Parton Branching TMDs

- PB TMDs
  - parton branching algorithm to solve evolution equations
  - determination of TMD densities at NLO from HERA DIS data

TMD – what is it and how to determine it ?

- TMDs (Transverse Momentum Dependent parton distribution)
  - Transverse momentum effects are naturally coming from intrinsic k_t and parton showers

- New: Parton Branching Method
  - determine integrated PDF from parton branching solution of evolution eq.
    - check consistency with standard evolution on integrated PDFs
      - at LO, NLO and NNLO
      - advantages of Parton Branching Method
  - determine TMD:
    - since each branching is generated explicitly, energy-momentum conservation is fulfilled and transverse momentum distributions can be obtained

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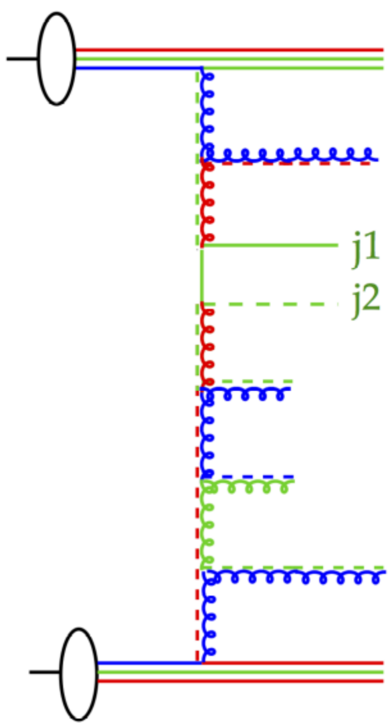
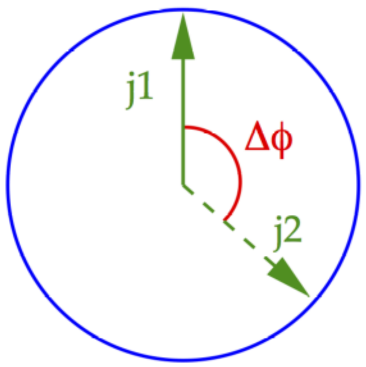
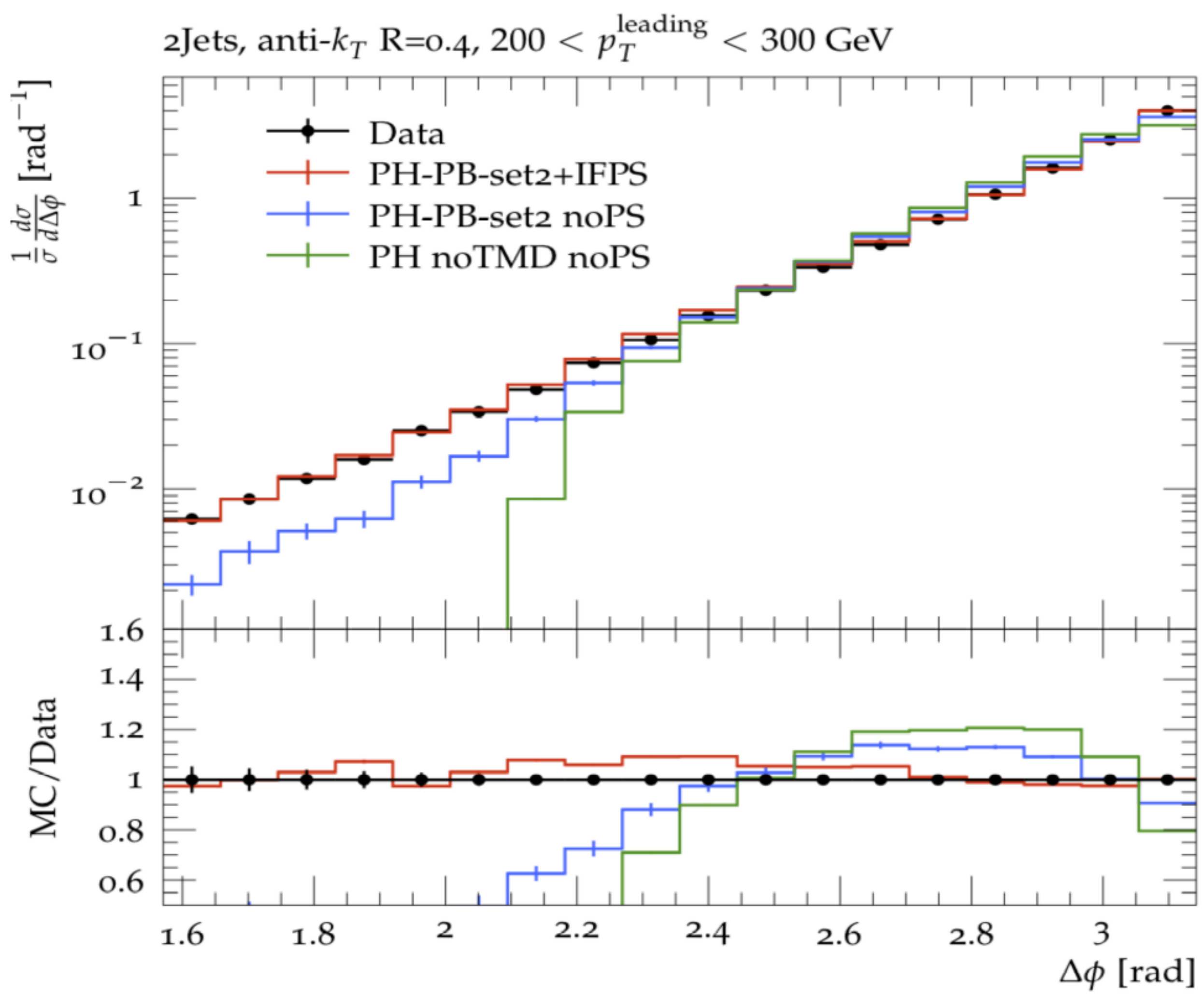
[1] F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Soft-gluon resolution scale in QCD evolution equations. Phys. Lett., B772:446–451, 2017.
[2] F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Collinear and TMD Quark and Gluon Densities from Parton Branching Solution of QCD Evolution Equations. JHEP, 01:070, 2018.
[3] A. Bermudez Martinez, P. Connor, F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Collinear and TMD parton densities from fits to precision DIS measurements in the parton branching method. DESY-18-042, arXiv 1804.11152

Why TMDs ?

- Measurements with $p_T > 200$ GeV
  - at least 2 jets







- NLO-dijet (Powheg) w/o PS cannot describe small $\Delta\,\phi$
- NLO-dijet (Powheg) with TMDs describes spectrum at small and large $\Delta\,\phi$
- Region of higher order emissions described by TMDs

DGLAP evolution – solution with parton branching method

- differential form: $$\mu^2 \frac{\partial}{\partial \mu^2} f(x, \mu^2) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) \, f\left(\frac{x}{z}, \mu^2\right)$$

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$$\Delta_s(\mu^2) = \exp\left(- \int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P^{(R)}(z) \right)$$

- differential form using f/Δ_s with

$$\mu^2 \frac{\partial}{\partial \mu^2} \frac{f(x,\mu^2)}{\Delta_s(\mu^2)} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{P^{(R)}(z)}{\Delta_s(\mu^2)} f\left(\frac{x}{z},\mu^2\right)$$

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- integral form

$$f(x,\mu^2) = f(x,\mu_0^2)\Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z},\mu'^2\right)$$

no – branching probability from $\mu^2{}_0$ to μ^2

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- solve integral equation via iteration:

$$f_0(x,\mu^2) \quad = \quad f(x,\mu_0^2)\Delta(\mu^2)$$

DGLAP evolution – solution with parton branching method

$$f(x,\mu^2) = f(x,\mu_0^2)\Delta_s(\mu^2) + \int^{z_M} \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z},\mu'^2\right)$$

- solve integral equation via iteration:

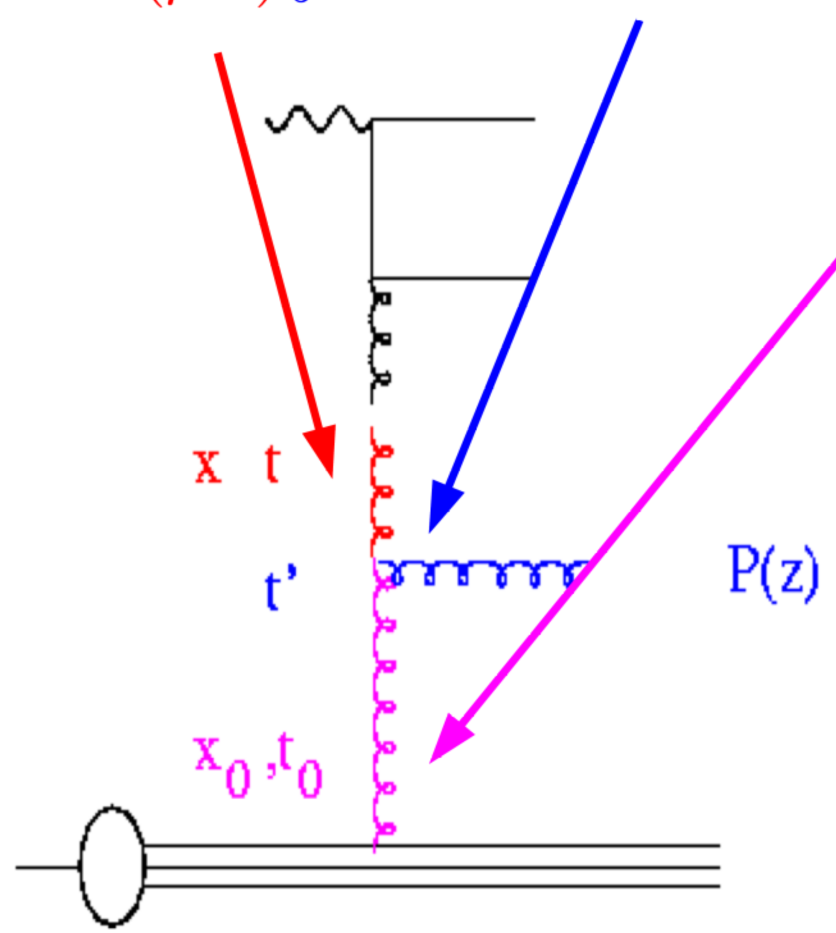
from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$f_0(x,\mu^2) = f(x,\mu_0^2)\Delta(\mu^2)$$

$$f_1(x,\mu^2) = f(x,\mu_0^2)\Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta(\mu^2)}{\Delta(\mu'^2)} \int^{z_M} \frac{dz}{z} P^{(R)}(z) f(x/z,\mu_0^2)\Delta(\mu'^2)$$



DGLAP evolution – solution with parton branching method

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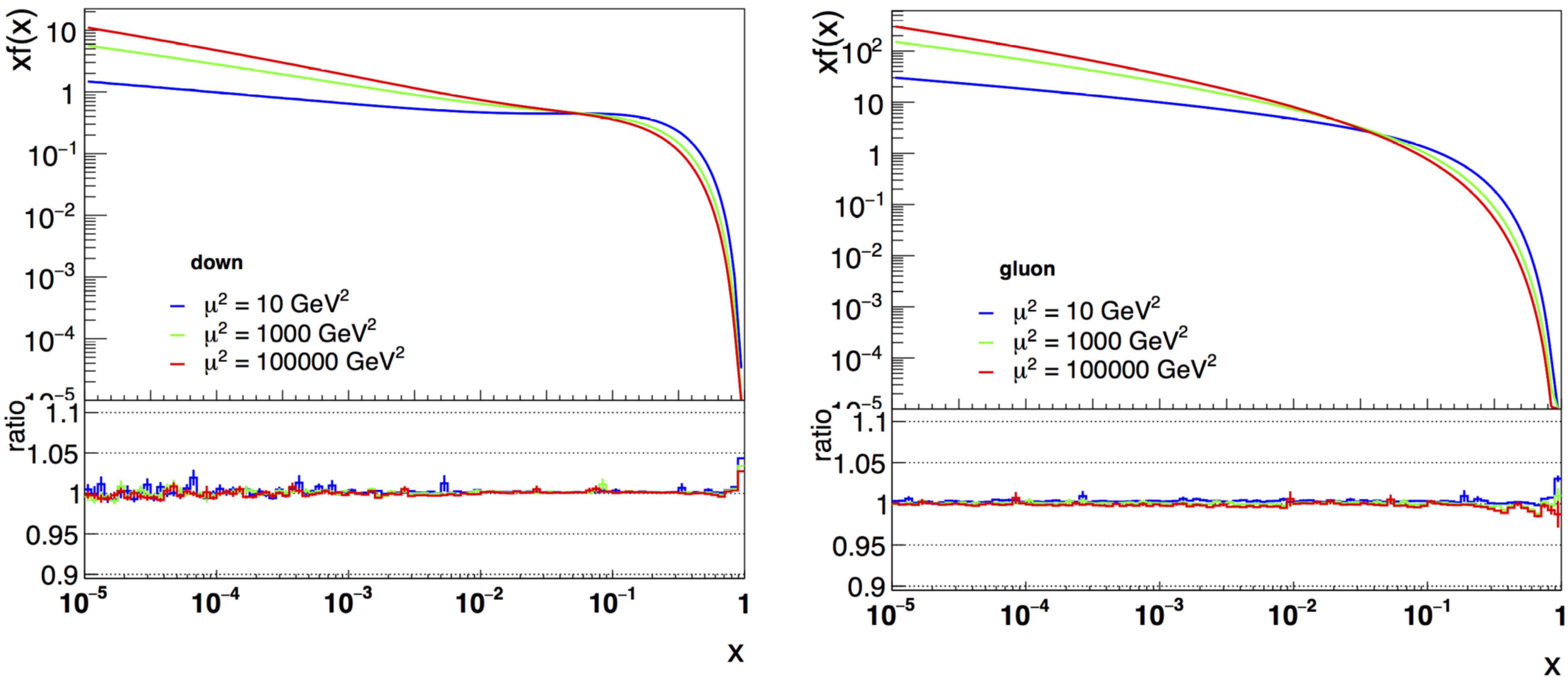
- with $P_{ab}^{(R)}(z)$ real emission probability (without virtual terms)
  - z_M introduced to separate real from virtual and non-emission probability
  - reproduces DGLAP up to $\mathcal{O}(1-z_M)$

- make use of momentum sum rule to treat virtual corrections
  - use Sudakov form factor for non-resolvable and virtual corrections

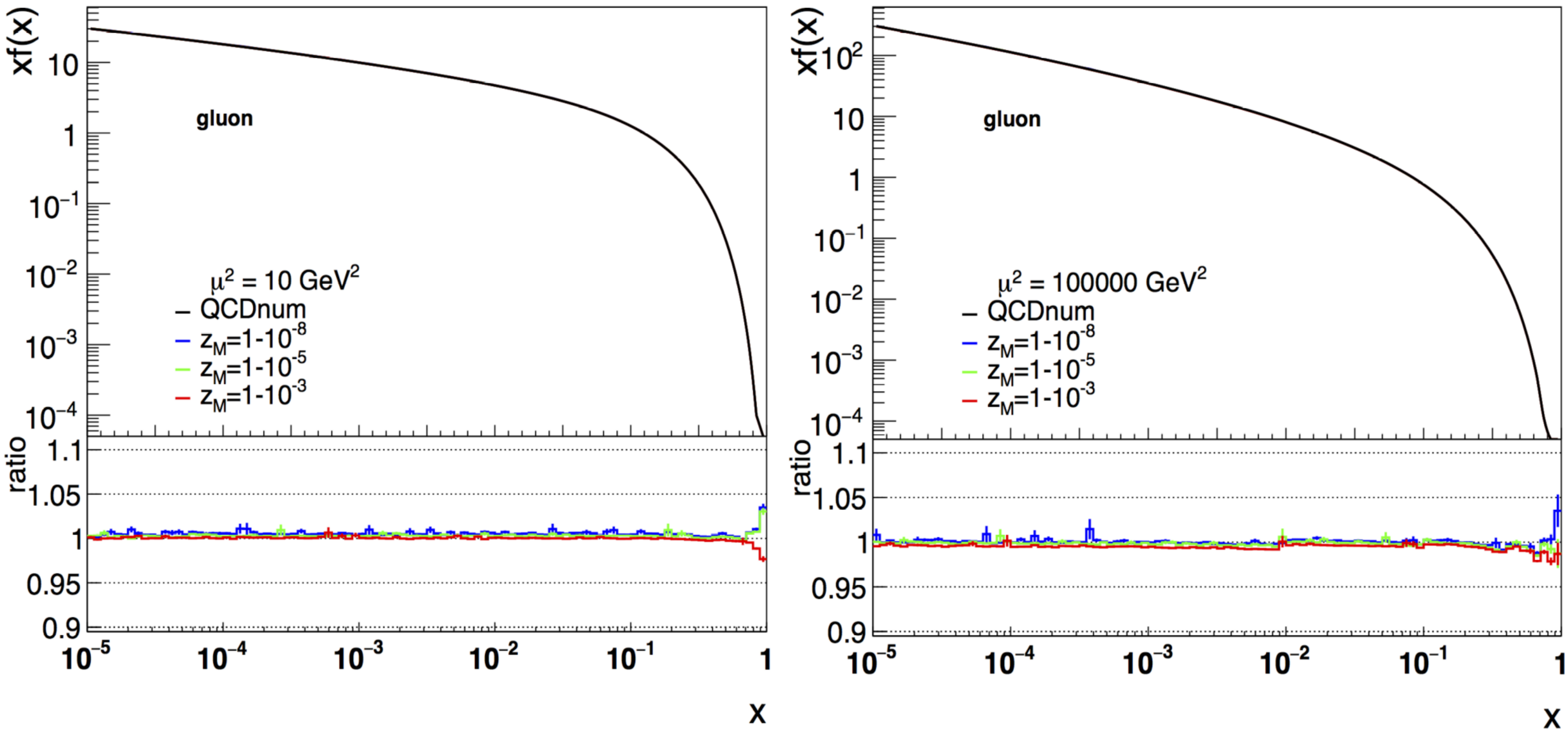
$$\Delta_a(z_M,\mu^2,\mu_0^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\ z\ P_{ba}^{(R)}(\alpha_s), z)\right)$$

Validation of method with QCDnum at NLO



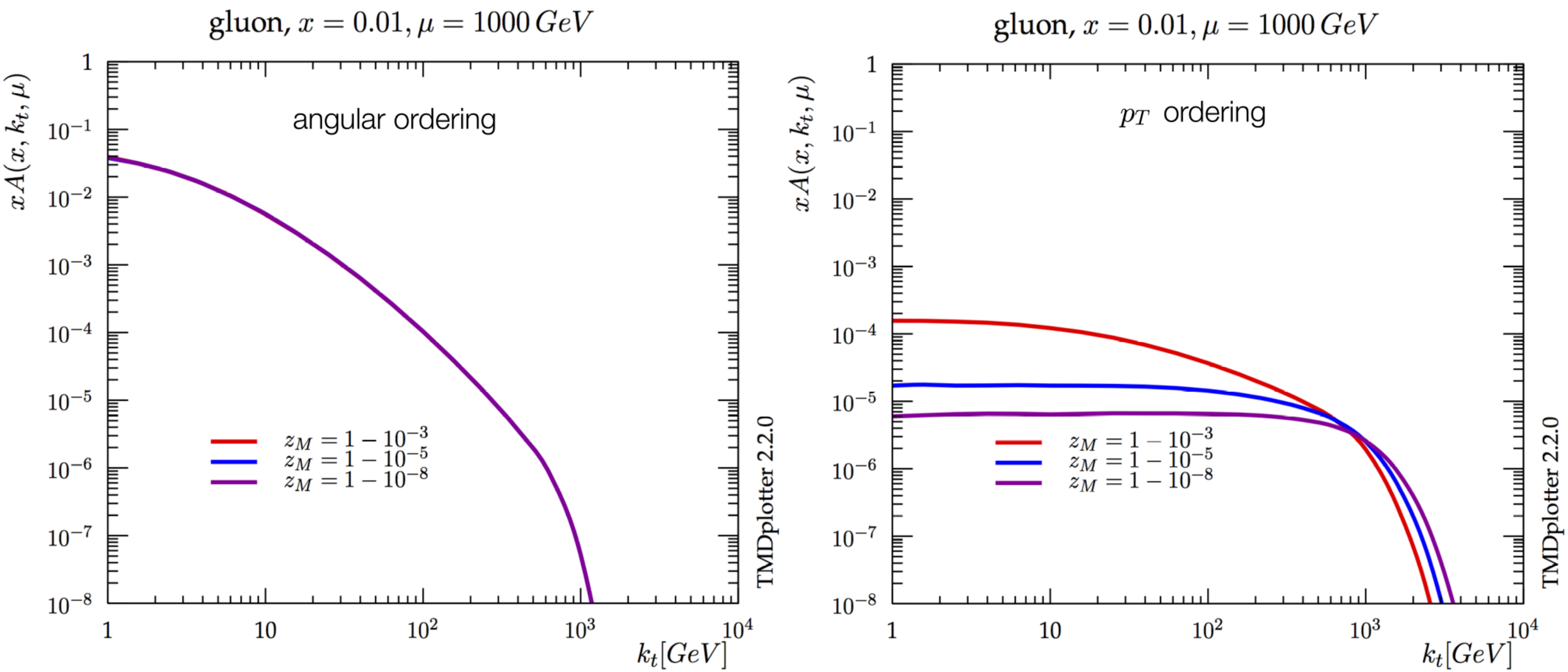
- Very good agreement with NLO - QCDnum over all x and μ^2
  - the same approach works also at NNLO !

Validation of method at NLO: z_M - dependence



- No dependence on z_M if z_M is large enough:
  - approximation is of $\mathcal{O}(1 - z_M)$
- Very good agreement with NLO - QCDnum

Transverse Momentum: z_M - dependence

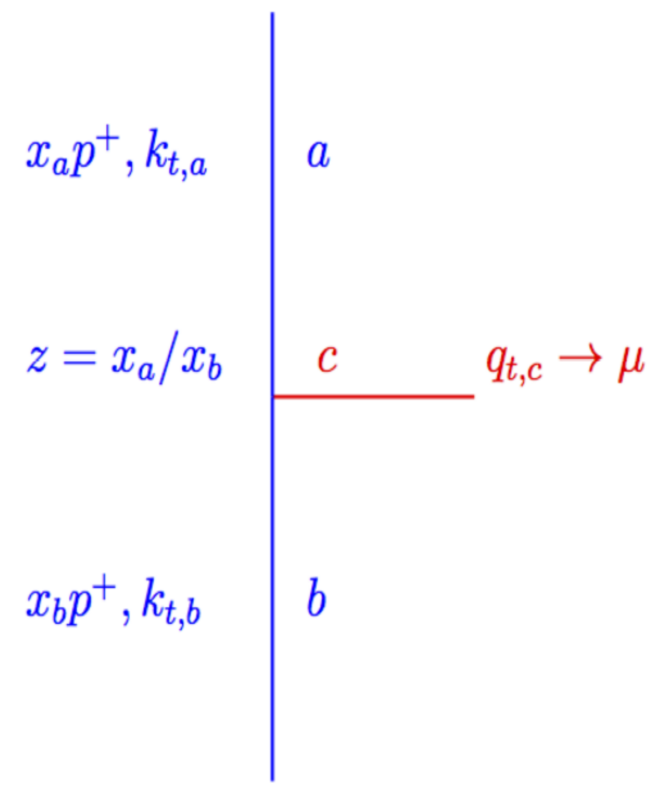


- p_T ordering ($\mu = q_T$) shows significant dependence on z_M: unstable result because of soft gluon contribution
- angular ordering ($\mu = q_T/(1-z)$) is independent of z_M: stable results since soft gluons are suppressed

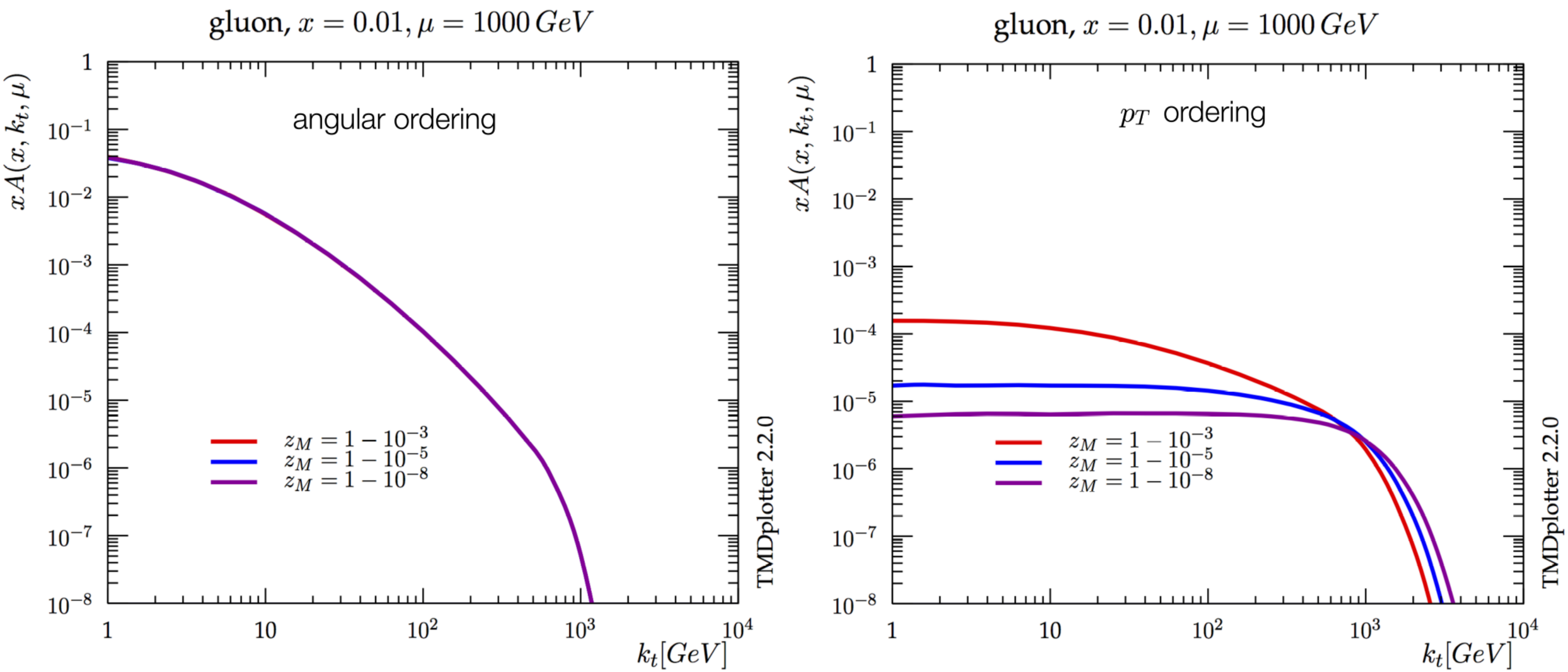
The new thing: Transverse Momentum Dependence

- Parton Branching evolution generates every single branching:
  - kinematics can be calculated at every step
- Give physics interpretation of evolution scale:
  - angular ordering:

    $\mu = q_T/(1-z)$



Transverse Momentum: z_M - dependence



- p_T – ordering ($\mu = q_T$) shows significant dependence on z_M: unstable result because of soft gluon contribution
- angular ordering ($\mu = q_T/(1-z)$) is independent of z_M: stable results since soft gluons are suppressed (angular ordering)

PDFs from Parton Branching method: fit to HERA data

- Convolution of kernel with starting distribution

$$x f_a(x,\mu^2) = x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b\left(x'',\mu^2\right) \delta(x'x''-x)$$
$$= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \; \tilde{\mathcal{A}}_a^b\left(\frac{x}{x'},\mu^2\right)$$

- Fit performed using xFitter frame (with collinear Coefficient functions at NLO)
  - using full HERA 1+2 inclusive DIS (neutral current, charged current) data
    - in total 1145 data points
      - $3.5 < Q^2 < 50000 \text{ GeV}^2$
      - $4 \cdot 10^{-5} < x < 0.65$
      - using starting distribution as in HERAPDF2.0
      - $\chi^2/ndf = 1.2$

➔ Can be easily extended to include any other measurement for fit !

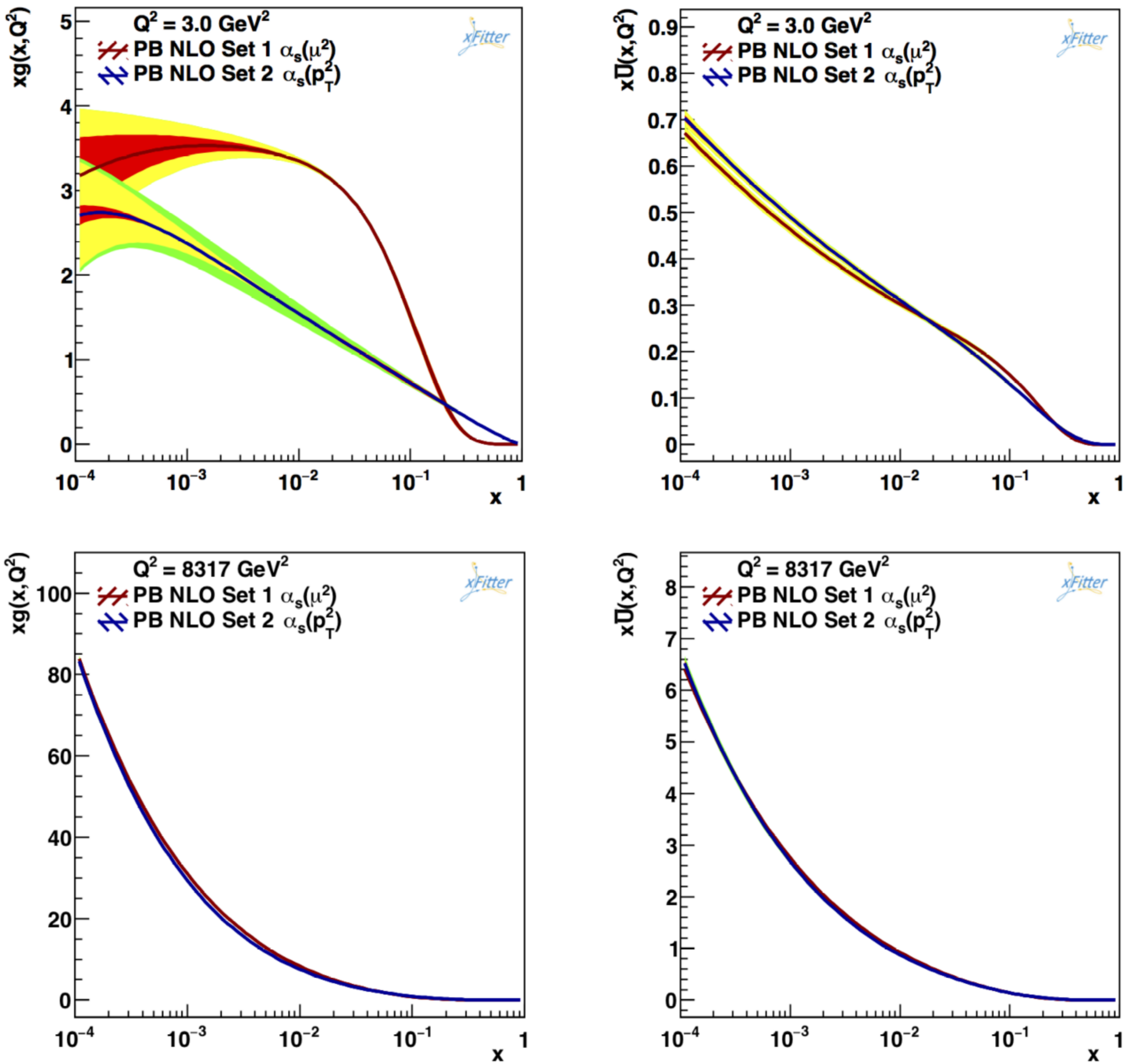
Advantages of parton branching method

- DGLAP equation:

$$\mu^2 \frac{\partial}{\partial \mu^2} f(x,\mu^2) = \int \frac{dz}{z} \frac{\alpha_s(\mu_r)}{2\pi} P_+(z)\, f\left(\frac{x}{z},\mu^2\right)$$

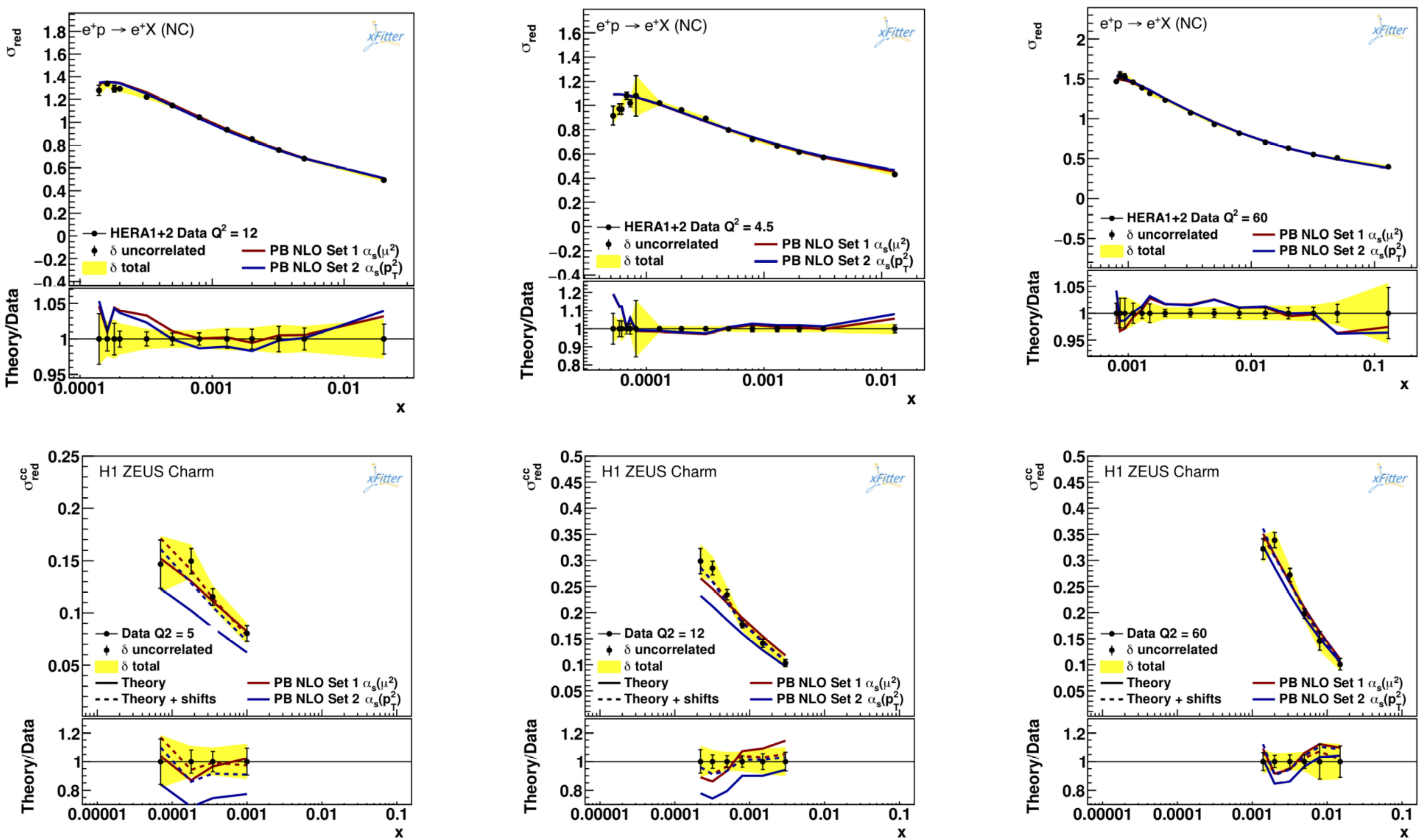
- Advantages of parton branching method for collinear PDFs:
  - access to all kinematic variables and combinations between them
    - full freedom of choosing:
      - renormalisation scale: $\alpha_s(\mu_r)$
      - evolution scale: μ_f
  - studies of different ordering conditions possible for the first time
    - angular ordering with $\alpha_s\,(q)$
    - but angular ordering suggests that renormalization scale is p_T and not angle
      - angular ordering with $\alpha_s\,(p_T)$ → $\alpha_s(q(1{-}z))$
      - repeat fits with changed renormalisation scale in pdf (but not yet in coefficient fct)

Fit with different scale in α_s

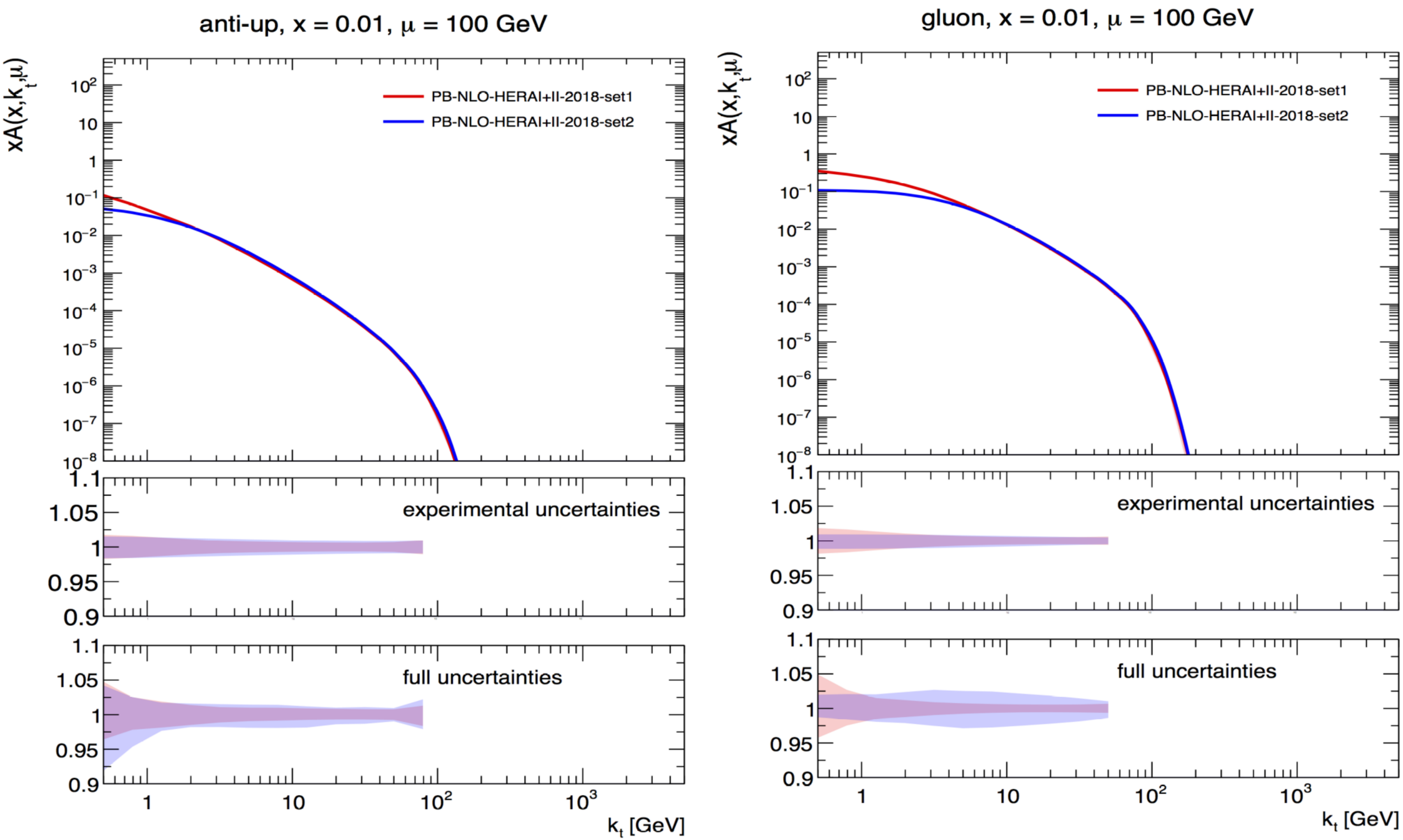


- fit 1 with $\alpha_s(q)$
  - as good as HERAPDF2.0 $\chi^2/ndf = 1.2$
- fit 2 with $\alpha_s(q(1-z))$
  - $\chi^2/ndf = 1.21$
- very different gluon distribution obtained at small Q^2

Fits to DIS x-section at NLO: F_2 and F_2^c



TMD distributions from fit to HERA data

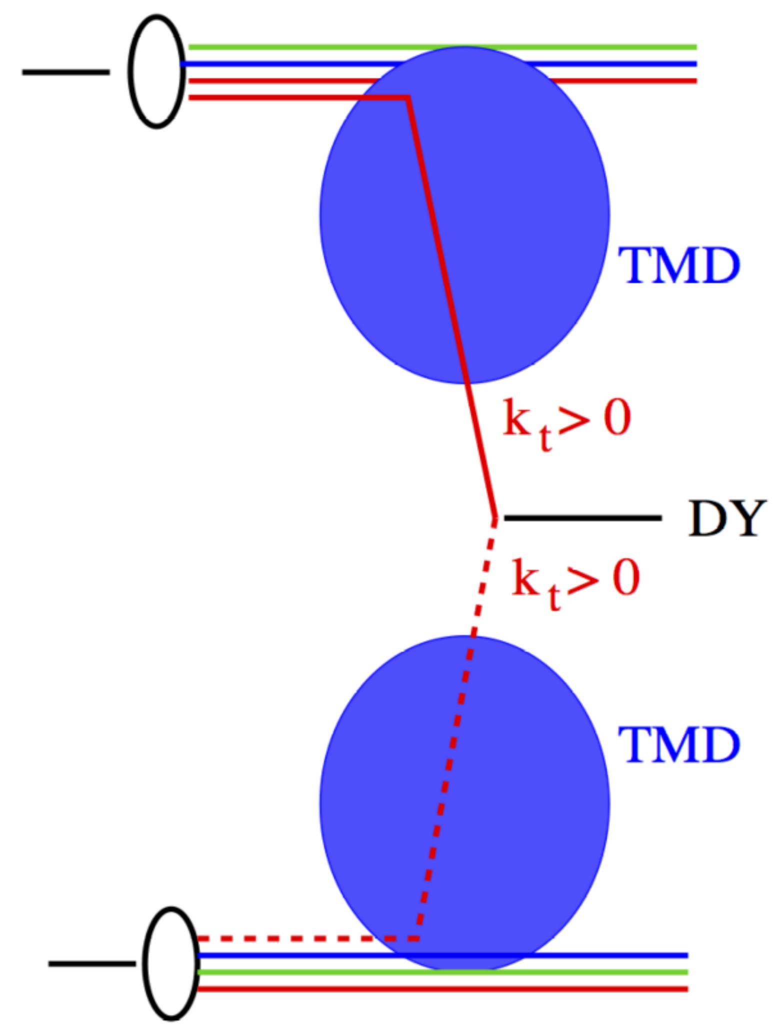


- model dependence larger than experimental uncertainties

Application to Drell – Yan production

Drell -Yan production: q_T - spectrum

- DY production
  - $q\bar{q} \to Z_0$
  - add k_t for each parton as function of x and μ according to TMD
  - keep final state mass fixed:
    - x_1 and x_2 (light-cone fraction) are different after adding k_t
  - use NLO calculations: MC@NLO



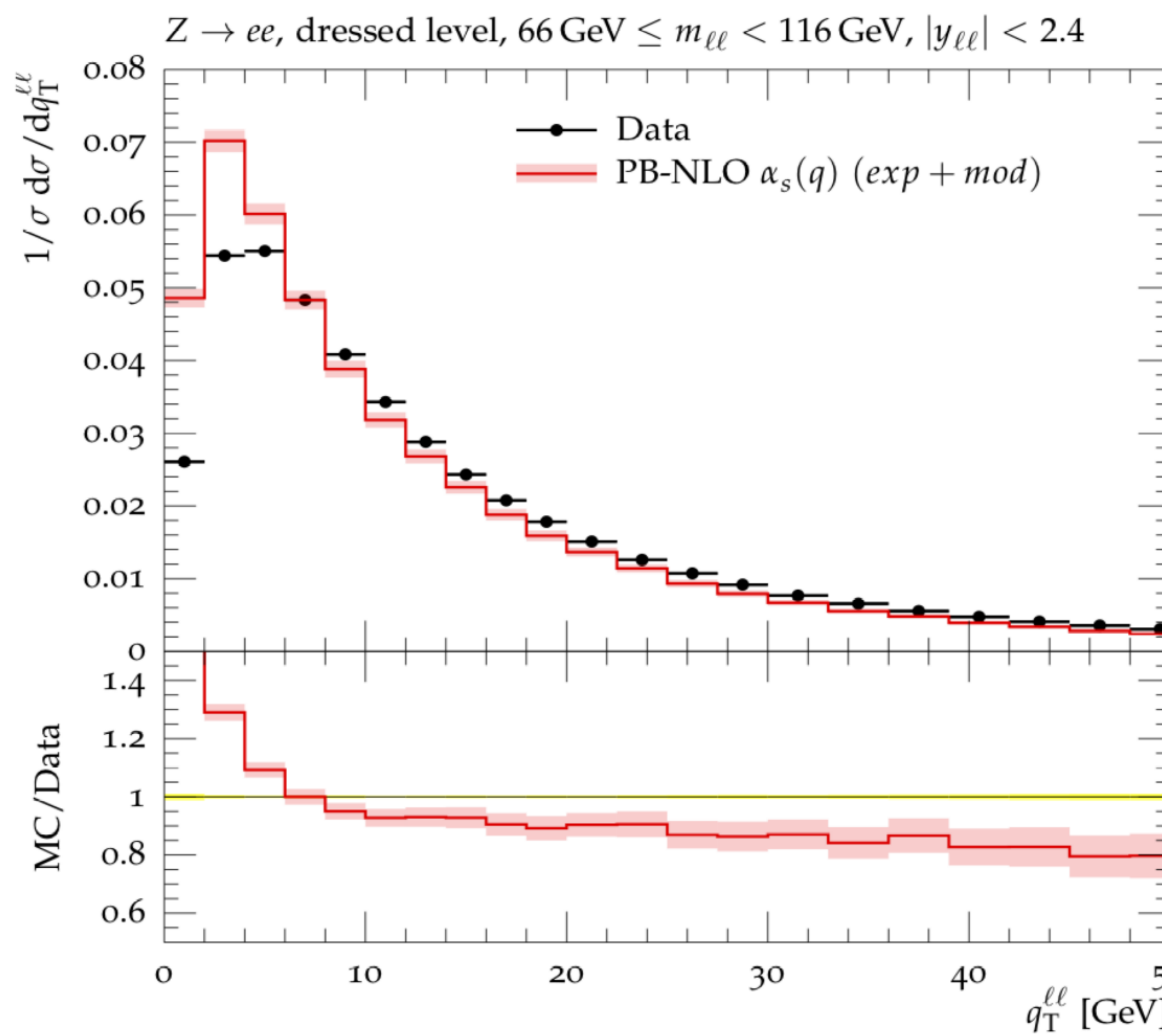
Application to DY q_T - spectrum

- Use LO DY production

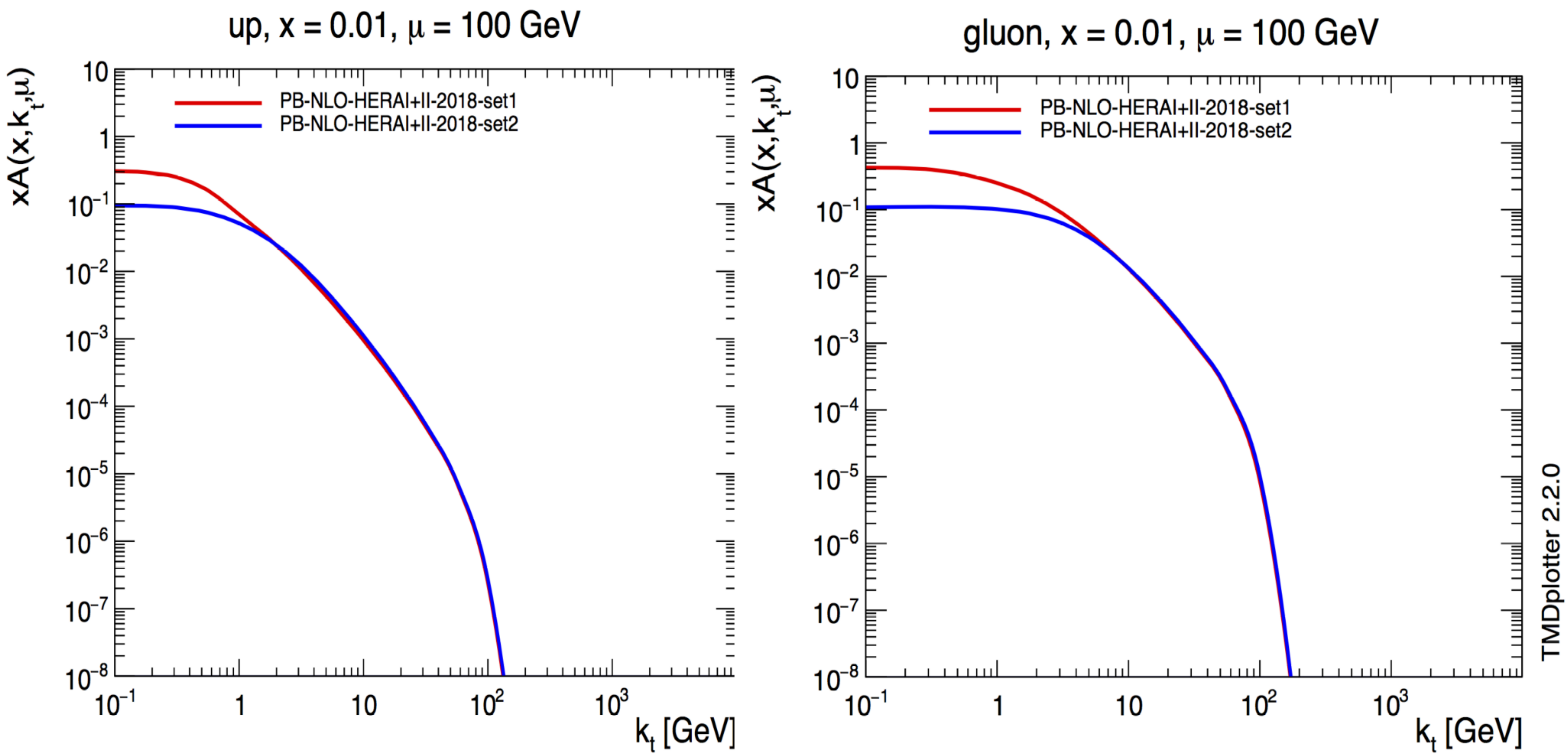
  $q\bar{q} \rightarrow Z_0$

- TMD with angular ordering including $\alpha_s(q)$

ATLAS Collaboration Eur. Phys. J. C76 (2016), 291 [arXiv:1512.02192]



TMD distributions

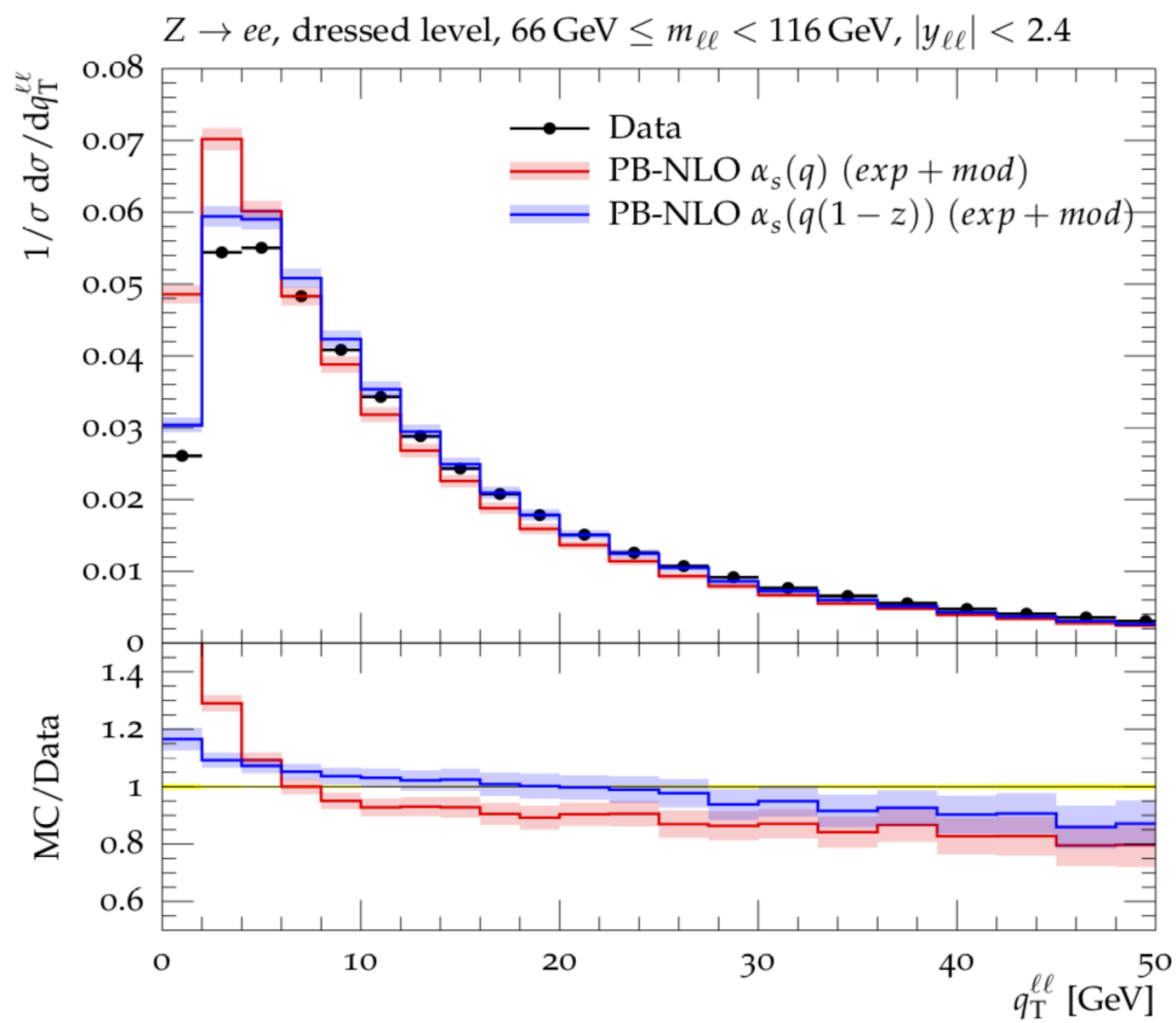


- Differences essentially in low k_T region
  - introducing q_T instead of q, suppresses further soft gluons at low k_T !

Application to DY q_T - spectrum

ATLAS Collaboration Eur. Phys. J. C76 (2016), 291 [arXiv:1512.02192]

- Use LO DY production
- TMD with angular ordering including $\alpha_s(q)$
- TMD with angular ordering including $\alpha_s(p_T)$ much better !



- Additional issues:
  - resolvable branching
  - freeze α_s
  - intrinsic k_T

Where to find TMDs ?   TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of REF workshop and developed since
- combine and collect different ansaetze and approaches:

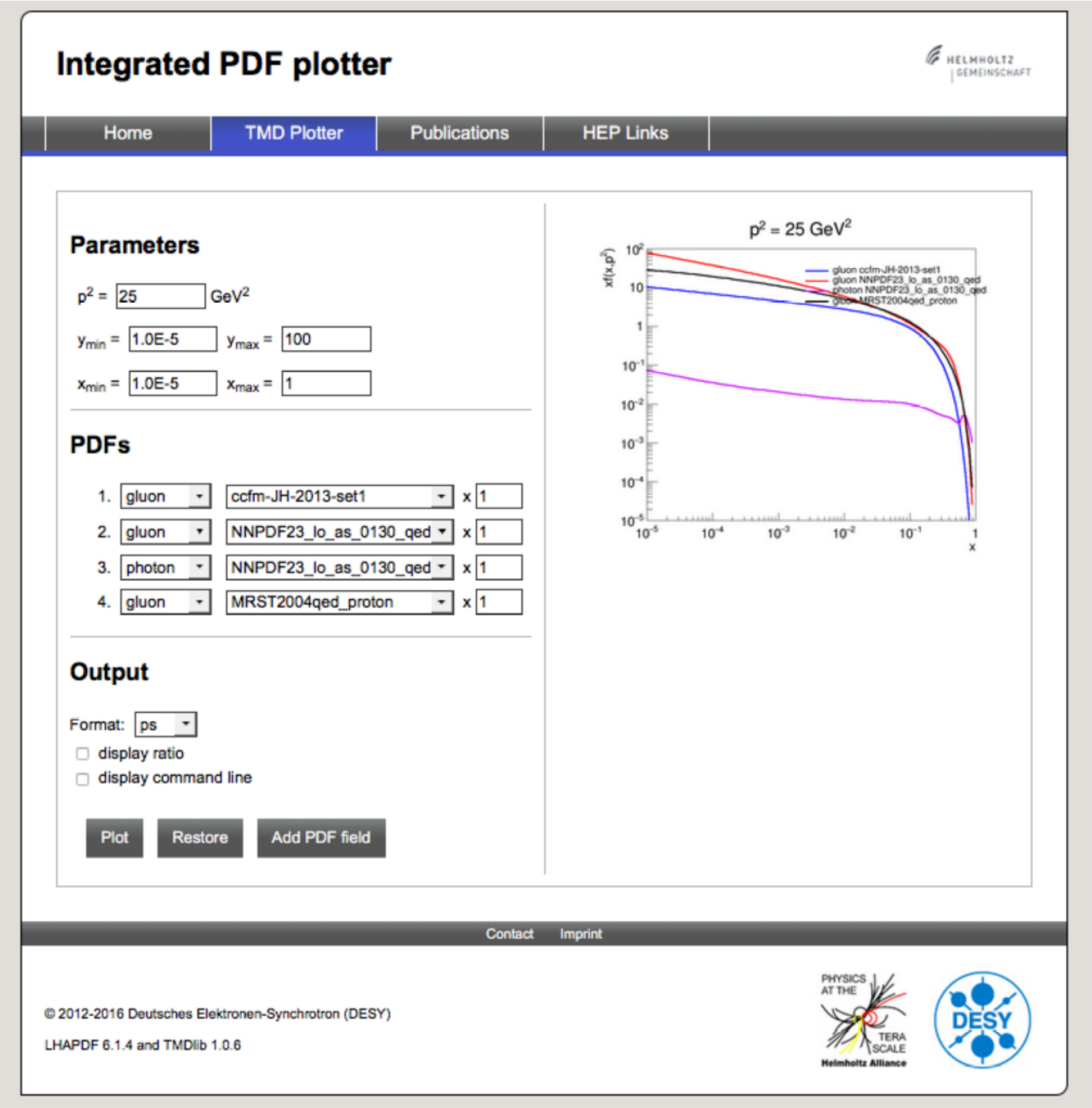
  http://tmd.hepforge.org/ and
  http://tmdplotter.desy.de

→ TMDlib: a library of parametrization of different TMDs and uPDFs (similar to LHApdf)

→ https://tmdlib.hepforge.org

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions, *F. Hautmann et al.* arXiv 1408.3015, Eur. Phys. J., C 74(12):3220, 2014.

→ Also integrated pdfs (including photon pdf are available via LHAPDF)



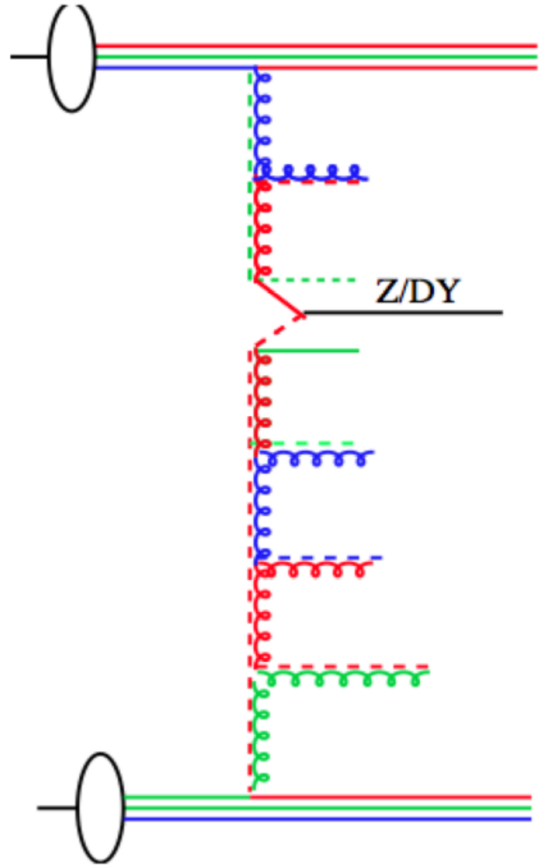
Conclusion

- Parton Branching method to solve DGLAP equation at LO, NLO and NNLO
  - ➔ consistence for collinear (integrated) PDFs shown
  - ➔ advantages of Parton Branching method !
- method directly applicable to determine k_t distribution (as would be done in PS)
  - ➔ TMD distributions for all flavors determined at LO and NLO, without free parameters
- PB method offers many new applications, and some of them will be studied during the exercises

Appendix

What happens at small m_{DY} and small $\sqrt{s}$?

- at low mass
  - p_T of DY is dominated by **intrinsic k_T** and by **soft gluons**, which need to be resummed :)
    - latest measurement: PHENIX (PhysRevD.99.072003) at $\sqrt{s} = 200$ GeV for $4.6 \leq m_{DY} \leq 8.2$ GeV
    - other measurements (older)
      - R209 (1982) PhysRevLett.48.302 at $\sqrt{s} = 62$ GeV (data read from plot in paper)
      - NUSEA (2003) hep-ex/0301031 at $\sqrt{s} = 38$ GeV (unpublished)



- Can PB method with McatNLO be applied to small $\sqrt{s}$?
  - Is there a small p_T crisis ?

The difficulties at small q_T and small $\sqrt{s}$

PHYSICAL REVIEW D **100,** 014018 (2019)

Difficulties in the description of Drell-Yan processes at moderate invariant mass and high transverse momentum

Alessandro Bacchetta,[1,2,*] Giuseppe Bozzi,[1,2,†] Martin Lambertsen,[3,‡] Fulvio Piacenza,[1,2,§] Julius Steiglechner,[3,‖] and Werner Vogelsang[3,¶]

[1]*Dipartimento di Fisica, Università di Pavia, via Bassi 6, I-27100 Pavia, Italy*
[2]*INFN Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy*
[3]*Institute for Theoretical Physics, Tübingen University, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

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Both regimes, $q_T \ll Q$ and $q_T \sim Q$, as well as their matching, must be under theoretical control in order to have a proper understanding of the physics of the Drell-Yan process. In the present work, we study the process at fixed-target energies for moderate values of the invariant mass Q and in the region $q_T \lesssim Q$. We focus on the predictions based on collinear factorization and examine their ability to describe the experimental data in this regime. We find, in fact, that the predicted cross sections fall significantly short of the available data even at the highest accessible values of q_T. We investigate possible sources of uncertainty in the predictions based on collinear factorization, and two extensions of the collinear framework: the resummation of high-q_T threshold logarithms, and transverse-momentum smearing. None of these appear to lead to a satisfactory agreement with the data. We argue that these findings also imply that the Drell-Yan cross section in the "matching regime" $q_T \lesssim Q$ is presently not fully understood at fixed-target energies.

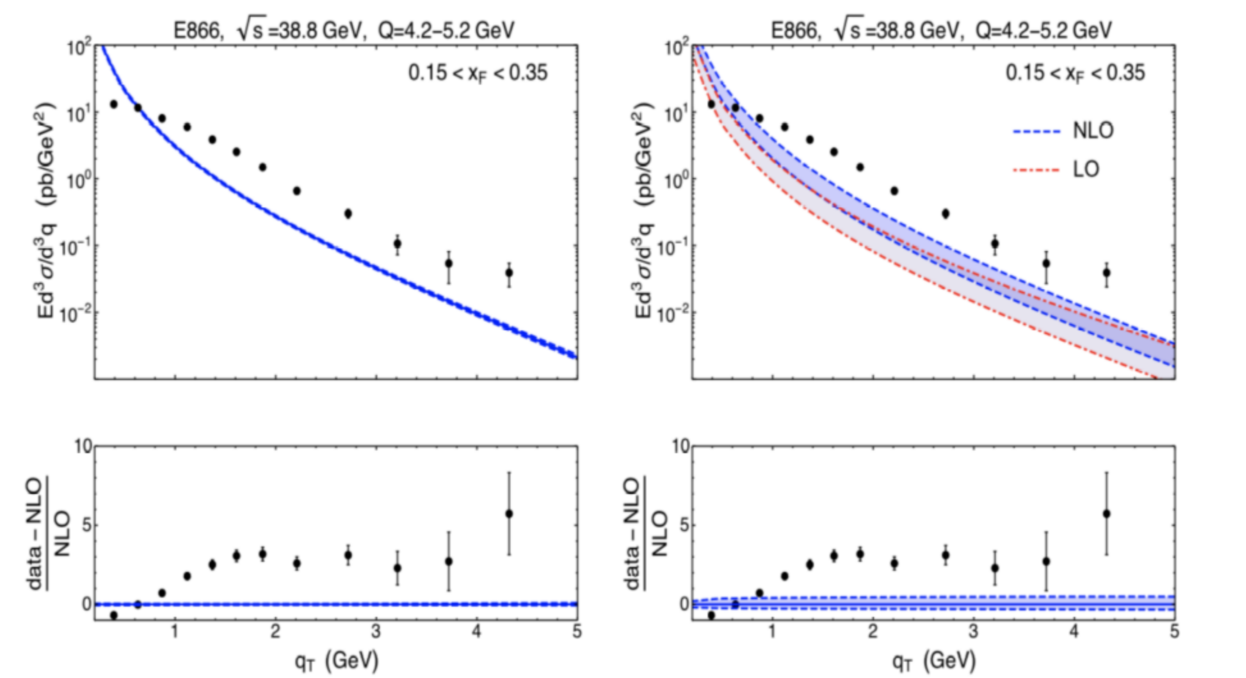
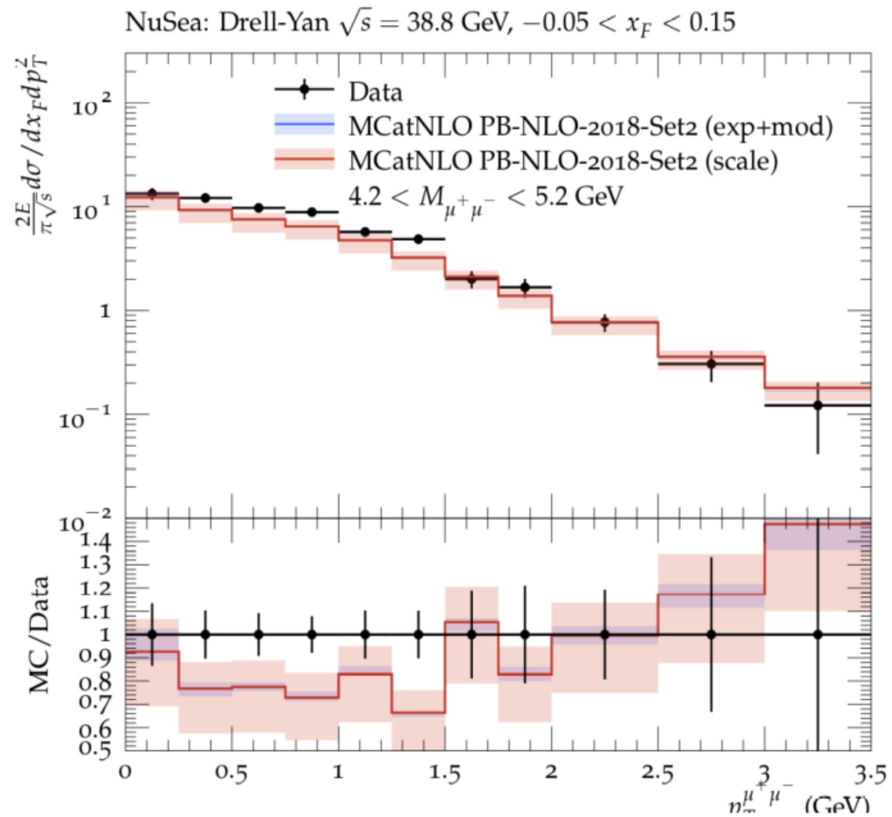
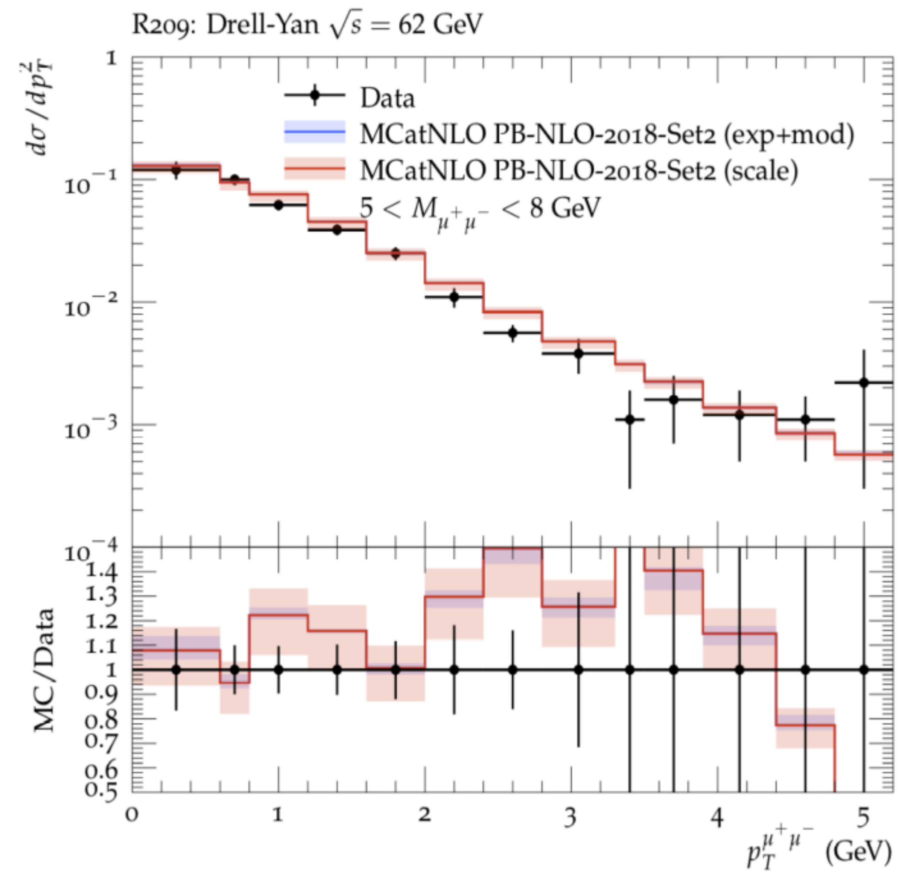
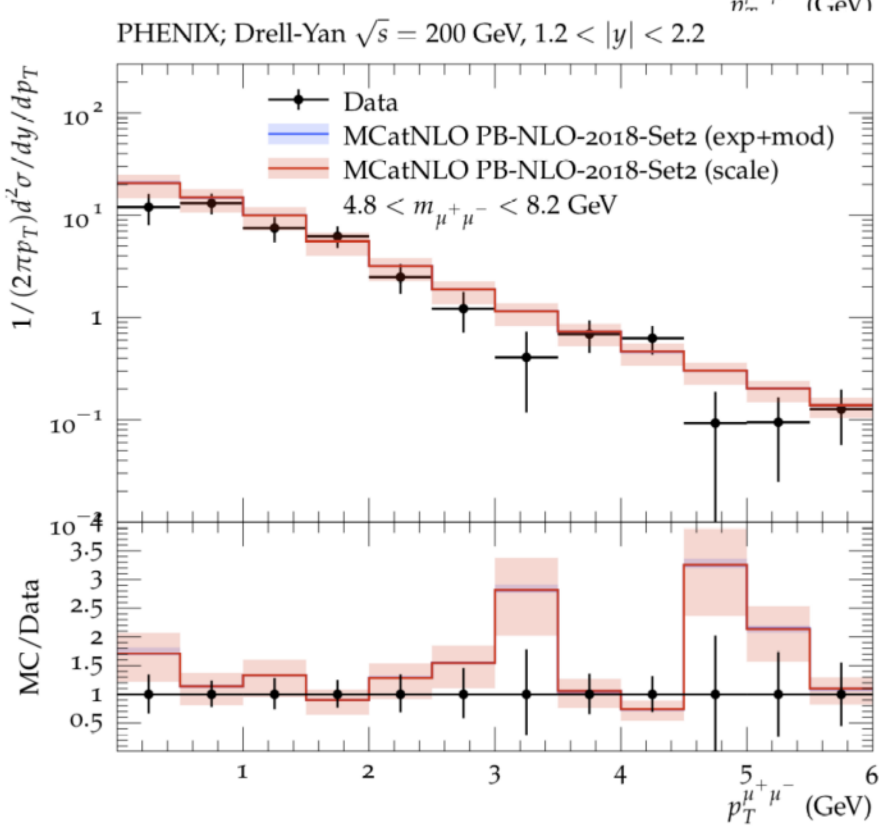


FIG. 2. Transverse-momentum distribution of Drell-Yan dimuon pairs at $\sqrt{s} = 38.8$ GeV in a selected invariant mass range and Feynman-x range: experimental data from Fermilab E866 (hydrogen target) [41] compared to LO QCD and NLO QCD results. (Left panels) NLO QCD $[\mathcal{O}(\alpha_s^2)]$ calculation with central values of the scales $\mu_R = \mu_F = Q = 4.7$ GeV, including a 90% confidence interval from the CT14 PDF set [39]. (Right panels) LO QCD and NLO QCD theoretical uncertainty bands obtained by varying the renormalization and factorization scales independently in the range $Q/2 < \mu_R, \mu_F < 2Q$.

Transverse momentum distributions: q_T crisis ?







- How good is description ?

χ^2/ndf

Qs	NuSea	R209	PHENIX
0.5	1.08	1.27	1.04

Is there " q_T – crisis" ?

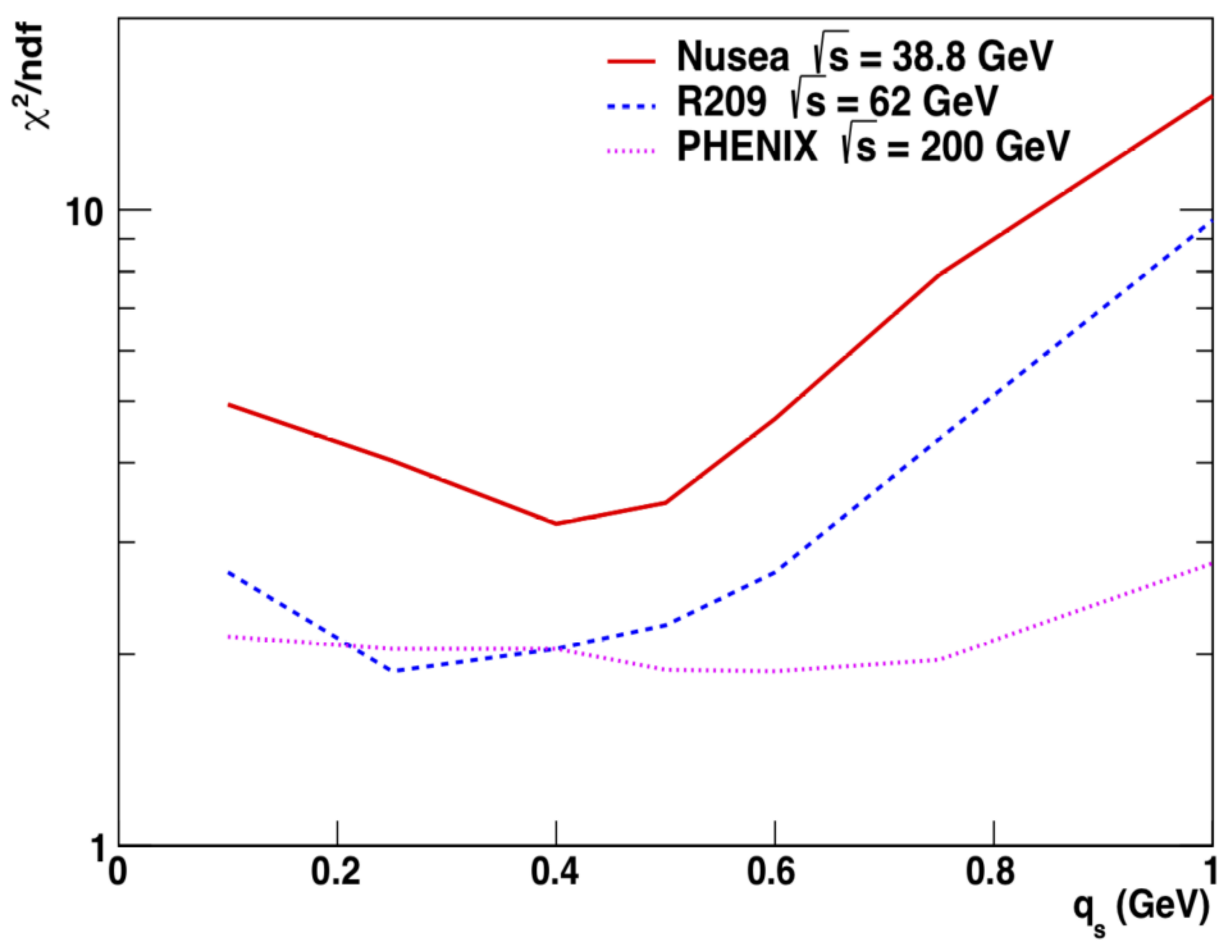
- data are perfectly well described by NLO calculation with PB TMDs

Constraints on intrinsic k_T

- Intrinsic k_T is included in starting distribution, for simplicity Gauss is assumed

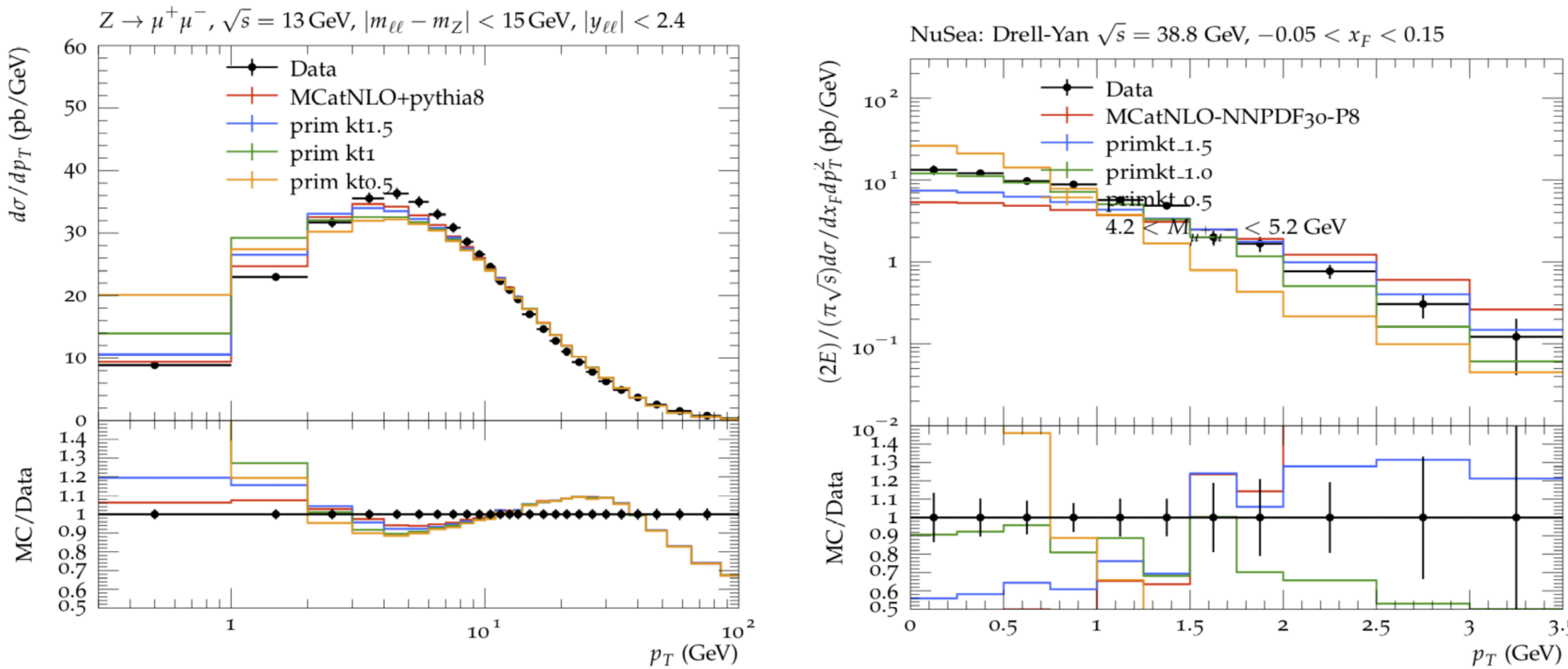
$$\mathcal{A}_{0,b}(x, k_T^2, \mu_0^2) = f_{0,b}(x, \mu_0^2) \cdot \exp\left(-|k_T^2|/2\sigma^2\right)/(2\pi\sigma^2)$$

- change width $\sigma^2 = q^2{}_s/2$ of Gauss distribution (default $q_s = 0.5\, GeV$)



- Only at low energies, sensitivity to intrinsic Gauss observed....

Predictions from MCatNLO+PYTHIA8



- differences observed using Monash tune in P8
  - intrinsic kt in P8 cannot be simply tuned to describe both high and low energy data