Multiple parton interactions: some theoretical considerations

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MPI 10, Hamburg, 14 September 2010

Multiparton interactions

- phenomenology based on simple, physically intuitive formula

$$\text{cross section} = \text{multiparton distributions} \times \text{hard-scattering cross sections}$$

cf. e.g. Paver, Treleani 1982, 1984; Mekhfi 1985

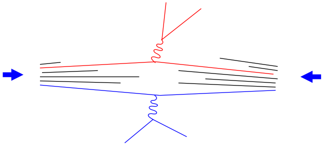
  questions:
  - to which extent can this formula be derived in QCD?
  - where and how does it need to be modified?
  - can a factorization theorem for multiparton interactions be formulated and proven?

- no definitive answers in present talk
  but some steps in this direction

MD, Ostermeier, Schäfer, in preparation

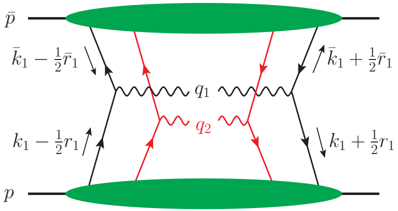
Theoretical framework

- require all interactions to have hard scale
  $\rightsquigarrow$ predictive power from factorization and pert. theory
- consider gauge boson pair production (pairs of γ^*, W, Z)



  - jet production highly relevant in practice
    but gluon exchange with spectator partons $\gg$ complicated
    for single dijet production cf. Mulders, Rogers 2010
- keep transverse momentum of bosons differential
  - are interested in final-state details (distributions, cuts etc)
  - need k_T dependent parton distributions
    basic formalism by Collins, Soper 1982
    recent work by Ji, Ma, Yuan 2004; Collins, Rogers, Stasto 2007

Basic structure: momentum



- longitudinal parton momenta $x_i p$, $\bar{x}_i \bar{p}$ fixed by final-state bosons
- transverse momentum balance:

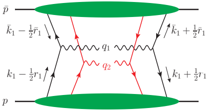
  $$\boldsymbol{q}_1 = (\boldsymbol{k}_1 \pm \tfrac{1}{2}\boldsymbol{r}_1) + (\bar{\boldsymbol{k}}_1 \pm \tfrac{1}{2}\bar{\boldsymbol{r}}_1) \quad \Rightarrow \quad \boldsymbol{r}_1 + \bar{\boldsymbol{r}}_1 = \boldsymbol{0}$$

  ⇝ transverse parton momenta not the same on left and right of final-state cut
- Fourier transform to impact parameter: $\boldsymbol{r}_1 \to \boldsymbol{y}$ and $\bar{\boldsymbol{r}}_1 \to \bar{\boldsymbol{y}}$

  $\boldsymbol{r}_1 + \bar{\boldsymbol{r}}_1 = \boldsymbol{0}$ implies $\boldsymbol{y} = \bar{\boldsymbol{y}}$

fully analogous for partons with index 2

Basic structure: cross section



- get cross section formula

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\;\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \left[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i \bar{x}_i s\big)\right]$$

$$\times \left[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i)\right] \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, \bar{F}(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

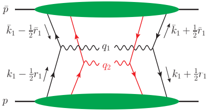
$\hat{\sigma}_i =$ parton-level cross section

$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = k_T$ dependent two-parton distribution

- result follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2$ in cross sect. $\rightarrow$ collinear distributions

$$F(x_i, \boldsymbol{y}) = \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$$

  recover usual cross section formula (slide 2)



Operator definitions

- k_T dependent distribution

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \int \frac{dz_2^-\, d^2\boldsymbol{z}_2}{(2\pi)^3}\, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2 \boldsymbol{k}_2} \int \frac{dz_1^-\, d^2\boldsymbol{z}_1}{(2\pi)^3}\, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1 \boldsymbol{k}_1}$$

$$\times\, 2p^+ \int dy^-\, \langle p|\bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2\, q\big(\tfrac{1}{2}z_2\big)\, \bar{q}\big(y - \tfrac{1}{2}z_1\big)\Gamma_1\, q\big(y + \tfrac{1}{2}z_1\big)|p\rangle\,\Big|_{z_i^+ = y^+ = 0}$$

- collinear distributions

$$F(x_i, \boldsymbol{y}) = \int \frac{dz_2^-}{2\pi}\, e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi}\, e^{ix_1 z_1^- p^+}$$

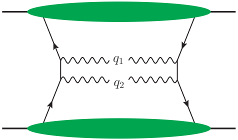
$$\times\, 2p^+ \int dy^-\, \langle p|\bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2\, q\big(\tfrac{1}{2}z_2\big)\, \bar{q}\big(y - \tfrac{1}{2}z_1\big)\Gamma_1\, q\big(y + \tfrac{1}{2}z_1\big)|p\rangle\,\Big|_{\substack{z_i^+ = y^+ = 0\\ \boldsymbol{z}_i = \boldsymbol{0}}}$$

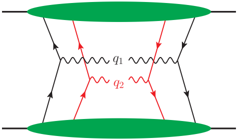
still $\boldsymbol{y} \neq \boldsymbol{0}$ $\Rightarrow$ fields at different transverse positions
$\Rightarrow$ not a twist-four operator
but product of two twist-two operators

Power behavior: single versus double hard scattering

- from scattering formulae readily find

$$s \frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\;\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2\Lambda^2} \qquad Q^2 \sim q_i^2,\, \Lambda^2 \sim \mathrm{GeV}^2$$

for both 

and 

$\Rightarrow$ double scattering not power suppressed

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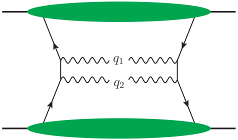
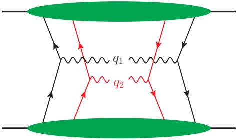
for both 

and 

$\Rightarrow$ double scattering not power suppressed

- but if integrate over $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ then

single: $$s \frac{d\sigma}{dx_1\, d\bar{x}_1\;\, dx_2\, d\bar{x}_2} \sim 1$$ since $\int d^2(\boldsymbol{q}_1 + \boldsymbol{q}_2) \sim \Lambda^2$ and $\int d^2(\boldsymbol{q}_1 - \boldsymbol{q}_2) \sim Q^2$

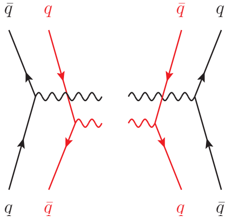
double: $$s \frac{d\sigma}{dx_1\, d\bar{x}_1\;\, dx_2\, d\bar{x}_2} \sim \frac{\Lambda^2}{Q^2}$$ since $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2 \sim \Lambda^4$

i.e. single hard scattering has larger phase space for transv. momenta previous formulae glossed over important details:

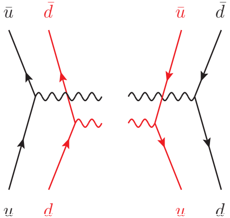
- quark flavors and quarks vs. antiquarks
- parton spin
- color
- scale dependence/evolution
- additional gluon exchange
  essential for factorization
  situation similar to single hard scattering, not discussed here

Interference effects

- so far: distributions with operators $\bar{q}_2 q_2\ \bar{q}_1 q_1$ $\rightsquigarrow$ double parton densities
  indices 1 and 2 refer to momentum fractions x_1, x_2
- but also have interference contributions (no probability interpretation)



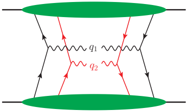
operator: $\bar{q}_2 \bar{q}_2\ q_1 q_1$



operator: $\bar{u}_2 d_2\ \bar{d}_1 u_1$

- must be included in cross section formula
  but is discarded in existing estimates

Spin structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \int \frac{dz_2^-\, d^2\boldsymbol{z}_2}{(2\pi)^3}\, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2 \boldsymbol{k}_2} \int \frac{dz_1^-\, d^2\boldsymbol{z}_1}{(2\pi)^3}\, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1 \boldsymbol{k}_1}$$

$$\times\, 2p^+ \int dy^- \;\langle p|\bar{q}(-\tfrac{1}{2}z_2)\Gamma_2\, q(\tfrac{1}{2}z_2)\; \bar{q}(y-\tfrac{1}{2}z_1)\Gamma_1\, q(y+\tfrac{1}{2}z_1)|p\rangle$$

- at leading twist for quarks: $\Gamma_i = \frac{1}{2}\gamma^+,\ \frac{1}{2}\gamma^+\gamma_5,\ \frac{1}{2}\sigma^{+\alpha}$
- spin correlations even in unpolarized target, e.g.

  $$\Gamma_1 = \Gamma_2 = \tfrac{1}{2}\gamma^+\gamma_5 \quad \Leftrightarrow \quad q_1^{\uparrow} q_2^{\uparrow} + q_1^{\downarrow} q_2^{\downarrow} - q_1^{\uparrow} q_2^{\downarrow} - q_1^{\downarrow} q_2^{\uparrow}$$

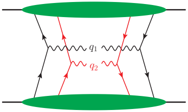
  note: not suppressed by hard scattering in double Drell-Yan
- transverse spin correlations from $\Gamma_1 = \Gamma_2 = \frac{1}{2}\sigma^{+\alpha}$

  $\rightsquigarrow$ $\cos 2\phi$ modulation between decay planes of the two bosons

  at low $\boldsymbol{q}_1, \boldsymbol{q}_2$ single hard scattering does not give $\cos 2\phi$ term

  in general: correlated decay planes

Spin structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{i x_2 z_2^- p^+ - i \boldsymbol{z}_2 \boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{i x_1 z_1^- p^+ - i \boldsymbol{z}_1 \boldsymbol{k}_1}$$
$$\times \, 2p^+ \int dy^- \, \langle p | \bar{q}(-\tfrac{1}{2} z_2) \Gamma_2 \, q(\tfrac{1}{2} z_2) \, \bar{q}(y - \tfrac{1}{2} z_1) \Gamma_1 \, q(y + \tfrac{1}{2} z_1) | p \rangle$$

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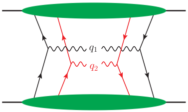
  note: not suppressed by hard scattering in double Drell-Yan
- may naively expect spin effects to decrease for small x_1, x_2, but
  - will see counter-example on slide 15
  - for $x_1 \sim x_2 \ll 1$ spin correlations may be
    weak a parton and the proton (far away in rapidity)
    important between two partons (close in rapidity)

Color structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2 \boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1 \boldsymbol{k}_1}$$
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- operators $\bar{q}_2 q_2$ and $\bar{q}_1 q_1$ can couple to color singlet or octet:

$$F_1 \longrightarrow (\bar{q}_2 \mathbb{1} q_2) \, (\bar{q}_1 \mathbb{1} q_1)$$
$$F_8 \longrightarrow (\bar{q}_2 t^a q_2) \, (\bar{q}_1 t^a q_1) = \tfrac{1}{2} (\bar{q}_2 \mathbb{1} q_1) \, (\bar{q}_1 \mathbb{1} q_2) - \tfrac{1}{2N_c} \, (\bar{q}_2 \mathbb{1} q_2) \, (\bar{q}_1 \mathbb{1} q_1)$$

- relative weight in gauge boson pair production:

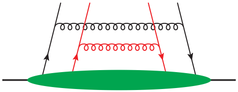
$$\tfrac{1}{N_c^2} \, F_1 \bar{F}_1 + \tfrac{4}{N_c^2 - 1} \, F_8 \bar{F}_8$$

- color octet distributions essentially unknown
  (no probability interpretation as a guide)

spin and color correlations discussed by M Mekhfi, PRD32 (1985)
but apparently not followed up in literature

High q_T: more predictive power

- consider region $\Lambda \ll q_T \ll Q$, with $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- k_T dependent distr'n = hard scattering $\otimes$ collinear distr'n
  hard scattering closely related to DGLAP splitting functions
- case 1: $|\boldsymbol{r}_1| \sim \Lambda$, i.e. $|\boldsymbol{y}| \sim 1/\Lambda$ of hadronic size
  $\rightsquigarrow$ independent hard scatters for pair 1 and 2



- color factors: C_F^2 for singlet F_1
  $\left(\frac{1}{2N_c}\right)^2$ for octet F_8
- singlet also favored in gluon sector

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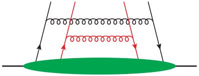
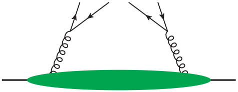
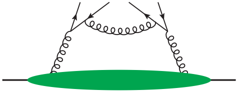


- case 2: $|\boldsymbol{r}_1| \sim |\boldsymbol{q}_i|$, i.e. $|\boldsymbol{y}| \ll 1/\Lambda$ small





  longitudinal q and $\bar{q}$ spins fully correlated
  also find sizeable transverse spin correlation
- find: case 1 is power suppressed by Λ^2/q_T^2 compared with case 2
  but has small-x enhancement

Evolution of collinear distributions

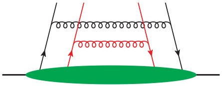
consider only color singlet combination F_1, situation for F_8 more complicated

$$F(x_i, \boldsymbol{y}) = \int \frac{dz_2^-}{2\pi}\, e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi}\, e^{ix_1 z_1^- p^+}$$
$$\times\, 2p^+ \int dy^- \,\langle p|\bar{q}(-\tfrac{1}{2}z_2)\Gamma_2\, q(\tfrac{1}{2}z_2)\, \bar{q}(y - \tfrac{1}{2}z_1)\Gamma_1\, q(y + \tfrac{1}{2}z_1)|p\rangle\,\Big|_{z_i^+ = y^+ = 0,\ \boldsymbol{z}_i = \boldsymbol{0}}$$

- $F(x_i, \boldsymbol{y})$ for $\boldsymbol{y} \neq \boldsymbol{0}$:

  separate DGLAP evolution for pair 1 and 2

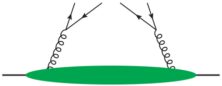
  $$\frac{d}{d\log\mu} F(x_i, \boldsymbol{y}) = P \otimes_{x_1} F + P \otimes_{x_2} F$$



- $\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})$:

  extra term from $2 \to 4$ parton transition

  Kirschner 1979; Shelest, Snigirev, Zinovev 1982
  recent study: Gaunt and Stirling

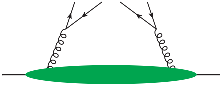


Evolution of collinear distributions: a closer look

- recall: in process cross sections need $\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})\, \bar{F}(\bar{x}_i, \boldsymbol{y})$
- $2 \to 4$ splitting graphs give contribution

$$F(x_i, \boldsymbol{y})\big|_{2\to 4} \propto 1/\boldsymbol{y}^2$$

logarithmic UV divergence only when $\int d^2\boldsymbol{y}$



- $\rightsquigarrow$ consistency problem for ansatz

$$F(x_i, \boldsymbol{y}; \mu) = G(\boldsymbol{y}) \left[\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y}) \right]_{\mu}$$

if on r.h.s. include $2 \to 4$ term in evolution

- may want to define modified $F(x_i, \boldsymbol{y})$
  where short-distance $2 \to 4$ splitting is removed altogether
  but has not been worked out

Sudakov factors

- cross section differential in $\boldsymbol{q}_i$ contains Sudakov logarithms
- can adapt Collins-Soper-Sterman formalism
  (originally developed for single Drell-Yan and similar proc's)
  $\rightsquigarrow$ include and resum Sudakov logs in k_T dependent parton distr's
- at leading double log accuracy: same Sudakov factor
  for singlet and octet distr's F_1 and F_8
- for $q_T \sim \Lambda$
  - Sudakov factors mix singlet and octet distr's
    mixing only suppressed by $1/N_c$
  - generically Sudakov factors of same size for singlet and octet
- for $q_T \gg \Lambda$ and $|\boldsymbol{y}| \sim 1/\Lambda$
  - singlet F_1 decouples from octet F_8
  - octet distr'n has extra suppression by factor

$$\sim \exp\Big[\text{const.}\, \alpha_s \log \frac{|\boldsymbol{z}_i|}{|\boldsymbol{y}|} \log(Q|\boldsymbol{z}_i|)\Big]$$

with $|\boldsymbol{z}_i|/|\boldsymbol{y}| \sim \Lambda/q_T$ and $Q|\boldsymbol{z}_i| \sim Q/q_T$

Summary

- multiple hard interactions not power suppressed
  for cross section differential in $|\boldsymbol{q}_i| \ll Q$
- nontrivial spin and color structure
  interference in fermion number and quark flavor
  size of these effects presently unknown
- some simplification for transv. mom. $|\boldsymbol{q}_i| \gg \Lambda$
  - collinear distributions as input
  - enhancement of color singlet combinations

  further studies needed to make this quantitative
- need multi-parton distr's depending on transverse distance between partons
- for finite $\boldsymbol{y}$ (not $\int d^2\boldsymbol{y}$) multi-parton distr's evolve with simple sum of ordinary DGLAP kernels