PHYSICS AT THE TERA SCALE

Helmholtz Alliance

Monte Carlo simulation and calculations for high-energy physics

http://www.terascale.de

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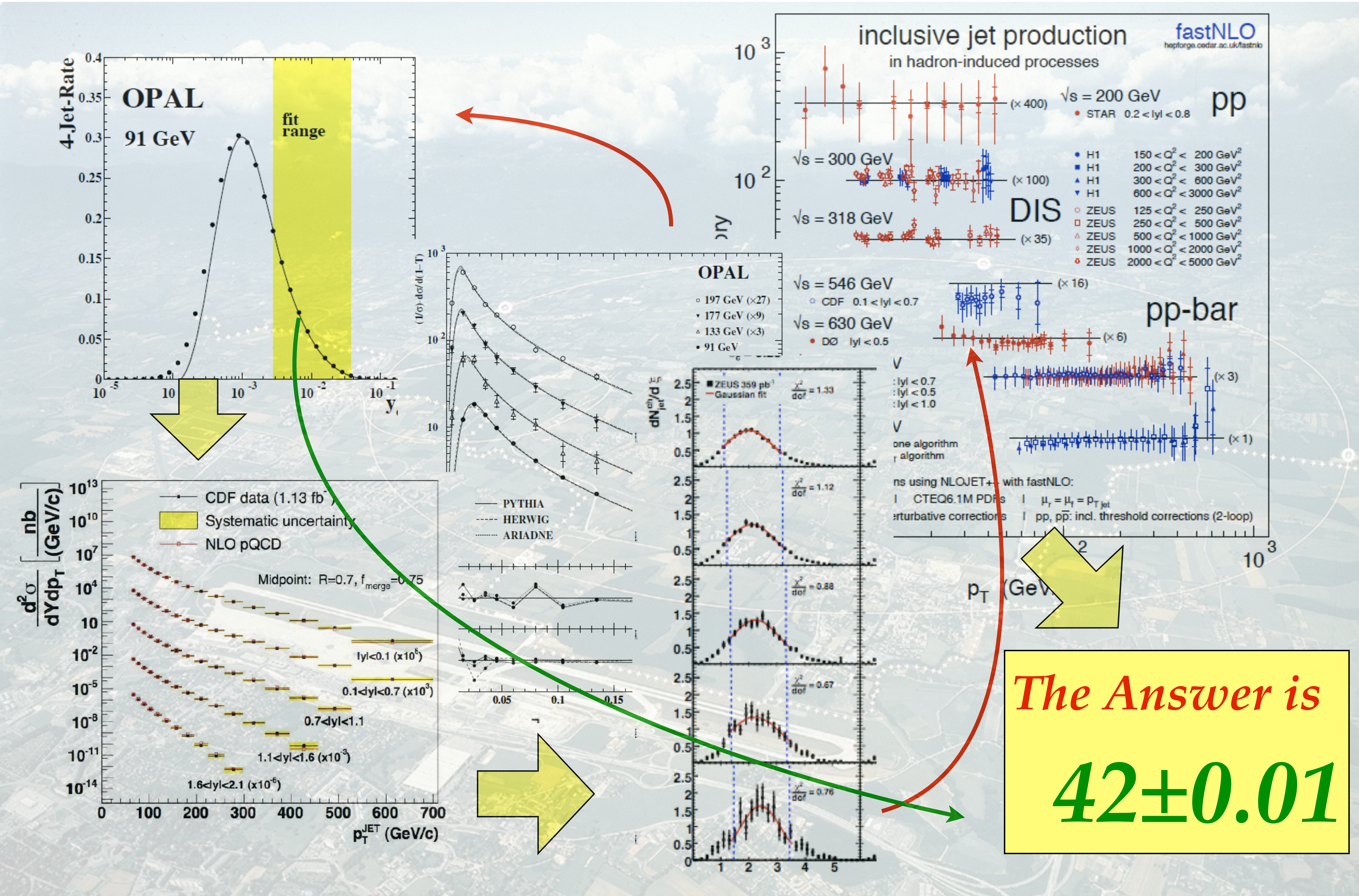
HEP School

February 23, 2011, Hamburg

Experimentalists' lectures

Warnings



Warnings

Theorists' lectures

$$\mathcal{H}_S \sim -\sum_{\substack{l,k\\ l\neq k}} \frac{\hat{p}_l\cdot\varepsilon(s)\,\hat{p}_k\cdot\varepsilon(s')}{\hat{p}_l\cdot\hat{p}_{m+1}\,\hat{p}_k\cdot\hat{p}_{m+1}}\, t_l\otimes t_k^\dagger$$

$$\mathcal{H}_C \sim \sum_l t_l\otimes t_l^\dagger\, V_{ij}(s_i,s_j)\otimes V_{ij}^\dagger(s_i',s_j') \Leftrightarrow \frac{\alpha_s}{2\pi}\sum_l \frac{1}{p_i\cdot p_j} P_{f_l,f_i}(z)+\dots$$

$$I=\lim_{\epsilon\to 0}\left[\int_0^1 \frac{dx}{x}\,x^{-\epsilon} f(x)+\frac{1}{\epsilon}f(0)\right]$$

$$\sigma[F]=\sum_m \int [d\{p,f\}_m]\,\overbrace{f_{a/A}(\eta_a,\mu_F^2)\,f_{b/B}(\eta_b,\mu_F^2)}^{\text{parton distributions}}\,\frac{1}{2\eta_a\eta_b p_A\cdot p_B}$$

$$\times\,\langle\mathcal{M}(\{p,f\}_m)|\underbrace{F(\{p,f\}_m)}_{\text{observable}}\underbrace{|\mathcal{M}(\{p,f\}_m)\rangle}_{\text{matrix element}}$$

$$\mathcal{N}(t',t)=\mathbb{T}\exp\left\{-\int_t^{t'} d\tau\,\mathcal{V}_I(\tau)\right\}$$

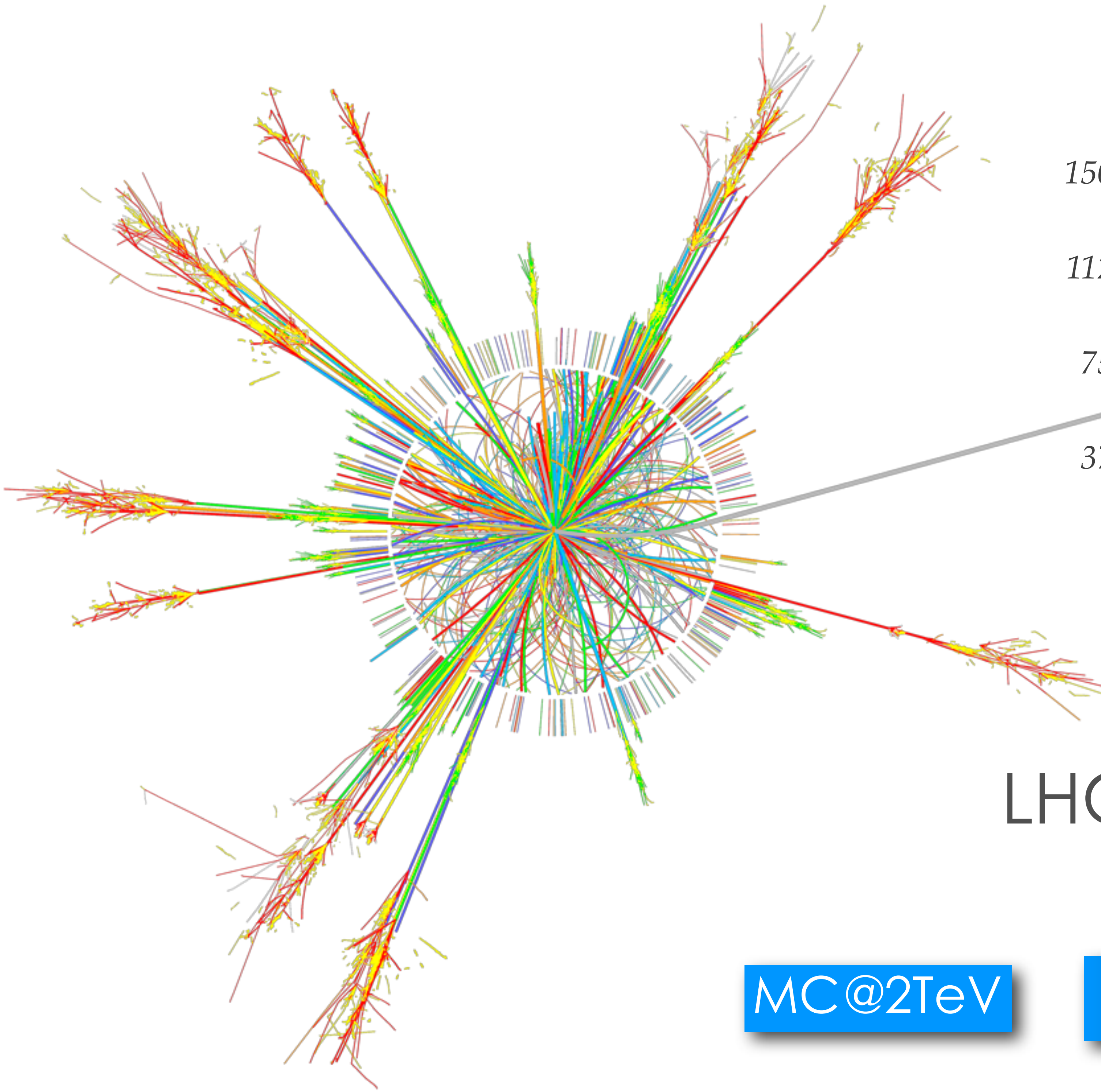
$$|\rho_\infty^{\mathrm{V}})\approx -\int_t^\infty d\tau\,\mathcal{V}_I^{(\epsilon)}(\tau)\,|\rho(t))$$

The Answer is

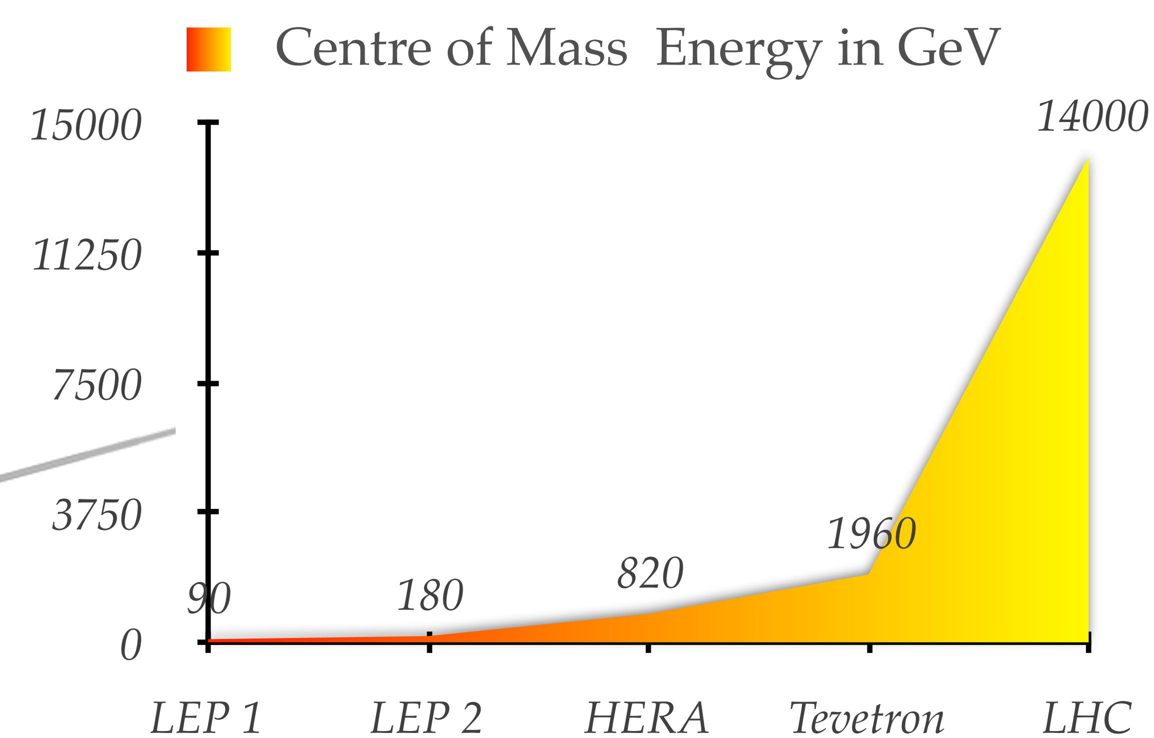
42

Introduction

The LHC is running and we will have to deal with the data soon.



Picture: ATLAS simulation

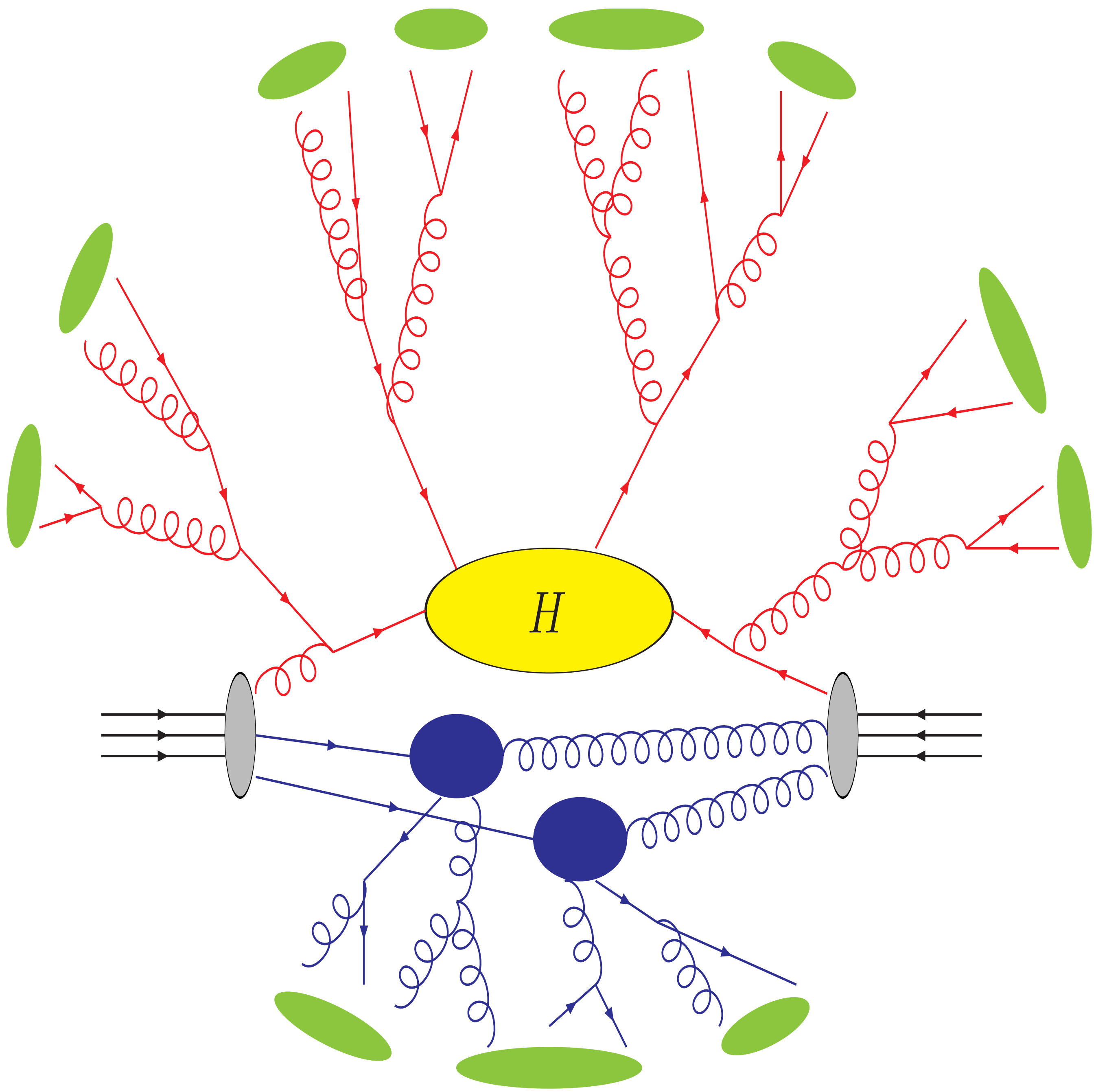


LHC = QCD + ε

MC@2TeV → *Extrapolation* → MC@14TeV

Introduction

The structure of the Monte Carlo event generators



1. Incoming hadron (gray bubbles)
   - Parton distribution function

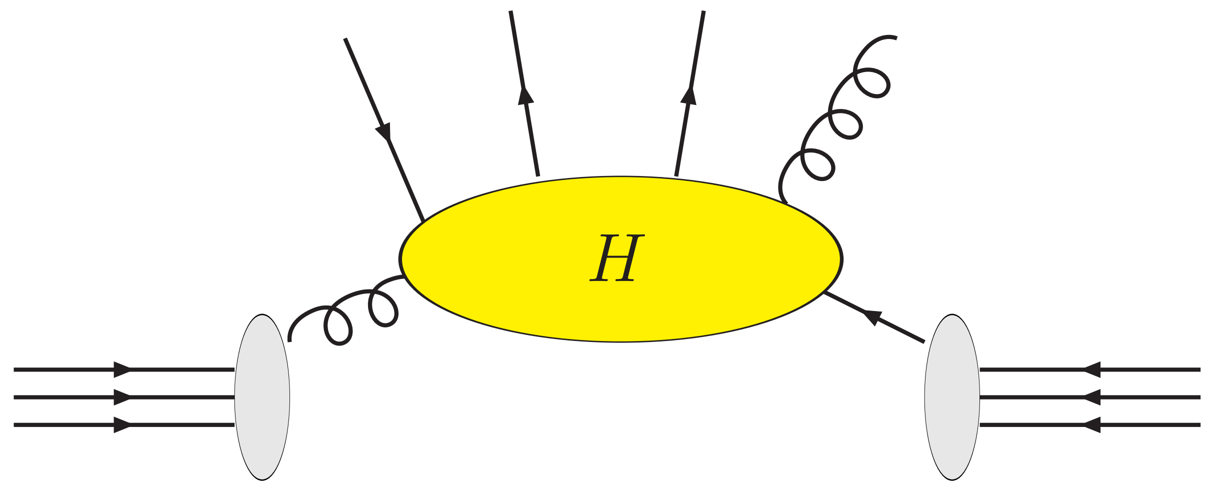
2. Hard part of the process (yellow bubble)
   - Matrix element calculation, cross sections at LO, NLO, NNLO level

3. Radiations (red graphs)
   - Parton shower calculation
   - Matching to the hard part

4. Underlying event (blue graphs)
   - Models based on multiple interaction

5. Hardonization (green bubbles)
   - Universal models

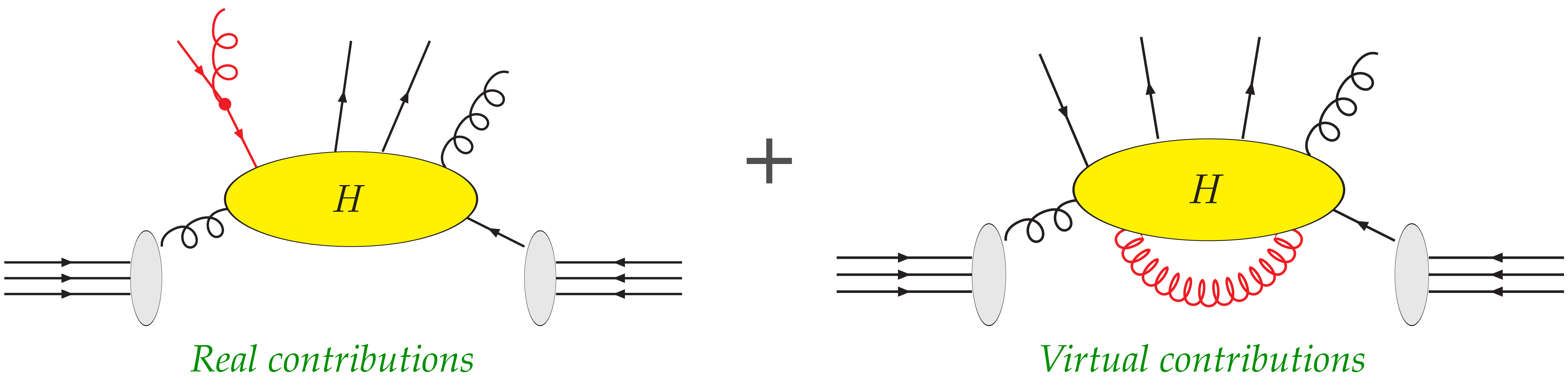
Born Level Calculation



$$\sigma[F_J] = \int_m d\Gamma^{(m)}(\{p\}_m)|\mathcal{M}(\{p\}_m)|^2 F_J(\{p\}_m)$$

- ✓ Easy to calculate, no IR singularities. Several matrix element generators are available (Alpgen, Helac, MadGraph, Sherpa)
- ✗ Strong dependence on the unphysical scales (renormalization and factorization scales)
- ✗ Exclusive quantities suffer on large logarithms
- ✗ Every jet is represented by a single parton
- ✗ No quantum corrections
- ✗ No hadronization

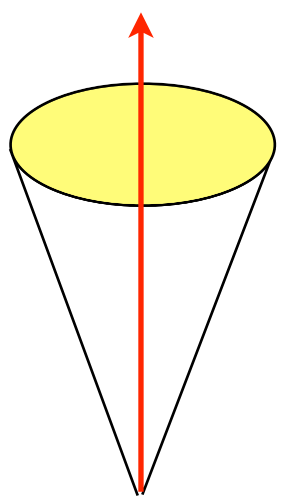
NLO Level Calculation



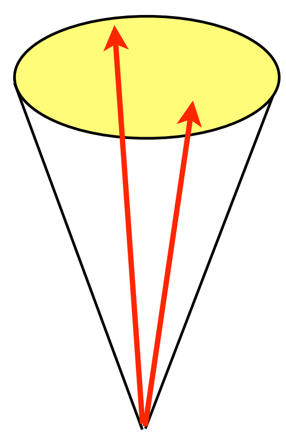
- ✓ Includes quantum corrections, in most of the cases it significantly reduces the unphysical scale dependences
- ✓ One of the jets consists of two partons (still very poor)
- ✓ Hard to calculate, the most complicated available processes are $2 \to 3$ (NLOJET++, MCFM, PHOX,..., even automated tools are available)
- ✗ Exclusive quantities suffer on large logarithms
- ✗ No hadronization

NLO Jet Structure

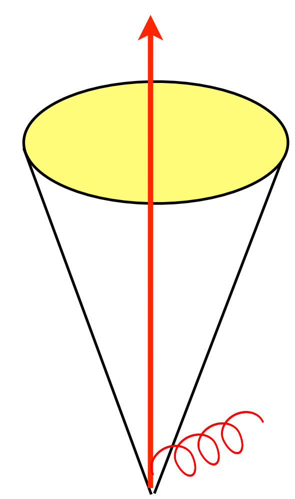
At Born level every jet is represented by one parton.



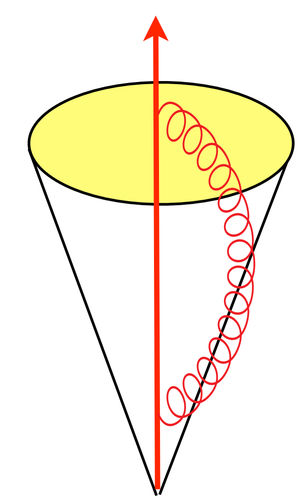
The collinear pair or the soft gluon is unresolvable, we have to integrate out these radiations. The observable is insensitive for these type of radiations.



Collinear radiation



Soft gluon radiation

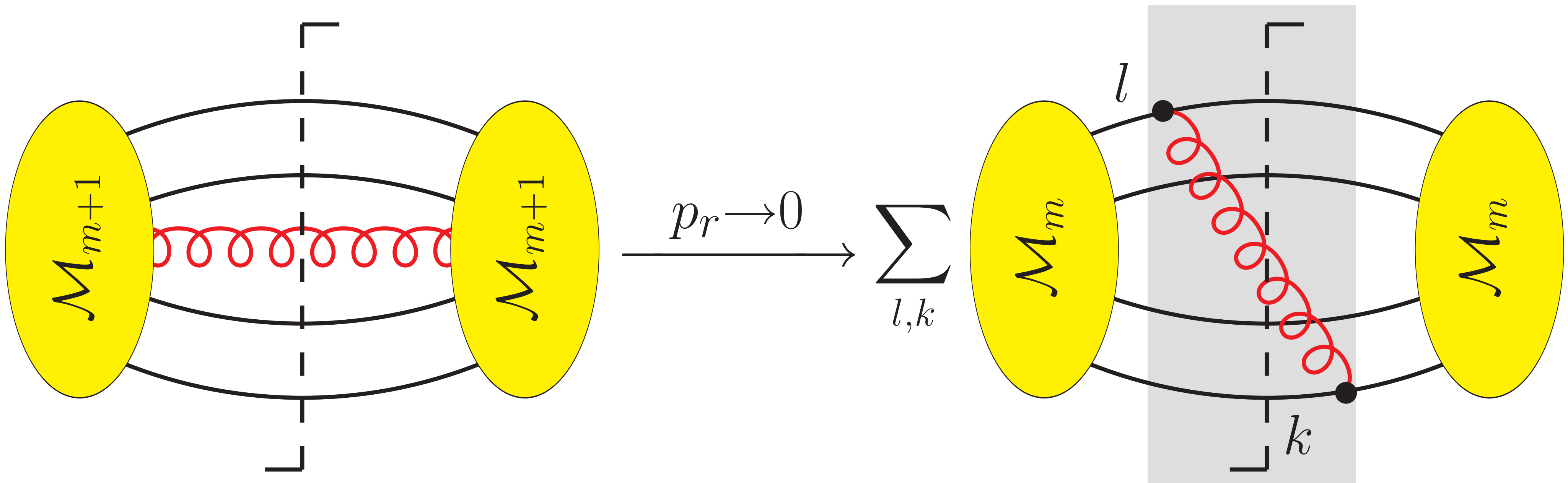


Virtual radiation

At NLO level one of the jets consists of two partons or one parton with virtual radiation.

Soft Singularities

The QCD matrix elements have universal factorization property when an external gluon becomes soft

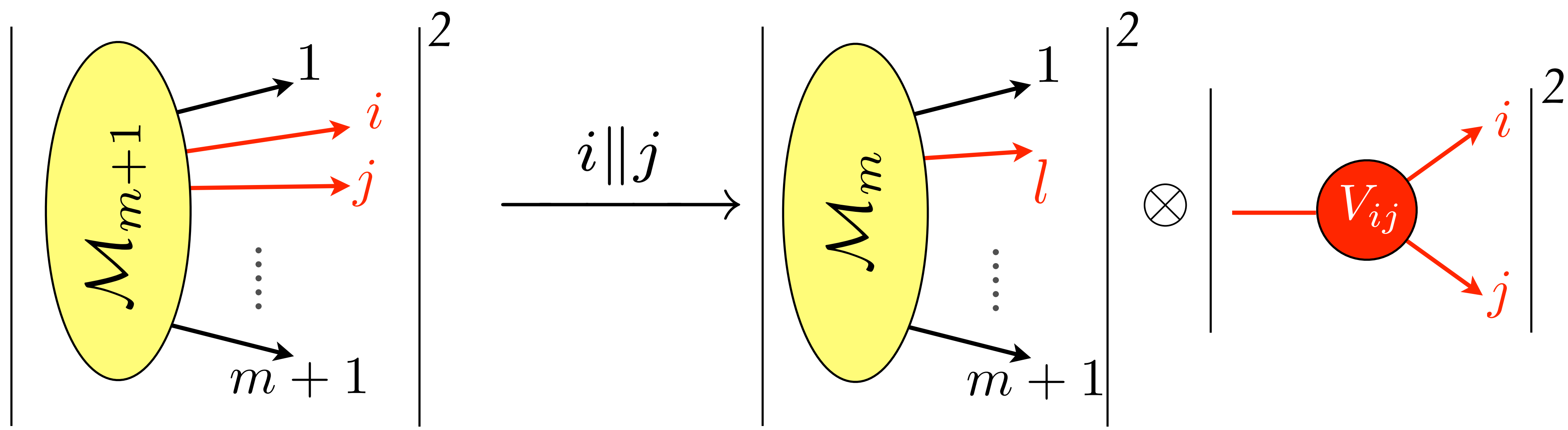


$$\mathcal{H}_S \sim -\sum_{\substack{l,k \\ l \neq k}} \frac{\hat{p}_l \cdot \varepsilon(s)\, \hat{p}_k \cdot \varepsilon(s')}{\hat{p}_l \cdot \hat{p}_{m+1}\, \hat{p}_k \cdot \hat{p}_{m+1}}\, t_l \otimes t_k^\dagger$$

Soft gluon connects everywhere and the color structure is not diagonal; quantum interferences in the color space.

Collinear Singularities

The QCD matrix elements have universal factorization property when two external partons become collinear



$$\mathcal{H}_C \sim \sum_l t_l \otimes t_l^\dagger \, V_{ij}(s_i, s_j) \otimes V_{ij}^\dagger(s_i', s_j') \Leftrightarrow \frac{\alpha_s}{2\pi} \sum_l \frac{1}{p_i \cdot p_j} P_{f_l, f_i}(z) + \dots$$

Altarelli-Parisi splitting kernels

1D NLO Problem

We want to calculate the following integral numerically

$$I = \lim_{\epsilon \to 0} \left[\int_0^1 \frac{dx}{x} x^{-\epsilon} f(x) + \frac{1}{\epsilon} f(0) \right]$$

We regularize the first term by a subtraction term that has the same singularity structure but it is a simpler function.

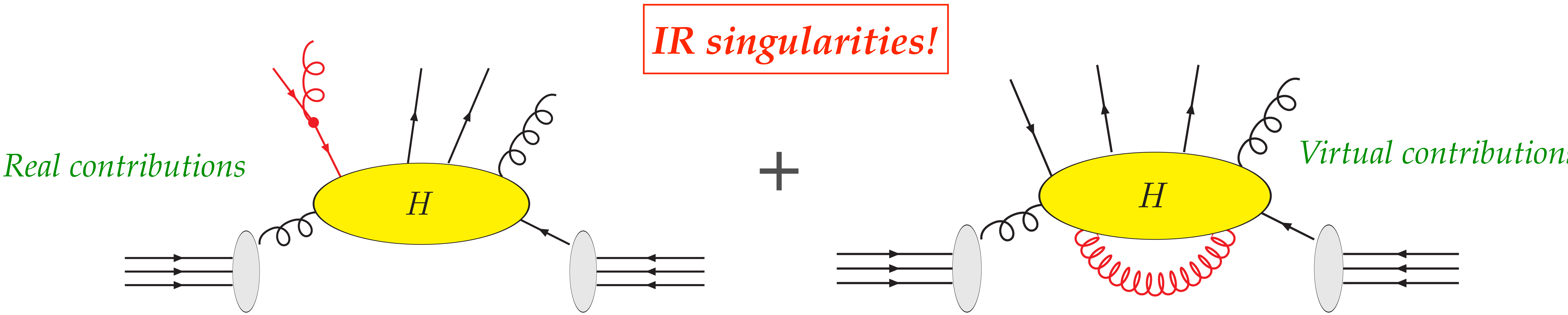
This is finite and can be done numerically.

$$I = \lim_{\epsilon \to 0} \left\{ \int_0^1 \frac{dx}{x} x^{-\epsilon} [f(x) - f(0)] + f(0) \int_0^1 \frac{dx}{x} x^{-\epsilon} + \frac{1}{\epsilon} f(0) \right\}$$

This is simple and can be done analytically.

$$I = \int_0^1 \frac{dx}{x} [f(x) - f(0)]$$

NLO Subtraction Scheme



$$\sigma_{\rm NLO} = \int_N d\sigma^B + \int_{N+1} \left[d\sigma^R - d\sigma^A\right]_{\epsilon=0} + \int_N \left[d\sigma^V + \int_1 d\sigma^A\right]_{\epsilon=0}$$

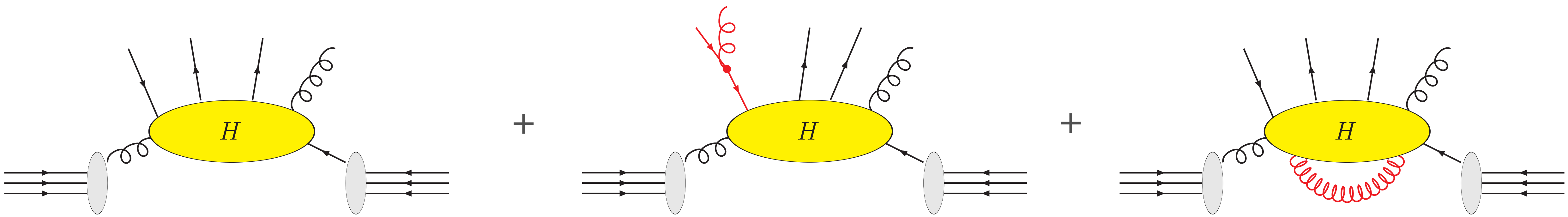
$$d\sigma^A \sim d\Gamma(\{p\}_{N+1})\, \underbrace{V \otimes |\mathcal{M}(\{\tilde{p}\}_N)|^2}_{\text{Based on soft and collinear factorization}} F_J(\{\tilde{p}\}_N)$$

Experimenter's NLO Wish List

Single boson	*Diboson*	*Triboson*	*Heavy Flavor*
Run II Monte Carlo Workshop, April 2001			
$V + \leq 5 jets$ $V + bb + \leq 3 jets$ $V + cc + \leq 3 jets$	$VV + \leq 5 jets$ $VV + bb + \leq 3 jets$ $VV + cc + \leq 3 jets$ $WZ + \leq 5 jets$ $WZ + bb + \leq 3 jets$ $WZ + cc + \leq 3 jets$ $W\gamma + \leq 3 jets$ $Z\gamma + \leq 3 jets$	$WWW + \leq 3 jets$ $WWW + bb + \leq 3 jets$ $WWW + cc + \leq 3 jets$ $Z\gamma\gamma + \leq 3 jets$ $WZZ + \leq 3 jets$ $ZZZ + \leq 3 jets$	$tt + \leq 3 jets$ $bb + \leq 3 jets$ $tt + V + \leq 2 jets$ $tt + H + \leq 2 jets$ $tb + \leq 2 jets$
Les Houches Workshop 2005			
$V + 3 jets$ $H + 2 jets$	$VV + \leq 2 jets$ $VV + bb$	ZZZ	$tt + 2 jets$ $tt + bb$
$V \in \{W, Z, \gamma\}$			

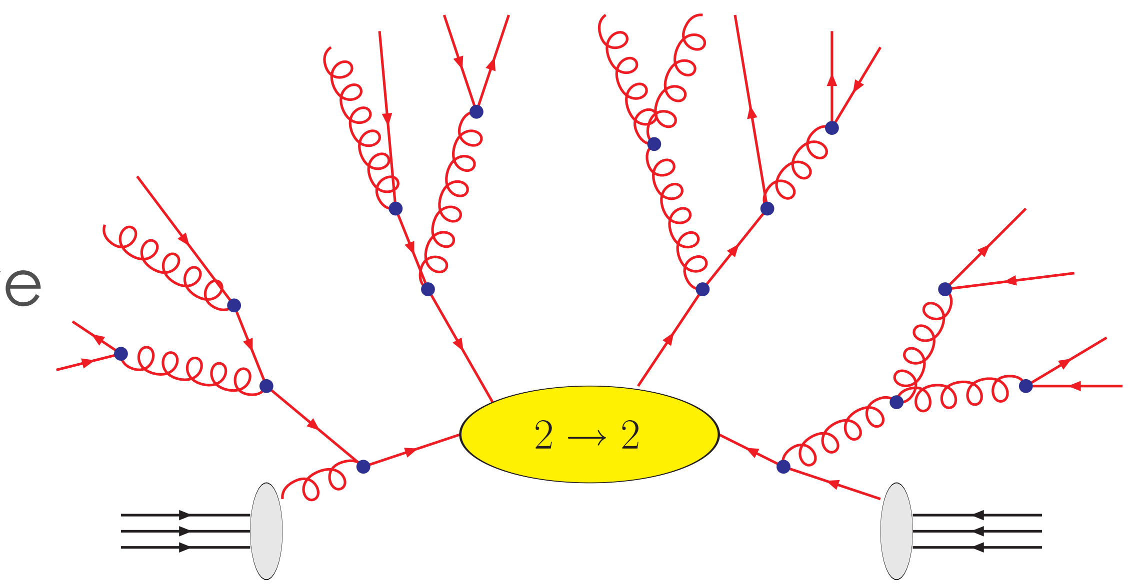
Why are these calculations so hard?

Why do we need parton shower?



Born contributions *Real contributions* *Virtual contributions*

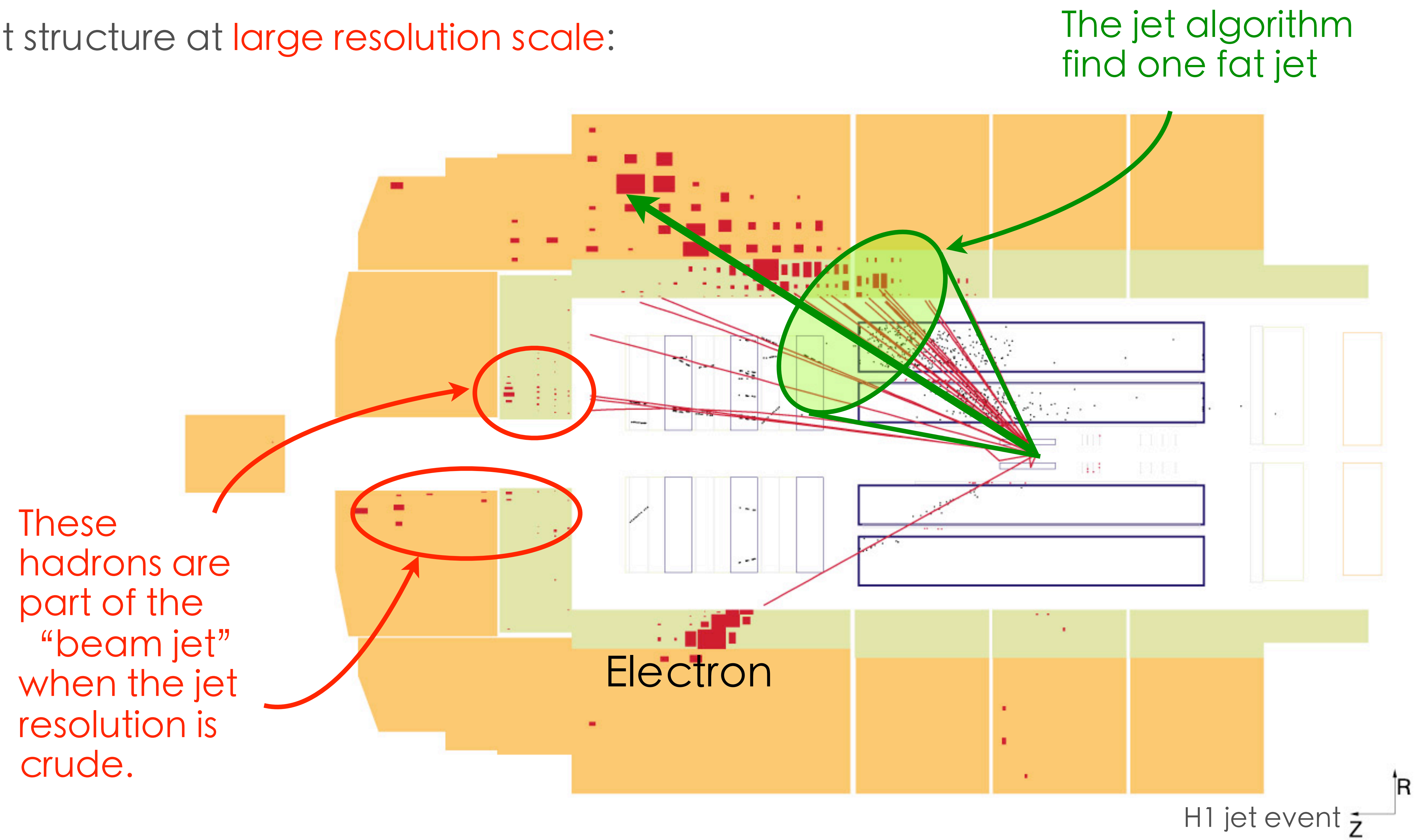
- We need predictions for LHC and Tevatron.
- LO and NLO perturbation theory can give predictions only for very inclusive cross sections.



- We use parton shower to get prediction for the complete final state approximately right.

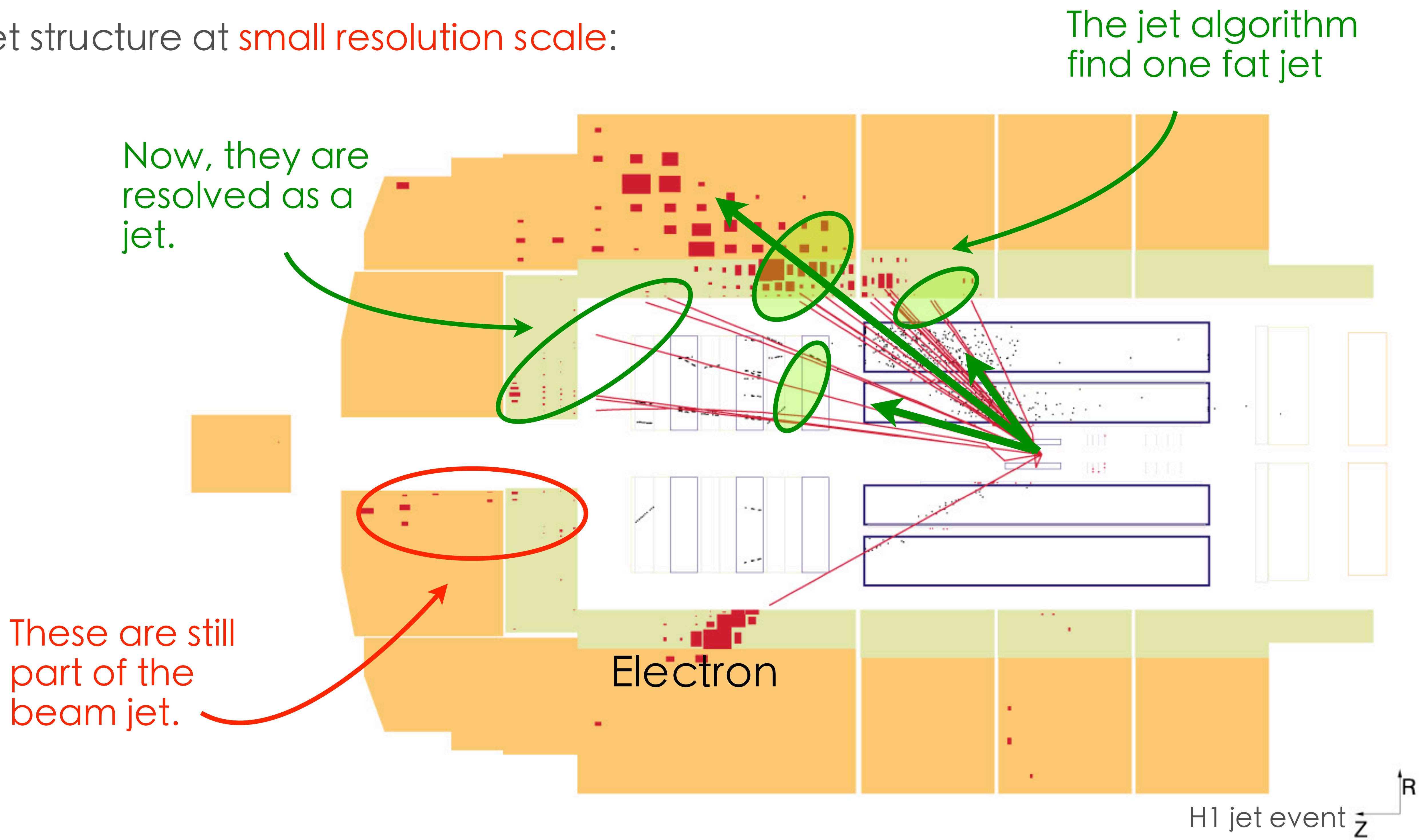
Jet event in DIS process

Jet structure at large resolution scale:



Jet event in DIS process

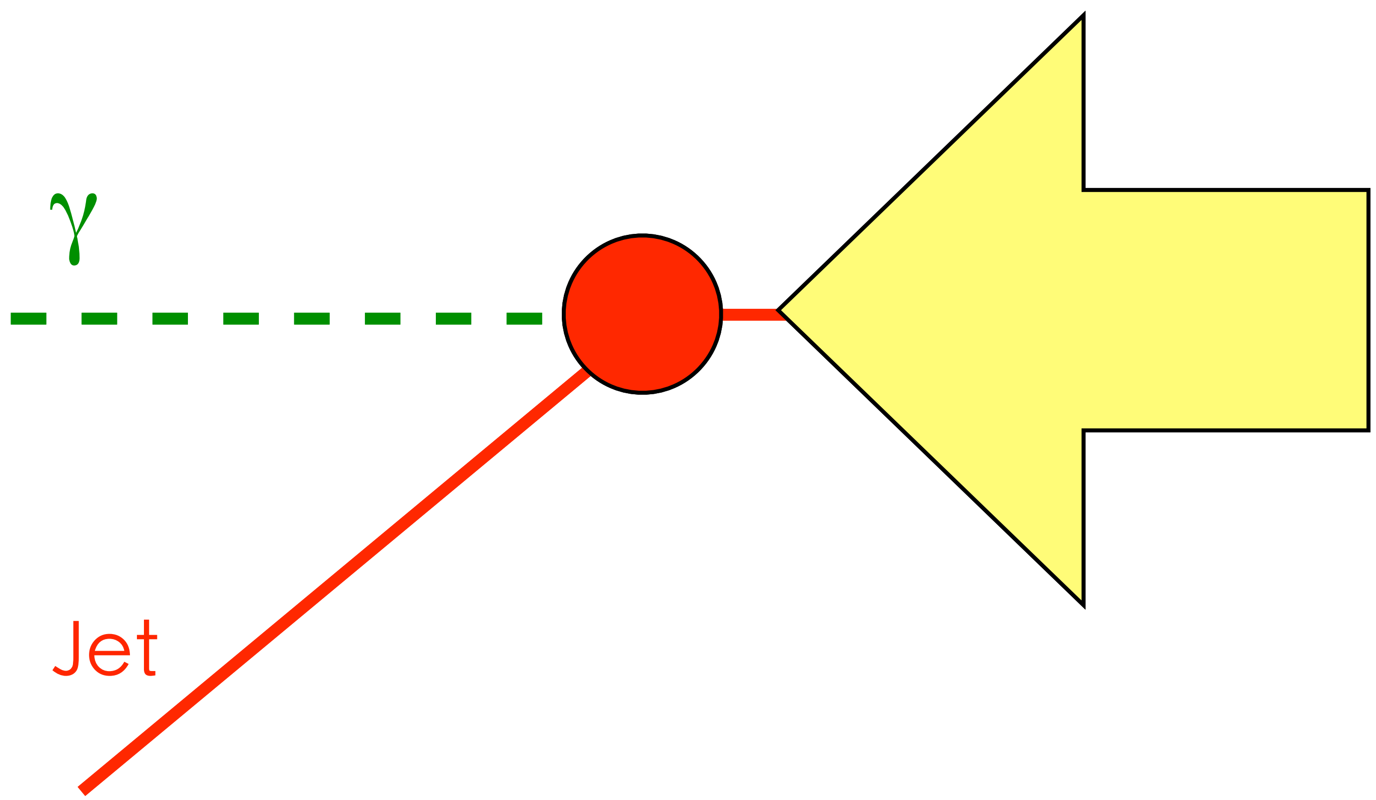
Jet structure at small resolution scale:



Deep Inelastic Scattering

Let us focus on the initial state radiations

$$\mu = 100\,\mathrm{GeV}$$

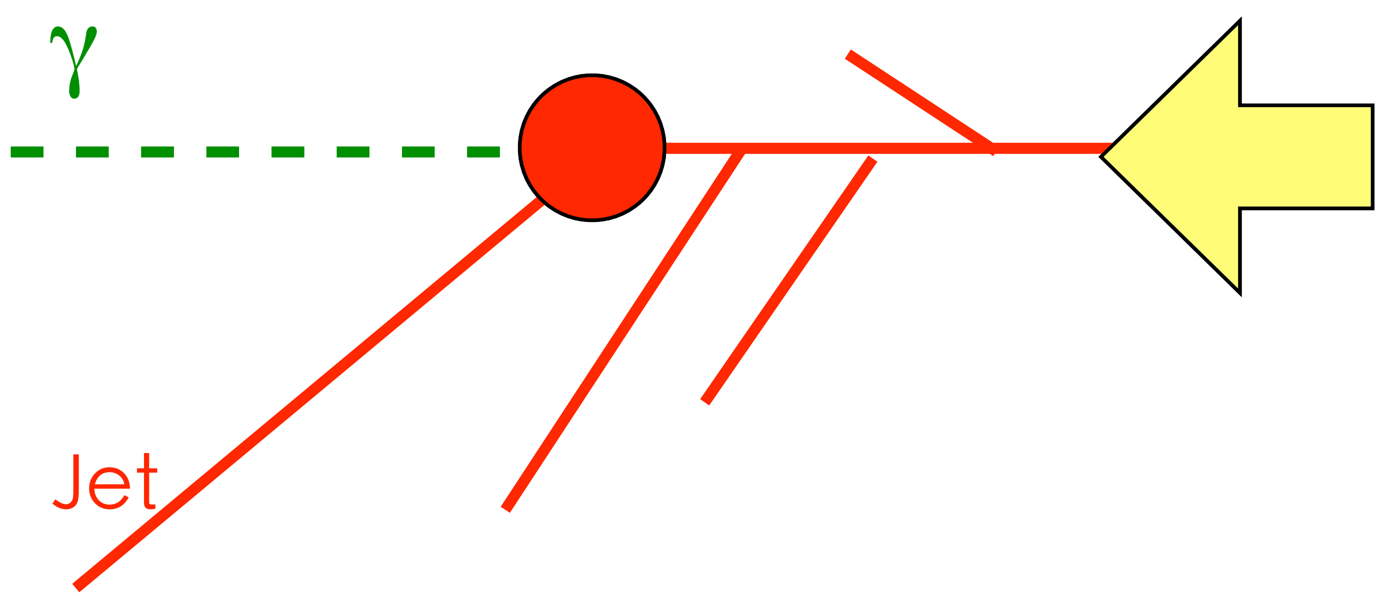


Every measurement has a typical resolution scale. Decreasing this resolution scale we can see more partonic (hadronic) activity and finer structures. Increasing this resolution scale we see fatter jets or cruder structures.

Deep Inelastic Scattering

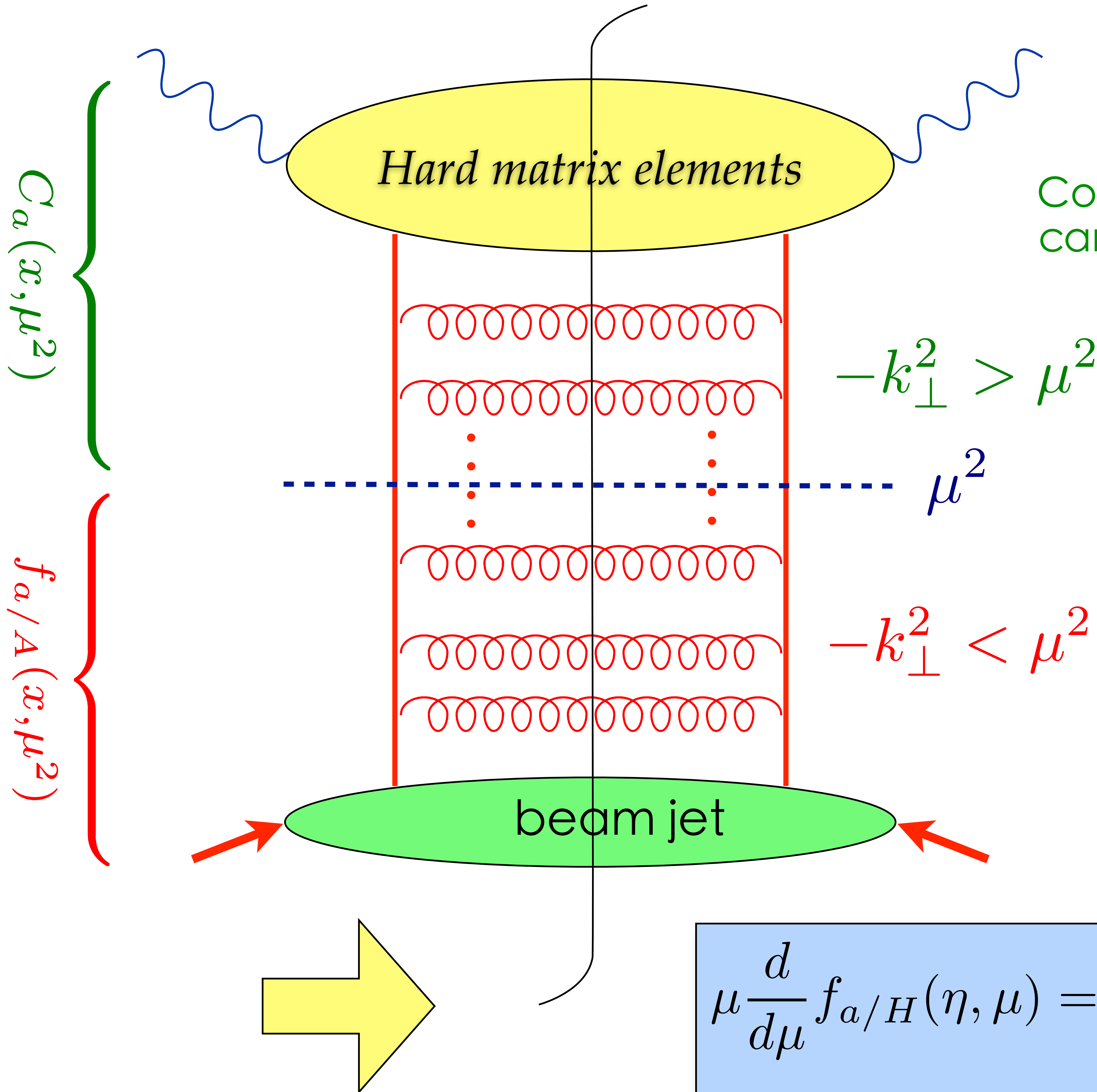
Let us focus on the initial state radiations

$$\mu = 25\,\mathrm{GeV}$$



Every measurement has a typical resolution scale. Decreasing this resolution scale we can see more partonic (hadronic) activity and finer structures. Increasing this resolution scale we see fatter jets or cruder structures.

Factorization



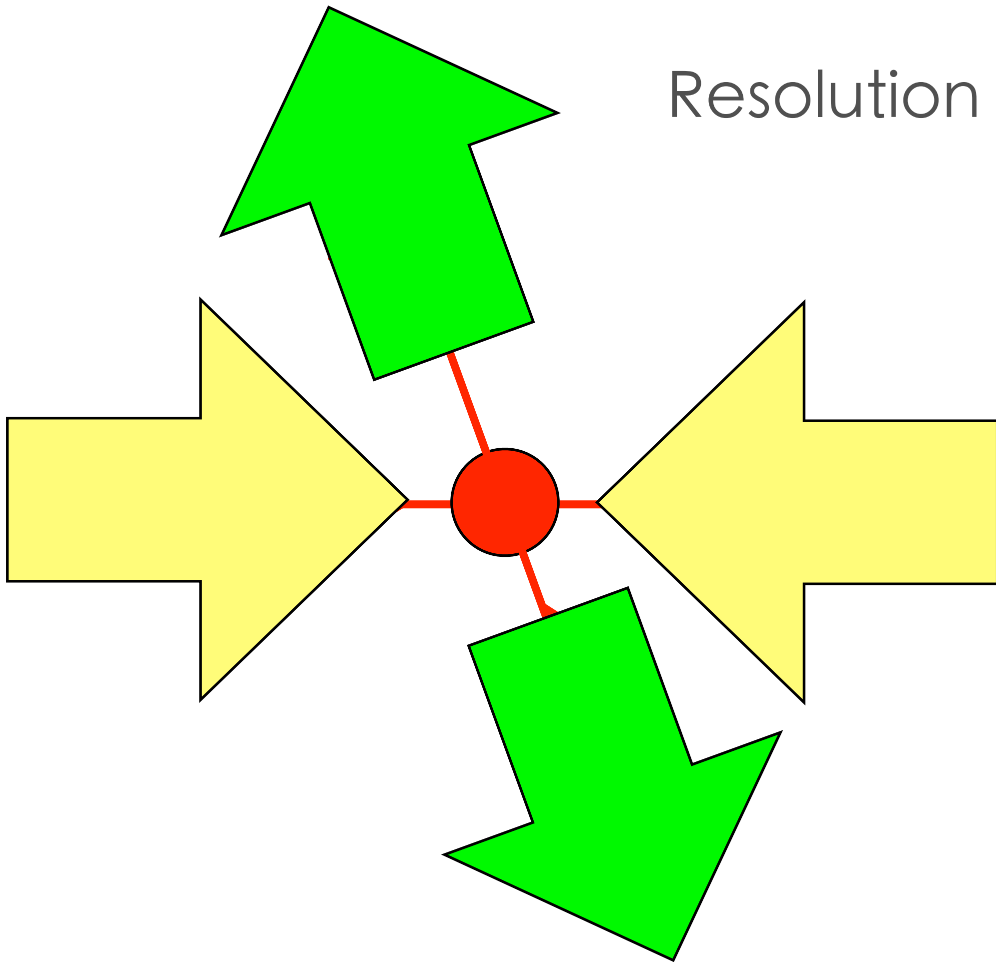
Coefficient function can be calculated in pQCD

PDF function, only the running can be calculated in pQCD

$$\mu \frac{d}{d\mu} f_{a/H}(\eta, \mu) = \sum_b [P_{a,b} \otimes f_{b/H}](x, \mu)$$

Hadron-hadron Collision

In hardon-hadron collision the picture is more complicated.



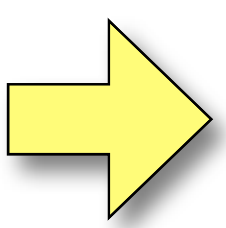
Resolution scale: 400 GeV

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

Important observation: The total cross section is independent of the resolution of the measurement (or detector).

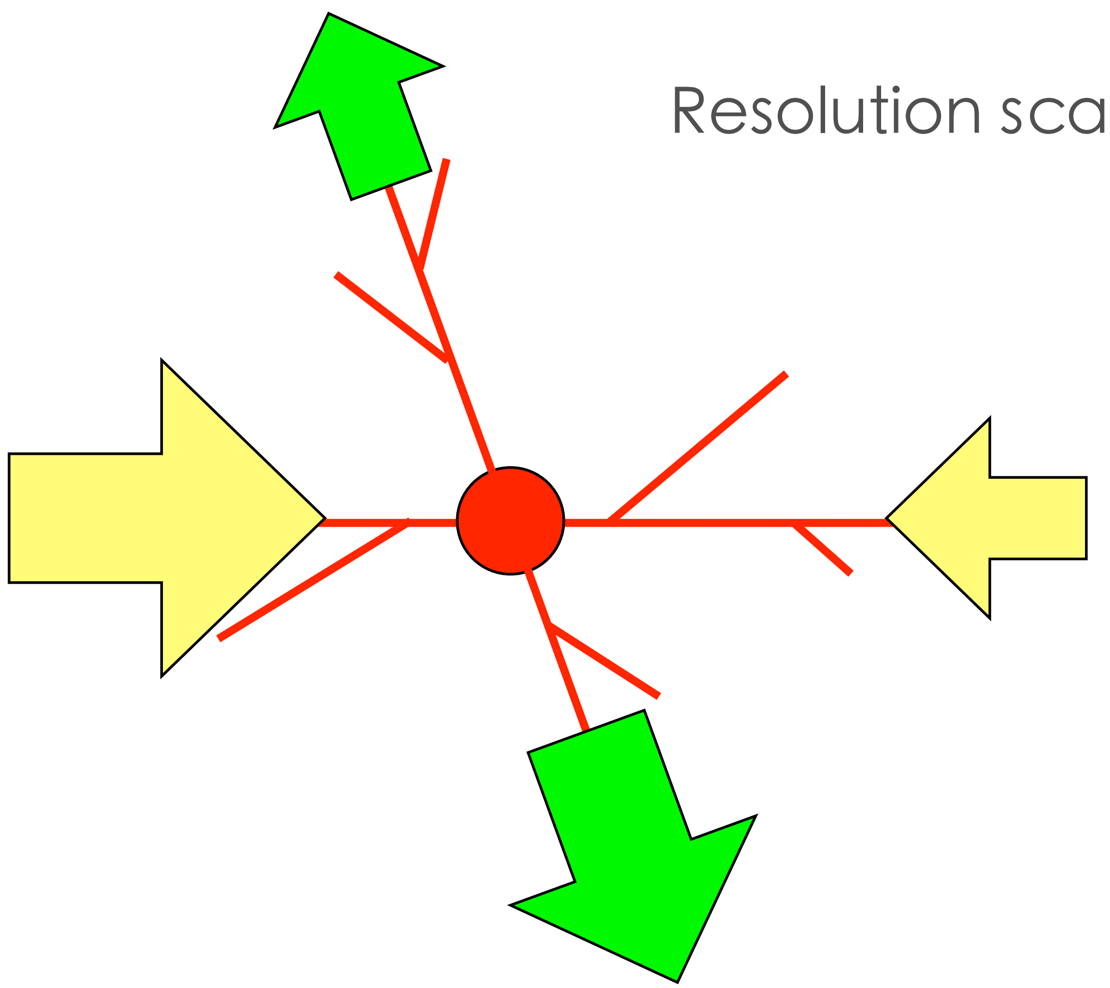
We have to also consider the evolution of the final state jets.

Does perturbative QCD support this nice intuitive picture?



Hadron-hadron Collision

In hardon-hadron collision the picture is more complicated.



Resolution scale: 100 GeV

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

Important observation: The total cross section is independent of the resolution of the measurement (or detector).

We have to also consider the evolution of the final state jets.

Does perturbative QCD support this nice intuitive picture?

Cross section

The cross section is a phase space integral of all the possible matrix elements and the a convolution to the parton distribution functions.

$$\sigma[F] = \sum_m \int [d\{p,f\}_m] \overbrace{f_{a/A}(\eta_a, \mu_F^2)\, f_{b/B}(\eta_b, \mu_F^2)}^{\textit{parton distributions}} \frac{1}{2\eta_a \eta_b p_A \cdot p_B}$$

$$\times \big\langle \mathcal{M}(\{p,f\}_m) \big| \underbrace{F(\{p,f\}_m)}_{\textit{observable}} \underbrace{\big| \mathcal{M}(\{p,f\}_m) \big\rangle}_{\textit{matrix element}}$$

- ✗ This is formally an all order expression and it is impossible to calculate out.
- ✗ We can do it at LO, NLO and in some cases NNLO level.
- ✗ Lots of complication with IR singularities.
- ✗ Lots of complication with spin and colors.
- ✓ The idea is to approximate the matrix elements using factorization properties of the QCD matrix element.
  - ⇨ We need a general formalism to describe parton shower evolution.

Statistical Space

Introducing the density operator, the cross section is

$$\sigma[F] = \sum_m \int [d\{p,f\}_m]\; \mathrm{Tr}\{\underbrace{\rho(\{p,f\}_m)}_{\text{density operator in color} \otimes \text{spin space}} F(\{p,f\}_m)\}$$

density operator in color ⊗ spin space

where the density operator is

$$\begin{aligned}\rho(\{p,f\}_m) &= \big|\mathcal{M}(\{p,f\}_m)\big\rangle \frac{f_{a/A}(\eta_\mathrm{a},\mu_F^2) f_{b/B}(\eta_\mathrm{b},\mu_F^2)}{2\eta_\mathrm{a}\eta_\mathrm{b} p_A \cdot p_B} \big\langle \mathcal{M}(\{p,f\}_m)\big| \\ &= \sum_{s,c,s',c'} \big|\{s',c'\}_m\big\rangle \big(\{p,f,s',c',s,c\}_m\big|\rho\big) \big\langle\{s,c\}_m\big|\end{aligned}$$

In the statistical space it is represented by a vector

$$|\rho) = \sum_m \frac{1}{m!} \int [d\{p,f,s',c',s,c\}_m]\; \underbrace{\big|\{p,f,s',c',s,c\}_m\big)}_{\text{Basis vector in the statistical space}} \big(\{p,f,s',c',s,c\}_m\big|\rho\big)$$

Basis vector in the statistical space

States

Basis: A state with m final state parton with momenta p, color c and c' and spin s', s is $\big|\{p,f,s',c',s,c\}_m\big)$

Physical state: $\big|\rho\big)$

Fully exclusive cross section to have m parton in the final state with fixed quantum numbers is $\big(\{p,f,...\}_m\big|\rho\big)$

Completeness relation :

$$1 = \sum_m \int [d\{p,f,s',c',s,c\}_m]\, \big|\{p,f,s',c',s,c\}_m\big)\big(\{p,f,s',c',s,c\}_m\big|$$

where

$$\int [d\{p,f,s',c',s,c\}_m] \equiv \int [d\{p,f\}_m] \sum_{s_\mathrm{a},s'_\mathrm{a},c_\mathrm{a},c'_\mathrm{a}} \sum_{s_\mathrm{b},s'_\mathrm{b},c_\mathrm{b},c'_\mathrm{b}} \prod_{i=1}^{m} \left\{ \sum_{s_i,s'_i,c_i,c'_i} \right\}$$

Orthonormal basis:

$$\big(\{p,f,s',c',s,c\}_m\big|\{\tilde{p},\tilde{f},\tilde{s}',\tilde{c}',\tilde{s},\tilde{c}\}_{\tilde{m}}\big) = \delta_{m,\tilde{m}}\ \delta(\{p,f,s',c',s,c\}_m;\{\tilde{p},\tilde{f},\tilde{s}',\tilde{c}',\tilde{s},\tilde{c}\}_{\tilde{m}})$$

Measurement function

Measurement operators can be also represented by vectors in the statistical space

$$\big|F\big) = \sum_m \frac{1}{m!}\int \big[d\{p,f,s',c',s,c\}_m\big]\,\big|\{p,f,s',c',s,c\}_m\big)\,F(\{p,f\}_m)$$

E.g.: Total cross section

$$\big|1\big) \Leftrightarrow F(\{p,f\}_m) = 1$$

Transverse momentum
in Drell-Yan:

$$\big|\boldsymbol{p}_\perp\big) \Leftrightarrow F(\{p,f\}_m) = \delta(\boldsymbol{p}_\perp - \boldsymbol{p}_{\perp,Z})$$

The cross section is

$$\sigma[F] = \big(F\big|\rho\big) = \sum_m \frac{1}{m!}\int \big[d\{p,f,\dots\}_m\big]\,F(\{p,f\}_m)\,\big(\{p,f,\dots\}_m\big|\rho\big)$$

Now, we have to generate the physical states.

Approx. of the Density Operator

The $m+1$ parton physical state is represented by density operator in the quantum space and by the statistical state in the statistical space.

$$\rho\big(\{p,f\}_{m+1}\big) \Leftrightarrow \big|\rho(\{p,f\}_{m+1})\big)$$

This is based on the $m+1$ parton matrix elements. They are very complicated (especially the loop matrix elements). We try to approximate them by using their soft collinear factorization properties. For this we introduce operators in the statistical space:

Wide angle soft contributions

$$\big|\rho(\{\hat{p},\hat{f}\}_{m+1})\big) \approx \int_{t_m}^{\infty} dt\, \big[\underbrace{\mathcal{H}_C(t)} + \overbrace{\mathcal{H}_S(t)}\big]\big|\rho(\{p,f\}_m)\big)$$

Collinear and soft-collinear contribution

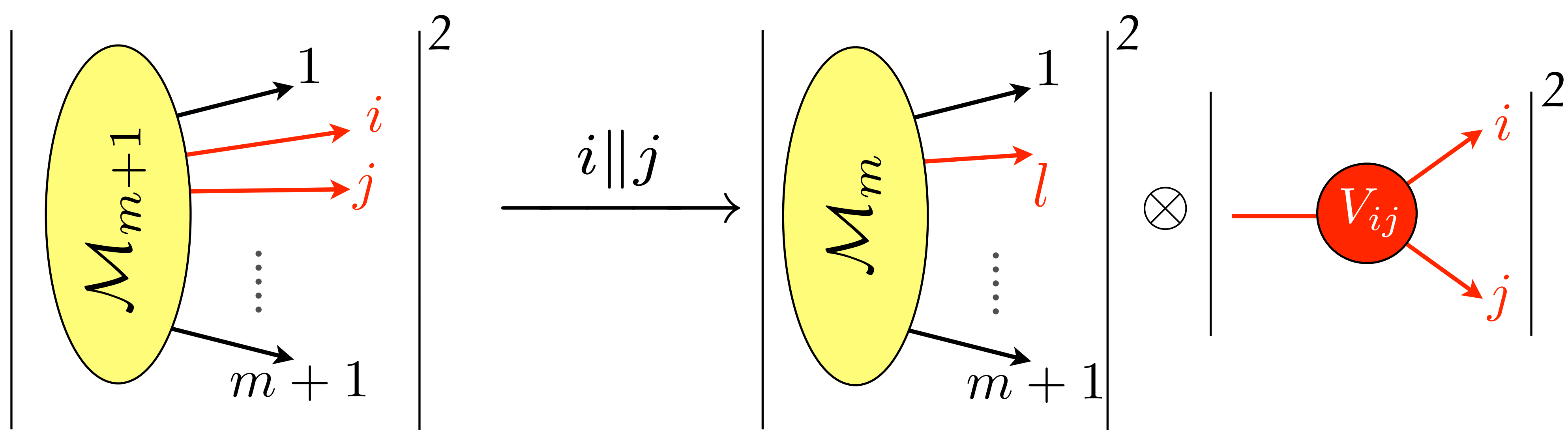
This parameter represents the hardness of the splitting. We will call it shower time.

The total splitting operator is

$$\mathcal{H}_I(t) = \mathcal{H}_C(t) + \mathcal{H}_S(t)$$

Collinear Singularities

The QCD matrix elements have universal factorization property when two external partons become collinear

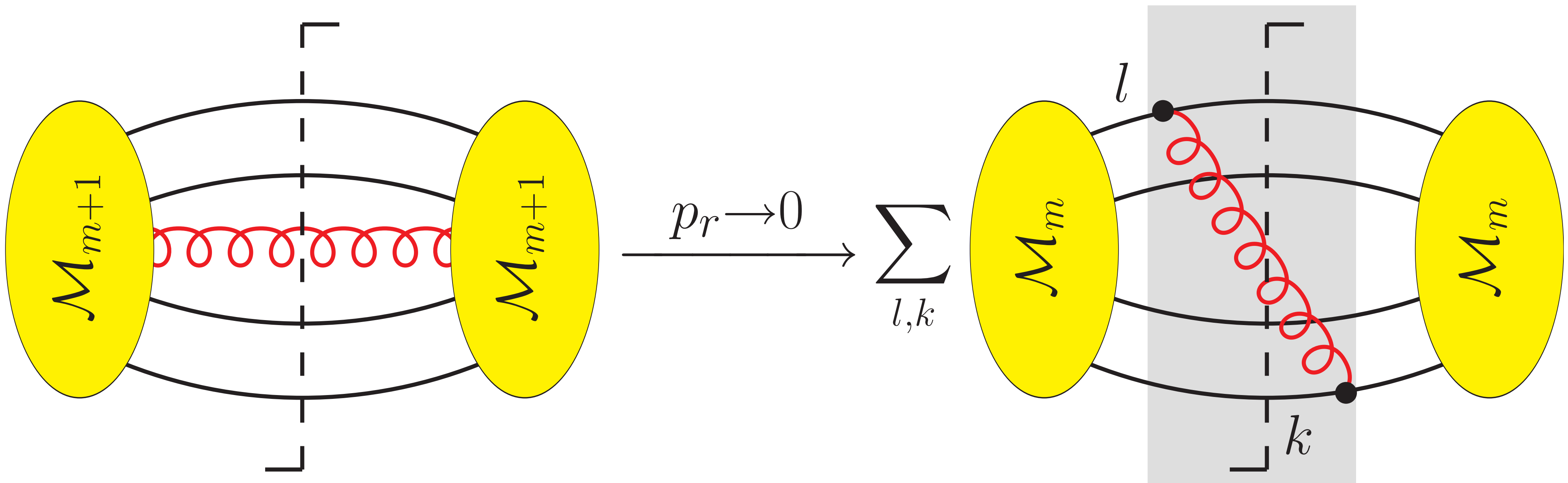


$$\mathcal{H}_C \sim \sum_l t_l \otimes t_l^\dagger \, V_{ij}(s_i, s_j) \otimes V_{ij}^\dagger(s'_i, s'_j) \Leftrightarrow \frac{\alpha_s}{2\pi} \sum_l \frac{1}{p_i \cdot p_j} P_{f_l, f_i}(z) + \ldots$$

Altarelli-Parisi splitting kernels

Soft Singularities

The QCD matrix elements have universal factorization property when an external gluon becomes soft



$$\mathcal{H}_S \sim -\sum_{\substack{l,k \\ l \neq k}} \frac{\hat{p}_l \cdot \varepsilon(s)\, \hat{p}_k \cdot \varepsilon(s')}{\hat{p}_l \cdot \hat{p}_{m+1}\, \hat{p}_k \cdot \hat{p}_{m+1}}\, t_l \otimes t_k^\dagger$$

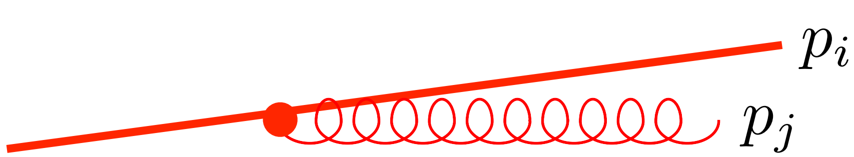
Soft gluon connects everywhere and the color structure is not diagonal; quantum interferences in the color space.

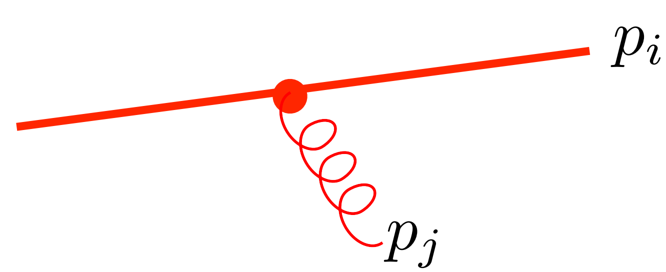
Shower Time

Now, we should define the t shower time. It is related to the hardness of the radiation. Its main purpose is to control the goodness of the approximation. We simply use the virtuality of the splitting,

$$t = \log \frac{Q_0^2}{2p_i \cdot p_j} \qquad\qquad 0 < t < \infty$$

For collinear and soft splitting





$$t \to \infty$$

The shower time dependence of the splitting operator is

$$\mathcal{H}_I(t) = \sum_l \mathcal{S}_l \, \delta\left(t - \log \frac{Q_0^2}{2p_i \cdot p_j}\right)$$

Other Choices

Some people prefers to use the transverse momentum as evolution variable

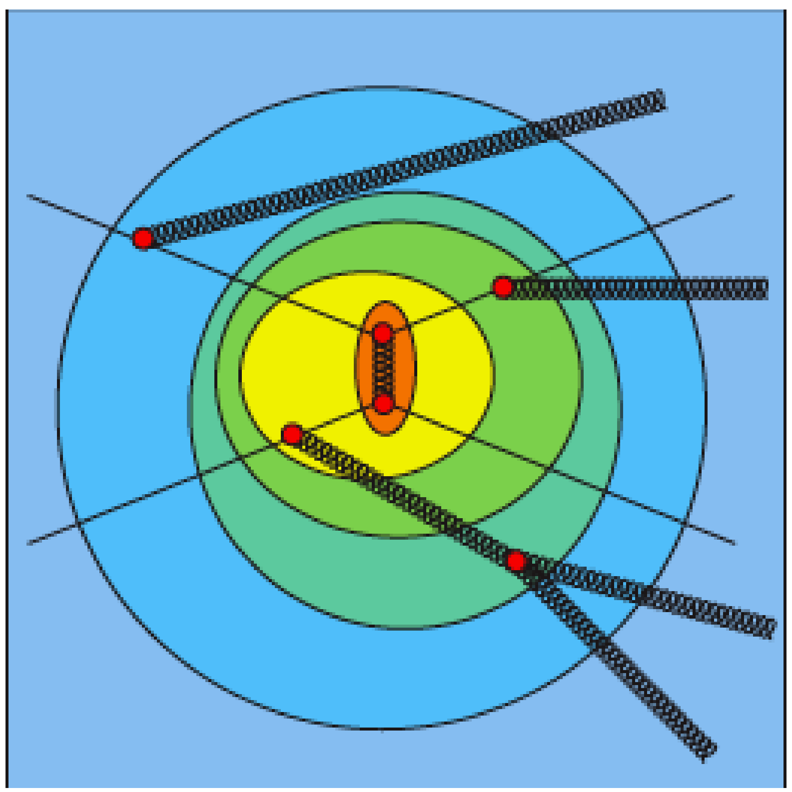
$$\mathcal{H}_I(t) = \sum_l \mathcal{S}_l \, \delta\left(t - \log\frac{Q_0^2}{-k_\perp^2}\right)$$

HERWIG uses the emission angle with transverse momentum veto

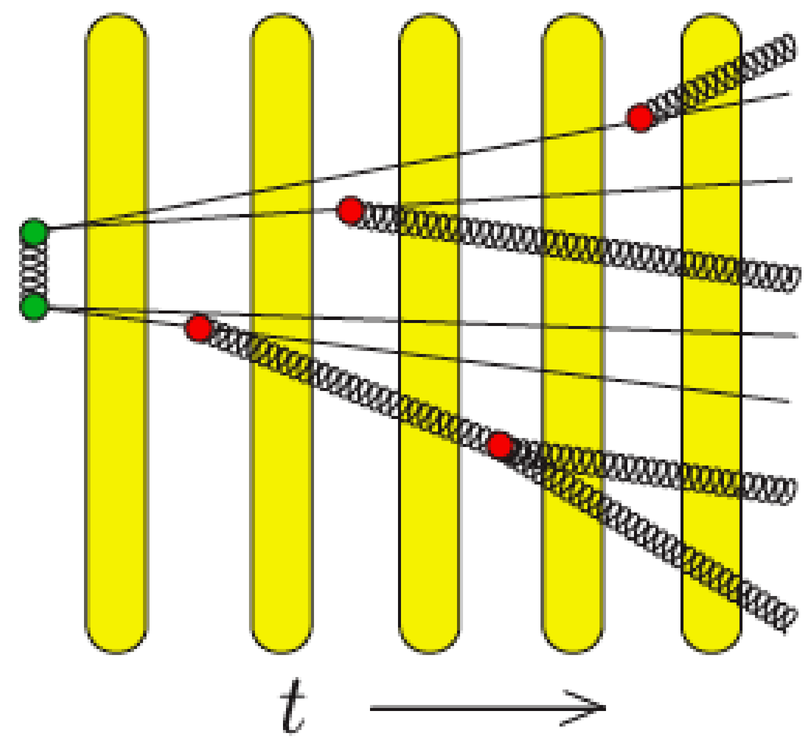
$$\mathcal{H}_I(t) = \sum_l \mathcal{S}_l \, \delta\left(t - \log\frac{2}{1-\cos\vartheta_{ij}}\right) \theta\left(-k_\perp^2 > 1\mathrm{GeV}^2\right)$$

Shower Time

Think of shower branching as developing in a "time" that goes from most virtual to least virtual.



Real time picture



Shower time picture

Resolvable Splittings

Let us consider a physical state at shower time t, $|\rho(t))$. This means every parton is resolvable at this time (this scale). Now, we apply the splitting operator:

$\mathcal{H}_I(t)$ operator changes

- the number of the partons, $m \to m+1$
- the color and spin structure
- flavors and momenta

$$|\rho^{\mathrm{R}}_\infty) = \int_t^\infty d\tau\, \mathcal{H}_I(\tau)\, |\rho(t))$$

This is good approximation if we allow only softer radiations than t, $\tau > t$

Now, let us consider a measurement with a resolution scale which correspond to shower time t'

$$|\rho^{\mathrm{R}}_\infty) \approx \underbrace{\int_t^{t'} d\tau\, \mathcal{H}_I(\tau)\, |\rho(t))}_{\text{Resolved radiations}} + \underbrace{\int_{t'}^\infty d\tau\, \mathcal{V}_I^{(\epsilon)}(\tau)\, |\rho(t))}_{\text{Unresolved radiations}}$$

Resolved radiations

Unresolved radiations

This is a singular contribution

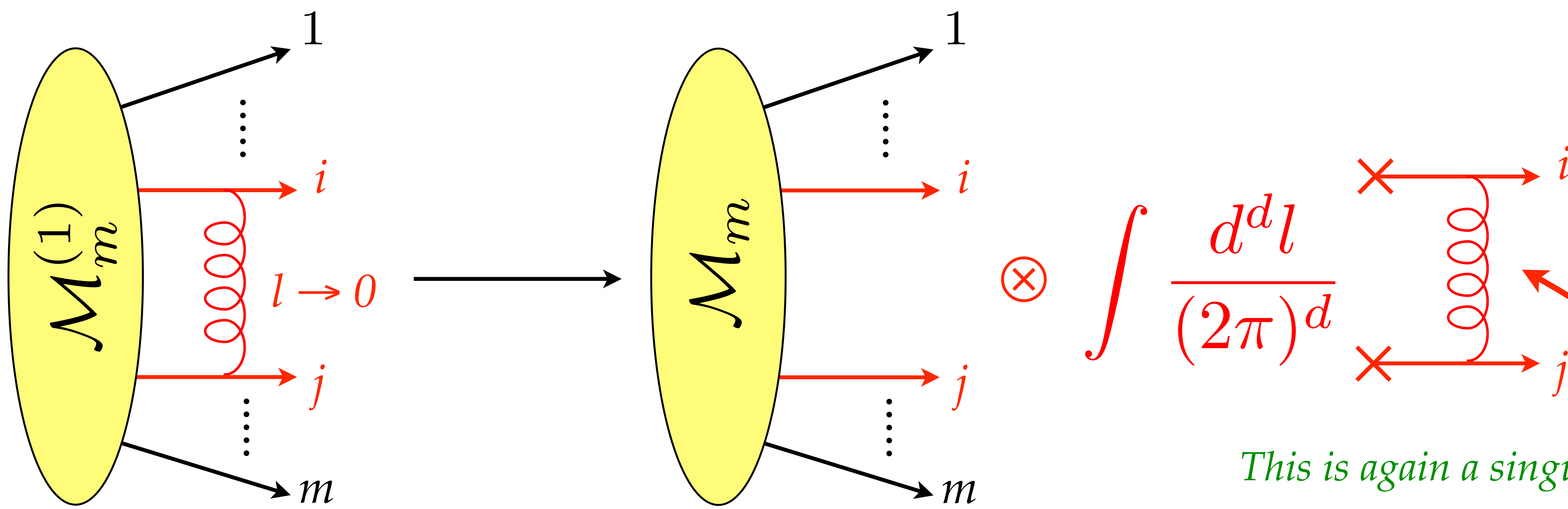
$\mathcal{V}_I(t)$ operator

- changes only the color structure
- $(1|\mathcal{V}_I(t) = (1|\mathcal{H}_I(t)$

What can we do about it?

Virtual Contributions

There is another type of the unresolvable radiation, the virtual (loop graph) contributions. We have universal factorization properties for the loop graphs. E.g.: in the soft limit, when the loop momenta become soft we have



This is again a singular operator only in the color space.

We can use this factorization to dress up partonic states with virtual radiation. After careful analysis one can found that the virtual contribution can be approximated by

$$\big|\rho_\infty^{\rm V}\big) \approx - \int_t^\infty d\tau\, \mathcal{V}_I^{(\epsilon)}(\tau)\, \big|\rho(t)\big)$$

Same structure like in the real unresolved case but here with opposite sign.

Physical States

Combining the real and virtual contribution we have got

$$\big|\rho_\infty^{\mathrm{R}}\big) + \big|\rho_\infty^{\mathrm{V}}\big) = \int_t^{t'} d\tau\, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]\big|\rho(t)\big)$$

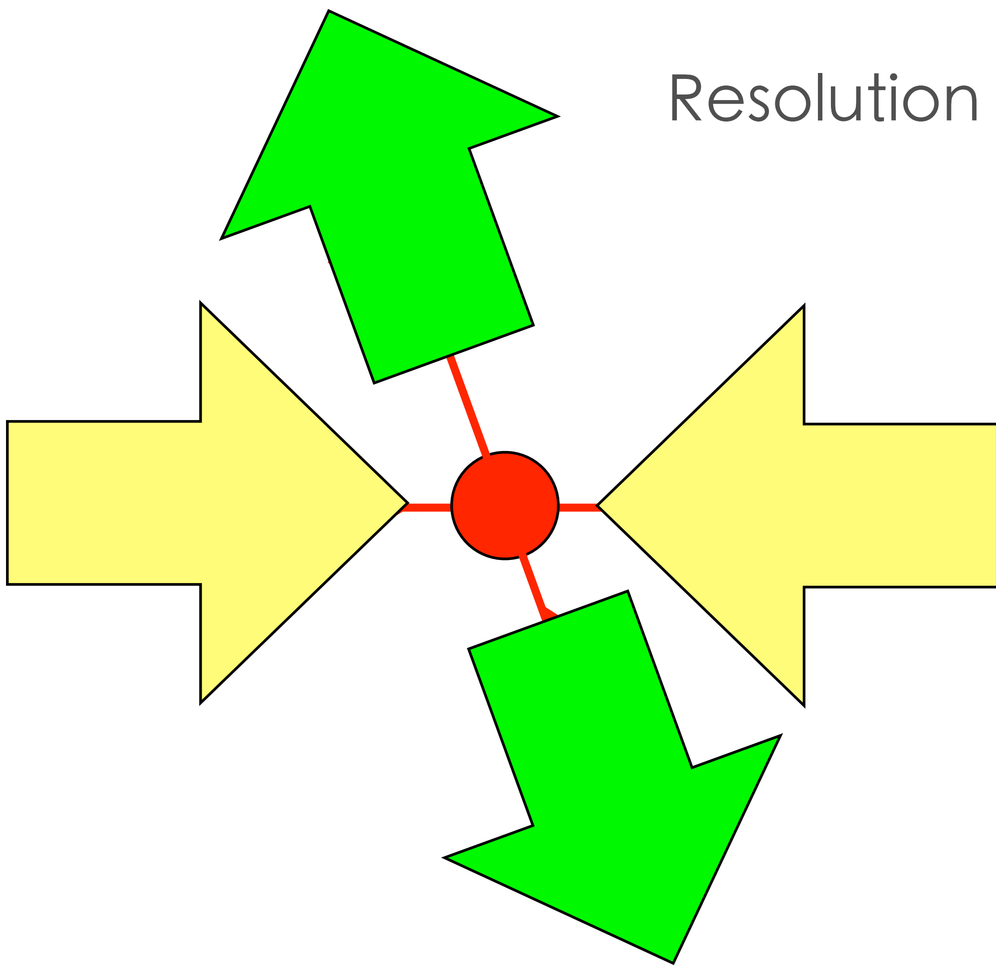
This operator dresses up the physical state with one real and virtual radiations that is softer or more collinear than the hard state. Thus the emissions are ordered. Now we can use this to build up physical states by considering all the possible way to go from t to t'.

$$\begin{aligned}\big|\rho(t')\big) &= \big|\rho(t)\big) \\ &\quad + \int_t^{t'} d\tau\, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]\big|\rho(t)\big) \\ &\quad + \int_t^{t'} d\tau_2\, [\mathcal{H}_I(\tau_2) - \mathcal{V}_I(\tau_2)] \int_t^{\tau_2} d\tau_1\, [\mathcal{H}_I(\tau_1) - \mathcal{V}_I(\tau_1)]\big|\rho(t)\big) \\ &\quad + \cdots \\ &= \underbrace{\mathbb{T}\exp\left\{\int_t^{t'} d\tau\, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]\right\}}_{\mathcal{U}(t',t)\ \textit{shower evolution operator}}\big|\rho(t)\big)\end{aligned}$$

$$\Rightarrow \quad \big|\rho(t')\big) = \mathcal{U}(t',t)\big|\rho(t)\big)$$

Evolution Operator

Back to our cartoon .....



Resolution scale: 400 GeV

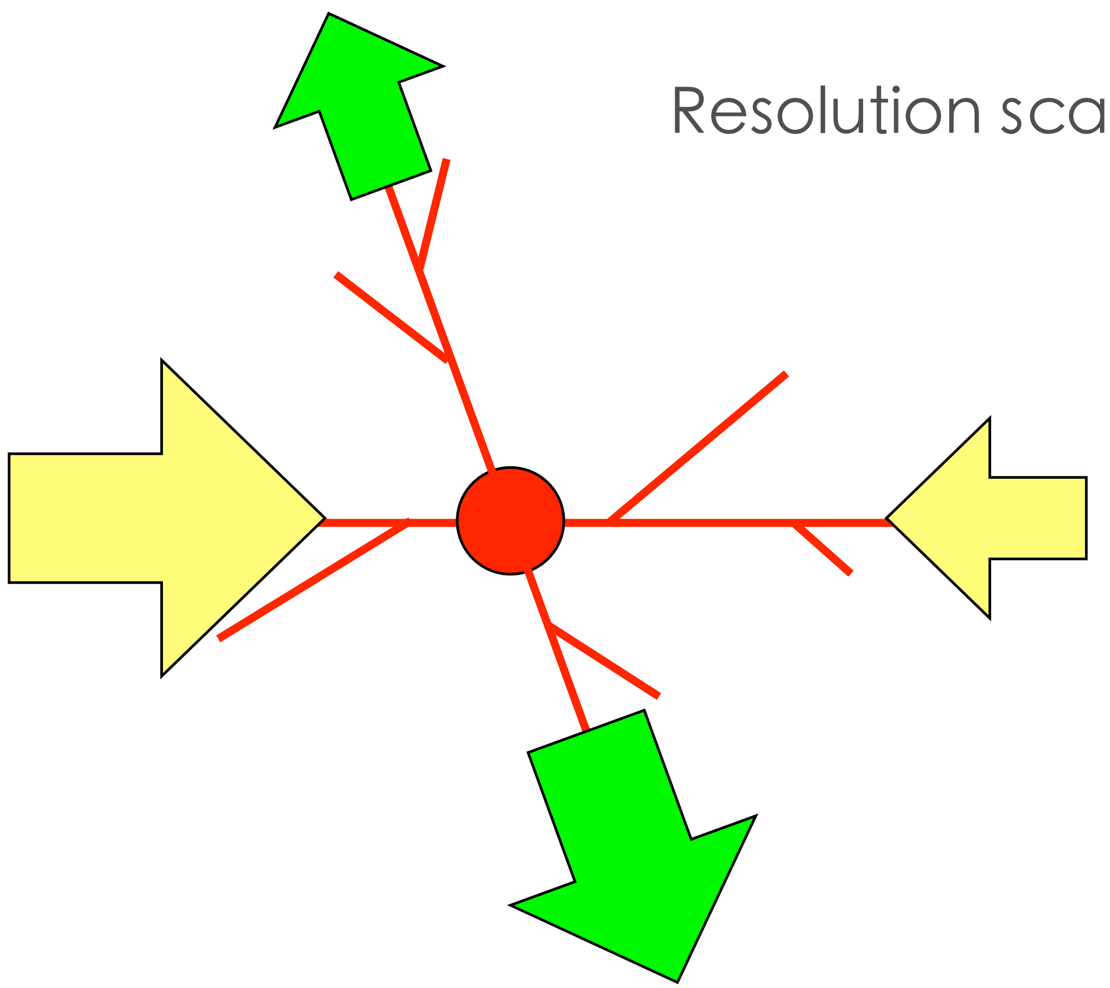
Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

Important observation: The total cross section is independent of the resolution of the measurement (or detector).

Does perturbative QCD support this nice intuitive picture?

Evolution Operator

Back to our cartoon .....



Resolution scale: 100 GeV

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

Important observation: The total cross section is independent of the resolution of the measurement (or detector).

Does perturbative QCD support this nice intuitive picture?

Shower Evolution

Unitarity:

Shower evolution operator satisfy the following equation

$$\frac{d}{dt}\mathcal{U}(t,t') = \left[\mathcal{H}_I(t) - \mathcal{V}_I(t)\right]\mathcal{U}(t,t')$$

From $\big(1\big|\mathcal{V}_I(t) = \big(1\big|\mathcal{H}_I(t)$ one can see that the shower preserve the total cross section

$$\big(1\big|\mathcal{U}(t,t') = \big(1\big|$$

Group decomposition property:

$$\mathcal{U}(t,t')\,\mathcal{U}(t',t'') = \mathcal{U}(t,t'')$$

Let us have a physical state evolved to t and consider a measurement F with the typical resolution $t_F < t$. For soft or collinear splittings we have

$$\int_{t_F}^{t} d\tau\,\big(F\big|\mathcal{H}_I(\tau) = \int_{t_F}^{t} d\tau\,\big(F\big|\mathcal{V}_I(t)$$

Now the cross section is

$$\big(F\big|\rho(t)\big) = \big(F\big|\mathcal{U}(t,t_F)\big|\rho(t_F)\big) = \big(F\big|\rho(t_F)\big)$$

The measurement is insensitive for the finer structure, thus they are integrated out to 1.

→ This is depicted in our cartoon!

Shower Evolution

Unitarity:

Shower evo

From $(1|\mathcal{V}_I(t$ ... otal cross section

Group deco

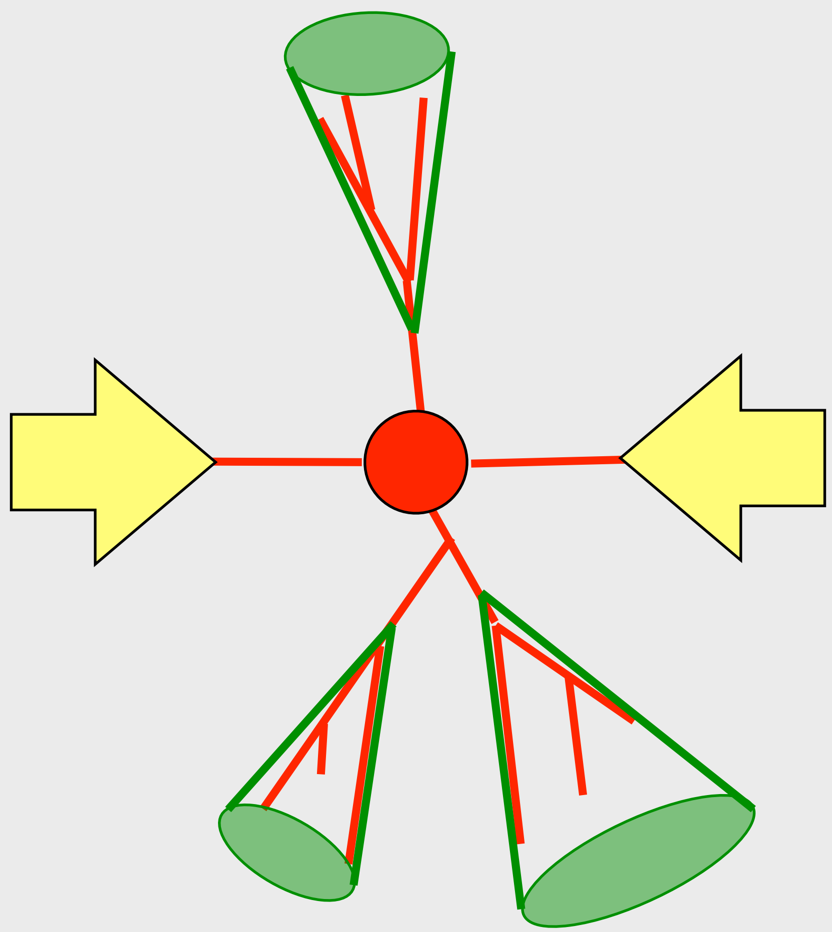
Let us have ... ment F

with the typ ... ve

Now the cro

The measu ... the finer str ... integrated ... d in our cartoon!

$$(F|\rho(t)) = (F|\mathcal{U}(t,t_F)|\rho(t_F)) = (F|\rho(t_F))$$



In this measurement we resolve jets. The resolution is represented by the "green cones"

The measurement is insensitive what is happening inside the "green cones". In the parton shower everything inside the cones are integrated out to 1.

Summary

- ✓ We have found the the hadronic final states can be understood in a very intuitive way (Wilsonian renormalization approach).
- ✓ We derived parton shower algorithm based on the soft and collinear approximation of the QCD tree and 1-loop matrix elements.
- ✓ This algorithm supports the intuitive picture.
- ✓ This is probably the most general theory of the parton shower algorithms. The available implementations (PYTHIA, HERWIG, ARIADNE) are based on this with some additional approximation and special choice of "ingredients".
- ? Some of you might be suspicious because we have not defined the Sudakov factor.

Evolution Equation

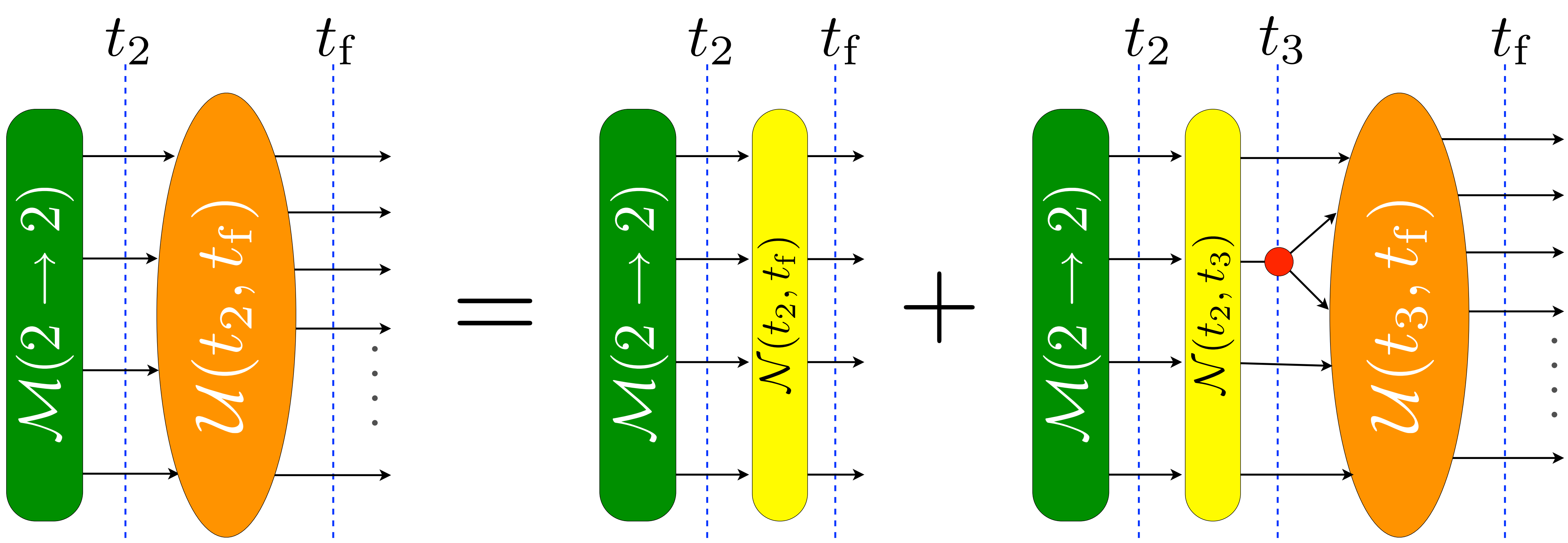
We can write the evolution equation in an integral equation form

$$\mathcal{U}(t_\mathrm{f}, t_2) = \underbrace{\mathcal{N}(t_\mathrm{f}, t_2)}_{\text{"Nothing happens"}} + \overbrace{\int_{t_2}^{t_\mathrm{f}} dt_3\, \mathcal{U}(t_\mathrm{f}, t_3) \mathcal{H}_I(t_3) \mathcal{N}(t_3, t_2)}^{\text{"Something happens"}}$$

where the non-splitting operator is

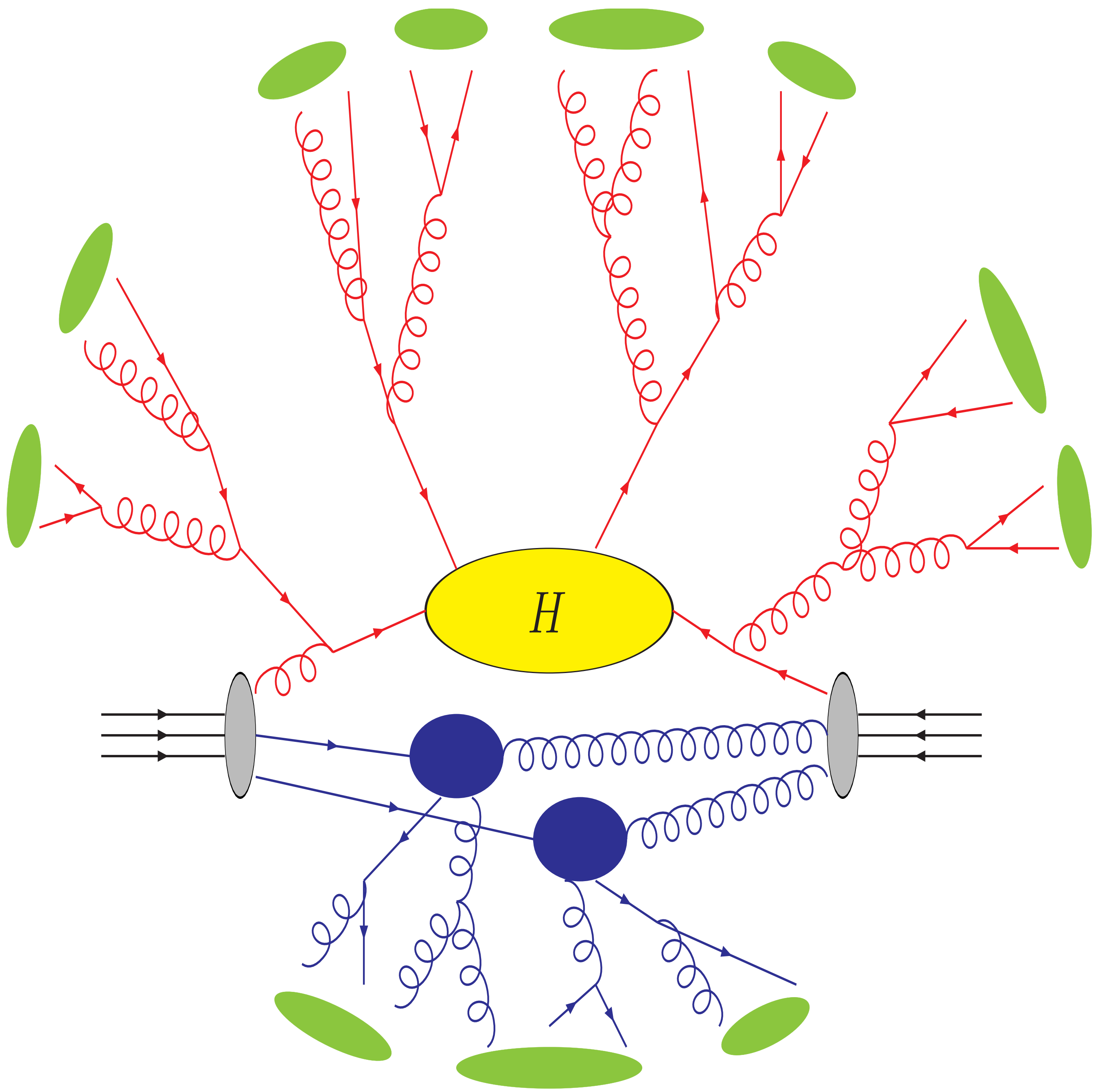
$$\mathcal{N}(t', t) = \mathbb{T} \exp\left\{ -\int_t^{t'} d\tau\, \mathcal{V}_I(\tau) \right\}$$

Sudakov operator



Monte Carlo Tools

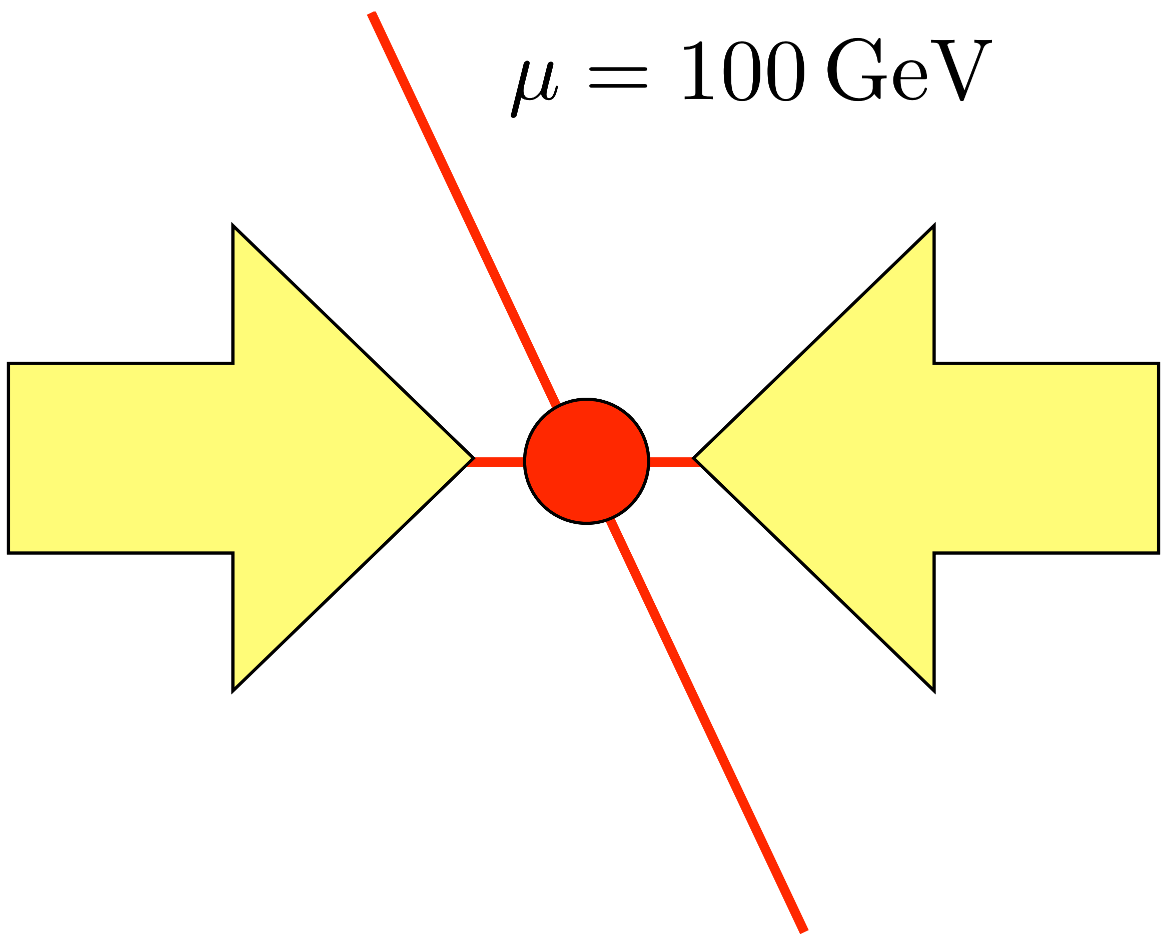
The structure of the Monte Carlo event generators



1. Incoming hadron (gray bubbles)
   - Parton distribution function
2. Hard part of the process (yellow bubble)
   - Matrix element calculation, cross sections at LO, NLO, NNLO level
3. Radiations (red graphs)
   - Parton shower calculation
   - Matching to the hard part
4. Underlying event (blue graphs)
   - Models based on multiple interaction
5. Hardonization (green bubbles)
   - Universal models

Multiple Interaction

Let us see how it looks at hadron collider



In hadron-hadron collision the parton distribution function also absorbs the contribution of the secondary interactions.

This is a more complicated evolution than in the DIS case.

- Is there factorization or can we define in a systematic way?
- If yes, how does it work?
- What is the evolution equation?

The evolution operator should be something like

$$\mathcal{U}_{I+MI}(t,t') = \mathbb{T}\exp\left\{\int_t^{t'} d\tau\, \Big[\overbrace{\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)}^{\text{Single radiations}} + \underbrace{\mathcal{H}_{MI}(\tau) - \mathcal{V}_{MI}(\tau)}_{\text{Multiple interaction}}\Big]\right\}$$

Multiple Interaction

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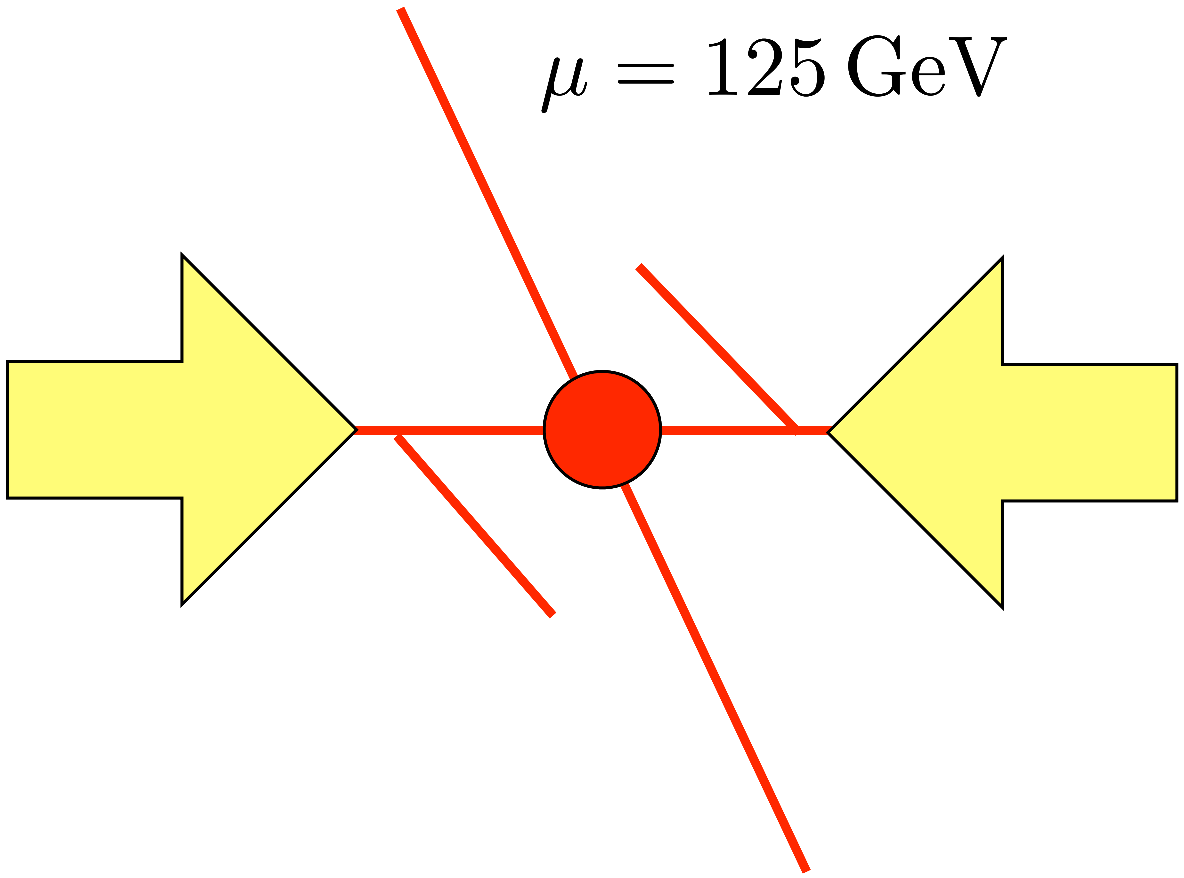

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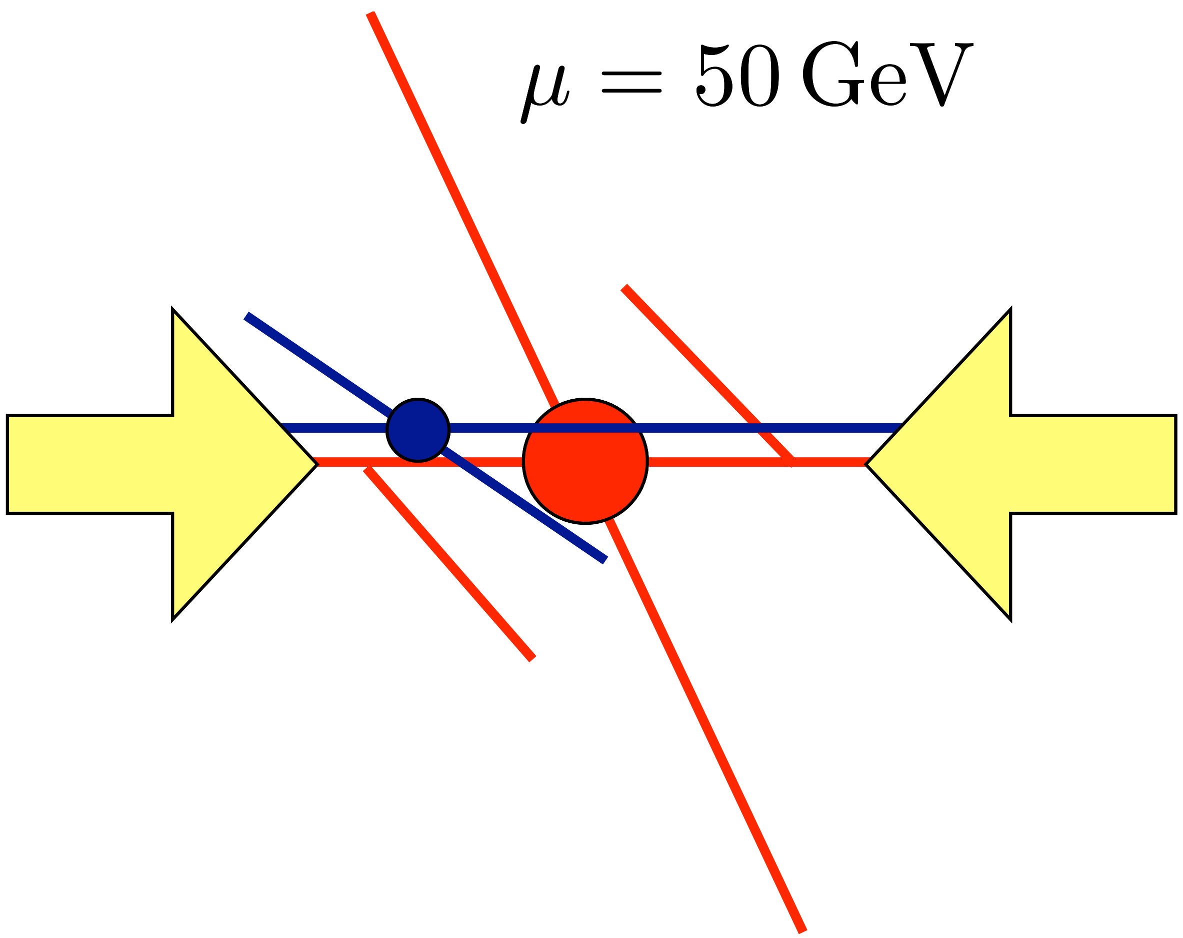

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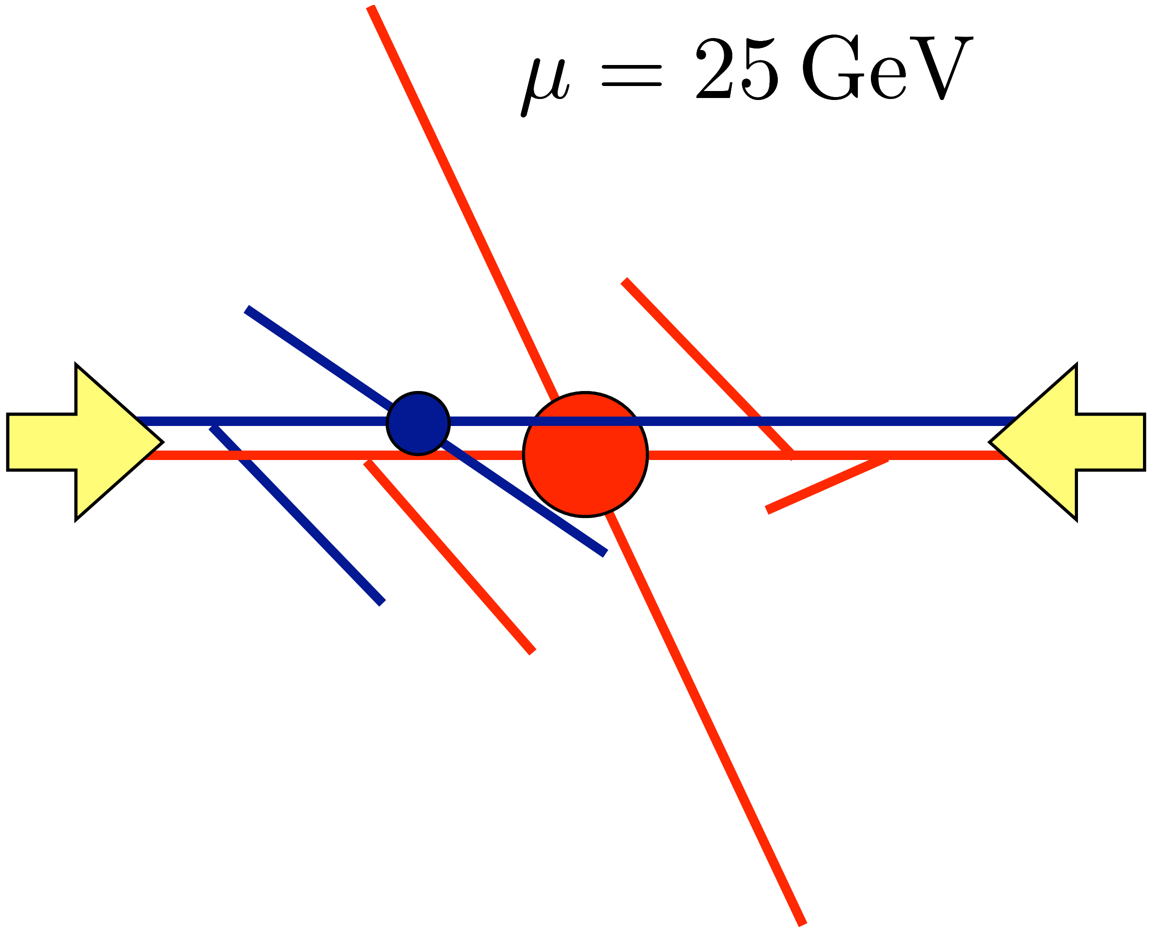

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I haven't talked about....

- Angular ordering (HERWIG)
- Leading color approximations (PYTHIA, ARIADNE, ...)
- Implementations
- Spin averaging
- Coulomb gluons
- Summation of large logarithm
- Matching at LO and NLO level
- ....
- Hardonization
- Underlaying event, multi parton interactions
- Hadronic decays
- Tuning and validation
- Other approaches (kT factorization)
- ....

Conclusion

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Please use the Monte Carlos wisely!