

4 **Impact of jet production data on the next-to-next-to-leading order**  
5 **determination of HERAPDF2.0 parton distributions**

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8 **Abstract**

9 The HERAPDF2.0 ensemble of parton distribution functions (PDFs) was introduced in  
10 2015. Presented is the final stage, a next-to-next-to-leading order (NNLO) analysis of the  
11 HERA data on inclusive deep inelastic  $ep$  scattering together with jet data as published by  
12 H1 and ZEUS. A pQCD fit to the data with free  $\alpha_s(M_Z^2)$  and free PDFs was used to determine  
13  $\alpha_s(M_Z^2)$  with the result  $\alpha_s(M_Z^2) = 0.1156 \pm 0.0011$  (exp)  $^{+0.0001}_{-0.0002}$  (model + parameterisation)  $\pm$   
14  $0.0029$  (scale). The PDF sets of HERAPDF2.0Jets NNLO were determined with fits using  
15 fixed the fixed values of  $\alpha_s(M_Z^2) = 0.1155$  and  $\alpha_s(M_Z^2) = 0.118$ . The latter value was al-  
16 ready chosen for the published HERAPDF2.0 NNLO analysis based on inclusive data only.  
17 The different sets of PDFs are presented and compared. The similarity of the PDFs demon-  
18 strates the consistency of inclusive and jet-production cross-section data. Predictions based  
19 on HERAPDF2.0Jets NNLO agree very well with the jet-production data used in the fits.

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# 1 Introduction

Data from deep inelastic scattering (DIS) of electrons<sup>1</sup> on protons,  $ep$ , at centre-of-mass energies of up to  $\sqrt{s} \approx 320$  GeV at HERA have been central to the exploration of proton structure and quark–gluon dynamics as described by perturbative Quantum Chromo Dynamics (pQCD) [1].

The combination of H1 and ZEUS data on inclusive  $ep$  scattering and the subsequent pQCD analysis, introducing the ensemble of parton density functions (PDFs) known as HERAPDF2.0, were milestones for the exploitation [2] of the HERA data. The HERAPDF analyses are based on pQCD fits to the HERA DIS data in the DGLAP [3–7] formalism in the  $\overline{\text{MS}}$  scheme [8].

The sets of PDFs presented in this work complete the HERAPDF2.0 ensemble [2] of PDFs. They were determined with a next-to-next-leading order (NNLO) analysis of HERA inclusive and selected jet-production data as published separately by the H1 and ZEUS collaborations [9–14]. An analysis of jet data at NNLO was not possible at the time of the introduction of the HERAPDF2.0 ensemble. It became possible when predictions of jet cross-section at NNLO [15–23] for  $ep$  became available.

The strategy chosen for the analysis presented here follows that of the previous pQCD [2] Jets analysis, which was performed only at NLO. Jet cross section data are included in the pQCD analysis to constrain the gluon PDF which, however, is correlated with the value of the strong coupling,  $\alpha_s(M_Z^2)$ . Thus, the PDFs and the value of  $\alpha_s(M_Z^2)$  were fit simultaneously, and then the resulting  $\alpha_s(M_Z^2)$  was used to refit the PDFs with  $\alpha_s(M_Z^2)$  fixed to this value in order to determine the uncorrelated uncertainties at this value of  $\alpha_s(M_Z^2)$ . The PDFs were also determined for  $\alpha_s(M_Z^2) = 0.118$ , the PDG18 value [40].

The calculation of jet cross-sections at NNLO is based on jets starting from massless partons. The inclusive data, on the other hand, are treated within the RTOPT [25–27] Variable Flavour Number Scheme (VFNS), which requires values of the parameters for the charm and beauty masses,  $M_c$  and  $M_b$ , as input. These parameters were optimised via QCD fits using both the cross sections for charm and beauty production that were published as combined data by the H1 and ZEUS collaborations [28] and the inclusive data. However, the heavy-quark data were not explicitly included in the pQCD fits that included jet data because of the different treatment of the mass parameters in the two data sets.

The results presented here are based entirely on HERA data, i.e. inclusive and jet-production data. The HERA inclusive data represent a single, highly consistent data set. Furthermore, the jet data have been found to be very consistent with the inclusive data at NLO [2]; the analysis presented here also tests their consistency at NNLO. In addition, PDF fits to LHC data might be biased by any physics Beyond the Standard Model (BSM) whose effects have so far escaped detection, thereby reducing the sensitivity of searches for BSM due to biased background predictions. Thus, the HERAPDF2.0 ensemble of PDFs provides a benchmark to which PDFs including data from LHC colliders may be compared. This could reveal BSM effects or the need for an extension of the QCD analyses for some processes.

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<sup>1</sup>From here on, the word “electron” refers to both electrons and positrons, unless otherwise stated.

## 2 Data

Data taken by the H1 and ZEUS collaborations from 1993 to 2007 were combined to form a coherent set of inclusive HERA  $ep$  DIS cross sections [2], which was used as input to the determinations of all previous members of the HERAPDF2.0 ensemble. The HERAPDF2.0Jets analysis at NLO, in addition, used selected data [9–12,14] on inclusive jet and dijet production from H1 and ZEUS, which were again used for the present analysis at NNLO. In addition, new data [13], published by the H1 collaboration on jet production in lower  $Q^2$  events, where  $Q^2$  is the four-momentum-transfer squared, together with six new high- $Q^2$  points at low  $p_T$ , where  $p_T$  is the transverse energy of the jet and which were published by H1 [14], were added as input to the NNLO analysis. A summary on the data of jet production used is provided in Table 1. For all data sets, the jets were identified with the  $k_T$  algorithm with the  $R$  parameter set to one.

The new treatment of inclusive jet and dijet production at NNLO was, however, only applicable to a slightly reduced phase space compared to HERAPDF2.0Jets NLO. All data points with  $\mu = \sqrt{\langle p_T^2 \rangle + Q^2} \leq 10.0 \text{ GeV}$  had to be excluded in order to ensure the convergence of the perturbative series and to limit the NNLO scale uncertainties of the theoretical predictions to below 10 % compared to below 24 % at NLO. This requirement on  $\mu$  also ensured that  $\mu$  was larger than the b-quark mass, which is necessary because the jets are built from massless partons in the calculation of the NNLO predictions. In addition, for each  $Q^2$  bin, the six data points with the lowest  $\langle p_T \rangle$  were excluded from the ZEUS dijet data set because the available NNLO predictions for these points were judged to be incomplete when considering the kinematic cuts<sup>2</sup>. The resulting reduction of data points is detailed in Table 1. In addition, the trijet data [14] which were used as input to HERAPDF2.0Jets NLO were excluded as no NNLO treatment was available.

The inclusive charm data [29], which were included in the analysis at NLO [2], were not explicitly used in the PDF fits of the analysis presented here, since complete NNLO predictions were not available. Heavy quark data [28] were only used to optimise the mass parameter values for charm,  $M_c$ , and beauty,  $M_b$ , which are needed as input to the adopted RTOPT [27] NNLO approach to the fitting of the inclusive data.

## 3 QCD Analysis

The analysis presented here was performed along the same lines as all previous HERAPDF2.0 analyses [2]. Only cross sections for  $Q^2$  starting at  $Q_{min}^2 = 3.5 \text{ GeV}^2$  were used in the analysis. The  $\chi^2$  definition was taken from equation 32 of the previous paper [2]. The value of the starting scale for the evolution was taken as  $\mu_{f0}^2 = 1.9 \text{ GeV}^2$ . The parameterisation and choice of free parameters also followed the prescription for the HERAPDF2.0Jets NLO fit, see Section 3.1.

All fits were performed using the programme QCDNUM [30] within the xFitter (formerly HERAFitter) framework [31] and were cross-checked with an independent programme, which was already used for cross-checks in the HERAPDF2.0 analysis. The results obtained using the

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<sup>2</sup>Due to the kinematic cuts used in selecting the dijet data, the LO prediction for the cross sections is zero. Thus, the NNLO term is only the second non-zero term.

96 two programmes, as previously for all HERAPDF2.0 fits [2], were in excellent agreement, i.e.  
 97 well within fit uncertainties. All numbers presented here were obtained using xFitter.

98 The light-quark coefficient functions were calculated in QCDNUM. The heavy-quark coeffi-  
 99 cient functions were calculated in the general-mass variable-flavour-number scheme RTOPT [25],  
 100 with recent modifications [26,27].

101 The analysis presented here became possible due to the newly available treatment of jet  
 102 production at NNLO, using the zero-mass scheme. This is expected to be a reasonable ap-  
 103 proximation when the relevant QCD scales are significantly above the charm- and beauty-quark  
 104 masses. The jet data were included in the fits at full NNLO using predictions for the jet cross  
 105 sections calculated using NNLOJET [15–17], which was interfaced to the fast interpolation grid  
 106 codes, fastNLO [18–20] and APPLgrid [21,22] using the APPLfast framework [23], in order  
 107 to achieve the required speed for the convolutions needed in an iterative PDF fit. The NNLO  
 108 jet predictions were provided in the massless scheme and were corrected for hadronisation and  
 109  $Z^0$  exchange before they were used in the fits. A running electromagnetic  $\alpha$  as implemented in  
 110 the 2012 version of the programme EPRC [32] was used in the treatment of the jet cross sec-  
 111 tions. **The predictions included uncertainties, which were taken into account in all fits as 50 %**  
 112 **correlated and 50 % uncorrelated between processes and bins.**

113 The choice of scales for the jet data had to be adjusted for the NNLO analysis. At NLO, the  
 114 factorisation scale was chosen as for the inclusive data, i.e.  $\mu_f^2 = Q^2$ , while the renormalisation  
 115 scale was linked to the transverse momenta,  $p_T$ , of the jets as  $\mu_r^2 = (Q^2 + p_T^2)/2$ . For the NNLO  
 116 analysis,  $\mu_f^2 = \mu_r^2 = Q^2 + p_T^2$  was used. This resulted in an improved  $\chi^2$  for the fits. Scale  
 117 variations were also considered and are discussed in Sections 4.1 and 4.2. **In general, scale**  
 118 **variations are used to estimate the uncertainties due to missing higher order contributions.**

### 119 3.1 Choice of parameterisation and model parameters

120 The PDFs were parameterised as a function of  $x$  at the input scale by the generic form

$$121 \quad xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2). \quad (1)$$

122 The PDF of the gluon was an exception, for which an additional term of the form  $A'_g x^{B'_g} (1-x)^{C'_g}$   
 123 was subtracted<sup>3</sup>. This choice of parameterisation follows the original concept of HERAPDF2.0,  
 124 for which all details were previously published [2]. The parameterisation is an effective way to  
 125 store the information derived from many data points in a limited set of numbers.

126 The parameterised PDFs are the gluon distribution  $xg$ , the valence-quark distributions  $xu_v$ ,  
 127  $xd_v$ , and the  $u$ -type and  $d$ -type anti-quark distributions  $x\bar{U}$ ,  $x\bar{D}$ , where  $x\bar{U} = x\bar{u}$  and  $x\bar{D} =$   
 128  $x\bar{d} + x\bar{s}$  at the chosen starting scale. The parameterisation for the central fit was determined  
 129 by initially fixing the  $D$ ,  $E$  and  $A'_g$  parameters to zero. This resulted in 10 free parameters.  
 130 The extra parameters were introduced one at a time until the  $\chi^2$  of the fit could not be further  
 131 improved [2,33]. This is also called the  $\chi^2$  saturation method. This resulted in a 14 parameter fit  
 132 which satisfied the criteria that all PDFs and all predicted cross sections were positive throughout

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<sup>3</sup>The parameter  $C'_g = 25$  was fixed since the fit is not sensitive to this value, provided it is high enough ( $C'_g > 15$ ) ensuring that the term does not contribute at large  $x$ .

133 the kinematic region probed by the data entering the fit. The suitability of the parameterisation  
 134 was, thus, also verified for the selection of jet data.

135 The final parameterisation was

$$136 \quad xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g}, \quad (2)$$

$$137 \quad xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2), \quad (3)$$

$$138 \quad xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}, \quad (4)$$

$$139 \quad x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x), \quad (5)$$

$$140 \quad x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}. \quad (6)$$

141 The normalisation parameters,  $A_g, A_{u_v}, A_{d_v}$ , were constrained by the quark-number and momen-  
 142 tum sum rules. The  $B$  parameters,  $B_{\bar{U}}$  and  $B_{\bar{D}}$ , were set equal,  $B_{\bar{U}} = B_{\bar{D}}$ , such that there was a  
 143 single  $B$  parameter for the sea distributions.

144 The strange-quark distribution was expressed as an  $x$ -independent fraction,  $f_s$ , of the  $d$ -type  
 145 sea,  $x\bar{s} = f_s x\bar{D}$  at  $Q_0^2$ . The central value  $f_s = 0.4$  was chosen to be a compromise between the  
 146 determination of a suppressed strange sea from neutrino-induced di-muon production [34,35]  
 147 and the determination of an unsuppressed strange sea from the ATLAS collaboration [36]. The  
 148 further constraint  $A_{\bar{U}} = A_{\bar{D}}(1-f_s)$ , together with the requirement  $B_{\bar{U}} = B_{\bar{D}}$ , ensured that  $x\bar{u} \rightarrow x\bar{d}$   
 149 as  $x \rightarrow 0$ .

## 150 3.2 Model and parameterisation uncertainties

151 Model and parameterisation uncertainties on the PDFs determined by a central fit were evaluated  
 152 with fits with modified input assumptions. The central values of the model parameters and their  
 153 variations are summarised in Table 2. The uncertainties on the PDFs obtained from variations  
 154 of  $M_c, M_b, f_s, Q_{min}^2$  were added in quadrature, separately for positive and negative uncertainties,  
 155 and represent the model uncertainty.

156 The uncertainty obtained from the variation of  $\mu_{f_0}^2$  was added to the parameterisation uncer-  
 157 tainty. A variation of the number of terms in the polynomial  $(1 + Dx + Ex^2)$  was considered  
 158 for each of the parton distributions listed in Eqs. 2–6. For this, all 15-parameter fits which have  
 159 one more non-zero free  $D$  or  $E$  parameter were considered as possible variants and the resulting  
 160 PDFs compared to the PDF from the 14-parameter central fit. The only significant change in the  
 161 PDFs was observed for the addition of a  $D_{u_v}$  parameter. The uncertainties on the central fits from  
 162 the parameterisation variations were stored as an envelope representing the maximal deviation  
 163 at each  $x$  value.

164 The total uncertainties on the PDFs were obtained by adding experimental, i.e. fit, model  
 165 and parameterisation uncertainties in quadrature.

## 166 3.3 Optimisation of $M_c$ and $M_b$

167 The RTOPT scheme used to calculate predictions for the inclusive data requires the charm- and  
 168 beauty-mass parameters,  $M_c$  and  $M_b$ , as input. The optimal values of these parameter were

reevaluated using the standard procedure [2,33], applied to the new combined HERA data on heavy quarks [28] together with the combined inclusive data [2]. The procedure comprises multiple pQCD fits with varying choices of the  $M_c$  and  $M_b$  parameters. The parameter values resulting in the lowest  $\chi^2$  values of the fit were chosen. This was done both at NNLO and NLO to provide consistent sets of  $M_c$  and  $M_b$  for future pQCD analyses. The one standard-deviation uncertainties of the mass parameters were determined by fitting the  $\chi^2$  values with a quadratic function and finding the mass-parameter values corresponding to  $\Delta\chi^2 = 1$ .

At NNLO (NLO), the fits for the optimisation were performed with fixed values of  $\alpha_s = 0.1155$ <sup>4</sup> ( $\alpha_s = 0.118$ )<sup>5</sup>. As a first iteration at NNLO (NLO),  $M_c$  was varied with fixed  $M_b = 4.5$  GeV (4.5 GeV) and  $M_b$  was varied with fixed  $M_c = 1.43$  GeV (1.47 GeV), i.e. the mass-parameter values used for HERAPDF2.0 NNLO (NLO) were used as fix-points. In every iteration, the mass-parameter values as obtained within the previous iteration were used as new fixed-points. The iterations were ended once values stable within 0.1 % for  $M_c$  and  $M_b$  were observed. The final  $\chi^2$  scans at NNLO are shown in Figs. 1 a) and c) and at NLO in Figs. 1 b) and d). The resulting values at NNLO are  $M_c = 1.41 \pm 0.04$  GeV and  $M_b = 4.20 \pm 0.10$  GeV, quite close to the values determined for HERAPDF2.0 NNLO, with slightly reduced uncertainties. The values at NLO are  $M_c = 1.46 \pm 0.04$  GeV and  $M_b = 4.30 \pm 0.10$  GeV. The minima in  $\chi^2$  for  $M_b$  demonstrate the power of the method. The minimum in  $\chi^2$  for the parameter  $M_c$  at NNLO is observed close to the technical limit of the fitting procedure.

The part of the model uncertainty concerning the heavy-flavour mass parameters would nominally have involved varying the value of  $M_c$  to the minimum and maximum of its one standard-deviation uncertainty. However, for  $M_c$ , the downward variation created a conflict with  $\mu_{f0}$ , which has to be less than  $M_c$  in the RTOPT scheme, such that charm can be generated perturbatively. Thus, only an upward variation of  $M_c$  was considered and the resulting uncertainty on the PDFs was symmetrised. In addition, the condition  $\mu_{f0} < M_c$  created a conflict with the variation of  $\mu_{f0}^2$ . The normal procedure would have included an upward variation of  $\mu_{f0}^2$  to 2.2 GeV<sup>2</sup> but  $\mu_{f0}$  would have become larger than the upper end of the uncertainty interval of  $M_c$ <sup>6</sup>. Thus,  $\mu_{f0}^2$  was only varied downwards to 1.6 GeV<sup>2</sup>, and the resulting uncertainty on the PDFs was again symmetrised. The suitability of the chosen central parameterisation was re-verified for the new settings for  $M_c$  and  $M_b$  using the  $\chi^2$  saturation method as described in Section 3.1.

Since predictions at NNLO for the jet data were only available in the zero-mass scheme, and results for the treatment of the inclusive data in different VFNS and FFNS schemes were consistent [2], no other heavy-flavour schemes were investigated.

### 3.4 Hadronisation uncertainties

For the jet-data analysis, it was also necessary to consider hadronisation and the effect of the uncertainties on hadronisation corrections. The uncertainties on the hadronisation corrections, which were supplied in the original publications, were reviewed for this analysis. The H1 uncertainties were used as published, while for technical reasons, those for the ZEUS data were

<sup>4</sup>A cross-check was performed with the fixed value of  $\alpha_s = 0.118$  and no significant difference in the resulting  $M_c$  and  $M_b$  values were observed.

<sup>5</sup>The value 0.118 was used in the pQCD analysis of heavy quark data [28].

<sup>6</sup>In previous HERAPDF analyses, the uncertainty on  $M_c$  was large enough to accommodate the upward  $\mu_{f0}^2$  variation.

207 increased to the maximum value quoted in the publications, 2 %. It was verified that this change  
 208 made no significant difference to any of the results presented here.

209 In the HERAPDF2.0Jets NLO analysis, hadronisation uncertainties were applied using the  
 210 offset method, i.e. performing separate fits with the hadronisation corrections set to their maxi-  
 211 mal and minimal values. This resulted in a hadronisation uncertainty on  $\alpha_s(M_Z^2)$  of  $\pm 0.0012$  [2].

212 The current procedure is different from that used previously. The uncertainties on the hadro-  
 213 nisation corrections were included as input to the HERAPDF2.0 Jets NNLO fits. They were  
 214 treated as **systematic uncertainties, 50 % correlated and 50 % uncorrelated between bins and**  
 215 **data sets**. Thus, their contribution became part of the overall experimental, i.e. fit, uncertainties.  
 216 For fits with fixed  $\alpha_s(M_Z^2)$ , their contribution was negligible. For fits with free  $\alpha_s(M_Z^2)$ , their con-  
 217 tribution to the experimental uncertainty on  $\alpha_s(M_Z^2)$  was  $\pm 0.0006$ . This represents a significant  
 218 reduction of the influence of the hadronisation uncertainties compared to previous analyses.

## 219 4 HERAPDF2.0Jets NNLO – results

### 220 4.1 Simultaneous determination of $\alpha_s(M_Z^2)$ and PDFs

221 Jet-production data are essential for the determination of the strong coupling constant,  $\alpha_s(M_Z^2)$ .  
 222 In pQCD fits to inclusive DIS data alone, the gluon PDF is determined via the DGLAP equations,  
 223 using the observed scaling violations. This results in a strong correlation between the shape of  
 224 the gluon distribution and the value of  $\alpha_s(M_Z^2)$ . Data on jet and dijet production cross-sections  
 225 provide an independent constraint on the gluon distribution and are also directly sensitive to  
 226  $\alpha_s(M_Z^2)$ . Thus, such data are essential for an accurate simultaneous determination of  $\alpha_s(M_Z^2)$  and  
 227 the gluon distribution.

228 When determining  $\alpha_s(M_Z^2)$ , it is necessary to consider so-called “scale uncertainties”. **They**  
 229 **serve as an approximate proxy for the uncertainty due to the unknown influence of higher orders**  
 230 **in the perturbation expansion**. This uncertainty was evaluated by varying the renormalisation  
 231 and factorisation scales by a factor of two, both separately and simultaneously<sup>7</sup>, and selecting  
 232 the maximal positive and negative deviations of the result as the “de facto” scale uncertainty.  
 233 These were observed for  $(2.0\mu_r, 1.0\mu_f)$  and  $(0.5\mu_r, 1.0\mu_f)$ , respectively.

234 The HERAPDF2.0Jets NNLO fit with free  $\alpha_s(M_Z^2)$  results in

$$235 \alpha_s(M_Z^2) = 0.1156 \pm 0.0011 \text{ (exp)} \begin{matrix} +0.0001 \\ -0.0002 \end{matrix} \text{ (model + parameterisation)} \pm 0.0029 \text{ (scale)} , \quad (7)$$

236 where “exp” denotes the experimental uncertainty, which was taken as the fit uncertainty, in-  
 237 cluding the contribution from hadronisation uncertainties. The value of  $\alpha_s(M_Z^2)$  and the size of  
 238 the experimental uncertainty were confirmed by the the result of a so-called  $\chi^2$  scan in  $\alpha_s(M_Z^2)$ ,  
 239 which is shown in Fig. 2 a). Numerous fits with varying  $\alpha_s(M_Z^2)$  were performed and the clear  
 240 minimum observed in  $\chi^2$  coincides with the value of  $\alpha_s(M_Z^2)$  determined with the fit. The width  
 241 of the minimum in  $\chi^2$  confirms the fit uncertainty. The combined model and parameterisation

<sup>7</sup>This procedure is often called 9-point variation, where the nine variations are  $(0.5\mu_r, 0.5\mu_f)$ ,  $(0.5\mu_r, 1.0\mu_f)$ ,  
 $(0.5\mu_r, 2.0\mu_f)$ ,  $(1.0\mu_r, 0.5\mu_f)$ ,  $(1.0\mu_r, 1.0\mu_f)$ ,  $(1.0\mu_r, 2.0\mu_f)$ ,  $(2.0\mu_r, 0.5\mu_f)$ ,  $(2.0\mu_r, 1.0\mu_f)$ ,  $(2.0\mu_r, 2.0\mu_f)$ .

242 uncertainty shown in Fig. 2 a) was determined by performing similar scans, for which the values  
 243 of the model parameters and the parameterisation were varied as described in Section 3.1.

244 Figure 2 a) also shows the scale uncertainty, which dominates the total uncertainty. The  
 245 scale uncertainty as listed in Eq. 7 was evaluated under the assumption of 100 % correlated un-  
 246 certainties between bins and data sets. The previously published result at NLO [2] had scale  
 247 uncertainties calculated under the assumption of 50 % correlated and 50 % uncorrelated uncer-  
 248 tainties between bins and data sets. A strong motivation to determine  $\alpha_s(M_Z^2)$  at NNLO was the  
 249 hope of a substantial reduction in the scale uncertainty. Therefore, the analysis was repeated  
 250 for these assumptions in order to be able to compare the NNLO to the NLO scale uncertainties.  
 251 The reevaluated NNLO scale uncertainty of ( $\pm 0.0022$ ) is indeed significantly lower than the  
 252 ( $+0.0037, -0.0030$ ) previously observed in the HERAPDF2.0Jets NLO analysis.

253 The HERAPDF2.0Jets NNLO fit with free  $\alpha_s(M_Z^2)$  was based on 1363 data points and had  
 254 a  $\chi^2/\text{d.o.f.} = 1614/1348 = 1.197$ . This can be compared to the  $\chi^2/\text{d.o.f.} = 1363/1131 = 1.205$   
 255 for HERAPDF2.0 NNLO based on inclusive data only [2]. The similarity of the  $\chi^2/\text{d.o.f.}$  values  
 256 indicates that the data on jet production do not introduce any additional tension to the fit. The  
 257 jet data are fully consistent with the inclusive data.

258 The question of whether data with relatively low  $Q^2$  bias the determination of  $\alpha_s(M_Z^2)$  arose  
 259 within the context of the HERAPDF2.0 analysis [2]. Figure 2 b) shows the result of  $\alpha_s(M_Z^2)$   
 260 scans with  $Q_{min}^2$  for the inclusive data set to  $3.5 \text{ GeV}^2$ ,  $10 \text{ GeV}^2$  and  $20 \text{ GeV}^2$ . Clear minima are  
 261 visible which coincide within uncertainties. Figure 2 c) shows the result of similar scans with  
 262 only the inclusive data used as input [2]. The inclusive data alone cannot sufficiently constrain  
 263  $\alpha_s(M_Z^2)$ .

264 To verify that the use of the  $A'_g$  term in the gluon parameterisation does not bias the determi-  
 265 nation of  $\alpha_s(M_Z^2)$ , cross-checks were made with two modified gluon parameterisations. These are  
 266  $A'_g = 0$  and  $xg(x) = A_g x^{B_g} (1-x)^{C_g}$  as well as the alternative gluon parameterisation, AG [2], for  
 267 which  $A'_g = 0$  and  $xg(x) = A_g x^{B_g} (1-x)^{C_g} (1 + D_g x)$ . A value of  $\alpha_s(M_Z^2) = 0.1151 \pm 0.0010(\text{exp})$   
 268 was obtained for both modifications of the parameterisation, which is in agreement with the  
 269 result for the standard parameterisation. The value of  $D_g$  in the AG parameterisation was con-  
 270 sistent with zero. These results demonstrate that the present  $\alpha_s(M_Z^2)$  determination is not very  
 271 sensitive to the details of the gluon parameterisation.

272 Other determinations of  $\alpha_s(M_Z^2)$  at NNLO using jet data as published by H1 [37] and NNLO-  
 273 JET authors and their collaborators [38] used fixed PDFs for their fits to determine  $\alpha_s(M_Z^2)$ .  
 274 While this is a common procedure, it could bias the resulting value of  $\alpha_s(M_Z^2)$  [24]. Thus, the  
 275 values of  $\alpha_s(M_Z^2)$  should not be directly compared. However, both analyses were performed with  
 276 a cut on  $\mu$  of  $\mu > 2M_b$ , which is quite similar to the  $\mu > 10.0 \text{ GeV}$  cut used for this analysis.  
 277 Thus, the scale uncertainties can be compared. The H1 result is based on H1 data only and  
 278 the quoted scale uncertainty is  $\pm 0.0039$ . The scale uncertainty published by NNLOjet using  
 279 H1 and ZEUS data  $\pm 0.0033$ . This can be compared to the  $\pm 0.0029$  obtained for the analysis  
 280 presented here. The somewhat reduced scale uncertainty for the present analysis could be due  
 281 to the correlation between PDFs and  $\alpha_s(M_Z^2)$  such that the evolution of the fixed PDFs increase  
 282 the dependence of  $\alpha_s(M_Z^2)$  on the chosen scales.

283 The H1 collaboration provided one simultaneous fit of  $\alpha_s(M_Z^2)$  and PDFs using a ZMVFN  
 284 scheme. It was based on H1 inclusive and jet data with  $Q_{min}^2 = 10 \text{ GeV}^2$ . For comparison, the  
 285 analysis presented here was modified by also setting  $Q_{min}^2 = 10 \text{ GeV}^2$ . The value of  $\alpha_s(M_Z^2)$

published by H1 is  $\alpha_s(M_Z^2) = 0.1147 \pm 0.0011(\text{exp}) \pm 0.0002\text{model} \pm 0.0003(\text{parameterisation}) \pm 0.0023(\text{scale})$  while the current modified analysis resulted in  $\alpha_s(M_Z^2) = 0.1156 \pm 0.0011(\text{exp}) \pm 0.0002(\text{model/parameterisation}) \pm 0.0021(\text{scale})$ . These values agree within uncertainties. Overall, the various determinations of  $\alpha_s(M_Z^2)$  provide a very consistent picture up to NNLO.

## 4.2 The PDFs of HERAPDF2.0Jets NNLO obtained for fixed $\alpha_s(M_Z^2)$

The value of  $\alpha_s(M_Z^2) = 0.1155$  was used for the determination of the PDFs in the HERAPDF2.0Jets NNLO analysis. The value listed in PDG12 [39], 0.118, which was also the value determined in the HERAPDF2.0Jets NLO analysis, was used for the original HERAPDF2.0 analyses at NNLO based on inclusive data only. Therefore, the PDFs of HERAPDF2.0Jets NNLO are shown in Fig. 3 a) and b) for both, fixed  $\alpha_s(M_Z^2) = 0.1155$  and fixed  $\alpha_s(M_Z^2) = 0.118$ , respectively, together with their uncertainties, at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties shown are the experimental, i.e. fit, uncertainties as well as the model and parameterisation uncertainties as defined in Section 3.2. The parameterisation uncertainty dominates the uncertainties and is itself dominated by the introduction of the parameter  $D_{u_v}$  as a variation. Details on the two sets of PDFs as released are listed in Appendix A.

As the PDFs were derived with fixed  $\alpha_s(M_Z^2)$  values, uncertainties on the PDFs from varying the scales in the fit procedure were not considered, because, in this case, a quantification of the influence of higher orders by varying the renormalisation and factorisation scales in the fit becomes questionable. Any variation of the renormalisation scale effectively amounts, in its numerical effect, to a modification of the value of  $\alpha_s(M_Z^2)$ , since the compensation with the explicit scale-dependent terms in the NLO and NNLO coefficients is incomplete. If a fit is performed with a fixed value of  $\alpha_s(M_Z^2)$ , it might thus not reach a local minimum. However, such a local minimum is required to estimate the unknown amount of influence of higher orders by varying the scales. Nevertheless, a cross-check with scale variations as described in Section 4.1 was made. The impact on the resulting PDFs was found to be negligible compared to the other uncertainties presented in Fig. 3.

A comparison between the PDFs obtained for  $\alpha_s(M_Z^2) = 0.1155$  and  $\alpha_s(M_Z^2) = 0.118$  is provided in Figs. 4 and 5 for the scales  $\mu_f = 10 \text{ GeV}^2$  and  $\mu_f = M_Z^2$ , respectively. Here, only total uncertainties are shown. At the lower scale, a significant difference is observed between the gluon distributions; the distribution for  $\alpha_s(M_Z^2) = 0.1155$  is above the distribution for  $\alpha_s(M_Z^2) = 0.118$  for  $x$  less than  $\approx 10^{-2}$ . This correlation between the value of  $\alpha_s(M_Z^2)$  and the shape of the gluon PDF is as expected from QCD evolution. At the scale of  $M_Z^2$ , the differences become negligible in the visible range of  $x$  due to QCD evolution.

A comparison of the PDFs obtained for  $\alpha_s(M_Z^2) = 0.118$  by HERAPDF2.0Jets NNLO to the PDFs of HERAPDF2.0 NNLO, based on inclusive data only, is provided in Fig. 6. These two sets of PDFs do not show any significant difference in the central values. However, there is a significant reduction of the uncertainties on the gluon PDFs as shown in Fig. 7 at the scale of  $\mu_f = 10 \text{ GeV}^2$  and in Fig. 8 at the scale of  $\mu_f = M_Z^2$ . The reductions in the uncertainties for HERAPDF2.0Jets NNLO for  $\alpha_s(M_Z^2) = 0.1155$  compared to  $\alpha_s(M_Z^2) = 0.118$  are shown in Fig. 9 and Fig. 10. At high  $x$  and  $\mu_f = M_Z^2$ , the parameterisation uncertainties become important as can be seen by comparing Fig. 10 b) and 10 c).

The reduction in model and parameterisation uncertainty for  $x < 10^{-3}$ , compared to HERAPDF2.0 NNLO, is mostly due to the improved procedure to estimate this uncertainty. The ranges,

329 in which  $M_c$  and  $M_b$  were varied, were reduced but this had only little effect. The major effect  
 330 came from symmetrising the results of the variations of  $\mu_{f0}^2$  and  $M_c^2$  as discussed in Section 3.3.  
 331 This removed a double counting of sources of uncertainty that had been present in the original  
 332 HERAPdf2.0 procedure. On the other hand, the reduction of experimental as well as model  
 333 and parameterisation uncertainties for  $x > 10^{-3}$ , is due to the influence of the jet data. This  
 334 is also demonstrated in Fig. 11, which shows ratios of the uncertainties with respect to the total  
 335 uncertainties of HERAPDF2.0 NNLO based on inclusive data only. Shown are the contributions  
 336 of the experimental, the experimental plus model, and the experimental plus parameterisation  
 337 uncertainties, with respect to the total uncertainties of HERAPDF2.0 NNLO, and the respective  
 338 reductions for HERAPDF2.0Jets NNLO. Selected other ratio plots are provided in Appendix B.

### 339 4.3 Comparisons of HERAPDF2.0Jets NNLO predictions to jet data

340 Comparisons of the predictions based on HERAPDF2.0Jets NNLO with fixed  $\alpha_s(M_Z^2) = 0.1155$   
 341 to the data on jet production used as input to the fit are shown in Figs. 12 to 19. Each figure  
 342 presents in a) a direct comparison of the cross sections and in b) the respective ratios.

343 The uncertainties on the NNLO predictions as calculated by NNLOJET were taken into  
 344 account in all HERAPDF2.0Jets NNLO fits. The predictions based on the HERAPDF2.0Jets  
 345 NNLO PDFs were computed using the assumption of massless jets, i.e. the transverse energy,  
 346  $E_T$ , and the transverse momentum of a jet,  $p_T$ , were assumed to be equivalent. For the inclusive  
 347 jet analyses, each jet  $p_T$  was entered separately. For dijet analyses, the average of the transverse  
 348 momenta,  $\langle p_T \rangle$  was used. In these cases,  $\langle p_T \rangle$  was also used to set the factorisation and  
 349 renormalisation scales to  $\mu_f^2 = \mu_r^2 = Q^2 + \langle p_T \rangle^2$  for calculating predictions. Scale uncertainties  
 350 were not considered [16] for the comparisons to data. The predictions based on the PDFs of  
 351 HERAPDF2.0Jets NNLO clearly fit the data on jet production used as input very well, showing  
 352 that the inclusive data and jet production data both used as input to the NNLO QCD fit are fully  
 353 consistent.

## 354 5 Summary

355 The HERA data set on inclusive  $ep$  scattering as published by the H1 and ZEUS collabora-  
 356 tions [2], together with selected data on jet production, published separately by the two collab-  
 357 orations, were used as input to a pQCD analysis at NNLO.

358 An analysis was performed where  $\alpha_s(M_Z^2)$  and the PDFs were fitted simultaneously. This  
 359 resulted in a value of  $\alpha_s(M_Z^2) = 0.1156 \pm 0.0011$  (exp) $_{-0.0002}^{+0.0001}$  (model/ /parameterisation)  $\pm$   
 360 0.0029 (scale). This result on  $\alpha_s(M_Z^2)$  is compatible with the world average [40] and it is compet-  
 361 itive in comparison with other determinations at NNLO. The scale uncertainties were calculated  
 362 under the assumption of fully correlated uncertainties between bins and data sets. They would  
 363 decrease to  $\pm 0.0022$  under the assumption of 50 % correlated and 50 % uncorrelated uncer-  
 364 tainties which is the value that can be directly compared to the previously published [2] scale  
 365 uncertainties of (+0.0037,-0.0030) observed in the HERAPDF2.0Jets NLO analysis.

366 Two sets of PDFs were determined for HERAPDF2.0Jets NNLO for fixed  $\alpha_s(M_Z^2) = 0.1155$   
 367 and  $\alpha_s(M_Z^2) = 0.118$ . They are available to the community. Comparisons between the PDFs of

368 HERAPDF2.0Jets NNLO obtained for the two values of  $\alpha_s(M_Z^2)$  were shown, as well as com-  
369 parisons to HERAPDF2.0 NNLO, for which jet data were not used as input to the fit. All these  
370 PDFs are very similar, showing the consistency of the inclusive and the jet production data.  
371 On balance, the inclusion of the jet data had two consequences: i) a lower value of  $\alpha_s(M_Z^2)$  is  
372 favoured; ii) the uncertainty on the gluon PDF was reduced. Predictions based on the PDFs of  
373 HERAPDF2.0Jets NNLO were compared to the jet production data used as input. The predic-  
374 tions describe the data very well.

375 The PDFs of HERAPDF2.0Jets NNLO complete the HERAPDF2.0 ensemble of parton dis-  
376 tribution functions. This ensemble of PDFs, extracted from HERA data alone, presents a con-  
377 sistent picture in the framework of pQCD. It is one of the legacies of HERA.

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## References

- 388
- 389 [1] A. Cooper-Sarkar and R. Devenish, *Deep inelastic Scattering*, Oxford Univ. Press (2011),  
390 ISBN 978-0-19-960225-4.
- 391 [2] H. Abramowicz *et al.*, [H1 and ZEUS Collaborations], *Eur. Phys. J. C* **75**, 580 (2015),  
392 [arXiv:1506.06042].
- 393 [3] V. N. Gribov and L. N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972).
- 394 [4] V. N. Gribov and L. N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 675 (1972).
- 395 [5] L. N. Lipatov, *Sov. J. Nucl. Phys.* **20**, 94 (1975).
- 396 [6] Y. L. Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977).
- 397 [7] G. Altarelli and G. Parisi, *Nucl. Phys. B* **126**, 298 (1977).
- 398 [8] B. Fanchiotti, S. Kniehl and A. Sirlin, *Phys. Rev. D* **48**, 307 (1993), [hep-ph/9803393].
- 399 [9] A. Aktas *et al.* [H1 Collaboration], *Phys. Lett. B* **653**, 134 (2007), [arXiv:0706.3722].
- 400 [10] F. Aaron *et al.* [H1 Collaboration], *Eur. Phys. J. C* **67**, 1 (2010), [arXiv:0911.5678].
- 401 [11] S. Chekanov *et al.* [ZEUS Collaboration], *Phys. Lett. B* **547**, 164 (2002), [hep-  
402 ex/0208037].
- 403 [12] H. Abramowicz *et al.* [ZEUS Collaboration], *Eur. Phys. J. C* **70**, 965 (2010),  
404 [arXiv:1010.6167].
- 405 [13] V. Andreev *et al.* [H1 Collaboration], *Eur. Phys. J. C* **77**, 215 (2017), [Erratum: *Eur. Phys.*  
406 *J. C* **81**, 739 (2021)], [1611.03421].
- 407 [14] V. Andreev *et al.* [H1 Collaboration], *Eur. Phys. J. C* **65**, 2 (2015), [arXiv:1406.4709].
- 408 [15] J. Currie, T. Gehrmann, and J. Niehues, *Phys. Rev. Lett.* **117**, 042001 (2016),  
409 [arXiv:1606.03991].
- 410 [16] J. Currie, T. Gehrmann, A. Huss, and J. Niehues, *JHEP* **07**, 018 (2017), [Erratum: *JHEP*  
411 **12**, 042 (2020)], [1703.05977].
- 412 [17] T. Gehrmann *et al.*, in *The Proceedings of the 13th International Symposium on Radiative*  
413 *Corrections (RADCOR2017), St. Gilgen, Austria* (2017), vol. 1707, [arXiv:1801.06415].
- 414 [18] T. Kluge, K. Rabbertz, and M. Wobisch (2006), [hep-ph/0609285].
- 415 [19] Britzger.D *et al.*, in *20th International Workshop on Deep-Inelastic Scattering and Related*  
416 *Subjects (DIS 2012): Bonn, Germany* (2012), p. 217, [arXiv:1208.3641].
- 417 [20] D. Britzger *et al.*, at *DIS 2014* (2014), URL [http://indico.cern.ch/event/258017/  
418 session/1/contribution/202](http://indico.cern.ch/event/258017/session/1/contribution/202).
- 419 [21] T. Carli, G. Salam, and F. Siegert (2005), [arXiv:0510324].

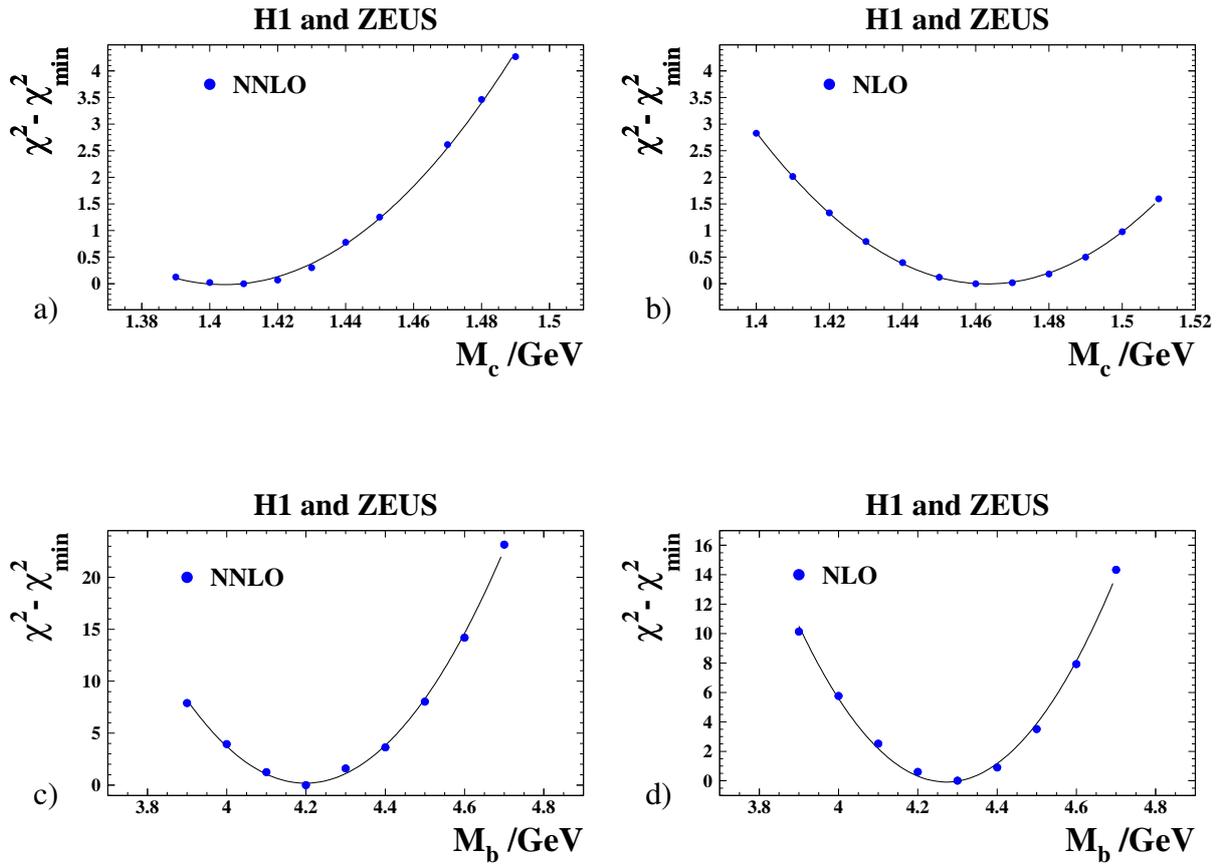
- 420 [22] T. Carli *et al.*, Eur. Phys. J. C **66**, 503 (2010), [arXiv:0911.2985].
- 421 [23] V. Andreev *et al.* [H1 Collaboration], Eur. Phys. J. C **77**, 791 (2017).
- 422 [24] S. Forte and Z. Kassabov, Eur. Phys. J. C **80**, 182 (2020), [arXiv:2001.04986].
- 423 [25] R. S. Thorne and R. G. Roberts, Phys. Rev. D **57**, 6871 (1998), [hep-ph/9709442].
- 424 [26] R. S. Thorne, Phys. Rev. D **73**, 054019 (2006), [hep-ph/0601245].
- 425 [27] R. S. Thorne, Phys. Rev. D **86**, 074017 (2012), [arXiv:1201.6180].
- 426 [28] H. Abramowicz *et al.*, [H1 and ZEUS Collaborations], Eur. Phys. J. C **78**, 473 (2018),  
427 [arXiv:1804.01019].
- 428 [29] F. D. Aaron *et al.*, [H1 and ZEUS Collaborations], Eur. Phys. J. C **73**, 2311 (2013),  
429 [arXiv:1211.1182].
- 430 [30] M. Botje, Comp. Phys. Comm. **182**, 490 (2011), [arXiv:1005.1481].
- 431 [31] S. Alekhin *et al.* (2014), [arXiv:1410.4412].
- 432 [32] H. Spiesberger, in *Proc. of Future Physics at HERA*, edited by G. Ingelman, A. De Roeck  
433 and R. Klanner (1995), p. 227.
- 434 [33] F. Aaron *et al.*, [H1 and ZEUS Collaborations], JHEP **1001**, 109 (2010),  
435 [arXiv:0911.0884].
- 436 [34] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009),  
437 [arXiv:0901.0002].
- 438 [35] P. M. Nadolsky *et al.*, Phys. Rev. D **78**, 013004 (2008), [arXiv:0802.0007].
- 439 [36] M. Aaboud *et al.*, Eur. Phys. J. C **77**, 367 (2017), [arXiv:1612.03016].
- 440 [37] V. Andreev *et al.* [H1 Collaboration], Eur. Phys. J. C **77**, 791 (2017), [Erratum: Eur. Phys.  
441 J. C **81**, 738 (2021)], [1709.07251].
- 442 [38] D. Britzger *et al.* [NNLOJet and Applfast Collaboration], Eur. Phys. J. C **79**, 845 (2019),  
443 [arXiv:1906.05303].
- 444 [39] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
- 445 [40] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
- 446 [41] The combined data together with the full correlation information and the grids for HERA-  
447 PDF2.0 are provided at URL <http://www.desy.de/h1zeus/herapdf20/>.

Data Set	taken		$Q^2$ [GeV <sup>2</sup> ] range		$\mathcal{L}$ pb <sup>-1</sup>	$e^+/e^-$	$\sqrt{s}$ GeV	norma- lised	all points	used points	Ref.
	from	to	from	to							
H1 HERA I normalised jets	1999	2000	150	15000	65.4	$e^+p$	319	yes	24	24	[9]
H1 HERA I jets at low $Q^2$	1999	2000	5	100	43.5	$e^+p$	319	no	28	20	[10]
H1 normalised inclusive jets at high $Q^2$	2003	2007	150	15000	351	$e^+p/e^-p$	319	yes	30	30	[13,14]
H1 normalised dijets at high $Q^2$	2003	2007	150	15000	351	$e^+p/e^-p$	319	yes	24	24	[14]
H1 normalised inclusive jets at low $Q^2$	2005	2007	5.5	80	290	$e^+p/e^-p$	319	yes	48	37	[13]
H1 normalised dijets at low $Q^2$	2005	2007	5.5	80	290	$e^+p/e^-p$	319	yes	48	37	[13]
ZEUS inclusive jets	1996	1997	125	10000	38.6	$e^+p$	301	no	30	30	[11]
ZEUS dijets	1998–2000 &	2004–2007	125	20000	374	$e^+p/e^-p$	318	no	22	16	[12]

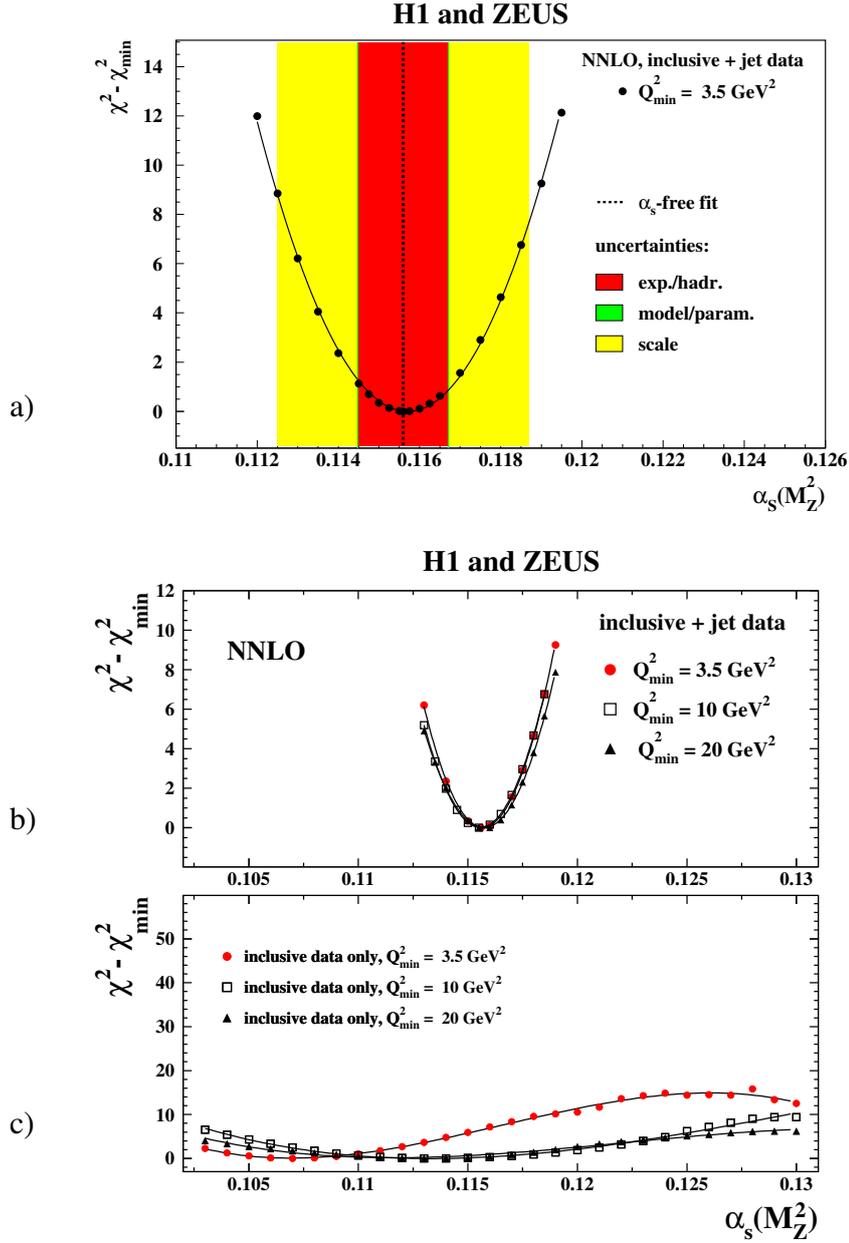
**Table 1:** The data sets on jet production from H1 and ZEUS used for the HERAPDF2.0Jets NNLO fits. The term normalised indicates that all cross sections are normalised to the respective NC inclusive cross sections.

Parameter	Central Value	Downwards variation	Upwards variation
$Q_{min}^2$ [GeV <sup>2</sup> ]	3.5	2.5	5.0
$f_s$	0.4	0.3	0.5
$M_c$ [GeV]	1.41	1.37*	1.45
$M_b$ [GeV]	4.20	4.10	4.30
$\mu_{f0}^2$ [GeV <sup>2</sup> ]	1.9	1.6	2.2*

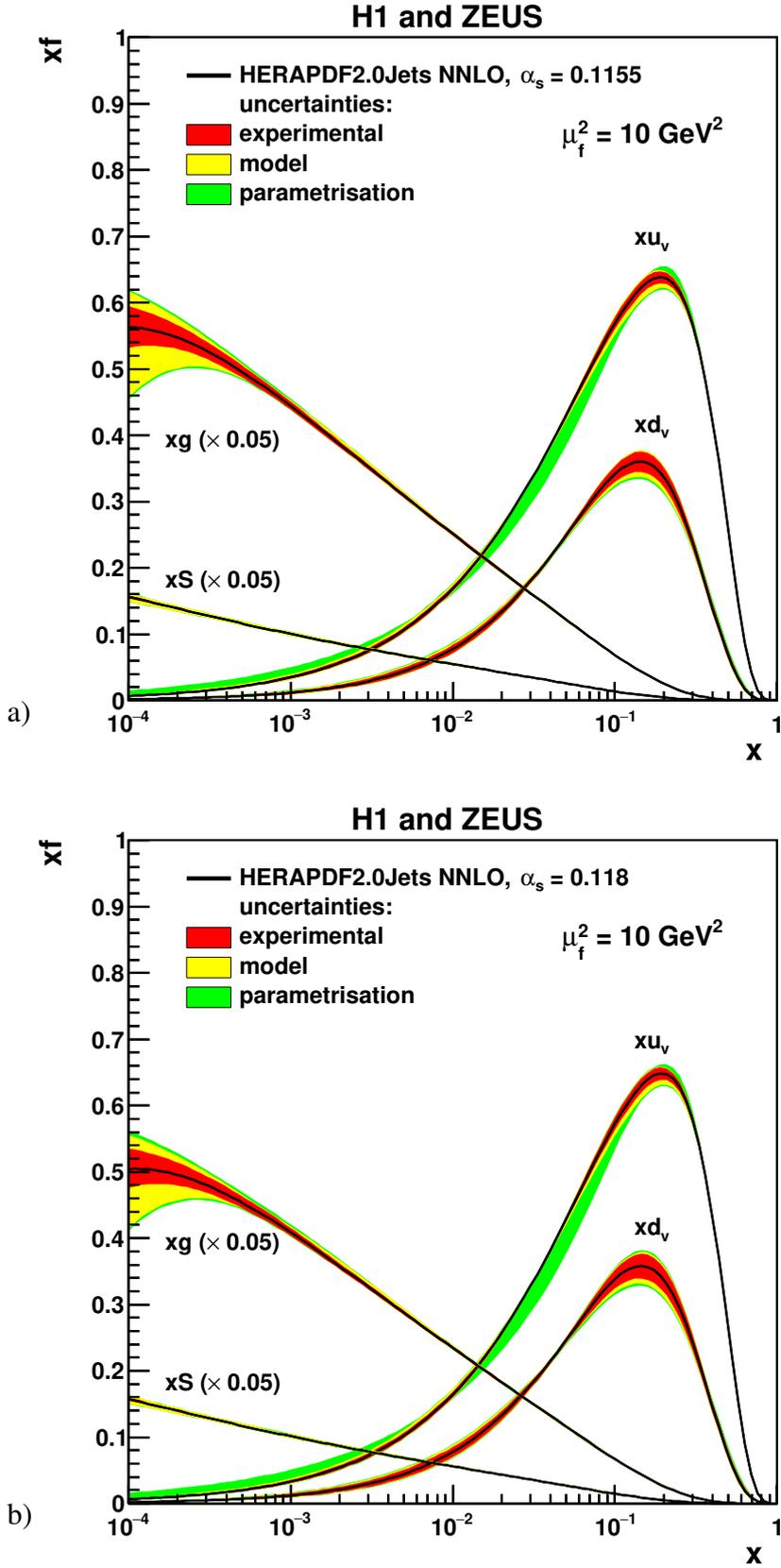
**Table 2:** Central values of model input parameters and their one-sigma variations. It was not possible to implement the variations marked \* because  $\mu_{f0} < M_c$  is required, see Section 3.3. In these cases, the uncertainty on the PDF obtained from the other variation was symmetrised.



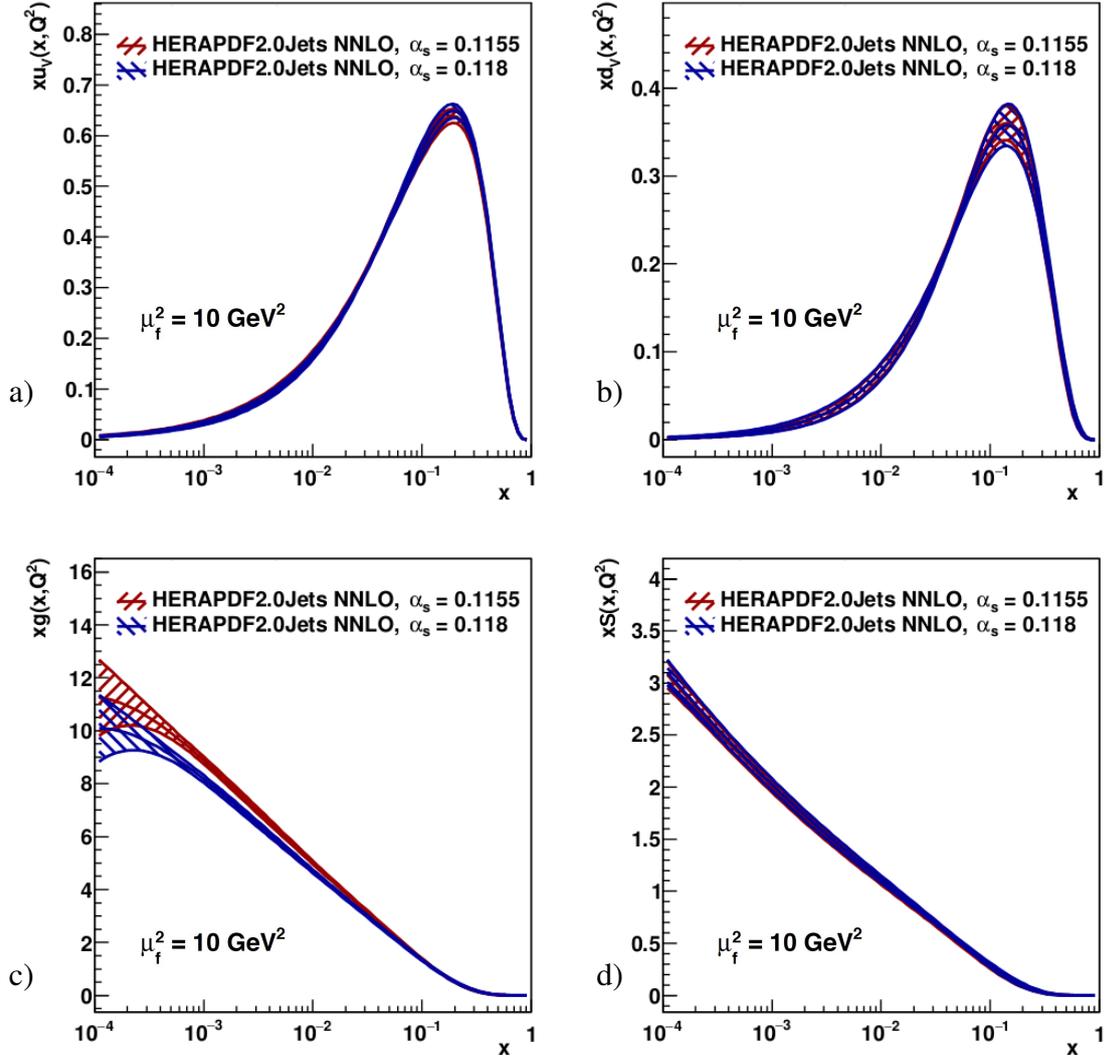
**Figure 1:**  $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$  vs. a) and b)  $M_c$  with  $M_b = 4.2$  GeV, and c) and d)  $M_b$  with  $M_c = 1.41$  GeV for a) and c) HERAPDF2.0Jets NNLO fits with fixed  $\alpha_s(M_Z^2) = 0.1155$  and b) and d) the corresponding NLO fits for  $M_c = 1.46$  GeV,  $M_b = 4.3$  GeV and  $\alpha_s(M_Z^2) = 0.118$ .



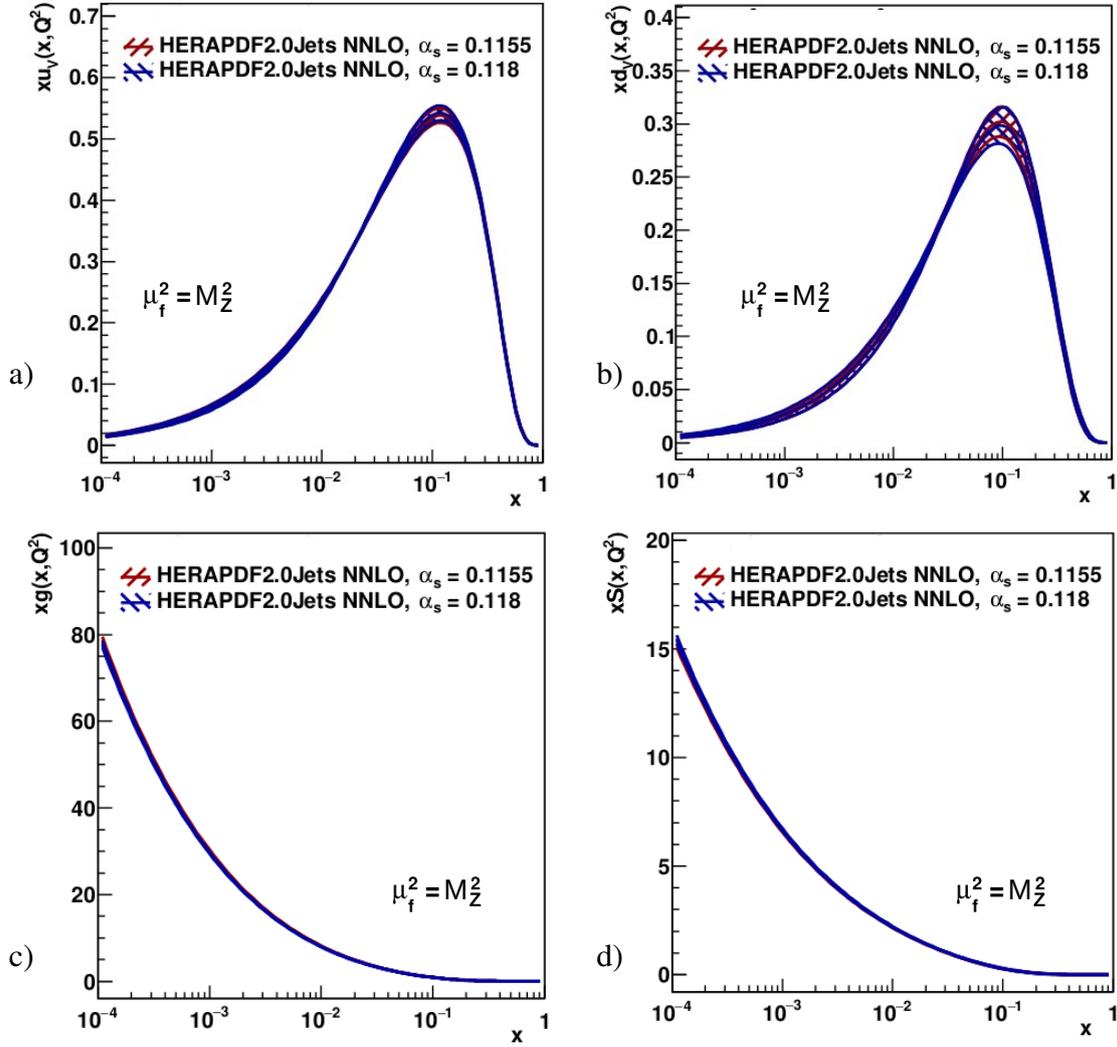
**Figure 2:**  $\Delta\chi^2 = \chi^2 - \chi_{\min}^2$  vs.  $\alpha_s(M_Z^2)$  for HERAPDF2.0Jets NNLO fits with fixed  $\alpha_s(M_Z^2)$  with a) the standard  $Q_{\min}^2$  of  $3.5 \text{ GeV}^2$  b) with  $Q_{\min}^2$  set to  $3.5 \text{ GeV}^2$ ,  $10 \text{ GeV}^2$  and  $20 \text{ GeV}^2$  for the inclusive data. In a), the result and all uncertainties determined for the HERAPDF2.0Jets NNLO fit with free  $\alpha_s(M_Z^2)$  are also shown, added in quadrature. In b), not all scan points for  $Q_{\min}^2$  of  $3.5 \text{ GeV}^2$  are plotted for better visibility. c) For comparison, the situation for fits to only inclusive data is shown, taken from [2].



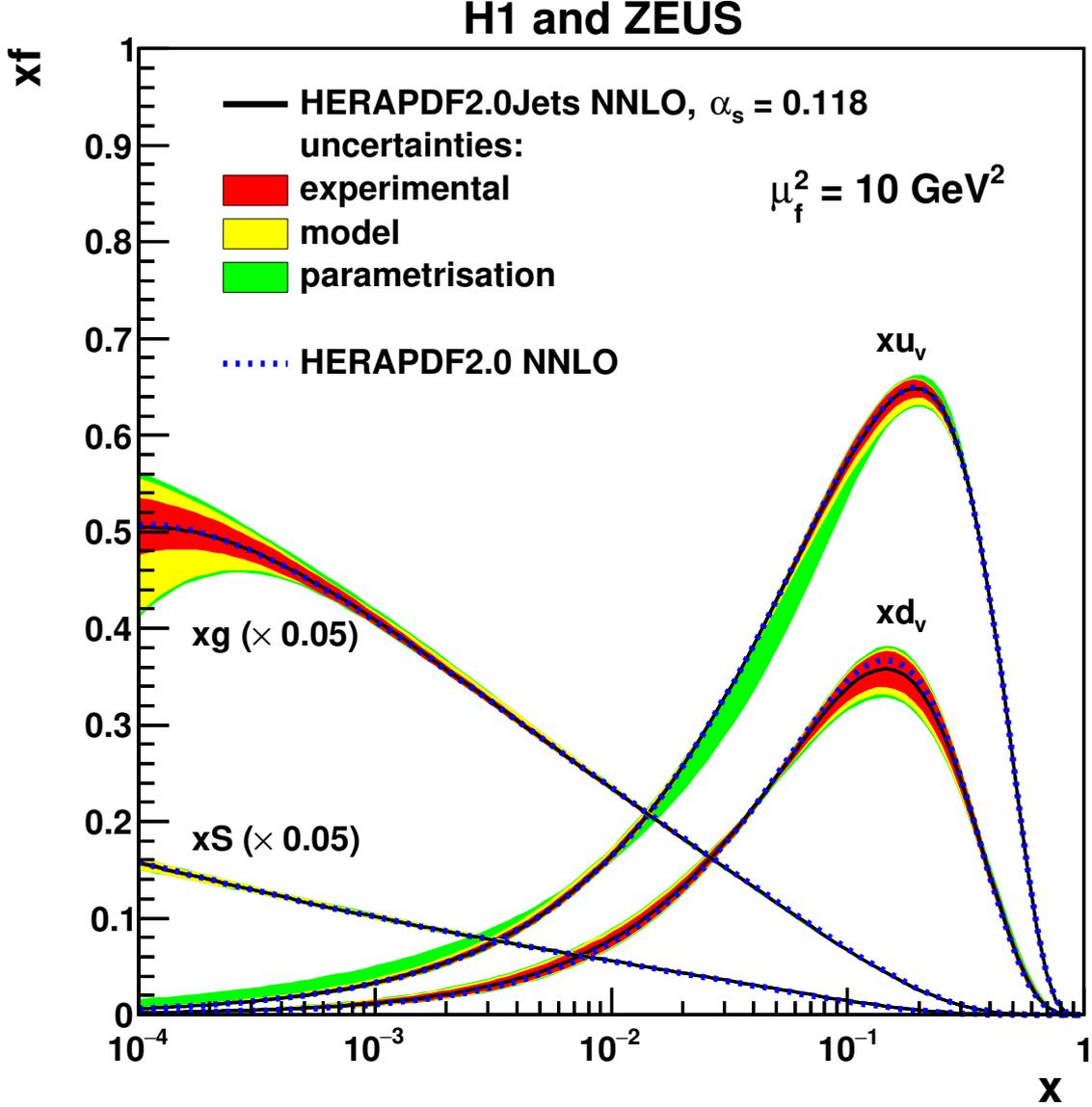
**Figure 3:** The parton distribution functions  $xu_v$ ,  $xd_v$ ,  $xg$  and  $xS = x(\bar{U} + \bar{D})$  of HERAPDF2.0Jets NNLO, with a)  $\alpha_s(M_Z^2)$  fixed to 0.1155 and b)  $\alpha_s(M_Z^2)$  fixed to 0.118 at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties are shown as differently shaded bands.



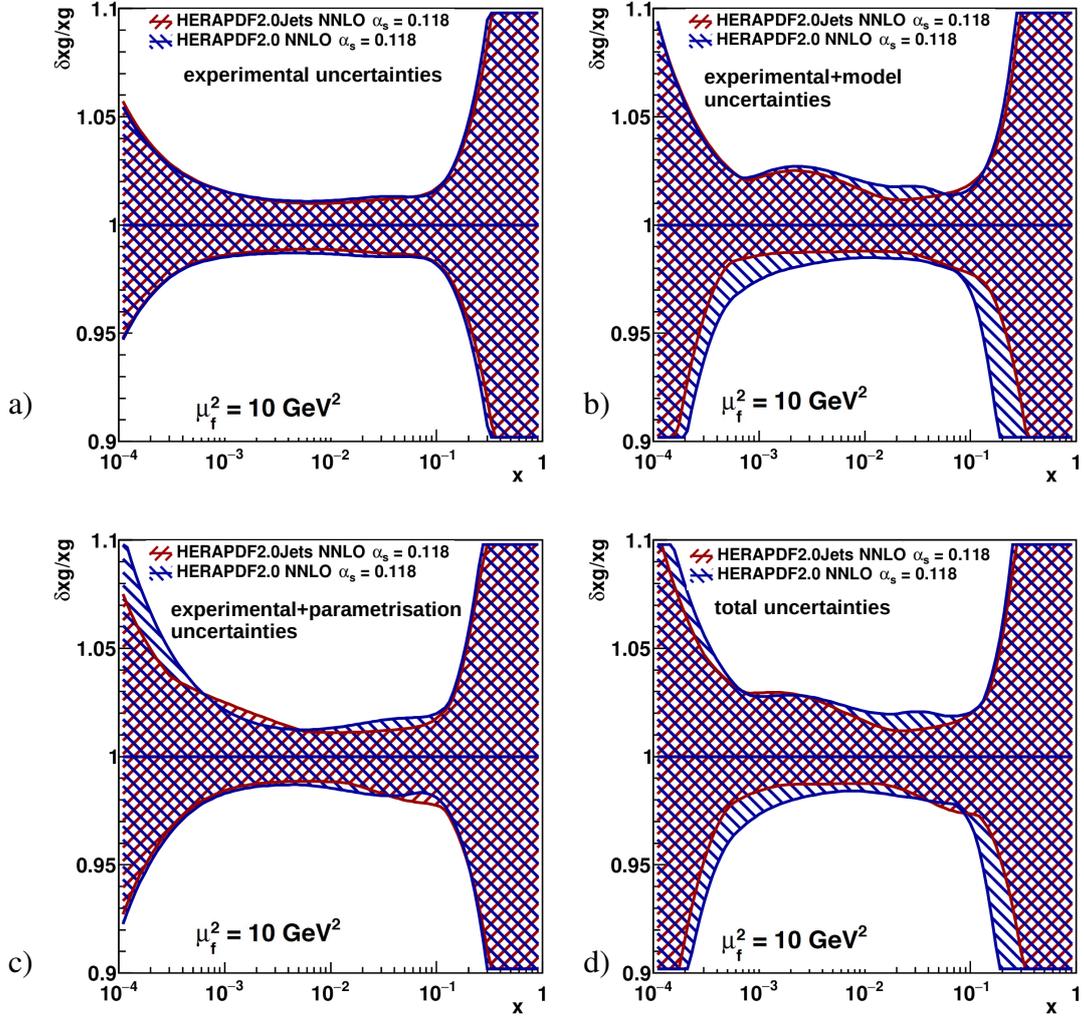
**Figure 4:** Comparison of the parton distribution functions a)  $xu_v$ , b)  $xd_v$ , c)  $xg$  and d)  $xS = x(\bar{U} + \bar{D})$  of HERAPDF2.0Jets NNLO with fixed  $\alpha_s(M_Z^2) = 0.1155$  and  $\alpha_s(M_Z^2) = 0.118$  at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The total uncertainties are shown as differently hatched bands.



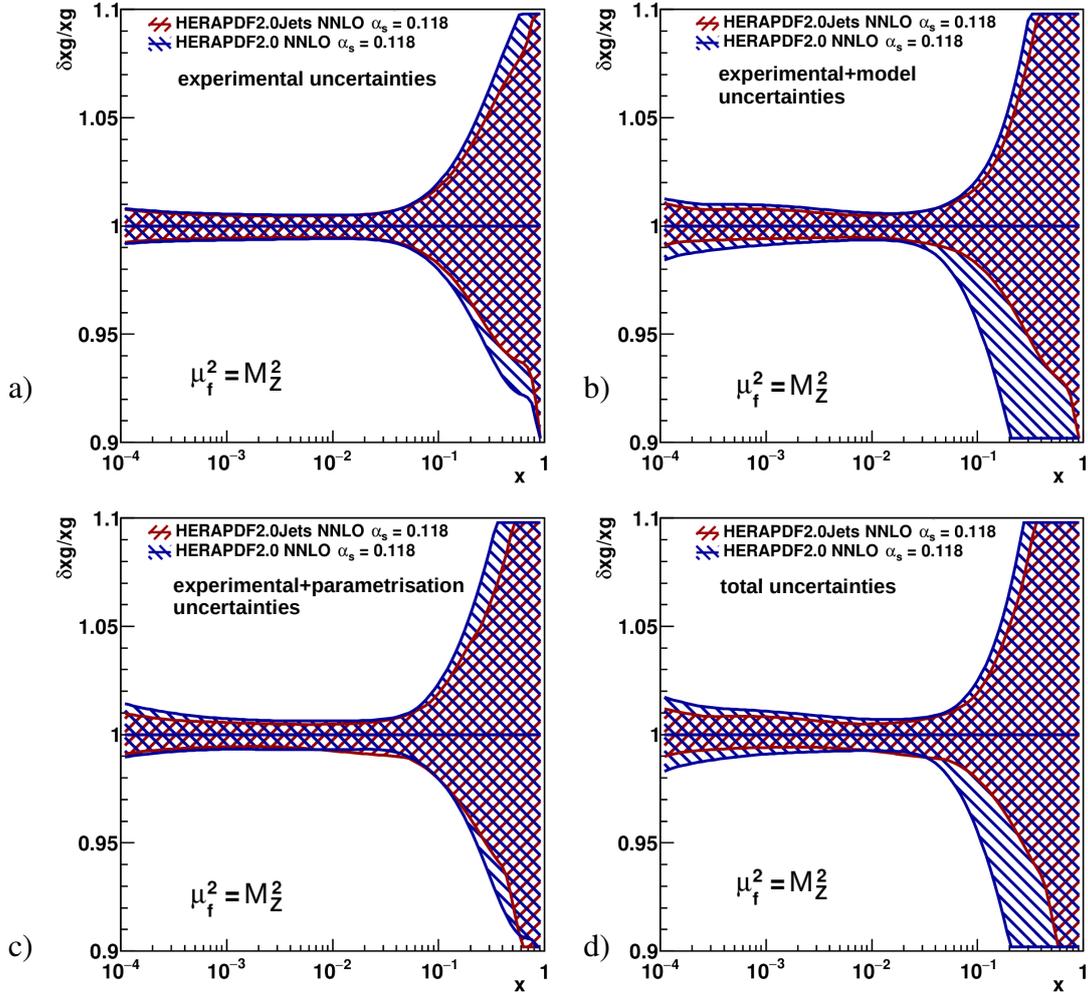
**Figure 5:** Comparison of the parton distribution functions a)  $xu_v$ , b)  $xd_v$ , c)  $xg$  and d)  $xS = x(\bar{U} + \bar{D})$  of HERAPDF2.0Jets NNLO with fixed  $\alpha_s(M_Z^2) = 0.1155$  and  $\alpha_s(M_Z^2) = 0.118$  at the scale  $\mu_f^2 = M_Z^2$  with  $M_Z = 91.19$  GeV [40]. The total uncertainties are shown as differently hatched bands.



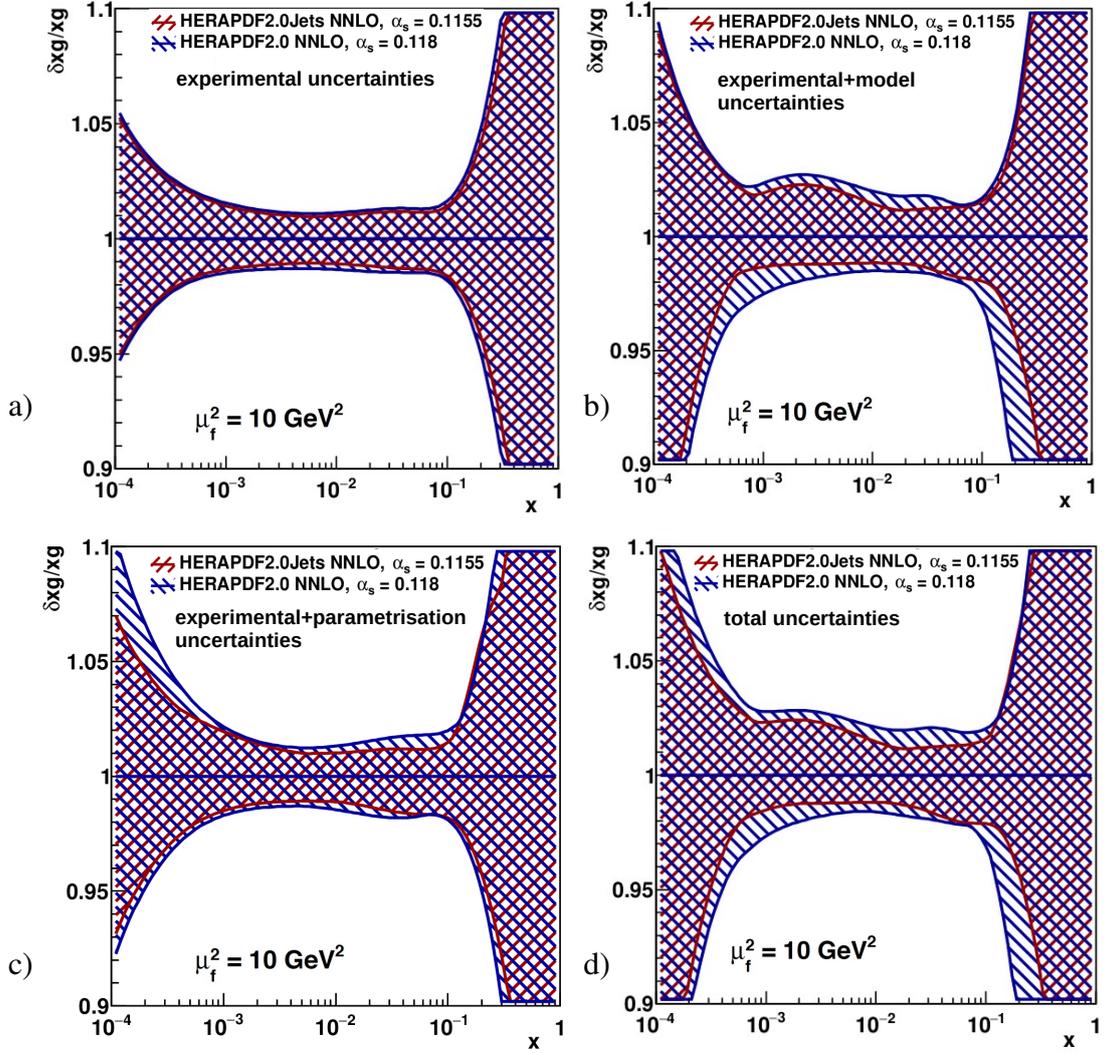
**Figure 6:** Comparison of the parton distribution functions  $xu_v$ ,  $xd_v$ ,  $xg$  and  $xS = x(\bar{U} + \bar{D})$  of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO, which was based on inclusive data only, both with fixed  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The full uncertainties of HERAPDF2.0Jets NNLO are shown as differently shaded bands and the central value of HERAPDF2.0 NNLO is shown as a dotted line.



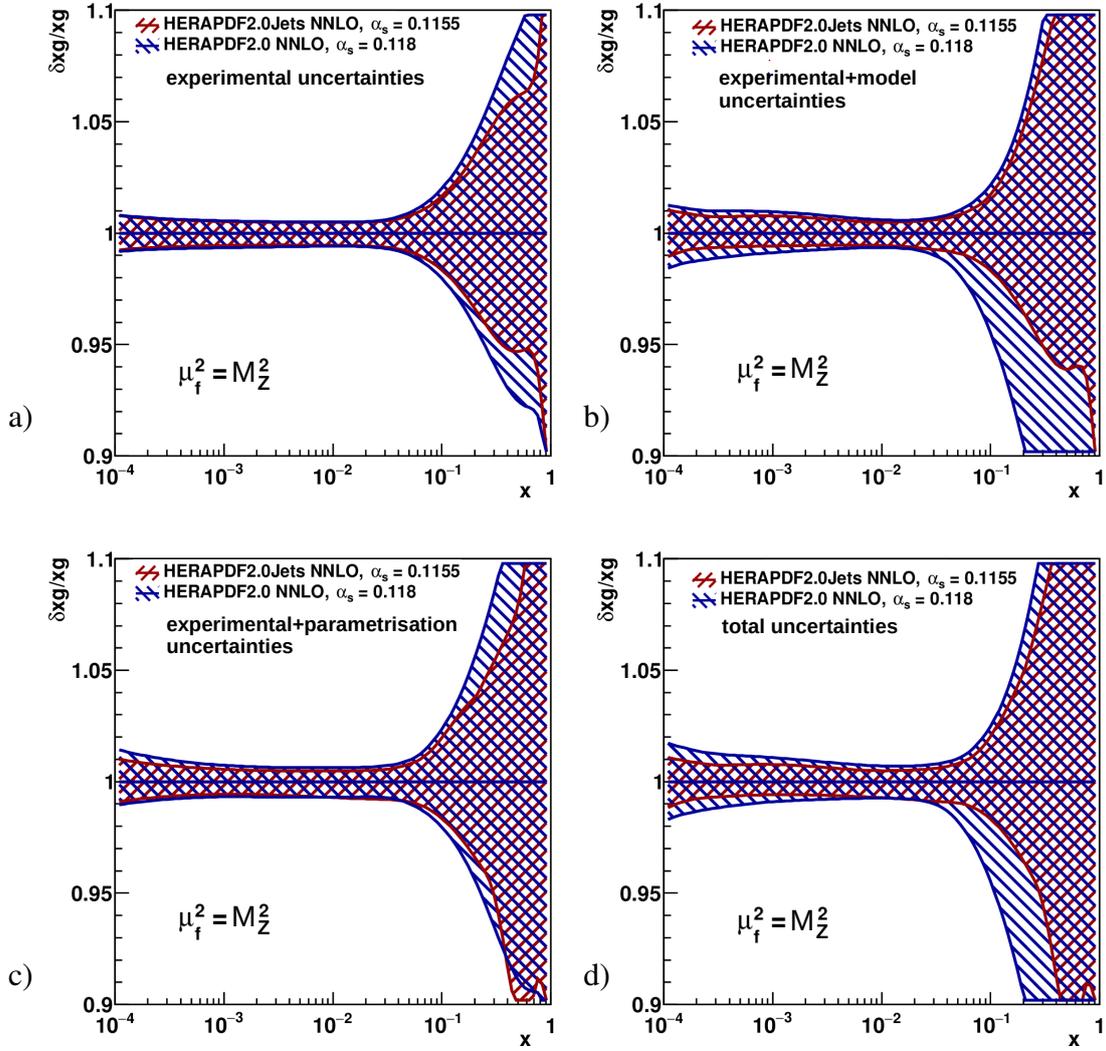
**Figure 7:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO for a) experimental, i.e. fit, b) experimental plus model, c) experimental plus parameterisation, d) total uncertainties at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties on both gluon distributions are shown as differently hatched bands.



**Figure 8:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO for a) experimental, i.e. fit, b) experimental plus model, c) experimental plus parameterisation, d) total uncertainties at the scale  $\mu_f^2 = M_Z^2$ . The uncertainties on both gluon distributions are shown as differently hatched bands.

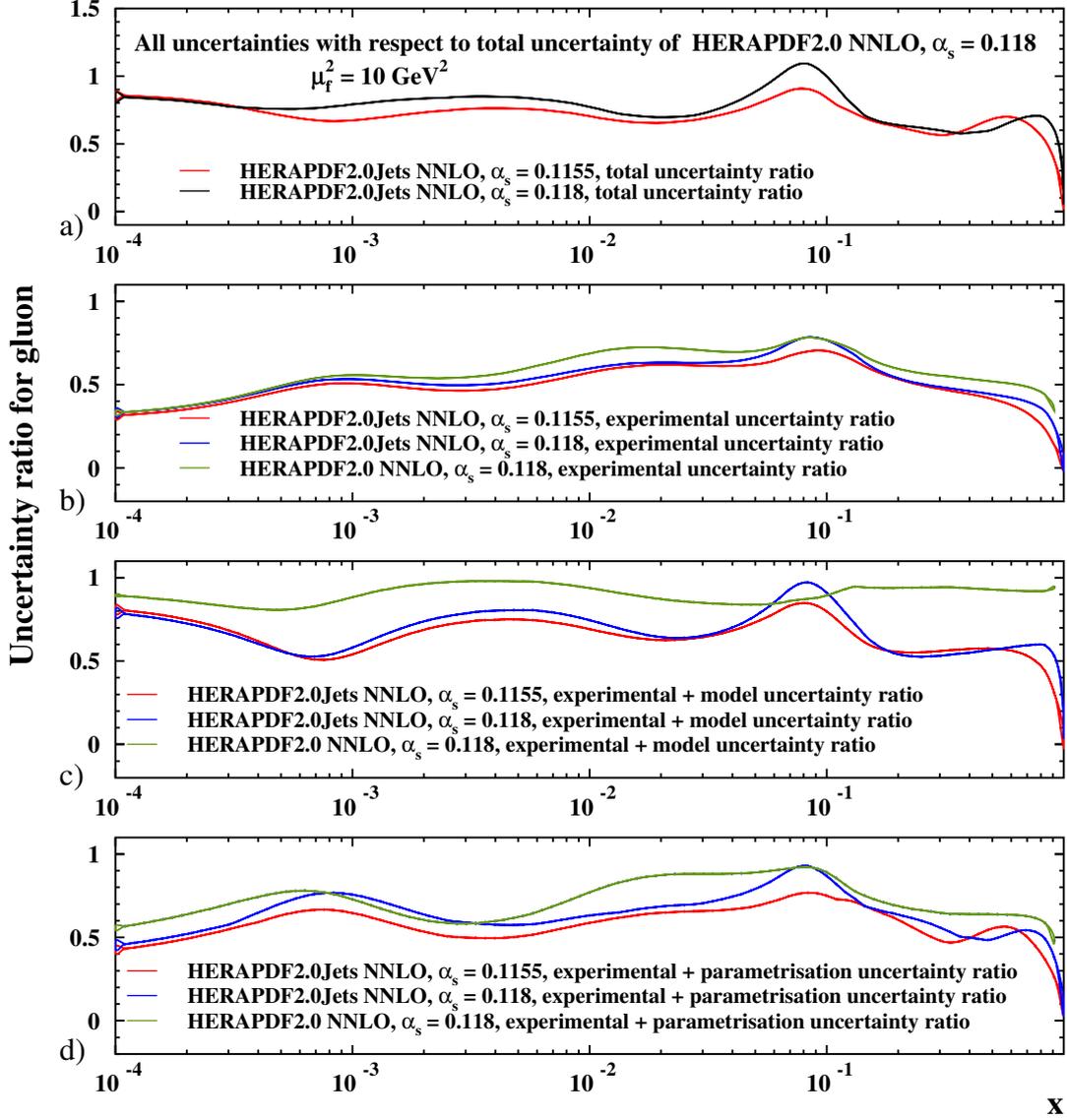


**Figure 9:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO for a) experimental, i.e. fit, b) experimental plus model, c) experimental plus parameterisation, d) total uncertainties at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties on both gluon distributions are shown as differently hatched bands.

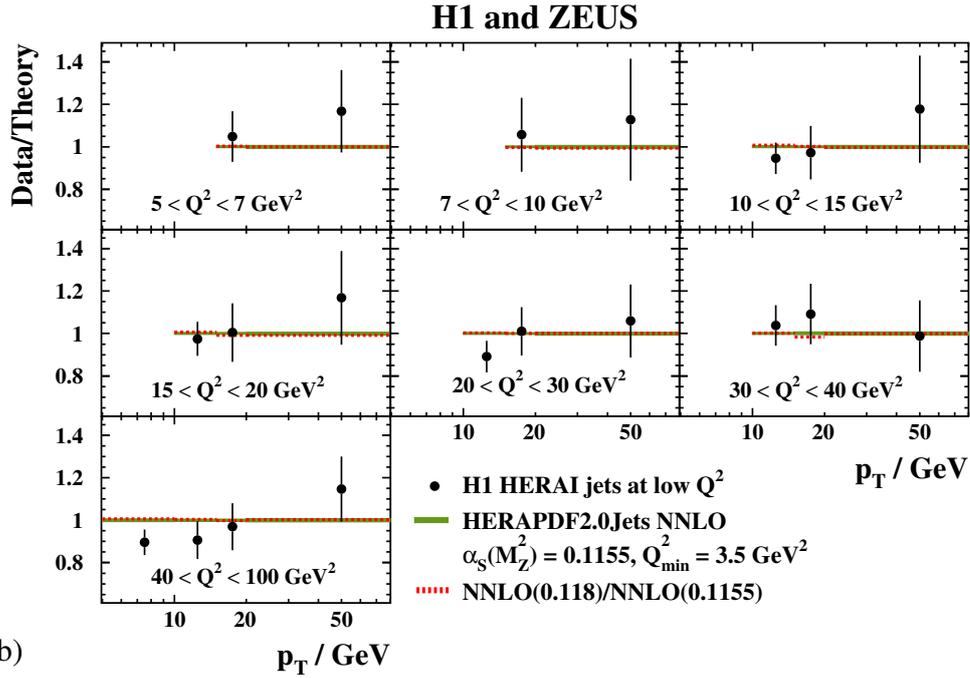
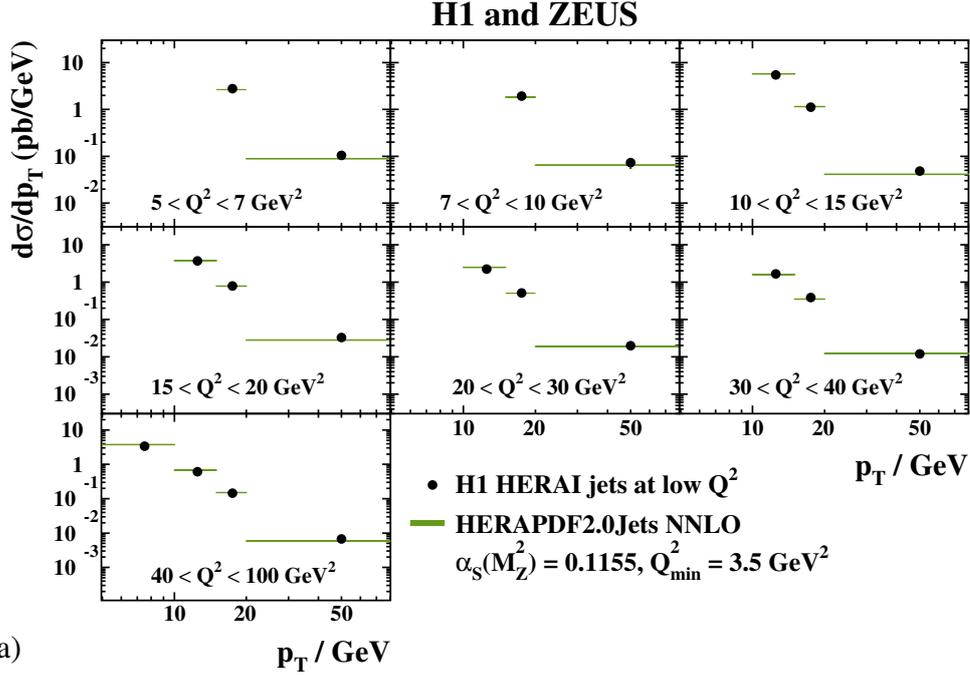


**Figure 10:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO and HERAPDF2.0 NNLO for a) experimental, i.e. fit, b) experimental plus model, c) experimental plus parameterisation, a) total uncertainties at the scale  $\mu_f^2 = M_Z^2$ . The uncertainties on both gluon distributions are shown as differently hatched bands.

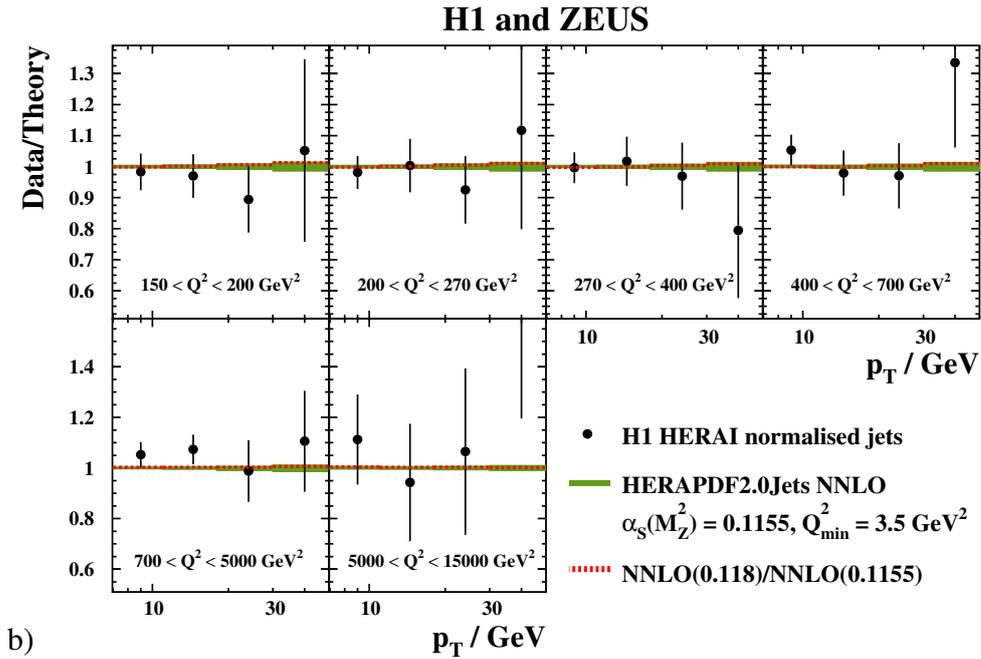
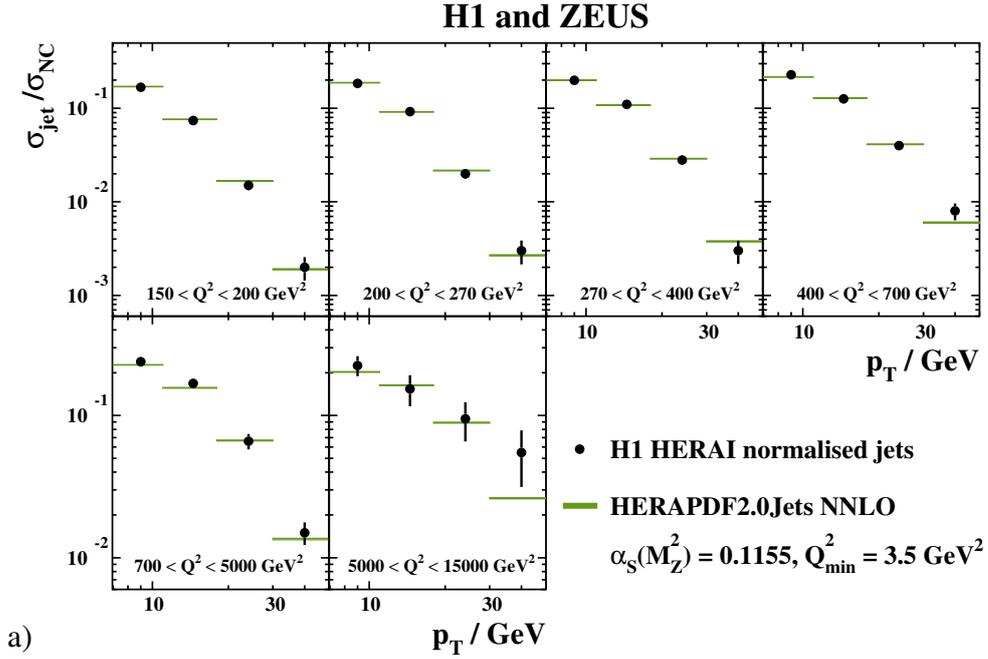
## H1 and ZEUS



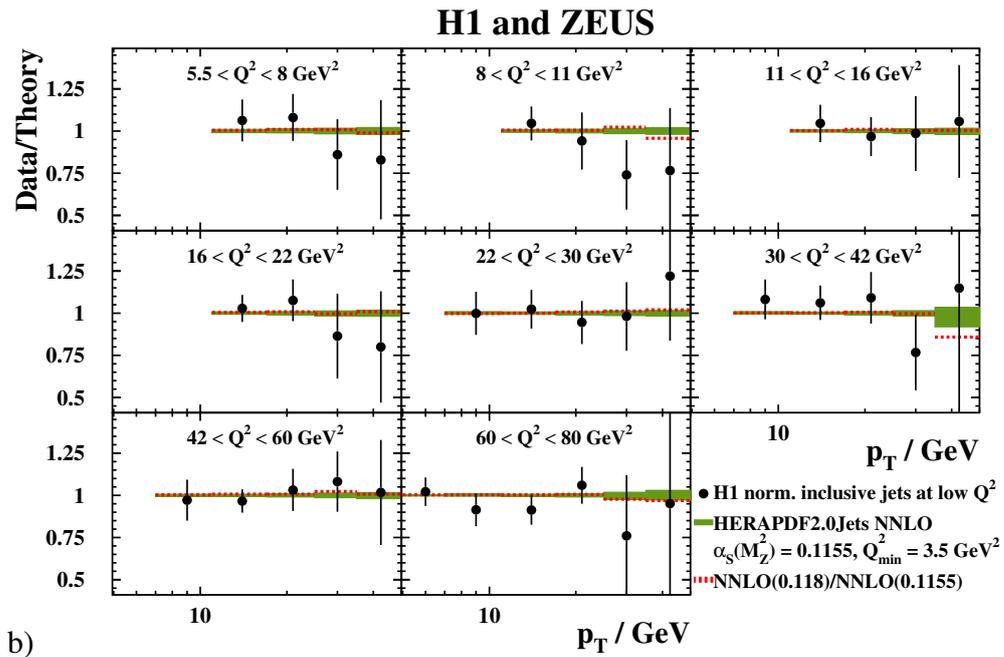
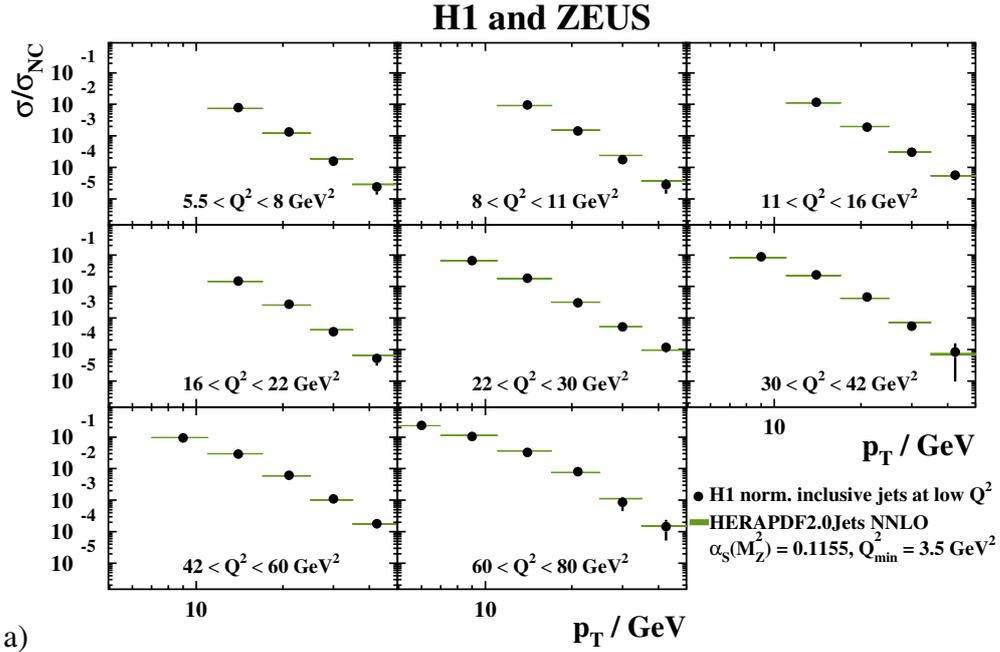
**Figure 11:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  a) total, b) experimental, c) experimental plus model, d) experimental plus parameterisation uncertainties for HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  and  $\alpha_s(M_Z^2) = 0.1155$  at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ .



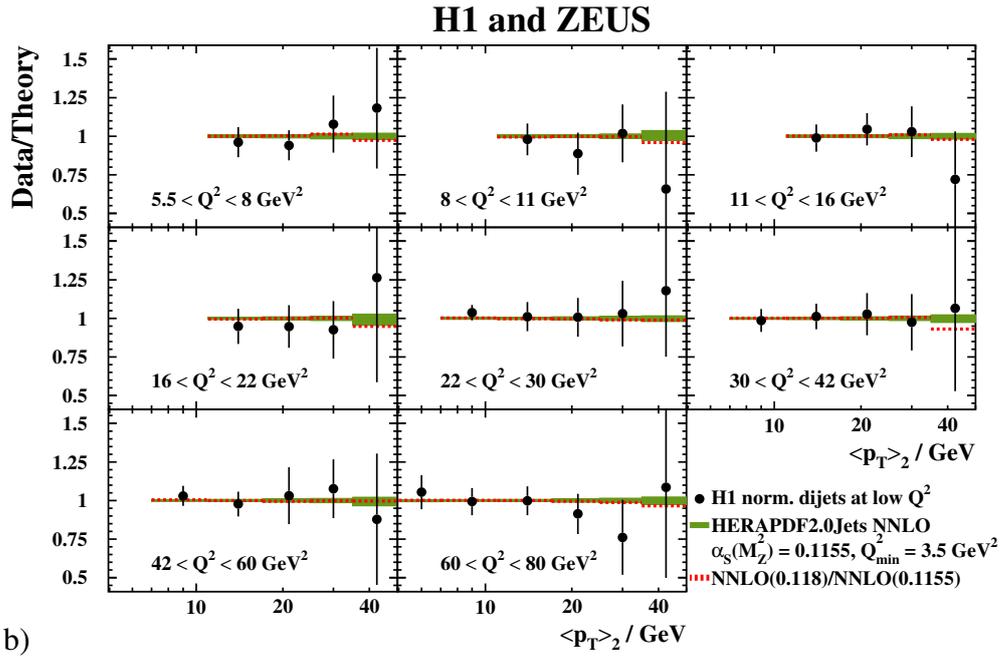
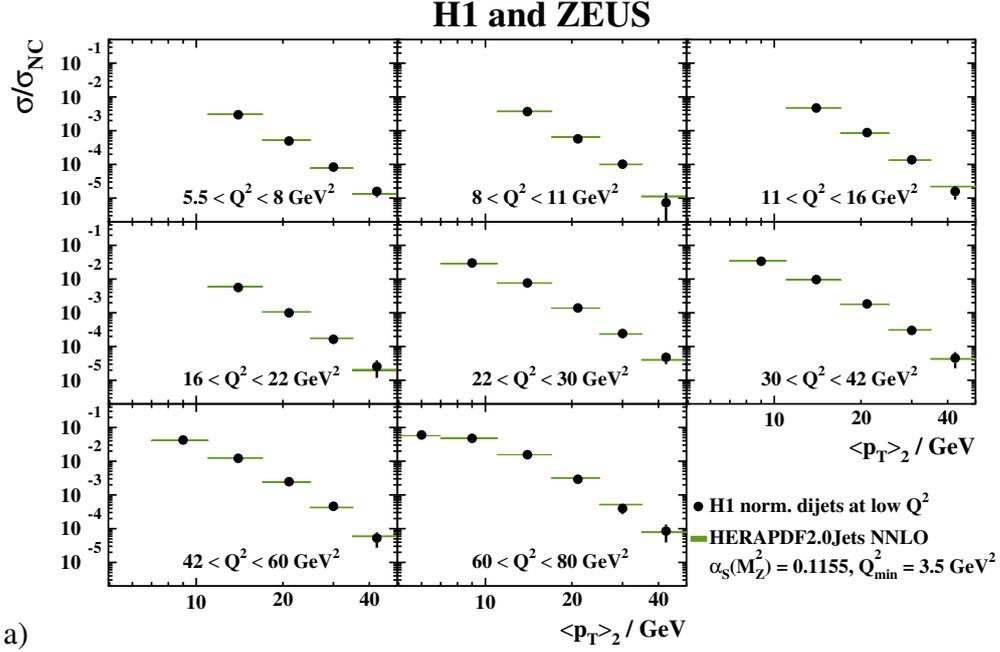
**Figure 12:** a) Differential jet cross sections,  $d\sigma/dp_T$ , in bins of  $Q^2$  between 5 and 100  $\text{GeV}^2$  as measured by H1. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties, the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections divided by predictions based on HERAPDF2.0Jets NNLO.



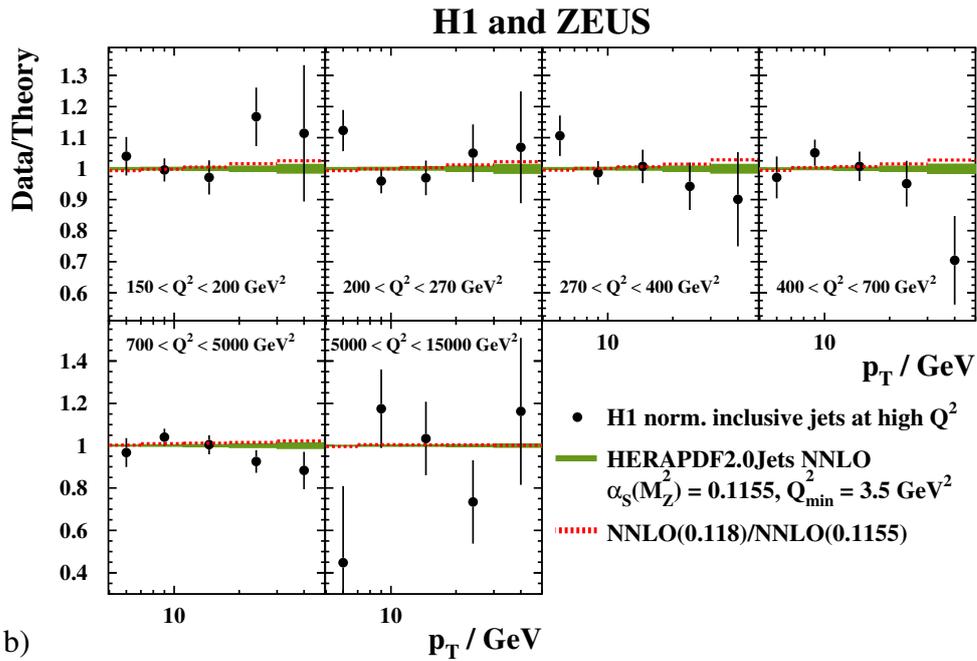
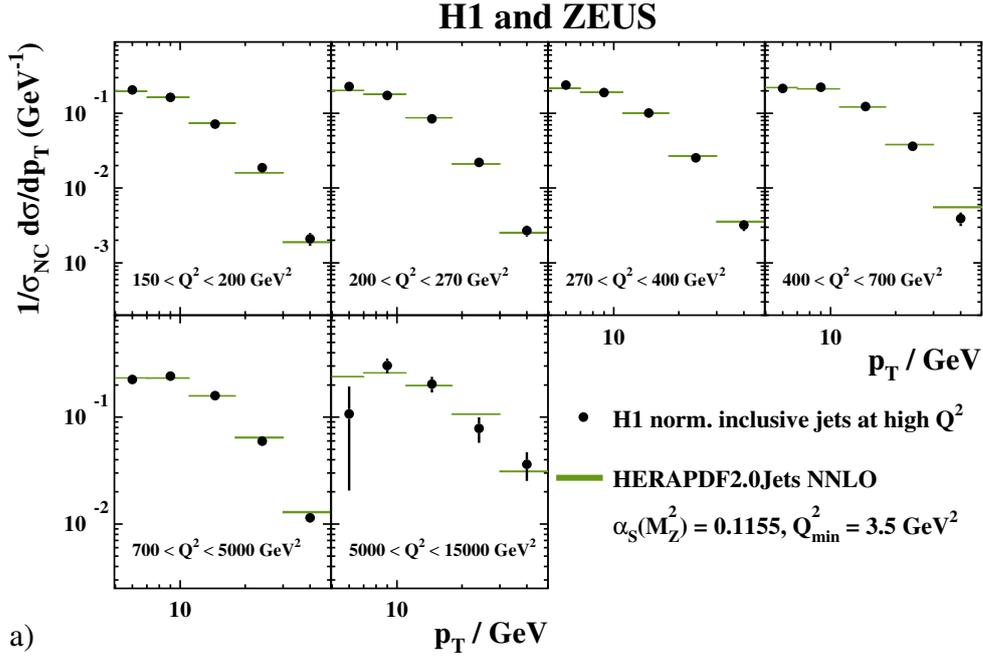
**Figure 13:** a) Differential jet cross sections,  $d\sigma/dp_T$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 150 and 15000  $\text{GeV}^2$  as measured by H1. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections divided by predictions based on HERAPDF2.0Jets NNLO.



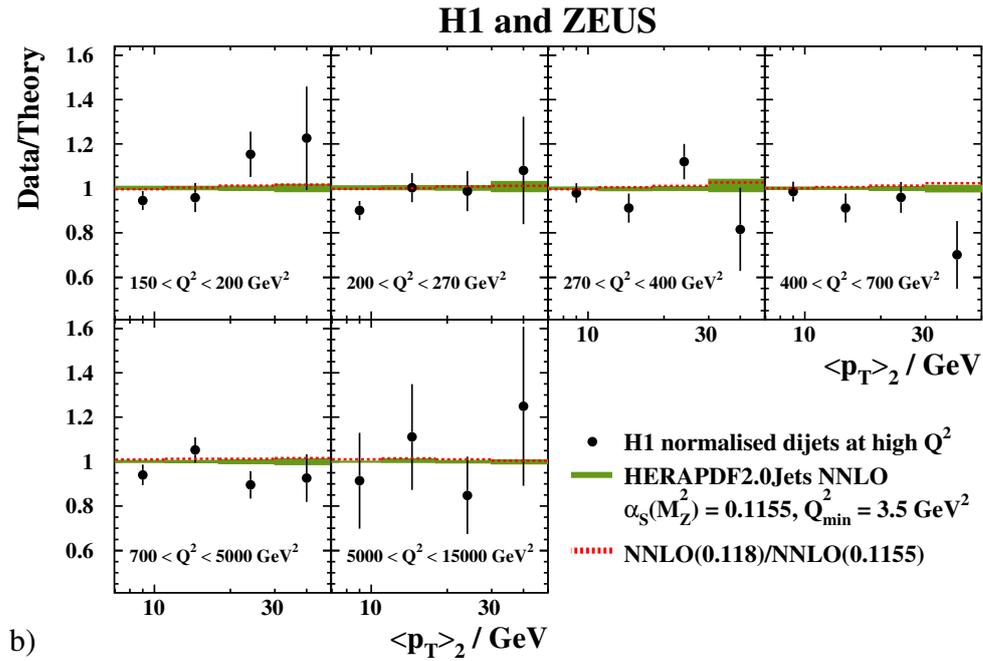
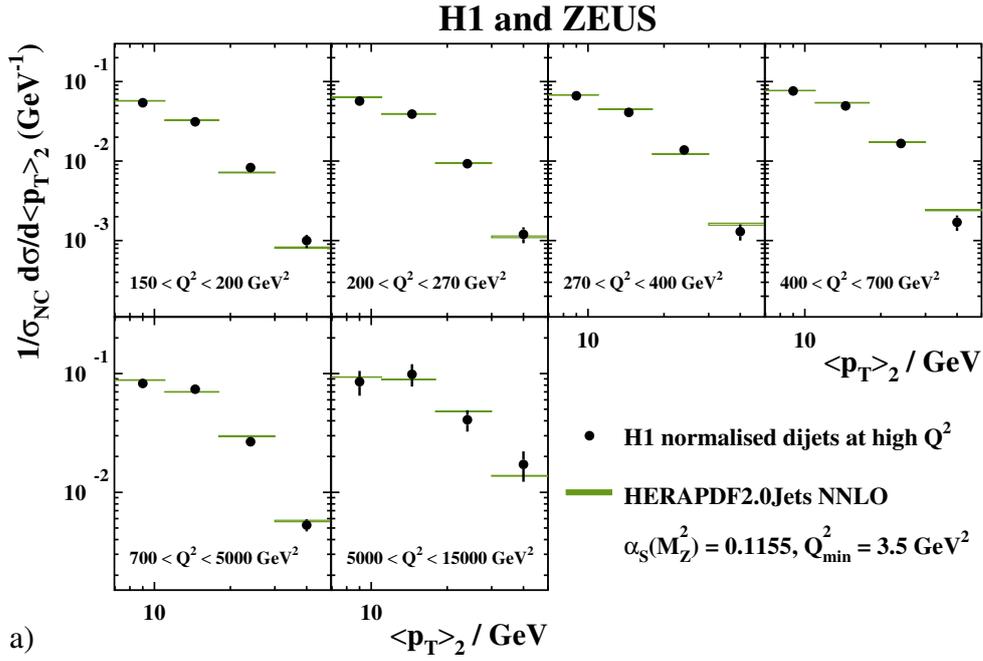
**Figure 14:** a) Differential inclusive jet cross sections,  $d\sigma/dp_T$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 5 and 80  $\text{GeV}^2$  as measured by H1. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections divided by predictions based on HERAPDF2.0Jets NNLO.



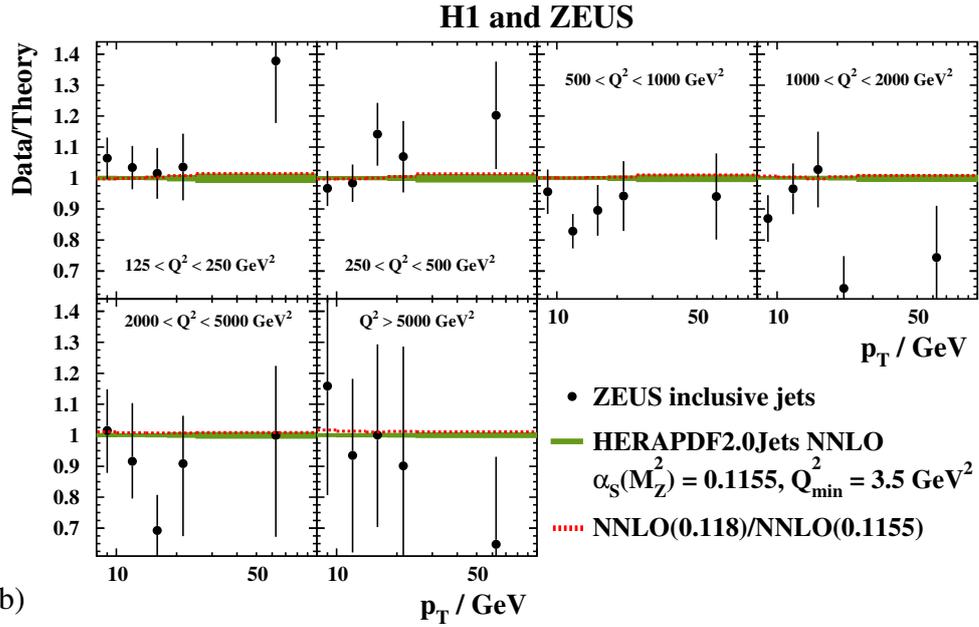
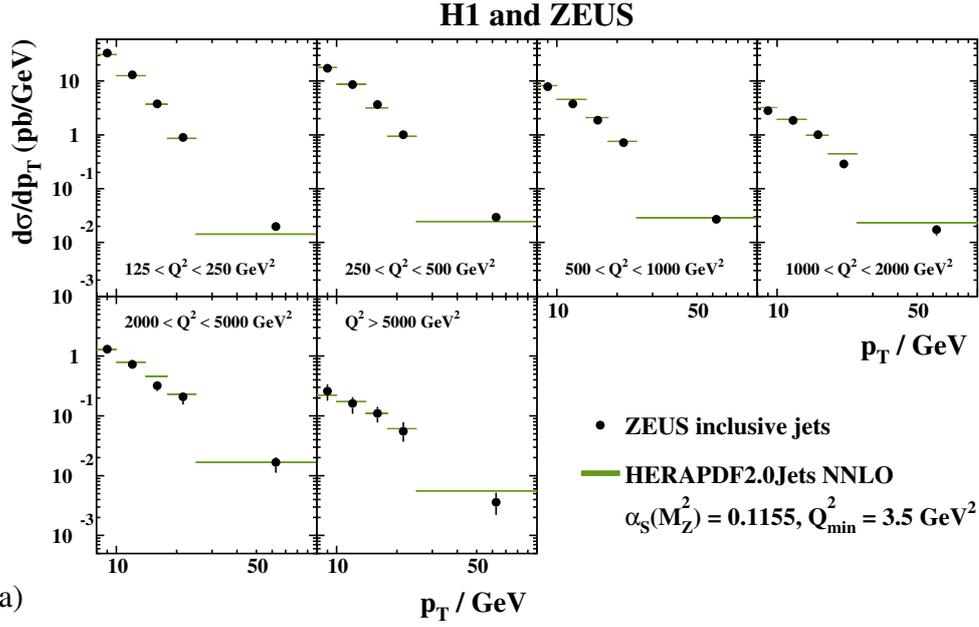
**Figure 15:** a) Differential dijet normalised cross sections,  $d\sigma/d\langle p_T \rangle_2$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 5 and 80  $\text{GeV}^2$  as measured by H1. The variable  $\langle p_T \rangle_2$  denotes the average  $p_T$  of the two jets. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections divided by predictions based on HERAPDF2.0Jets NNLO.



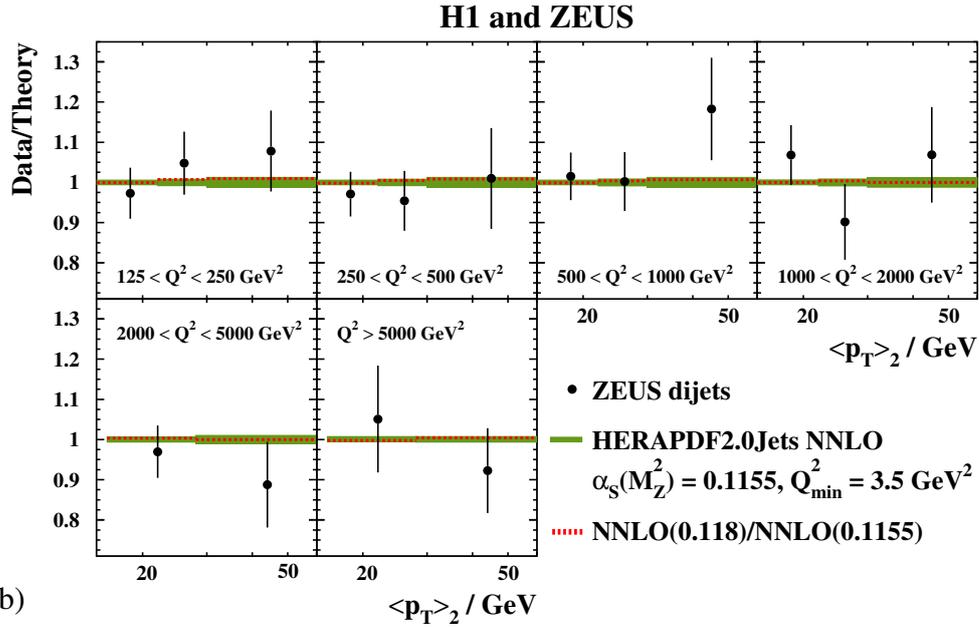
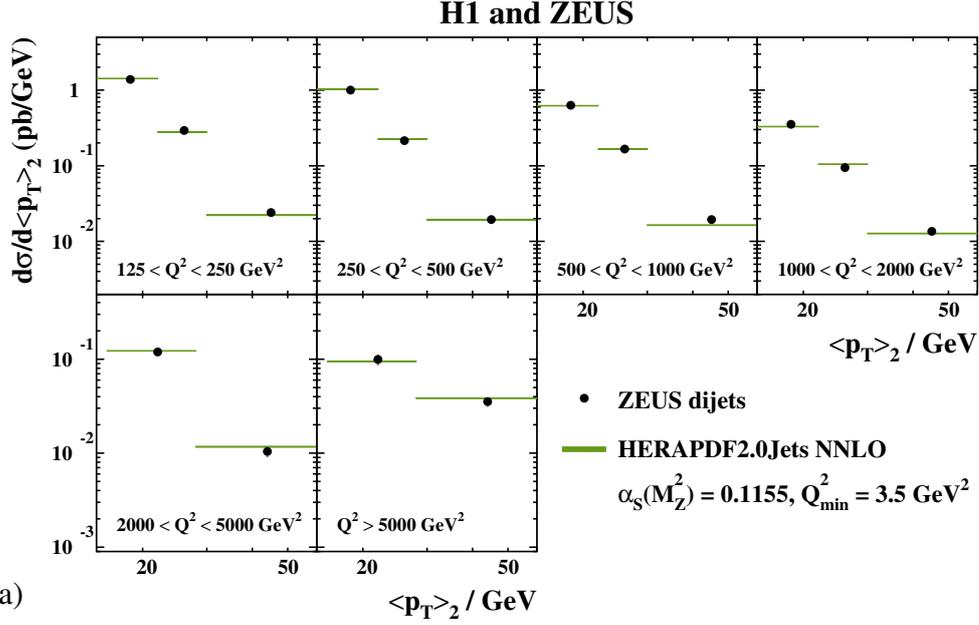
**Figure 16:** a) Differential inclusive jet cross sections,  $d\sigma/dp_T$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 150 and 15000  $\text{GeV}^2$  as measured by H1. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections divided by predictions based on HERAPDF2.0Jets NNLO.



**Figure 17:** a) Differential dijet cross sections,  $d\sigma/d\langle p_T \rangle_2$ , normalised to NC inclusive cross sections, in bins of  $Q^2$  between 150 and 15000  $\text{GeV}^2$  as measured by H1. The variable  $\langle p_T \rangle_2$  denotes the average  $p_T$  of the two jets. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainties on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections divided by predictions based on HERAPDF2.0Jets NNLO.



**Figure 18:** a) Differential jet cross sections,  $d\sigma/dp_T$ , in bins of  $Q^2$  between 125 and 10000  $\text{GeV}^2$  as measured by ZEUS. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainty on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections divided by predictions based on HERAPDF2.0Jets NNLO.



**Figure 19:** a) Differential dijet cross sections,  $d\sigma/d\langle p_T \rangle_2$ , in bins of  $Q^2$  between 125 and 20000  $\text{GeV}^2$  as measured by ZEUS. The variable  $\langle p_T \rangle_2$  denotes the average  $p_T$  of the two jets. Also shown are predictions based on HERAPDF2.0Jets NNLO. The bands represent the total uncertainty on the predictions excluding scale uncertainties; the bands are mostly invisible. Only data used in the fit are shown. b) Measured cross sections divided by predictions based on HERAPDF2.0Jets NNLO.

# Appendix A:

## PDF sets released

The following two sets of PDFs are released [41] and available on LHAPDF:

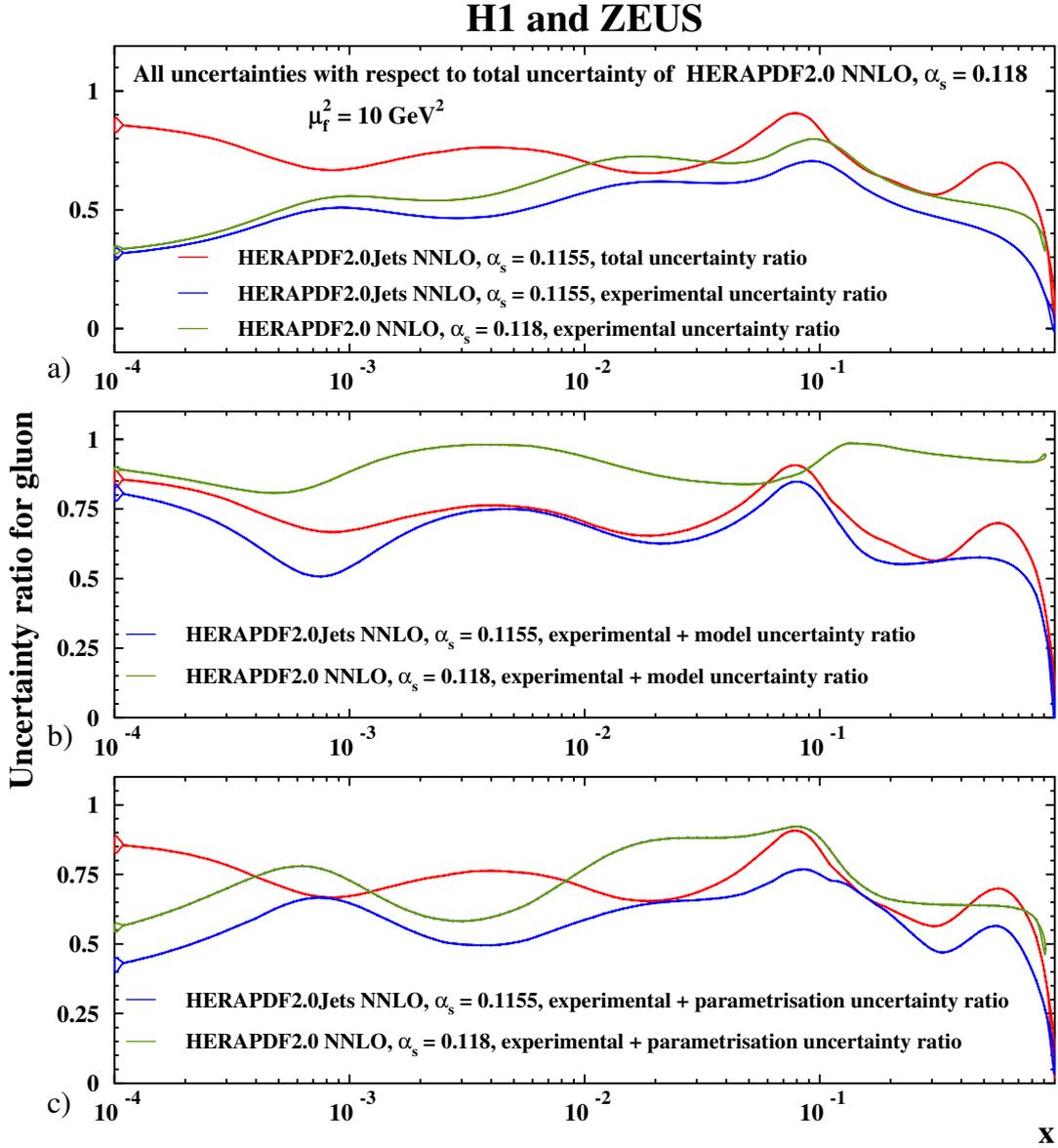
(<https://lhapdf.hepforge.org/pdfsets.html>).

- HERAPDF2.0Jets NNLO

- based on the combination of inclusive data from the H1 and ZEUS collaborations and selected data on jet production;
- with  $Q_{\min}^2 = 3.5 \text{ GeV}^2$ ;
- using the RTOPT variable-flavour-number scheme;
  - \* with fixed value of  $\alpha_s(M_Z^2) = 0.01155$ ;
  - \* with fixed value of  $\alpha_s(M_Z^2) = 0.0118$ ;
- 14 eigenvector pairs give Hessian experimental (fit) uncertainties including hadronisation uncertainties;
- grids of 14 variations are released to describe the model and parameterisation uncertainties.

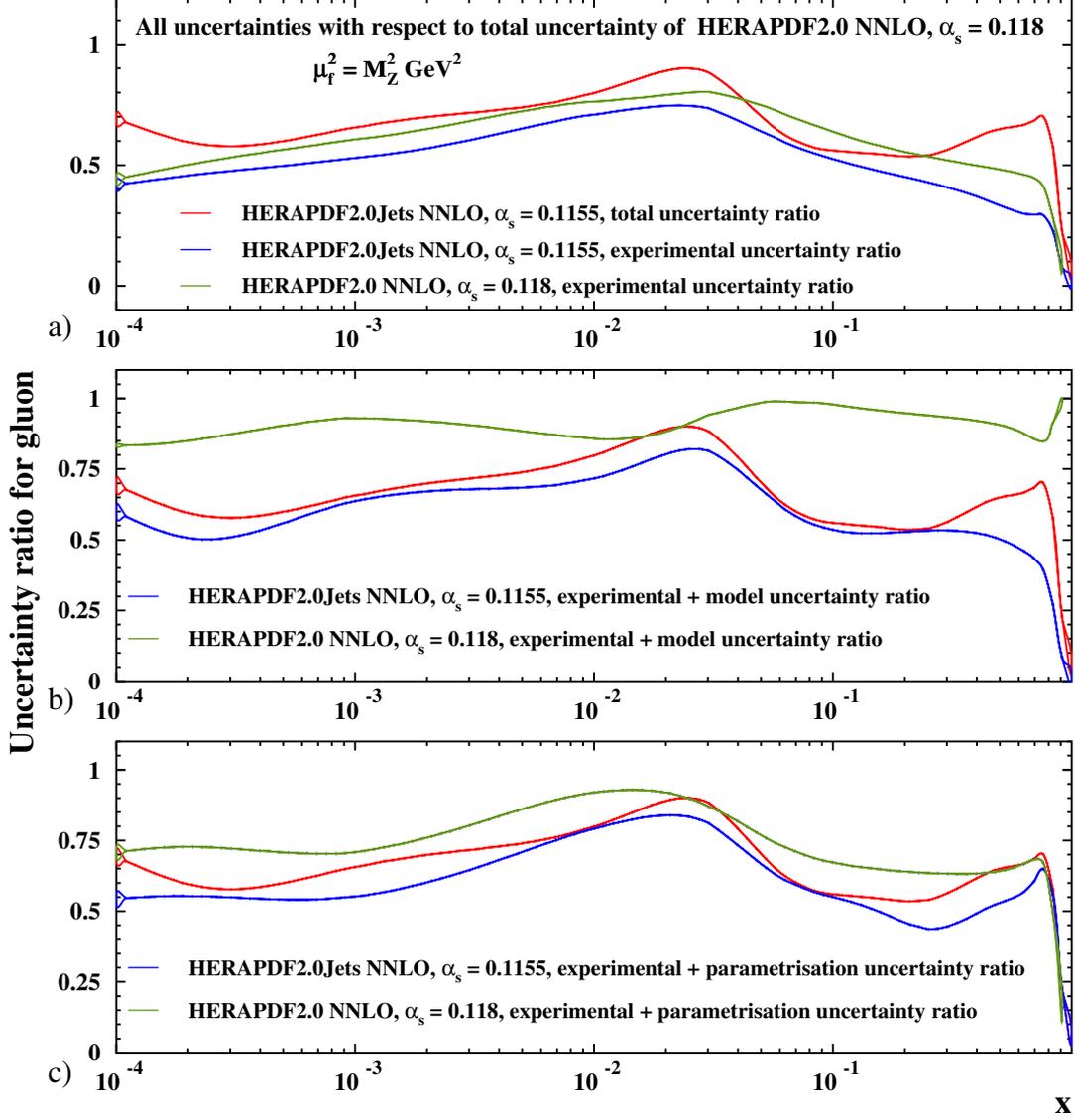
# Appendix B:

## Additional ratio plots on gluon PDF uncertainties



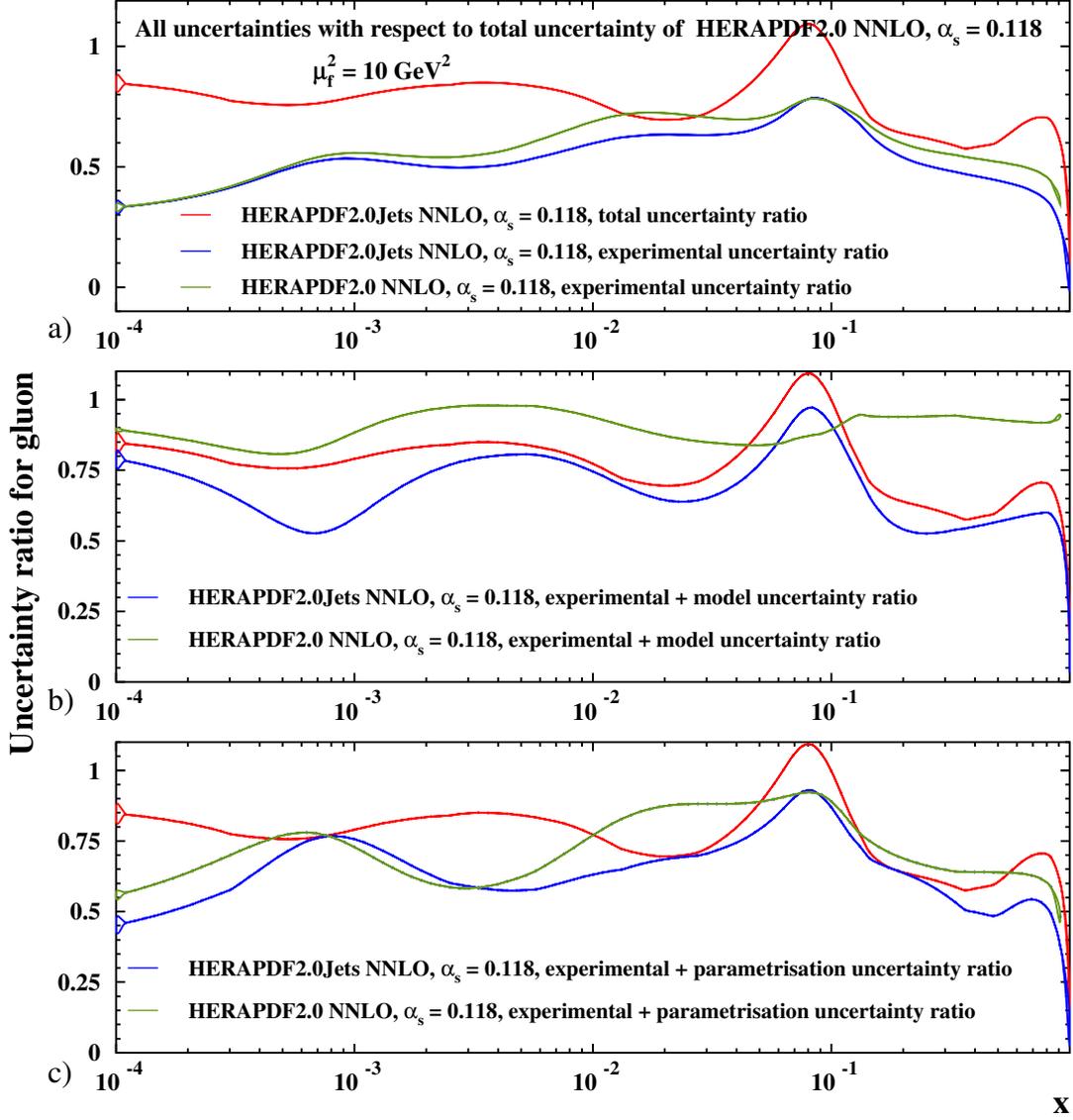
**Figure 20:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  for the total uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  and the a) experimental uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus model uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ .

## H1 and ZEUS



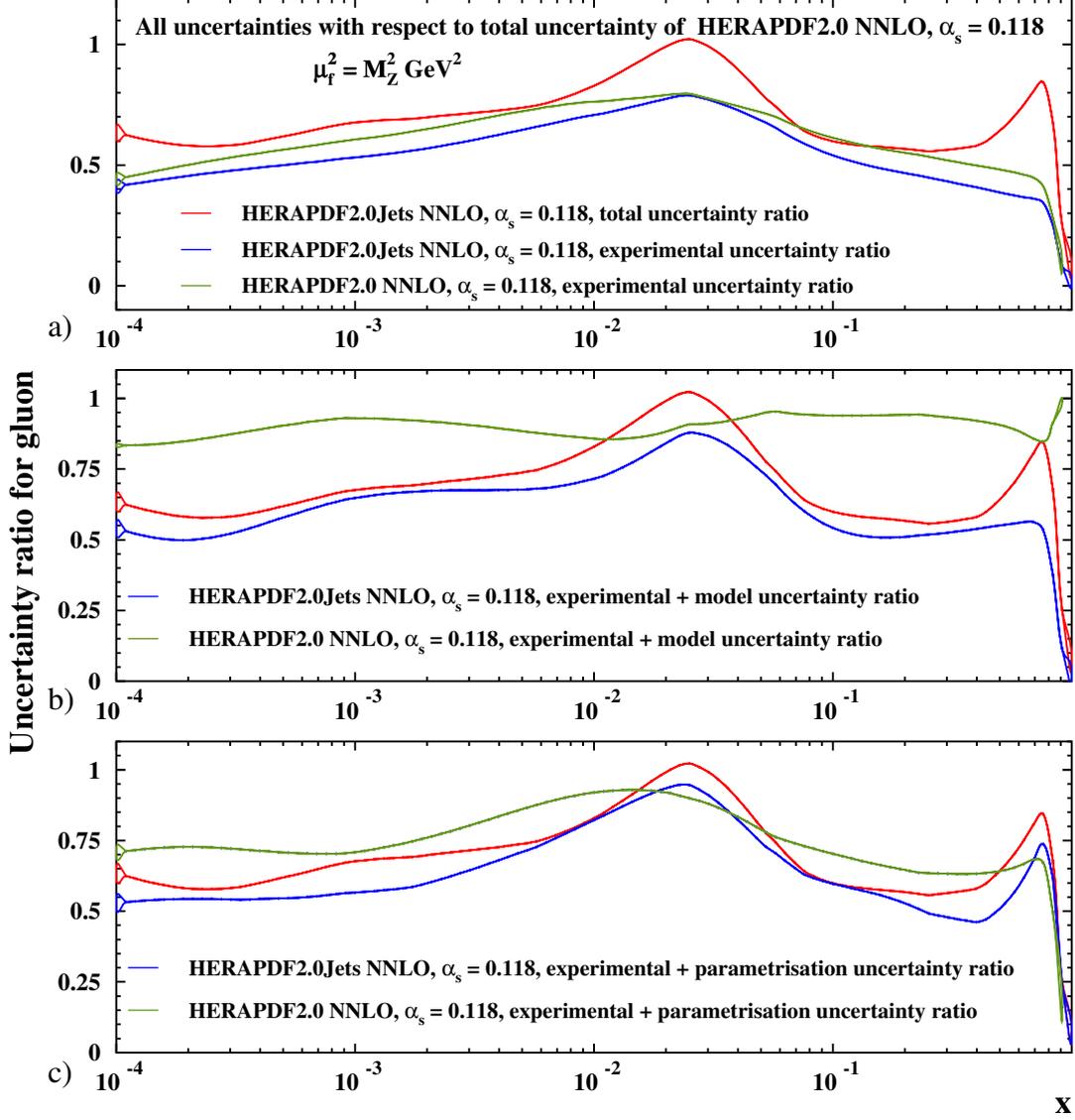
**Figure 21:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  for the total uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  and the a) experimental uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus model uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.1155$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = M_Z^2$ .

## H1 and ZEUS



**Figure 22:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  for the total uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  and the a) experimental uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental plus model uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ .

## H1 and ZEUS



**Figure 23:** Ratios of uncertainties relative to the total uncertainties of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$  for the total uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  and the a) experimental uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , b) experimental plus model uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental plus model uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , c) experimental plus parameterisation uncertainty of HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2) = 0.118$  as well as the experimental plus parameterisation uncertainty of HERAPDF2.0 NNLO  $\alpha_s(M_Z^2) = 0.118$ , at the scale  $\mu_f^2 = M_Z^2$ .

# Internal extra material:

## Comparison of results on $\alpha_s(M_Z^2)$ determined at NLO and NNLO:

A more detailed comparison between the NLO and NNLO results must account for the following differences:

- the choice of scale was different;
- the NLO result did not include the recently published H1 low- $Q^2$  inclusive and dijet data [13];
- the NLO result did not include the newly published low  $p_T$  points from the H1 high- $Q^2$  inclusive data;
- the NNLO result does not include trijet data;
- the NNLO result does not include the low  $p_T$  points from the ZEUS dijet data;
- the NNLO analysis imposes a stronger kinematic cut  $\mu > 10.0$  GeV;
- the treatment of hadronisation uncertainty differs.

All these changes with respect to the NLO analysis had to be made to create a consistent environment for a fit at NNLO. At the same time, an NLO fit cannot be done under exactly the same conditions as the NNLO fit since the H1 low  $Q^2$  data cannot be well fitted at NLO. However, an NLO and an NNLO fit can be done under the common conditions:

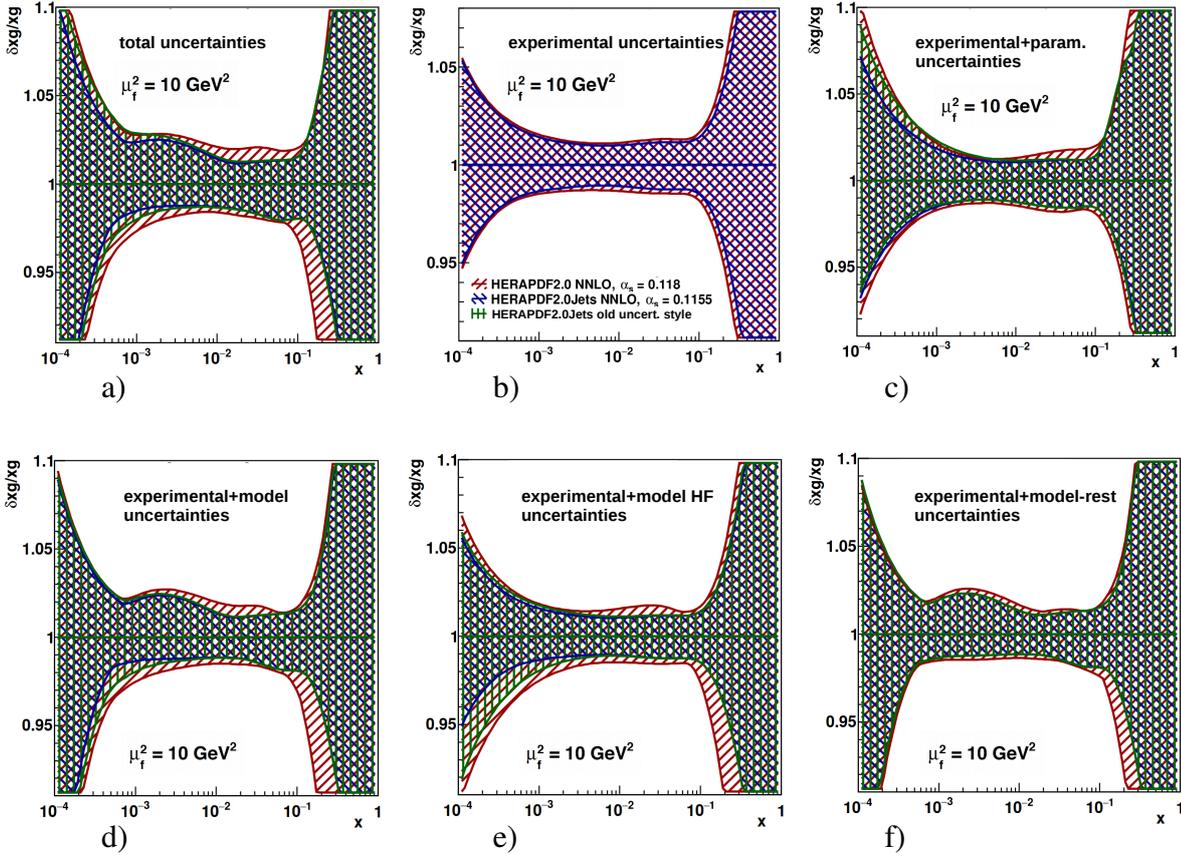
- choice of scale,  $\mu_r^2 = \mu_f^2 = Q^2 + p_T^2$ ;
- exclusion of the H1 low- $Q^2$  inclusive and dijet data;
- exclusion of the low- $p_T$  points from the H1 high- $Q^2$  inclusive jet data;
- exclusion of trijet data;
- exclusion of low- $p_T$  points from the ZEUS dijet data;
- exclusion of data with  $\mu < 10.0$  GeV;
- hadronisation uncertainties treated as correlated systematic uncertainties as done in the NNLO analysis.

In this case, the values obtained were  $\alpha_s(M_Z^2) = 0.1186 \pm 0.0014(\text{exp})$  at NLO and  $\alpha_s(M_Z^2) = 0.1144 \pm 0.0013(\text{exp})$  at NNLO. The new NLO value of  $\alpha_s(M_Z^2)$  agrees with the published [2] value of 0.1183. The change of the NNLO result from the preferred value of 0.1156 is mostly due to the exclusion of the H1 low  $Q^2$  data and the low- $p_T$  points at high  $Q^2$ .

# Internal extra material:

More detailed information concerning the source of uncertainties at a scale of  $10 \text{ GeV}^2$ : The green band represents HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2)=0.1155$  as obtained for the old procedure, i.e. with double counting.

This shows that the improvement is mainly due to jet data.



**Figure 24:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO, HERAPDF2.0 NNLO and HERAPDF2.0Jets NNLO with old procedure on uncertainties for a) total, b) experimental, i.e. fit, c) experimental plus parameterisation, d) experimental plus model, e) experimental plus model due to heavy flavour f) experimental plus all model but heavy flavour uncertainties at the scale  $\mu_f^2 = 10 \text{ GeV}^2$ . The uncertainties on the three gluon distributions are shown as differently hatched bands.

# Internal extra material:

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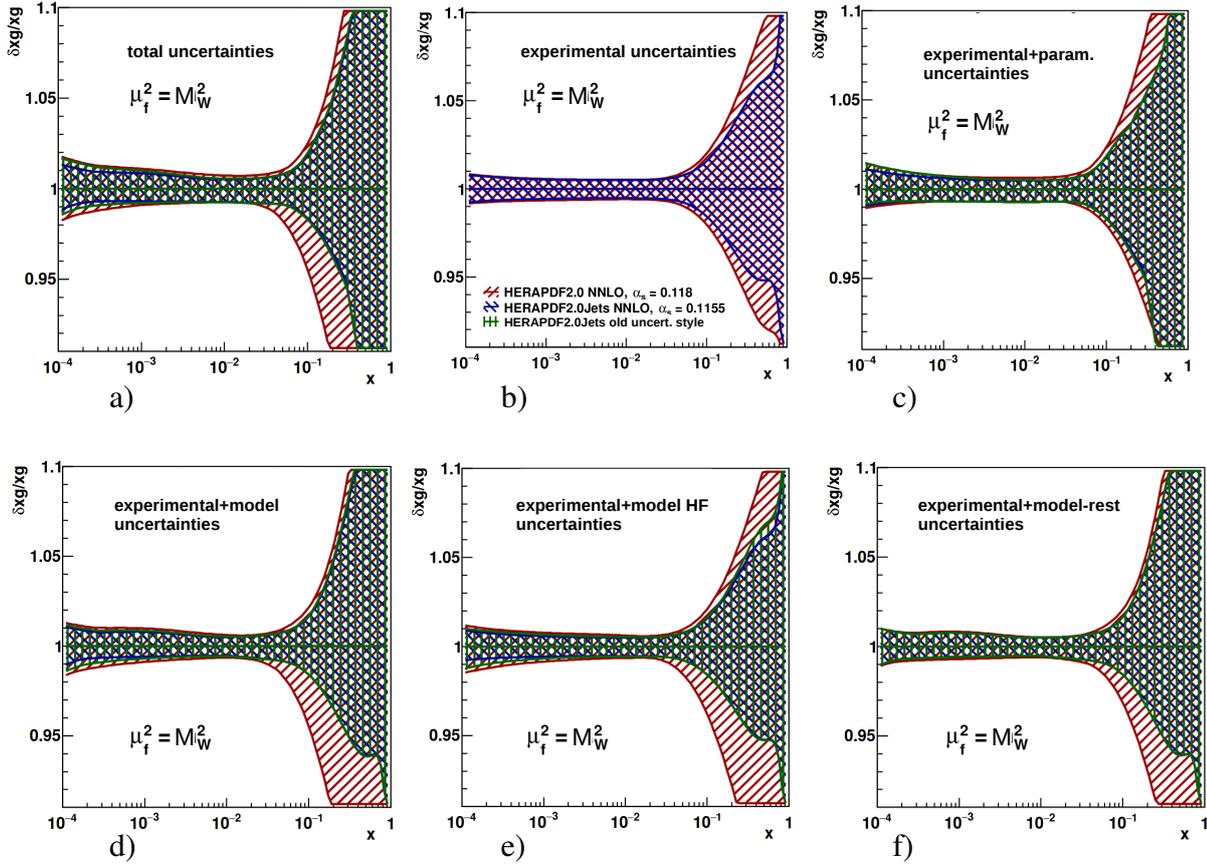
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More detailed information concerning the source of uncertainties at a scale of  $M_W^2$ : The green band represents HERAPDF2.0Jets NNLO  $\alpha_s(M_Z^2)=0.1155$  as obtained for the old procedure, i.e. with double counting.

This shows that the improvement is mainly due to jet data.



**Figure 25:** Comparison of the normalised uncertainties on the gluon PDFs of HERAPDF2.0Jets NNLO, HERAPDF2.0 NNLO and HERAPDF2.0Jets NNLO with old procedure on uncertainties for a) total, b) experimental, i.e. fit, c) experimental plus parameterisation, d) experimental plus model, e) experimental plus model due to heavy flavour f) experimental plus all model but heavy flavour uncertainties at the scale  $\mu_f^2 = M_W^2$ . The uncertainties on the three gluon distributions are shown as differently hatched bands.

# Internal extra material:

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Parameters as determined by the fits and their correlations

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PARAMETER CORRELATION COEFFICIENTS

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NO. GLOBAL 2 3 7 8 12 13 15 22 23 33 34 41 42 43

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600 7 0.99943 -0.891-0.325 1.000 0.920 0.109-0.063-0.112 0.034 0.029 0.014 0.004 0.028 0.032-0.025
601 8 0.99712 -0.656-0.056 0.920 1.000 0.231-0.111-0.221 0.092 0.076 0.012-0.013 0.010-0.027-0.035
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604 15 0.99429 -0.031-0.053-0.112-0.221-0.734 0.713 1.000-0.203-0.171 0.161 0.344-0.373-0.296-0.148
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607 33 0.99858 0.023 0.171 0.014 0.012 0.379 0.418 0.161 0.404 0.169 1.000 0.940-0.017-0.033 0.192
608 34 0.99841 0.033 0.230 0.004-0.013 0.134 0.433 0.344 0.331 0.115 0.940 1.000-0.223-0.229 0.228
609 41 0.96869 -0.145-0.374 0.028 0.010 0.442-0.118-0.373 0.265 0.233-0.017-0.223 1.000 0.953 0.287
610 42 0.97473 -0.204-0.465 0.032-0.027 0.340-0.092-0.296 0.220 0.196-0.033-0.229 0.953 1.000 0.264
611 43 0.98749 -0.027-0.372-0.025-0.035 0.171-0.132-0.148 0.625 0.530 0.192 0.228 0.287 0.264 1.000

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618 3 0.99467 0.584 1.000-0.086 0.184 0.146-0.004-0.071-0.148-0.192 0.160 0.233-0.432-0.517-0.453
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622 13 0.98045 -0.002-0.004-0.095-0.142-0.176 1.000 0.712-0.219-0.278 0.360 0.389-0.112-0.079-0.150
623 15 0.99461 -0.029-0.071-0.183-0.288-0.777 0.712 1.000-0.258-0.208 0.089 0.264-0.354-0.270-0.188
624 22 0.99185 -0.005-0.148 0.071 0.115 0.302-0.219-0.258 1.000 0.920 0.351 0.291 0.287 0.238 0.666
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630 43 0.99021 -0.067-0.453-0.030-0.053 0.216-0.150-0.188 0.666 0.556 0.135 0.157 0.337 0.312 1.000
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