Jet Physics: Tutorial

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Analysis framework

Software:

- ► FastJet (v3.0alpha2)
- ► HepMC (v2.06.04)
- ▶ ROOT (v5.28.00a)

Extra tools:

- ▶ SimpleHist.hh provides class that we will use for histogramming

Analysis framework

Data files (thanks to Simon Plätzer, Frank Siegert and Stefan Prestel!)

LHC
$$\sqrt{s} = 7 \text{ TeV}$$

- dijets-no-ue.hepmc.gz (Pythia 8.1.4.5, no MPI, $p_{t,\min}^{\text{hard proc}} = 60 \text{ GeV}$)
- ▶ dijets-with-ue.hepmc.gz (Pythia 8.1.4.5, UE tune 3C, $p_{t,\min}^{\text{hard proc}} = 60 \text{ GeV}$)
- ▶ hz-bbar.hepmc.gz (Herwig++ 2.5, with NLO matching, $Z \rightarrow e^+e^-$, $p_{t,H} > 120$ GeV, $m_H = ????$ GeV [we will try to find out])
- $ilde{ t z}$ -plus-jets.hepmc.gz (Sherpa 1.2.3, LO with CKKW merging, $Z o e^+ e^-$)

Example programs for this tutorial

/mc-school/jets-tutorial

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Open README there and follow the instructions

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 $\verb|algorithms.cc|| jets.cc|| ue.cc|| subtraction.cc|| boostedhiggs.cc||$

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In this tutorial, we will write the following programs:

 $\verb|algorithms.cc|| \verb|jets.cc|| ue.cc|| subtraction.cc|| boosted higgs.cc||$

A lot more can be found

- ▶ in the FastJet source directory fastjet-VERSION/example
- online http://www.fastjet.fr (Manual, Doxygen documentation)

Seeing jets

Pythia 8, LHC, $\sqrt{s} = 7 \text{ TeV}$

▶ a single dijet event (no. 21) from dijets-with-ue.hepmc.gz

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 JetDefinition jet_def(antikt_algorithm,0.6);

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AreaDefinition area_def(active_area_explicit_ghosts);
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```

cluster and get the list of all jets

```
ClusterSequenceArea cs(input_particles, jet_def, area_def);
vector<PseudoJet> jets = sorted_by_pt(cs.inclusive_jets());
```

Print list of jets (our own function defined in EventRecord.hh)

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```

produce lego plot with showjets.C

```
root
root> .L showjets.C+
root> showjets("dijetevent-akt.out")
```

Play a bit with parameters and look how results change

• get inclusive jets with $p_t > ptmin$

```
vector<PseudoJet> jets = sorted_by_pt(cs.inclusive_jets(5.0));
```

- \blacktriangleright see what happens when you go to larger jet radius, e.g. R= 1.2
- try other native algorithms: kt_algorithm, cambridge_algorithm
- ▶ try an external algorithm: SISCone
 - ▶ it comes as a plugin therefore we need to modify the way we set jet definition

```
JetDefinition::Plugin *plugin;
plugin = new SISConePlugin(0.6, 0.75);
JetDefinition jet_def(plugin);
```

▶ for speed reasons, it is good to use large area of ghosts

```
GhostedAreaSpec gaspec(8.0, 1, 0.05);
AreaDefinition area_def(active_area_explicit_ghosts,gaspec);
```

Seeing more jets

Selectors

```
Selector select_pt = SelectorPtMin(70.0);
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Selector full_selection = select_pt && select_rapidity;
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  JetDefinition jet_def(cambridge_algorithm, 0.6);
 ClusterSequence cs(input_particles, jet_def);
 vector<PseudoJet> jets = cs.inclusive_jets();
 vector<PseudoJet> selected_jets
        = sorted_by_pt(full_selection(jets));
  if (selected_jets.size() == 0) continue;
 PseudoJet hardest_selected_jet = selected_jets[0];
```

Add to histogram

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hisjetspthardest.add_entry(hardest_selected_jet.perp()); Code that you find in jets.cc below // write writes the histogram of p_t of the hardest jet into a file.
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- ▶ set some suffix for the output file, e.g. string suffix = "rap02";
- ▶ compile the program, run it and plot the histogram from the .out file

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When the above is done...

- change rapidity selection from the range [0, 2] to [2, 4]
- change suffix
- run the program and compare results from the two above selections of rapidity ranges

Next, we will try to get mass distribution of the hardest jet

modify the rapidity selection to

```
Selector select_rapidity = SelectorAbsRapMax(4.0);
```

- ▶ in analogy with *p_t* histograms: add all code needed for writing histograms of the mass of hardest jet
- \triangleright produce mass histograms with two p_t cuts: at 50 GeV and 200 GeV

Seeing more than jets: underlying event

Jet based techniques can be also used to measure the underlying event!

area/median approach [Cacciari, Salam, Soyez (2008)]

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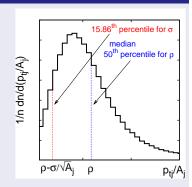
For each event

- cluster particles with an infrared safe jet finding algorithm (all particles are clustered so we have set of jets ranging from hard to soft)
- 2. from the list of all jets (no cuts required!) determine

$$\rho = \text{median}\left[\left\{\frac{p_{t,j}}{A_j}\right\}\right]$$

and its uncertainty σ

- ▶ median gives a typical value of p_t/A for a given event
- using median is a way to dynamically separate hard and soft parts of the event



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Add rho to the histogram histogram

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Compile and let it run for a while...

Take a look at the ρ -histogram

- the distribution is broad! underlying event fluctuates a lot from one event to another
- there is always a contribution to "zeroth bin", it comes from quiet events with pure ghost jets constituting more than half of all jets

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Change the rapidity range

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Modify the name of the output file, produce results for rho and sigma and compare them with those from the rapidity range [0, 2].

ue.cc

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ightharpoonup
ho distribution dependence on the rapidity range where the UE is measured

Seeing jets better

What else can we do with ρ and σ ?

ightharpoonup
ho may be used e.g. to correct hard jet transverse momentum

$$p_{t,j}^{\mathrm{(sub)}} = p_{t,j} - \rho A_j \pm \sigma \sqrt{A_j}$$

since jet area measures the jet susceptibility to the soft radiation

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Run the program and produce the results for p_t distribution of the leading jet in the rapidity range [-4, 4] in the case without the underlying event.

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Go for the clustering with area

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AreaDefinition area_def(active_area_explicit_ghosts);
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```

Determine rho on the event by event basis

Subtract the UE contamination and add the result for corrected p_t to the histogram

```
PseudoJet vect_area = hardest_selected_jet.area_4vector();
PseudoJet jet_cor = hardest_selected_jet - rho*vect_area;
hispthardest_cor.add_entry(jet_cor.perp());
```

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- compare results for jets without UE with the uncorrected results with UE
- add the corrected result with UE

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- compare results for jets without UE with the uncorrected results with UE
- add the corrected result with UE

We see that the correction brings the spectrum from the events with UE close to that from the events without UE.

result still can be improved, e.g. by optimizing R and rapidity range in which UE is determined

Seeing more inside jets: an example of subjet analysis

The file /mc-school/data/hz-bbar.hepmc.gz provides 50 000 events for $pp \to ZH \to I^+I^- + b\bar{b}$.

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Get hadrons and leptons from this file (handled by EventRecord)

```
vector<PseudoJet> input_particles = evtrec.hadrons();
vector<PseudoJet> leptons = sorted_by_pt(evtrec.leptons());
```

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```
vector<PseudoJet> input_particles = evtrec.hadrons();
vector<PseudoJet> leptons = sorted_by_pt(evtrec.leptons());
```

Select only events with $p_{t,Z} > 200\,\mathrm{GeV}$ and $80\,\mathrm{GeV} < m_{I^+I^-} < 100\,\mathrm{GeV}$

```
PseudoJet Zboson;
if (leptons.size()>=2)
  Zboson = leptons[0]+leptons[1];
  if (Zboson.perp() < 200 || Zboson.m() < 80 || Zboson.m() >100)
  continue;
```

Cluster hadrons and accept only events with $p_{t, \text{hardest jet}} > 200\,\text{GeV}$

```
ClusterSequence cs(input_particles, jet_def);
vector<PseudoJet> jets = sorted_by_pt(cs.inclusive_jets(30.0));
if (jets.size()==0) continue;
PseudoJet hardest_jet = jets[0];
if (hardest_jet.perp() < 200) continue;</pre>
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```

Remember that our jets were defined with R=1.2. By selecting only the events with high transverse momenta of the lepton pair (i.e. Z boson) and the leading jets we choose configurations with highly collimated products of the Higgs decay $H \to b\bar{b}$. Hence, we believe that b and \bar{b} should end up in our large, leading jet.

Add entry to the histogram

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hismasshardest.add_entry(hardest_jet.m());
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Compile the program, run it and take a look at the mass distribution...

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$$m_{j_1} < \mu m_j$$
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$$\frac{\min(p_{t,j_1}, p_{t,j_2})}{\max(p_{t,j_1}, p_{t,j_2})} \simeq \frac{\min(p_{t,j_1}^2, p_{t,j_2}^2)\Delta_{12}^2}{m_j^2} > y_{\text{cut}}$$

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5. if the two conditions are satisfied close, exit the loop, if not redefine j to be equal to j_1 and go back to step 1

To see how it works in practice, uncomment the following lines in the source file

```
// mass drop analysis
double mass_drop_threshold = 0.667;
double rtycut
                            = 0.3:
PseudoJet parent1, parent2;
bool had_parents;
while ((had_parents = hardest_jet.has_parents(parent1,parent2))) {
  if (parent1.m() < parent2.m()) swap(parent1,parent2);</pre>
  if (parent1.m() < mass_drop_threshold * hardest_jet.m() &&</pre>
      parent1.kt_distance(parent2) > pow(rtycut,2) * hardest_jet.m2()) {
    break:
  } else {
    hardest_jet = parent1;
  } }
```

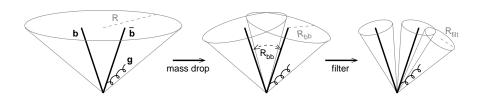
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    break:
  } else {
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  } }
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Change the name of the output file, compile the program, run it and compare that result with that without mass drop.

To remove contamination from the underlying event and further improved the quality of the peak we add the procedure of *filtering*

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[from J. M. Butterworth, A. R. Davison, M. Rubin and G. P. Salam, Phys. Rev. Lett. **100** (2008) 242001]

Define the new filter radius

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double Rzqq = sqrt( parent1.squared_distance(parent2) );
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Filter the hardest jet

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PseudoJet filtered_jet = filter(hardest_jet);
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Filter the hardest jet

```
PseudoJet filtered_jet = filter(hardest_jet);
```

Check the *ptmin* criterion for the filtered jet and, if satisfied, add entry to the histogram

```
if (filtered_jet.perp() < 200 ) continue;
hismasshardest_filt.add_entry(filtered_jet.m());</pre>
```

▶ get the same three histograms for Z + j

```
string proc = "z-plus-jets";
```

▶ compare how H + Z looks on the top of Z + j background be careful: the first process is in nb the second in pb