<sup>2</sup> following H1prelim-19-041, ZEUS-prel-19-001

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# Impact of jet production data on the next-to-next-to-leading order determination of HERAPDF2.0 parton distributions

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#### Abstract

The HERAPDF2.0 ensemble of parton distribution functions (PDFs) was introduced in 9 2015. Presented is the final stage, a next-to-next-to-leading order analysis of the HERA 10 data on inclusive deep inelastic ep scattering together with jet data as published by H1 11 and ZEUS. A pQCD fit to the data with free  $\alpha_s(M_Z^2)$  and free PDFs was used to determine 12  $\alpha_s(M_Z^2)$  with the result  $\alpha_s(M_Z^2) = 0.1156 \pm 0.0011$  (exp)  $^{+0.0001}_{-0.0002}$  (model + parameterisation)  $\pm$ 13 0.0029 (scale). The HERAPDF2.0Jets NNLO sets of parton density functions from fits with 14 fixed  $\alpha_s(M_Z^2) = 0.1155$  and  $\alpha_s(M_Z^2) = 0.118$ , the value used for the published HERA-15 PDF2.0 NNLO analysis based on inclusive data only, are presented and compared. The 16 PDFs of HERAPDF2.0Jets NNLO for fixed  $\alpha_s(M_7^2) = 0.118$  are also compared to the PDFs 17 of HERAPDF2.0 NNLO. The similarity of the PDFs demonstrates the consistency of inclu-18 sive and jet-production cross-section data. Predictions based on HERAPDF2.0Jets NNLO 19 agree very well with the jet-production data used in the fits. 20

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## <sup>22</sup> 1 Introduction

<sup>23</sup> Data from deep inelastic scattering (DIS) of electrons<sup>1</sup> on protons, *ep*, at centre-of-mass energies <sup>24</sup> of up to  $\sqrt{s} \approx 320 \text{ GeV}$  at HERA have been central to the exploration of proton structure and <sup>25</sup> quark–gluon dynamics as described by perturbative Quantum Chromo Dynamics (pQCD) [1].

The combination of H1 and ZEUS data on inclusive *ep* scattering and the subsequent pQCD analysis, introducing the ensemble of parton density functions (PDFs) known as HERAPDF2.0, were milestones for the exploitation [2] of the HERA data. The HERAPDF analyses are based on pQCD fits to the HERA DIS data in the DGLAP [3–7] formalism in the MS scheme [8].

The sets of PDFs presented in this work complete the HERAPDF2.0 ensemble [2] of PDFs.

<sup>31</sup> They were determined with an NNLO analysis of HERA inclusive and selected jet-production

data as published separately by the H1 and ZEUS collaborations [9–14]. An analysis of jet data

at NNLO was not possible at the time of the introduction of the HERAPDF2.0 ensemble. It became possible when predictions of jet cross-section at NNLO [15–23] for ep became available.

The strategy of the analysis follows the strategy of the of the original and verified pQCD [2] analysis at NLO. As the value of the strong coupling constant,  $\alpha_s(M_Z^2)$ , cannot be separated from the PDFs resulting from any pQCD fit, a suitable value of  $\alpha_s(M_Z^2)$  has to be determined first by fitting the PDFs and  $\alpha_s(M_Z^2)$  simultaneously. This avoids biases on  $\alpha_s(M_Z^2)$  as would be introduced by fitting  $\alpha_s(M_Z^2)$  with fixed PDFs [24]. In a second step, the PDFs are refined by a

fit with  $\alpha_s(M_Z^2)$  fixed to the optimised value.

<sup>35</sup> The calculation of jet cross-sections at NNLO constructs jets starting from massless partons.

<sup>36</sup> The inclusive data, on the other hand, are treated within the RTOPT [25–27] Variable Flavour

<sup>37</sup> Number Scheme (VFNS), which requires values of the parameters for the charm and bottom

masses,  $M_c$  and  $M_b$ , as input. These parameters were optimised using cross sections for charm

<sup>39</sup> and bottom production, which were published as combined data by the H1 and ZEUS collabora-

tions together with a pQCD analysis [28]. An inclusion of the heavy-quark data in the pQCD fit

including jets is considered inappropriate due to the different treatment of heavy-quark masses

<sup>42</sup> for the predictions on inclusive and jet data.

The results presented here are based entirely on HERA data, i.e. inclusive and jet-production 43 data. The HERA inclusive data represent a single, highly consistent data set. Furthermore, the 44 jet data have been found to be very consistent with the inclusive data at NLO [2]; the analysis 45 presented here also tests consistency at NNLO. In addition, DIS is the only process for which 46 the factorisation theorem is fully established. It is only a standard assumption that it is also valid 47 for hadron-hadron interaction processes. However, even if this assumption is valid, PDF fits 48 to LHC data would be biased by any physics Beyond the Standard Model (BSM) whose effects 49 have so far escaped detection, thereby reducing the sensitivity of searches for BSM due to biased 50 background predictions. Thus, the HERAPDF2.0 ensemble of PDFs provides a benchmark to 51 which PDFs including data from LHC colliders may be compared. This could reveal BSM 52 effects or the need for an extension of the QCD analyses for some processes. 53

<sup>&</sup>lt;sup>1</sup>From here on, the word "electron" refers to both electrons and positrons, unless otherwise stated.

### 54 **2 Data**

Data taken by the H1 and ZEUS collaborations from 1993 to 2007 were combined to form 55 a coherent set of inclusive HERA ep DIS cross sections [2], which was used as input to the 56 determinations of all previous members of the HERAPDF2.0 ensemble. The HERAPDF2.0Jets 57 analysis at NLO, in addition, used selected data [9-12,14] on inclusive jet and dijet production 58 from H1 and ZEUS, which were again used for the present analysis at NNLO. In addition, new 59 data [13], published by the H1 collaboration on jet production in lower  $Q^2$  events, where  $Q^2$ 60 is the four-momentum-transfer squared, together with six new high- $Q^2$  points at low  $p_T$ , where 61  $p_T$  is the transverse energy of the jet, which were published by H1 in the same publication to 62 complete the previously published high- $Q^2$  data set [14], were added as input to the NNLO 63 analysis. A summary on the data of jet production used is provided in Table 1. For all data sets, 64 the jets were identified with the  $k_T$  algorithm with the R parameter set to one. 65 The new treatment of inclusive jet and dijet production at NNLO was, however, only ap-66 plicable to a slightly reduced phase space compared to HERAPDF2.0Jets NLO. All data points 67

with  $\mu = \sqrt{\langle p_T^2 \rangle + Q^2} \le 10.0 \,\text{GeV}$  had to be excluded in order to ensure the convergence of the perturbative series and to limit the NNLO scale uncertainties of the theoretical predictions 68 69 to below 10 % compared to below 24 % at NLO. This requirement on  $\mu$  also ensured that  $\mu$  was 70 larger than the b-quark mass, which is necessary because the jets are assumed to be built from 71 massless partons in the calculation of the NNLO predictions. In addition, for each  $Q^2$  bin, the 72 six data points with the lowest  $\langle p_T \rangle$  had to be excluded from the ZEUS dijet data set because 73 the available NNLO predictions for these points were judged to be incomplete considering the 74 kinematic cuts  $^2$ . The resulting reduction of data points is detailed in Table 1. In addition, the 75 trijet data [14] which were used as input to HERAPDF2.0Jets NLO had to be excluded as no 76 NNLO treatment was available. 77

The inclusive charm data [29], which were included in the analysis at NLO [2] were not explicitly used in the PDF fits of the analysis presented here, since complete NNLO predictions were not available. Heavy quark data [28] were only used to optimise the mass parameter values for charm,  $M_c$ , and beauty,  $M_b$ , which are needed as input to the adopted RTOPT [27] NNLO approach to the fitting of the inclusive data.

## **3 QCD Analysis**

The analysis presented here was done along the same lines as all previous HERAPDF2.0 analyses [2]. Only cross sections for  $Q^2$  starting at  $Q_{min}^2 = 3.5 \text{ GeV}^2$  were used in the analysis. The  $\chi^2$  definition was taken from equation 32 of the previous paper [2]. The value of the starting scale for the evolution was taken as  $\mu_{f0}^2 = 1.9 \text{ GeV}^2$ . The parameterisation and choice of free parameters also followed the prescription for the HERAPDF2.0Jets NLO fit, see Section 3.1 below.

All fits were performed using the programme QCDNUM [30] within the xFitter, formerly HERAFitter, framework [31] and were cross-checked with an independent programme, which

<sup>&</sup>lt;sup>2</sup>Due to the kinematic cuts used in selecting the dijet data, the LO prediction for the cross sections is zero. Thus, the NNLO term is only the second non-zero term.

was already used as a second programme in the HERAPDF2.0 analysis. The results obtained
 using the two programmes, as previously for all HERAPDF2.0 fits [2], were in excellent agree ment, i.e. well within fit uncertainties. All numbers presented here were obtained using xFitter.

The light-quark coefficient functions were calculated in QCDNUM. The heavy-quark coefficient functions were calculated in the general-mass variable-flavour-number scheme RTOPT [25], with recent modifications [26,27].

The analysis presented here became possible due to the newly available treatment of jet 98 production at NNLO, using the zero-mass scheme. This is expected to be a reasonable ap-99 proximation when the relevant QCD scales are significantly above the charm- and beauty-quark 100 masses. The jet data were included in the fits at full NNLO using predictions for the jet cross 101 sections calculated using NNLOJET [15–17], which was interfaced to the fast interpolation grid 102 codes, fastNLO [18-20] and APPLgrid [21,22] using the APPLfast framework [23], in order to 103 achieve the required speed for the convolutions needed in an iterative PDF fit. The NNLO jet 104 predictions were provided in the massless scheme and were corrected for hadronisation and  $Z^0$ 105 exchange before they were used in the fits. A running electro magnetic  $\alpha$  as implemented in the 106 2012 version of the programme EPRC [32] was used in the treatment of the jet cross sections. 107 The predictions were provided with fully correlated uncertainties, which were taken into account 108 in all fits. 109

The choice of scales for the jet data had to be adjusted for the NNLO analysis. At NLO, the factorisation scale was chosen as for the inclusive data, i.e.  $\mu_f^2 = Q^2$ , while the renormalisation scale was linked to the transverse momenta,  $p_T$ , of the jets as  $\mu_r^2 = (Q^2 + p_T^2)/2$ . For the NNLO analysis,  $\mu_f^2 = \mu_r^2 = Q^2 + p_T^2$  was used. This resulted in an improved  $\chi^2$  for the fits. Scale variations were also considered and are discussed in Sections 4.1 and 4.2.

#### **3.1** Choice of parameterisation and model parameters

The PDFs were parameterised as a function of x at the input scale by the generic form

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$$xf(x) = Ax^{B}(1-x)^{C}(1+Dx+Ex^{2}).$$
(1)

The PDF of the gluon was an exception, for which an additional term of the form  $A'_g x^{B'_g} (1-x)^{C'_g}$ was subtracted <sup>3</sup>. This choice of parameterisation follows the original concept of HERAPDF2.0, for which all details were previously published [2]. The parameterisation is an effective way to store the information derived from many data points in a limited set of numbers.

The parameterised PDFs are the gluon distribution xg, the valence-quark distributions  $xu_v$ ,  $xd_v$ , and the *u*-type and *d*-type anti-quark distributions  $x\overline{U}$ ,  $x\overline{D}$ , where  $x\overline{U} = x\overline{u}$  and  $x\overline{D} =$  $x\overline{d} + x\overline{s}$  at the chosen starting scale. The parameterisation for the central fit was determined by initially fixing the *D*, *E* and  $A'_g$  parameters to zero. This resulted in 10 free parameters. The extra parameters were introduced one at a time until the  $\chi^2$  of the fit could not be further improved [2,33]. This is also called the  $\chi^2$  saturation method. This resulted in a 14 parameter fit which satisfied the criteria that all PDFs and all predicted cross sections were positive throughout

<sup>&</sup>lt;sup>3</sup>The parameter  $C'_g = 25$  was fixed since the fit is not sensitive to this value, provided it is high enough ( $C'_g > 15$ ) ensuring that the term does not contribute at large x.

the kinematic region probed by the data entering the fit. The suitability of the parameterisation
was, thus, also verified for the selection of jet data.

<sup>131</sup> The final parameterisation was

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$$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g},$$
(2)

133 
$$xu_{\nu}(x) = A_{\mu} x^{B_{\mu}} (1-x)^{C_{\mu}} \left(1+E_{\mu} x^{2}\right), \qquad (3)$$

134 
$$xd_{v}(x) = A_{d_{v}}x^{B_{d_{v}}}(1-x)^{C_{d_{v}}},$$
 (4)

135 
$$x\bar{U}(x) = A_{\bar{U}}x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}(1+D_{\bar{U}}x),$$
 (5)

$$x\bar{D}(x) = A_{\bar{D}}x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.$$
(6)

<sup>137</sup> The normalisation parameters,  $A_g$ ,  $A_{u_v}$ ,  $A_{d_v}$ , were constrained by the quark-number and momen-<sup>138</sup> tum sum rules. The *B* parameters,  $B_{\bar{U}}$  and  $B_{\bar{D}}$ , were set equal,  $B_{\bar{U}} = B_{\bar{D}}$ , such that there was a <sup>139</sup> single *B* parameter for the sea distributions.

The strange-quark distribution was expressed as an *x*-independent fraction,  $f_s$ , of the *d*-type sea,  $x\bar{s} = f_s x\bar{D}$  at  $Q_0^2$ . The central value  $f_s = 0.4$  was chosen to be a compromise between the determination of a suppressed strange sea from neutrino-induced di-muon production [34,35] and the determination of an unsuppressed strange sea from the ATLAS collaboration [36]. The further constraint  $A_{\bar{U}} = A_{\bar{D}}(1-f_s)$ , together with the requirement  $B_{\bar{U}} = B_{\bar{D}}$ , ensured that  $x\bar{u} \to x\bar{d}$ as  $x \to 0$ .

#### **3.2** Model and parameterisation uncertainties

Model and parameterisation uncertainties on the PDFs determined by a central fit were evaluated with fits with modified input assumptions. The central values of the model parameters and their variations are summarised in Table 2. The value of  $\alpha_s(M_Z^2)$  is either fixed to the input value or free for the simultaneous fit of  $\alpha_s(M_Z^2)$  and the PDFs.

The uncertainties on the PDFs obtained from variations of  $M_c$ ,  $M_b$ ,  $f_s$ ,  $Q_{min}^2$  were added in quadrature, separately for positive and negative uncertainties, and represent the model uncertainty.

The uncertainty obtained from the variation of  $\mu_{f0}^2$  was added to the parameterisation uncer-154 tainty. A variation of the number of terms in the polynomial  $(1 + Dx + Ex^2)$  was considered 155 for each of the parton distributions listed in Eqs. 2-6. For this, all 15-parameter fits which have 156 one more non-zero free D or E parameter were considered as possible variants and the resulting 157 PDFs compared to the PDF from the 14-parameter central fit. The only significant change in the 158 PDFs was observed for the addition of a  $D_{u_n}$  parameter. The uncertainties on the central fits from 159 the parameterisation variations were stored as an envelope representing the maximal deviation 160 at each x value. 161

The total uncertainties on the PDFs were obtained by adding experimental, i.e. fit, model and parameterisation uncertainties in quadrature.

#### <sup>164</sup> **3.3** Optimisation of $M_c$ and $M_b$

The RTOPT scheme used to calculate predictions for the inclusive data requires the charm- and 165 beauty-mass parameters,  $M_c$  and  $M_b$ , as input. The optimal values of these parameter were 166 reevaluated using new combined HERA data, which became available [28], superseding the 167 previously published combination of charm data [29] and the data published separately by H1 168 and ZEUS on beauty production. The optimisation was done using the standard procedure [?] 169 through fits to the inclusive HERA data together with the new combined heavy-flavour data with 170 varying choices of the mass parameter values. The values resulting in the lowest  $\chi^2$  values of the 171 fit were chosen for the jet analysis. This was done both at NLO to facilitate the pQCD analysis at 172 NLO published previously [28] and at NNLO for the analysis presented here. The one standard-173 deviation uncertainties of the mass parameters were determined by fitting the  $\chi^2$  values with a 174 quadratic function and finding the mass-parameter values corresponding to  $\Delta \chi^2 = 1$  values. 175

At NNLO, the fits for the optimisation were performed using the fixed value of  $\alpha_s = 0.1155^4$ ; 176 at NLO,  $\alpha_s = 0.118$  was used. As a first iteration at NNLO (NLO),  $M_c$  was varied with fixed 177  $M_b = 4.5 \,\text{GeV}$  (4.5 GeV) and  $M_b$  was varied with fixed  $M_c = 1.43 \,\text{GeV}$  (1.47 GeV), i.e. the 178 mass-parameter values used for HERAPDF2.0 NNLO (NLO) were used as fix-points. In every 179 iteration, the mass-parameter values as obtained in the previous iteration were used as new fix-180 points. The iteration was ended once values stable to 0.1 % for  $M_c$  and  $M_b$  were observed. The 181 final  $\chi^2$  scans at NNLO are shown in Figs. 1 a) and c) and at NLO Figs. 1 b) and d). The resulting 182 values at NNLO are  $M_c = 1.41 \pm 0.04$  GeV and  $M_b = 4.20 \pm 0.10$  GeV, quite close to the values 183 determined for HERAPDF2.0 NNLO, with slightly reduced uncertainties. The values at NLO 184 are  $M_c = 1.46 \pm 0.04 \,\text{GeV}$  and  $M_b = 4.30 \pm 0.10 \,\text{GeV}$ . The minima in  $\chi^2$  for  $M_b$  demonstrate 185 the power of the method. The minimum in  $\chi^2$  for the parameter  $M_c$  at NNLO is observed close 186 to the technical limit of the fitting procedure. 187

The part of the model uncertainty concerning the heavy-flavour mass parameters would nom-188 inally have involved varying the value of  $M_c$  to the minimum and maximum of its one standard-189 deviation uncertainty. However, for  $M_c$ , the downward variation created a conflict with  $\mu_{f0}$ , 190 which has to be less than  $M_c$  in the RTOPT scheme, such that charm can be generated perturba-191 tively. Thus, only an upward variation of  $M_c$  was considered and the resulting uncertainty on the 192 PDFs was symmetrised. In addition, the condition  $\mu_{f0} < M_c$  created a conflict with the variation 193 of  $\mu_{f0}^2$ . The normal procedure would have included an upward variation of  $\mu_{f0}^2$  to 2.2 GeV<sup>2</sup> but 194  $\mu_{f0}$  would have become larger than the upper end of the uncertainty interval of  $M_c$ <sup>5</sup>. Thus,  $\mu_{f0}^2$ 195 was only varied downwards to 1.6 GeV<sup>2</sup>, and the resulting uncertainty on the PDFs was again 196 symmetrised. The suitability of the chosen central parameterisation was re-verified for the new 197 settings for  $M_c$  and  $M_b$  using the  $\chi^2$  saturation method as described in Section 3.1. 198

Since predictions at NNLO for the jet data were only available in the zero-mass scheme, and results for the treatment of the inclusive data in different VFNS and FFNS schemes were consistent [2], no other heavy-flavour schemes were investigated.

<sup>&</sup>lt;sup>4</sup>A cross-check was performed with the fixed value of  $\alpha_s = 0.118$  and no significant difference in the resulting  $M_c$  and  $M_b$  values were observed.

<sup>&</sup>lt;sup>5</sup>In previous HERAPDF analyses, the uncertainty on  $M_c$  was large enough to accommodate the upward  $\mu_{f0}^2$  variation.

#### 3.4 Hadronisation uncertainties 202

For the jet-data analysis, it was also necessary to consider hadronisation and the effect of the 203 uncertainties on hadronisation corrections. The uncertainties on the hadronisation corrections, 204 which were supplied in the original publications, were reviewed for this analysis. The H1 un-205 certainties were used as published, while for technical reasons, those for the ZEUS data were 206 increased to the maximum value quoted in the publications, 2 %. It was checked that this change 207 made no significant difference to any of the results presented below. 208

In the HERAPDF2.0Jets NLO analysis, hadronisation uncertainties were applied using the 209 offset method, i.e. performing separate fits with the hadronisation corrections set to their maxi-210 mal and minimal values. This resulted in a hadronisation uncertainty on  $\alpha_s(M_{\pi}^2)$  of  $\pm 0.0012$  [2]. 211

The current procedure is different from this previously used procedure. The uncertainties on 212 the hadronisation corrections were included as input to the HERAPDF2.0 Jets NNLO fits. They 213 were treated as systematic uncertainties correlated between all data sets. Thus, their contribution 214 became part of the overall experimental, i.e. fit, uncertainties. For fits with fixed  $\alpha_s(M_7^2)$ , their 215 contribution was negligible. For fits with free  $\alpha_s(M_z^2)$ , their contribution to the experimental 216 uncertainty on  $\alpha_s(M_z^2)$  was ±0.0006. This represents a significant reduction of the influence of 217 the hadronisation uncertainties. 218

#### **HERAPDF2.0Jets NNLO – results** 4 219

#### Simultaneous determination of $\alpha_s(M_{\chi}^2)$ and PDFs 4.1 220

Jet-production data are essential for the determination of the strong coupling constant,  $\alpha_s(M_{\pi}^2)$ . 221 In pQCD fits to inclusive DIS data alone, the gluon PDF is determined via the DGLAP equations, 222 using the observed scaling violations. This results in a strong correlation between the shape of 223 the gluon distribution and the value of  $\alpha_s(M_7^2)$ . Data on jet and dijet production cross-sections 224 provide an independent constraint on the gluon distribution and are also directly sensitive to 225  $\alpha_s(M_Z^2)$ . Thus, such data are essential for an accurate simultaneous determination of  $\alpha_s(M_Z^2)$  and 226 the gluon distribution. 227

When determining  $\alpha_s(M_7^2)$ , it is necessary to consider so-called "scale uncertainties". They 228 approximate the uncertainty due to the influence of higher orders in the perturbation extension. 229 This uncertainty was evaluated by varying the renormalisation and factorisation scales by a 230 factor of two, both separately and simultaneously<sup>6</sup>, and selecting the maximal positive and 231 negative deviations of the result as the "de facto" scale uncertainty. These were observed for 232  $(2.0\mu_{\rm r}, 1.0\mu_{\rm f})$  and  $(0.5\mu_{\rm r}, 1.0\mu_{\rm f})$ , respectively. 233

2

The HERAPDF2.0Jets NNLO fit with free 
$$\alpha_s(M_Z^2)$$
 results in

 $\alpha_s(M_Z^2) = 0.1156 \pm 0.0011 \text{ (exp)} \stackrel{+0.0001}{_{-0.0002}} \text{ (model + parameterisation)} \pm 0.0029 \text{ (scale)}$ , (7)235

<sup>&</sup>lt;sup>6</sup>This procedure is often called 9-point variation, where the nine variations are  $(0.5\mu_r, 0.5\mu_f), (0.5\mu_r, 1.0\mu_f),$  $(0.5\mu_{\rm r}, 2.0\mu_{\rm f}), (1.0\mu_{\rm r}, 0.5\mu_{\rm f}), (1.0\mu_{\rm r}, 1.0\mu_{\rm f}), (1.0\mu_{\rm r}, 2.0\mu_{\rm f}), (2.0\mu_{\rm r}, 0.5\mu_{\rm f}), (2.0\mu_{\rm r}, 1.0\mu_{\rm f}), (2.0\mu_{\rm r}, 2.0\mu_{\rm f}).$ 

where "exp" denotes the experimental uncertainty, which was taken as the fit uncertainty, in-236 cluding the contribution from hadronisation uncertainties. The value of  $\alpha_s(M_Z^2)$  and the size of 237 the experimental uncertainty were confirmed by the the result of a so-called  $\chi^2$  scan in  $\alpha_s(M_{\pi}^2)$ , 238 which is shown in Fig. 2 a). Numerous fits with varying  $\alpha_s(M_Z^2)$  were performed and the clear 239 minimum observed in  $\chi^2$  coincides with the value of  $\alpha_s(M_z^2)$  determined with the fit. The width 240 of the minimum in  $\chi^2$  confirms the fit uncertainty. The combined model and parameterisation 241 uncertainty shown in Fig. 2 a) was determined by performing similar scans, for which the values 242 of the model parameters and the parameterisation were varied as described in Section 3.1. 243

Figure 2 a) also shows the scale uncertainty, which dominates the uncertainties. The scale 244 uncertainty as listed in Eq. 7 was evaluated under the assumption of 100 % correlated uncertain-245 ties between bins and data sets. The previously published result at NLO [2] had scale uncer-246 tainties calculated under the assumption of 50% correlated and 50% uncorrelated uncertainties 247 between bins and data sets. A strong motivation to determine  $\alpha_s(M_z^2)$  at NNLO was the hope to 248 substantially reduce scale uncertainties. Therefore, the analysis was repeated for these assump-249 tions in order to be able to compare the NNLO to the NLO scale uncertainties. The reevaluated 250 NNLO scale uncertainty of  $(\pm 0.0022)$  is indeed significantly lower than the  $(\pm 0.0037, -0.0030)$ 251 previously observed in the HERAPDF2.0Jets NLO analysis. 252

The HERAPDF2.0Jets NNLO fit with free  $\alpha_s(M_Z^2)$  was based on 1363 data points and had a  $\chi^2/d.o.f. = 1614/1348 = 1.197$ . This can be compared to the  $\chi^2/d.o.f. = 1363/1131 = 1.205$ for HERAPDF2.0 NNLO based on inclusive data only [2]. The similarity of the  $\chi^2/d.o.f.$  values indicates that the data on jet production do not introduce any additional tension to the fit. The jet data are fully consistent with the inclusive data.

The question whether data with relatively low  $Q^2$  bias the determination of  $\alpha_s(M_Z^2)$  arose within the context of the HERAPDF2.0 analysis [2]. Figure 2 b) shows the result of  $\alpha_s(M_Z^2)$ scans with  $Q_{min}^2$  for the inclusive data set to 3.5 GeV<sup>2</sup>, 10 GeV<sup>2</sup> and 20 GeV<sup>2</sup>. Clear minima are visible which coincide within uncertainties. Figure 2 c) shows the result of similar scans with only the inclusive data used as input [2]. The inclusive data alone cannot sufficiently constrain  $\alpha_s(M_Z^2)$ .

It has also been suggested that the use of the  $A'_q$  term, in the gluon parameterisation could 264 bias the determination of  $\alpha_s(M_Z^2)$ . Thus cross-checks were made with two modified gluon pa-265 rameterisations,  $A'_g = 0$  and  $xg(x) = A_g x^{B_g} (1-x)^{C_g}$  as well as the alternative gluon para-266 meterisation, AG [2], for which  $A'_g = 0$  and  $xg(x) = A_g x^{B_g} (1-x)^{C_g} (1+D_g x)$ . A value of 267  $\alpha_s(M_z^2) = 0.1151 \pm 0.0010(\exp)$  was obtained for both modifications of the parameterisation, 268 which is in agreement with the result for the standard parameterisation. The value of  $D_a$  in 269 the AG parameterisation was consistent with zero. These results demonstrate that the present 270  $\alpha_s(M_7^2)$  determination is not sensitive to the details of the gluon parameterisation. 271

The result presented here cannot be directly compared to an H1 result [37] and a result published by the NNLOJET authors and their collaborators [38] because a previous version of the theoretical predictions were used for these analyses. The groups have to tell me what to compare to. I could write something about the same version, but as I expect errata, I would prefer to compare to what will come or has come. Decisions and info during EB meeting, please. The following text is tentative.

Other determinations of  $\alpha_s(M_Z^2)$  at NNLO using jet data as published by H1 [37] and NNLO-JET authors and their collaborators [38] used fixed PDFs for their fits to determine  $\alpha_s(M_Z^2)$ . Therefore, the values of  $\alpha_s(M_Z^2)$  should not be directly compared. However, both analyses were performed with a cut on  $\mu$  of  $\mu > 2M_b$ , which is quite similar to the  $\mu > 10.0$  GeV cut used for this analysis. Thus, the scale uncertainties can be compared. The H1 result is based on H1 data only and the quoted scale uncertainty of  $\pm 0.0042$  can be compared to the  $\pm 0.0029$  obtained for the analysis presented here based on H1 and ZEUS data. The scale uncertainty published by NNLOjet is  $\pm 0.0036$ .

The H1 collaboration provided one simultanous fit of  $\alpha_s(M_Z^2)$  and PDFs, based on H1 inclusive and jet data only, and with  $Q_{min}^2 = 10 \text{ GeV}^2$ . For comparison, the analysis presented here was modified by also setting  $Q_{min}^2 = 10 \text{ GeV}^2$ . The value of  $\alpha_s(M_Z^2)$  published by H1 is  $\alpha_s(M_Z^2) =$  $0.1142 \pm 0.0011(\exp) \pm 0.0003(\text{model/parameterisation}) \pm 0.0026(\text{scale})$  while the current modified analysis resulted in  $\alpha_s(M_Z^2) = 0.1156 \pm 0.0011(\exp) \pm 0.0002(\text{model/parameterisation}) \pm 0.0021(\text{scale})$ .

#### <sup>292</sup> 4.2 The PDFs of HERAPDF2.0Jets NNLO obtained for fixed $\alpha_s(M_{\tau}^2)$

The value of  $\alpha_s(M_7^2) = 0.1155$  was used for the determination of the PDFs in the HERA-293 PDF2.0Jets NNLO analysis. The value listed in PDG12 [39], 0.118, which was also the value 294 determined in the HERAPDF2.0Jets NLO analysis, was used for the original HERAPDF2.0 295 analyses at NNLO based on inclusive data only. Therefore, the PDFs of HERAPDF2.0Jets 296 NNLO are shown in Fig. 3 a) and b) for both, fixed  $\alpha_s(M_Z^2) = 0.1155$  and fixed  $\alpha_s(M_Z^2) = 0.118$ , 297 respectively, together with their uncertainties, at the scale  $\mu_f^2 = 10 \,\text{GeV}^2$ . The uncertainties 298 shown are the experimental, i.e. fit, uncertainties as well as the model and parameterisation 299 uncertainties as defined in Section 3.2. The parameterisation uncertainty dominates the uncer-300 tainties and is itself dominated by the introduction of the parameter  $D_{u_v}$  as a variation. Details 301 on the two sets of PDFs as released are listed in Appendix A. 302

As the PDFs were derived with a fixed  $\alpha_s(M_Z^2)$  value scale uncertainties on the PDFs were 303 not considered, because, in this case, a quantification of theory uncertainties through a variation 304 of the renormalisation and factorisation scales in the fit becomes questionable. Even after the 305 compensation of explicit scale-dependent terms in the NLO and NNLO coefficients, a variation 306 of the renormalisation scale effectively amounts, in its numerical effect, to a modification of the 307 value of  $\alpha_s(M_z^2)$ . Fixing the value of  $\alpha_s(M_z^2)$  externally amounts to forcing the fit away from 308 a local minimum, where a variation of the scales could map out the putative uncertainty from 309 missing higher orders. Therefore, scale variations cannot be used as a proxy for uncertainties 310 on the PDF extraction due to missing higher orders. Nevertheless, a cross-check with scale 311 variations as described in Section 4.1 for the fit with free  $\alpha_s(M_7^2)$  was made. The impact on 312 the resulting PDFs was found to be negligible compared to the other uncertainties presented in 313 Fig. 3. 314

A comparison between the PDFs obtained for  $\alpha_s(M_Z^2) = 0.1155$  and  $\alpha_s(M_Z^2) = 0.118$  is provided in Figs. 4 and 5 for the scales  $\mu_f = 10 \text{ GeV}^2$  and  $\mu_f = M_Z^2$ , respectively. Here, only total uncertainties are shown. At the lower scale, a significant difference is observed between the gluon distributions; the distribution for  $\alpha_s(M_Z^2) = 0.1155$  is above the distribution for  $\alpha_s(M_Z^2) =$ 0.118 for x less than  $\approx 10^{-2}$ . This correlation between the value of  $\alpha_s(M_Z^2)$  and the shape of the gluon PDF is as expected from QCD evolution. At the scale of  $M_Z^2$ , the differences become negligible in the visible range of x due to QCD evolution. A comparison of the PDFs obtained for  $\alpha_s(M_Z^2) = 0.118$  by HERAPDF2.0Jets NNLO to the PDFs of HERAPDF2.0 NNLO, based on inclusive data only, is provided in Fig. 6. These two sets of PDFs do not show any significant difference in the central values. However, there is a significant reduction of the uncertainties on the gluon PDFs as shown in Fig. 7 at the scale of  $\mu_f = 10 \text{ GeV}^2$  and in Fig. 8 at the scale of  $\mu_f = M_Z^2$ . The reductions in the uncertainties for HERAPDF2.0Jets NNLO for  $\alpha_s(M_Z^2) = 0.1155$  compared to  $\alpha_s(M_Z^2) = 0.118$  are shown in Fig. 9 and Fig. 10. At high x and  $\mu_f = M_Z^2$ , the parameterisaton uncertainties become important as can be seen by comparing Fig. 10 b) and 10 c).

The reduction in model and parameterisation uncertainty for  $x < 10^{-3}$  compared to HERA-330 PDF2.0 NNLO is mostly due to the improved procedure to estimate this uncertainty. The ranges, 331 in which  $M_c$  and  $M_b$  were varied were reduced, but but this had basically no effect on the uncer-332 tainties but for the following effect. As discussed Section 3.3, it was necessary to symmetrise 333 the downward variation of  $\mu_{f0}^2$  rather than allowing both upward and downward variations. This 334 had the positive effect of removing a slight double-counting of sources of uncertainty that could 335 not be avoided in the original HERAPDF2.0 NNLO procedure. The reduction in the model and 336 parameterisation uncertainties for  $x < 10^{-3}$  is mostly due to this effect, whereas the reduction 337 in experimental as well as model and parameterisation uncertainties for  $x > 10^{-3}$  is due to the 338 influence of the jet data. This is also demonstrated in Fig. 11, which shows ratios of uncer-339 tainties with respect to the total uncertainties of HERAPDF2.0 NNLO based on inclusive data 340 only. Shown are the contributions of the experimental, the experimental plus model and the 341 experimental plus parameterisation uncertainties to the the total uncertainties of HERAPDF2.0 342 NNLO and the respective reductions for HERAPDF2.0Jets NNLO. Further such ratio plots are 343 provided in Appendix B. 344

#### **4.3** Comparisons of HERAPDF2.0Jets NNLO predictions to jet data

<sup>346</sup> Comparisons of the predictions based on HERAPDF2.0Jets NNLO with fixed  $\alpha_s(M_Z^2) = 0.1155$ <sup>347</sup> to the data on jet production used as input to the fit are shown in Figs. 12 to 19. Each figure <sup>348</sup> presents in a) a direct comparison of the cross sections and in b) the respective ratios.

The uncertainties on the NNLO predictions as provided by applfast were taken into account 349 in all HERAPDF2.0Jets NNLO fits. The predictions based on the HERAPDF2.0Jets NNLO 350 PDFs were computed using the assumption of massless jets, i.e. the transverse energy,  $E_T$ , and 351 the transverse momentum of a jet,  $p_T$ , were assumed to be equivalent. For the inclusive jet 352 analyses, each jet  $p_T$  was entered separately. For dijet analyses, the average of the transverse 353 momenta,  $\langle p_T \rangle$  was used. In these cases,  $\langle p_T \rangle$  was also used to set the factorisation and 354 renormalisation scales to  $\mu_{\rm f}^2 = \mu_{\rm r}^2 = Q^2 + \langle p_T \rangle^2$  for calculating predictions. Scale uncertainties 355 were not considered [16] for the comparisons to data. The predictions based on the PDFs of 356 HERAPDF2.0Jets NNLO clearly fit the data on jet production used as input very well, showing 357 that the inclusive data and jet production data both used as input to the NNLO QCD fit are fully 358 consistent. 359

#### **360 5 Summary**

The HERA data set on inclusive *ep* scattering as published by the H1 and ZEUS collaborations [2], together with selected data on jet production, published separately by the two collab<sup>363</sup> orations, were used as input to a pQCD analysis at NNLO.

An analysis was performed where  $\alpha_s(M_Z^2)$  and the PDFs were fitted simultaneously. This 364 resulted in a value of  $\alpha_s(M_Z^2) = 0.1156 \pm 0.0011 \,(\exp)^{+0.0001}_{-0.0002} \,(\text{model}/\ /\text{parameterisation}) \pm 0.0011 \,(\exp)^{+0.0001}_{-0.0002} \,(\text{model}/\ /\text{parameterisation})$ 365 0.0029 (scale). This result on  $\alpha_s(M_z^2)$  is compatible with the world average [40] and it is compet-366 itive in comparison with other determinations at NNLO. The scale uncertainties were calculated 367 under the assumption of fully correlated uncertainties between bins and data sets. They would 368 decrease to ±0.0021 under the assumption of 50% correlated and 50% uncorrelated uncer-369 tainties which is the value that can be directly compared to the previously published [2] scale 370 uncertainties of (+0.0037,-0.0030) observed in the HERAPDF2.0Jets NLO analysis. 371

Two sets of PDFs were determined for HERAPDF2.0Jets NNLO for fixed  $\alpha_s(M_Z^2) = 0.1155$ 372 and  $\alpha_s(M_Z^2) = 0.118$ . They are available to the community. Comparisons between the PDFs of 373 HERAPDF2.0Jets NNLO obtained for the two values of  $\alpha_s(M_7^2)$  were shown, as well as com-374 parisons to HERAPDF2.0 NNLO, for which jet data were not used as input to the fit. All these 375 PDFs are very similar, showing the consistency of the inclusive and the jet production data. 376 On balance, the inclusion of the jet data had two consequences: i) a lower value of  $\alpha_s(M_z^2)$  is 377 favoured; ii) the uncertainty on the gluon PDF was reduced. Predictions based on the PDFs of 378 HERAPDF2.0Jets NNLO were compared to the jet production data used as input. The predic-379 tions describe the data very well. 380

The PDFs of HERAPDF2.0Jets NNLO complete the HERAPDF2.0 ensemble of parton distribution functions. This ensemble of PDFs, extracted from HERA data alone, presents a consistent picture in the framework of pQCD. It is on of the legacies of HERA.

#### **384** 6 Acknowledgements

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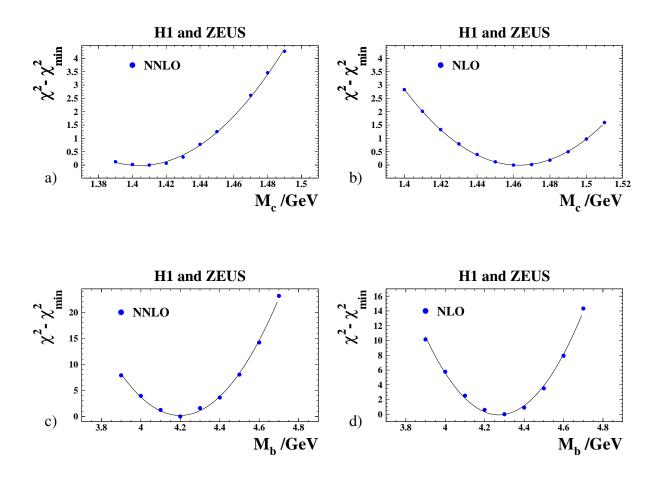
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| Data Set                            | taken                 | $Q^2$ [GeV <sup>2</sup> ] range |       | L                | $e^{+}/e^{-}$ | $\sqrt{s}$ | norma- | all    | used   | Ref.    |
|-------------------------------------|-----------------------|---------------------------------|-------|------------------|---------------|------------|--------|--------|--------|---------|
|                                     | from to               | from                            | to    | pb <sup>-1</sup> |               | GeV        | lised  | points | points |         |
| H1 HERA I normalised jets           | 1999 - 2000           | 150                             | 15000 | 65.4             | $e^+p$        | 319        | yes    | 24     | 24     | [9]     |
| H1 HERA I jets at low $Q^2$         | 1999 - 2000           | 5                               | 100   | 43.5             | $e^+p$        | 319        | no     | 28     | 20     | [10]    |
| H1 normalised inclusive jets at his | igh $Q^2$ 2003 – 2007 | 150                             | 15000 | 351              | $e^+ p/e^- p$ | 319        | yes    | 30     | 30     | [13,14] |
| H1 normalised dijets at high $Q^2$  | 2003 - 2007           | 150                             | 15000 | 351              | $e^+ p/e^- p$ | 319        | yes    | 24     | 24     | [14]    |
| H1 normalised inclusive jets at lo  | $Q^2 = 2005 - 2007$   | 5.5                             | 80    | 290              | $e^+ p/e^- p$ | 319        | yes    | 48     | 37     | [13]    |
| H1 normalised dijets at low $Q^2$   | 2005 - 2007           | 5.5                             | 80    | 290              | $e^+ p/e^- p$ | 319        | yes    | 48     | 37     | [13]    |
| ZEUS inclusive jets                 | 1996 – 1997           | 125                             | 10000 | 38.6             | $e^+p$        | 301        | no     | 30     | 30     | [11]    |
| ZEUS dijets 1998 –2                 | 000 & 2004 - 2007     | 125                             | 20000 | 374              | $e^+ p/e^- p$ | 318        | no     | 22     | 16     | [12]    |

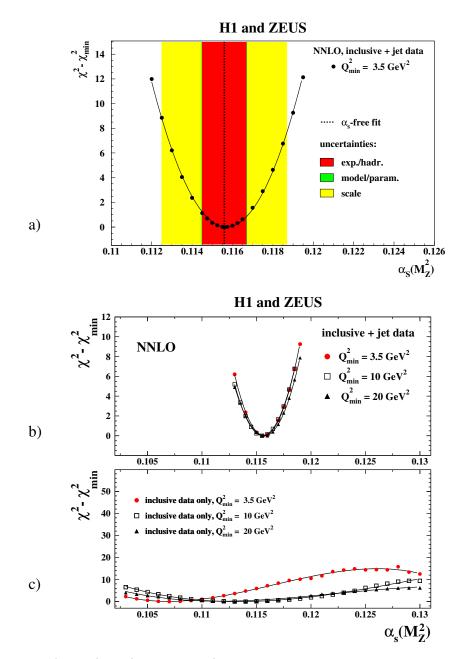
**Table 1:** The data sets on jet production from H1 and ZEUS used for the HERAPDF2.0Jets NNLO its. The term normalised indicates that all cross sections are normalised to the respective NC inclusive cross sections.

| Parameter    |           | Central Value | Downwards variation | Upwards variation |  |  |  |
|--------------|-----------|---------------|---------------------|-------------------|--|--|--|
| $Q^2_{min}$  | $[GeV^2]$ | 3.5           | 2.5                 | 5.0               |  |  |  |
| $f_s$        |           | 0.4           | 0.3                 | 0.5               |  |  |  |
| $M_c$        | [GeV]     | 1.41          | 1.37*               | 1.45              |  |  |  |
| $M_b$        | [GeV]     | 4.20          | 4.10                | 4.30              |  |  |  |
| $\mu_{f0}^2$ | $[GeV^2]$ | 1.9           | 1.6                 | 2.2*              |  |  |  |

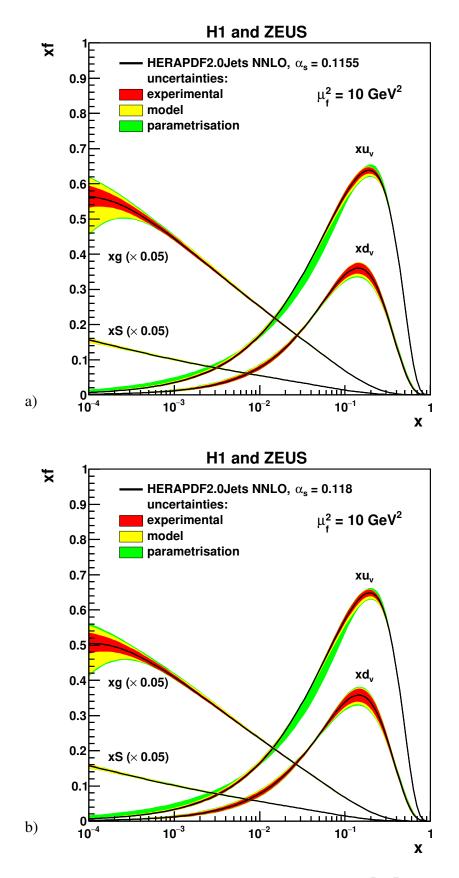
**Table 2:** Central values of model input parameters and their one-sigma variations. It was not possible to implement the variations marked \* because  $\mu_{f0} < M_c$  is required, see Section 3.3. In these cases, the uncertainty on the PDF obtained from the other variation was symmetrised.



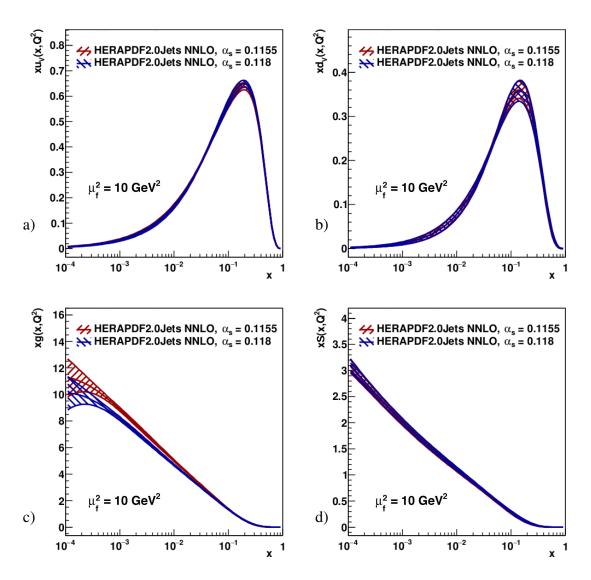
**Figure 1:**  $\Delta \chi^2 = \chi^2 - \chi^2_{min}$  vs. a) and b)  $M_c$  with  $M_b = 4.2$  GeV, and c) and d)  $M_b$  with  $M_c = 1.41$  GeV for a) and c) HERAPDF2.0Jets NNLO fits with fixed  $\alpha_s(M_Z^2) = 0.1155$  and the corresponding NLO fits for  $M_c = 1.46$  GeV,  $M_b = 4.3$  GeV and  $\alpha_s(M_Z^2) = 0.118$ .



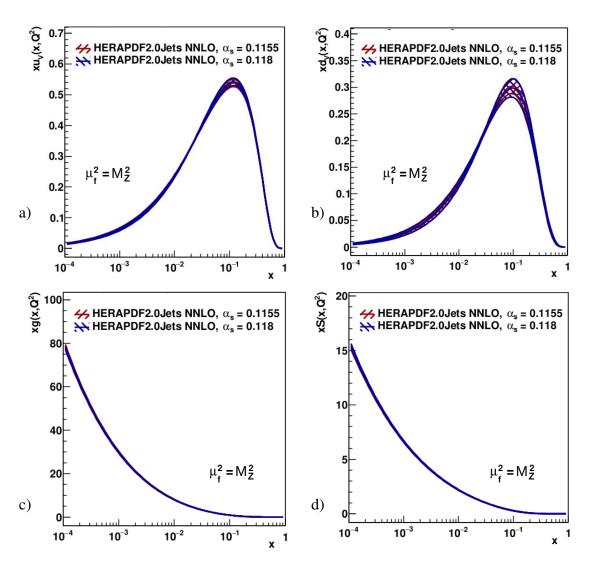
**Figure 2:**  $\Delta \chi^2 = \chi^2 - \chi^2_{\min}$  vs.  $\alpha_s(M_Z^2)$  for HERAPDF2.0Jets NNLO fits with fixed  $\alpha_s(M_Z^2)$  with a) the standard  $Q_{\min}^2$  of 3.5 GeV<sup>2</sup> b) with  $Q_{\min}^2$  set to 3.5 GeV<sup>2</sup>, 10 GeV<sup>2</sup> and 20 GeV<sup>2</sup> for the inclusive data. In a), the result and all uncertainties determined for the HERAPDF2.0Jets NNLO fit with free  $\alpha_s(M_Z^2)$  are also shown, added in quadrature. In b), not all scan points for  $Q_{\min}^2$  of 3.5 GeV<sup>2</sup> are plotted for better visibility. c) For comparison, the situation for fits to only inclusive data is shown, taken from [2].



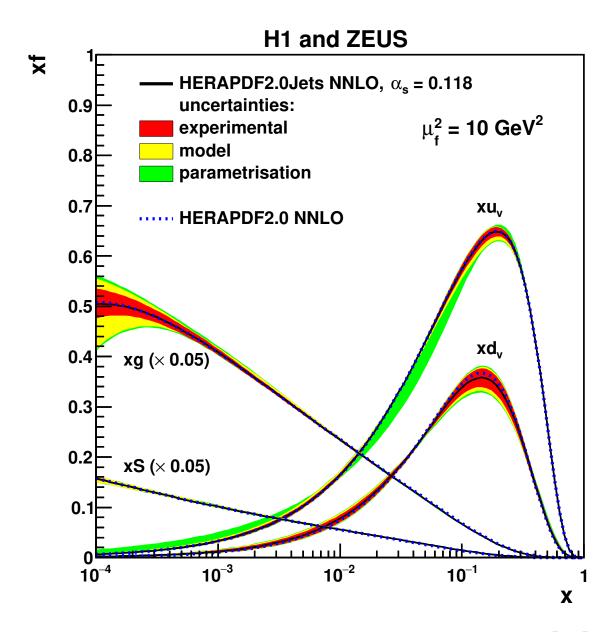
**Figure 3:** The parton distribution functions  $xu_v$ ,  $xd_v$ , xg and  $xS = x(\bar{U}+\bar{D})$  of HERAPDF2.0Jets NNLO, with a)  $\alpha_s(M_Z^2)$  fixed to 0.1155 and b)  $\alpha_s(M_Z^2)$  fixed to 0.118 at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties are shown as differently shaded bands.



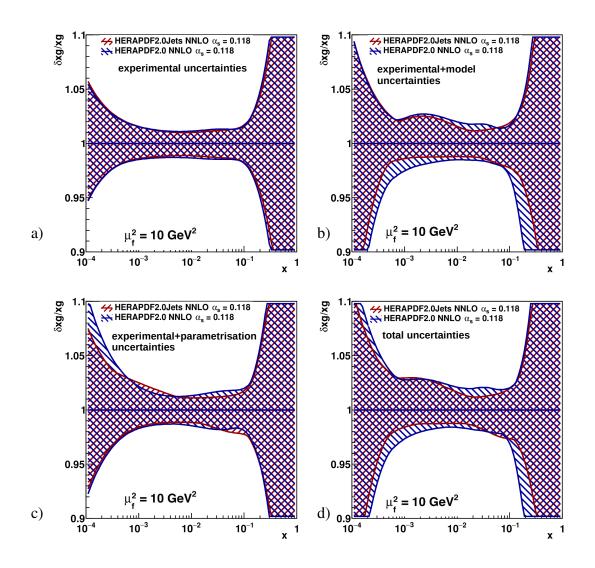
**Figure 4:** Comparison of the parton distribution functions a)  $xu_v$ , b)  $xd_v$ , c) xg and d)  $xS = x(\bar{U} + \bar{D})$  of HERAPDF2.0Jets NNLO with fixed  $\alpha_s(M_Z^2) = 0.1155$  and  $\alpha_s(M_Z^2) = 0.118$  at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The total uncertainties are shown as differently hatched bands.



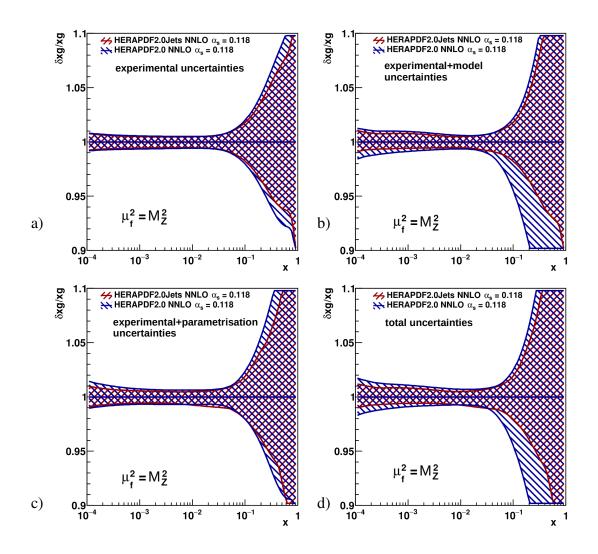
**Figure 5:** Comparison of the parton distribution functions a)  $xu_v$ , b)  $xd_v$ , c) xg and d)  $xS = x(\bar{U} + \bar{D})$  of HERAPDF2.0Jets NNLO with fixed  $\alpha_s(M_Z^2) = 0.1155$  and  $\alpha_s(M_Z^2) = 0.118$  at the scale  $\mu_f^2 = M_Z^2$  with  $M_Z = 91.19$  GeV [40]. The total uncertainties are shown as differently hatched bands.



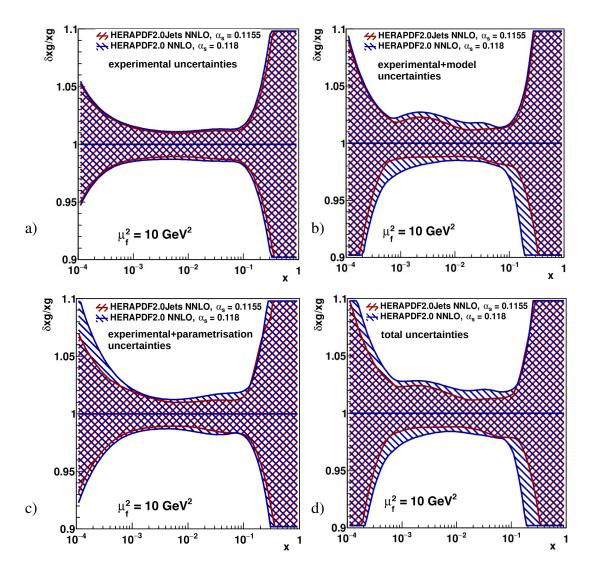
**Figure 6:** Comparison of the parton distribution functions  $xu_v$ ,  $xd_v$ , xg and  $xS = x(\bar{U} + \bar{D})$  of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO, which was based on inclusive data only, both with fixed  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The full uncertainties of HERA-PDF2.0Jets NNLO are shown as differently shaded bands and the central value of HERAPDF2.0 NNLO is shown as a dotted line.



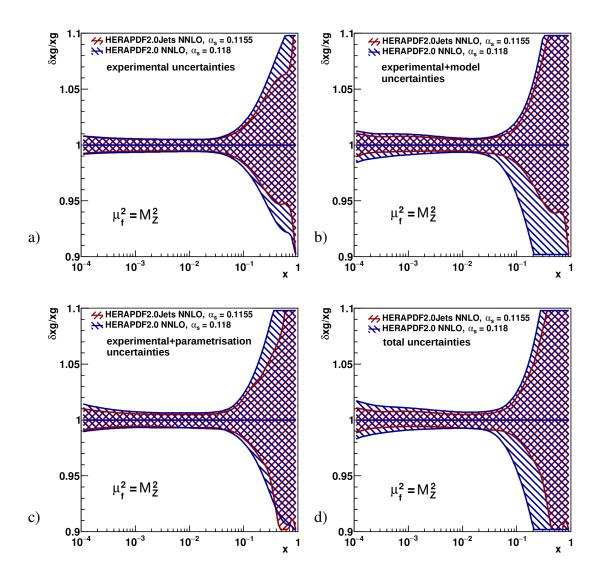
**Figure 7:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO for a) experimental, i.e. fit, b) experimental plus model, c) experimental plus parameterisation, d) total uncertainties at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties on both gluon distributions are shown as differently hatched bands.



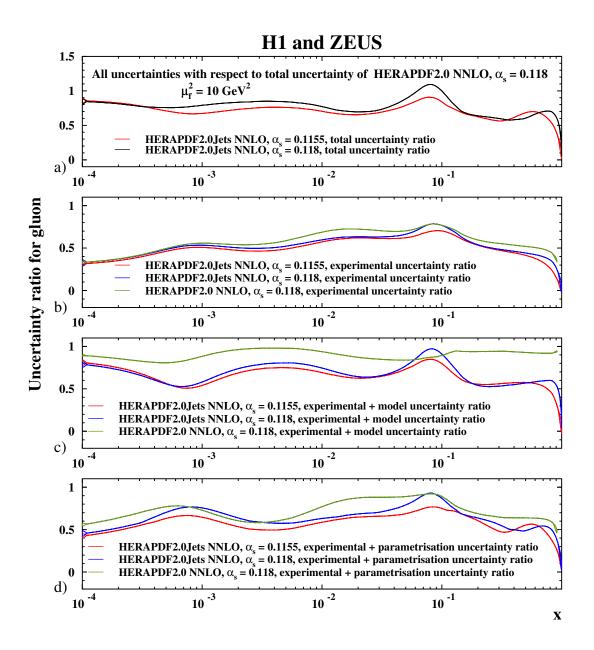
**Figure 8:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO for a) experimental, i.e. fit, b) experimental plus model, c) experimental plus parameterisation, d) total uncertainties at the scale  $\mu_f^2 = M_Z^2$ . The uncertainties on both gluon distributions are shown as differently hatched bands.



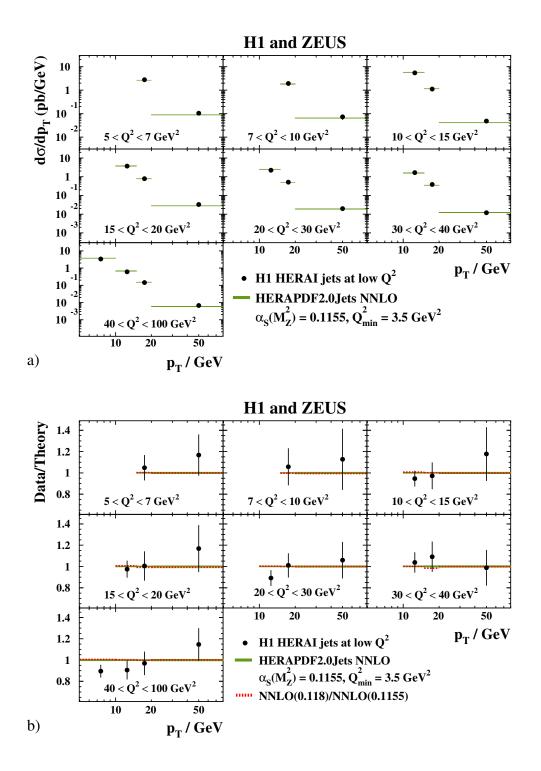
**Figure 9:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO for a) experimental, i.e. fit, b) experimental plus model, c) experimental plus parameterisation, d) total uncertainties at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties on both gluon distributions are shown as differently hatched bands.



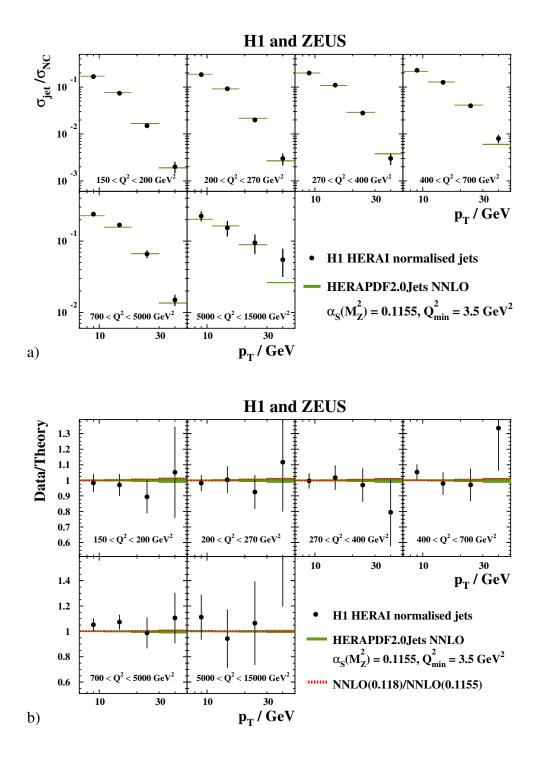
**Figure 10:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO for a) experimental, i.e. fit, b) experimental plus model, c) experimental plus parameterisation, a) total uncertainties at the scale  $\mu_f^2 = M_Z^2$ . The uncertainties on both gluon distributions are shown as differently hatched bands.



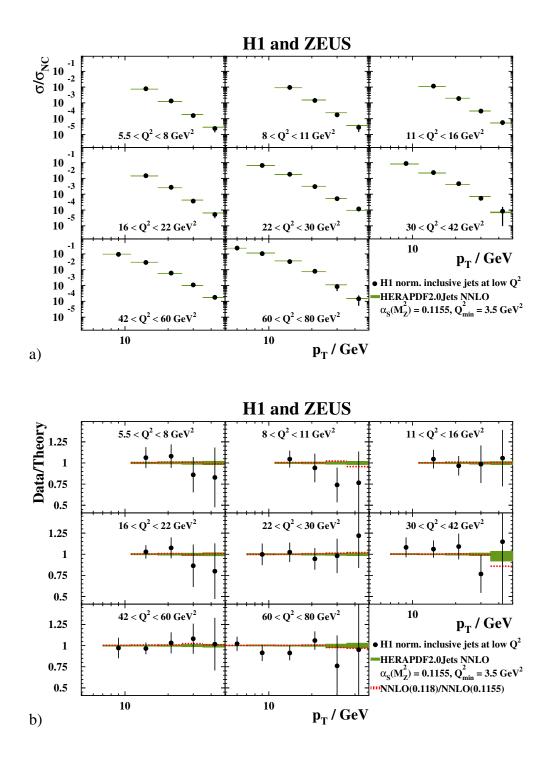
**Figure 11:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  a) total, b) experimental, c) experimental plus model, d) experimental plus parameterisation uncertainties for HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  and  $\alpha_s(M_Z^2) = 0.1155$  at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ .



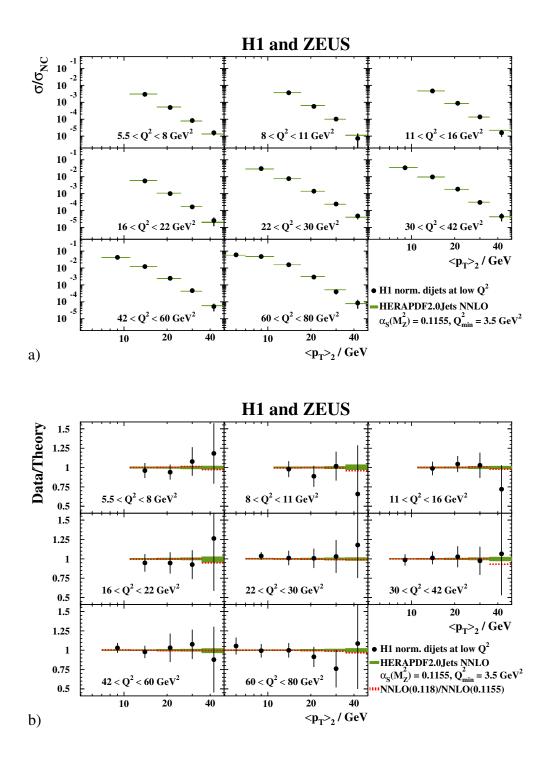
**Figure 12:** a) Differential jet cross sections,  $d\sigma/dp_T$ , in bins of  $Q^2$  between 5 and 100 GeV<sup>2</sup> as measured by H1. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties, the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections devided by predictions based on HERAPDF2.0Jets NNLO.



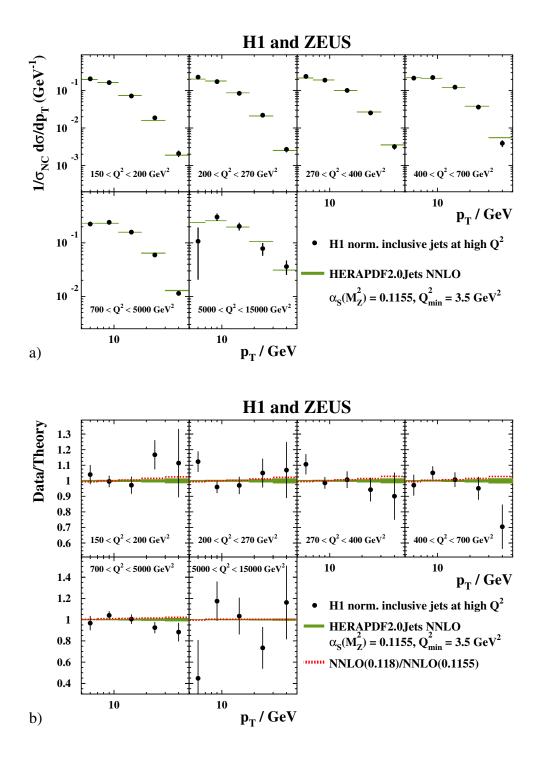
**Figure 13:** a) Differential jet cross sections,  $d\sigma/dp_T$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 150 and 15000 GeV<sup>2</sup> as measured by H1. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections devided by predictions based on HERAPDF2.0Jets NNLO.



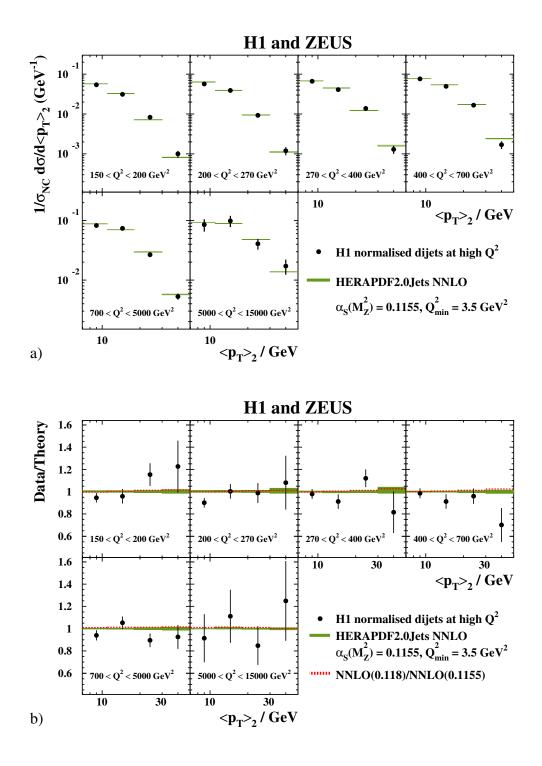
**Figure 14:** a) Differential inclusive jet cross sections,  $d\sigma/dp_T$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 5 and  $80 \text{ GeV}^2$  as measured by H1. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections devided by predictions based on HERAPDF2.0Jets NNLO.



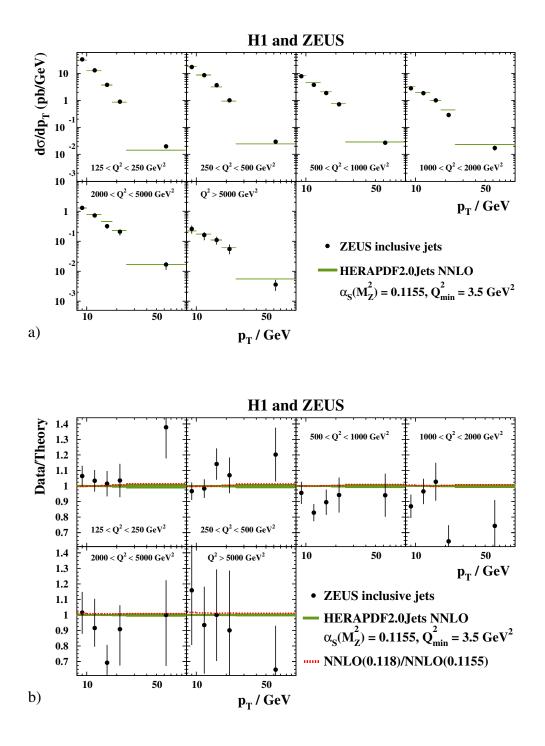
**Figure 15:** a) Differential dijet normalised cross sections,  $d\sigma/d\langle p_T \rangle_2$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 5 and 80 GeV<sup>2</sup> as measured by H1. The variable  $\langle p_T \rangle_2$  denotes the average  $p_T$  of the two jets. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections devided by predictions based on HERAPDF2.0Jets NNLO.



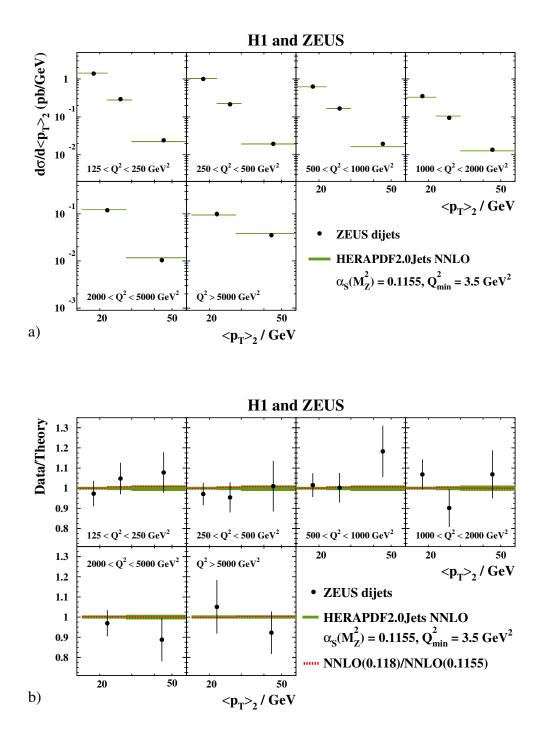
**Figure 16:** a) Differential inclusive jet cross sections,  $d\sigma/dp_T$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 150 and 15000 GeV<sup>2</sup> as measured by H1. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections devided by predictions based on HERAPDF2.0Jets NNLO.



**Figure 17:** a) Differential dijet cross sections,  $d\sigma/d\langle p_T \rangle_2$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 150 and 15000 GeV<sup>2</sup> as measured by H1. The variable  $\langle p_T \rangle_2$  denotes the average  $p_T$  of the two jets. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections devided by predictions based on HERAPDF2.0Jets NNLO.



**Figure 18:** a) Differential jet cross sections,  $d\sigma/dp_T$ , in bins of  $Q^2$  between 125 and 10000 GeV<sup>2</sup> as measured by ZEUS. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainty on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections devided by predictions based on HERAPDF2.0Jets NNLO.



**Figure 19:** a) Differential dijet cross sections,  $d\sigma/d\langle p_T \rangle_2$ , in bins of  $Q^2$  between 125 and 20000 GeV<sup>2</sup> as measured by ZEUS. The variable  $\langle p_T \rangle_2$  denotes the average  $p_T$  of the two jets. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainty on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections devided by predictions based on HERAPDF2.0Jets NNLO.

# **Appendix A:**

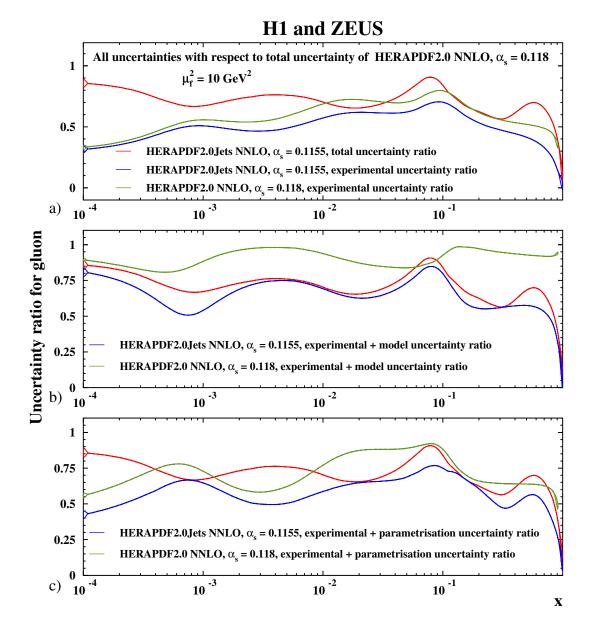
| 452        | PDF sets released   |
|------------|---|
| 453        | The following two sets of PDFs are released [41] and available on LHAPDF:   |
| 454        | (https://lhapdf.hepforge.org/pdfsets.html).   |
| 455        | • HERAPDF2.0Jets NNLO   |
| 456<br>457 | <ul> <li>based on the combination of inclusive data from the H1 and ZEUS collaborations<br/>and selected data on jet production;</li> </ul> |
| 458        | - with $Q_{\min}^2 = 3.5 \mathrm{GeV}^2$ ;  |
| 459        | <ul> <li>using the RTOPT variable-flavour-number scheme;</li> </ul>   |
| 460        | * with fixed value of $\alpha_s(M_Z^2) = 0.01155$ ;   |
| 461        | * with fixed value of $\alpha_s(M_Z^2) = 0.0118$ ;  |
| 462<br>463 | <ul> <li>14 eigenvector pairs give Hessian experimental (fit) uncertainties including hadroni-<br/>sation uncertainties;</li> </ul>         |
| 464<br>465 | <ul> <li>grids of 14 variations are released to describe the model and parameterisation uncer-<br/>tainties.</li> </ul>                     |

# **Appendix B:**

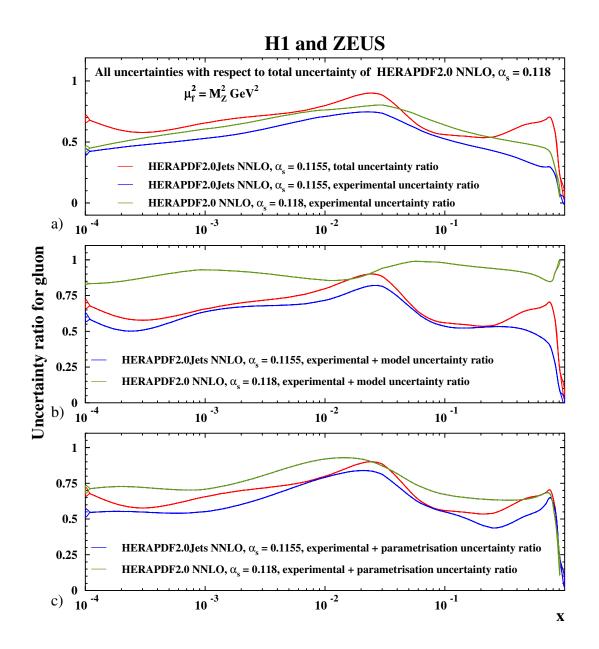


466

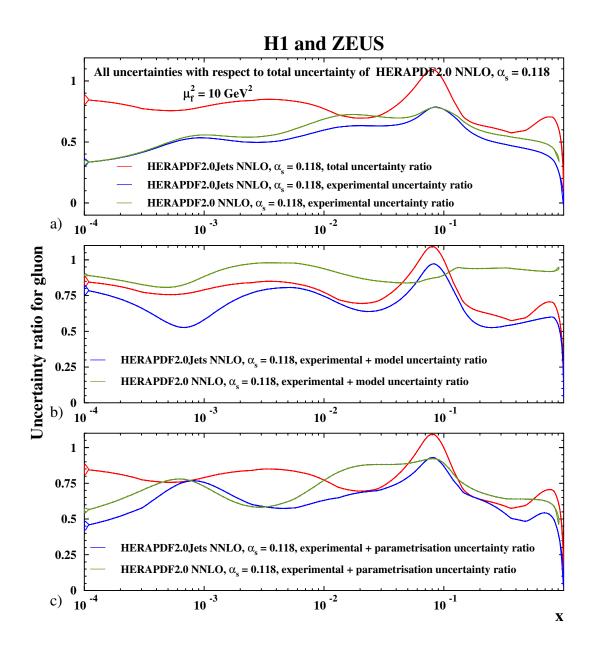
## Additional ratio plots on gluon PDF uncertainties



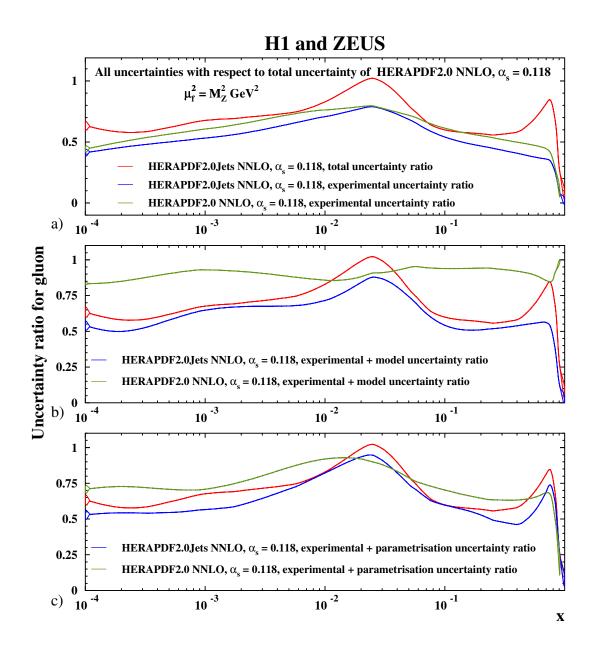
**Figure 20:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  for the total uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  and the a) experimental uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1185$  as well as the experimental plus model uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.1185$  as well as the experimental plus model uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.1185$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1185$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1185$ , at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ .



**Figure 21:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  for the total uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  and the a) experimental uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = M_Z^2$ .



**Figure 22:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  for the total uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  and the a) experimental uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ .



**Figure 23:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  for the total uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  and the a) experimental uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , as well as the experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = M_Z^2$ .

# **Internal extra material:**<sup>7</sup>

## 469 Comparison of results on $\alpha_s(M_7^2)$ determined at NLO and NNLO:

A more detailed comparison between the NLO and NNLO results must account for the following differences:

| 472        | • the choice of scale was different;  |
|------------|---|
| 473<br>474 | • the NLO result did not include the recently published H1 low-Q <sup>2</sup> inclusive and dijet data [13];  |
| 475<br>476 | • the NLO result did not include the newly published low $p_T$ points from the H1 high- $Q^2$ inclusive data; |
| 477        | • the NNLO result does not include trijet data;   |
| 478        | • the NNLO result does not include the low $p_T$ points from the ZEUS dijet data;                             |
| 479        | • the NNLO analysis imposes a stronger kinematic cut $\mu > 13.5$ GeV;  |
| 480        | • the treatment of hadronisation uncertainty differs.   |

All these changes with respect to the NLO analysis had to be made to create a consistent environment for a fit at NNLO. at the same time, an NLO fit cannot be done under exactly the same conditions as the NNLO fit since the H1 low  $Q^2$  data cannot be well fitted at NLO. However, an NLO and an NNLO fit can be done under the common conditions:

- choice of scale,  $\mu_{\rm f}^2 = \mu_{\rm r}^2 = Q^2 + p_T^2$ ;
- exclusion of the H1 low- $Q^2$  inclusive and dijet data;
- exclusion of the low- $p_T$  points from the H1 high- $Q^2$  inclusive jet data;
- exclusion of trijet data;
- exclusion of low- $p_T$  points from the ZEUS dijet data;
- exclusion of data with  $\mu < 13.5 \,\text{GeV}$ ;
- hadronisation uncertainties treated as correlated systematic uncertainties as done in the
   NNLO analysis.

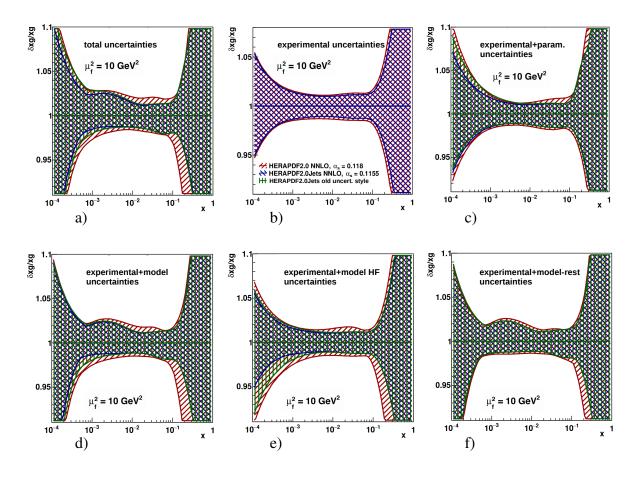
In this case, the values obtained were  $\alpha_s(M_Z^2) = 0.1186 \pm 0.0014(\exp)$  at NLO and  $\alpha_s(M_Z^2) = 0.1144 \pm 0.0013(\exp)$  at NNLO. The new NLO value of  $\alpha_s(M_Z^2)$  agrees with the published [2] value of 0.1183. The change of the NNLO result from the preferred value of 0.1156 is mostly due to the exclusion of the H1 low  $Q^2$  data and the low- $p_T$  points at high  $Q^2$ .

<sup>&</sup>lt;sup>7</sup>This material was Appendix A and was moved to internal extra material according to an EB decision.

# **497** Internal extra material: 8

<sup>498</sup> More detailed information concerning the source of uncertainties at a <sup>499</sup> scale of 10 GeV<sup>2</sup>: The green band represents HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2)$ =0.1155 <sup>500</sup> as obtained for the old procedure, i.e. with double counting.

<sup>501</sup> This shows that the improvement is mainly due to jet data.



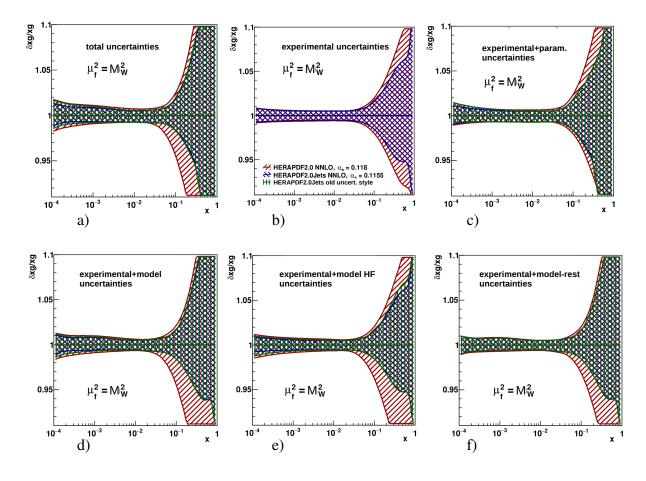
**Figure 24:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO, HERAPDF2.0 NNLO and HERAPDF2.0Jets NNLO with old procedure on uncertainties for a) total, b) experimental, i.e. fit, c) experimental plus parameterisation, d) experimental plus model, e) experimental plus model due to heavy flavour f) experimental plus all model but heavy flavour uncertainties at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties on the three gluon distributions are shown as differently hatched bands.

<sup>&</sup>lt;sup>8</sup>This material was public extra material and was moved to internal extra material according to an EB decision.

# <sup>502</sup> Internal extra material: <sup>9</sup>

<sup>503</sup> More detailed information concerning the source of uncertainties at a <sup>504</sup> scale of  $M_W^2$ : The green band represents HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2)$ =0.1155 <sup>505</sup> as obtained for the old procedure, i.e. with double counting.

<sup>506</sup> This shows that the improvement is mainly due to jet data.



**Figure 25:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO, HERAPDF2.0 NNLO and HERAPDF2.0Jets NNLO with old procedure on uncertainties for a) total, b) experimental, i.e. fit, c) experimental plus parameterisation, d) experimental plus model, e) experimental plus model due to heavy flavour f) experimental plus all model but heavy flavour uncertainties at the scale  $\mu_f^2 = M_W^2$ . The uncertainties on the three gluon distributions are shown as differently hatched bands.

<sup>&</sup>lt;sup>9</sup>This material was external extra material and was moved to internal extra material according to an EB decision.

# Internal extra material: 10

507

596

508 509 510 Parameters as determied by the fits and their correlations 511 512 PARAMETERS WITH UNCERTAINTIES: 513 514 515 as free 516 517 2 'Ba' -0.084608 0.071758 518 3 7 'Cg' 6.145485 0.553362 'Apria 519 0.148366 0.134036 520 521 8 'Bprig -0.408486 0.062832 'Cprig 25.000000 0.000000 fixed 9 522 523 12 'Buv' 0.782478 0.027706 'Cuv' 13 4.878155 0.083909 524 525 15 22 10.390885 1.352200 'Euv' 'Bdv' 0.983110 0.083080 526 527 23 33 'Cdv' 4.795152 0.383854 1.699099 'CUbar 7.123114 528 529 34 41 'DUbar' 1.995344 2.431042 0.262598 0.010781 'ADbar 530 531 42 'BDbar -0 128810 0.004899 43 'CDbar 9.094971 1.741850 532 533 101 'alphas' 0.115638 0.001142 534 535 as = 0.1155 536 537 'Bg' 'Cg' -0.085574 0.039648 2 538 3 6.171545 0.496131 539 'Aprig 0.147903 0.040820 8 540 'Bprig -0 409380 0 028287 541 25.000000 0.000000 fixed 9 'Cprig 542 543 0.781078 4.880050 12 13 'Buy' 0 025867 'Cuv' 0.080411 544 545 15 22 'Euv' 10.401539 1,289019 'Bdv' 0.983055 0.084572 546 23 'Cdv' 4.804735 0.380423 547 548 33 1.645404 'CUbar 7.125150 34 'DUbar 2.031948 2.222251 549 550 41 'ADbar 0.262191 0.010036 42 'BDbar -0.128934 0.004725 551 552 43 'CDbar' 9.161993 1.693978 553 554 as = 0.118 \_\_\_\_ 555 556 'Bg' 'Cg' 'Aprig' -0.070319 0.043016 2 557 558 3 7 5.670899 0.482567 0.161572 559 560 -0.391610 25.000000 0.027755 0.000000 fixed 8 9 'Bprig 'Cprig 0.806334 561 562 12 'Buv' 0.028281 'Cuv' 13 4.844608 0.081284 563 564 15 'Euv' 10.242348 1.441602 'Bdv' 22 0.981522 0.092135 565 566 23 33 'Cdv' 'CUbar 4.622768 7.137838 0.397334 1.347568 567 568 1.458837 0.269978 34 41 'DUbar' 1.614989 0.010673 'ADbar 569 570 571 572 42 'BDhar' -0.126504 0.004831 43 'CDbar 1.509073 8.036277 573 574 575 576 PARAMETER CORRELATION COFFEICIENTS 577 578 as free 579 579 580 581 582 583 NO. GLOBAL 12 13 15 22 23 33 34 41 1.000 0.544-0.880-0.627 0.112-0.024-0.040 0.030-0.015 0.024 0.019-0.090-0.166-0.066 0.135 0.99909 2 0.99544 0.544 1.000-0.294-0.077-0.034 0.078-0.036-0.095-0.060 0.141 0.242-0.452-0.503-0.226-0.386 7 0.99942 -0.880-0.294 1.000 0.914 0.101-0.067-0.115 0.033 0.028 0.010-0.001 0.025 0.028-0.026 0.002 584 -0.627-0.077 0.914 1.000 0.251-0.130-0.230 0.094 0.057 0.010-0.028 0.038-0.009-0.062 0.093 0.99710 585 0.112-0.034 0.101 0.251 1.000-0.208-0.711 0.254 0.050 0.326 0.036 0.524 0.400 0.021 0.418 12 0.99580 0.112-0.054 0.101 0.251 1.000-0.206-0.711 0.254 0.050 0.526 0.556 0.524 0.400 0.021 0.416 -0.024 0.078-0.067-0.130-0.208 1.000 0.708-0.193-0.212 0.374 0.410-0.168-0.124-0.089-0.183 -0.040-0.036-0.115-0.230-0.711 0.708 1.000-0.226-0.165 0.133 0.338-0.369-0.299-0.137-0.956 0.030-0.095 0.033 0.094 0.254-0.193-0.226 1.000 0.892 0.370 0.287 0.266 0.228 0.591 0.020 -0.015-0.060 0.028 0.057 0.050-0.212-0.165 0.892 1.000 0.151 0.114 0.154 0.147 0.553-0.197 586 587 13 0.98055 15 0.99428 588 589 22 0.99034 23 0.98232 590 591 0.024 0.141 0.010 0.010 0.326 0.374 0.133 0.370 0.151 1.000 0.923-0.006-0.020 0.160 0.002 0.019 0.242-0.001-0.028 0.036 0.410 0.338 0.287 0.114 0.923 1.000-0.253-0.252 0.228-0.108 33 0.99829 34 0.99812 592 593 -0.090-0.452 0.025 0.038 0.524-0.168-0.369 0.266 0.154-0.006-0.253 1.000 0.950 0.168 0.330 -0.166-0.503 0.028-0.009 0.400-0.124-0.299 0.228 0.147-0.020-0.252 0.950 1.000 0.188 0.220 41 0.97212 42 0.97595 594 595 43 0.98859 -0.066-0.226-0.026-0.062 0.021-0.089-0.137 0.591 0.553 0.160 0.228 0.168 0.188 1.000-0.291 0.135-0.386 0.002 0.093 0.418-0.183-0.056 0.020-0.197 0.002-0.108 0.330 0.220-0.291 1.000 101 0.99603

<sup>10</sup>The EB decided to have the information on parameters and their correlations in an ascci file for internal usage.

41

| 597 | as = 0.1155 |         |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
|-----|-------------|---------|---------|---------|--------|--------|--------|-------|--------|---------|--------|-------|--------|---------|---------|-------|
| 598 |             |         |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
| 599 |             |         |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
| 600 | NO.         | GLOBAL  | 2       | 3       | 7      | 8      | 12     | 13    | 15     | 22      | 23     | 33    | 34     | 41      | 42      | 43    |
| 601 | 2 (         | 0.99909 | 1.000   | 0.653-0 | .891-0 | .656   | 0.060  | 0.002 | -0.031 | 0.027   | 0.012  | 0.023 | 0.033- | 0.145-0 | 9.204-0 | .027  |
| 602 | 3 (         | 99467   | 0.653   | 1.000-0 | .325-0 | .056   | 0.160  | 0.023 | -0.053 | -0.078- | 0.144  | 0.171 | 0.230- | 0.374-0 | 0.465-0 | .372  |
| 603 | 7 (         | 0.99943 | -0.891- | 0.325 1 | .000 0 | .920   | 0.109- | 0.063 | -0.112 | 0.034   | 0.029  | 0.014 | 0.004  | 0.028   | 0.032-0 | .025  |
| 604 | 8 (         | 0.99712 | -0.656- | 0.056 0 | .920 1 | .000   | 0.231- | 0.111 | -0.221 | 0.092   | 0.076  | 0.012 | -0.013 | 0.010-0 | 0.027-0 | .035  |
| 605 | 12 (        | 0.99499 | 0.060   | 0.160 0 | .109 0 | .231   | 1.000- | 0.117 | -0.734 | 0.285   | 0.154  | 0.379 | 0.134  | 0.442 ( | 0.340 0 | .171  |
| 606 | 13 (        | 0.98052 | 0.002   | 0.023-0 | .063-0 | . 111- | 0.117  | 1.000 | 0.713  | -0.154- | 0.239  | 0.418 | 0.433- | 0.118-0 | 0.092-0 | .132  |
| 607 | 15 (        | 0.99429 | -0.031- | 0.053-0 | .112-0 | .221-  | 0.734  | 0.713 | 1.000  | -0.203- | 0.171  | 0.161 | 0.344- | 0.373-0 | 0.296-0 | .148  |
| 608 | 22 0        | 0.99053 | 0.027-  | 0.078 0 | .034 0 | .092   | 0.285- | 0.154 | -0.203 | 1.000   | 0.910  | 0.404 | 0.331  | 0.265 ( | 9.220 0 | .625  |
| 609 | 23 (        | 0.98154 | 0.012-  | 0.144 0 | .029 0 | .076   | 0.154- | 0.239 | -0.171 | 0.910   | 1.000  | 0.169 | 0.115  | 0.233 ( | 0.196 0 | .530  |
| 610 | 33 (        | 0.99858 | 0.023   | 0.171 0 | .014 0 | .012   | 0.379  | 0.418 | 0.161  | 0.404   | 0.169  | 1.000 | 0.940- | 0.017-0 | 0.033 0 | . 192 |
| 611 | 34 (        | 0.99841 | 0.033   | 0.230 0 | .004-0 | .013   | 0.134  | 0.433 | 0.344  | 0.331   | 0.115  | 0.940 | 1.000- | 0.223-0 | 0.229 0 | .228  |
| 612 | 41 (        | 0.96869 | -0.145- | 0.374 0 | .028 0 | .010   | 0.442- | 0.118 | -0.373 | 0.265   | 0.233- | 0.017 | -0.223 | 1.000 ( | 0.953 0 | .287  |
| 613 | 42 (        | 0.97473 | -0.204- | 0.465 0 | .032-0 | .027   | 0.340- | 0.092 | -0.296 | 0.220   | 0.196- | 0.033 | -0.229 | 0.953   | 1.000 0 | .264  |
| 614 | 43 (        | 0.98749 | -0.027- | 0.372-0 | .025-0 | .035   | 0.171- | 0.132 | -0.148 | 0.625   | 0.530  | 0.192 | 0.228  | 0.287 ( | 9.264 1 | .000  |
| 615 |             |         |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
| 616 | as = 0.118  | 3       |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
| 617 |             | -       |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
| 618 |             |         |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
| 619 | NO.         | GLOBAL  | 2       | 3       | 7      | 8      | 12     | 13    | 15     | 22      | 23     | 33    | 34     | 41      | 42      | 43    |
| 620 | 2 (         | 0.99830 | 1.000   | 0.584-0 | .794-0 | .507   | 0.052- | 0.002 | -0.029 | -0.005- | 0.017  | 0.025 | 0.045- | 0.188-0 | 0.238-0 | .067  |
| 621 | 3 (         | 0.99467 | 0.584   | 1.000-0 | .086 0 | .184   | 0.146- | 0.004 | -0.071 | -0.148- | 0.192  | 0.160 | 0.233- | 0.432-0 | 9.517-0 | .453  |
| 622 | 7 (         | 0.99906 | -0.794- | 0.086 1 | .000 0 | .917   | 0.190- | 0.095 | -0.183 | 0.071   | 0.059  | 0.029 | 0.015- | 0.019-0 | 0.048-0 | .030  |
| 623 | 8 (         | 0.99645 | -0.507  | 0.184 0 | .917 1 | .000   | 0.308- | 0.142 | -0.288 | 0.115   | 0.094  | 0.033 | 0.008- | 0.065-0 | 0.131-0 | .053  |
| 624 | 12 (        | 0.99521 | 0.052   | 0.146 0 | .190 0 | . 308  | 1.000- | 0.176 | -0.777 | 0.302   | 0.184  | 0.381 | 0.166  | 0.429 0 | 0.321 0 | .216  |
| 625 | 13 (        | 0.98045 | -0.002- | 0.004-0 | .095-0 | .142-  | 0.176  | 1.000 | 0.712  | -0.219- | 0.278  | 0.360 | 0.389- | 0.112-0 | 0.079-0 | .150  |
| 626 | 15 (        | 0.99461 | -0.029- | 0.071-0 | .183-0 | .288-  | 0.777  | 0.712 | 1.000  | -0.258- | 0.208  | 0.089 | 0.264- | 0.354-0 | 9.270-0 | .188  |
| 627 | 22 (        | 0.99185 | -0.005- | 0.148 0 | .071 0 | .115   | 0.302- | 0.219 | -0.258 | 1.000   | 0.920  | 0.351 | 0.291  | 0.287 ( | 0.238 0 | .666  |
| 628 | 23 (        | 0.98399 | -0.017- | 0.192 0 | .059 0 | .094   | 0.184- | 0.278 | -0.208 | 0.920   | 1.000  | 0.159 | 0.107  | 0.248 ( | 0.208 0 | .556  |
| 629 | 33 (        | 0.99867 | 0.025   | 0.160 0 | .029 0 | .033   | 0.381  | 0.360 | 0.089  | 0.351   | 0.159  | 1.000 | 0.948  | 0.010-0 | 0.006 0 | .135  |
| 630 | 34 (        | 0.99849 | 0.045   | 0.233 0 | .015 0 | .008   | 0.166  | 0.389 | 0.264  | 0.291   | 0.107  | 0.948 | 1.000- | 0.178-0 | 0.186 0 | .157  |
| 631 | 41 (        | 0.96829 | -0.188- | 0.432-0 | .019-0 | .065   | 0.429- | 0.112 | -0.354 | 0.287   | 0.248  | 0.010 | -0.178 | 1.000 0 | 0.953 0 | .337  |
| 632 | 42 0        | 0.97500 | -0.238- | 0.517-0 | .048-0 | .131   | 0.321- | 0.079 | -0.270 | 0.238   | 0.208- | 0.006 | -0.186 | 0.953   | 1.000 0 | .312  |
| 633 | 43 (        | 0.99021 | -0.067- | 0.453-0 | .030-0 | .053   | 0.216- | 0.150 | -0.188 | 0.666   | 0.556  | 0.135 | 0.157  | 0.337 ( | 9.312 1 | .000  |
| 634 |             |         |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
| 635 |             |         |         |         |        |        |        |       |        |         |        |       |        |         |         |       |
|     |             |         |         |         |        |        |        |       |        |         |        |       |        |         |         |       |