Broken Lines Trajectory Following Kalman Residuals

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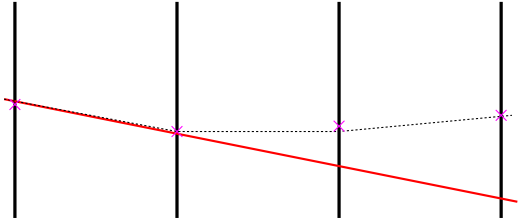
Outline

- Motivation
- Constructing the Trajectory
- χ^2 Comparison with Kalman

Reminder on Broken Lines

- Parameters $\boldsymbol{p}$:
  - offsets in curvilinear frame of each hit measurement,
  - q/p (or 9 parameters for TwoBodyDecayTrajectory).
- Measurements and uncertainties:
  - hit residuals,
  - multiple scattering kinks.
- The fit needs derivatives of the (current) prediction with respect to the parameters.

2D Illustration without B-Field



- **Hit**
- **Real Trajectory with Multiple Scattering**
- **ReferenceTrajectory: Extrapolation from First Layer**

- Real particle trajectory subject to multiple scattering.
- Current ReferenceTrajectory implementation simply extrapolates from prediction at first layer (with deterministic energy loss), i.e. measured kinks transferred to pede are all zero.
- Can lead to large residuals r.
- Problems (how large?):
  - Broken line track fit assumes small r ($\ll$ path length s [?]s).
  - Magnetic field effects sampled along wrong path.

Idea

Construct ReferenceTrajectory from

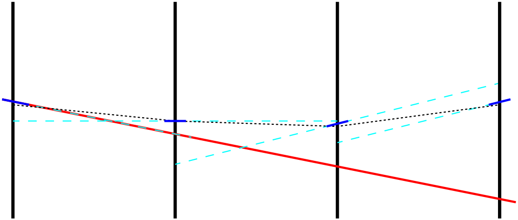
- Kalman hit residuals,
- multiple scattering angles as measured by Kalman fit.

Note

- Smoothed Kalman fit predicts trajectory state
  $T = (q/p, dx/dz, dy/dz, x, y)$ on each layer (local coordinates).
- Fit includes
  - effect of thin scatterers ($\Delta(dx/dz)$, $\Delta(dy/dz)$),
  - energy loss ($\Delta q/p$).

. . . due to bad understanding of Kalman fit procedure

- Extrapolation to next layer gives offsets $\Delta x \neq 0$ and $\Delta y \neq 0$ (like for thick scatterer).
- Sometimes energy gain, not loss (although $\langle p_{i+1} - p_i \rangle < 0$).



- **Kalman Predictions on Layers**
- **their Extrapolations**
- **Real Trajectory with Multiple Scattering**
- **ReferenceTrajectory: Extrapolation from First Kalman Prediction**

Explanation (Thanks Sascha!)

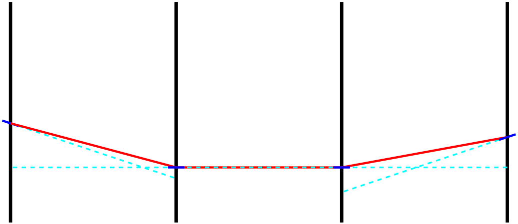
Although “process noise” comes from thin scatterer and E loss, **all five** trajectory parameters are updated.

Modified Approach

Broken lines fit for thin scatterers does not tolerate offsets: multiple scattering leads to kinks only.

Manipulate Kalman Predictions

- Hit residuals from pure Kalman predictions (refinement later).
- Multiple scattering kinks by
  - finding change of local direction
  - that makes the extrapolation
  - match Kalman position prediction at next layer.
- Before extrapolation adjust q/p according to energy loss in current layer (deterministically).



- **Kalman Predictions on Layers**
- **their Extrapolations**
- **Trajectory Following Kalman Position Predictions**

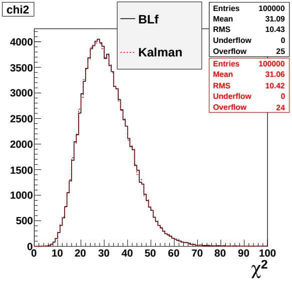
Implementation Details

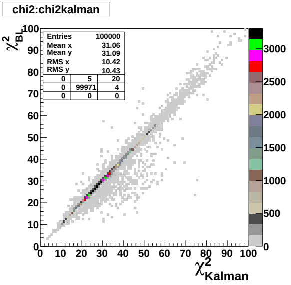
Manipulate Kalman Predictions

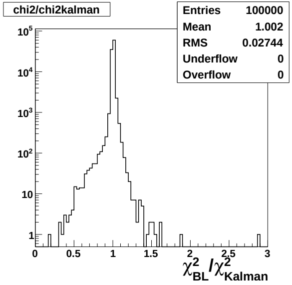
- Use “updated” TrajectoryStates, i.e. smoothed Kalman results.
- Extrapolations with RungeKutta propagator.
- Kink angles $\Delta(da/dz)$ calculated (iteratively) by solving
$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}_{next} = J \cdot \begin{pmatrix} \Delta(dx/dz) \\ \Delta(dy/dz) \end{pmatrix}_{current},$$
J sub-matrix of $\frac{\partial T_{next}}{\partial u_{next}} \cdot \frac{\partial u_{next}}{\partial u_{current}} \cdot \frac{\partial u_{current}}{\partial T_{current}}$,
$\frac{\partial u_{next}}{\partial u_{current}}$ from AnalyticalCurvilinearJacobian
u_i state in curvilinear frame at layer i.
- Stop iteration if $(\Delta x)^2 + (\Delta y)^2 < 0.02\ \mu m^2$.
- To get rid of inaccuracies below this level, calculate residuals with respect to the extrapolated state (should be done even if iteration does not converge - so far these few tracks are rejected?).
- No energy loss update going from first to second layer.

Kalman Fit vs BL Refit (in pede)

- Feed local y-residuals to pede for same $N(dof)$.
- Simulated isolated muons:
  - TkAlIsolatedMuon_Pt5
  - $p_t > 5$ GeV, $N(hit_{(2D)}) \geq 10(3)$.



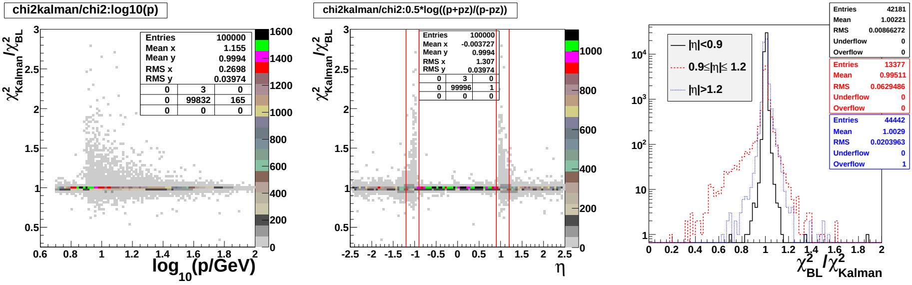




Kalman χ^2 Reproduced

- $\langle \chi^2_{BL} / \chi^2_{Kalman} \rangle$ off by only 2 per mille, RMS 2.7%.

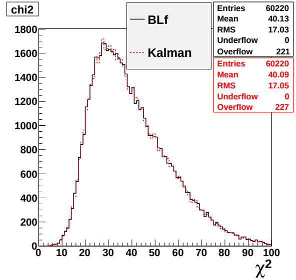
χ^2 Comparison vs. p_t, η

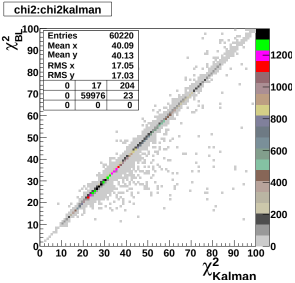


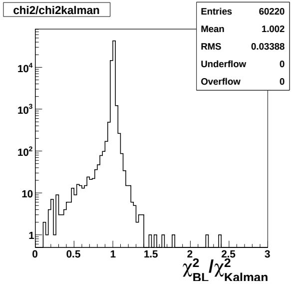
- Largest differences for $0.9 < |\eta| < 1.2$, i.e. transition from region barrel to endcap.
  - For $\log_{10}(p/\mathrm{GeV}) < 0.89$, i.e. $p < 7.8$ GeV: $|\eta| < 1$ in test sample.
- Assumption of thin scatterer probably not correct?
- Hardly see differences compared to current implementation.
  - Should compare for lower momenta.

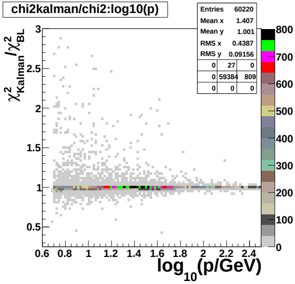
Fresh Results for Cosmics

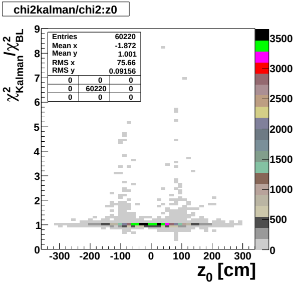
$p > 5$ GeV, $N(hit_{(2D)}) \geq 14(3)$











- Agrees well, too.
- RMS slightly larger: 3.4%.
- Problems again in transition region $|z_0| \approx 100$ cm.

Summary

- Current BrokenLines trajectory may not be perfect for low momenta.
- New approach constructs trajectory
  - following Kalman position predictions,
  - calculating scattering angles by enforcing vanishing multiple scattering offsets
  - and with deterministic energy loss treatment (like current implementation).
- Tested on isolated muons and cosmics,
  - generally good χ^2 agreement,
    i.e. $\chi^2_{BL}/\chi^2_{Kalman}$ with RMS = 2.7(3.4)% for isolated muons (cosmics)
  - largest RMS in transition region from barrel to endcap,
  - pretty similar for current trajectory.

Outlook

- Investigate cases that do not converge when finding kinks.
  - Accepting $\Delta x/y$ up to few μm gives still good χ^2 match.
- Test on samples with lower momenta.
- More detailed comparison with current implementation.
- Including MC alignment test.
- Extend to coarse BrokenLines.
- Provide CMSSW implementation:
  - keep current implementation configurable?
  - time scale?