Introduction to Event Generators

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Topics of the lectures

1. Lecture 1: *The Monte Carlo Principle*
2. Lecture 2: *Parton level event generation*
3. Lecture 3: *Dressing the Partons*
4. Lecture 4: *Modelling beyond Perturbation Theory & Improving the showers*

Thanks to

- My fellow MC authors, especially S.Gieseke, K.Hamilton, L.Lonnblad, F.Maltoni, M.Mangano, P.Richardson, M.Seymour, T.Sjostrand, B.Webber.
- the other Sherpas: J.Archibald, T.Gleisberg, S.Höche, S.Schumann, F.Siegert, M.Schönherr, and J.Winter.

Menu of lecture 3

- Prelude: Orientation
- Why we need parton showers
- An analogy
- The parton shower as a theory instrument
- Improving the accuracy
- New showers

Prelude: Orientation

Event generator paradigm

Divide event into stages, separated by different scales.

- Signal/background:

  Exact matrix elements.

- QCD-Bremsstrahlung:

  Parton showers (also in initial state).

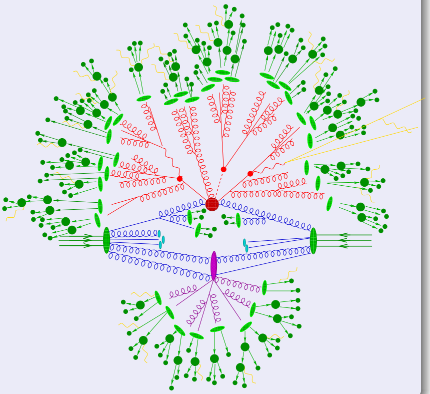
- Multiple interactions:

  Beyond factorisation: Modelling.

- Hadronisation:

  Non-perturbative QCD: Modelling.

Sketch of an event



Motivation: Why parton showers?

Common wisdom

- Well-known: Accelerated charges radiate
- QED: Electrons (charged) emit photons
  Photons split into electron-positron pairs
- QCD: Quarks (coloured) emit gluons
  Gluons split into quark pairs
- Difference: Gluons are coloured (photons are not charged)
  Hence: Gluons emit gluons!
- Cascade of emissions: Parton shower

Some more refined reasons

- Experimental definition of jets based on hadrons.
- But: Hadronisation through phenomenological models (need to be tuned to data).
- Wanted: Universality of hadronisation parameters (independence of hard process important).
- Link to fragmentation needed: Model softer radiation (inner jet evolution).
- Similar to PDFs (factorisation) just the other way around (fragmentation functions at low scale, parton shower connects high with low scale).

An analogy: Radioactive decays

The form of the solution

- Consider the radioactive decay of an unstable isotope with half-life τ. (Ignore factors of ln 2.)
- "Survival" probability after time t is given by
$$\mathcal{S}(t) = \mathcal{P}_{\rm nodec}(t) = \exp[-t/\tau].$$
(Note "unitarity relation": $\mathcal{P}_{\rm dec}(t) = 1 - \mathcal{P}_{\rm nodec}(t)$.)
- Probability for an isotope to decay at time t:
$$\frac{{\rm d}\mathcal{P}_{\rm dec}(t)}{{\rm d}t} = -\frac{{\rm d}\mathcal{P}_{\rm nodec}(t)}{{\rm d}t} = \frac{1}{\tau}\exp(-t/\tau).$$
- Now: Connect half-life with width $\Gamma = 1/\tau$.
- Probability for the isotope to decay at any fixed time t is determined by Γ.

Adding a non-trivial time dependence

- Rewrite Γt in the exponential as $\int\limits_0^t \mathrm{d}t'\Gamma$.

  (This allows to make life more interesting, see below.)

- Allows to have a time-dependent decay probability $\Gamma(t')$.
- Then decay-probability at a given time t is given by

$$\frac{\mathrm{d}\mathcal{P}_{\mathrm{dec}}(t)}{\mathrm{d}t} = \Gamma(t)\,\exp\left[-\int\limits_0^t \mathrm{d}t'\Gamma(t')\right] = \Gamma(t)\,\mathcal{P}_{\mathrm{nodec}}(t).$$

  (Unitarity strikes again: $\mathrm{d}\mathcal{P}_{\mathrm{dec}}(t)/\mathrm{d}t = -\mathrm{d}\mathcal{P}_{\mathrm{nodec}}(t)/\mathrm{d}t$.)

- Interpretation of l.h.s.:
  - First term is for the actual decay to happen.
  - Second term is to ensure that no decay before t

    $\Longrightarrow$ Conservation of probabilities.

    The exponential is called the Sudakov form factor.

A detour: The Altarelli-Parisi equation

The form of the equation for one parton type q

- AP describes the scaling behaviour of the parton distribution function: (which depends on Bjorken-parameter and scale Q^2)

$$\frac{\mathrm{d}q(x,Q^2)}{\mathrm{d}\ln Q^2} = \int\limits_x^1 \frac{\mathrm{d}y}{y}\left[\alpha_s(Q^2)P_q(x/y)\right] q(y,Q^2)$$

- Here the term in square brackets determines the probability that the parton emits another parton at scale Q^2 and Bjorken-parameter y. (after the splitting, $x \to yx + (1-y)x$.)
- Will be central in constructing Sudakov form factor.
- Driving term: Splitting function $P_q(x)$.
  Important property: Universal, process independent.

Splitting functions and large logarithms

$e^+e^- \to$ jets

- Differential cross section:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

Singular for $x_{1,2}\to 1$.

- Rewrite with opening angle θ_{qg} and gluon energy fraction $x_3 = 2E_g/E_{\mathrm{c.m.}}$:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}\cos\theta_{qg}\mathrm{d}x_3} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin^2\theta_{qg}}\frac{1+(1-x_3)^2}{x_3} - x_3\right]$$

Singular for $x_3\to 0$ ("soft"), $\sin\theta_{qg}\to 0$ ("collinear").

Collinear singularities

- Use

$$\frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta^2_{qg}}{\theta^2_{qg}} + \frac{\mathrm{d}\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}}$$

- Independent evolution of two jets (q and $\bar{q}$):

$$\mathrm{d}\sigma_{ee\to 3j} \approx \sigma_{ee\to 2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2_{jg}}{\theta^2_{jg}} P(z)\,,$$

where $P(z) = \frac{1+(1-z)^2}{z}$ (DGLAP splitting function)

Expressing the collinear variable

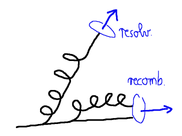
- Same form for any $t \propto \theta^2$:
- Transverse momentum $k_\perp^2 \approx z^2(1-z)^2E^2\theta^2$
- Invariant mass $q^2 \approx z(1-z)E^2\theta^2$

$$\frac{\mathrm{d}\theta^2}{\theta^2} \approx \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \approx \frac{\mathrm{d}q^2}{q^2}$$

- Parametrisation-independent observation: (Logarithmically) divergent expression for $t \to 0$.
- Practical solution: Cut-off t_0.
  $\Longrightarrow$ Divergence will manifest itself as $\log t_0$.
- Similar for $P(z)$: Divergence for $z \to 0$ cured by cut-off.

Parton resolution

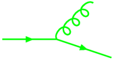
- What is a parton?
  Collinear pair/soft parton recombine!
- Introduce resolution criterion $k_\perp > Q_0$.



- Combine virtual contributions with unresolvable emissions:
  Cancels infrared divergences $\Longrightarrow$ Finite at $\mathcal{O}(\alpha_s)$

  (Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

- Unitarity: Probabilities add up to one
  $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1.$

 $+$  $+$  $=1.$

The Sudakov form factor

- Diff. probability for emission between q^2 and $q^2 + \mathrm{d}q^2$:

$$\mathrm{d}\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}\int\limits_{z_{\min}}^{z_{\max}} \mathrm{d}z P(z) =: \mathrm{d}q^2\,\Gamma(q^2)\,.$$

- $\Gamma(q^2)$ often dubbed "integrated splitting" function.

(Terms like $1/q^2$ may be pulled out in literature.)

- No-emission prob. P_{nodec} given by Sudakov form factor Δ.
- From radioactive example: Evolution equation for Δ

$$-\frac{\mathrm{d}\Delta(Q^2,q^2)}{\mathrm{d}q^2} = \Delta(Q^2,\,q^2)\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}q^2} = \Delta(Q^2,\,q^2)\Gamma(q^2)$$

$$\Longrightarrow\quad \Delta(Q^2,\,q^2) = \exp\left[-\int\limits_{q^2}^{Q^2}\mathrm{d}k^2\Gamma(k^2)\right].$$

The Sudakov form factor (cont'd)

- Remember: Sudakov form factor describes probabilities for (no) branchings.
- It has been derived here by analysing the structure of gluon radiation off a $q\bar{q}$ pair in the (collinear) approximation of large logarithms.

(In the splitting function we only took terms $\propto 1/z$ into account.)

- It can be shown that this structure factorises to all orders:

(c.f. proof of the AP equation)

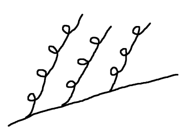
$$\mathrm{d}\sigma_{N+1} \approx \frac{\mathrm{d}k^2}{k^2}\frac{\mathrm{d}\phi}{2\pi}\mathrm{d}z\,\alpha_s P(z)\,\mathrm{d}\sigma_N$$

- This allows the resummation of all large logs.

Many emissions

- Iterate emissions (jets)

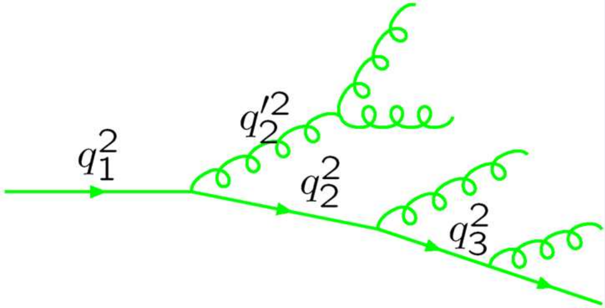
Maximal result for $t_1 > t_2 > \ldots t_n$:



$$d\sigma \propto \sigma_0 \int\limits_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int\limits_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \ldots \int\limits_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} \propto \log^n \frac{Q^2}{Q_0^2}$$

- How about Q^2? Process-dependent!

Ordering the emissions : Radiation pattern

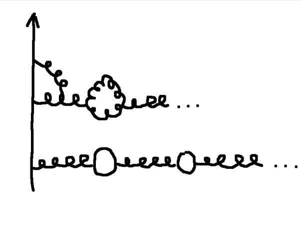


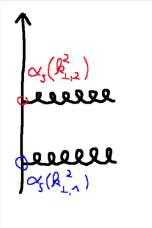
$$q_1^2 > q_2^2 > q_3^2,\ q_1^2 > q_2'^2$$

Improvement: Inclusion of quantum effects

Running coupling

- Effect of summing up higher orders (loops): $\alpha_s \to \alpha_s(k_\perp^2)$

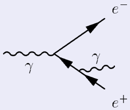




- Much faster parton proliferation, especially for small $k_\perp^2$.
- Must avoid Landau pole: $k_\perp^2 > Q_0^2 \gg \Lambda^2_{\rm QCD}$
  $\Longrightarrow Q_0^2 =$ physical parameter.

Soft logarithms : Angular ordering

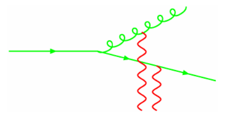
- Soft limit for single emission also universal
- Problem: Soft gluons come from all over (not collinear!) Quantum interference? Still independent evolution?
- Answer: Not quite independent.
  - Assume photon into e^+e^- at θ_{ee} and photon off electron at θ
  - Energy imbalance at vertex: $k_\perp^\gamma \sim zp\theta$, hence $\Delta E \sim k_\perp^2/zp \sim zp\theta^2$.
  - Time for photon emission: $\Delta t \sim 1/\Delta E$.
  - ee-separation: $\Delta b \sim \theta_e e\Delta t > \Lambda/\theta \sim 1/(zp\theta)$
  - Thus: $\theta_{ee}/(zp\theta^2) > 1/(zp\theta) \implies \theta_{ee} > \theta$



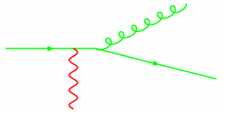
- Thus: Angular ordering takes care of soft limit.

Soft logarithms : Angular ordering

G.Marchesini and B.R.Webber, Nucl. Phys. B **238** (1984) 1.



$\Longrightarrow$

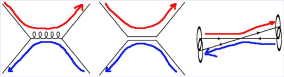


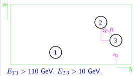
Gluons at large angle from combined colour charge!

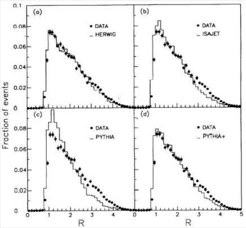
Soft logarithms : Angular ordering

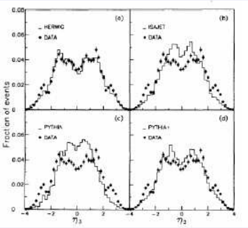
Experimental manifestation:

ΔR of 2nd & 3rd jet in multi-jet events in pp-collisions









Aside: Using the Sudakov form factor analytically

Resummed jet rates in $e^+e^- \to$ hadrons

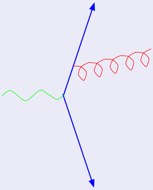
S.Catani *et al.* Phys. Lett. **B269** (1991) 432

- Use Durham jet measure ($k_\perp$-type):

$$k^2_{\perp,ij} = 2\min(E_i^2,\, E_j^2)(1-\cos\theta_{ij}) > Q^2_{\rm jet}\,.$$

- Remember prob. interpretation of Sudakov form factor.
- Then:

$$\begin{aligned}
\mathcal{R}_2(Q_{\rm jet}) &= \left[\Delta_q(E_{\rm c.m.},\,Q_{\rm jet})\right]^2\\
\mathcal{R}_3(Q_{\rm jet}) &= 2\Delta_q(E_{\rm c.m.},\,Q_{\rm jet})\\
&\quad\cdot\int {\rm d}q\left[\Gamma_q(q)\frac{\Delta_q(E_{\rm c.m.},\,Q_{\rm jet})}{\Delta_q(q,\,Q_{\rm jet})}\Delta_q(q,\,Q_{\rm jet})\Delta_g(q,\,Q_{\rm jet})\right]
\end{aligned}$$



Aside: Dipole shower(s)

First implemented in Ariadne (L.Lonnblad, Comput. Phys. Commun. **71**, 15 (1992)).

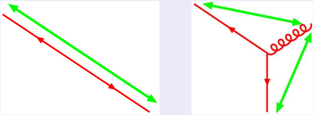
Upshot

- Essentially the same as parton shower (benefit: particles always on-shell)

$$\mathrm{d}\sigma = \sigma_0 \frac{C_F \alpha_s(k_\perp^2)}{2\pi} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \mathrm{d}y.$$

- Always colour-connected partners (recoil of emission)

$\Longrightarrow$ emission: 1 dipole $\to$ 2 dipoles.



Further developments of parton showers

Shower based on Catani-Seymour splitting kernels

First discussed in: Z.Nagy and D.E.Soper, JHEP **0510** (2005) 024.

Implemented in: S.Schumann and F.K., arXiv:0709.1027 [hep-ph];

M.Dinsdale, M.Ternick and S.Weinzierl, arXiv:0709.1026 [hep-ph]

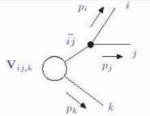
- Catani-Seymour dipole subtraction terms as universal framework for QCD NLO calculations.
- Factorization formulae for real emission process:
- Full phase space coverage & good approx. to ME.

Example: final-state final-state dipoles

splitting: $\tilde{p}_{ij} + \tilde{p}_k \to p_i + p_j + p_k$

variables: $y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$, $z_i = \frac{p_i p_k}{p_i p_k + p_j p_k}$

consider $\mathrm{q}_{ij} \to \mathrm{q}_i \mathrm{g}_j$: $\langle V_{\mathrm{q}_i \mathrm{g}_j,k}(\tilde{z}_i, y_{ij,k})\rangle = C_F \left\{ \frac{2}{1-\tilde{z}_i + \tilde{z}_i y_{ij,k}} - (1+\tilde{z}_i) \right\}$



Features of dipole showers

- Quantum coherence on similar grounds for angular and k_T-ordering, typical ordering in dipole showers by $k_\perp$.
- Many new shower formulations in past few years, many (nearly all) based on dipoles in one way or the other.
- Seemingly closer link to NLO calculations: Use subtraction kernels like antennae or Catani-Seymour kernels.
- Typically: First emission fully accounted for.
- Caveat: Must check some logarithmic terms

B.Webber pointed out some potential problems, when $k_\perp$-ordered shower implemented in naive way.

Further improvements

Could further enhance parton shower by:

- Adding angular correlations: $\mathrm{d}\phi$ not flat

  I.Knowles, Comput.Phys.Commun.**58** (1990), 271

  P.Richardson, JHEP **0111** (2001) 029

- adding further correlations (color)

  Z.Nagy & D.Soper, JHEP **0807** (2008) 025

- in all cases: need to generate density matrices of varying size, and reweight the shower emission
  $\longrightarrow$ may lead to "weighted showers"
  thus hampering the probabilistic structure.

Survey of existing showering tools

(in publicly available code)

Tools	evolution	dipole	AO/Coherence
Ariadne	$k_\perp$	yes	by construction (?)
Herwig	angular	no	by construction
Herwig++	improved angular	no	by construction
Pythia	old: virtuality	no	by hand
	new: $k_\perp$	pseudo	by construction (??)
Sherpa	$k_\perp$-ordering	CS	by construction (?)
Vincia	$k_\perp$-ordered	yes	by construction (?)

Summary of lecture 3

- Parton showers as simulation tools.
- Discussed theoretical background:
  Universal approximation to full matrix elements in the collinear limit.
- Highlighted some systematic improvements.
- Hinted at close relation to resummation.