RG of GR from on-shell amplitudes

with D. Haslehner, M. Ruhdorfer, J. Serra & A. Weiler

arXiv 2109.06191

Pietro Baratella, TUM

Outline

- Motivation (what do we do and what for?)
- Formalism (why amplitudes?)
- Results of the analysis:
    - (modified) helicity rules
    - non-renormalization theorems
    - computing the RG

RG of GR

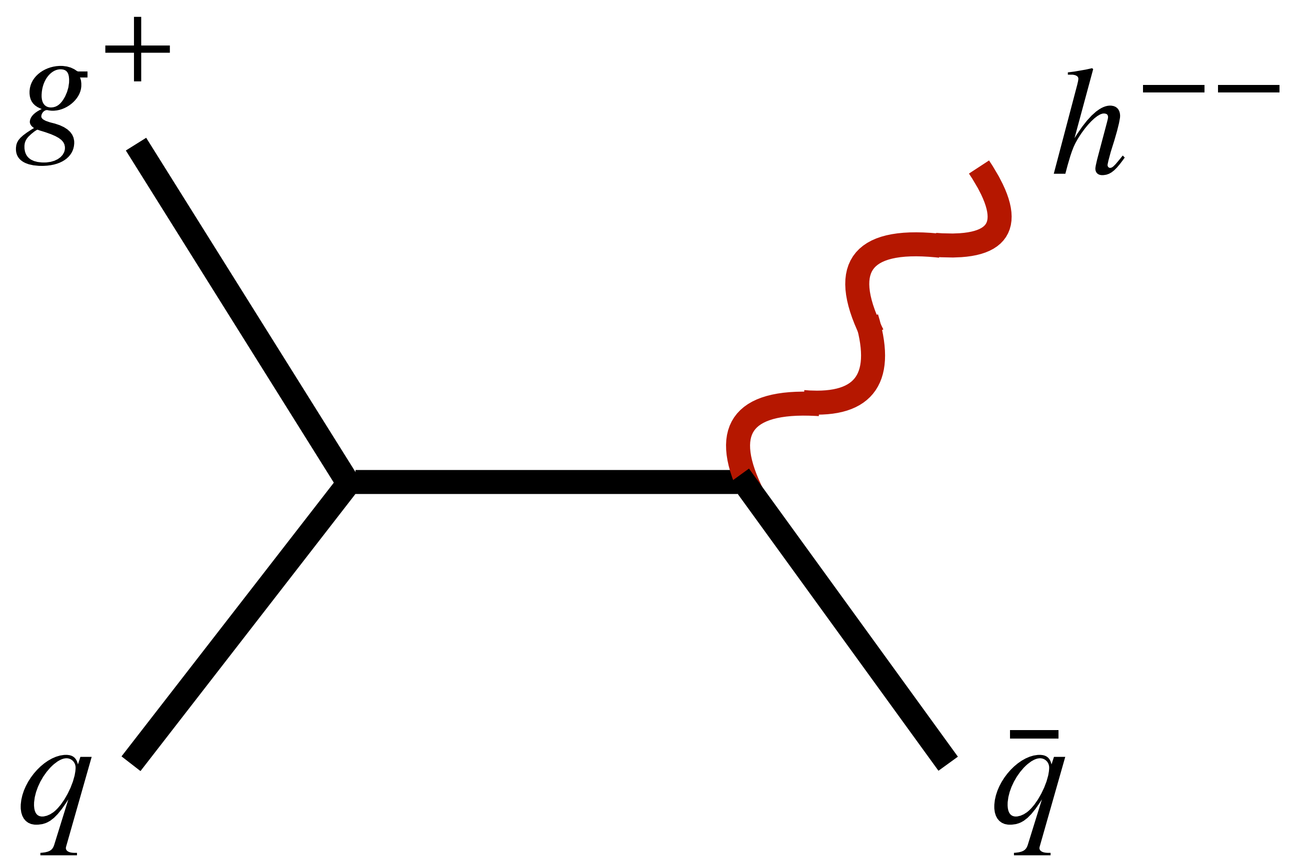
what do we do?

- we study the RG of effective theories that include gravity
    - encoded in β functions of couplings and UV anomalous dimensions γ_{UV} of operators
- work at the amplitude level, up to one loop and 4 external legs

RG of GR

but M_P is 'large'!

- A graviton is expected to pay M_P^{-1} to interact with stuff
  - all the effects of gravity that go like $(E/M_P)^{\#}$ are typically small, e.g. in collider experiments where $E \lesssim \sqrt{s_{LHC}}$



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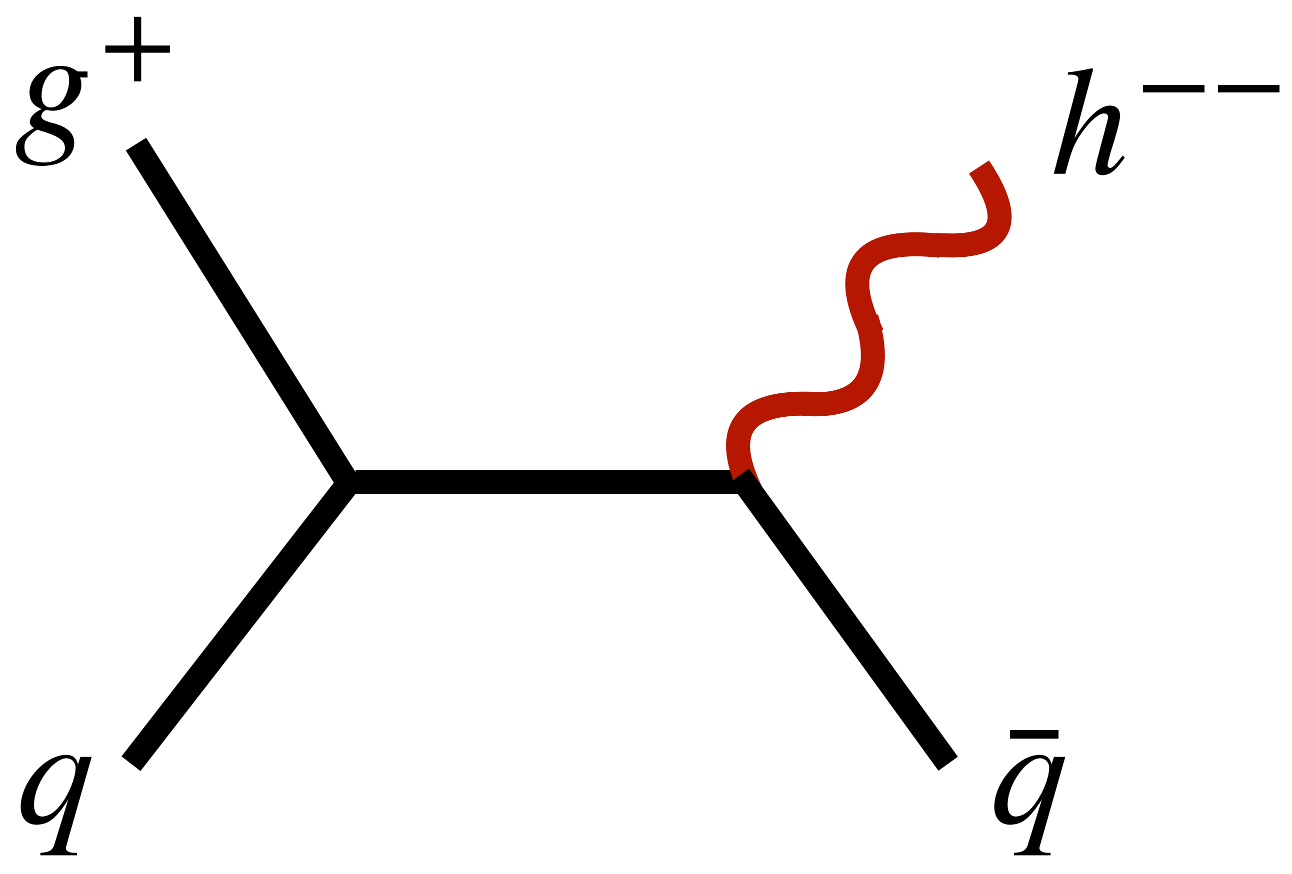
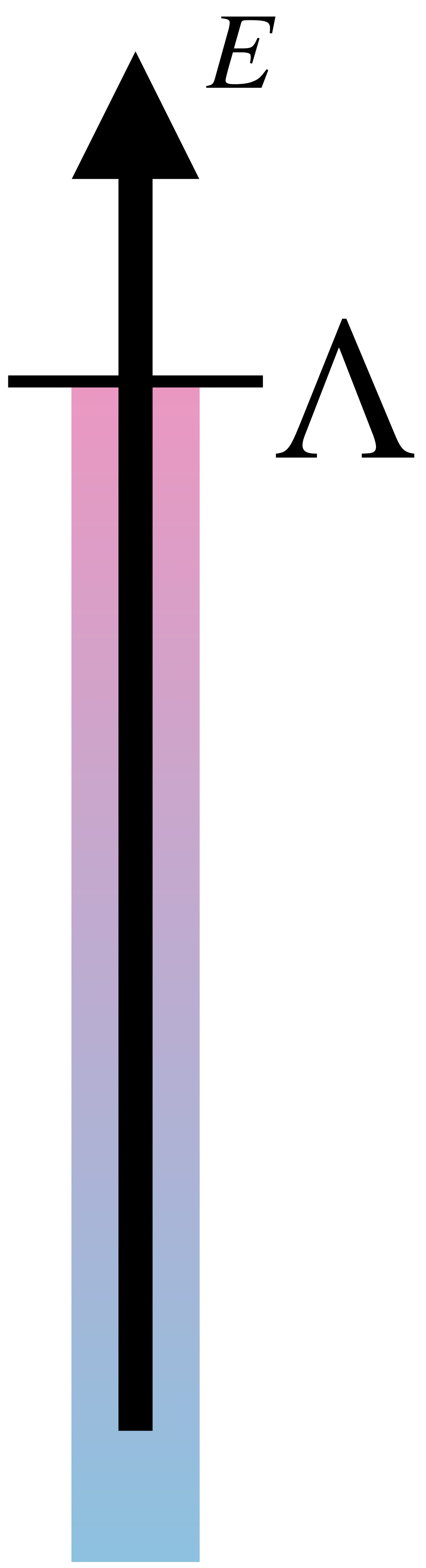


this is *not* what we have in mind

RG of GR

(how to read)

- gravity is a fundamental interaction, that we study with an EFT approach
- γ_{UV} of operators encode fundamental properties of the EFT of matter + gravity
- we provide methods to efficiently compute γ_{UV} (and compute some)



Example

arXiv 2109.13937 (Arkani-Hamed, Huang, Liu, Remmen)

- Einstein-Maxwell effective theory: study deviations from R + FF
    - encoded in higher-dimensional operators as $C_{\mathcal{O}}\mathcal{O}$
    - control M/Q of extremal black-hole solutions (deviation away from unity)
    - in the deep IR:

$$C_{\mathcal{O}} \sim \gamma_{\mathcal{O}} \ln(s/\mu^2)$$

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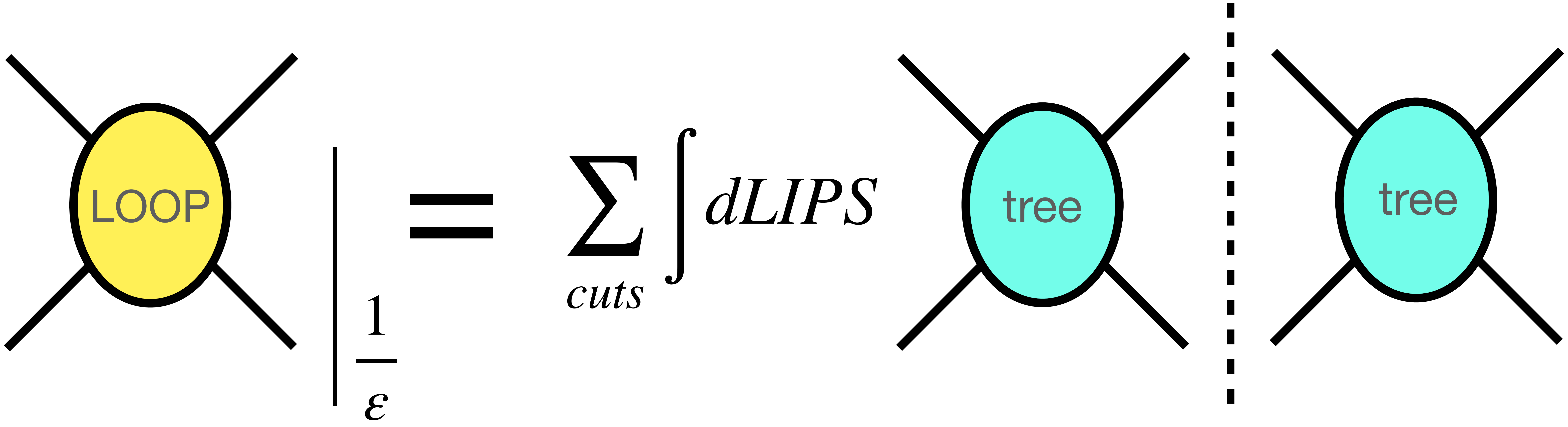
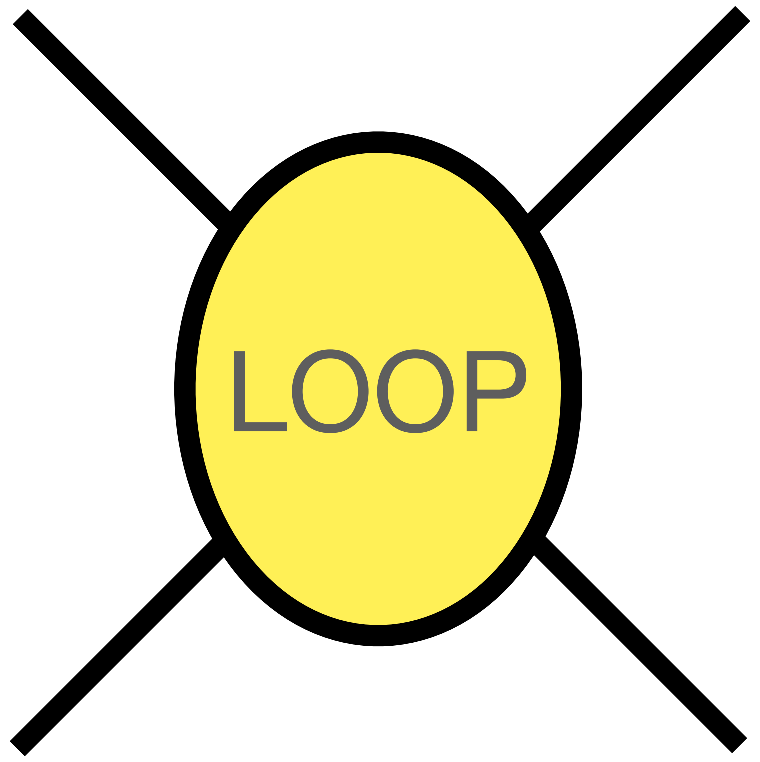
sign controlled by weak-gravity conjecture

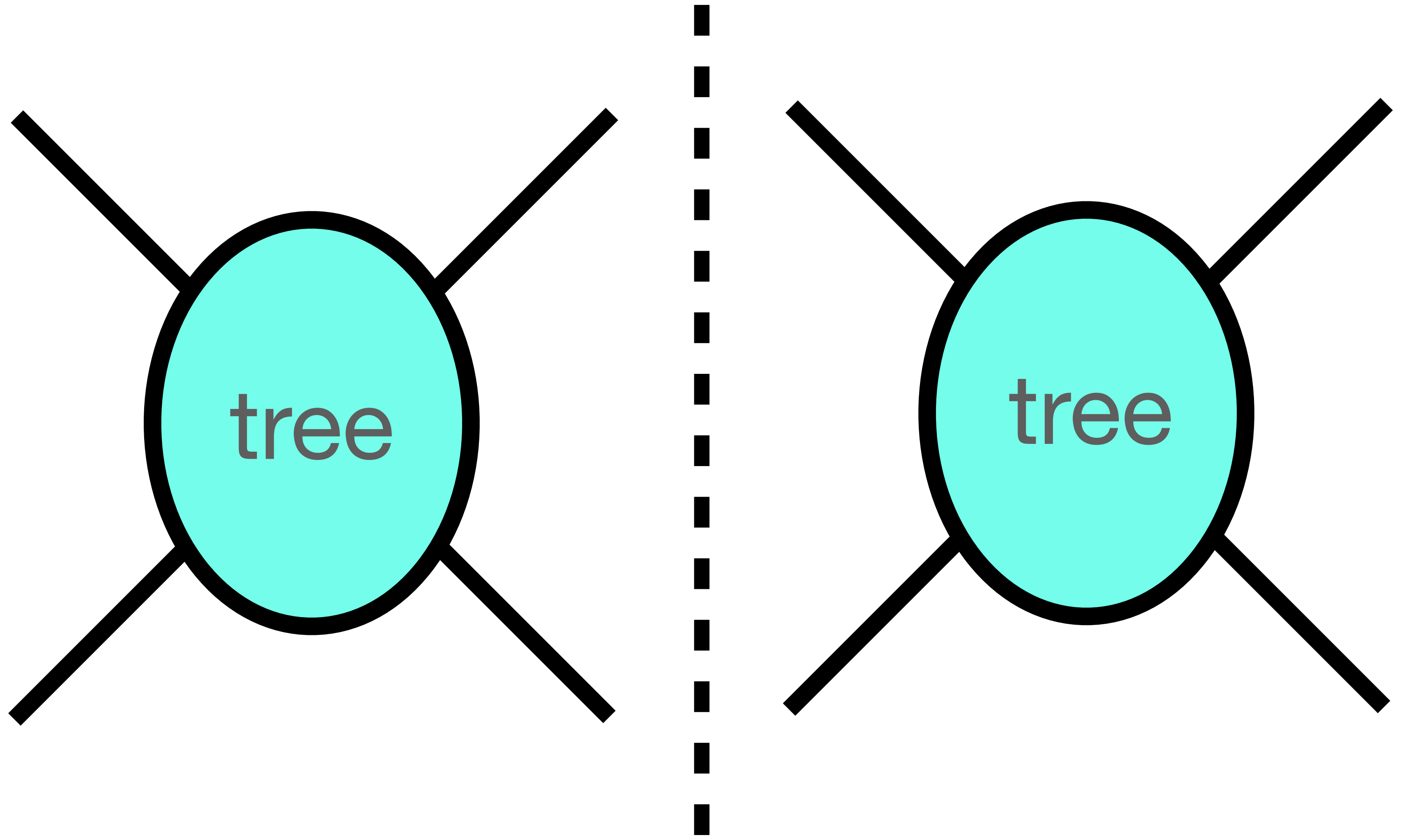
γ_{UV} from on-shell amplitudes



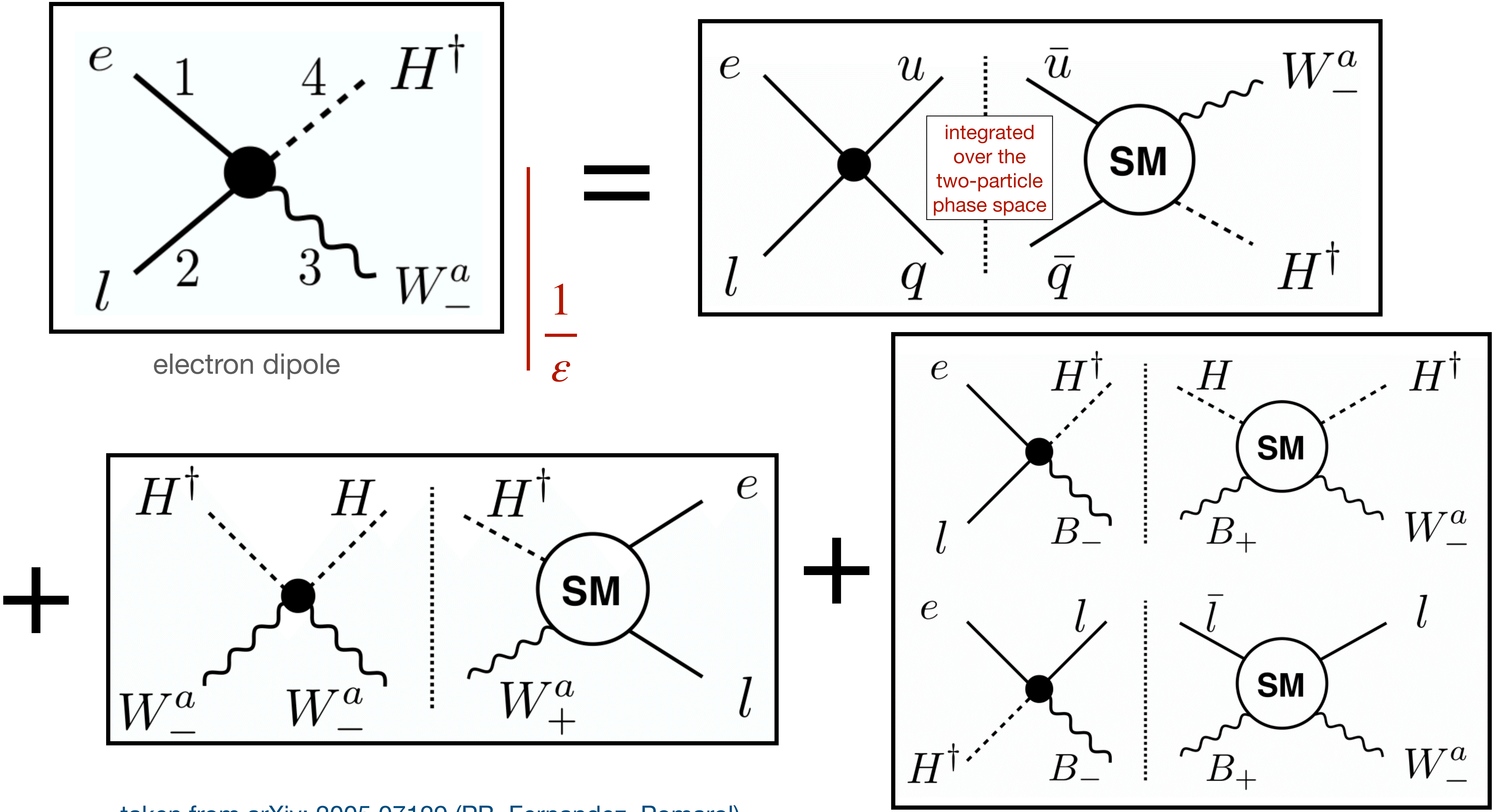
γ_{UV} from on-shell amplitudes



$$\Big|_{\frac{1}{\varepsilon}} = \sum_{cuts} \int dLIPS$$



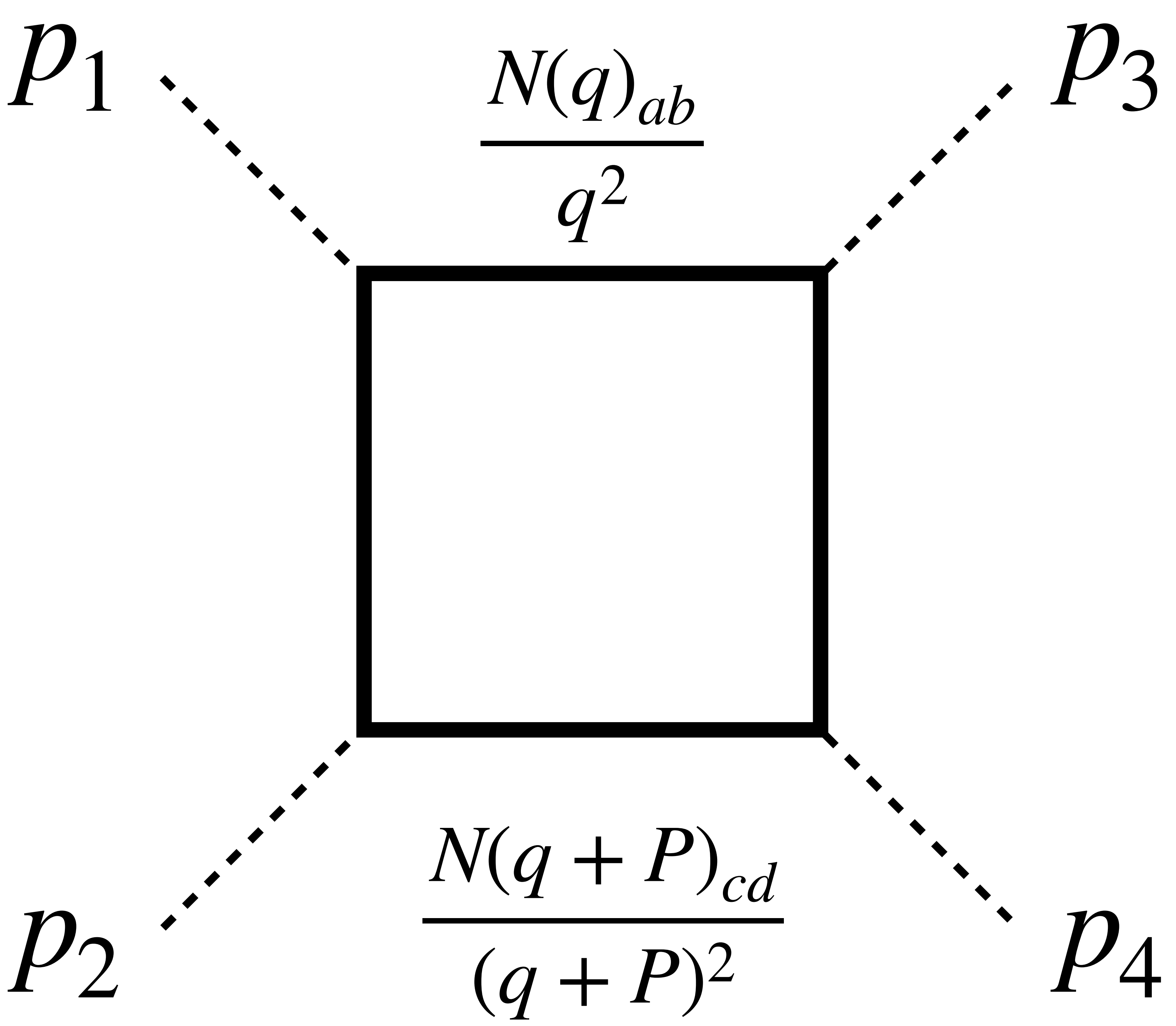
$$1/\varepsilon \sim \ln \mu \sim \mathrm{RG}$$



taken from arXiv: 2005.07129 (PB, Fernandez, Pomarol)

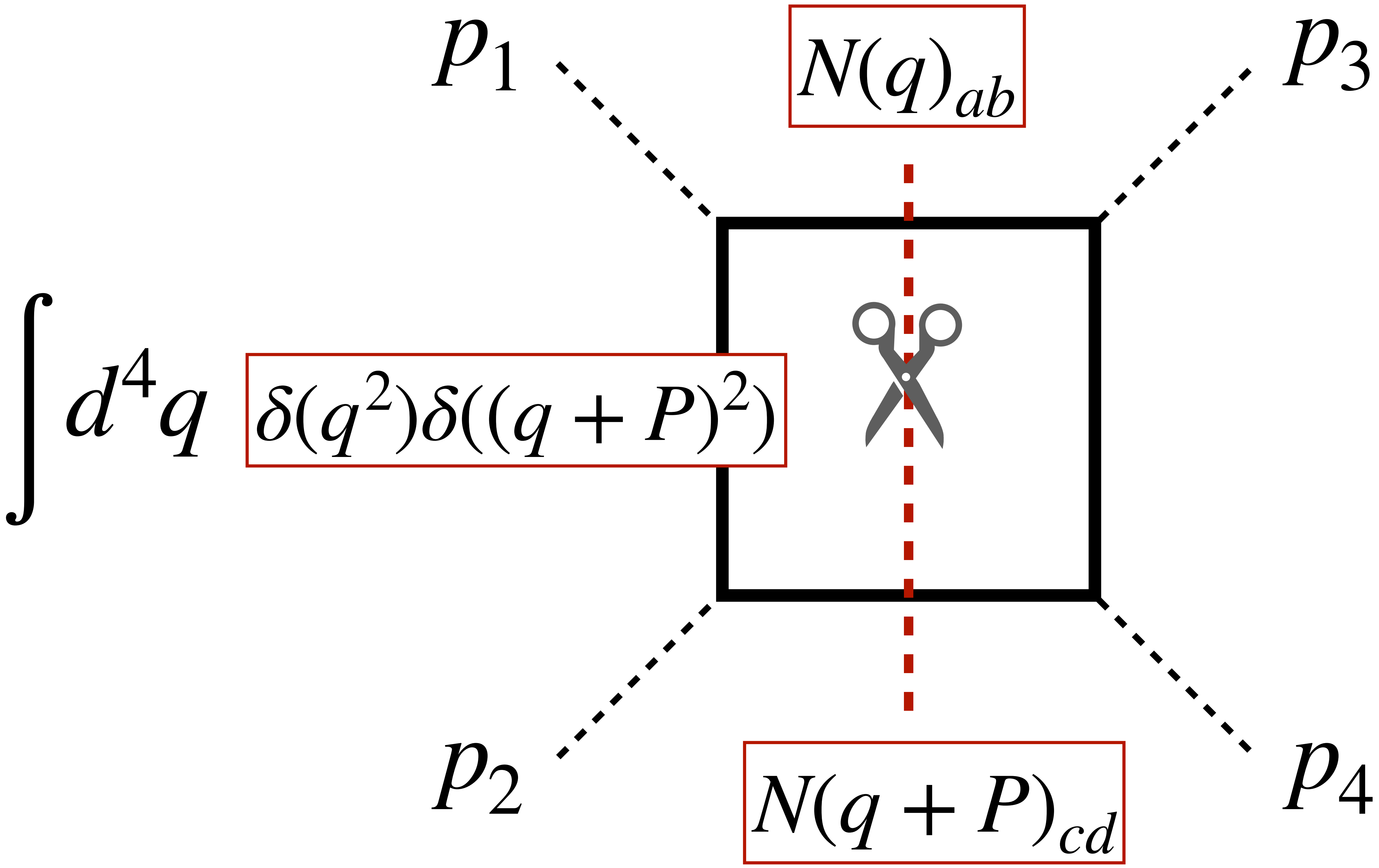
From loop to cut

$$\int d^4q \quad \times$$



$+$ other diagrams

From loop to cut



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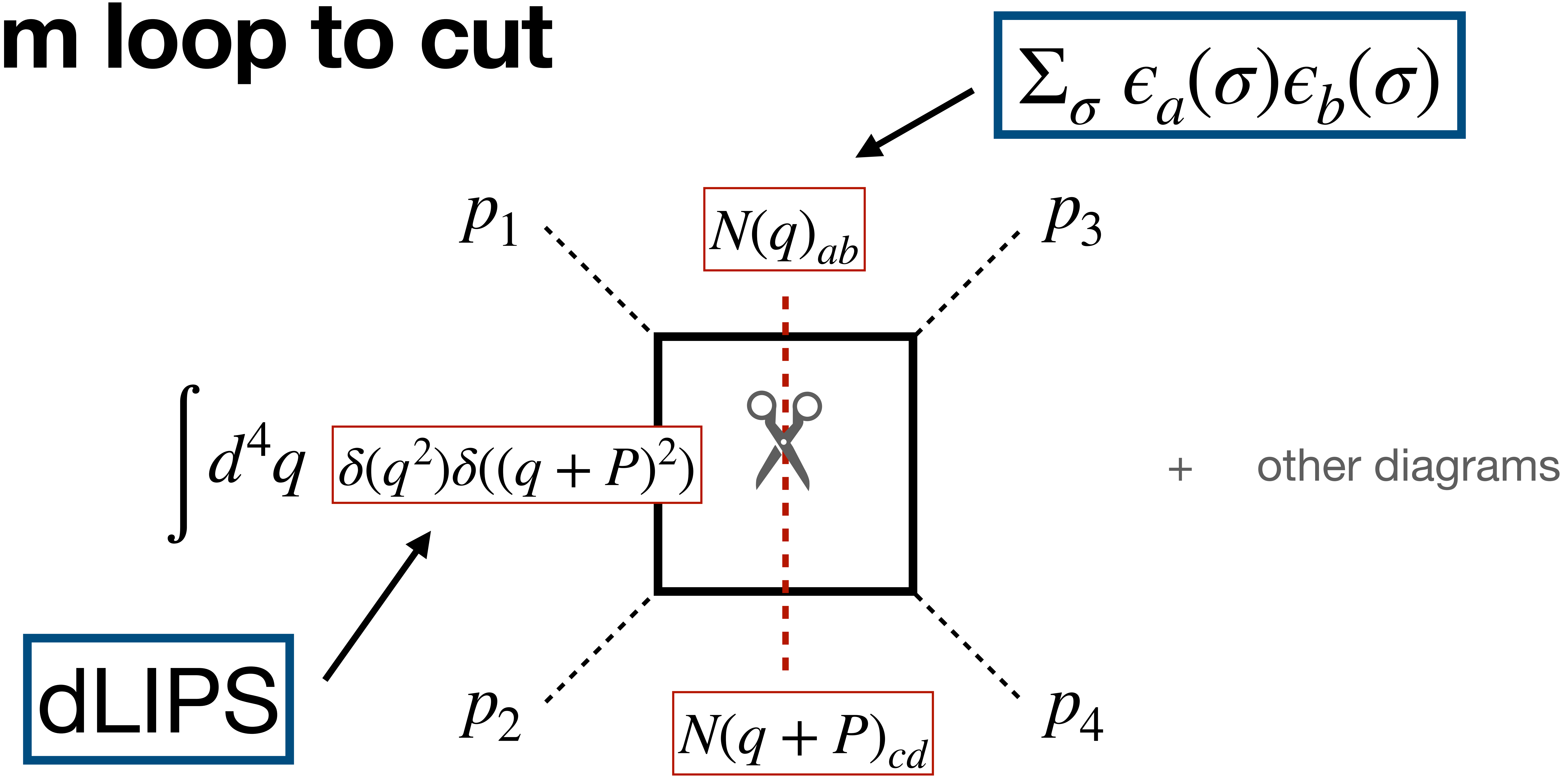
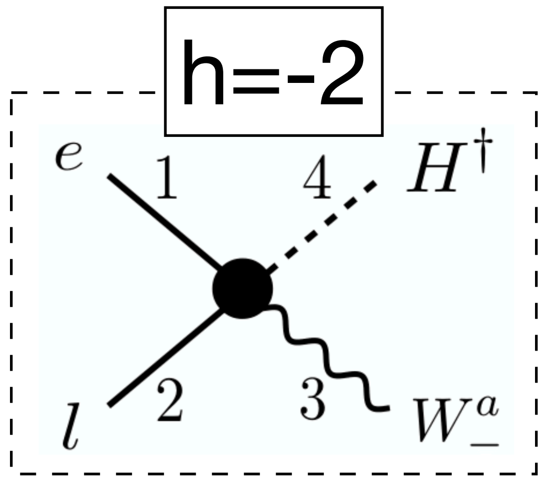


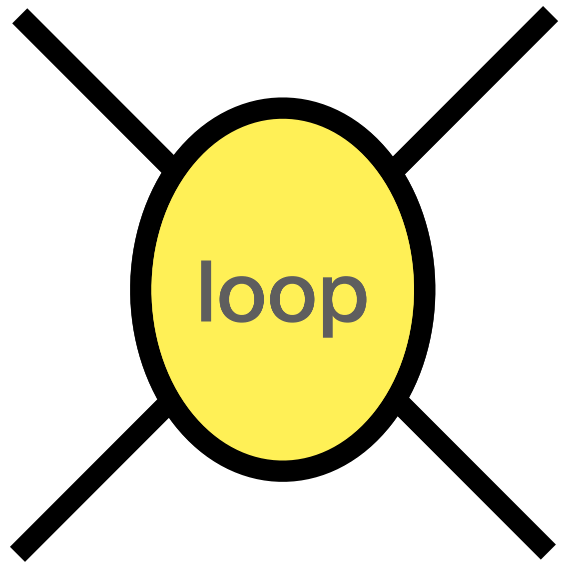
From loop to cut

- Well defined operation (cut) that sends a loop integral to a product of on-shell tree amplitudes with definite helicity, integrated over a phase space
  - **keeps all the information on the divergent (or lnμ) part**

1. on-shell helicity amplitudes: extremely convenient when dealing with massless particles with $h \geq 1$ (no gauge redundancies)
2. tree-level: helicity bounds on tree amplitudes allow to obtain non-renormalization theorems at loop level

Non-renormalization from helicity





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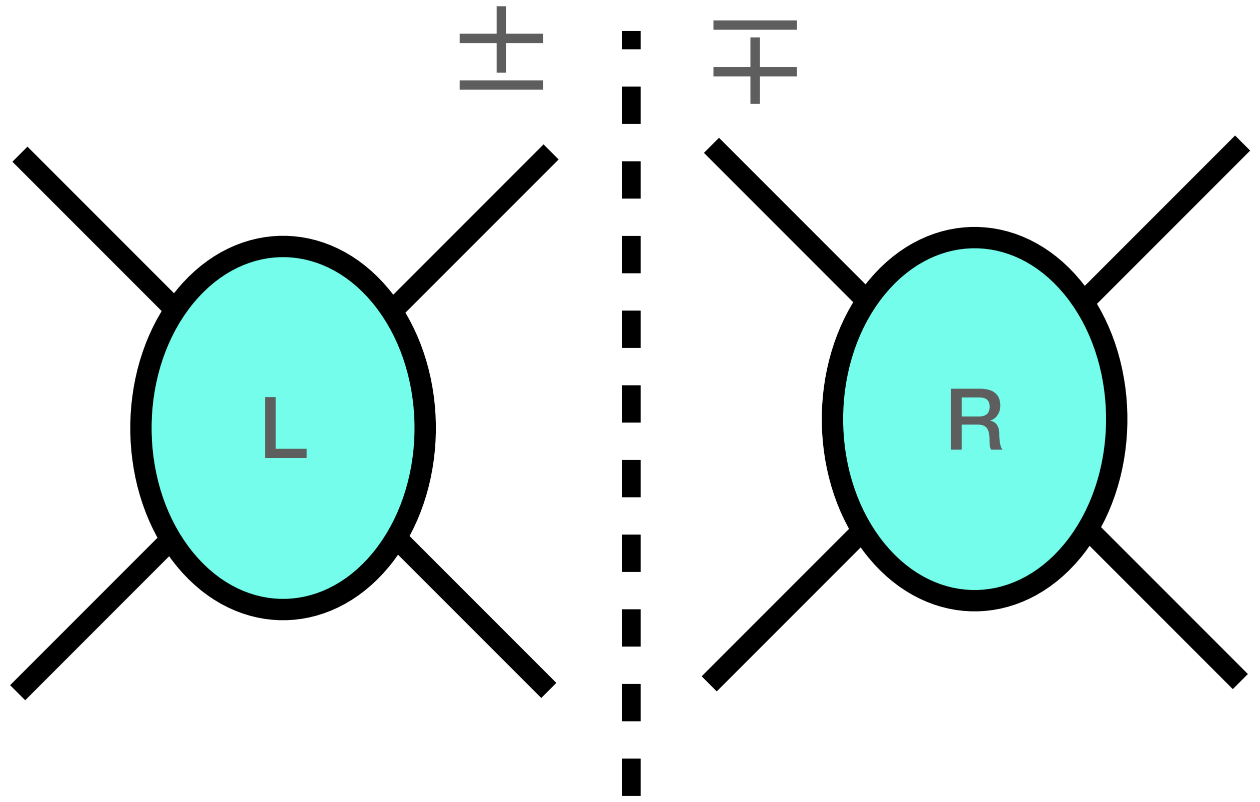

$$h_A \equiv \sum_i h_i$$

(all incoming)

$$h_{\text{loop}} = h_L + h_R$$

$$|\, h_{\text{loop}} \,| \leq |\, h_L \,| + |\, h_R \,|$$

(triangle inequality)

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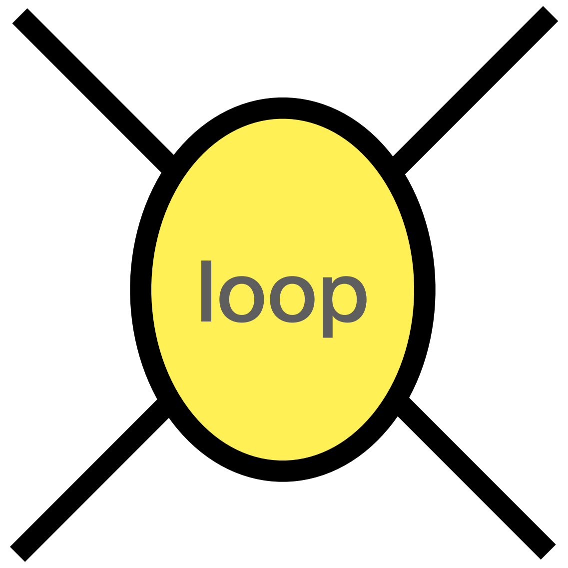

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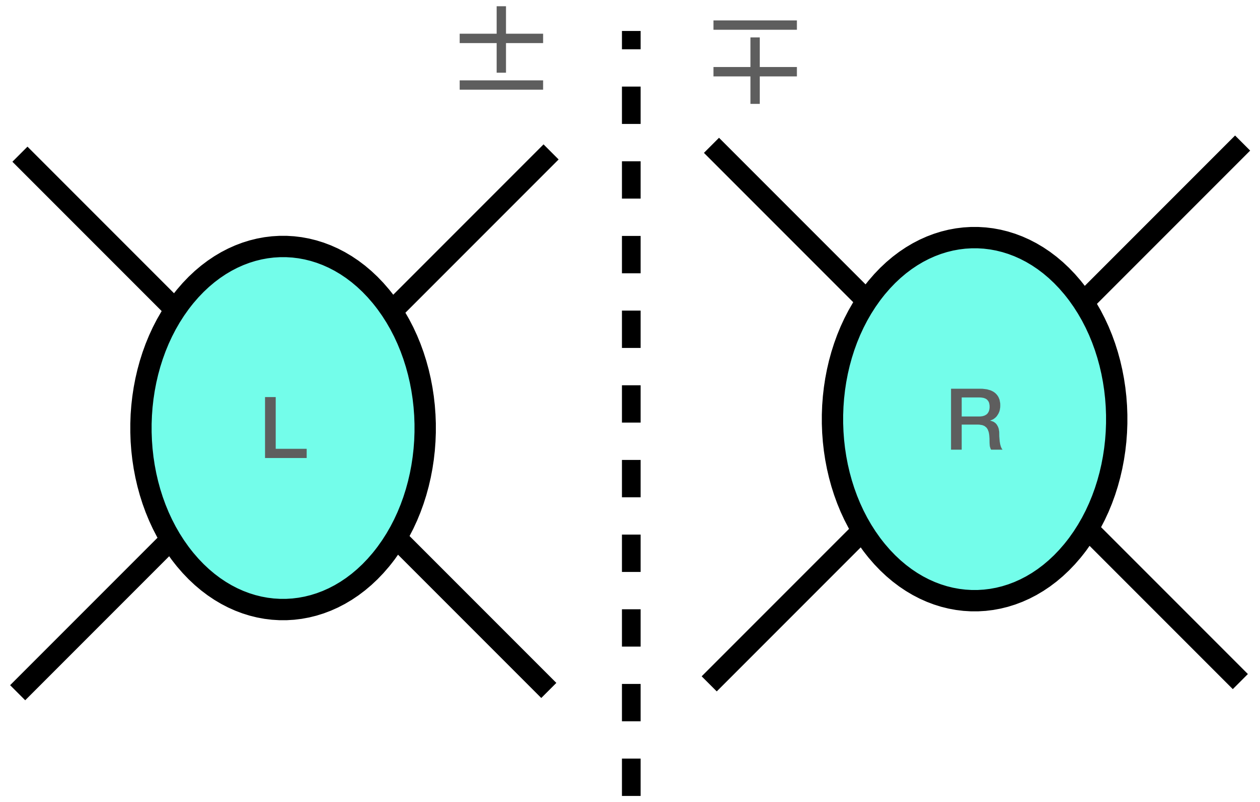

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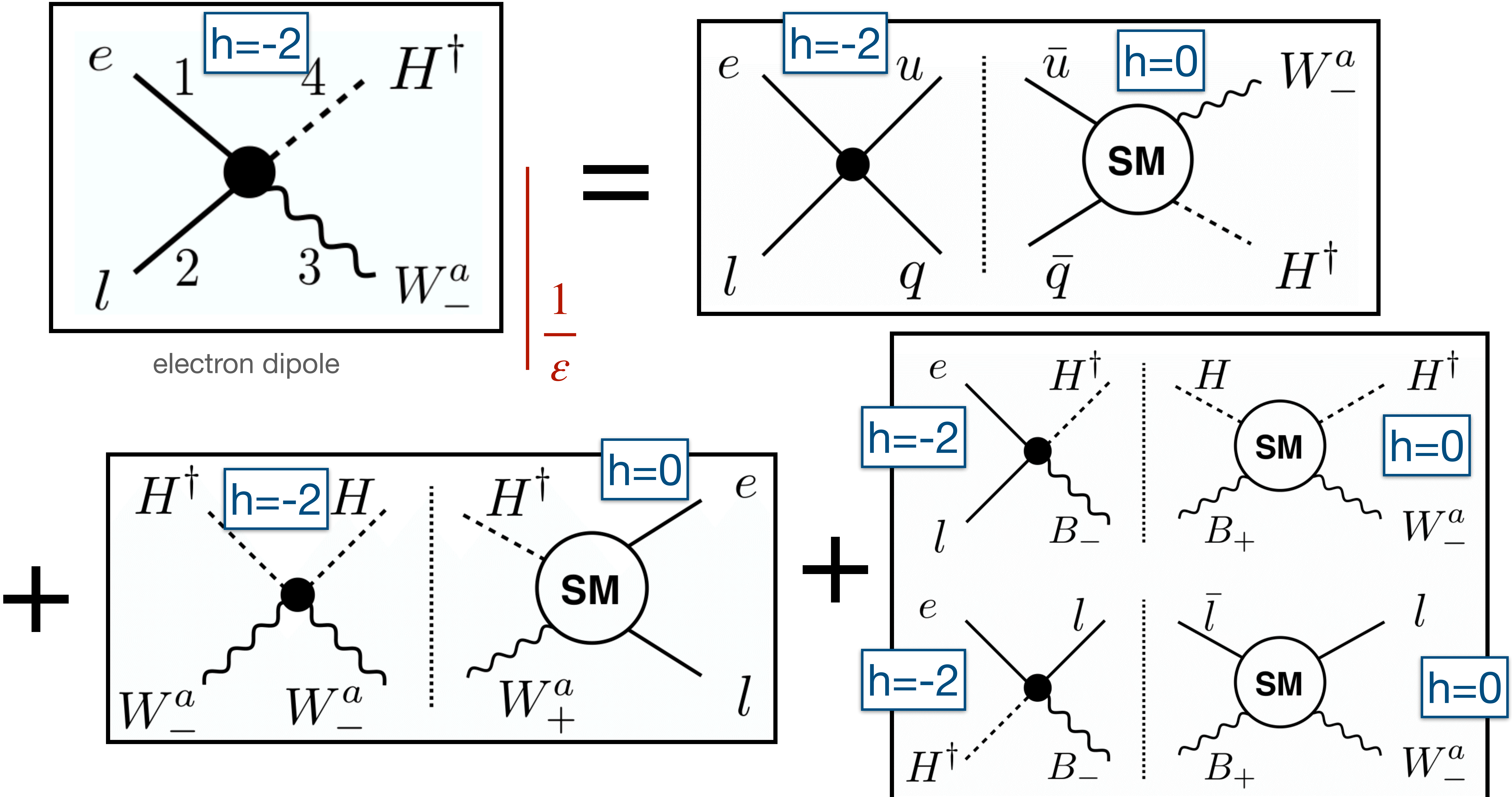
- Limits the way in which divergences can appear, <u>in a non-trivial way</u>
- arXiv 1505.01844 (non-renormalization without supersymmetry)



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Non-renormalization from helicity

- what makes this non-trivial?

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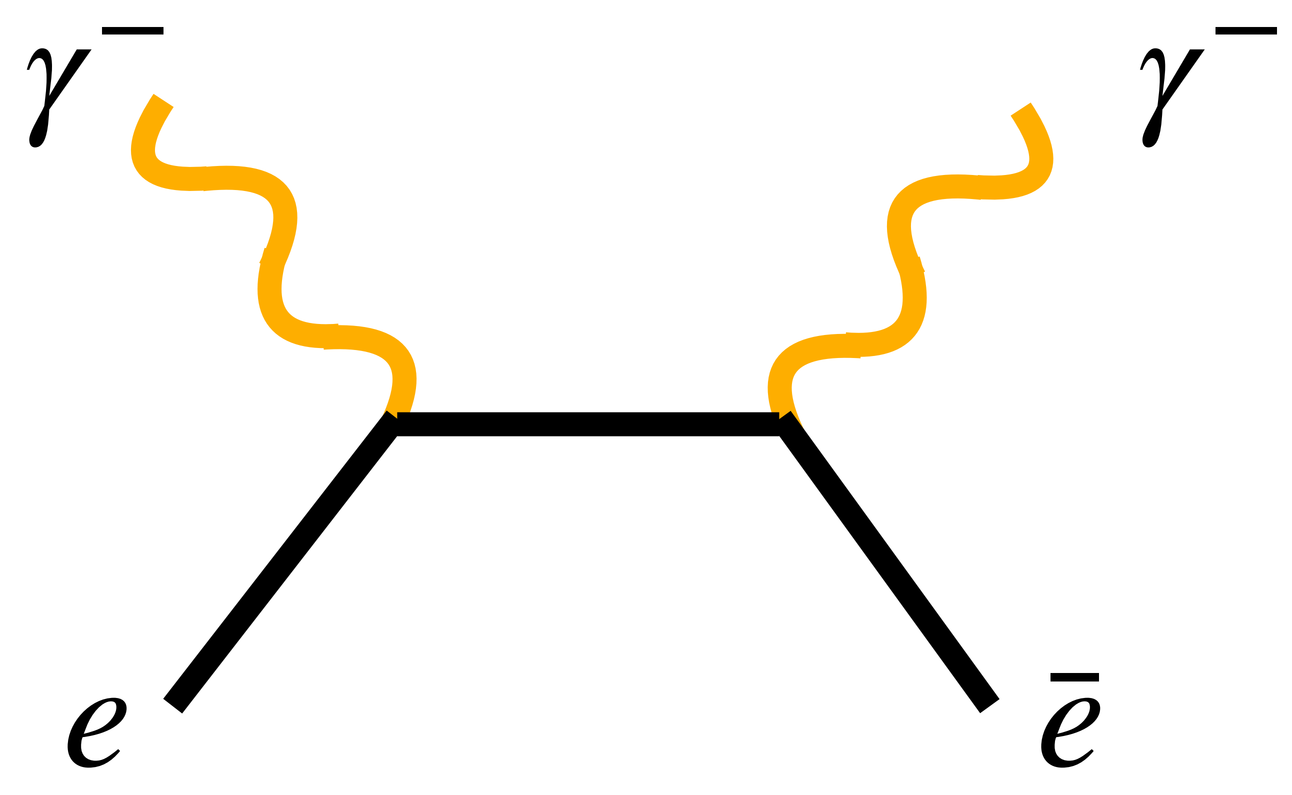
- surprisingly, 4-point tree-level amplitudes in a marginal theory have (almost) all *h = 0* (not obvious from Feynman diagrams)



$+$ crossing $=$ 0

Non-renormalization from helicity

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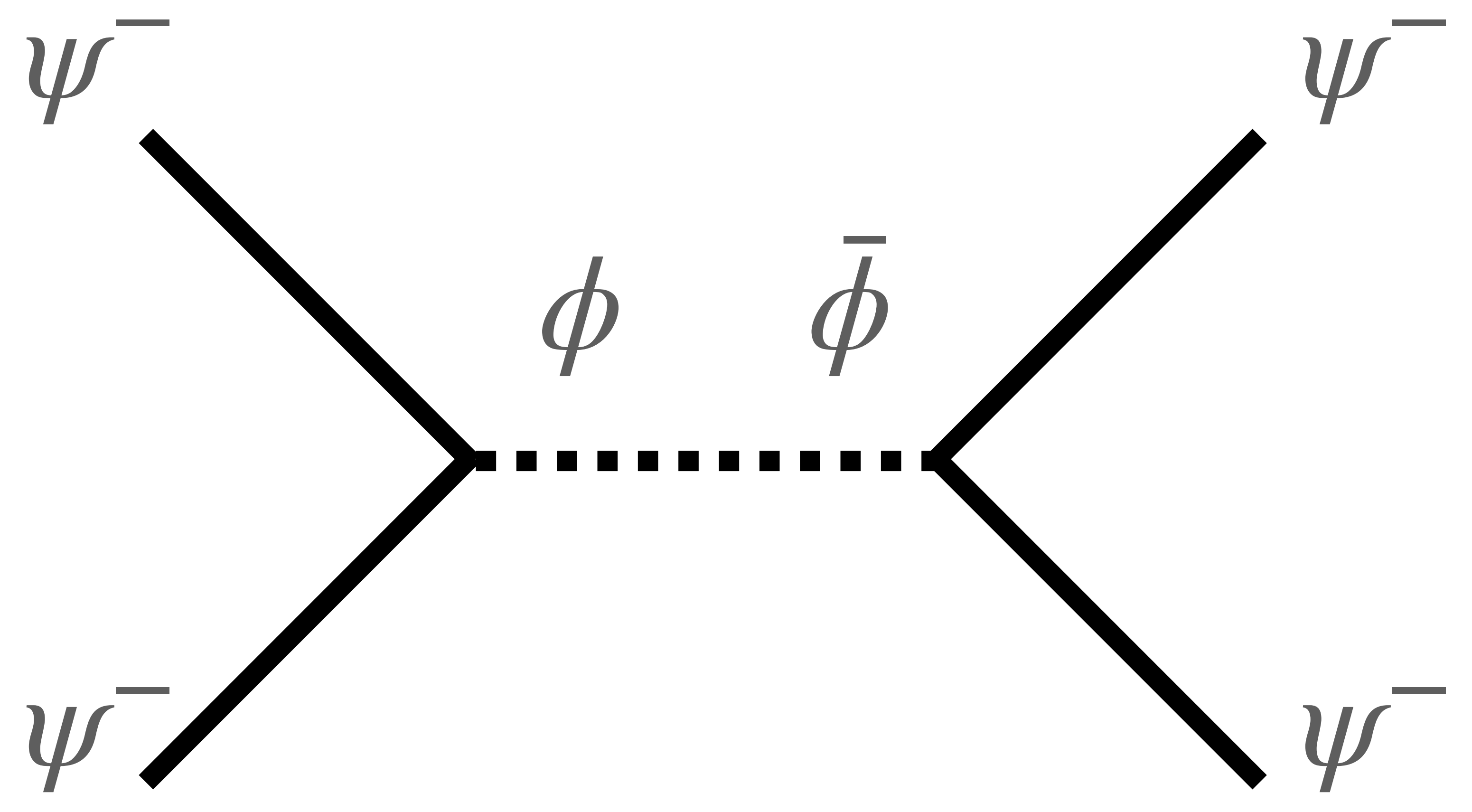
$$|h^{(4)}_{\text{loop}}| \leq |h^{(4)}_L| + |h^{(4)}_R| = 0$$

constraint on how infinities can appear in amplitudes with 4 external legs in a marginal theory

Helicity bounds on tree amplitudes

- Non-trivial bounds on total helicity of tree amplitudes (marginal couplings)
  - direct computation
  - supersymmetric Ward identities (arXiv: 1607.05236)



$$|h| = 2$$

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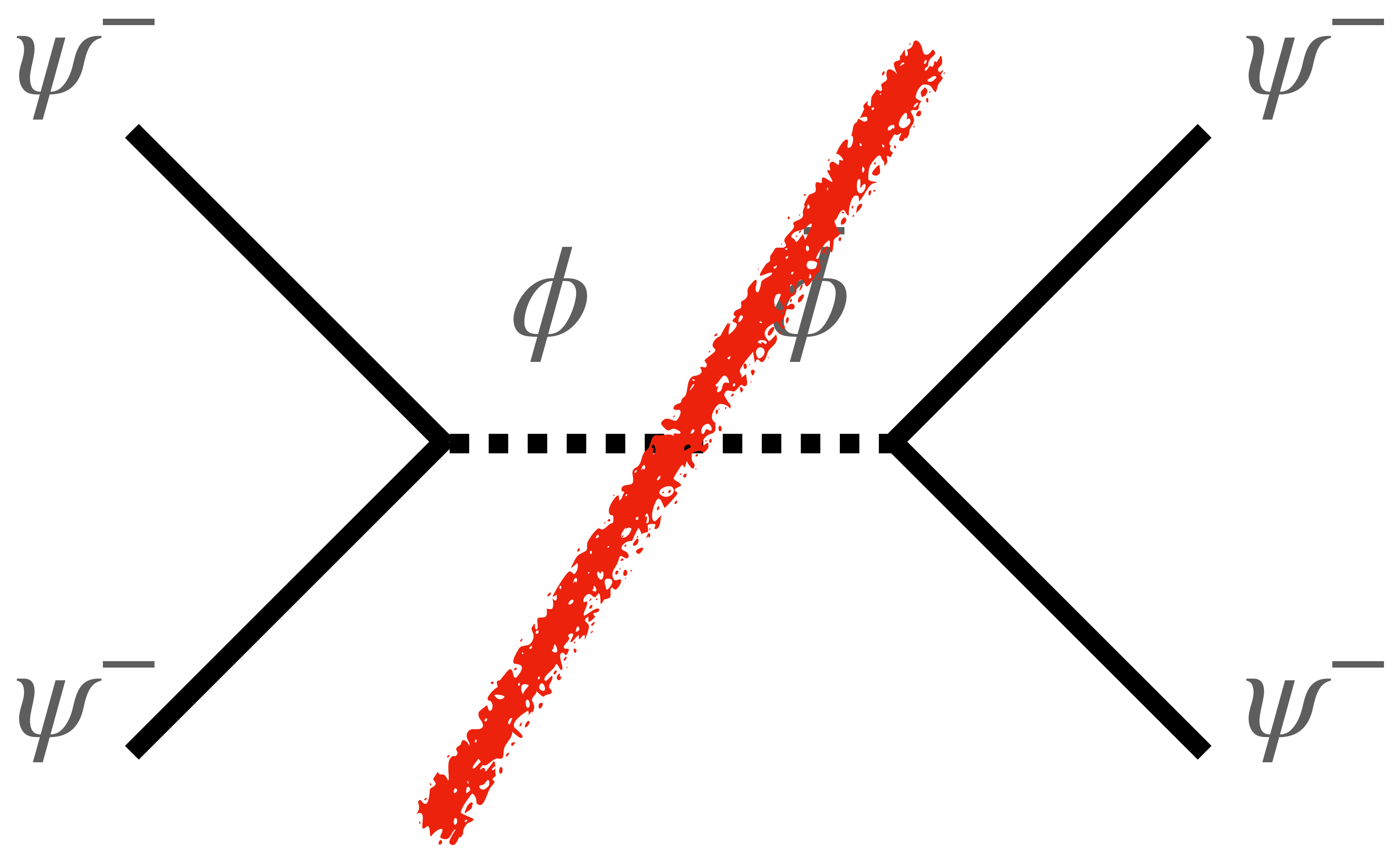
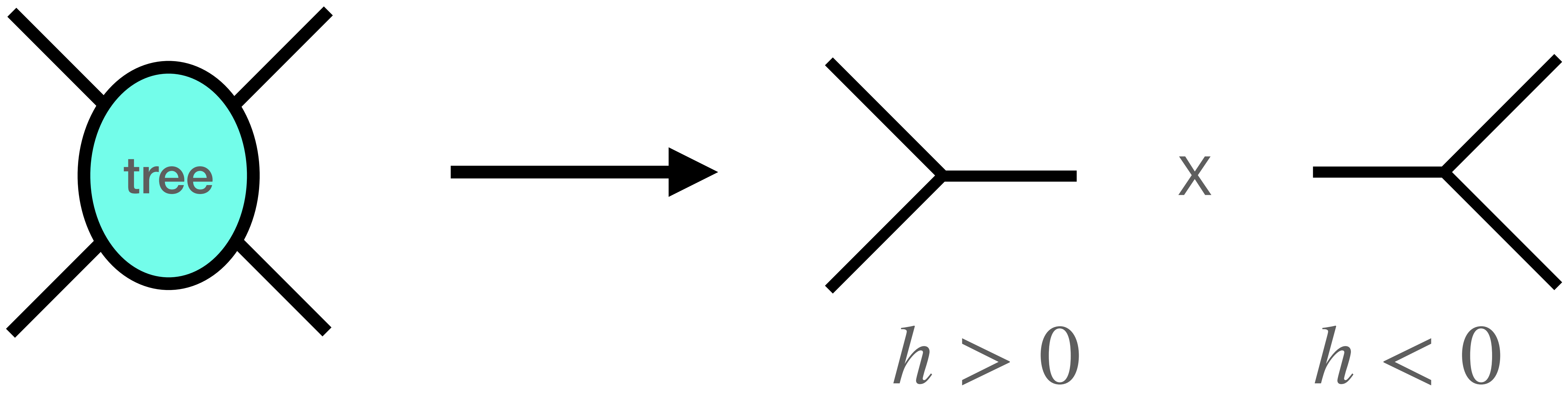


can not arise from a holomorphic potential

Helicity bounds on tree amplitudes

including minimally coupled gravity

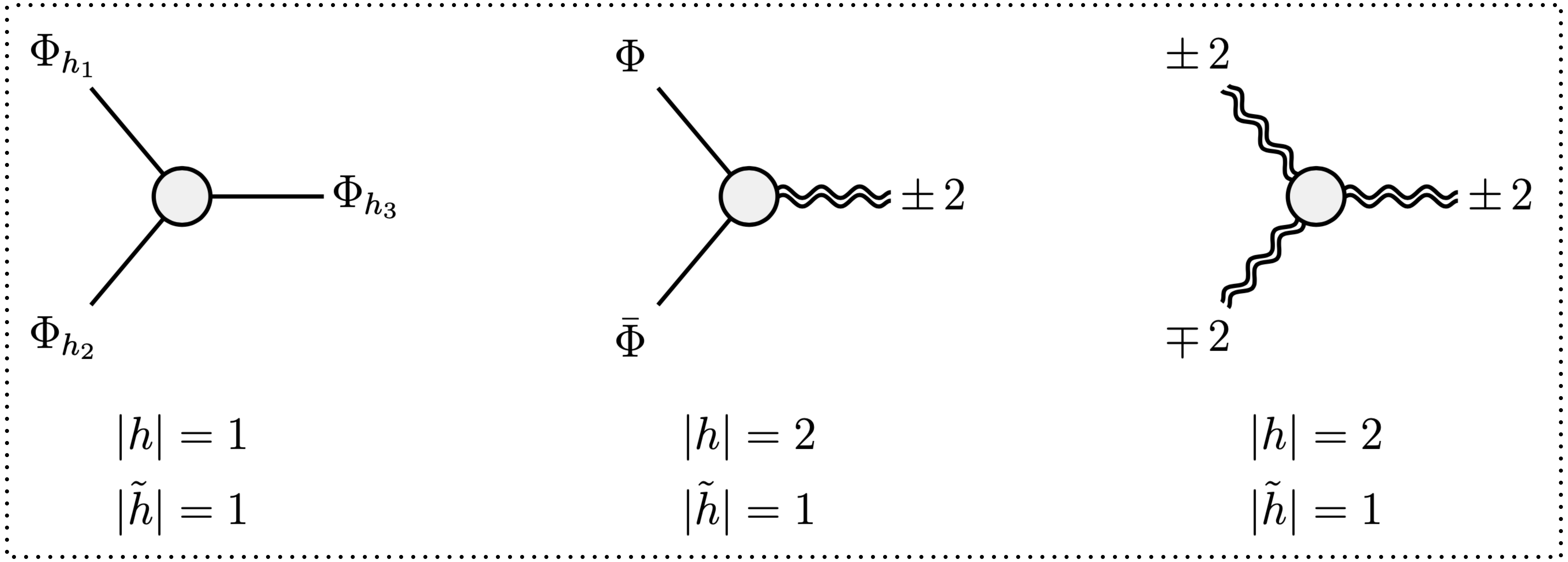
- simple rule of thumb (maybe more than just this): factorization into $A_3 \times A_3$



- in marginal theories, h_3=+1 or h_3=-1, implying h_4=0
- the rule also applies when including minimal coupling to gravity, but now $|h_3|$ =1,2 and h_4=0 no longer holds

Helicity bounds on tree amplitudes

including minimally coupled gravity



- modified helicity $\tilde{h}$, of which gravitons carry <u>*one*</u> unit instead of two
- all 3-point amplitudes (marginal + minimal) have $\tilde{h} = \pm 1$

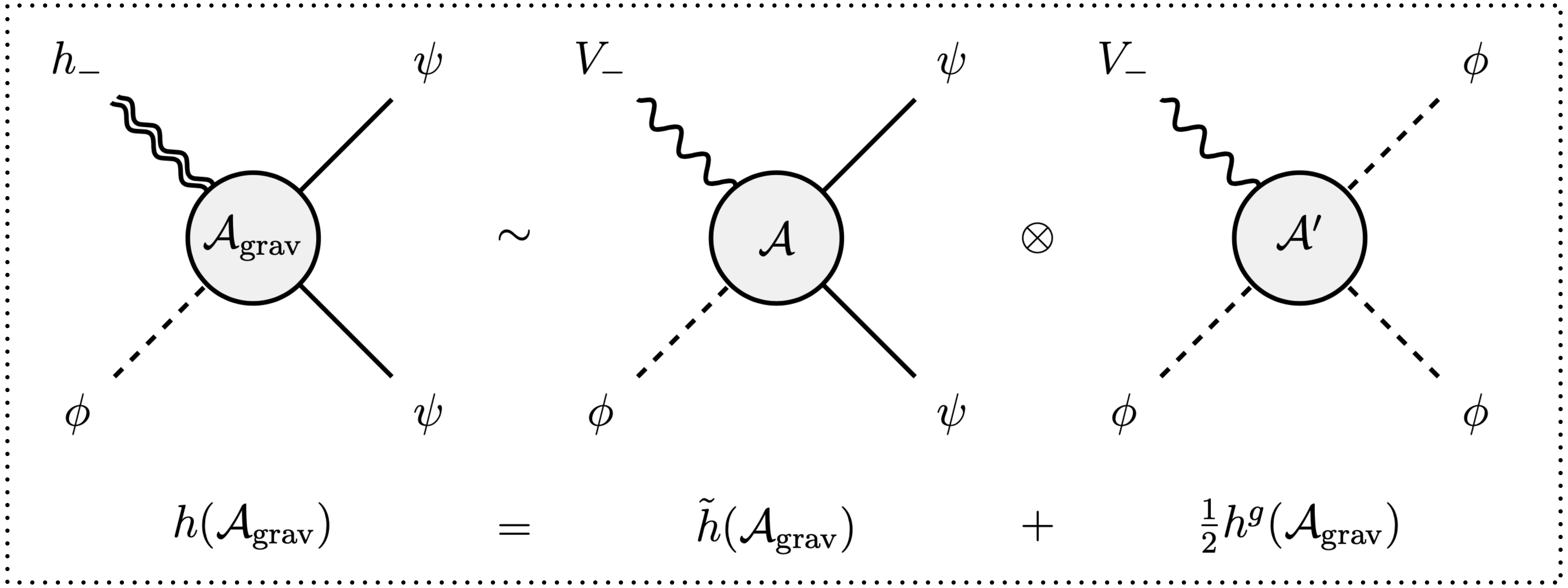
Helicity bounds on tree amplitudes

including minimally coupled gravity

- all 4-point amplitudes including minimally coupled gravitons that are factorizable (all except '$\lambda\phi^4$') can have $|\tilde{h}| = 0{,}2$
- it turns out that all those with $|\tilde{h}| = 2$ actually *vanish* (in line with the rule of thumb)
- helicity bound easily promoted by induction to $|\tilde{h}_n| \leq n - 4$
- $\tilde{h}$ extremely useful to express non-renormalization results including gravity (standard helicity does not allow to make clean statements)

Modified helicity

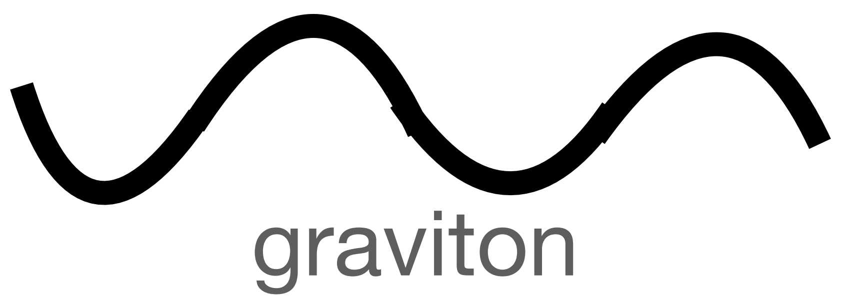
KLT relations



- modified helicity has a natural interplay with the KLT relations
- $\tilde{h} = 0$ can be seen as a consistency requirement coming from KLT (and the fact that h=0 in marginal theories)

Modified helicity

(summary)

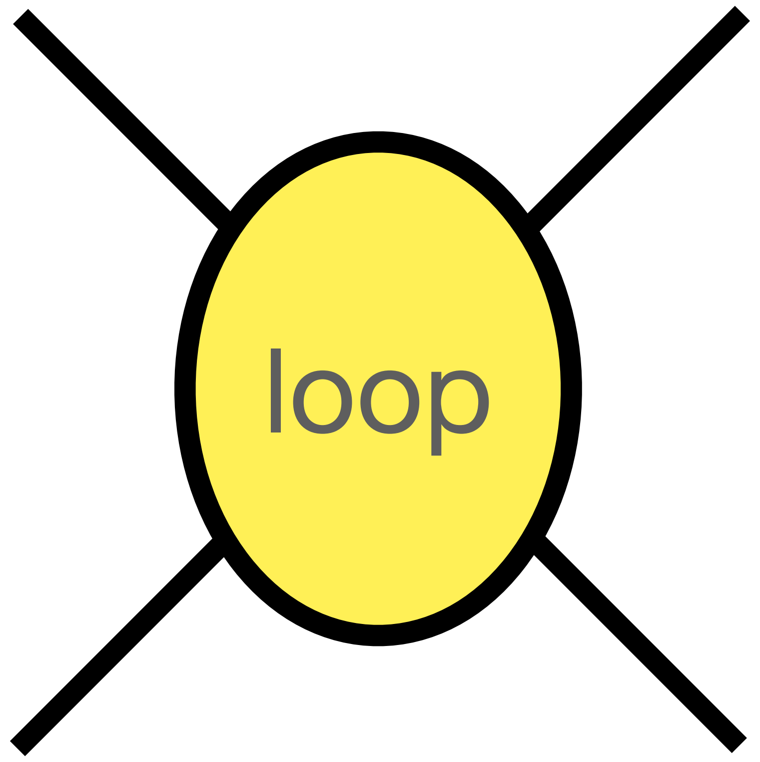
$\tilde{h}$ [

] = +1,-1

$\tilde{h}$ [

] = 0

Non-renormalization including gravity



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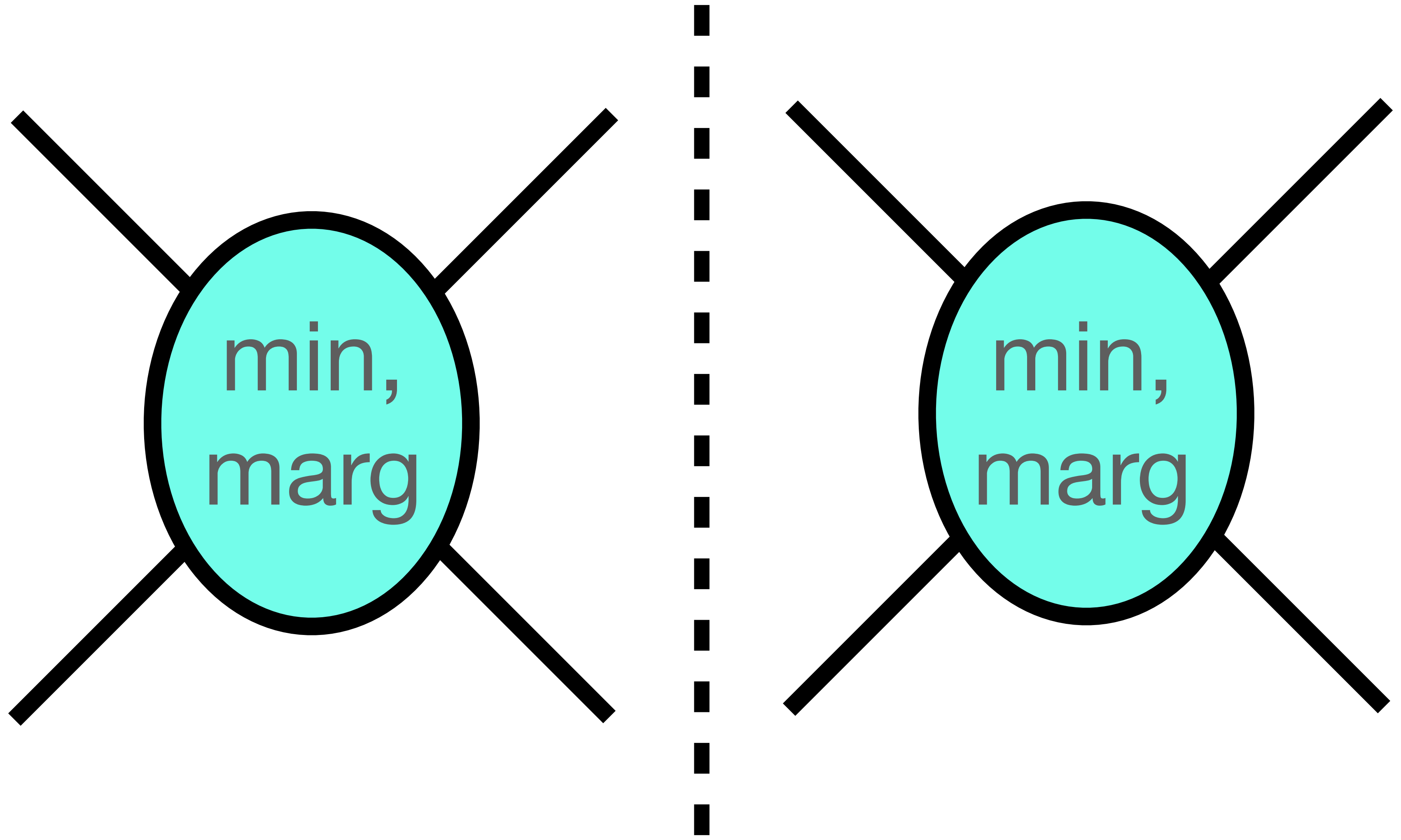


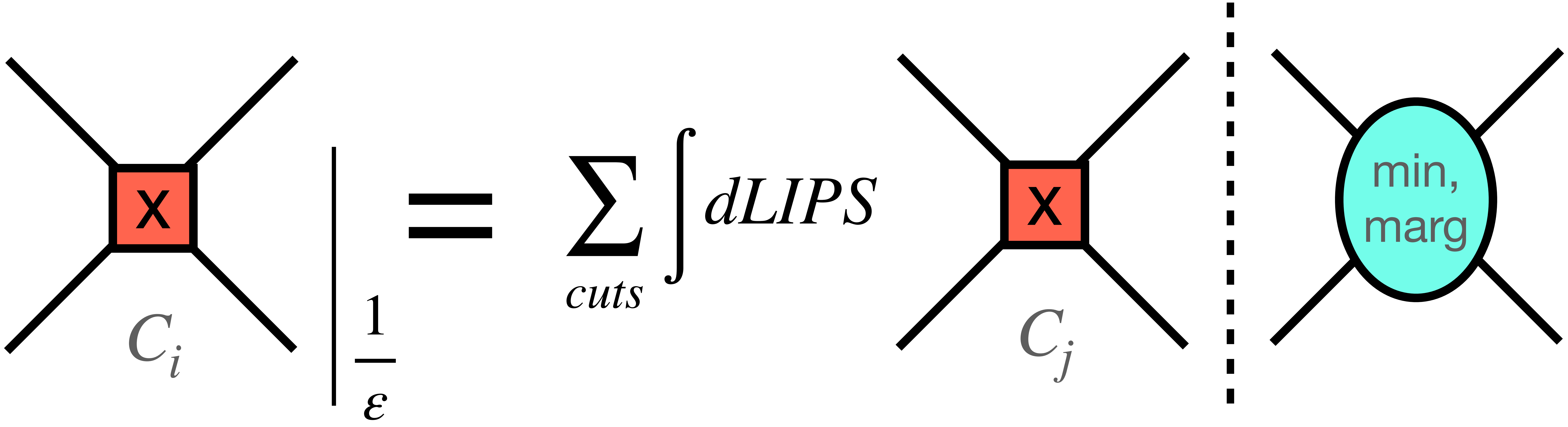
$$\tilde{h}_{\rm loop} = 0$$

At 4 points and any order in M_P^{-1}
in a minimally coupled marginal theory

Non-renormalization including gravity

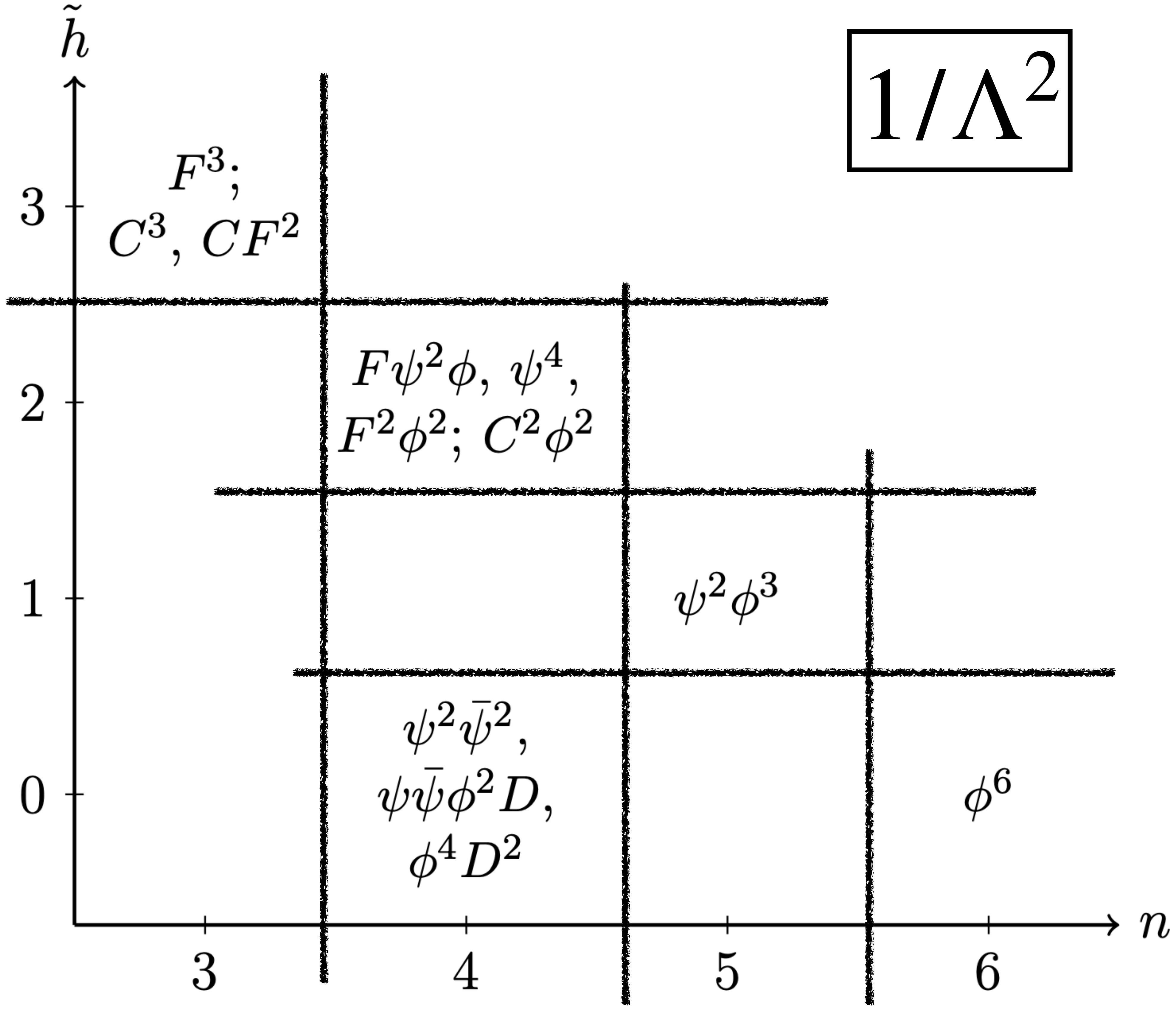
$$\gamma_{ij} = 0 \quad \text{unless} \quad \tilde{h}_i = \tilde{h}_j$$

in a 4 to 4 mixing, here including operators
and amplitudes containing gravitons



Non-renormalization including gravity

beyond four point



- previous discussion suggests to organise EFT operators according to n and modified helicity

Non-renormalization including gravity

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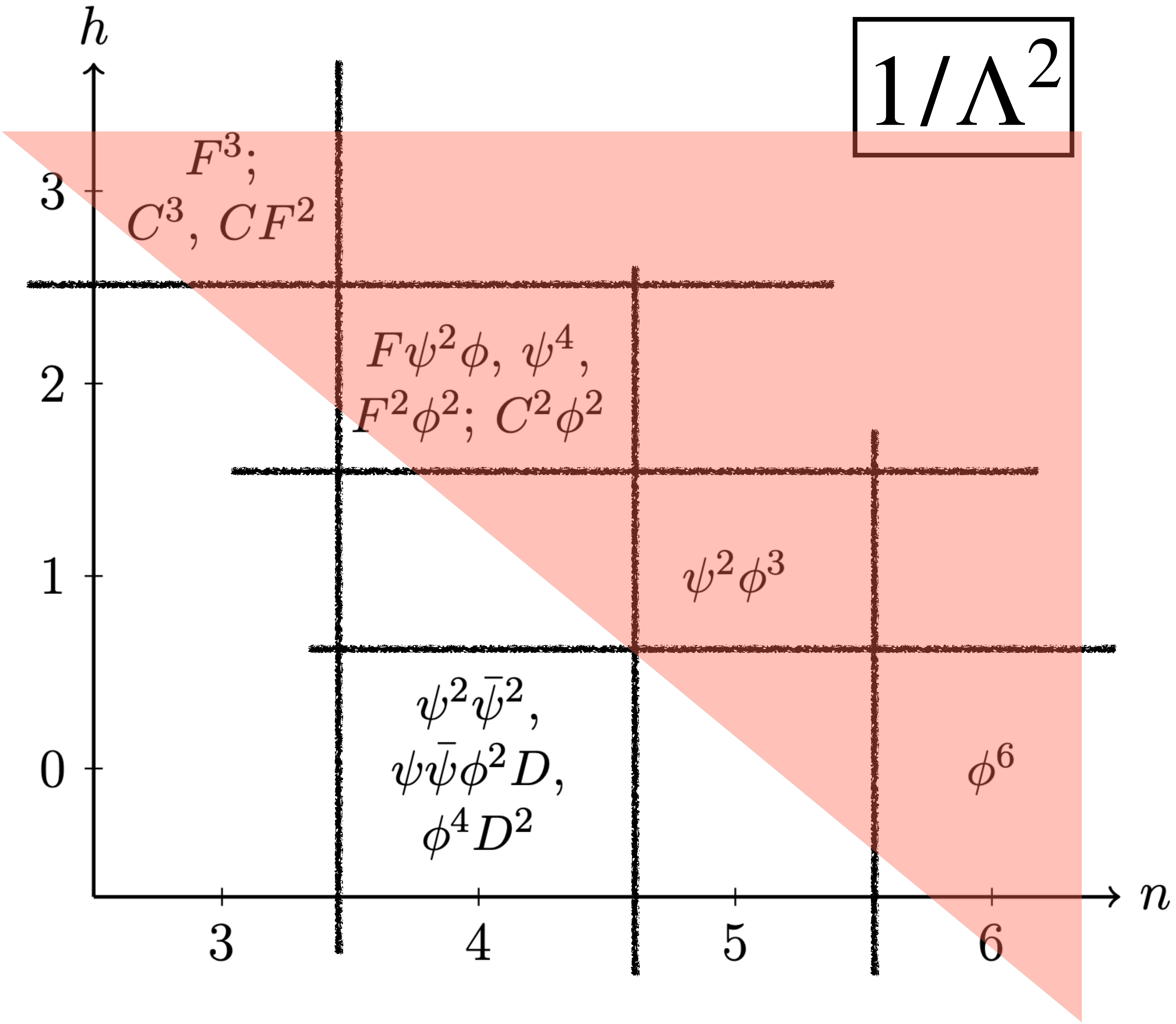

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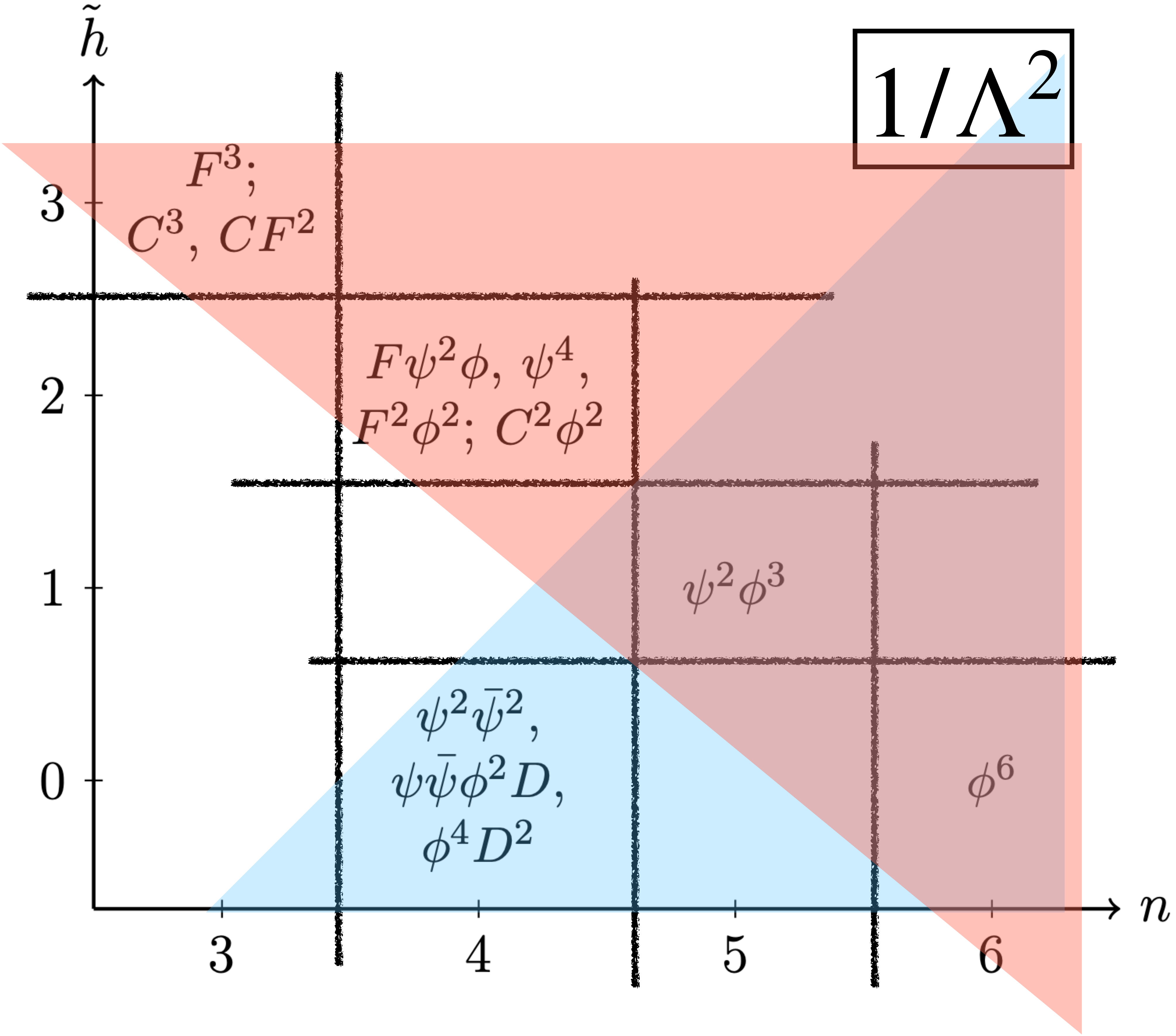
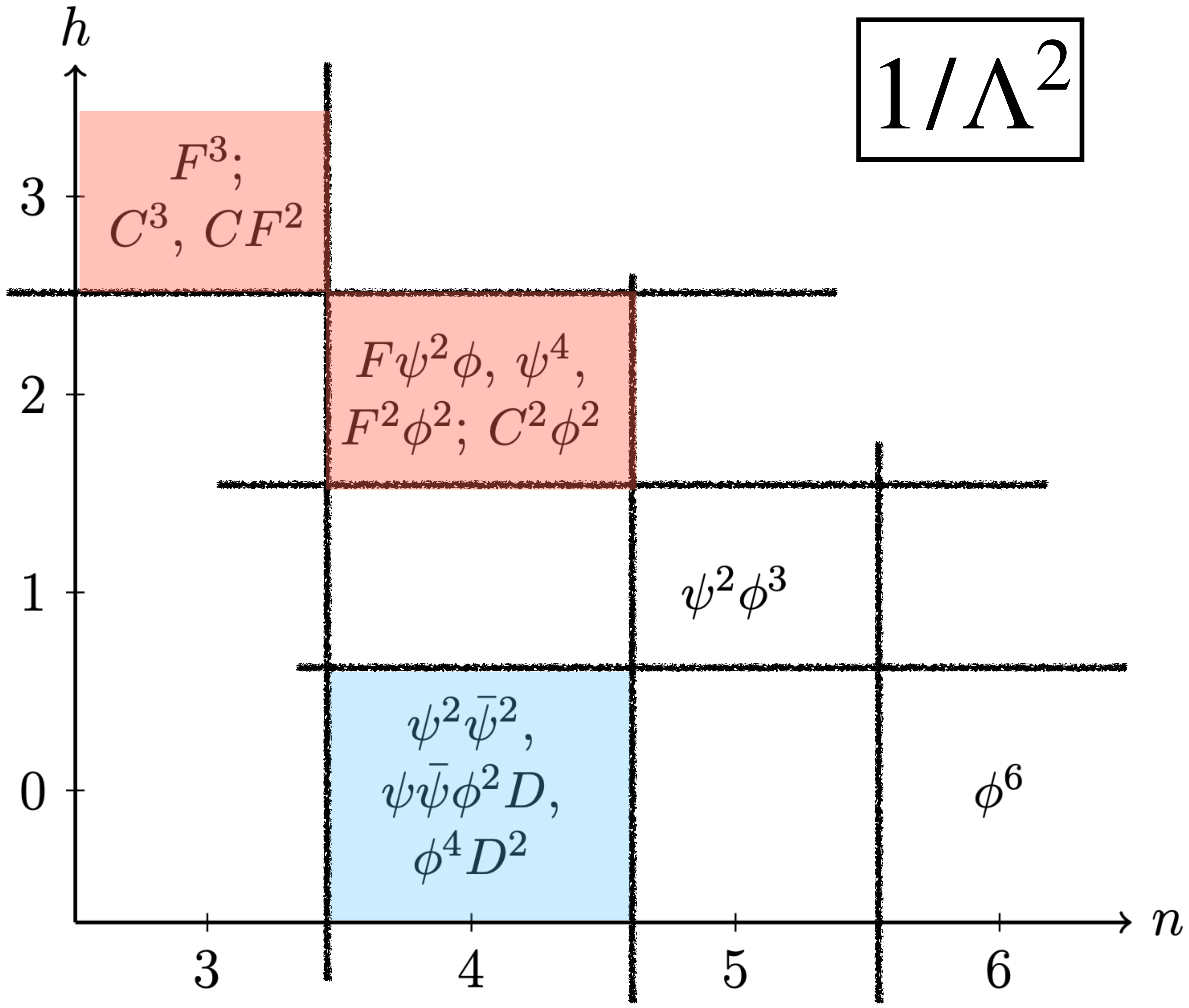


- previous discussion suggests to organise EFT operators according to n and modified helicity
- non-mixing result expressed with red cone
- possible divergences at order M_P^{-2} in a marginal theory minimally coupled to gravity must lie inside the blue cone

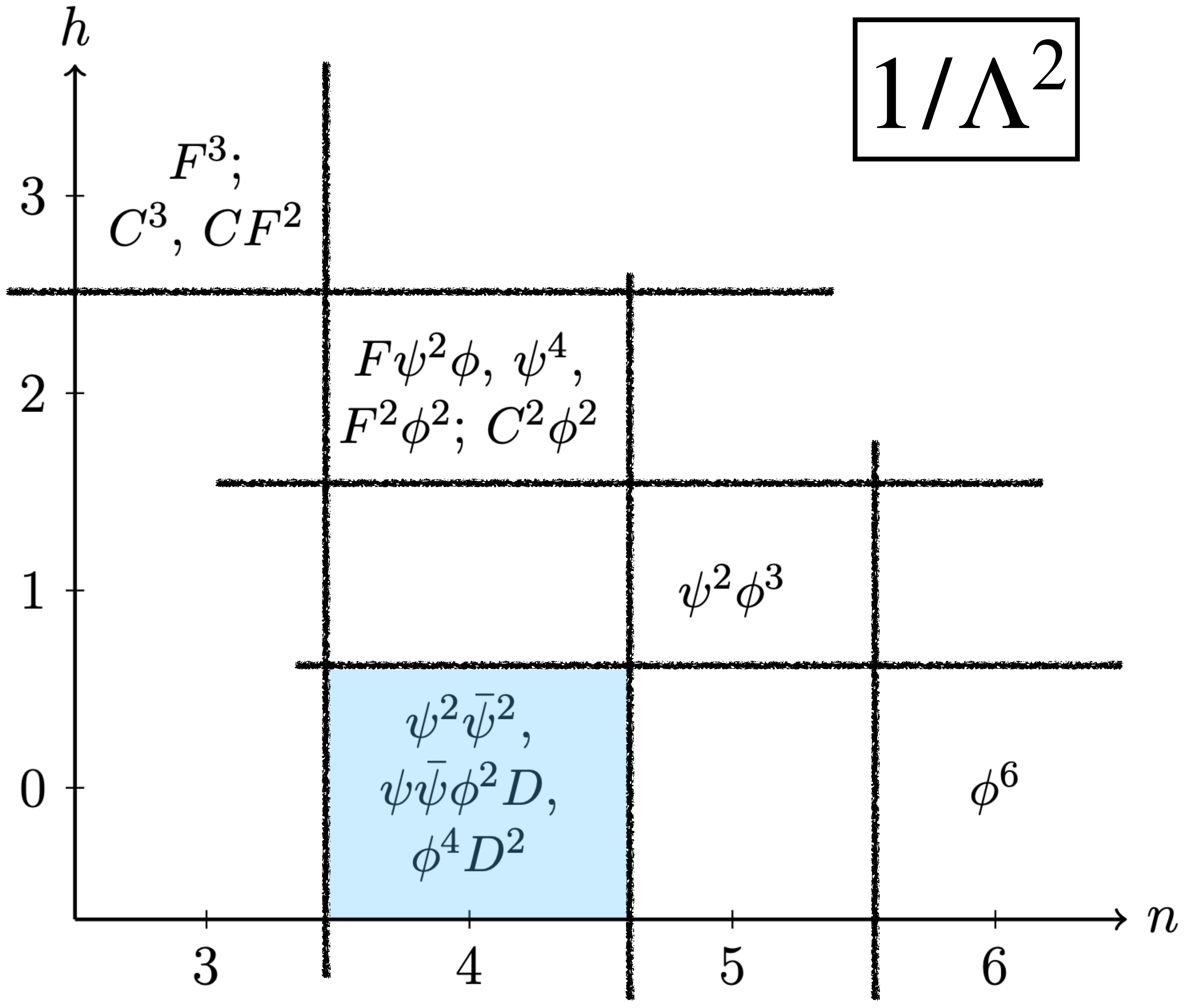
Computing the RG of GR



- mixing among operators including at least one graviton up to n=4 (red)
- divergences in generic minimally coupled theories at order M_P^{-2} (blue) and M_P^{-4}, up to four legs
- order M_P^{-4} anomalous dimensions are connected to positivity of Wilson coefficients at dimension 8

Computing the RG of GR

divergences @ O(M_P^{-2}) in any minimally coupled theory



- divergences at this order only can involve h=0,±1/2 particles as external states
- therefore M_P^{-2} can only come from an internal graviton propagating

Computing the RG of GR

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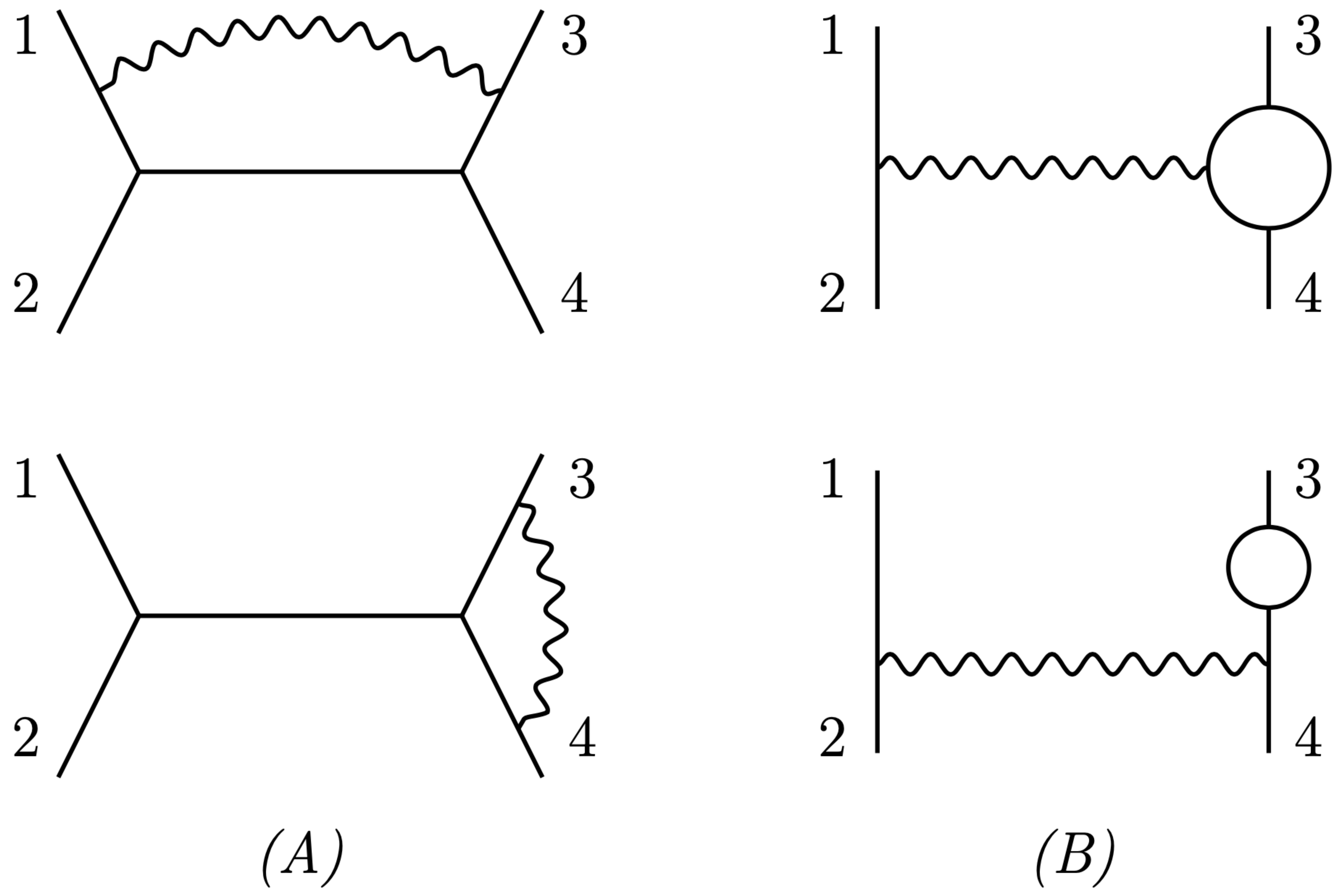

Computing the RG of GR

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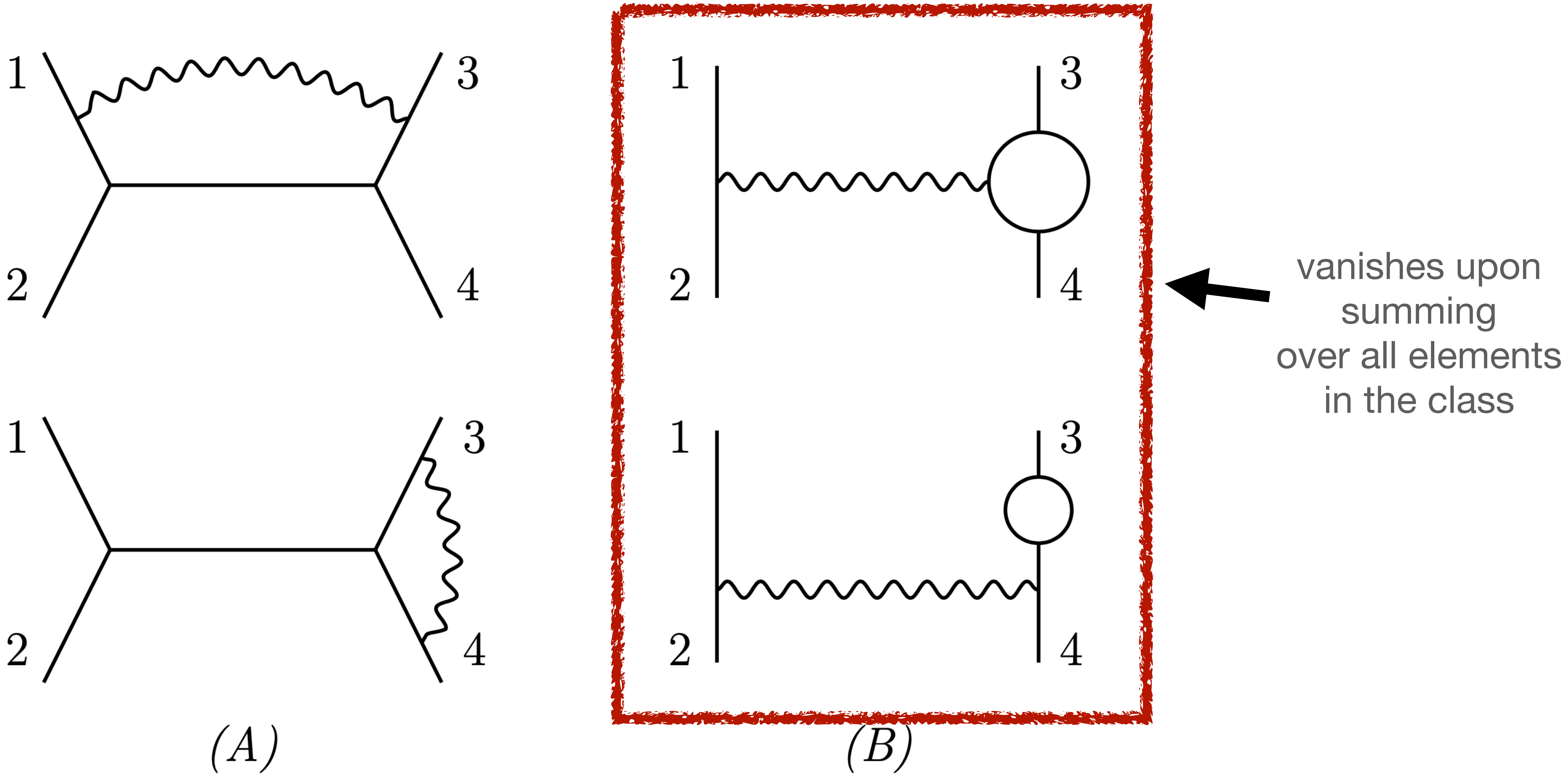
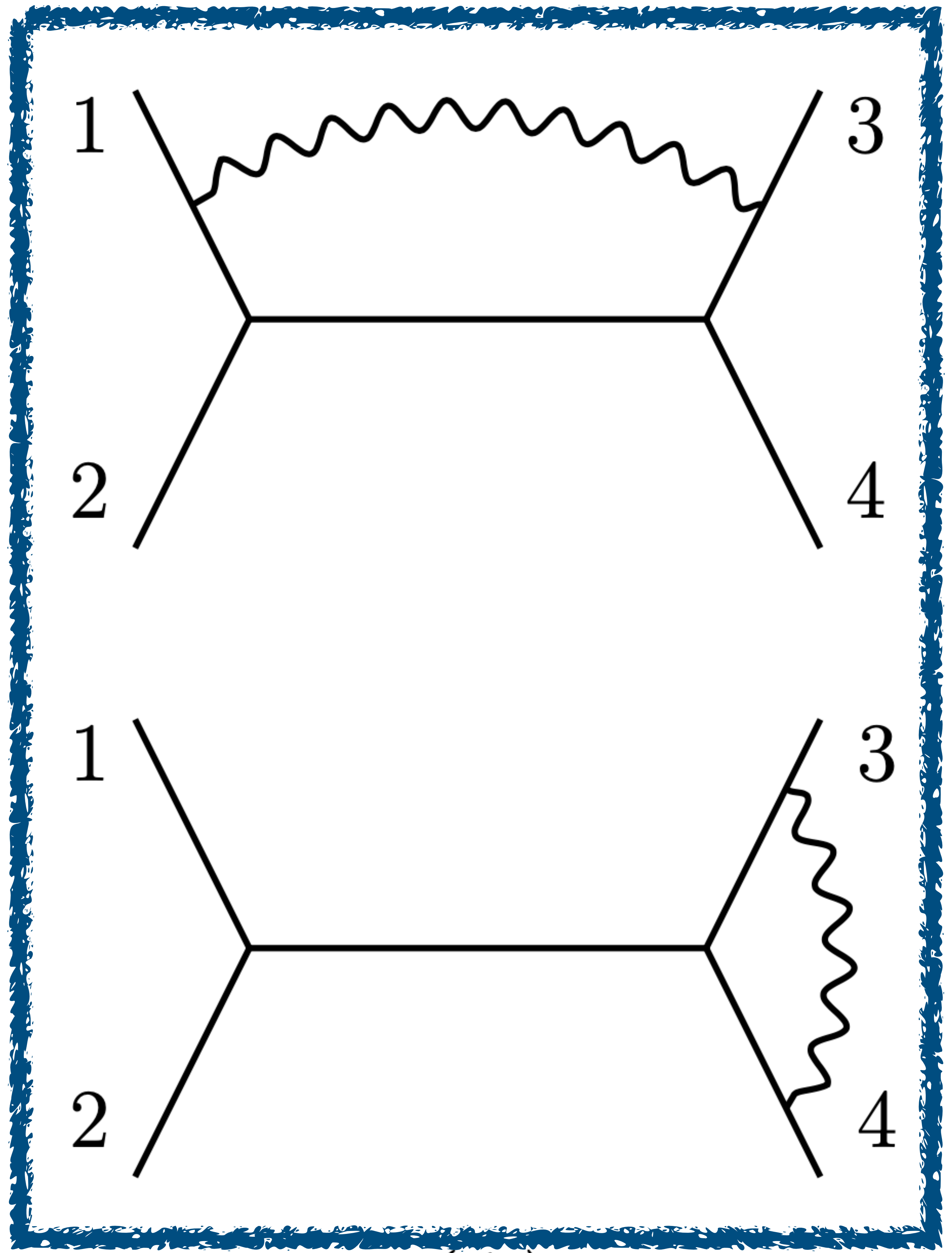


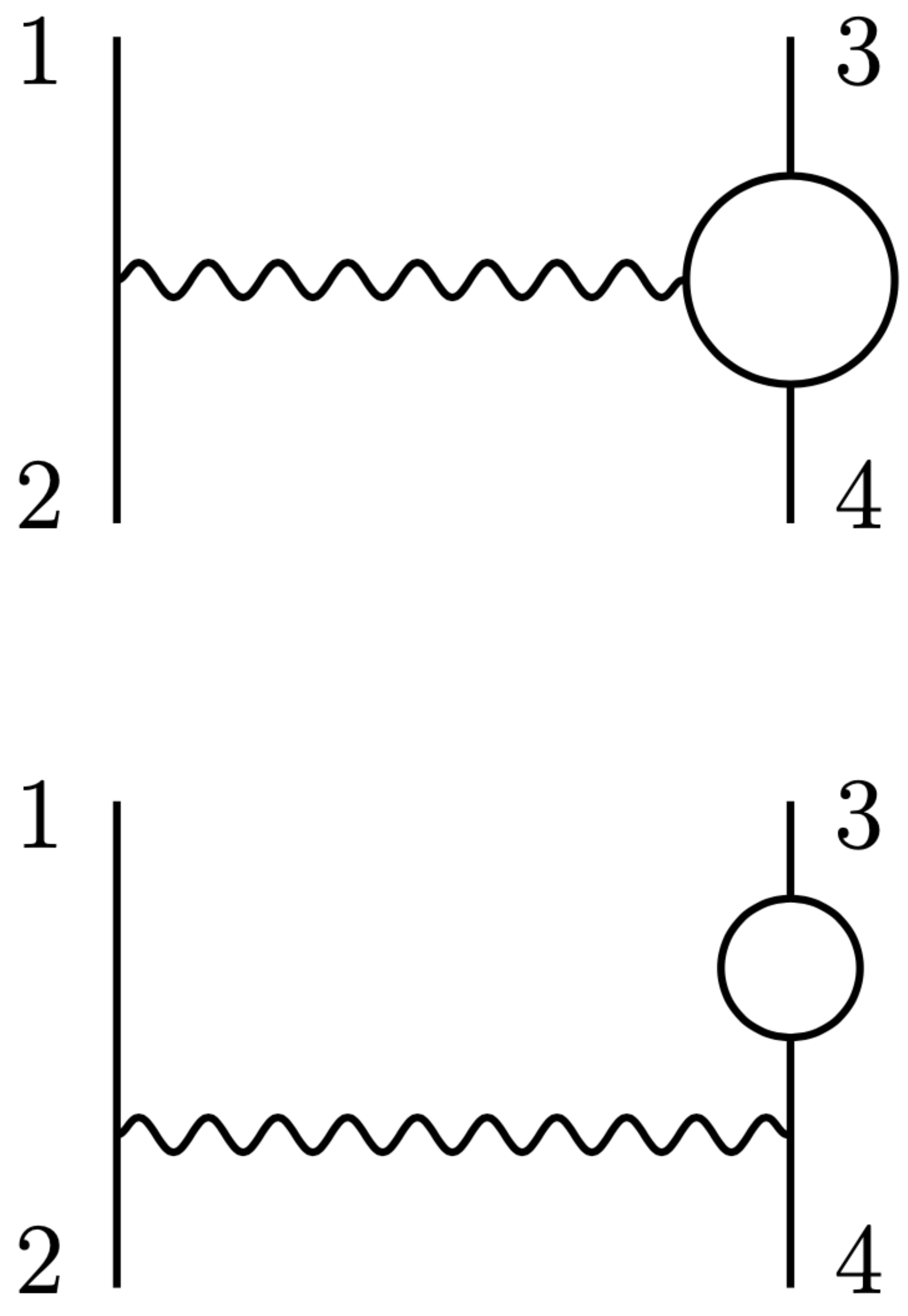
Computing the RG of GR

divergences @ O(M_P^{-2}) in any minimally coupled theory

flavor and color 'flow' as if the graviton was not there (helps in providing fully general results)



(A)



(B)

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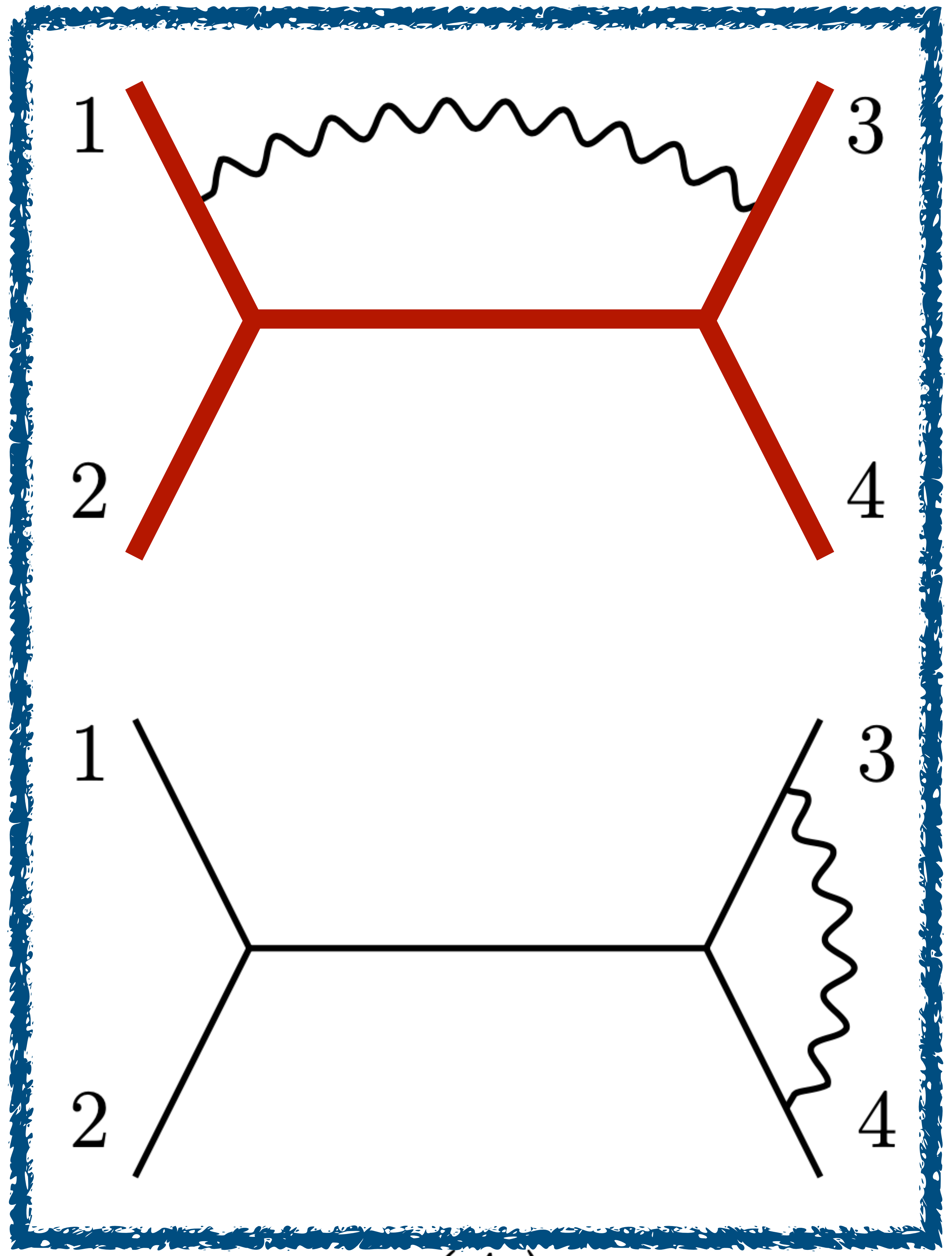

(A)

loop divergence as a 'function' of the corresponding tree amplitude (red)

$$\mathcal{A}_{\text{tree}}(1_{\bar{\psi}_1}, 2_{\psi_2}, 3_{\phi_3}, 4_{\phi_4}) = \left(\frac{T_s}{s} + \frac{Y_t}{t} + \frac{Y_u}{u}\right)\langle 13\rangle[23]\,,$$

Computing the RG of GR

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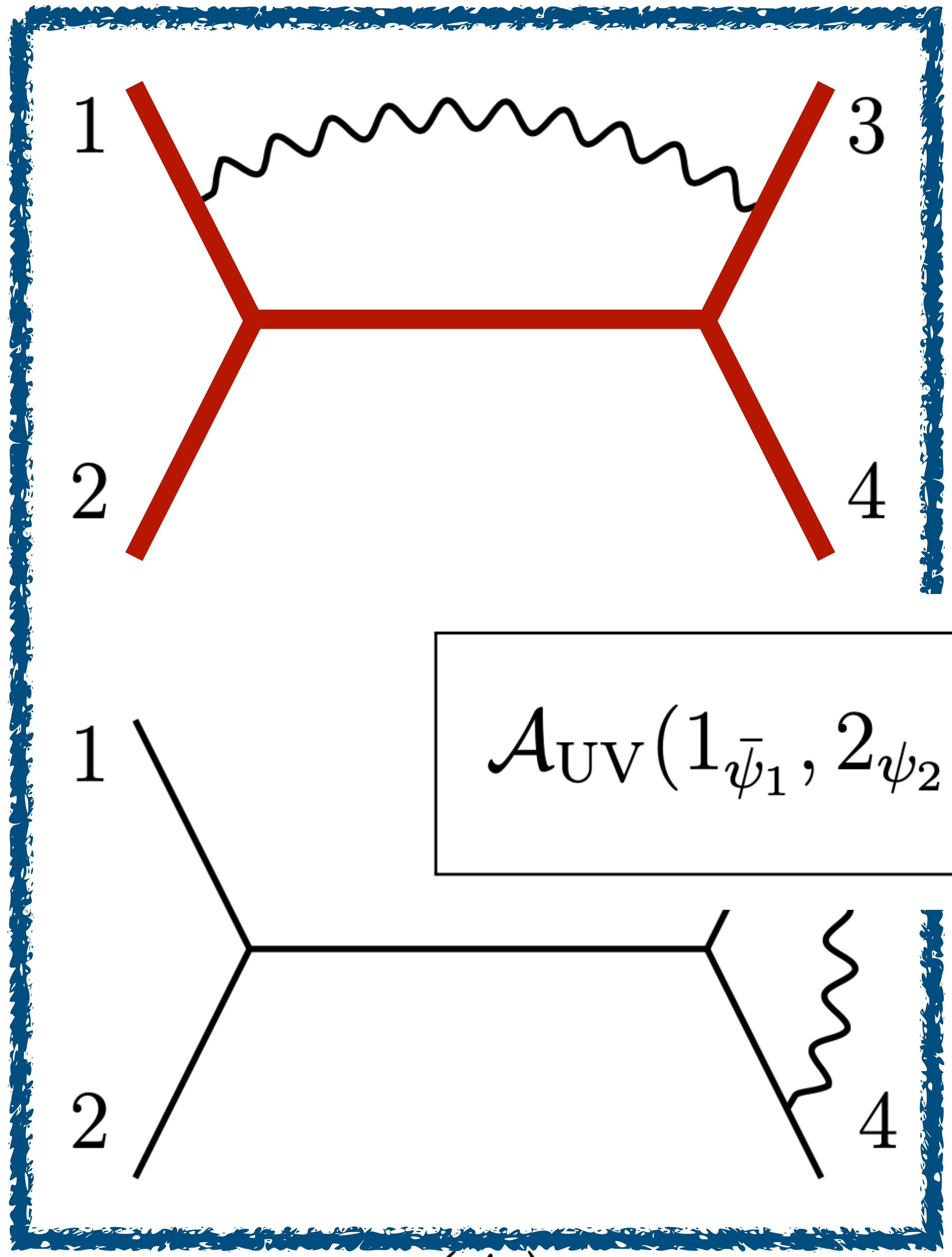

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$$\mathcal{A}_{\text{UV}}(1_{\bar{\psi}_1}, 2_{\psi_2}, 3_{\phi_3}, 4_{\phi_4}) = -\frac{7}{64\pi^2 M_{\text{Pl}}^2 \epsilon}\,(3\,T_s + Y_t + Y_u)\,\langle 13\rangle[23]\,.$$

Computing the RG of GR

divergences @ O(M_P^{-2}) in any minimally coupled theory

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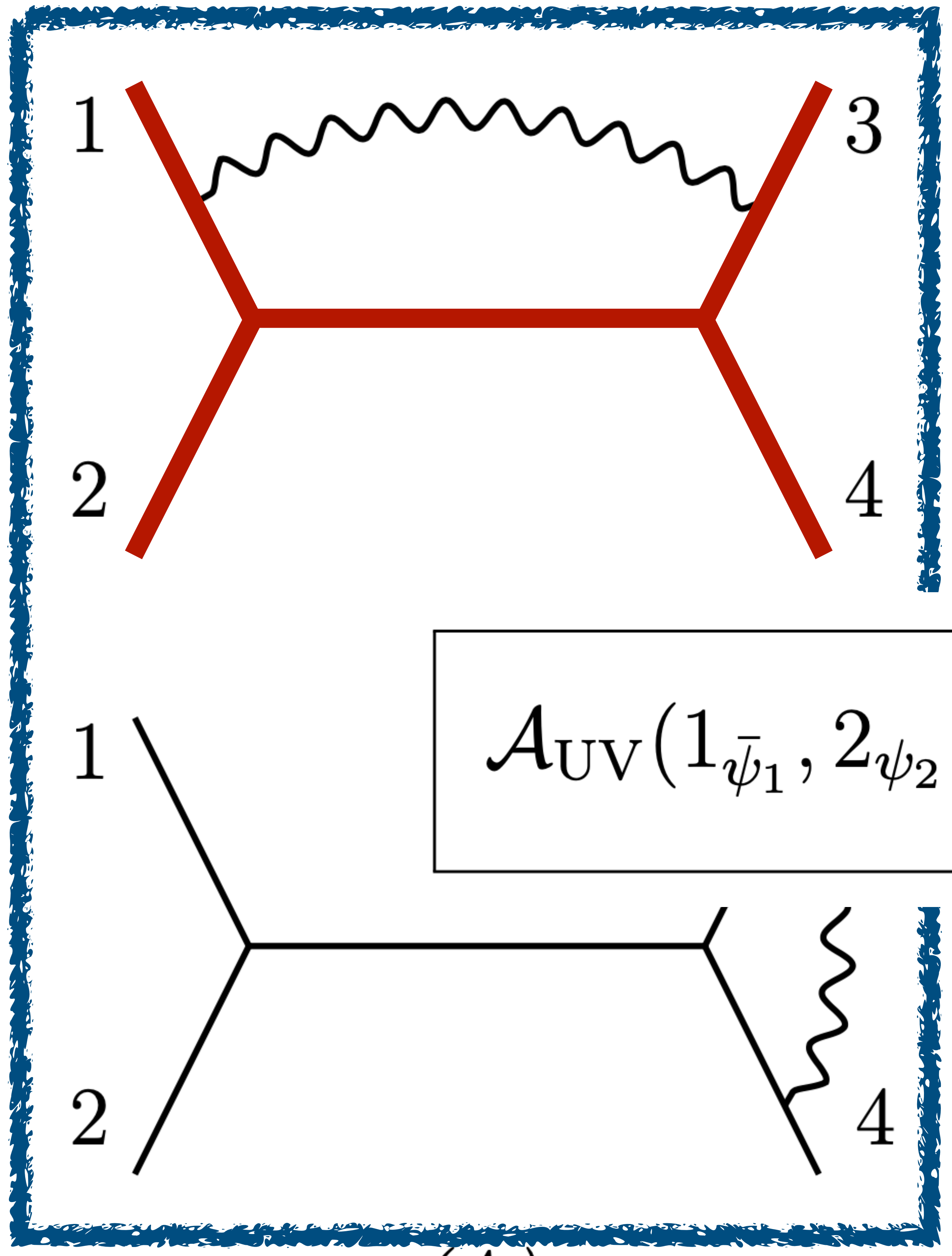

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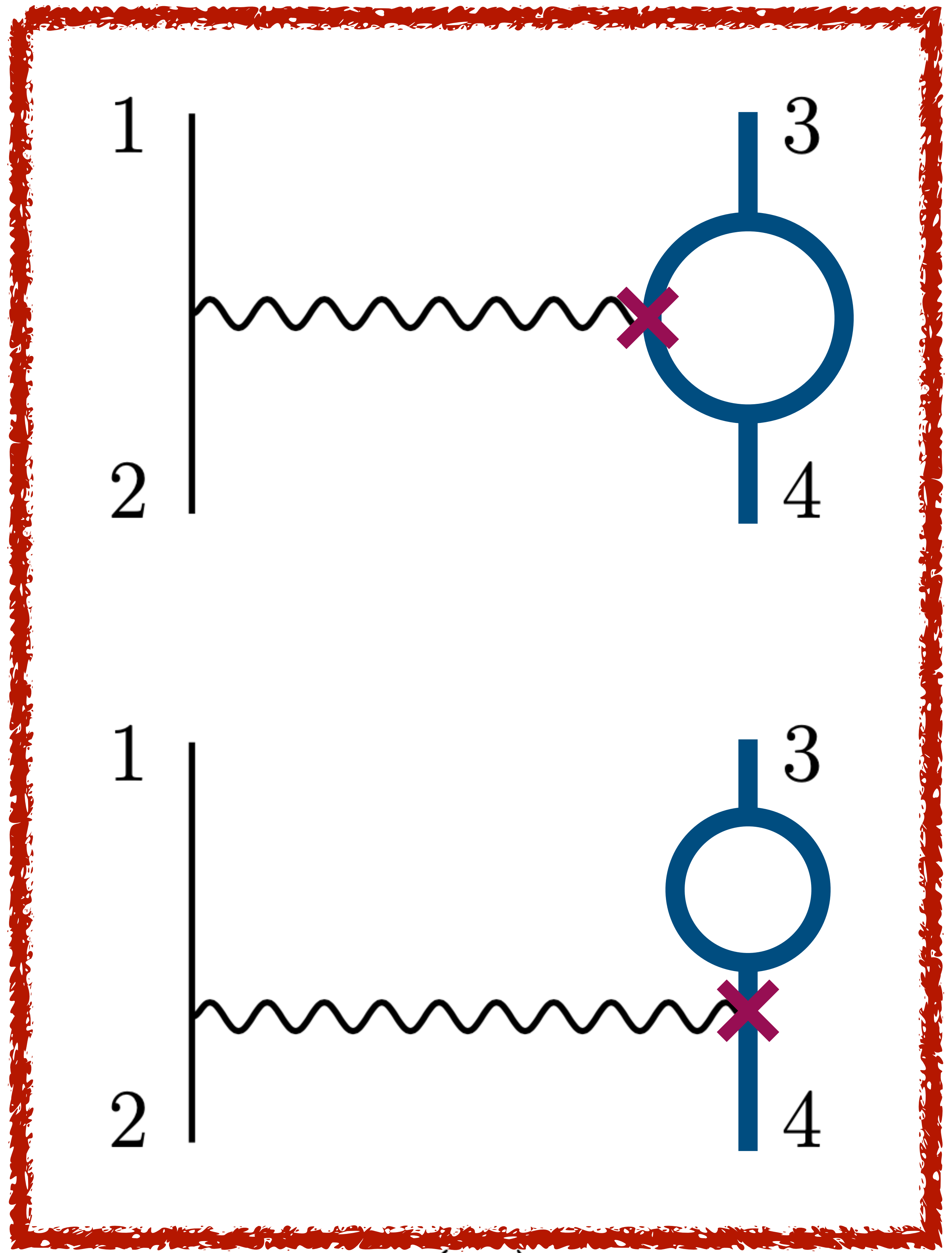
… and similarly for all other relevant helicity configurations

Computing the RG of GR

vanishing of 'class (B)'

connected to the non-renormalization of $T^{\mu\nu}$ (graviton couples to matter through $h_{\mu\nu}T^{\mu\nu}$)

$$\langle 0 | T^{\mu\nu} | 3,4 \rangle_{\text{div}} = 0$$



(B)

vanishes upon summing over all elements in the class

Computing the RG of GR

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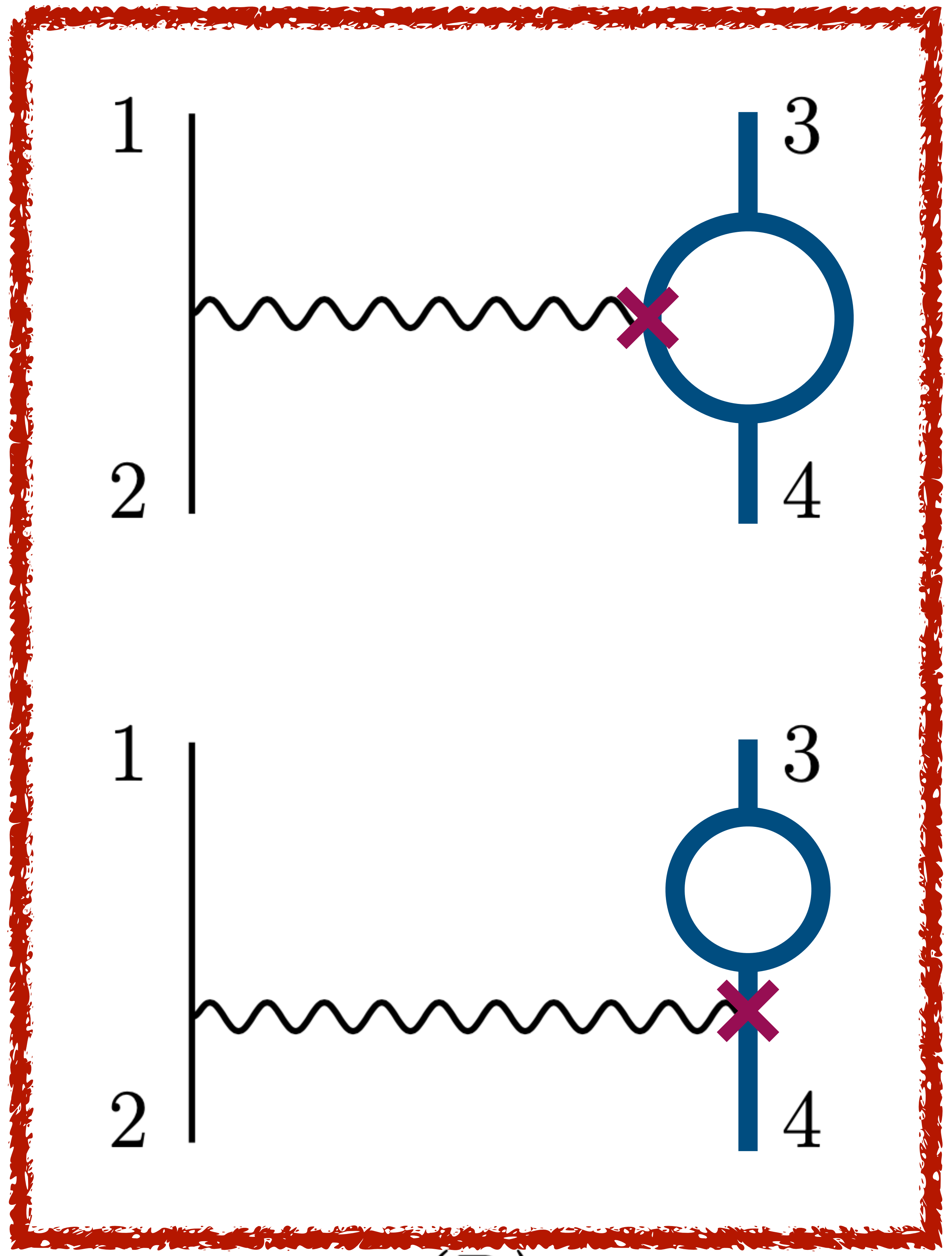
second diagram proportional to the <u>collinear anomalous dimension</u> of particle 3 (and its antiparticle 4)



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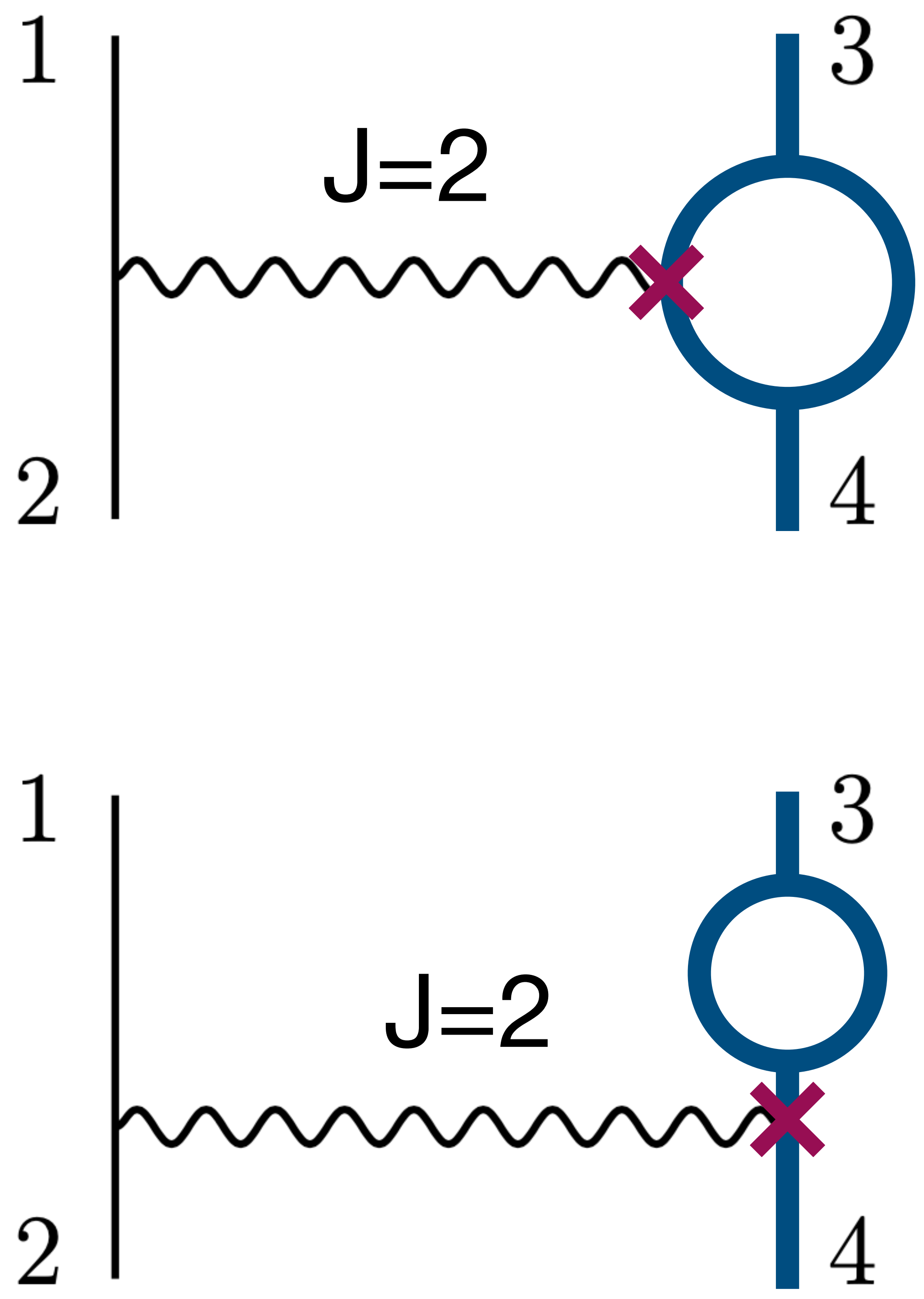
vanishes upon summing over all elements in the class

γ_{coll} from $T^{\mu\nu}$ non-renormalization

- angular analysis shows that $T^{\mu\nu}$ has J=2
- first diagram proportional to a sum over certain J=2 partial wave coefficients
- second diagram proportional to γ_{coll}
- from the vanishing of their sum we get a new formula to express collinear anomalous dimensions



(B)

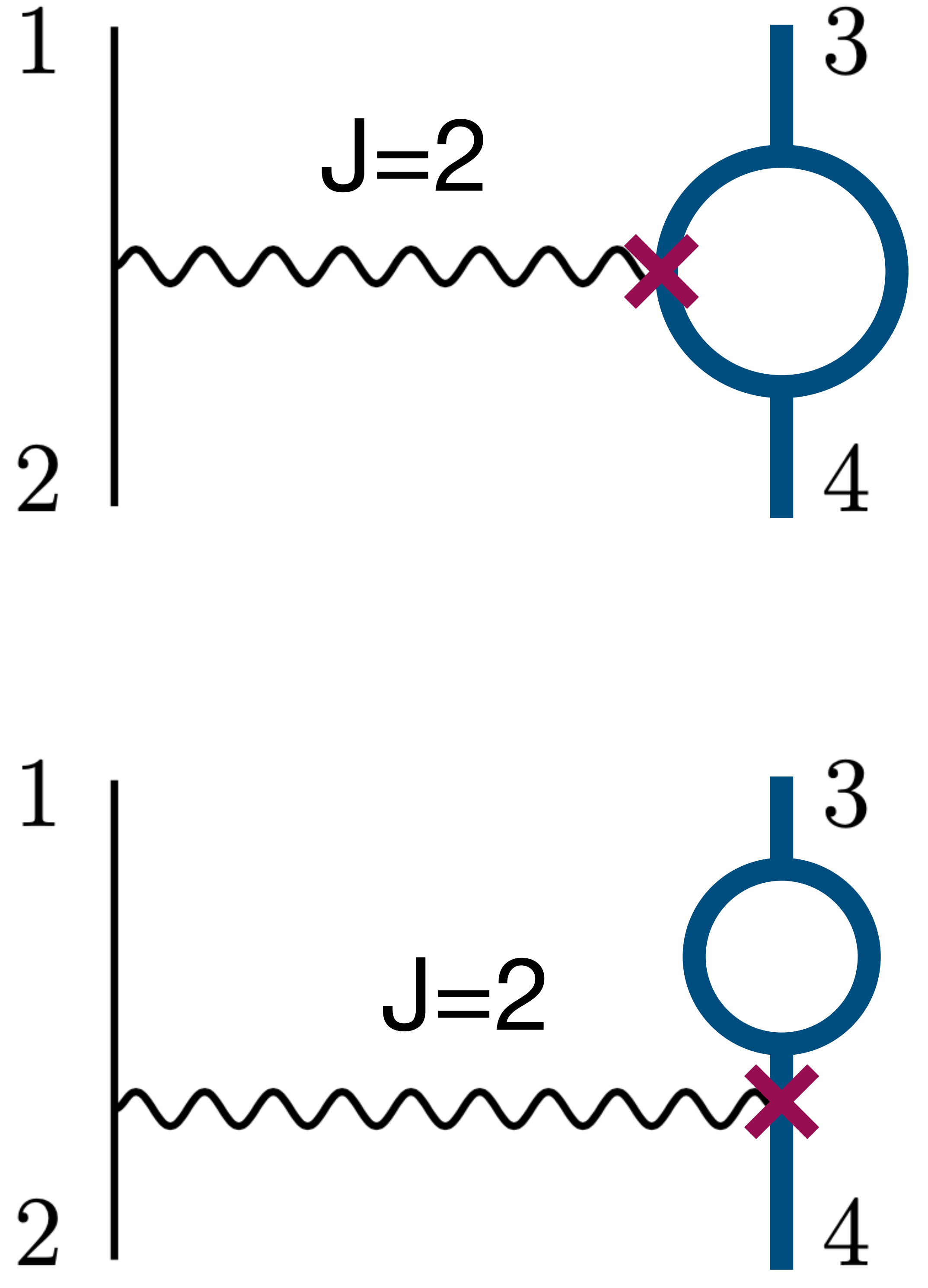
$\gamma_{\rm coll}$ from $T^{\mu\nu}$ non-renormalization

- new formula to express collinear anomalous dimensions in terms of partial wave coefficients (marginal couplings only)

$$\gamma^{(\Phi)}_{\rm coll} = \frac{1}{16\pi^2}\sum_{\Phi'}\frac{f_{\Phi'}}{f_{\Phi}}\, a^{(2)}_{\Phi'\bar\Phi'\to\Phi\bar\Phi}\Big|_{\rm reg}$$

2010.13809 (PB, Fernandez, von Harling, Pomarol)

$$f_\varphi = 1/\sqrt{6} \quad f_\psi = 1/2 \quad f_V = -1$$



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Mixing including gravity

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in a 4 to 4 mixing, here including operators
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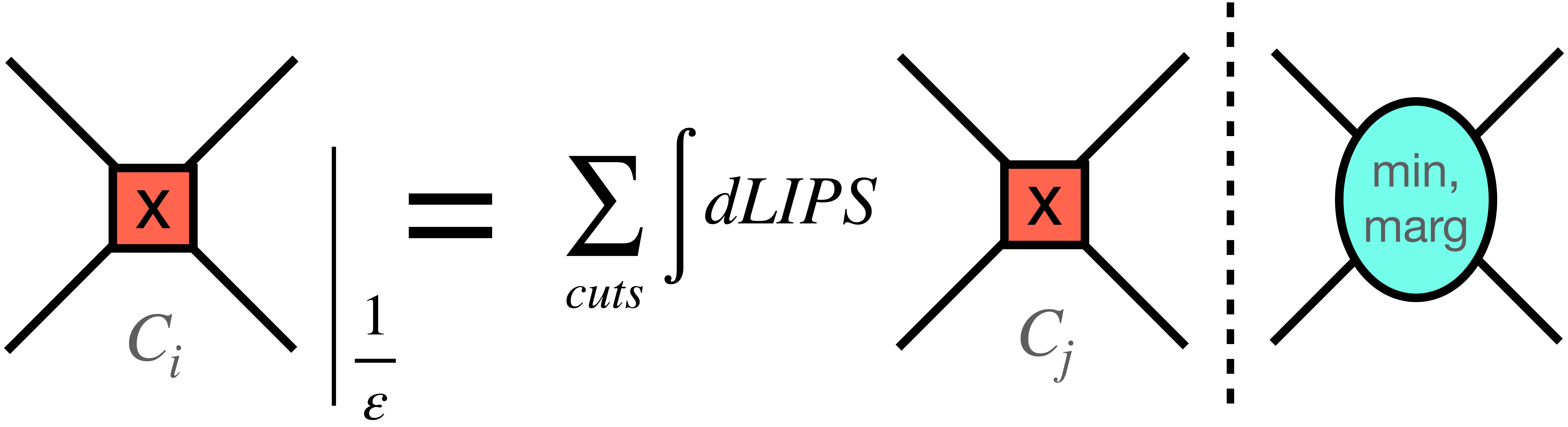
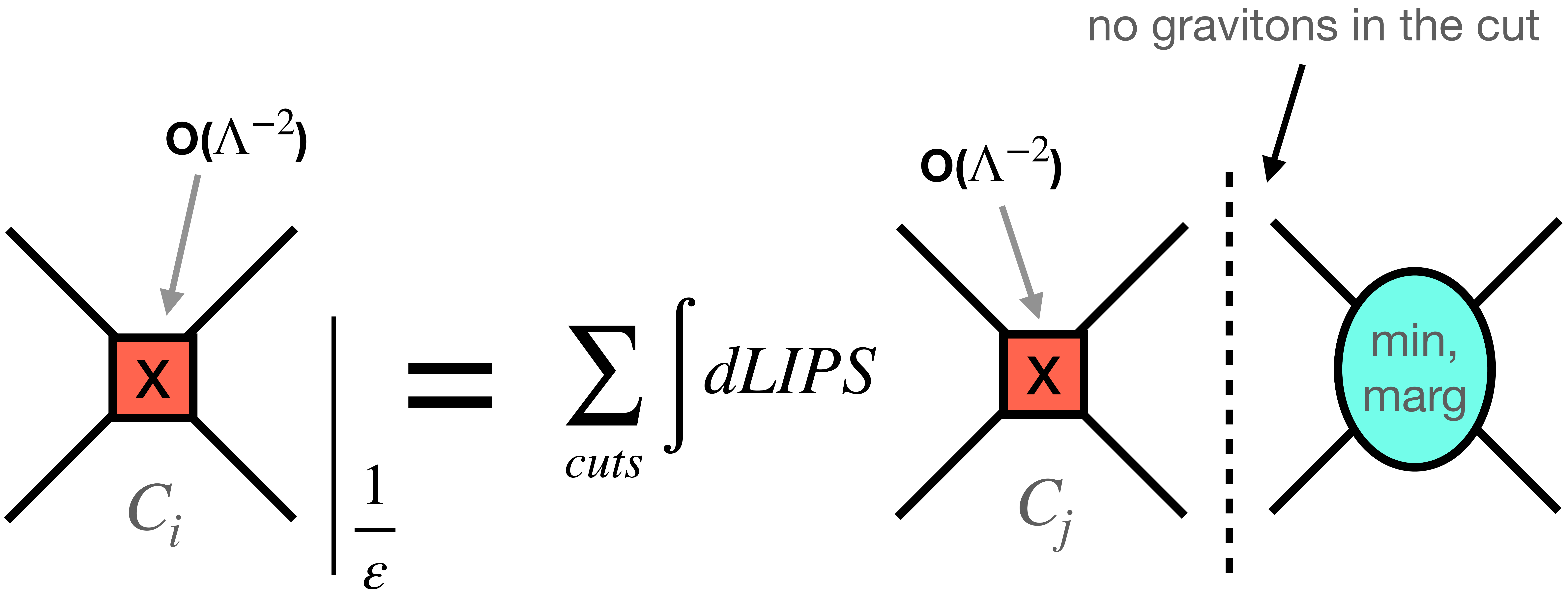


Mixing including gravity

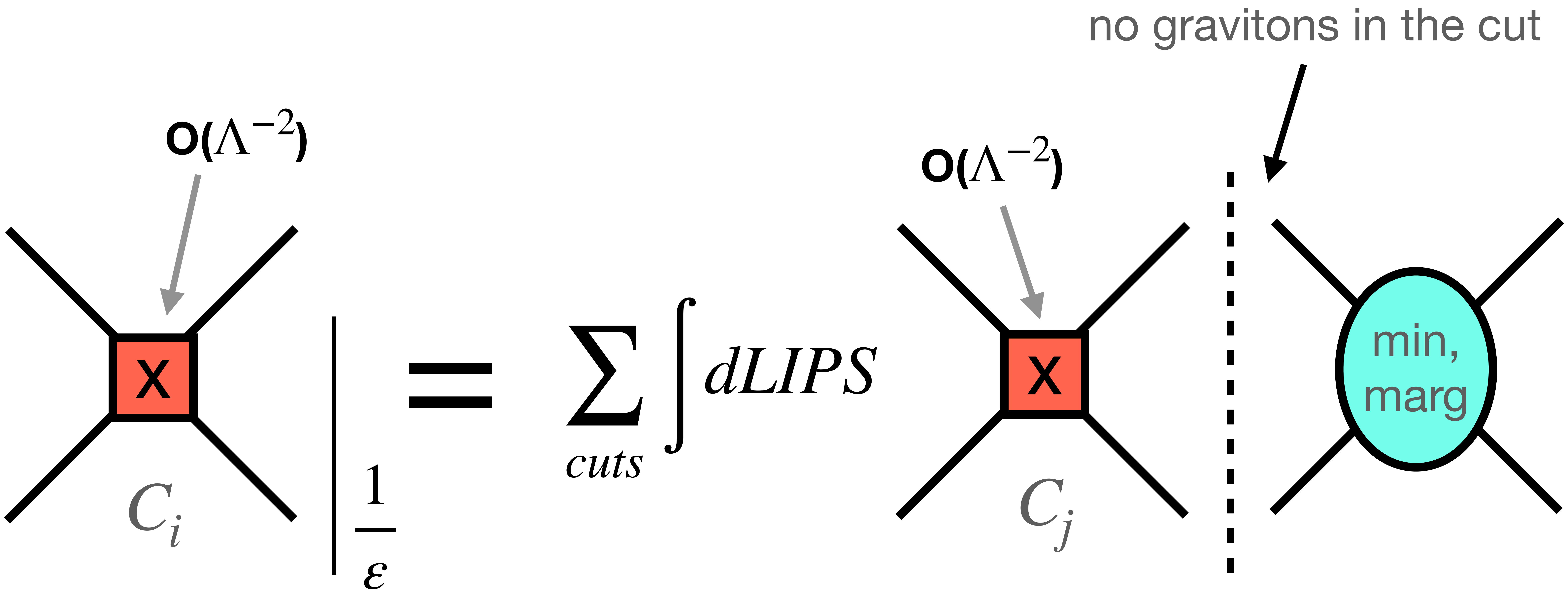
Leading order running of amplitudes with gravitons



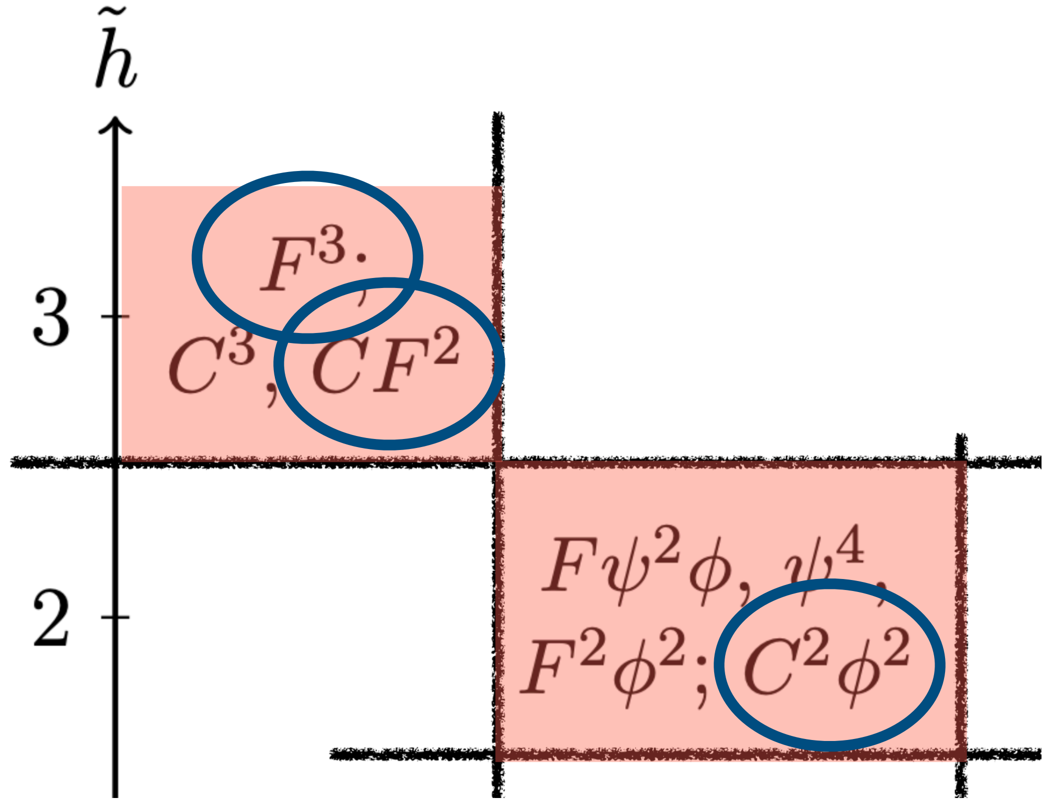
Mixing including gravity

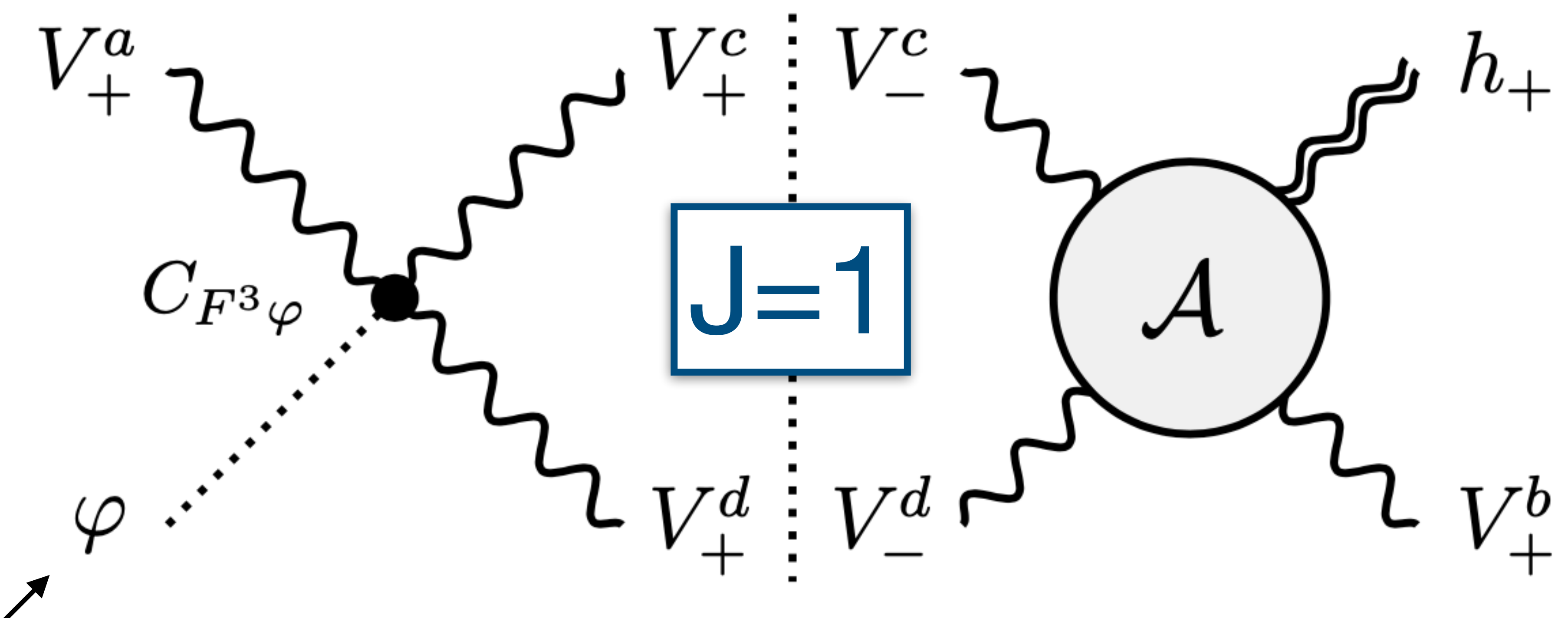
Leading order running of amplitudes with gravitons

- Only a handful of mixings (with at least one graviton) at O(Λ^{-2})

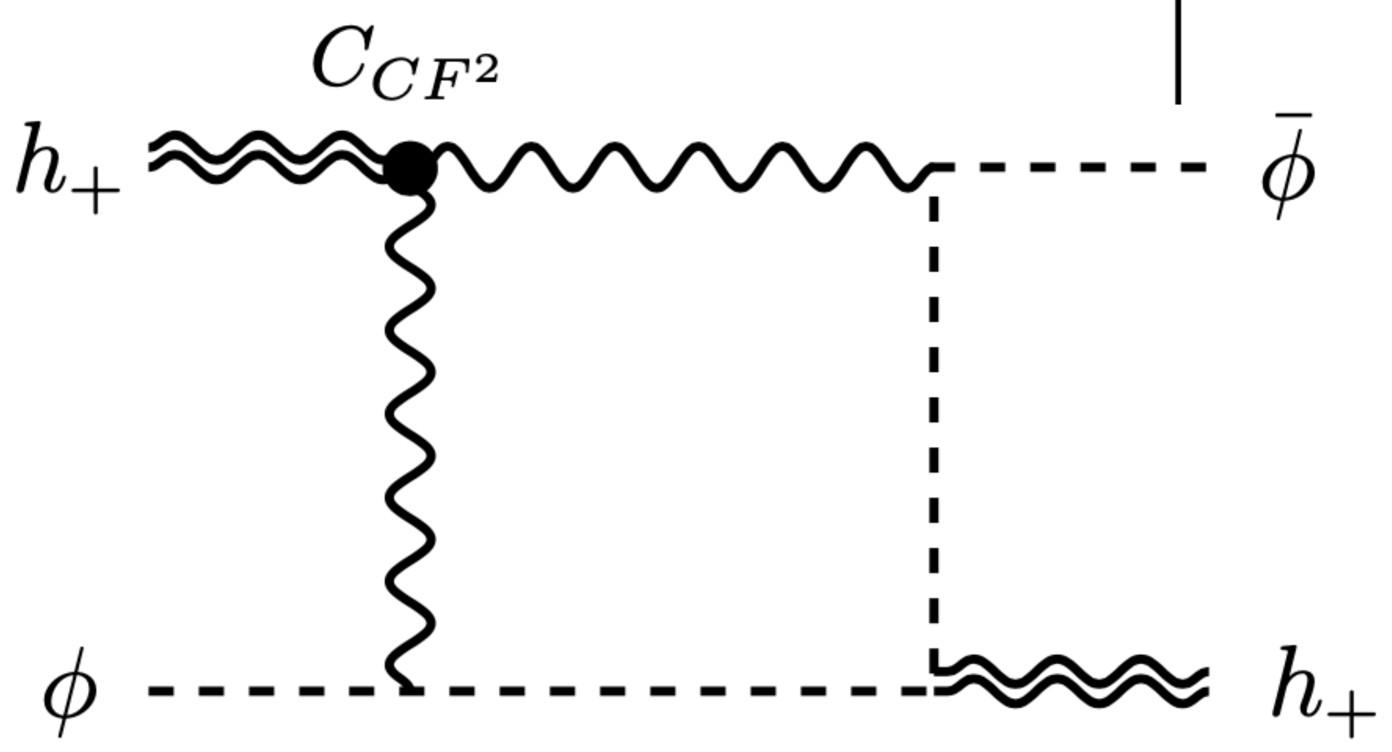


Mixing including gravity



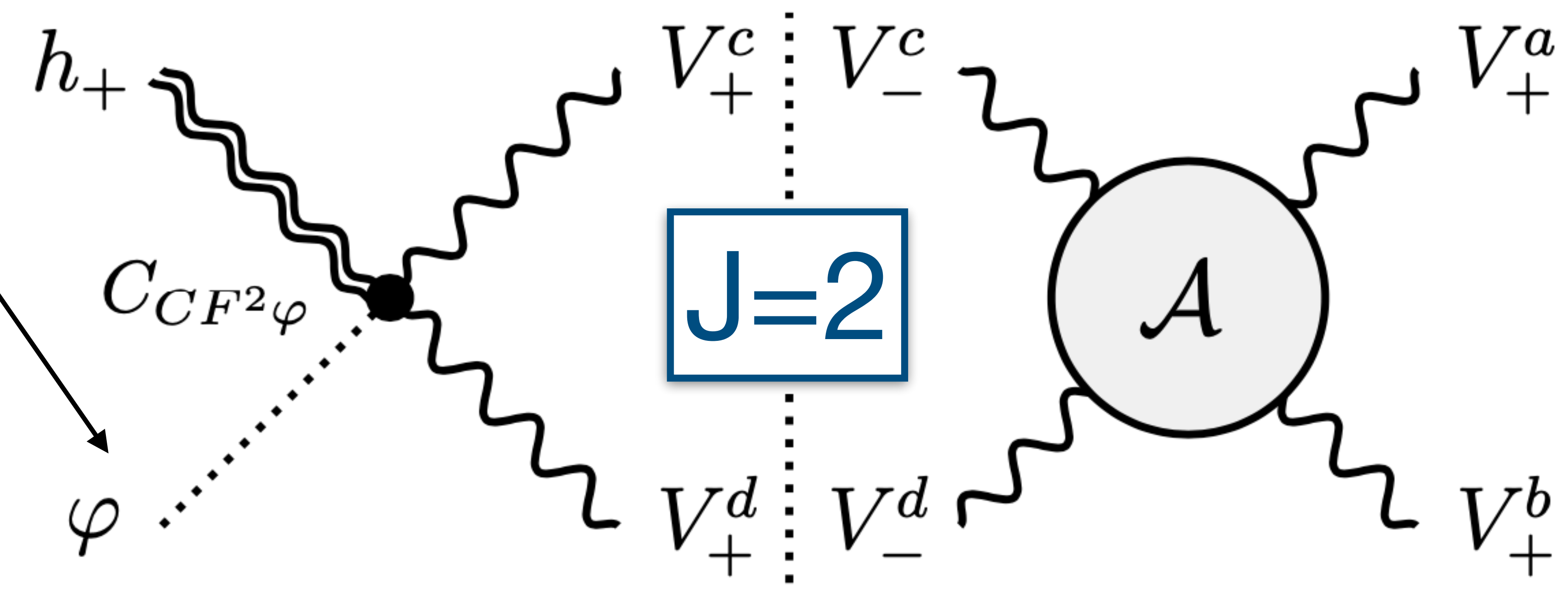


$C_{F^3} \rightarrow C_{CF^2}$

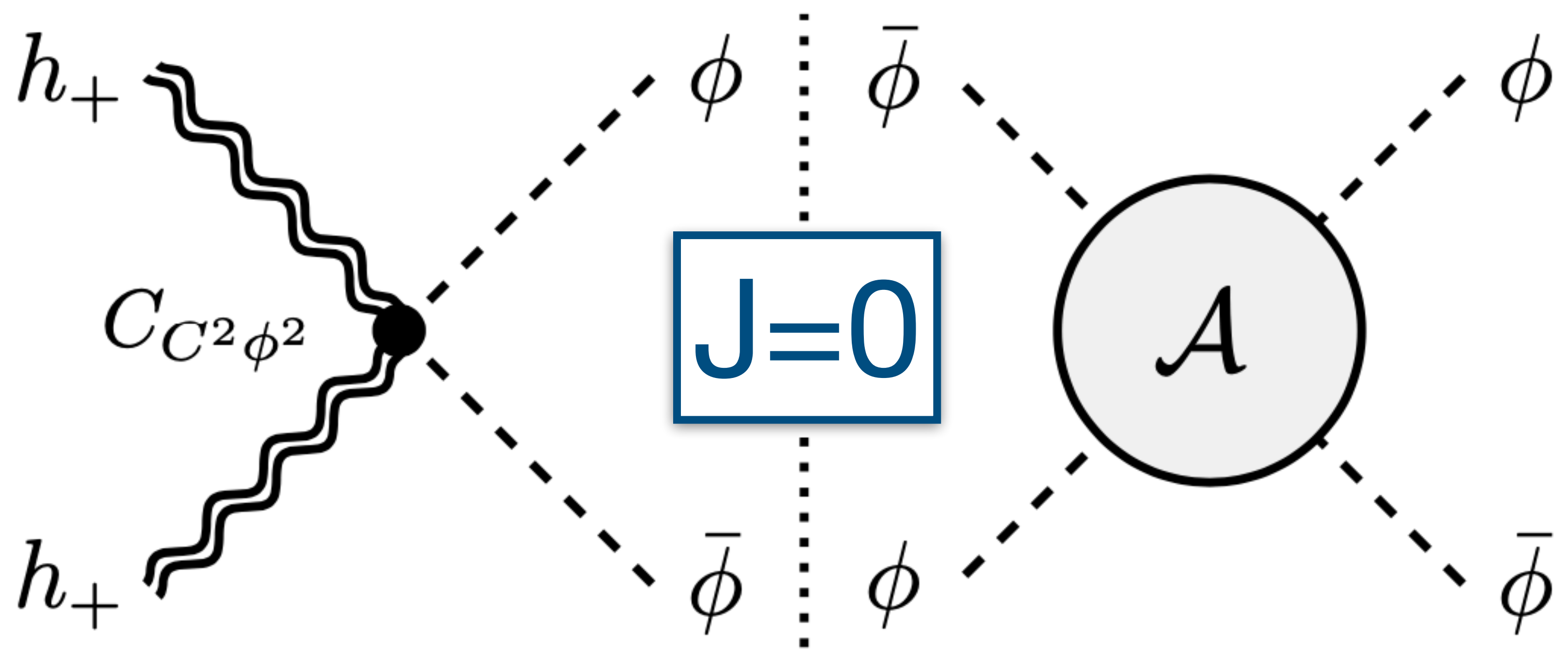


$C_{CF^2} \rightarrow C_{C^2\phi^2}$

auxiliary legs



$C_{CF^2} \rightarrow C_{CF^2}$



$C_{C^2\phi^2} \rightarrow C_{C^2\phi^2}$

Summary

- Motivation (γ_{UV} encode fundamental properties of gravity EFTs)
- γ_{UV} from on-shell amplitudes (gravity is included without effort)
- Non-renormalization from helicity considerations
- Bound on total helicity of tree-amplitudes (without and <u>with gravity</u>: $\tilde{h}$)
- Non-renormalization with gravity
- Computation of RG (very convenient procedure)