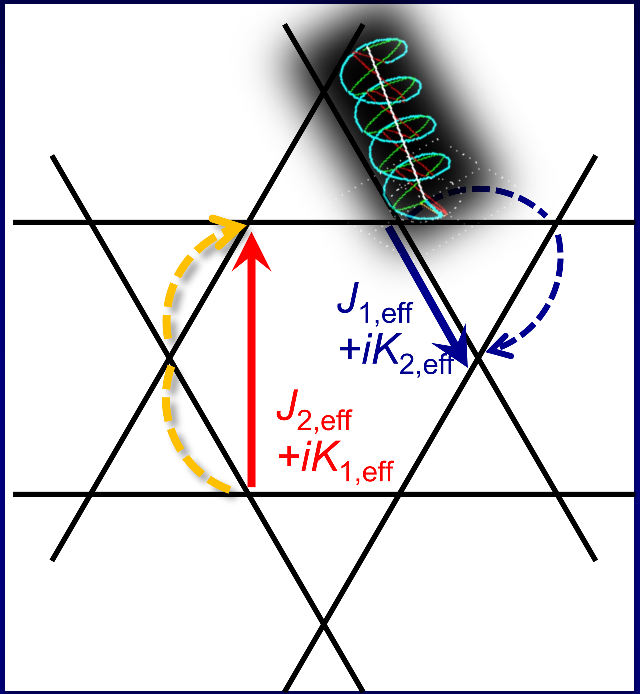


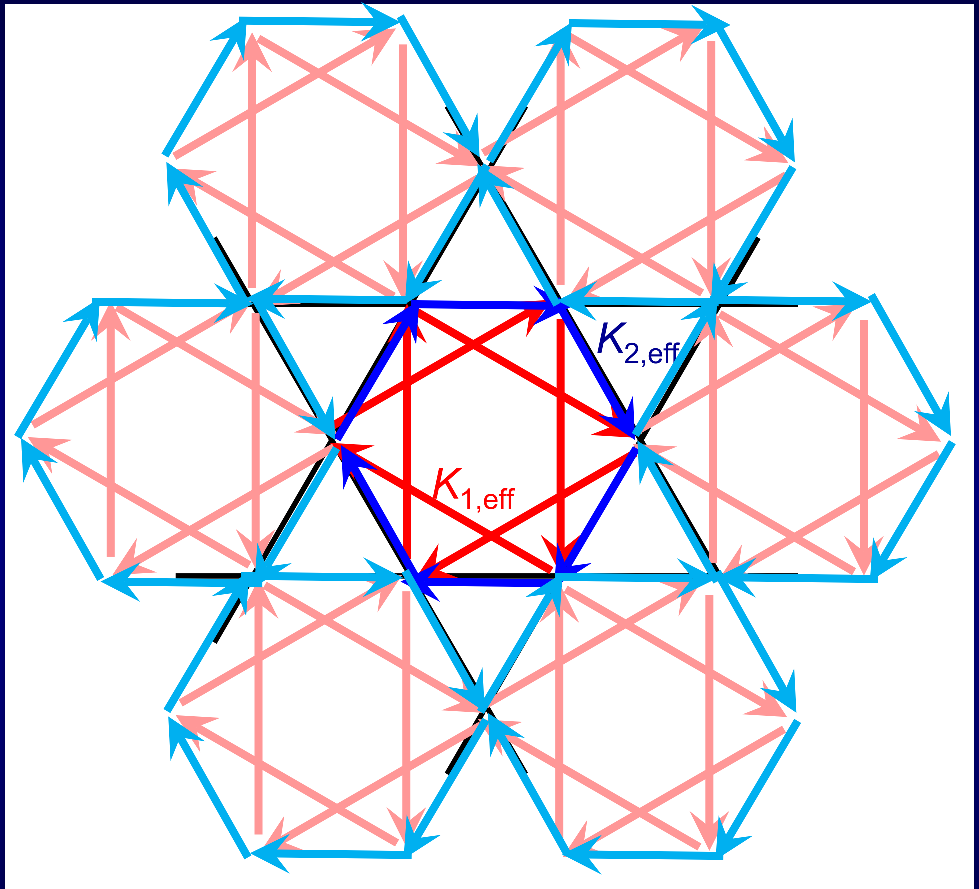


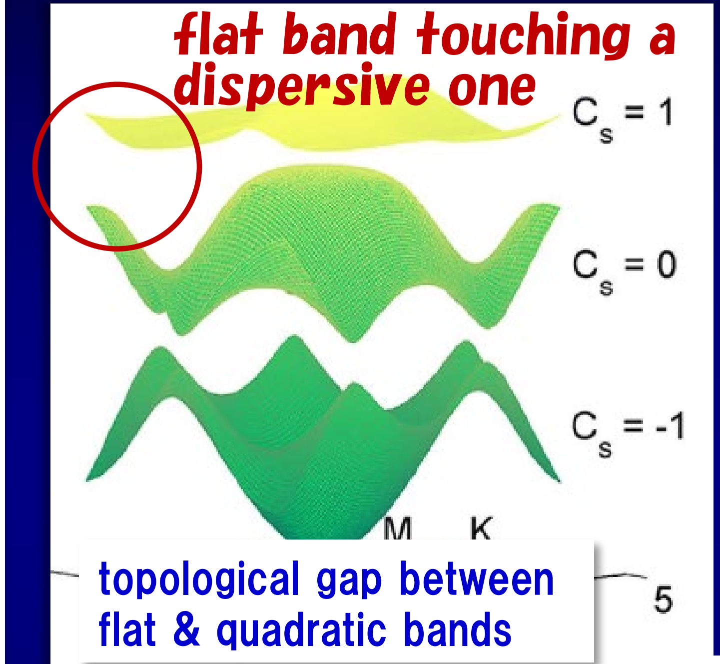


Another flat band: Kagome + CPL

(Mikami et al, PRB 2016)



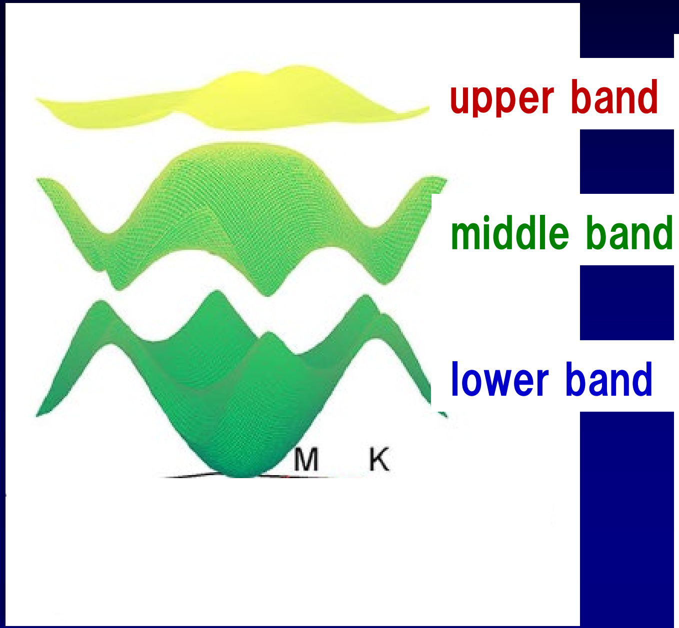


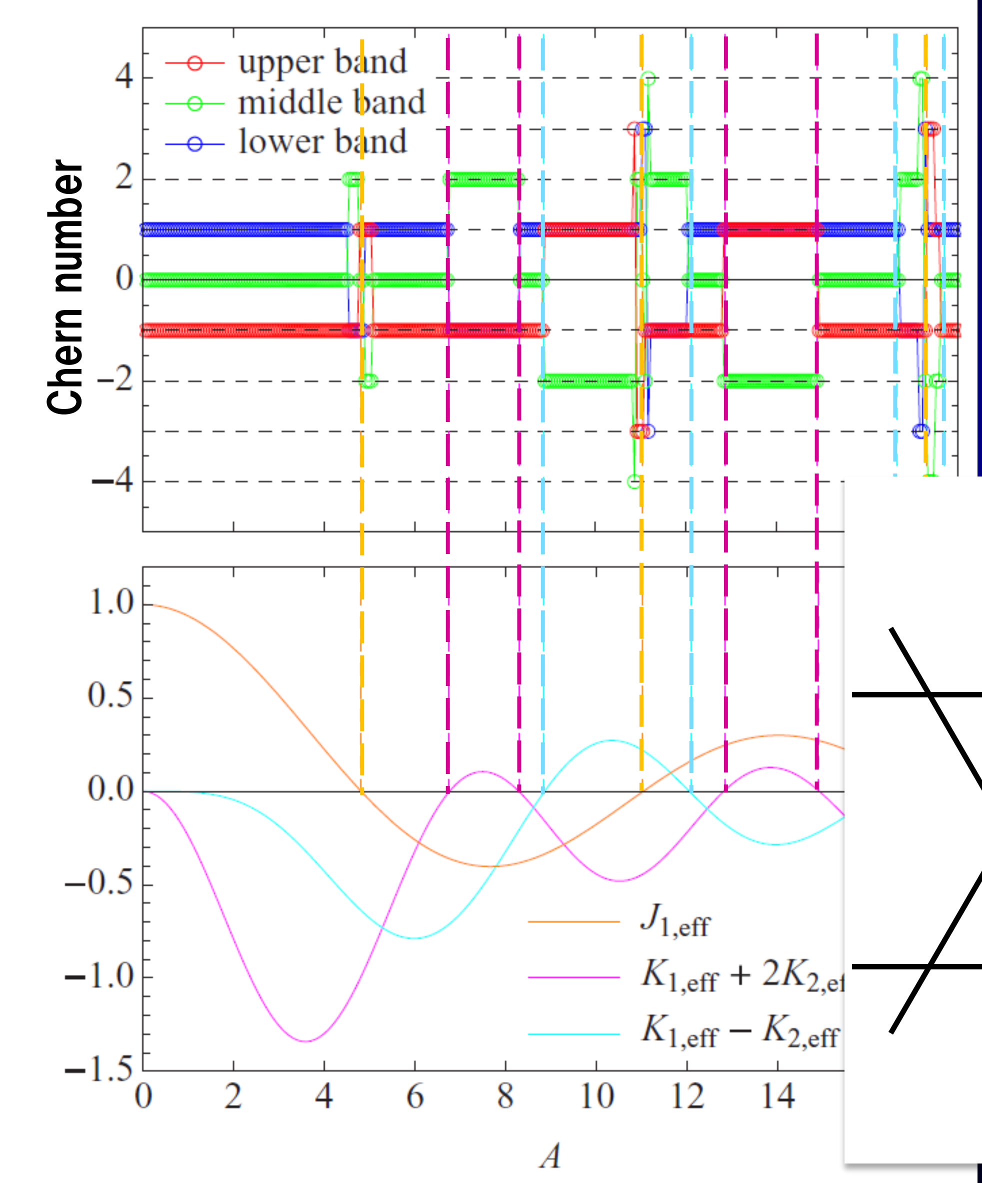


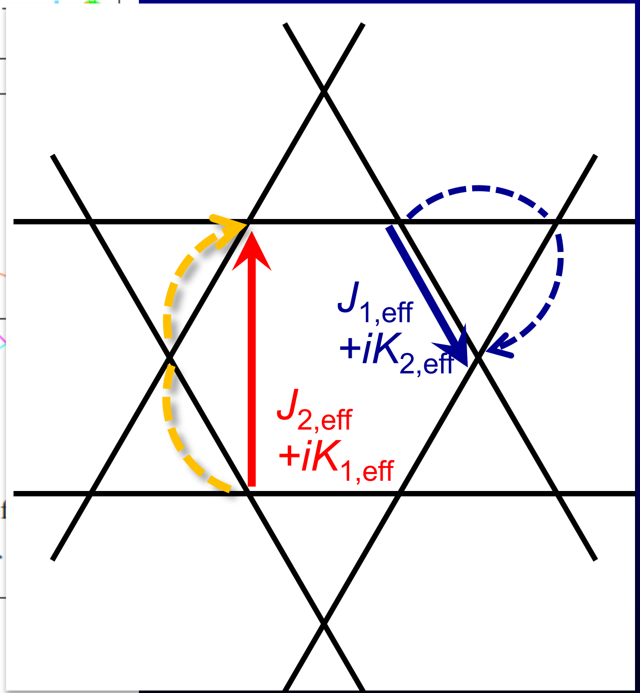
(see also Wang et al, Sci Rep 2017)

Floquet topological flat-bands (eg kagome) → Chern # behaves wildly

(Mikami et al, PRB 2016)





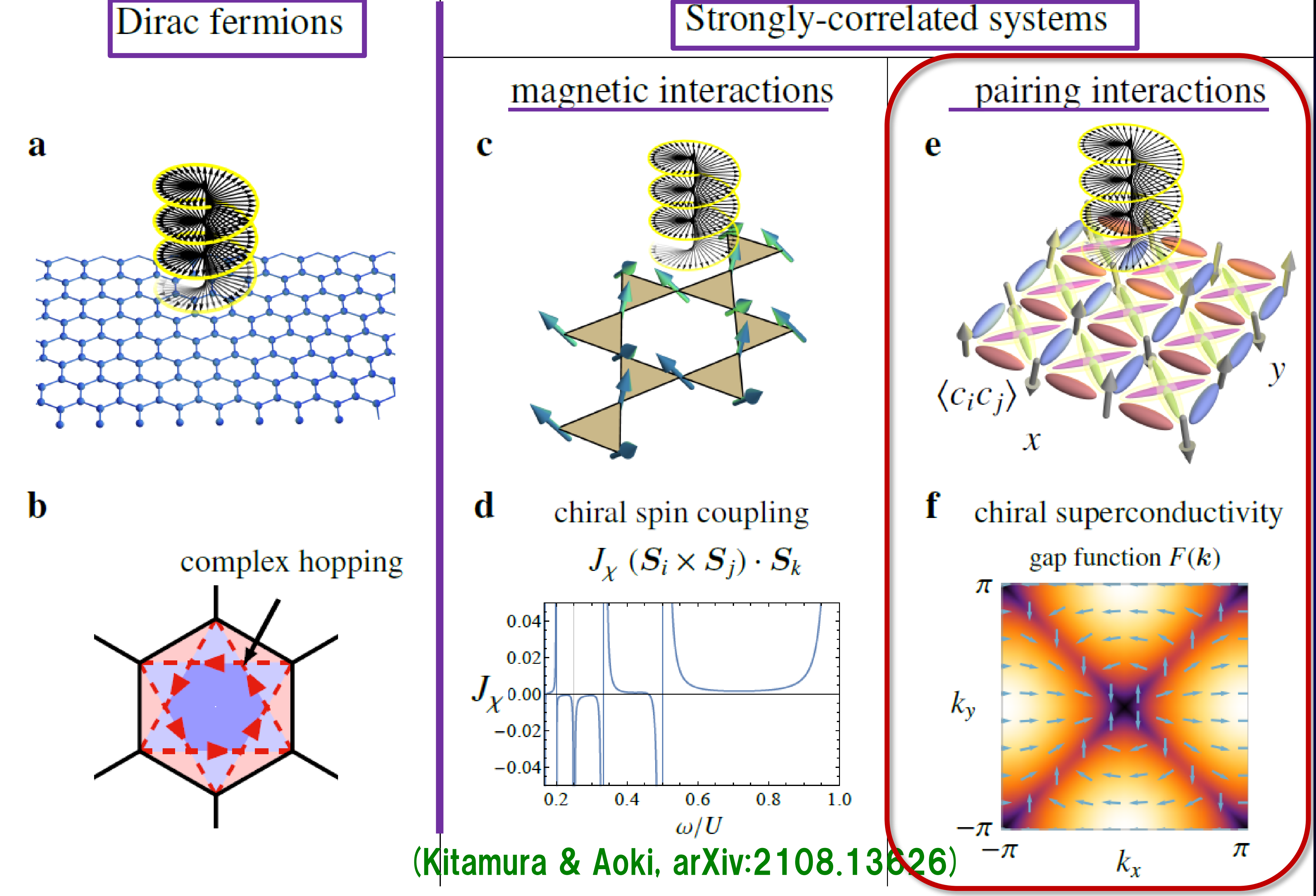


	Superconductivity (from repulsion)	Topological
Equilibrium	✔ Flat-band SC ✔ Non-Fermi liquid	Flat-band topological states
Non-equil	✔ Non-equil induced SC ?	✔ Floquet topological insulator

Topological classification
(Kitagawa, ..., Demler, PRB2010)

d-wave SC + circularly polarised light
→ "Floquet topological $d_{x^2-y^2} + id_{xy}$ SC"

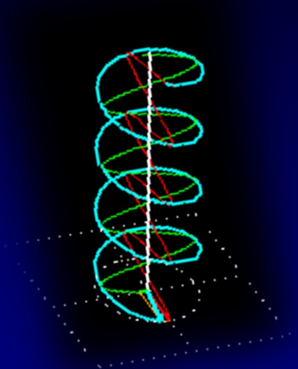
(Kitamura & Aoki, arXiv:2108.13626)

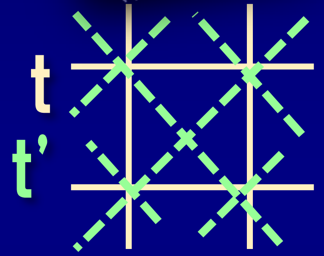


(Kitamura & Aoki, arXiv:2108.13626)

d-wave SC + circularly-polarised light → "Floquet topological SC"

(Kitamura & Aoki, arXiv:2108.13626)





Repulsive Hubbard model + CPL on a square lattice

$$\hat{H}(t) = -\sum_{ij\sigma} t_{ij} e^{-i\boldsymbol{A}(t)\cdot\boldsymbol{R}_{ij}} \hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + \frac{U}{2}\sum_{i}\hat{n}_i(\hat{n}_i - 1)$$

$t << U$ → * Equilibrium: t-J model

* Under CPL,

photon-induced correlated hopping

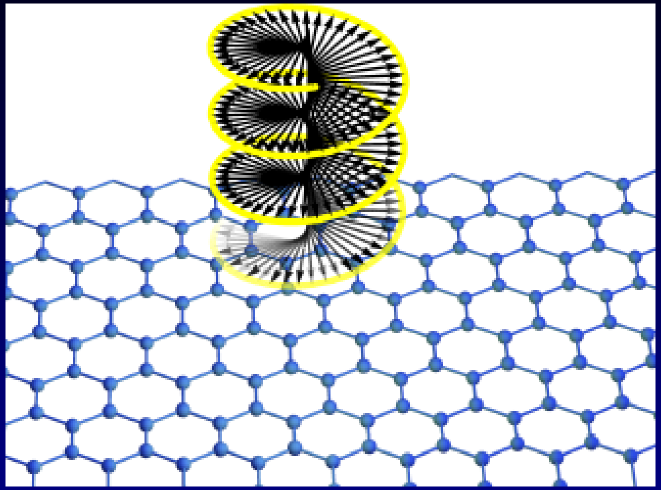
photon-modified kinetic exchange

$$\hat{H}_{\mathrm{F}} = -\sum_{ij\sigma}\tilde{t}_{ij}\hat{P}_G\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma}\hat{P}_G + \frac{1}{2}\sum_{ij}J_{ij}\left(\hat{\boldsymbol{S}}_i\cdot\hat{\boldsymbol{S}}_j - \frac{1}{4}\hat{n}_i\hat{n}_j\right)$$
$$+ \sum_{ijk\sigma\sigma'}\Gamma_{i,j;k}\hat{P}_G\left[(\hat{c}^{\dagger}_{i\sigma}\boldsymbol{\sigma}_{\sigma\sigma'}\hat{c}_{j\sigma'})\cdot\hat{\boldsymbol{S}}_k - \frac{1}{2}\delta_{\sigma\sigma'}\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma}\hat{n}_k\right]\hat{P}_G$$
$$+ \frac{1}{6}\sum_{ijk}J^{\chi}_{ijk}(\hat{\boldsymbol{S}}_i\times\hat{\boldsymbol{S}}_j)\cdot\hat{\boldsymbol{S}}_k, \qquad (5)$$

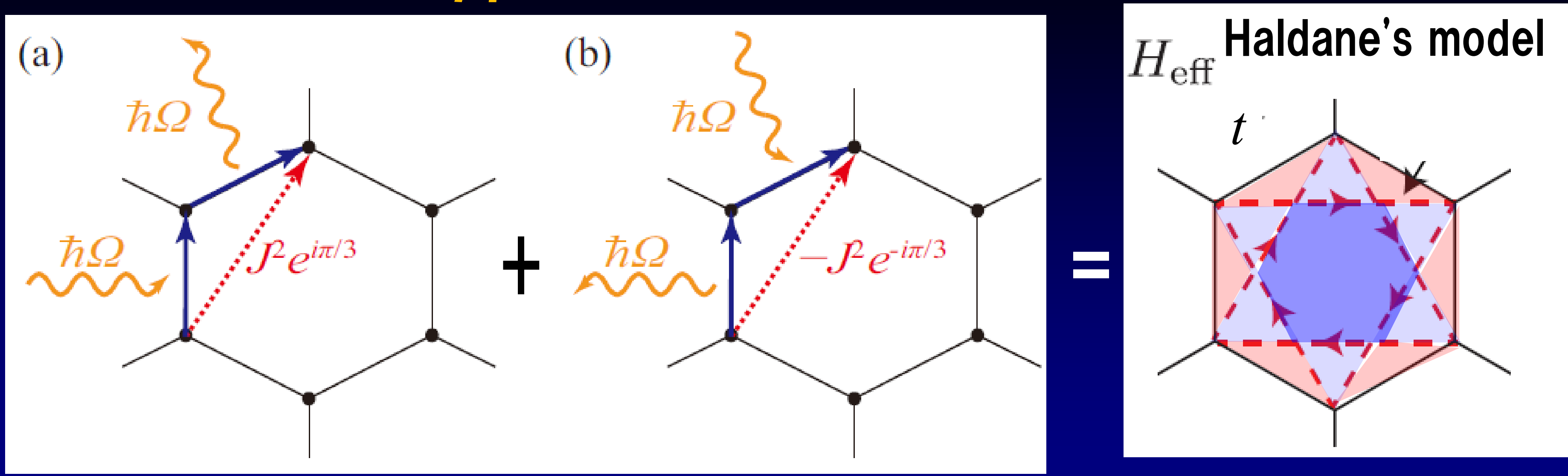
Floquet renormalised hopping

photon-induced chiral spin coupling

honeycomb

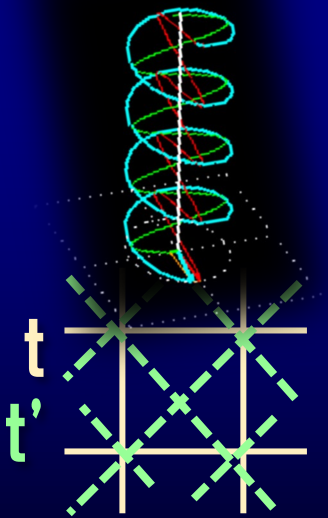


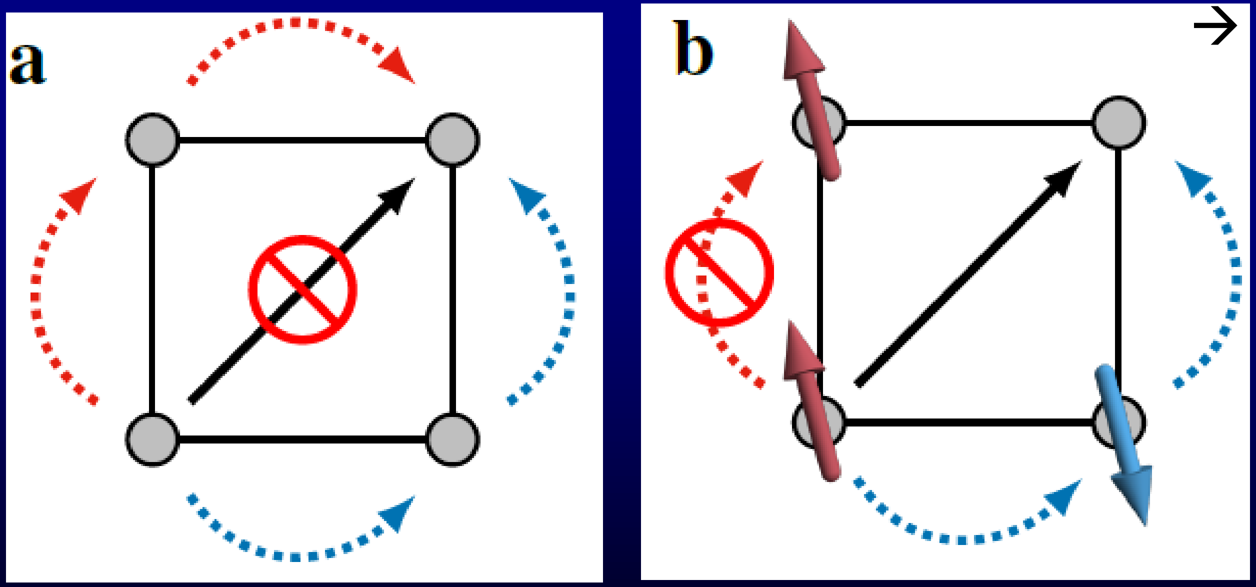
one-body problem



this is why graphene accommodates FTI

square

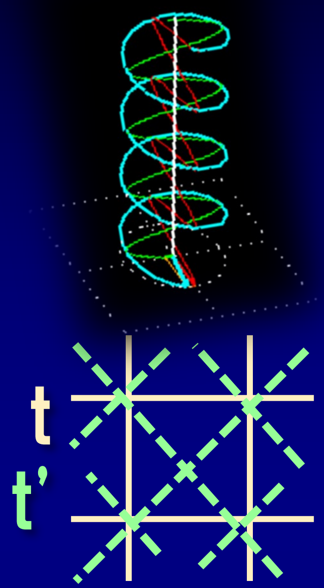




Strongly-correlated case
→ *square lattice already accommodates topology*

Floquet dynamics: d-wave SC → Floquet topological dx²−y² + idxy

(Kitamura & Aoki, arXiv:2108.13626)



Floquet Hamiltonian in Bogoliubov-de Gennes form

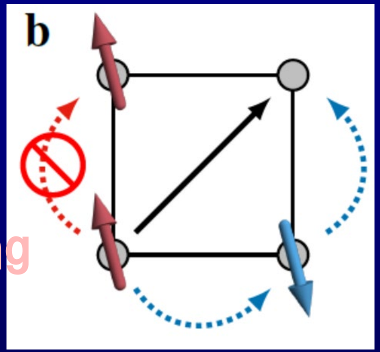
$$\hat{H}_{\rm F} = \sum_{\boldsymbol{k}} \begin{pmatrix} \hat{c}_{\boldsymbol{k}\uparrow} \\ \hat{c}^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}^\dagger \begin{pmatrix} \varepsilon(\boldsymbol{k}) & F(\boldsymbol{k}) \\ F(\boldsymbol{k})^* & -\varepsilon(-\boldsymbol{k}) \end{pmatrix} \begin{pmatrix} \hat{c}_{\boldsymbol{k}\uparrow} \\ \hat{c}^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}$$

$$= \sum_{\boldsymbol{k}} \begin{pmatrix} \hat{c}_{\boldsymbol{k}\uparrow} \\ \hat{c}^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}^\dagger \left[\sum_{\tau} \begin{pmatrix} \varepsilon_\tau & F_\tau \\ F^*_\tau & -\varepsilon_\tau \end{pmatrix} \cos \boldsymbol{k} \cdot \boldsymbol{R}_{i,i+\tau} \right] \begin{pmatrix} \hat{c}_{\boldsymbol{k}\uparrow} \\ \hat{c}^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}$$

acquires a *dxy* component ∝ sin (*kx*) sin (*ky*) involving

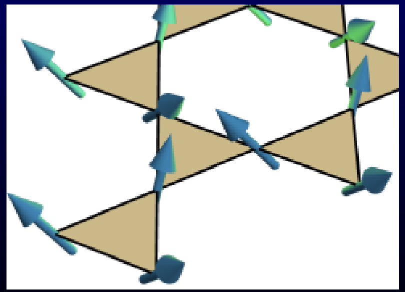
$$\gamma = \mathrm{Im}\,(\Gamma_{i-x,i;\,i+y} - \Gamma_{i-x-y,i+x;\,i}),$$

two-step correlated hopping



$$J_\chi = J^\chi_{i,i+y,i+x}, \quad J'_\chi = J^\chi_{i-x,i,i+x+y}.$$

chiral spin-coupling



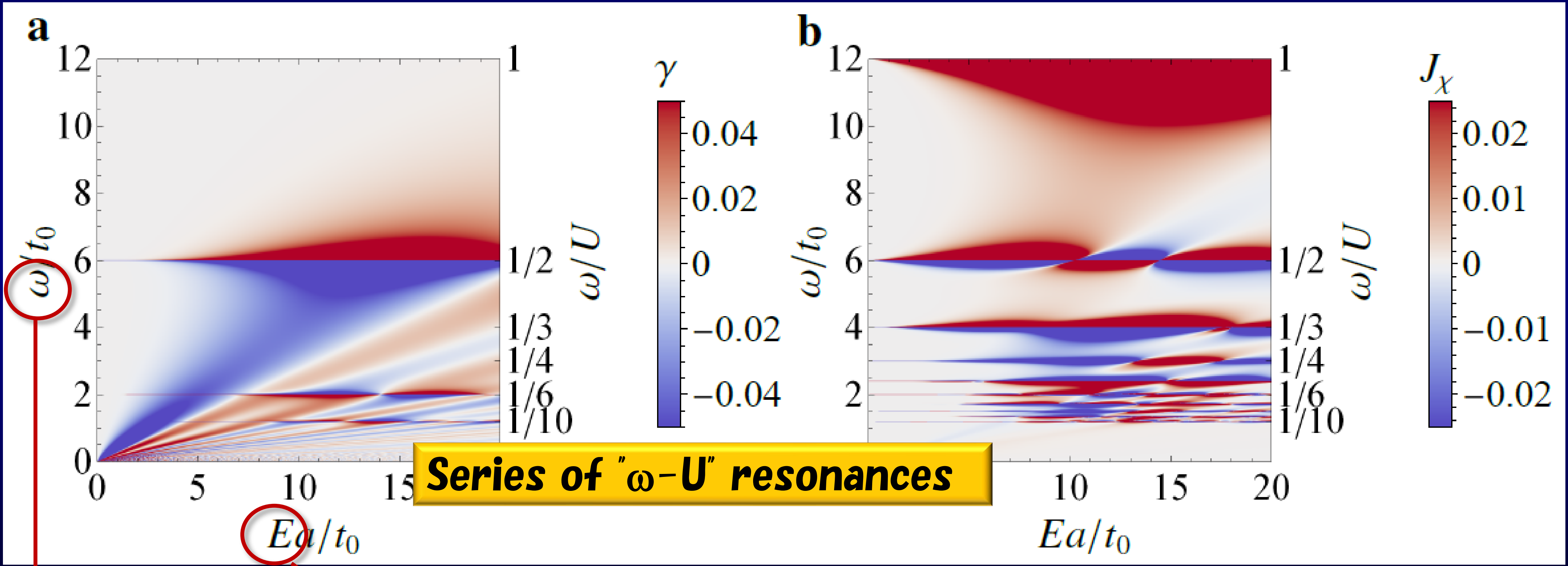
(Kitamura & Aoki, arXiv:2108.13626)

two-step correlated hopping

$$\gamma = \mathrm{Im}\,(\Gamma_{i-x,i;\,i+y} - \Gamma_{i-x-y,i+x;\,i}),$$

chiral spin-coupling

$$J_\chi = J^\chi_{i,i+y,i+x},\quad J'_\chi = J^\chi_{i-x,i,i+x+y}.$$

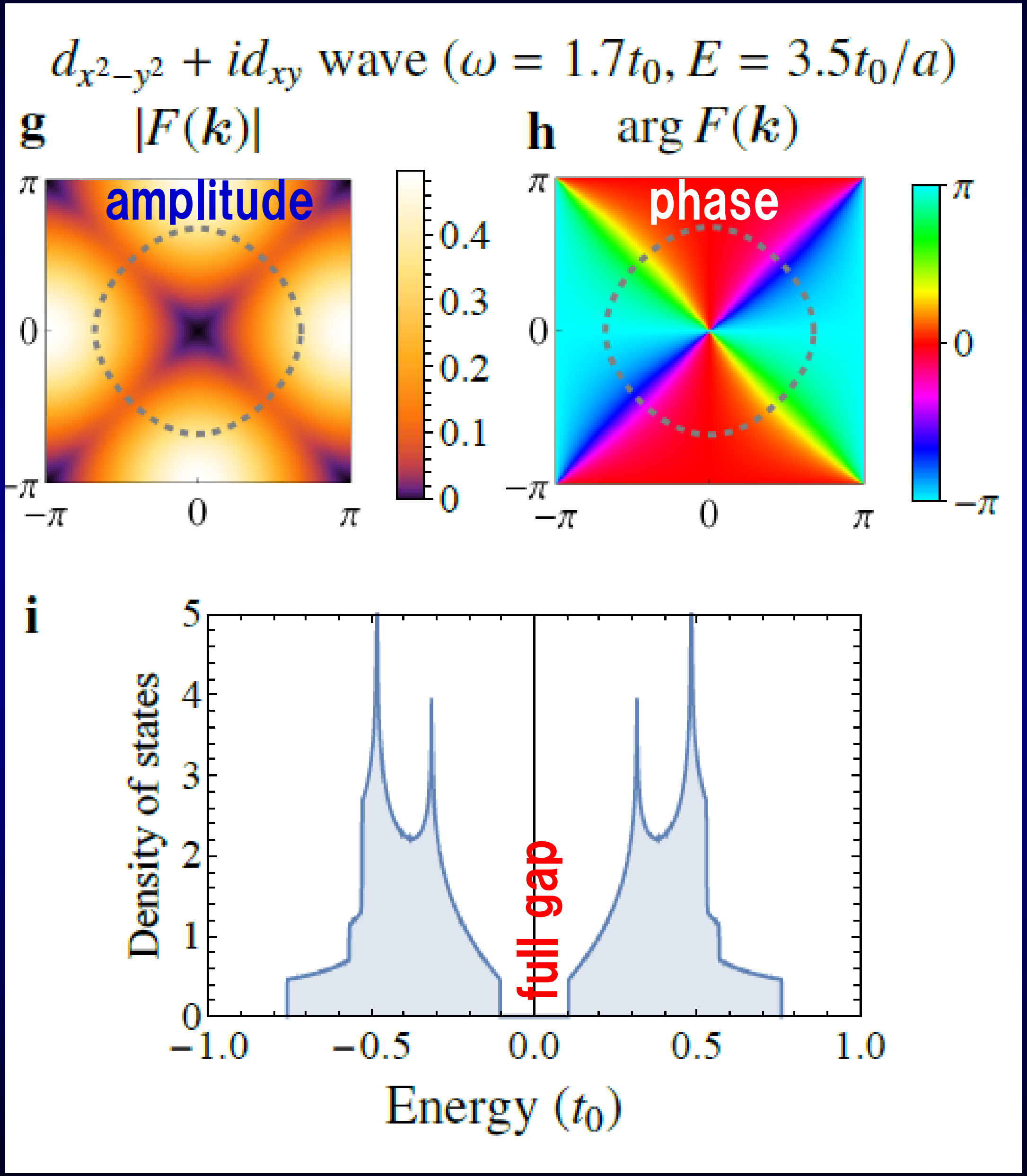


Series of "ω−U" resonances

laser frequency

laser field intensity

d-wave SC + CPL → Floquet topological $d_{x^2-y^2} + id_{xy}$



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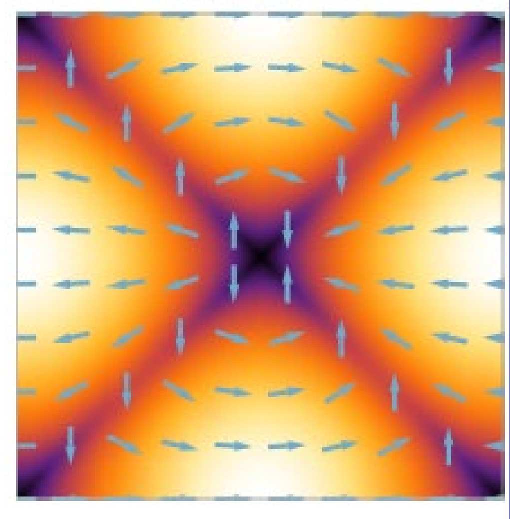
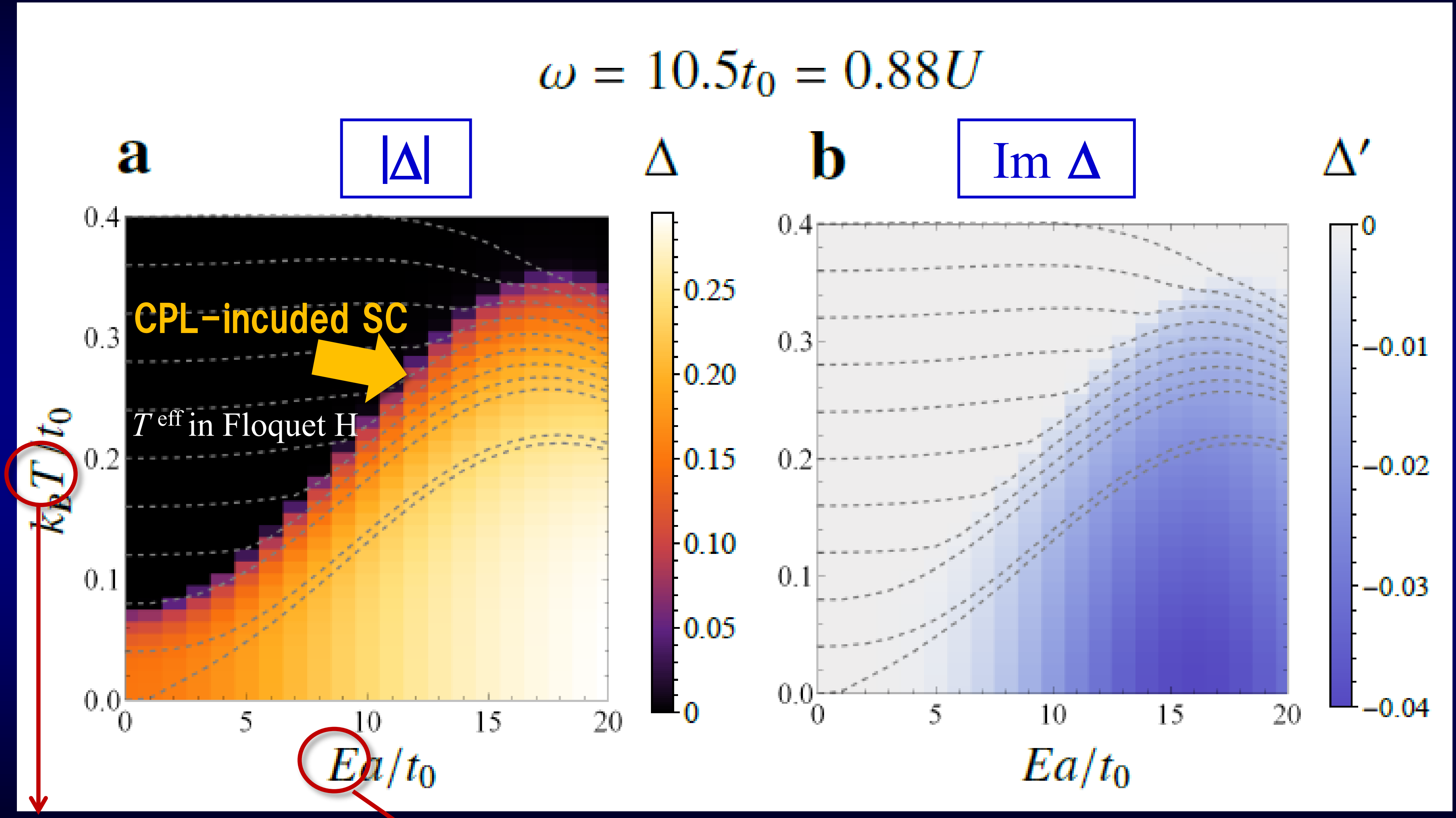


(Kitamura & Aoki, arXiv:2108.13626)

"Tc dome" against laser intensity for the (d + id) SC



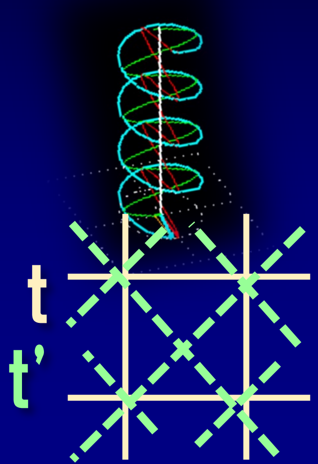
temperature

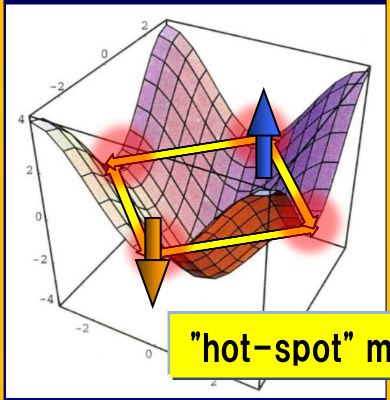
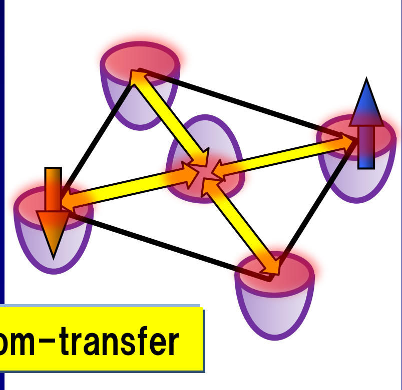
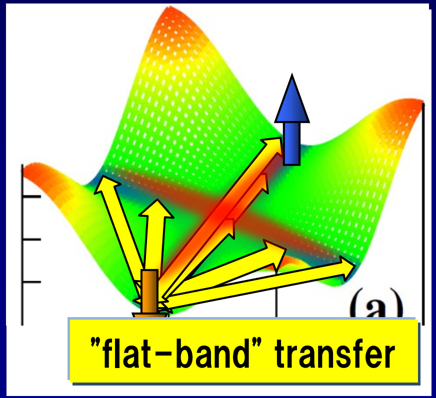
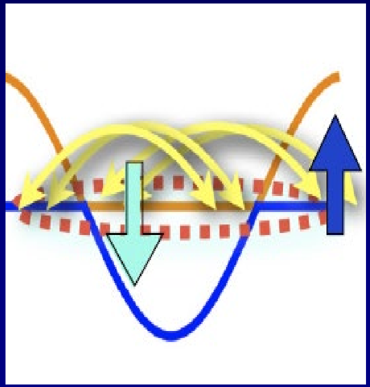
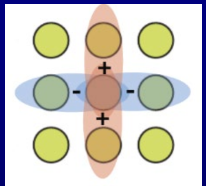
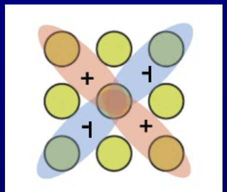
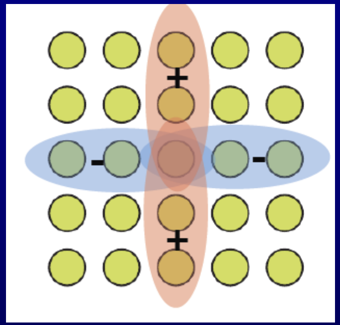
laser field intensity

(Kitamura & Aoki, arXiv:2108.13626)

Equilibrium

Floquet dynamics → topological SC



	single-orbital, one-band	multi-o, multi-b	flat-band: single-o, one-b	flat-band: single-o, multi-b
k-space				
real space	 d (as in cuprates)	 s+- (as in iron-based)		

My guess: a good place to look for topological SC (in eq)

Summary

	Superconductivity	Topological
Equilibrium	✓ 1-band vs multiband * Incipient-band SC * Flat-band SC * Nematic SC	✓ Topological states *Dispersive vs flat bands
Non-equil	✓ Floquet topological SC	✓ Floquet topological insulator *Dispersive vs flat bands

These indeed harbour unique opportunities for a host of quantum phases.
Keywords: incipient, flat, nematicity, Floquet, ...

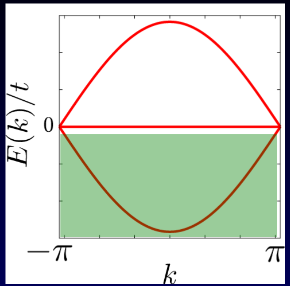
Future works

Details of topological I, SC

Details of non-Fermi liquid SC

Last but not least,

Eugene, congratulations !



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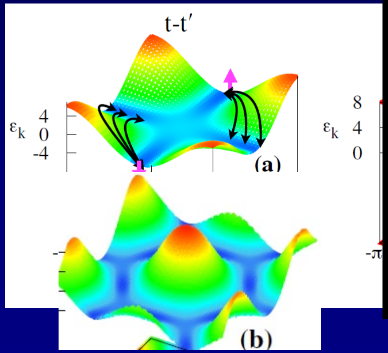

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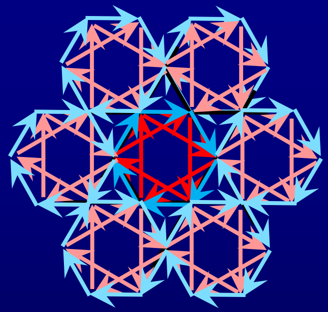

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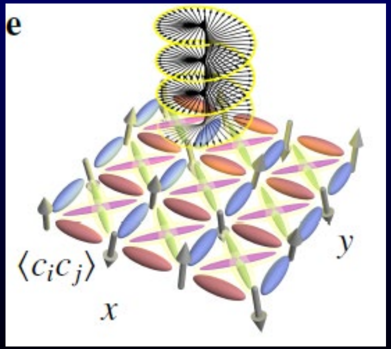

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