CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

How warm are non-thermal relics?
Out-of-equilibrium dark matter production

Mathias Pierre

DESY Theory Seminar

October 25$^{\text{th}}$ 2021

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Out-of-equilibrium dark matter production

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PhD 2015-18
Orsay - France

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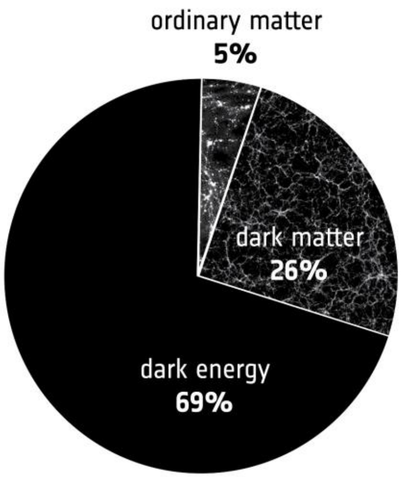


How dark matter is produced?

Can we probe the production mechanism?

➡ **Focus on out-of-equilibrium particle dark matter**

Based on [**arXiv:2011.13458**] – **JCAP 21**
with **G. Ballesteros** & **M. A. G. Garcia**

Outline

1. **Introduction - Motivation**
2. **Production of out-of-equilibrium dark matter**
   1. The dark matter phase space distribution
   2. Decay of condensate
   3. Ultra-violet freeze-in via scattering
   4. Freeze-in via decay
3. **Cosmological imprint**
   1. Non-Cold Dark Matter
   2. Matter power spectrum and Lyman-alpha forest

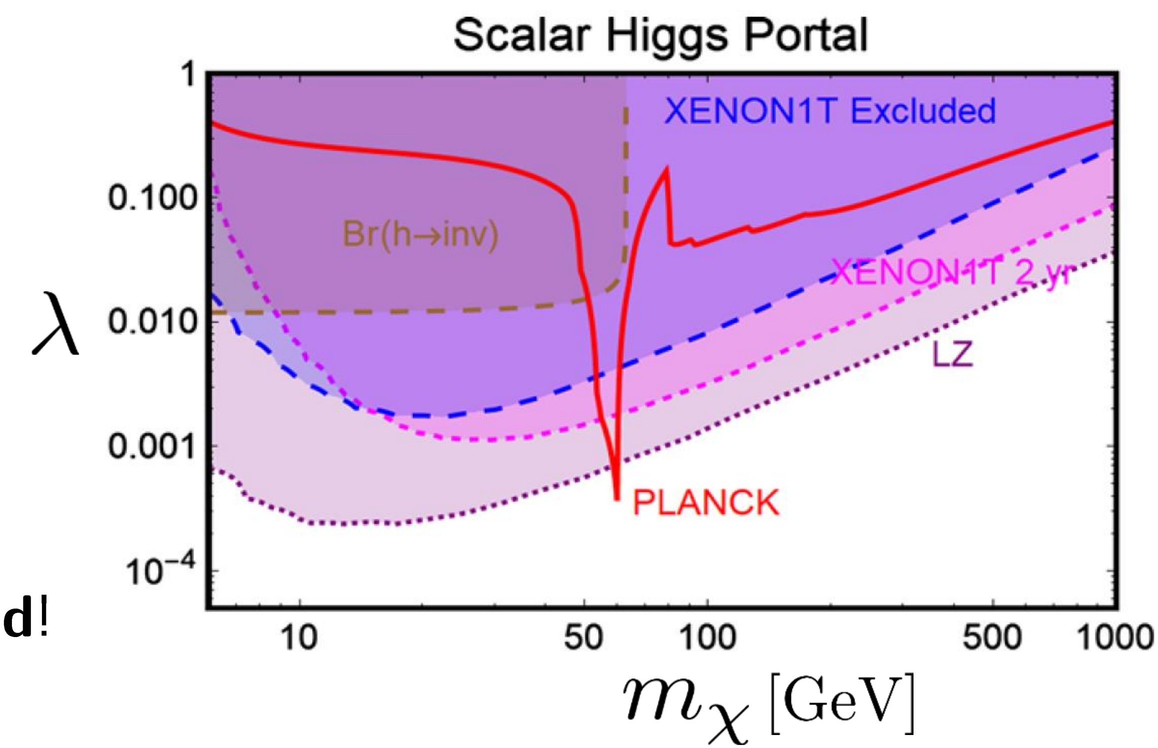
The waning of the WIMP?

- Most minimal Weakly Interacting Massive Particles (**WIMP**) models based on "**freeze-out**". Example : introduce **dark matter scalar** χ

$$\mathcal{L} = \lambda |\chi|^2 |H|^2$$

- **DM thermalizes** and **annihilate** to **SM**

- Sizable **coupling required**!



- **Minimalistic WIMP models under siege!**

[M. Escudero, A. Berlin, D. Hooper, M.-X. Lin - JCAP 12 (2016) 029]
[G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, **MP**, S. Profumo, F. S. Queiroz EPJC 78 (2018) 203]
[G. Arcadi, A. Djouadi, M. Raidal - Phys.Rept. 842 (2020) 1-180]

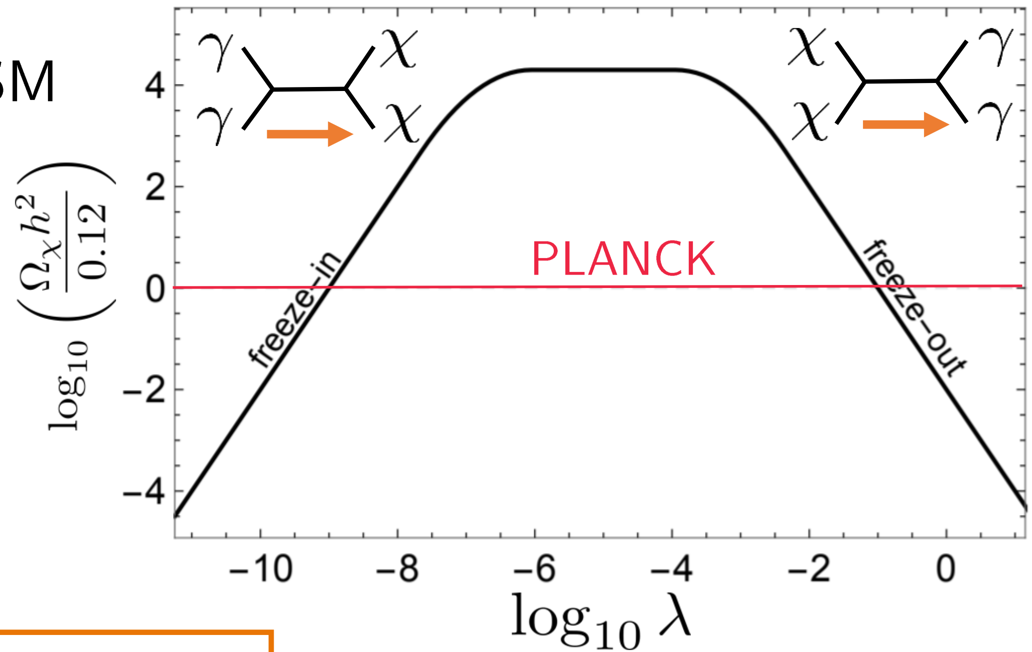
The dawn of FIMP?

- Feebly Interacting Massive Particles (**FIMP**) models based on "**freeze-in**" opens up **parameter space**!

[J. McDonald PRL 88 (2002) 091304 - K.-Y. Choi, L. Roszkowski AIP Conf.Proc. 805 (2005) 1, 30-36
Kusenko PRL 97 (2006) 241301 - K. Petraki, A. Kusenko PRD 77 (2008) 065014
L. J. Hall, K. Jedamzik, J. March-Russell, S. M. West JHEP 03 (2010) 080
N. Bernal, M. Heikinheimo, T. Tenkanen, K. Tuominen and V. Vaskonen – IJMP A 32 (2017) 27, 1730023]

- DM **never thermalizes** with SM

- **DM produced** from **SM annihilations**



χ : **Dark Matter (DM) particles**
γ : **Standard Model (SM) particles**

The dawn of FIMP?

- Feebly Interacting Massive Particles (**FIMP**) models based on "**freeze-in**" opens up **parameter space**!

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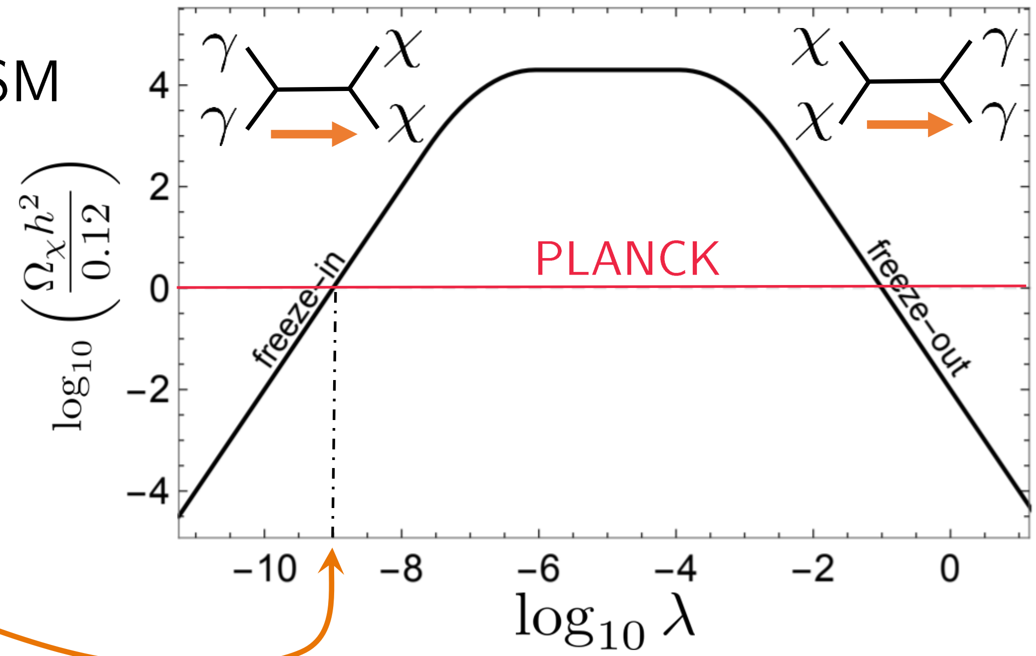
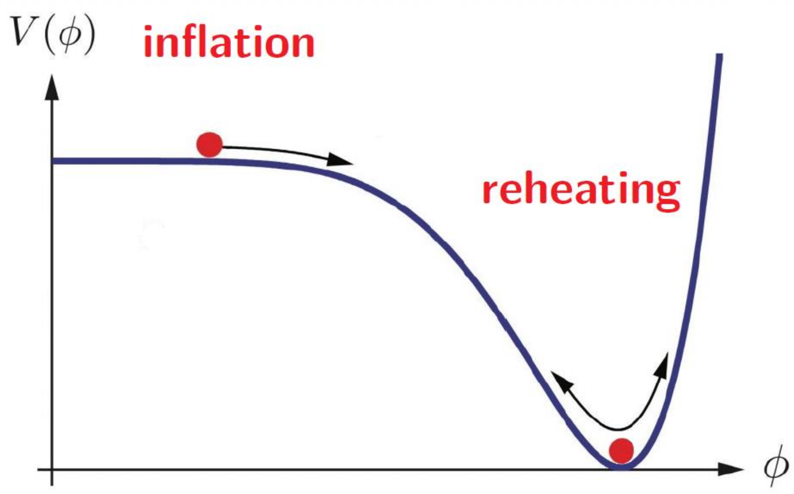
- DM **never thermalizes** with SM
- **DM produced** from **SM annihilations**
- Requires **small coupling**!



...introduce instead **large seclusion** scale between SM and DM?

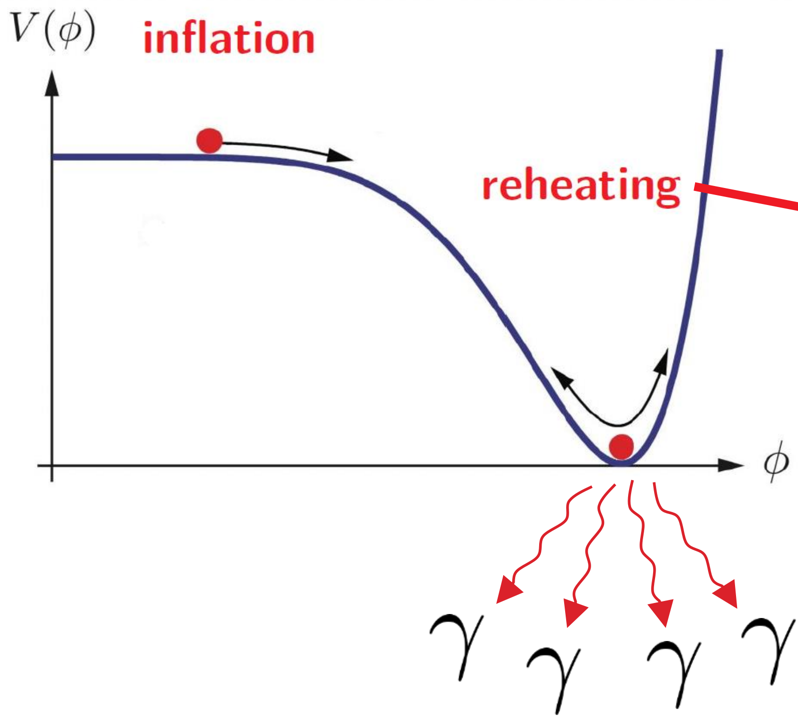
➡ **Ultra-violet (UV) freeze-in** ➡ post **inflation** production!

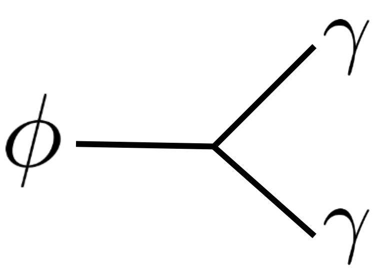
Reheating in a nutshell



ϕ : **Inflaton field**

Reheating in a nutshell





rate Γ_ϕ

For a **quadratic** potential

$$\frac{d\rho_\phi}{dt} + 3H\rho_\phi = -\Gamma_\phi \rho_\phi$$

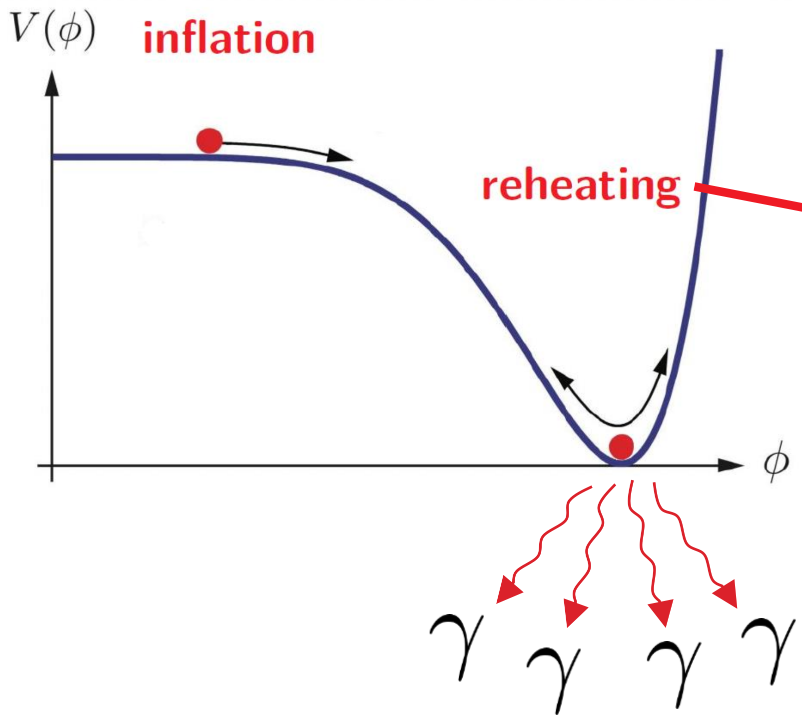
$$\frac{d\rho_\gamma}{dt} + 4H\rho_\gamma = \Gamma_\phi \rho_\phi$$

$$H^2 = \frac{1}{3M_{\rm Pl}}(\rho_\phi + \rho_\gamma)$$

For **non-quadratic** potentials:

[M. A. G. Garcia, K. Kaneta, Y. Mambrini, K. A. Olive PRD 101, 123507 (2020)]

Reheating in a nutshell



For a **quadratic** potential

$$\frac{\mathrm{d}\rho_\phi}{\mathrm{d}t} + 3H\rho_\phi = -\Gamma_\phi \rho_\phi$$

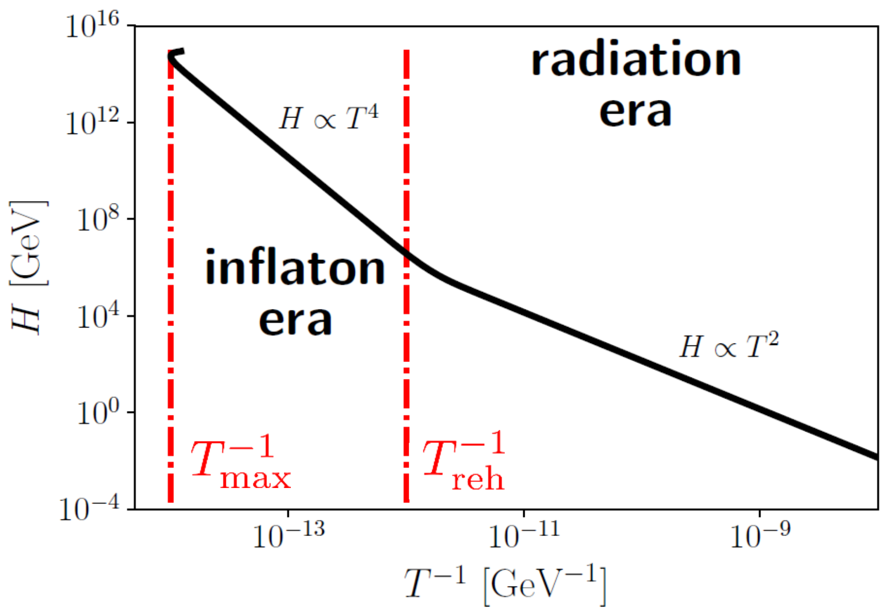
$$\frac{\mathrm{d}\rho_\gamma}{\mathrm{d}t} + 4H\rho_\gamma = \Gamma_\phi \rho_\phi$$

$$H^2 = \frac{1}{3M_{\mathrm{Pl}}}(\rho_\phi + \rho_\gamma)$$

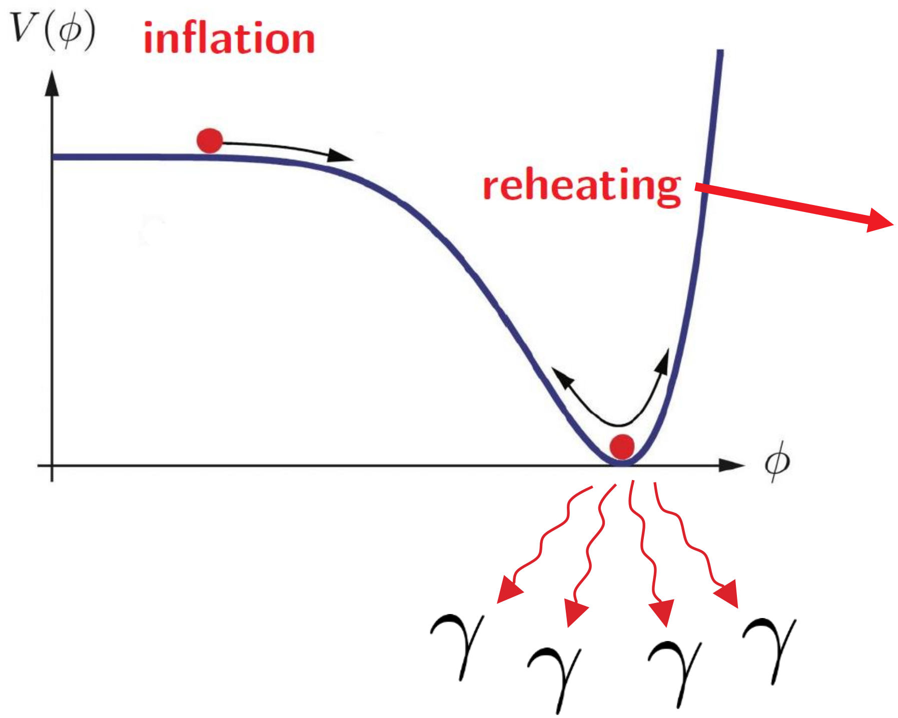
T_{reh} **: Reheating temperature**

$$\rho_\phi(T_{\mathrm{reh}}) = \rho_\gamma(T_{\mathrm{reh}})$$

$$(\Leftrightarrow t_{\mathrm{reh}} \simeq \Gamma_\phi^{-1})$$



Reheating in a nutshell



For a **quadratic** potential

$$\begin{cases} \dfrac{d\rho_\phi}{dt} + 3H\rho_\phi = -\Gamma_\phi \rho_\phi \\ \dfrac{d\rho_\gamma}{dt} + 4H\rho_\gamma = \Gamma_\phi \rho_\phi \\ H^2 = \dfrac{1}{3M_{\rm Pl}}(\rho_\phi + \rho_\gamma) \end{cases}$$

$T_{\rm reh}$: **Reheating temperature**

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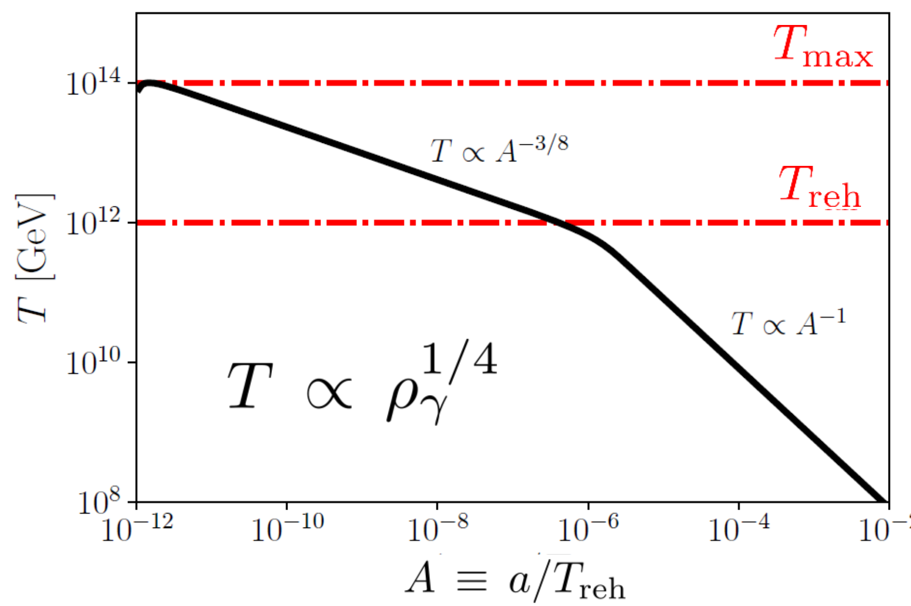
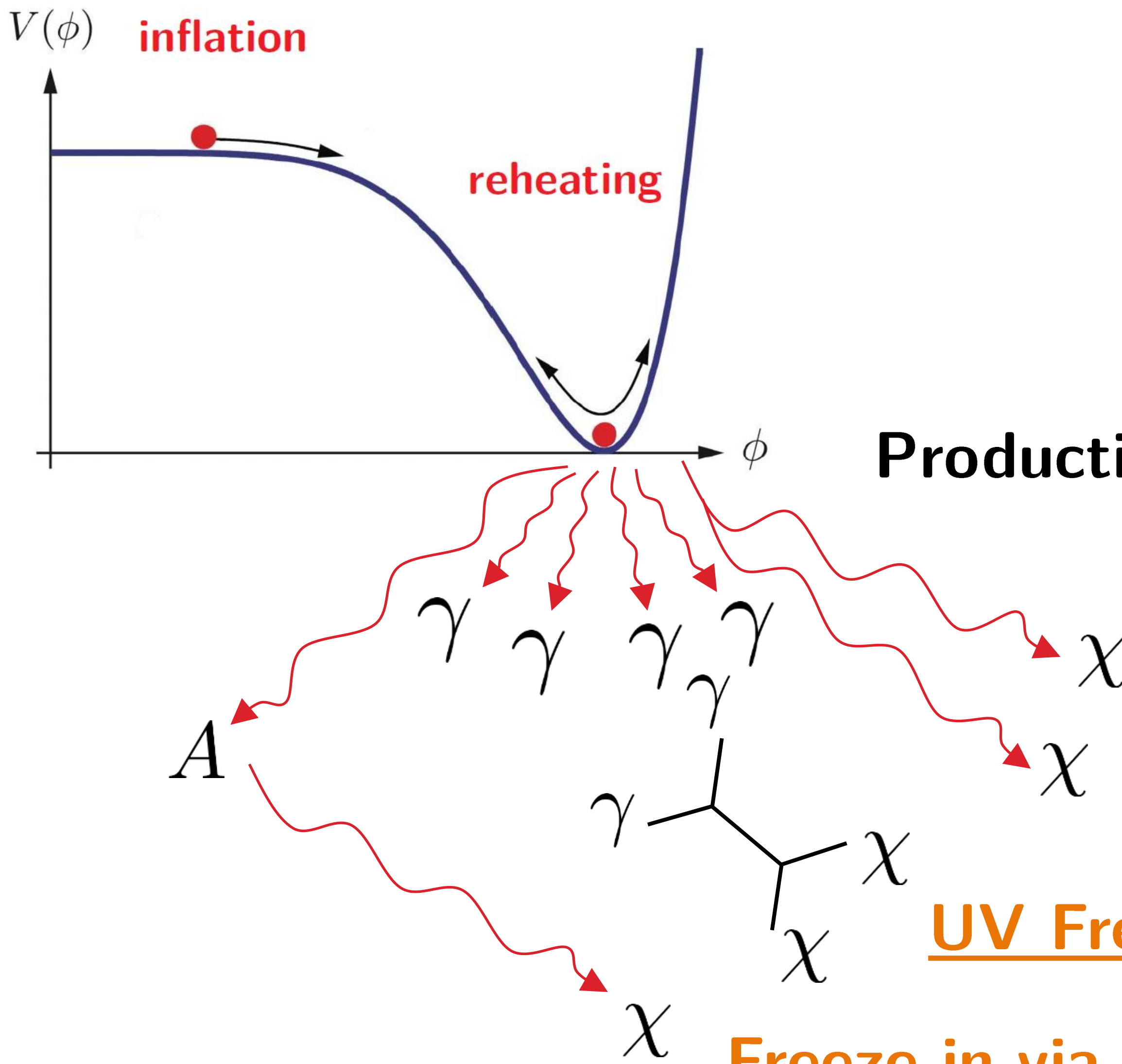


DM production during/after reheating



Production of dark matter χ

Decay of inflaton

UV Freeze-in via scattering

Freeze-in via decay

A : (SM or not) particle

Production of out-of-equilibrium dark matter

DM Phase space distribution

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{p}\, f_\chi(p_0,t)$$

number-density

$$\rho_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{p}\, p_0\, f_\chi(p_0,t)$$

energy-density

- Obtain **phase space distribution** by solving **Boltzmann** equation

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial|\boldsymbol{p}|} = \mathcal{C}[f_\chi(|\boldsymbol{p}|,t)]$$

- **Collision term** for processes $\chi + a + b + \cdots \longleftrightarrow i + j + \cdots$

$$\begin{aligned}\mathcal{C}\left[f_\chi\right] = -\frac{1}{2p_0}\int & \frac{g_a\mathrm{d}^3\boldsymbol{p}_a}{(2\pi)^3 2p_{a0}}\frac{g_b\mathrm{d}^3\boldsymbol{p}_b}{(2\pi)^3 2p_{b0}}\cdots\frac{g_i\mathrm{d}^3\boldsymbol{p}_i}{(2\pi)^3 2p_{i0}}\frac{g_j\mathrm{d}^3\boldsymbol{p}_j}{(2\pi)^3 2p_{j0}}\cdots \\ & \times (2\pi)^4\,\delta^{(4)}(p_\chi + p_a + p_b + \cdots - p_i - p_j - \cdots) \\ & \times \Big[|\mathcal{M}|^2_{\chi+a+b+\cdots\longrightarrow i+j+\cdots}\, f_a f_b\cdots f_\chi(1\pm f_i)(1\pm f_j)\cdots \\ & \quad - |\mathcal{M}|^2_{i+j+\cdots\longrightarrow \chi+a+b+\cdots}\, f_i f_j\cdots(1\pm f_a)(1\pm f_b)\cdots(1\pm f_\chi)\Big]\end{aligned}$$

DM Phase space distribution

- **Solution** to the Boltzmann equation gives

$$f_\chi(p_0, t) = \int_{t_i}^{t} \mathcal{C}[f_\chi]\left(\frac{a(t)}{a(t')}|\boldsymbol{p}|, t'\right) \mathrm{d}t'$$

- For $t > t_{\rm dec}$ after **decoupling** (when production stops)

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial |\boldsymbol{p}|} = 0 \quad \Rightarrow \quad f_\chi(|\boldsymbol{p}|, t) = \bar{f}\left(|\boldsymbol{p}|\frac{a(t)}{a_{\rm dec}}, t_{\rm dec}\right)$$

- After decoupling, distribution function only depends on the **comoving momentum**

$$q \equiv \frac{p\, a(t)}{T_\star} \qquad n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\frac{T_\star^3}{a^3}\int \mathrm{d}^3\boldsymbol{q}\, \bar{f}_\chi(q)$$

$T_\star \equiv T_{\rm NCDM}$ in **CLASS** [J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]

Production from inflaton decay

- Pair of **DM** or **SM** produced from **perturbative** inflaton decay

$$\substack{\gamma \\ \chi} \longleftarrow \phi \longrightarrow \substack{\gamma \\ \chi}$$

$$\mathcal{C}[f(p,t)] \; = \; \frac{8\pi^2}{g m_\phi^2} \Gamma_\phi \, \mathrm{Br} \, n_\phi(t) \, \delta(p - m_\phi/2)$$

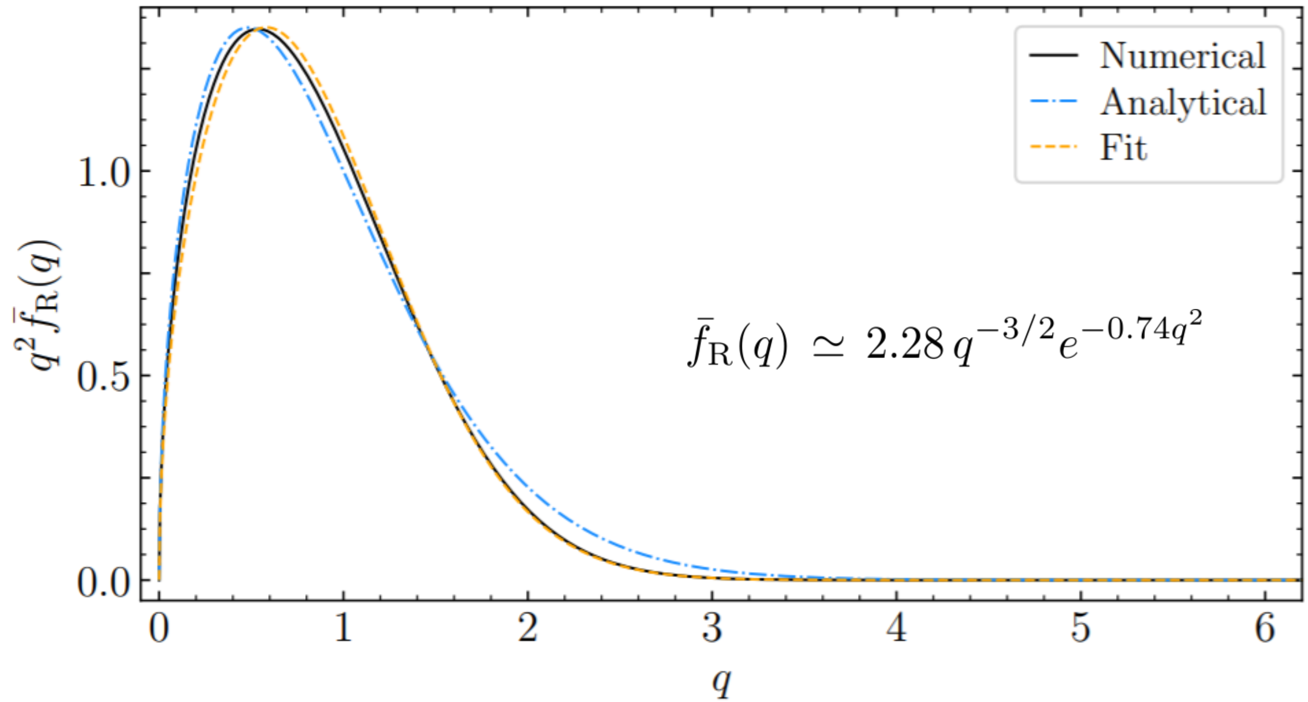
$$f(p,t) \; = \; \frac{16\pi^2 \Gamma_\phi \, \mathrm{Br} \, n_\phi(\hat{t})}{g m_\phi^3 H(\hat{t})} \theta(t - \hat{t}) \qquad \frac{a(t)}{a(\hat{t})} = \frac{m_\phi}{2p}$$

$t \ll t_{\mathrm{reh}}$

$$f(p,t) \simeq \frac{24\pi^2 n(t)}{g m_\phi^3} \left(\frac{m_\phi}{2p}\right)^{3/2} \theta(m_\phi/2 - p)$$

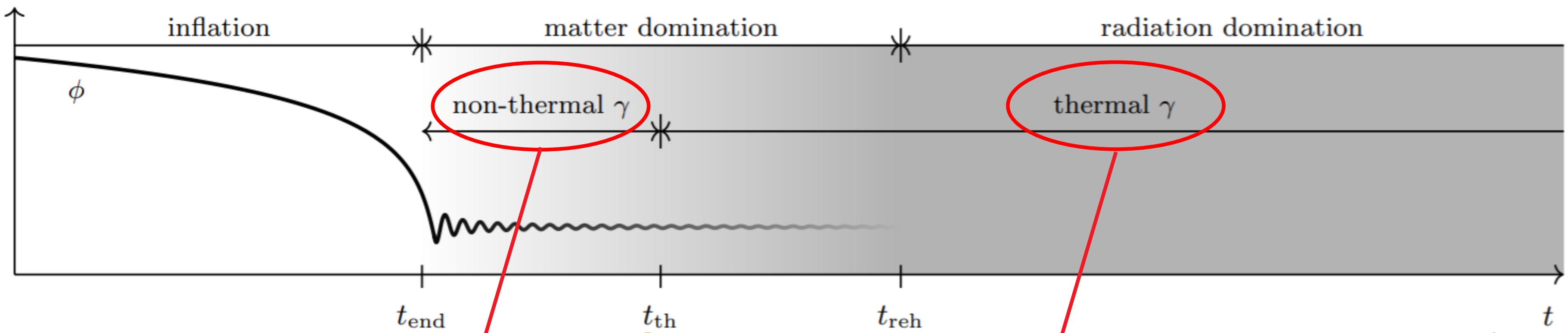
Inflaton decay : DM production

$$f_\chi(p,t)\,\mathrm{d}^3\boldsymbol{p} \;=\; \frac{4\pi^4 \mathrm{Br}_\chi g_{*s}^{\mathrm{reh}}}{5 g_\chi}\left(\frac{T_{\mathrm{reh}}}{m_\phi}\right)^4 \left(\frac{a_0}{a(t)}\right)^3 T_\star^3\,\bar{f}_{\mathrm{R}}(q)\,\mathrm{d}^3\boldsymbol{q} \qquad T_\star \;=\; \left(\frac{g_{*s}^0}{g_{*s}^{\mathrm{reh}}}\right)^{1/3}\frac{m_\phi}{2T_{\mathrm{reh}}}\,T_0$$

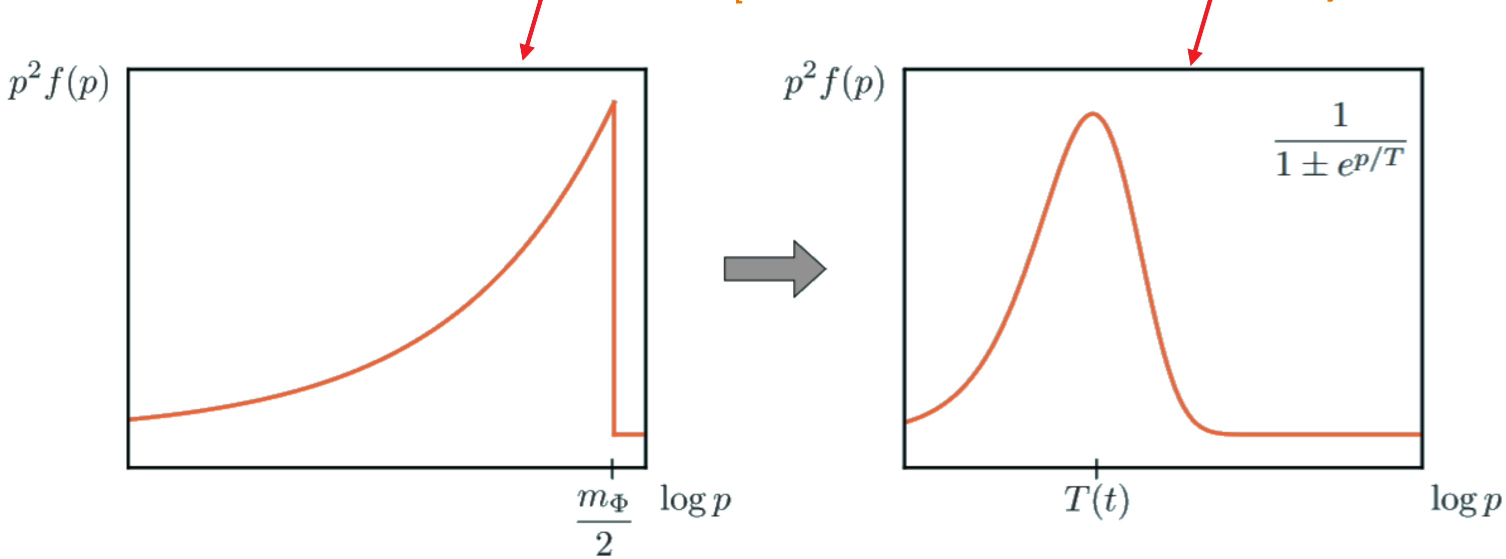


$$\Omega_\chi h^2 \;\simeq\; 0.1\left(\frac{\mathrm{Br}_\chi}{5.5\times 10^{-4}}\right)\left(\frac{m_\chi}{1\,\mathrm{MeV}}\right)\left(\frac{T_{\mathrm{reh}}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{3\times 10^{13}\,\mathrm{GeV}}{m_\phi}\right)$$

Inflaton decay : SM production



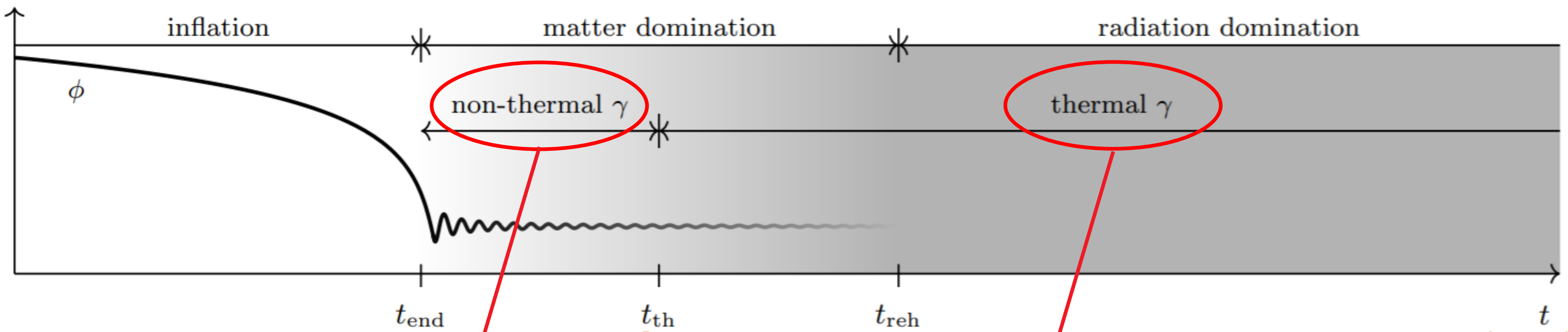
[M. A. G. Garcia & M. A. Amin Phys. Rev. D 98, 103504 (2018)]



$$\Gamma_\phi \, t_{\rm th} \simeq \alpha_{\rm SM}^{-16/5} \left(\frac{\Gamma_\phi m_\phi^2}{M_{\rm Pl}^3} \right)^{2/5}$$

[K. Mukaida & M. Yamada, JCAP 02 (2016) 003]
[K. Harigaya & K. Mukaida, JHEP 05 (2014) 006]

Inflaton decay : SM production



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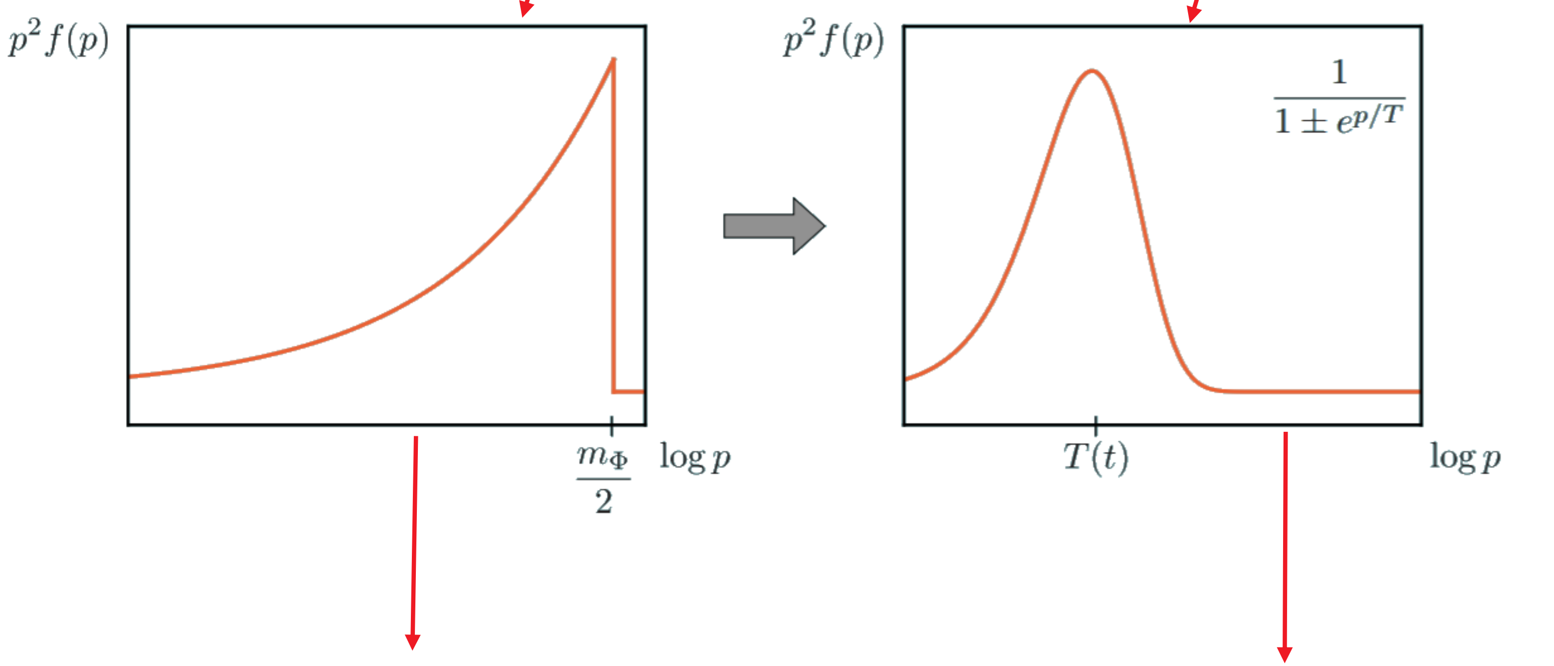
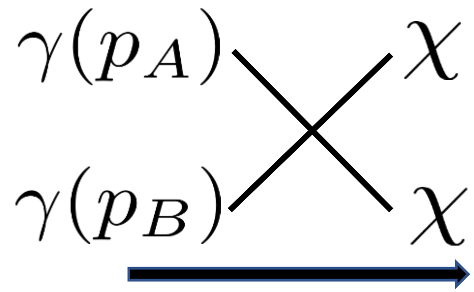


- **Non-thermal UV freeze-in**
- **Thermal UV freeze-in**

UV freeze-in from scattering

$$\int \mathrm{d}^3\boldsymbol{p} \quad \frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial |\boldsymbol{p}|} = \mathcal{C}[f_\chi(|\boldsymbol{p}|,t)]$$

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = R(t)$$



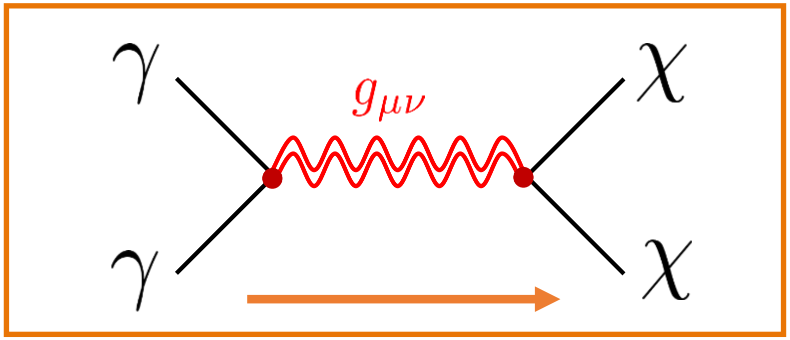
$$R(t) \equiv 2g_A g_B g_\chi^2 \int \frac{\mathrm{d}^3\boldsymbol{p}_A}{(2\pi)^3 2p_1^0}\frac{\mathrm{d}^3\boldsymbol{p}_B}{(2\pi)^3 2p_2^0}\, s\,\sigma(s)\, f_A(p_A) f_B(p_B)$$ **: Production rate**

- Assume **cross section** for $\gamma\gamma \to \chi\chi$: $\sigma(s) = \dfrac{s^{\frac{n}{2}}}{\Lambda^{n+2}}$
- If production from **thermal** SM : $R(T) \propto \dfrac{T^{n+6}}{\Lambda^{n+2}}$
- Production occurs around $t \lesssim t_{\mathrm{reh}}$ for $n > -1$

Λ **: High energy scale** s **: Center-of-mass energy**

UV freeze-in from scattering

- $n = 0$ **Low-scale SUSY** for **gravitinos** $\sigma \propto 1/M_{\rm Pl}^2$ or **axinos** $\sigma \propto 1/f_a^2$
  [V. Rychkov, A. Strumia, PRD 75 (2007) 075011 - A. Strumia, JHEP 06 (2010) 036]
- $n = 2$ **Heavy Z′** from gauge unification $\sigma \propto s/m_{Z'}^4$
  [Y. Mambrini, K. A. Olive, J. Quevillon, B. Zaldívar- PRL 110, 241306]
  Gravity mediated freeze-in $\sigma \propto s/M_{\rm Pl}^4$
  [M. Garny, M. Sandora, M. S. Sloth - PRL 116 (2016) 10, 101302
  N. Bernal, M. Dutra, Y. Mambrini, K. Olive, M. Peloso, **MP** - PRD 97 (2018) 11, 115020]
- $n = 4$ Non-SUSY **Spin-3/2 DM** + sterile neutrino $\sigma \propto s^2/(m_{3/2} m_R M_{\rm Pl})^2$
  [M A. G. Garcia, Y. Mambrini, K. A. Olive, S. Verner - PRD 102 (2020) 8, 083533]



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- $n = 6$ **High-scale SUSY** for **gravitinos** DM $\sigma \propto s^3/(m_{3/2} M_{\rm Pl})^4$
  [K. Benakli, Y. Chen, E. Dudas, Y. Mambrini - PRD 95 (2017) 9, 095002]

  Heavy **Spin-2 mediator** $\sigma \propto s^3/(m_{\tilde{h}} M_{\rm Pl})^4$
  [N. Bernal, M. Dutra, Y. Mambrini, K. Olive, M. Peloso, **MP** - PRD 97 (2018) 11, 115020]

  Energy-momentum tensor portal
  [P. Anastasopoulos, K. Kaneta, Y. Mambrini, **MP** - PRD 102 (2020) 5, 055019]

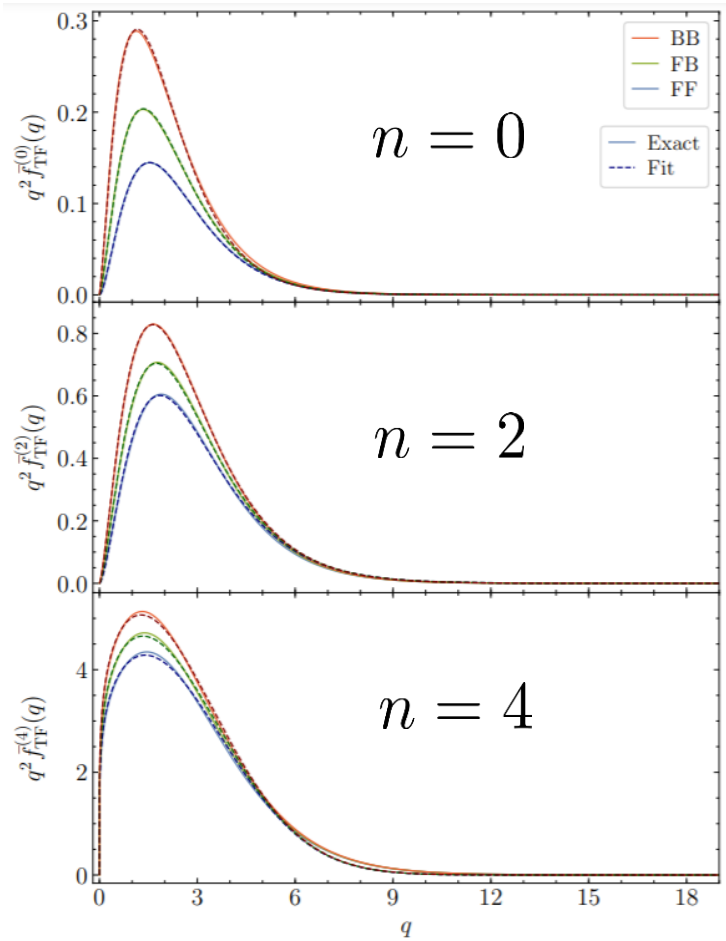
  Disformal dark matter [P. Brax, K. Kaneta, Y. Mambrini, **MP** - PRD 103 (2021) 5, 015028]

- $n > 6$ **Heavy Z′** and **non-abelian vector DM**
  [G. Bhattacharyya, M. Dutra, Y. Mambrini, **MP** - PRD (2018) 3, 035038]

UV freeze-in from scattering $(n < 6)$

$$f_\chi(p,t)\,\mathrm{d}^3\boldsymbol{p} \simeq \left(\frac{6b}{g_{*s}^{\mathrm{reh}}}\right)^{1/2} \frac{3\cdot 2^{n+6}\Gamma(\frac{n+4}{2}) g_A g_B g_\psi M_P T_{\mathrm{reh}}^{n+1}}{5(2\pi)^3 \Lambda^{n+2}} \left(\frac{a_0}{a(t)}\right)^3 T_\star^3\, \bar{f}_{\mathrm{TF}}^{(n)}(q)\,\mathrm{d}^3\boldsymbol{q}$$

$$T_\star = \left(\frac{g_{*s}^0}{g_{*s}^{\mathrm{reh}}}\right)^{1/3} T_0$$



- **Mild** dependence on progenitor **spin**
  B : Bose-Einstein
  F : Fermi-Dirac

- **Maxwell-Boltzmann $\simeq$ FB**

- Not thermal but well **fitted** by
  $$f(q) \propto q^\alpha \exp\left(-\beta\, q^\gamma\right)$$

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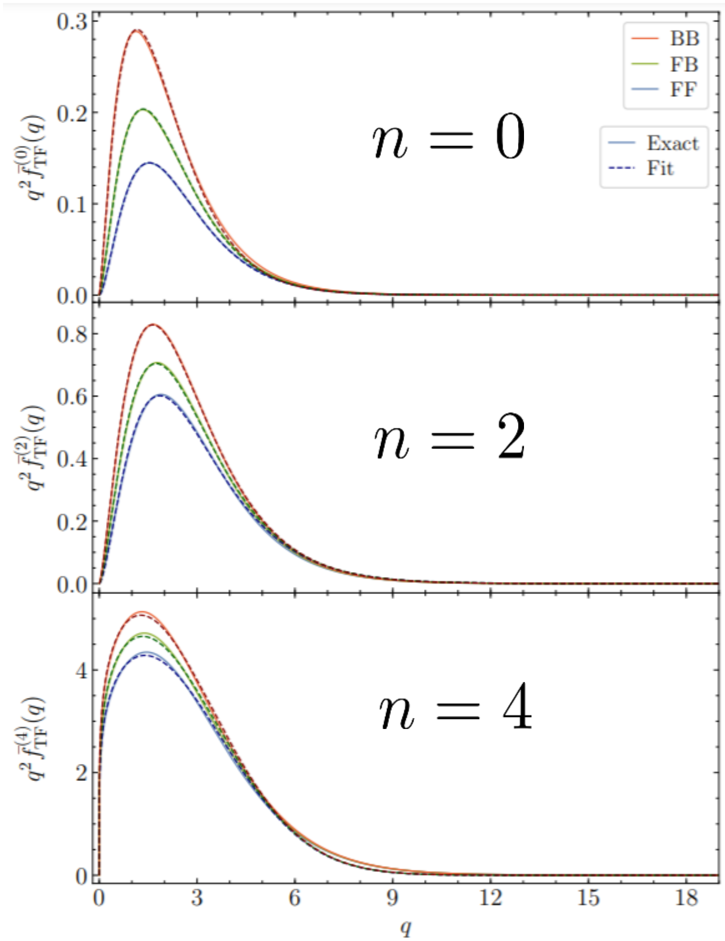
$$\Omega_\chi^{(n)} h^2 \simeq \frac{g_A g_B g_\psi g_\chi \sqrt{b}\, 2^{n+3}\Gamma(\frac{n}{2}+3)^2\zeta(\frac{n}{2}+3)^2\mathcal{S}(n)}{(6-n)(n+4)} \left(\frac{106.75}{g_{*s}^{\mathrm{reh}}}\right)^{3/2} \times \left(\frac{T_{\mathrm{reh}}}{\Lambda}\right)^{n+1} \left(\frac{10^{16}\,\mathrm{GeV}}{\Lambda}\right) \left(\frac{m_{\mathrm{NCDM}}}{1\,\mathrm{keV}}\right)$$



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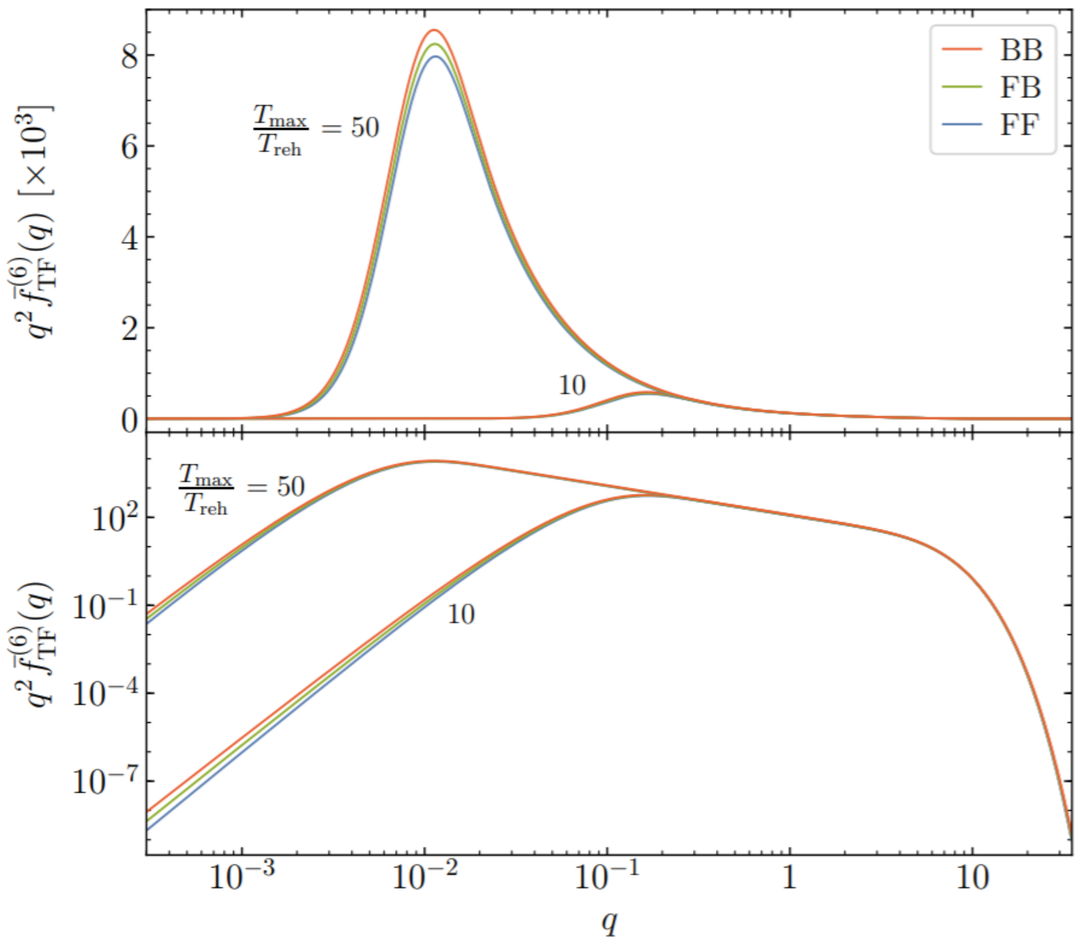
    $$f(q) \propto q^\alpha \exp(-\beta\, q^\gamma)$$

UV freeze-in from scattering $(n = 6)$

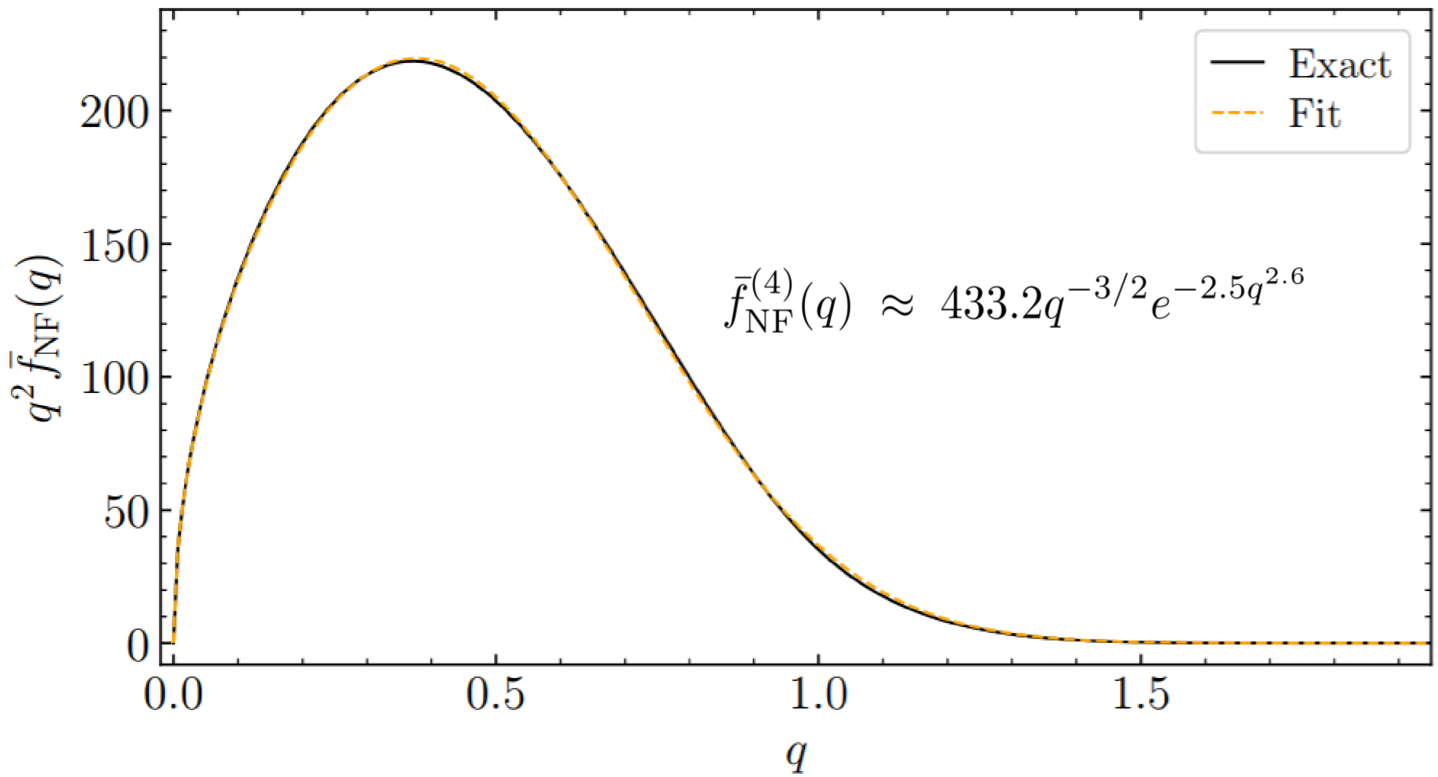
$$\Omega_\chi^{(6)} h^2 = g_A g_B g_\psi g_\chi \sqrt{b}\, \mathcal{S}(6) \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{3/2} \left(\frac{m_{\rm DM}}{1.2\,{\rm keV}}\right) \left(\frac{T_{\rm reh}}{10^6\,{\rm GeV}}\right)^7 \left(\frac{10^8\,{\rm GeV}}{\Lambda}\right)^8 \ln\left(\frac{T_{\rm max}}{T_{\rm reh}}\right)$$



Non-termal UV freeze-in $(n = 4)$

$$f_\chi(p,t)\,\mathrm{d}^3\boldsymbol{p} \simeq \frac{256\pi^2 g_\psi}{15015\Lambda^6}\left(\frac{\pi^2 b g_{*s}^{\mathrm{reh}}}{24}\right)^{13/10}\left(\frac{\alpha_{\mathrm{SM}}^{16} T_{\mathrm{reh}}^{26} M_P^{13}}{m_\phi^9}\right)^{1/5}\left(\frac{a_0}{a(t)}\right)^3 T_\star^3\,\bar{f}_{\mathrm{NF}}^{(4)}(q)\,\mathrm{d}^3\boldsymbol{q}\,,$$

$$T_\star = \frac{\alpha_{\mathrm{SM}}^{-32/15}}{2}\left(\frac{g_{*s}^0}{g_{*s}^{\mathrm{reh}}}\right)^{1/3}\left(\frac{\pi^2 b g_{*s}^{\mathrm{reh}}}{24}\right)^{2/15}\left(\frac{m_\phi}{T_{\mathrm{reh}}}\right)^{7/15}\left(\frac{m_\phi}{M_P}\right)^{16/15} T_0\,,$$



Freeze-in via decay $\phi \to A \to \chi$

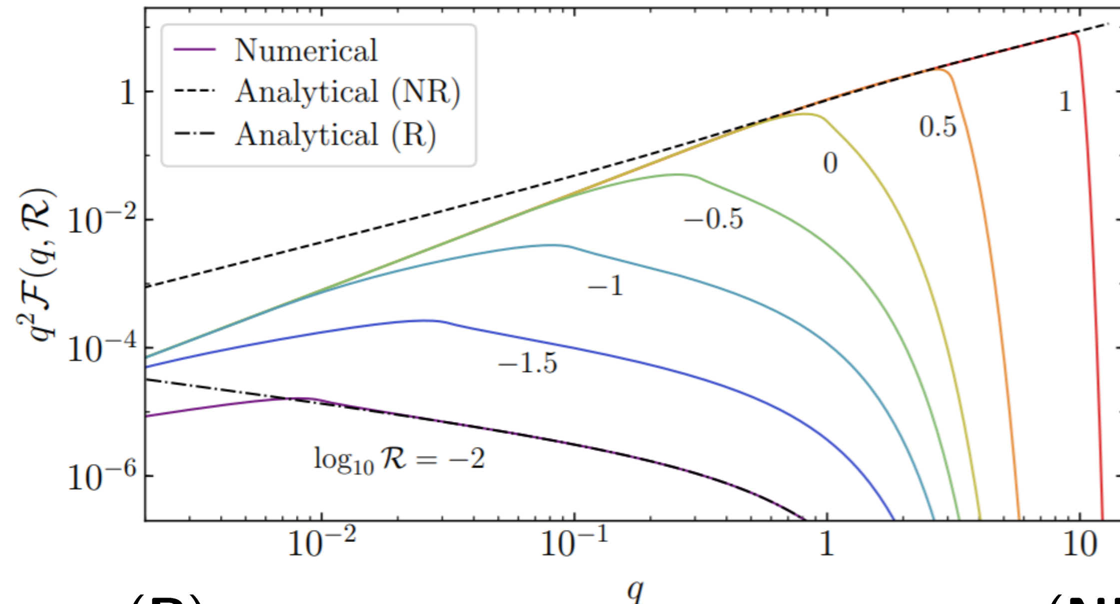
$$f_\chi(p,t)\,\mathrm{d}^3\boldsymbol{p} \;=\; \frac{24\pi^3\sqrt{10 g_{*s}^{\rm reh}}\,\mathrm{Br}_\chi \mathrm{Br}_A \Gamma_A M_P}{5 g_A m_A^2}\left(\frac{T_{\rm reh}}{m_\phi}\right)^2 \mathcal{F}(q,\mathcal{R})\left(\frac{a_0}{a(t)}\right)^3 T_\star^3\,\mathrm{d}^3\boldsymbol{q}$$

- **Relativistic** factor $\mathcal{R} \;=\; \left(\frac{g_{*s}^{\rm reh}}{g_{*s}^{\rm dec}}\right)^{1/3}\frac{m_A T_{\rm reh}}{m_\phi T_{\rm dec}} \propto \frac{m_A}{\langle p\rangle}$

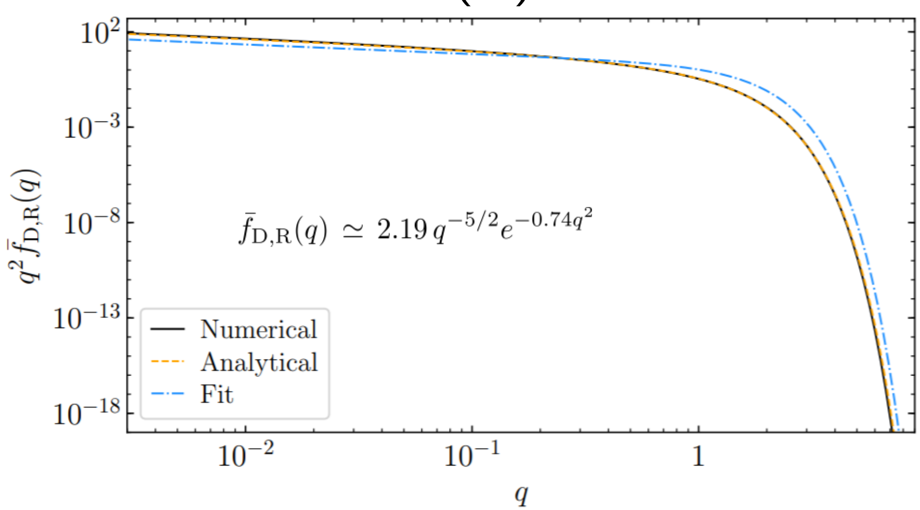
$$\mathcal{F}(q,\mathcal{R}) \;=\; q^{-2}\int_0^{\mathcal{R}}\mathrm{d}y\, y^2\int_{\left|q-\frac{y^2}{q}\right|}^{\infty}\frac{z\,\mathrm{d}z}{\sqrt{q^2+4y^2}}\,\bar{f}_R(z) \;\simeq\; \begin{cases}\bar{f}_{\rm D,NR}(q)\,, & \mathcal{R}\gg 1\,, \quad \text{Non-Relativistic } (\textbf{NR})\\ \frac{\mathcal{R}^3}{3}\bar{f}_{\rm D,R}(q)\,, & \mathcal{R}\ll 1\,. \quad \text{Relativistic } (\textbf{R})\end{cases}$$

$$n_\chi(t) \;\simeq\; g_{*s}^0\,\mathrm{Br}_\chi\mathrm{Br}_A\left(\frac{g_\chi}{g_A}\right)\left(\frac{T_{\rm reh}}{m_\phi}\right)\left(\frac{a_0}{a(t)}\right)^3 T_0^3 \times \begin{cases}\left(\frac{g_{*s}^{\rm reh}}{g_{*s}^{\rm dec}}\right)^{1/6}, & \mathcal{R}\gg 1\,,\\ \left(\frac{g_{*s}^{\rm reh}}{g_{*s}^{\rm dec}}\right)^{1/4}, & \mathcal{R}\ll 1\,.\end{cases}$$

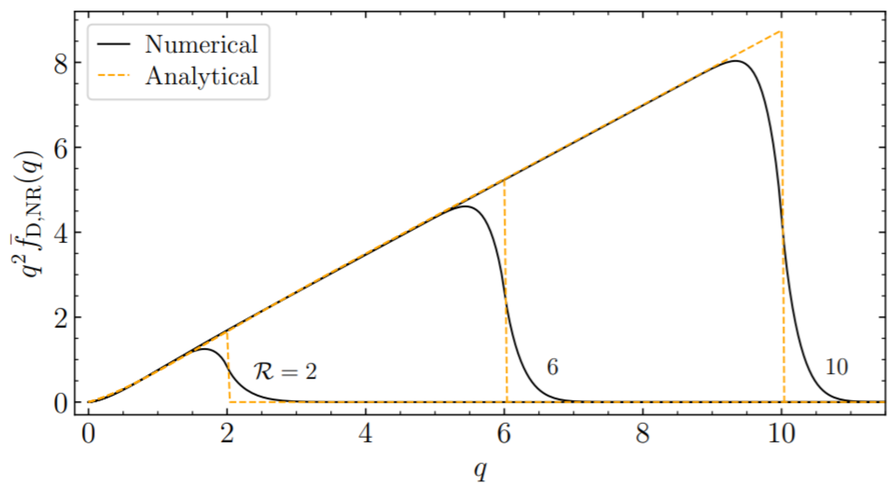
Freeze-in via decay $\phi \to A \to \chi$



(R)

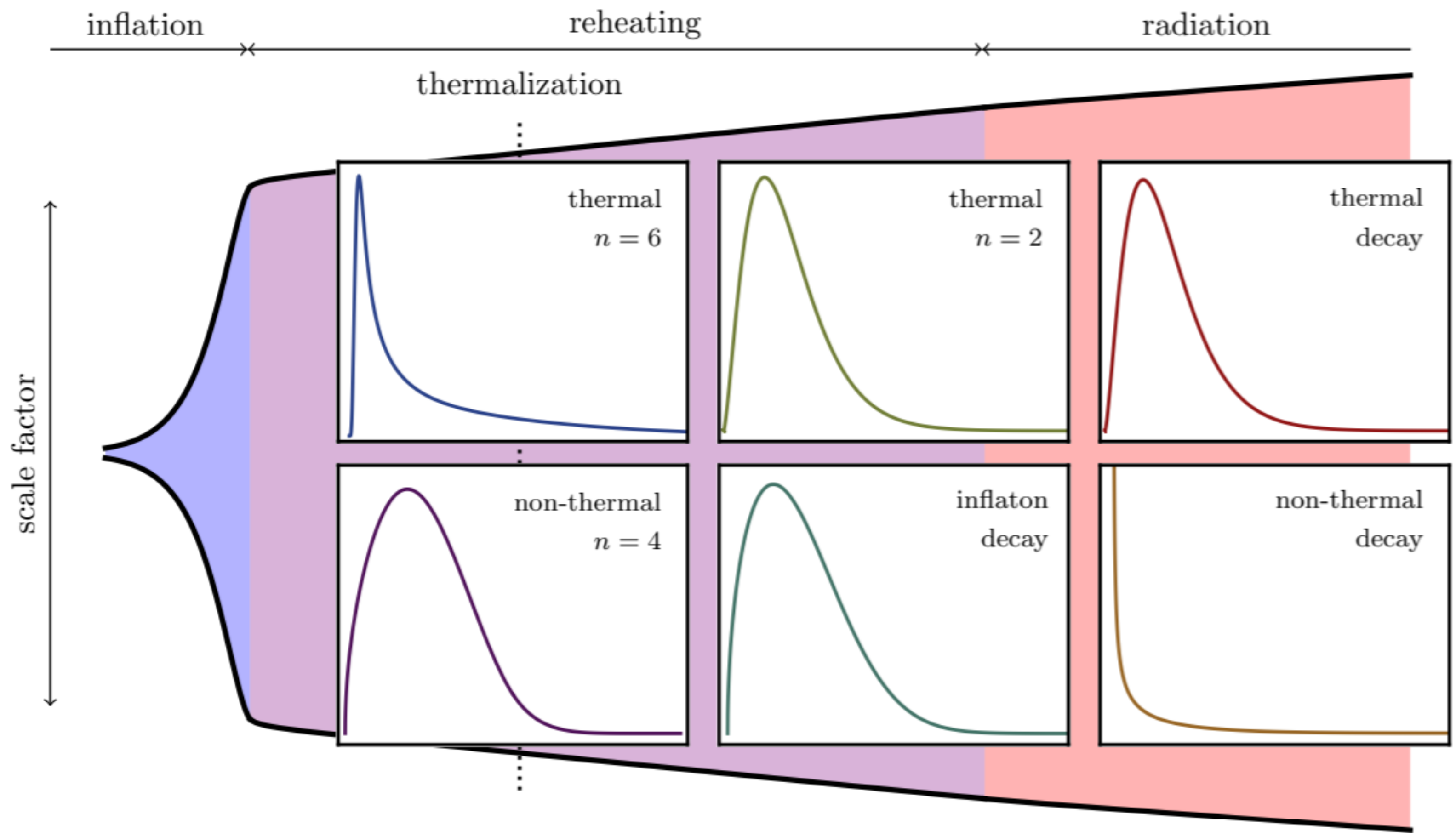


(NR)



Summary

- **Phase space distribution of out-of-equilibrium DM**



Cosmological imprint

Cosmological imprint

- **Cosmological role** of out-of-equilibrium dark matter via

$$\bar{\rho} = 4\pi \left(\frac{T_\star}{a}\right)^4 \int q^2 \epsilon \bar{f}(q)\, \mathrm{d}q$$

energy-density

$$\bar{P} = \frac{4\pi}{3} \left(\frac{T_\star}{a}\right)^4 \int q^2 \frac{q^2}{\epsilon} \bar{f}(q)\, \mathrm{d}q$$

pressure

$$q \equiv \frac{p\, a(t)}{T_\star}$$ **: comoving momentum**

$$\epsilon = \sqrt{q^2 + \left(\frac{m_{\mathrm{DM}}\, a}{T_\star}\right)^2}$$

- Define $w \equiv \bar{P}/\bar{\rho}$ **: equation-of-state parameter**
- In pure $\Lambda\mathrm{CDM}$: $w = 0$ precisely (**Cold = pressureless**)
- But $w \neq 0$! ➡ **Non-Cold Dark Matter** cosmology

Non-Cold Dark Matter $w \neq 0$

- Expanding quantities around **homogenous background**

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = \bar{f}(|\boldsymbol{p}|, \tau)[1 + \Psi(\boldsymbol{x}, \boldsymbol{p}, \tau)]$$

- In matter domination, matter **overdensities** δ follow

$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - \frac{k^2}{k_{\mathrm{FS}}^2}\right)\delta = 0 \qquad w \ll 1$$

where $k_{\mathrm{FS}}^2(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w(a)}$ is the **Free-Streaming wavenumber**

$\mathrm{d}\tau \equiv a\,\mathrm{d}t$: **Conformal time** τ

$\mathcal{H} \equiv a\,H$: **Conformal Hubble rate**

$w \equiv \bar{P}/\bar{\rho}$: **Equation-of-state parameter**

[C. Ma & E. Bertschinger. ApJ 455 (1995) 7-25]
[J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]
[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]
[G. Ballesteros, M. A. G. Garcia & **MP**, 2011.13458]

Non-Cold Dark Matter $w \neq 0$

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where $k_{\rm FS}^2(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w(a)}$ is the **Free-Streaming wavenumber**

- If $w = 0$ all modes grow "**democratically**" : **CDM** limit
  $w \neq 0$ **cutoff in power spectrum** at $k_{\rm H}(a) \equiv \left[\int_0^a \frac{1}{k_{\rm FS}(\tilde{a})} \frac{{\rm d}\tilde{a}}{\tilde{a}}\right]^{-1}$

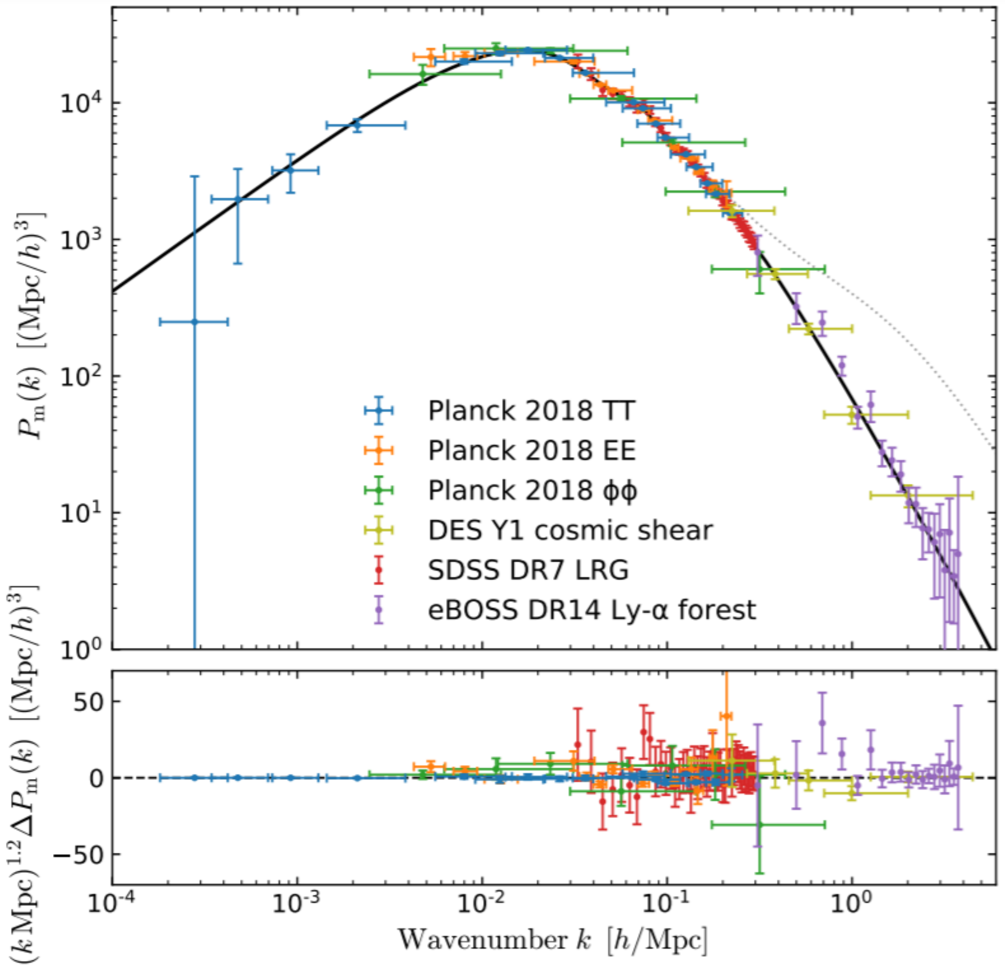
- **Only w controls the cutoff scale!**

$d\tau \equiv a\,dt$: **Conformal time τ**
$\mathcal{H} \equiv a\,H$: **Conformal Hubble rate**
$w \equiv \bar{P}/\bar{\rho}$: **Equation-of-state parameter**

[C. Ma & E. Bertschinger. ApJ 455 (1995) 7-25]
[J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]
[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]
[G. Ballesteros, M. A. G. Garcia & **MP**, 2011.13458]

Non-Cold Dark Matter $w \neq 0$

- Small scales of power spectrum probed by **Lyman-alpha forest**

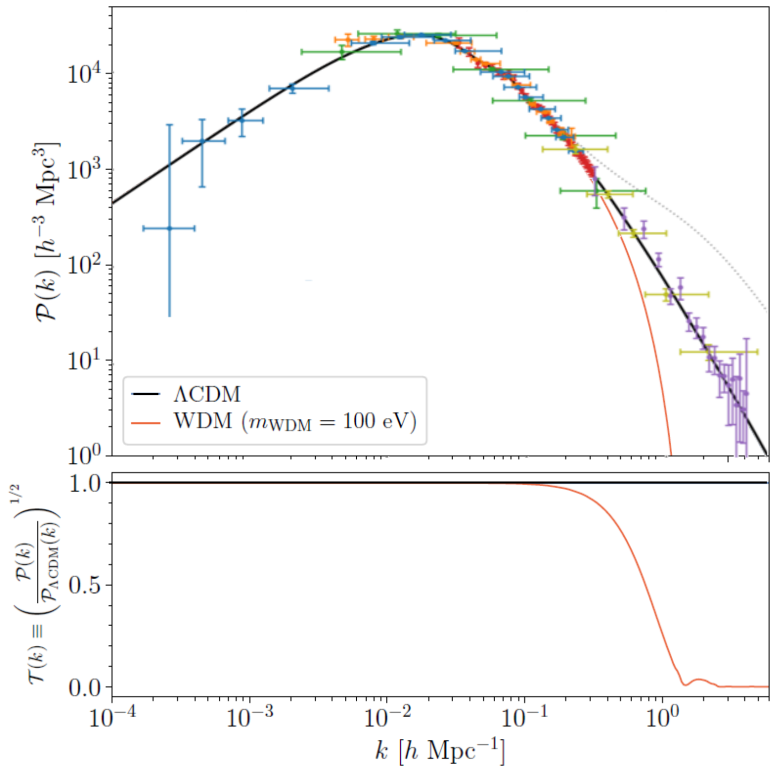


[S. Chabanier, M. Millea, N. Palanque-Delabrouille, MNRAS 489 (2019) 2, 2247-2253]

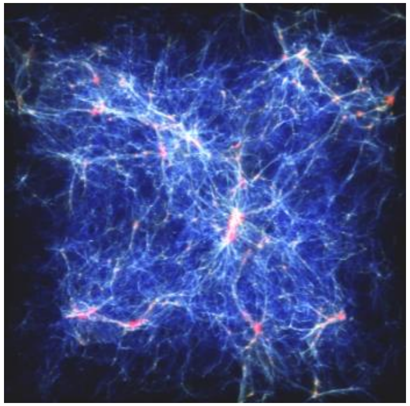
Non-Cold Dark Matter $w \neq 0$

- Lyman-alpha forest constraints Warm Dark Matter (**WDM**)

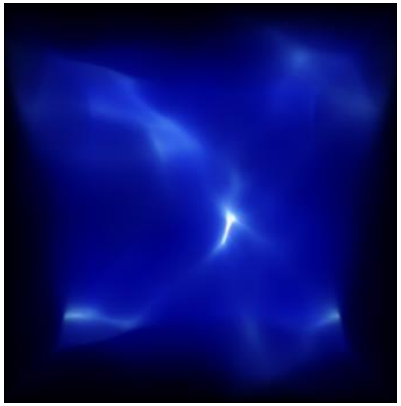
$$\bar{f}_{\rm WDM}(q) = \frac{1}{1 + e^{q/T_{\rm WDM}}} \Rightarrow \Omega_{\rm WDM} h^2 \simeq \left(\frac{m_{\rm WDM}}{94\ {\rm eV}}\right)\left(\frac{T_{\rm WDM}}{T_\nu}\right)^3 \simeq 0.12$$



ΛCDM



WDM



$m_{\rm WDM} = 100\,{\rm eV}$

[J. Baur, N. Palanque-Delabrouille, C. Yèche, C. Magneville, M. Viel, JCAP 08 (2016) 012]

How warm is Non-Cold Dark Matter?

- From **Lyman-alpha** forest $m_{\rm WDM}^{\rm Ly-\alpha} = (1.9 - 5.3)$ keV at 95% C.L.

[Braur et al. JCAP 08 (2016) 012 – Iršič et al. PRD 96 (2017) 2, 023522
Palanque Delabrouille et al. JCAP 04 (2020) 038 – Viel et al. PRD 88 (2013) 043502
Viel et al. PRD 71 (2005) 063534 – Narayanan et al. ApJ 543 (2000) L103-L106]

$$w_{\rm WDM}(a) \simeq 6\times 10^{-15}\, a^{-2} \left(\frac{\rm keV}{m_{\rm WDM}}\right)^{8/3} \Rightarrow w_{\rm WDM}(a=1) < 10^{-15}$$

- Constraints much **stronger** than **CMB**! $w_{\rm WDM}(a=1) < 10^{-10}$

[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]

- How **cold** are **WIMPs** ?

$$w(a) \simeq 10^{-29} \left(\frac{1}{a^2}\right)\left(\frac{20\,T_F}{m_{\rm DM}}\right)\left(\frac{100\ \rm GeV}{m_{\rm DM}}\right)^2\left(\frac{100}{g_*^F}\right)^{2/3} \Rightarrow$$

- **How to translate Lyman-alpha WDM bounds on any DM ?**

$$\boxed{w(m_{\rm DM}) = w_{\rm WDM}(m_{\rm WDM}^{\rm Ly-\alpha})}$$

[S. Colombi, S. Dodelson, L. M. Widrow ApJ. 458 (1996) 1 - Kamada, N. Yoshida, K. Kohri, T. Takahashi JCAP 03 (2013) 008
K. J. Bae, R. Jinno, A. Kamada, K. Yanagi JCAP 03 (2020) 042 - A. Kamada & K. Yanagi JCAP 1911 (2019) 029]

How warm is Non-Cold Dark Matter?

$w -$ matching

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_\star^2}{3 m_{\rm DM}^2} \frac{\langle q^2 \rangle}{a^2}$$

$$m_{\rm DM} = m_{\rm WDM}^{\rm Ly-\alpha} \left(\frac{T_\star}{T_{\rm WDM}} \right) \sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\rm WDM}}}$$

- Compute **2nd moment of distribution** + **determine** $T_\star$

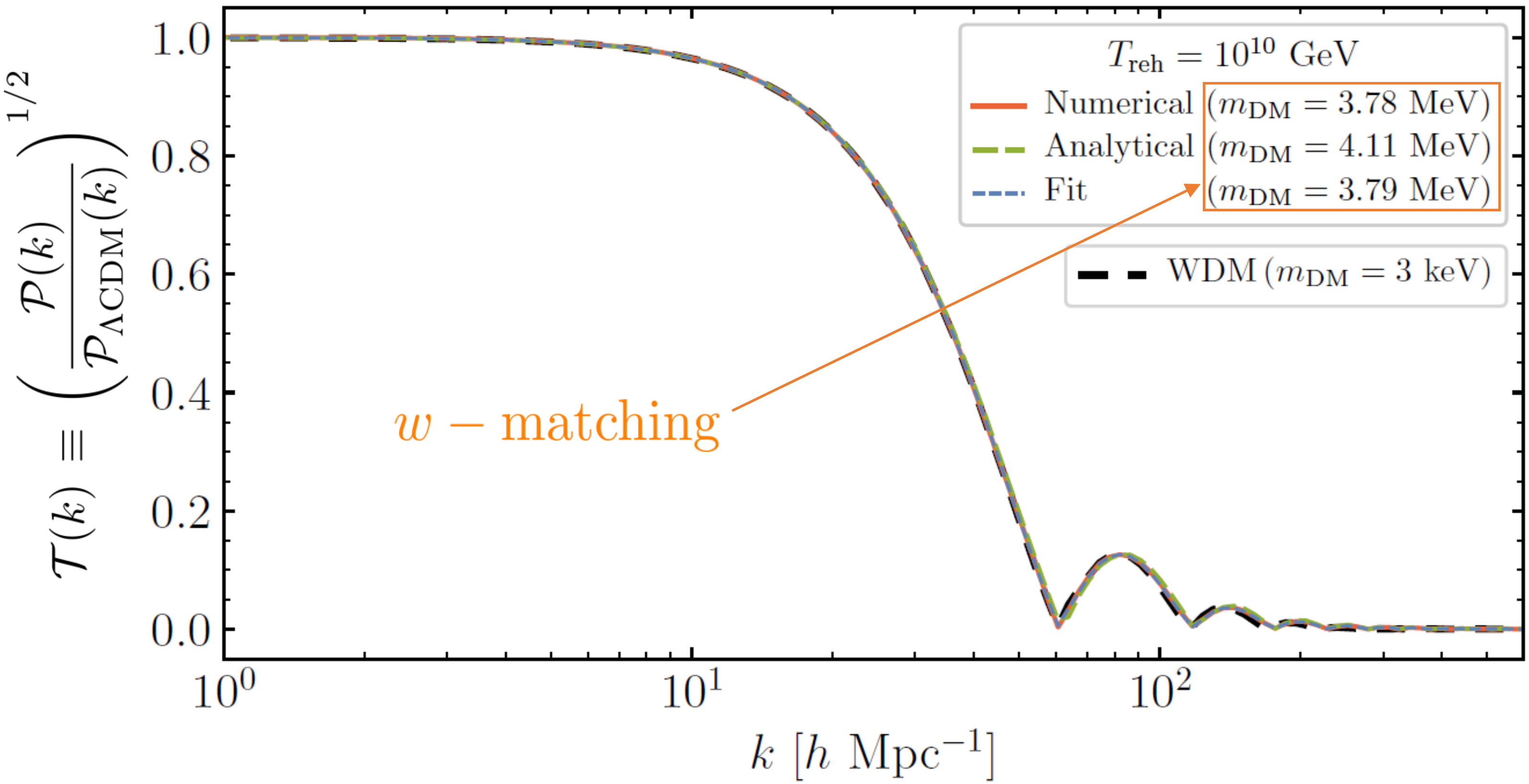
- If **distribution** can be fitted by $f(q) \propto q^\alpha \exp\left(-\beta\, q^\gamma\right)$

$w -$ matching ➡ $m_{\rm DM} \simeq 7.56 \text{ keV} \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3 \text{ keV}} \right)^{4/3} \left(\frac{\langle p \rangle_0}{T_0} \right) \sqrt{\frac{\Gamma\left(\frac{3+\alpha}{\gamma}\right) \Gamma\left(\frac{5+\alpha}{\gamma}\right)}{\Gamma^2\left(\frac{4+\alpha}{\gamma}\right)}}$

How warm is Non-Cold Dark Matter?

- Example: **inflaton decay** case computed using **CLASS**

[D. Blas, J. Lesgourgues & T. Tram JCAP 07 (2011) 034 - J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]



- **Excellent agreement** with w – matching for **all distributions**!

Inflaton decay

- **Lyman-alpha** bounds translate into

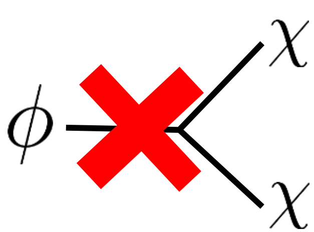
$$m_{\rm DM} \gtrsim \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ \rm keV}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \left(\frac{m_\phi}{3\times 10^{13}\ \rm GeV}\right) \left(\frac{10^{10}\ \rm GeV}{T_{\rm reh}}\right) \begin{cases} 3.78\ \rm MeV\,, & \text{Numerical}\,, \\ 4.11\ \rm MeV\,, & \text{Analytical}\,, \\ 3.79\ \rm MeV\,, & \text{Fit}\,. \end{cases}$$

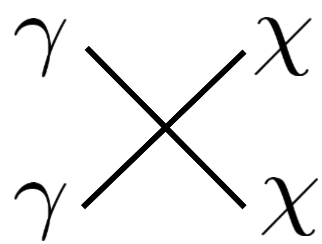
- For **low reheating temperature** $T_{\rm reh} \ll m_\phi$

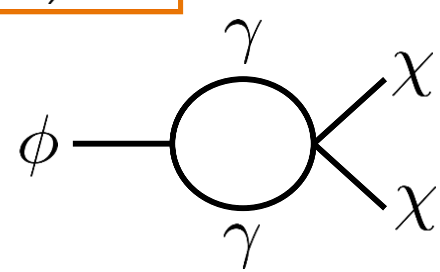
$$\boxed{m_{\rm DM} \gtrsim \rm EeV}$$

- **Combining** with **relic density** condition

$$\boxed{{\rm Br}_\chi < 1.5\times 10^{-4} \left(\frac{g_{*s}^{\rm reh}}{106.5}\right)^{1/3} \left(\frac{3\,\rm keV}{m_{\rm WDM}^{\rm Ly-\alpha}}\right)^{4/3}}$$

- Even if 

, since 

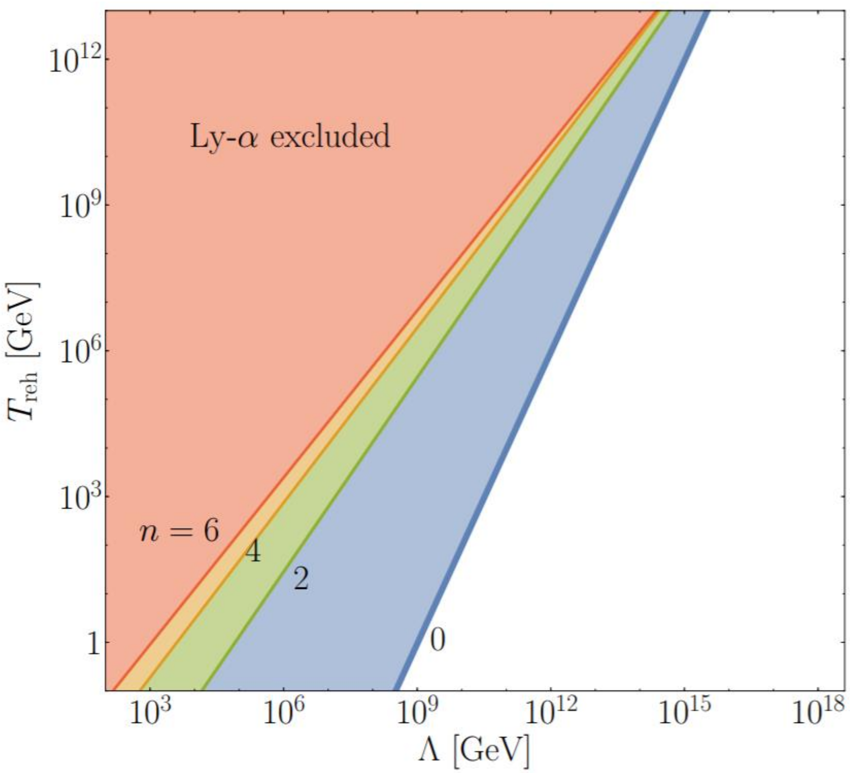
then 

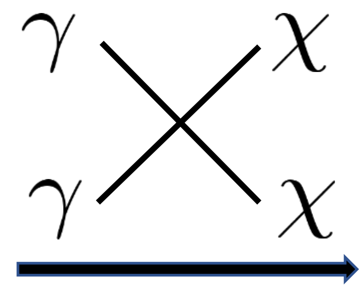
[K. Kaneta, Y. Mambrini & Keith A. Olive Phys.Rev.D 99 (2019) 6, 063508]

UV freeze-in via scattering

$$m_{\rm DM} \gtrsim \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ {\rm keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \begin{cases} 7.27\ (7.17)\ {\rm keV}\,, & {\rm FF} \quad {\rm Numerical\ (Fit)}\,, \quad n=0 \\ 8.48\ (8.73)\ {\rm keV}\,, & {\rm FF} \quad {\rm Numerical\ (Fit)}\,, \quad n=2 \\ 8.52\ (8.05)\ {\rm keV}\,, & {\rm FF} \quad {\rm Numerical\ (Fit)}\,, \quad n=4 \end{cases}$$

- **Combine** with **relic density** condition





DM production

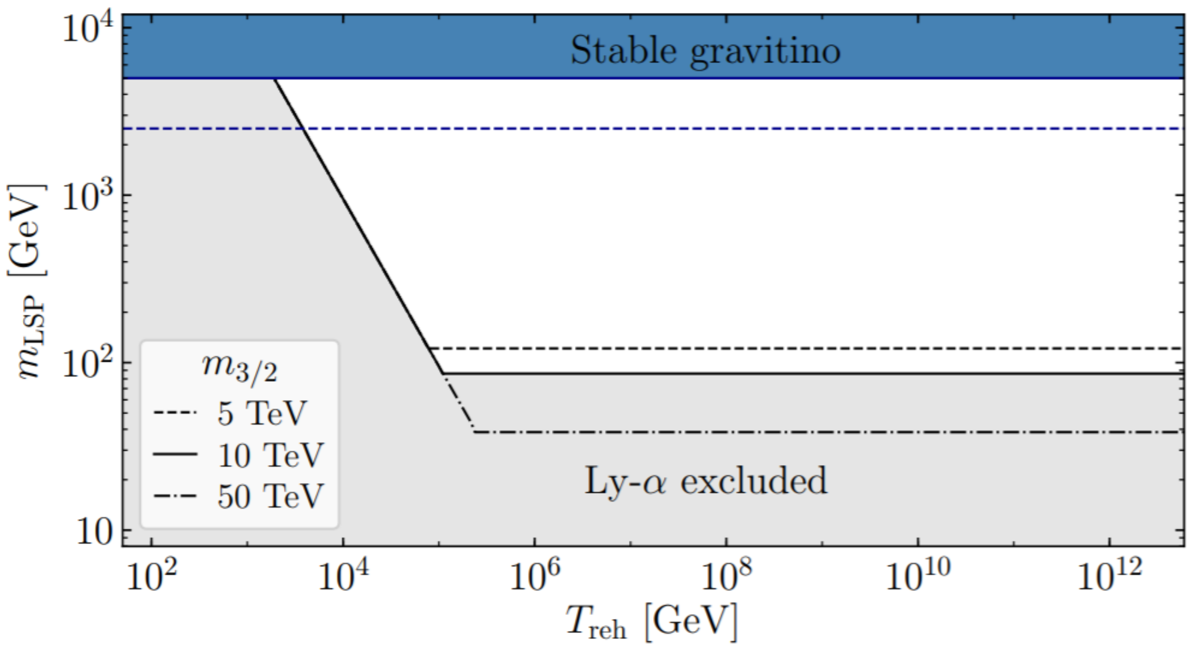
$$\sigma(s) = \frac{s^{\frac{n}{2}}}{\Lambda^{n+2}}$$

- **Apply to any UV freeze-in model!**

Freeze-in via decay $\phi \to A \to \chi$

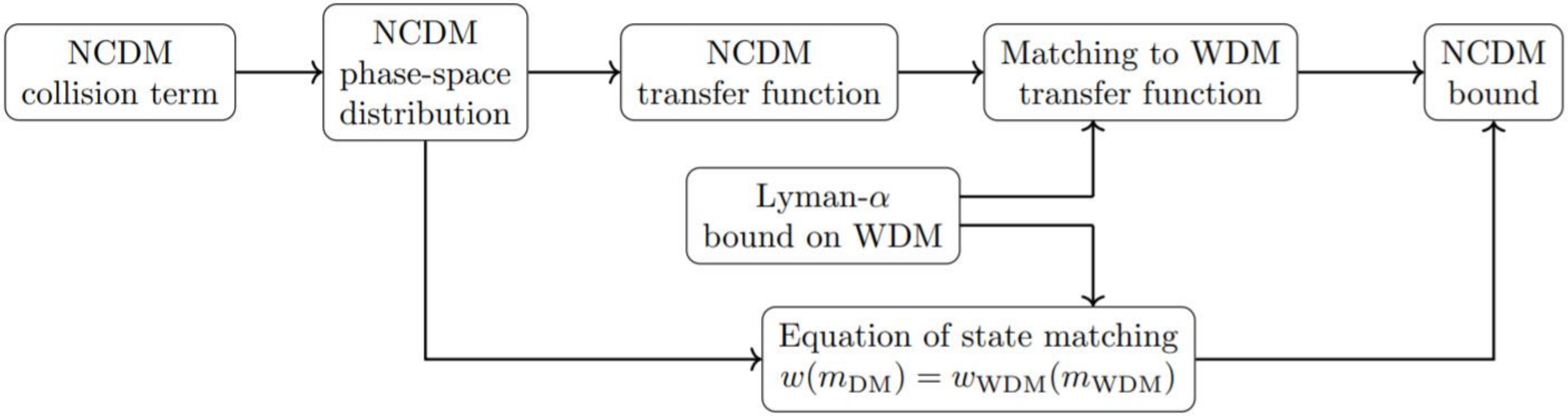
- **Example :** $\phi \to \Psi_{3/2} \to \mathrm{LSP}$

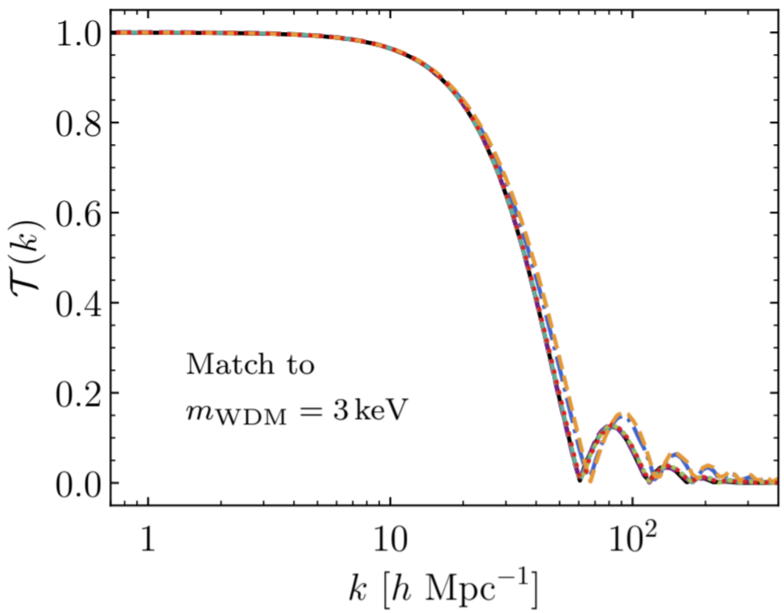
$$\Gamma_{3/2} = \frac{193}{384\pi}\frac{m_{3/2}^3}{M_P^2}\,.$$



$$m_{\rm LSP} \gtrsim \begin{cases} 86\,\mathrm{GeV}\left(\dfrac{m_{\rm WDM}}{3\,\mathrm{keV}}\right)^{4/3}\left(\dfrac{10\,\mathrm{TeV}}{m_{3/2}}\right)^{1/2}, & T_{\rm reh} \gg 10^5\,\mathrm{GeV}\left(\dfrac{m_{3/2}}{10\,\mathrm{TeV}}\right)^{1/2}, \\ 95\,\mathrm{GeV}\left(\dfrac{m_{\rm WDM}}{3\,\mathrm{keV}}\right)^{4/3}\left(\dfrac{10^5\,\mathrm{GeV}}{T_{\rm reh}}\right), & T_{\rm reh} \ll 10^5\,\mathrm{GeV}\left(\dfrac{m_{3/2}}{10\,\mathrm{TeV}}\right)^{1/2}. \end{cases}$$

Summary : cosmological imprint





Take home message

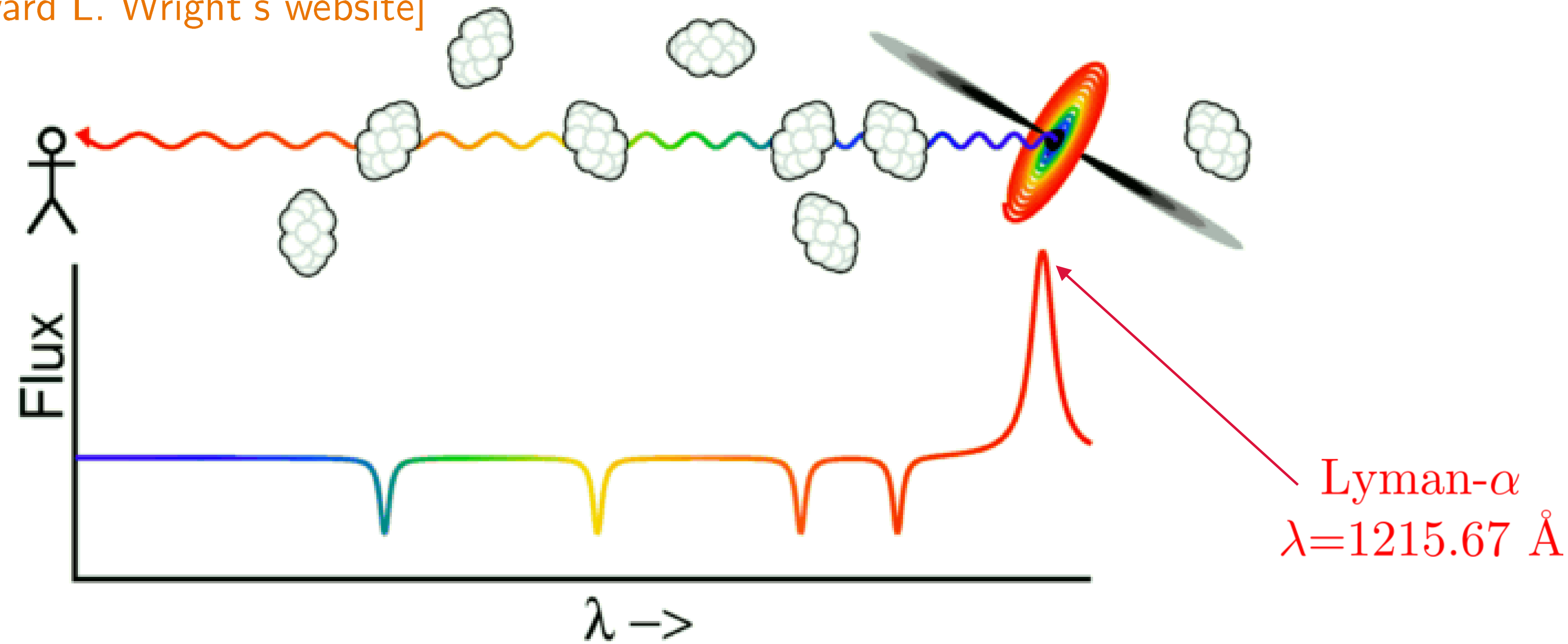
- **Out-of-equilibrium DM** can be produced **after inflation**
- Most **distributions** considered can be **fitted** by
$$f(q) \propto q^{\alpha} \exp\left(-\beta\, q^{\gamma}\right)$$
- Lyman-alpha is a **powerful tool** to **probe out-of-equilibrium dark matter** and **early universe dynamics**
- DM produced from modulus decay, from decay of thermalized particle + much more... **in the paper**!
- **Dark matter is cold**.

Thank you for your attention

Back-up Slides

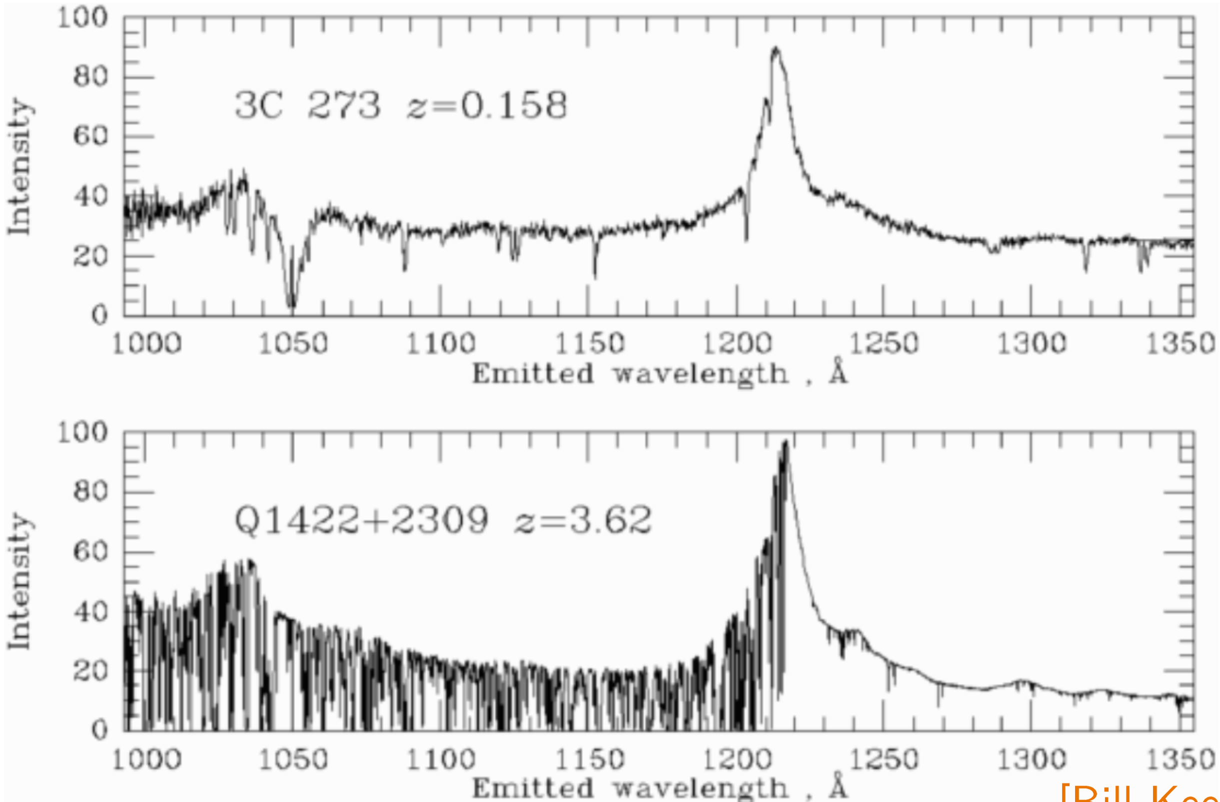
Lyman-alpha forest

[Edward L. Wright's website]



- Quasi-Stellar Objects (QSO) are luminous astrophysical objects powered by gas spiraling at high velocity into an massive black hole
- Light emitted by distant QSO is absorbed in foreground structures
- Allows for a 1D measure of overdensities along line of sight

Lyman-alpha forest



[Bill Keel's website]

- Comparison of QSO spectra at low and high redshift in QSO rest frame

NCDM Cosmology

- Expand around (homogenous) **background quantities**

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = \bar{f}(|\boldsymbol{p}|, \tau)[1 + \Psi(\boldsymbol{x}, \boldsymbol{p}, \tau)]$$

- Expand **fluctuations** in term of **Legendre polynomials**

$$\Psi(\boldsymbol{k}, \hat{\boldsymbol{n}}, q, \tau) = \sum_{\ell=0}^{\infty} (-i)^{\ell} (2\ell + 1) \Psi_{\ell}(\boldsymbol{k}, q, \tau) P_{\ell}(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}})$$

- Express **fluctuations** in terms of **Legendre coefficients**

$$\delta\bar{\rho} = 4\pi \left(\frac{T_\star}{a}\right)^4 \int q^2 \epsilon \bar{f}(q) \Psi_0 \, \mathrm{d}q, \qquad \text{energy density fluctuation}$$

$$\delta\bar{P} = \frac{4\pi}{3} \left(\frac{T_\star}{a}\right)^4 \int q^2 \frac{q^2}{\epsilon} \bar{f}(q) \Psi_0 \, \mathrm{d}q, \qquad \text{pressure (density) fluctuation}$$

$$(\bar{\rho} + \bar{P})\theta = 4\pi k \left(\frac{T_\star}{a}\right)^4 \int q^3 \bar{f}(q) \Psi_1 \, \mathrm{d}q, \qquad \text{velocity divergence}$$

$$(\bar{\rho} + \bar{P})\sigma = \frac{8\pi k}{3} \left(\frac{T_\star}{a}\right)^4 \int q^2 \frac{q^2}{\epsilon} \bar{f}(q) \Psi_2 \, \mathrm{d}q, \qquad \text{anisotropic stress.}$$

NCDM Cosmology

- The phase space distribution satisfies **collisionless Boltzmann equation**

$$\frac{\partial f}{\partial \tau} + \frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\partial f}{\partial x^i} + \frac{\mathrm{d}q}{\mathrm{d}\tau}\frac{\partial f}{\partial q} + \frac{\mathrm{d}n_i}{\mathrm{d}\tau}\frac{\partial f}{\partial n_i} = 0\,,$$

- **Plugging** distribution expansion in **Legendre polynomials** give

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 + \frac{1}{6}\dot{h}\frac{\mathrm{d}\ln\bar{f}}{\mathrm{d}\ln q}\,,$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}\Big(\Psi_0 - 2\Psi_2\Big)\,,$$

$$\dot{\Psi}_2 = \frac{qk}{5\epsilon}\Big(2\Psi_1 - 3\Psi_3\Big) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{\mathrm{d}\ln\bar{f}}{\mathrm{d}\ln q}\,,$$

$$\dot{\Psi}_\ell = \frac{qk}{(2\ell+1)\epsilon}\Big(\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}\Big)\,, \quad [\ell \geq 3]$$

$$\mathrm{d}s^2 = a(\tau)\big(-\mathrm{d}\tau^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\big)$$

in **synchronous** gauge $h \equiv h_{ii}$

For a non-relativistic species, higher multipoles are typically suppressed by (positive) powers of $q/\epsilon \sim p/m_{\mathrm{DM}}$, making any Ψ_ℓ with $\ell \geq 2$ much smaller than Ψ_0 and Ψ_1. In this case, the Boltzmann hierarchy can be truncated imposing $\Psi_\ell = 0$ for $\ell > 1$. In this (non-relativistic) case Ψ_0 depends only mildly on the variable q, and the integrals are dominated by the low $q \ll \epsilon$ regime so that we can identify $\delta P/\delta\rho \simeq \bar{P}/\bar{\rho} = w$.

NCDM Cosmology

- **Neglecting higher multipoles**, for very **non-relativistic** DM, integrating over momenta gives

$$\dot{\delta} = -(1+w)\left(\theta + \frac{\dot{h}}{2}\right) - 3\mathcal{H}\left(\hat{c}_s^2 - w\right)\delta + 9\mathcal{H}^2(1+w)\left(\hat{c}_s^2 - c_a^2\right)\frac{\theta}{k^2},$$

$$\dot{\theta} = -\mathcal{H}\left(1 - 3\hat{c}_s^2\right)\theta + \frac{\hat{c}_s^2}{1+w}k^2\delta,$$

- In **matter domination**, from **Einstein equations**, metric perturbation follow

$$\ddot{h} + \mathcal{H}\dot{h} + 3(1+3w)\mathcal{H}^2\delta = 0,$$

- Which can be translated to evolution of **matter density fluctuations**

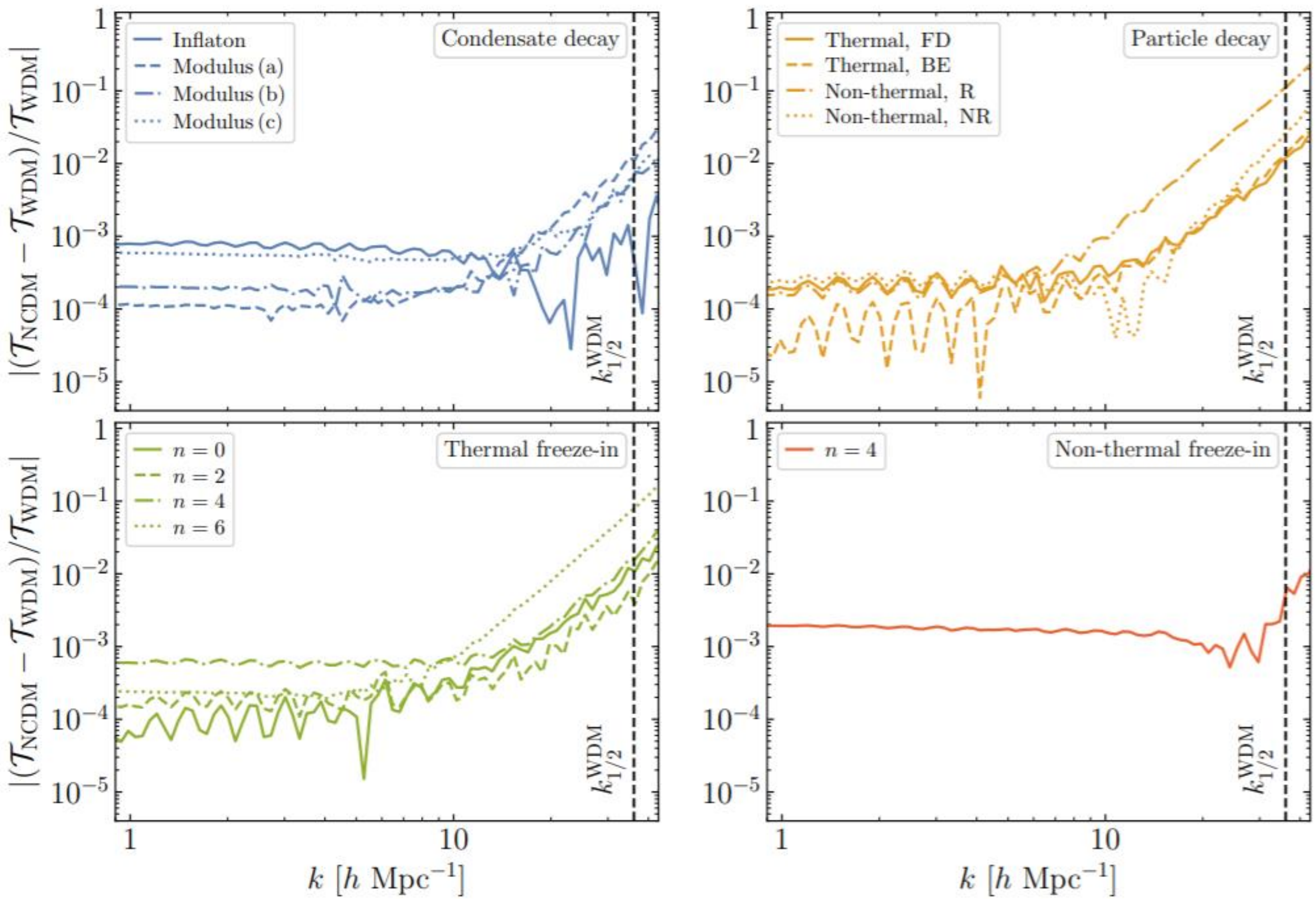
$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0.$$

General phase space distribution

$$f(q) \propto q^{\alpha} \exp\left(-\beta\, q^{\gamma}\right)$$

Scenario		α	β	γ
Inflaton decay		-3/2	0.74	1.00
Moduli decay	during reheating	-3/2	1.00	3/2
	after reheating	-1.00	1.00	2.00
Thermal decay		-1/2	1.00	1.00
Non-thermal decay	non-relativistic	-	-	-
	relativistic	-5/2	0.74	2.00
UV Freeze-in ($n=0$)	BB	0.70	1.13	1.00
	FB	0.51	1.10	1.00
	FF	0.29	1.11	1.00
UV Freeze-in ($n=2$)	BB	0.51	0.91	1.00
	FB	0.42	0.90	1.00
	FF	0.33	0.90	1.00
UV Freeze-in ($n=4$)	BB	0.21	0.06	1.98
	FB	0.21	0.06	2.04
	FF	0.21	0.05	2.10
UV Freeze-in ($n=6$)	BB	-	-	-
	FB	-	-	-
	FF	-	-	-
Non-thermal UV Freeze-in		-3/2	2.5	2.6

Precision on transfer functions



Contribution to N_{eff} ?

$$\Delta N_{\text{eff}} = \frac{8}{7}\left(\frac{T}{T_\nu}\right)^4 \frac{\rho_\chi - m_{\text{DM}} n_\chi}{\rho_\gamma}$$
$$= \frac{8\pi\Omega_\chi}{7\Omega_\gamma}\left(\frac{g_{*s}(T)}{g_{*s}^0}\right)^{4/3}\left(\frac{T}{T_\nu}\right)^4\left(\frac{T_\star}{m_{\text{DM}}}\right)$$
$$\times\left[\left\langle\sqrt{q^2+\left(\frac{g_{*s}^0}{g_{*s}(T)}\right)^{2/3}\left(\frac{m_{\text{DM}}}{T_\star}\right)^2\left(\frac{T_0}{T}\right)^2}\right\rangle - \left(\frac{g_{*s}^0}{g_{*s}(T)}\right)^{1/3}\left(\frac{m_{\text{DM}}}{T_\star}\right)\left(\frac{T_0}{T}\right)\right].$$

- **Saturating** the Lyman-alpha bound gives

$$\Delta N_{\text{eff,max}} \simeq \frac{1.4\times 10^{-4}}{\sqrt{\langle q^2\rangle}}\left(\frac{g_{*s}(T)}{g_{*s}^0}\right)^{4/3}\left(\frac{\Omega_\chi h^2}{0.1}\right)\left(\frac{3\,\text{keV}}{m_{\text{WDM}}}\right)^{4/3}\left(\frac{T}{T_\nu}\right)^4$$
$$\times\left[\left\langle\sqrt{q^2+\mu_*(T)^2}\right\rangle - \mu_*(T)\right],$$

$$\mu_*(T) \equiv \sqrt{\langle q^2\rangle}\left(\frac{g_{*s}^0}{g_{*s}(T)}\right)^{1/3}\left(\frac{3\,\text{keV}}{m_{\text{WDM}}}\right)^{4/3}\left(\frac{7.56\,\text{keV}}{T}\right).$$

$$\boxed{\Delta N_{\text{eff}}(T_{\text{BBN}}) \lesssim 5.4\times 10^{-4}\left(\frac{\langle q\rangle}{\sqrt{\langle q^2\rangle}}\right)\left(\frac{\Omega_\chi h^2}{0.1}\right)\left(\frac{3\,\text{keV}}{m_{\text{WDM}}}\right)^{4/3},}$$