EFT Lagrangian Morphing

Modelling physics distributions in the framework of an Effective Field Theory (EFT)

2nd Pan-European Advanced School on Statistics in High Energy Physics

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Outline

Effective field theory

- Motivation for EFT approach at the Large Hadron Collider
- Effective Lagrangian
- Observable dependence on EFT parameters

About the theoretical model

Effective Lagrangian Morphing

- Morphing physics distributions within EFT
- Interface between data and theory

Implementation within ROOT

- Modelling EFT observables with RooLagrangianMorphFunc
- Examples based on 2 and 3 EFT parameters

Describing data observables within model

References

Effective field theory

- Introduction to Effective Field Theories, *Manohar,* *Les Houches Lect. Notes 108(2020)*
- The Standard Model as an effective field theory, *Brivio, Trott*, *Physics Reports, Volume 793, 2019*

Effective Lagrangian Morphing

- A morphing technique for signal modelling in a multidimensional space of coupling parameters, *The ATLAS collaboration,* *ATL-PHYS-PUB-2015-047*
- Effective Lagrangian Morphing, *Balasubramanian, Burgard, Verkerke*, *arXiv:2202.13612*

Morphing within ROOT

- RooFit tutorials

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Effective field theory

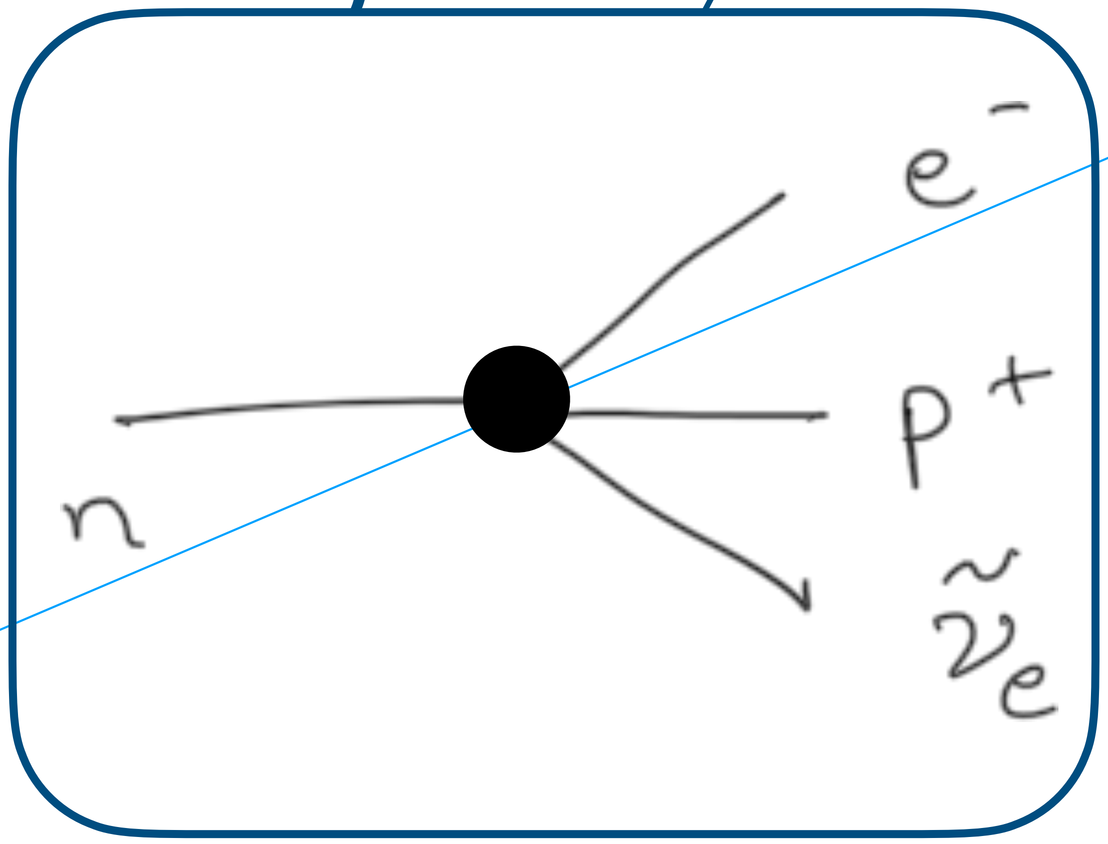
- Effective theories appear everywhere in physics !

- Based on the **principle of separation of scale**, physics at a given energy scales decouples from physics at another scale except for a few parameters

  Ex: Fermi theory sufficient to describe β decays, no description of electroweak Interactions required !

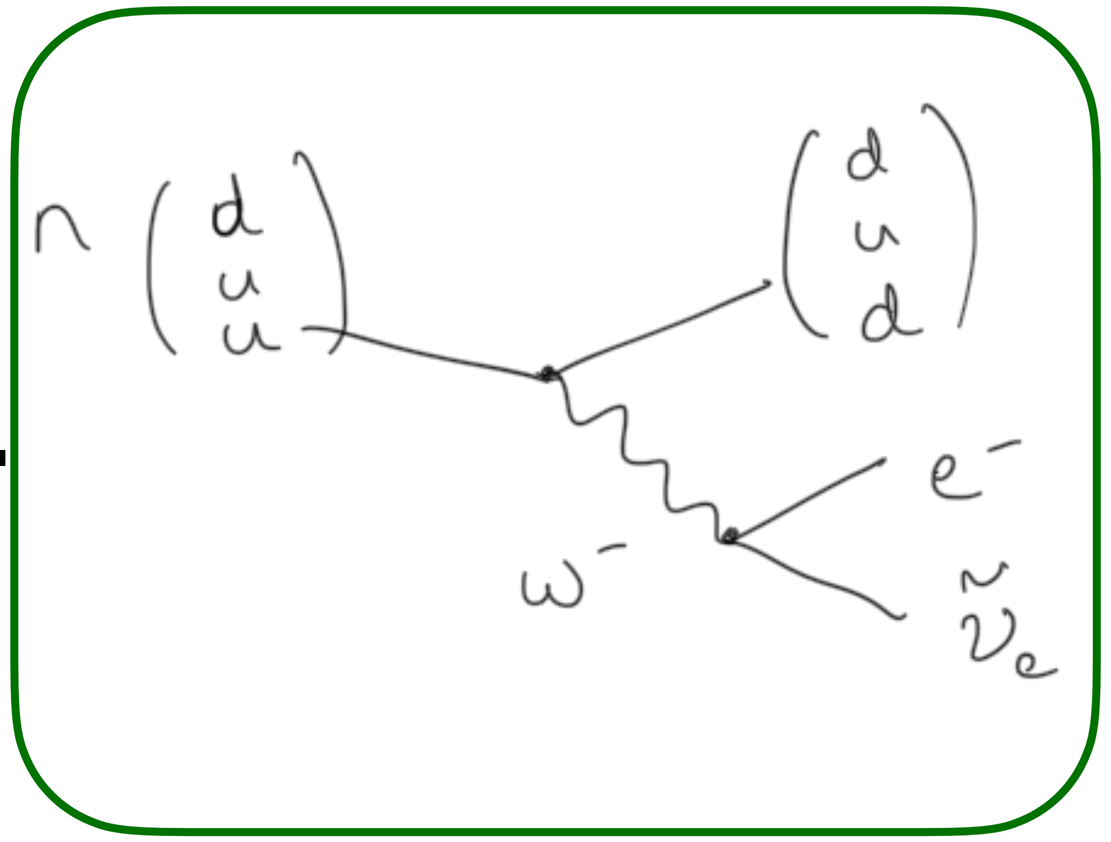
β decay

$\propto \frac{1}{m_W^2}$



$\xleftarrow{p \ll m_W}$

W mediated



$\propto \frac{1}{p^2 - m_W^2}$

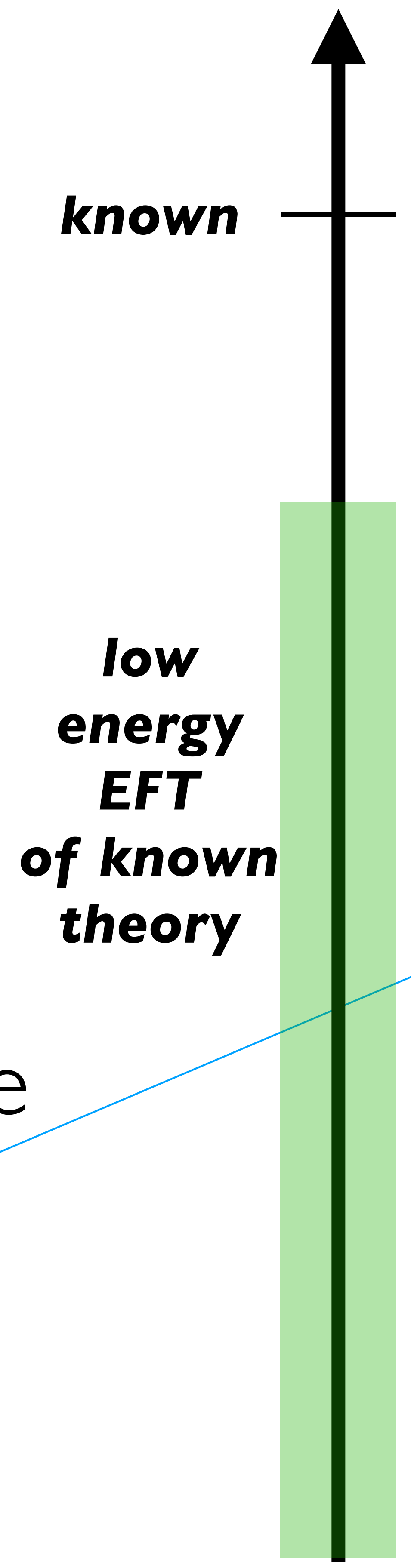
Two types of EFT

Top-down

- Theory at higher-energy scale known, can simplify the picture to describe phenomena at lower energy scales.

  ex.

  top quark (M_t = 173 GeV) is much heavier for QCD processes where the interactions are at the O(few) GeV scale

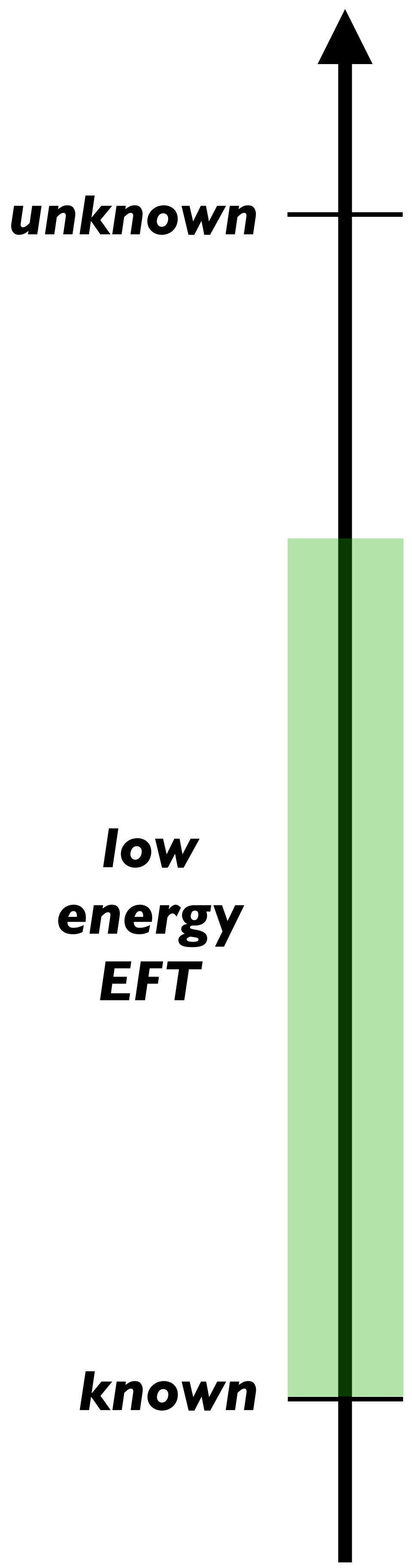


Bottom-up

- Valid theory valid theory known at a current energy scale, however no description of physics at higher scales.
- Current theory can be viewed as a low-energy effective approximation of a more fundamental theory

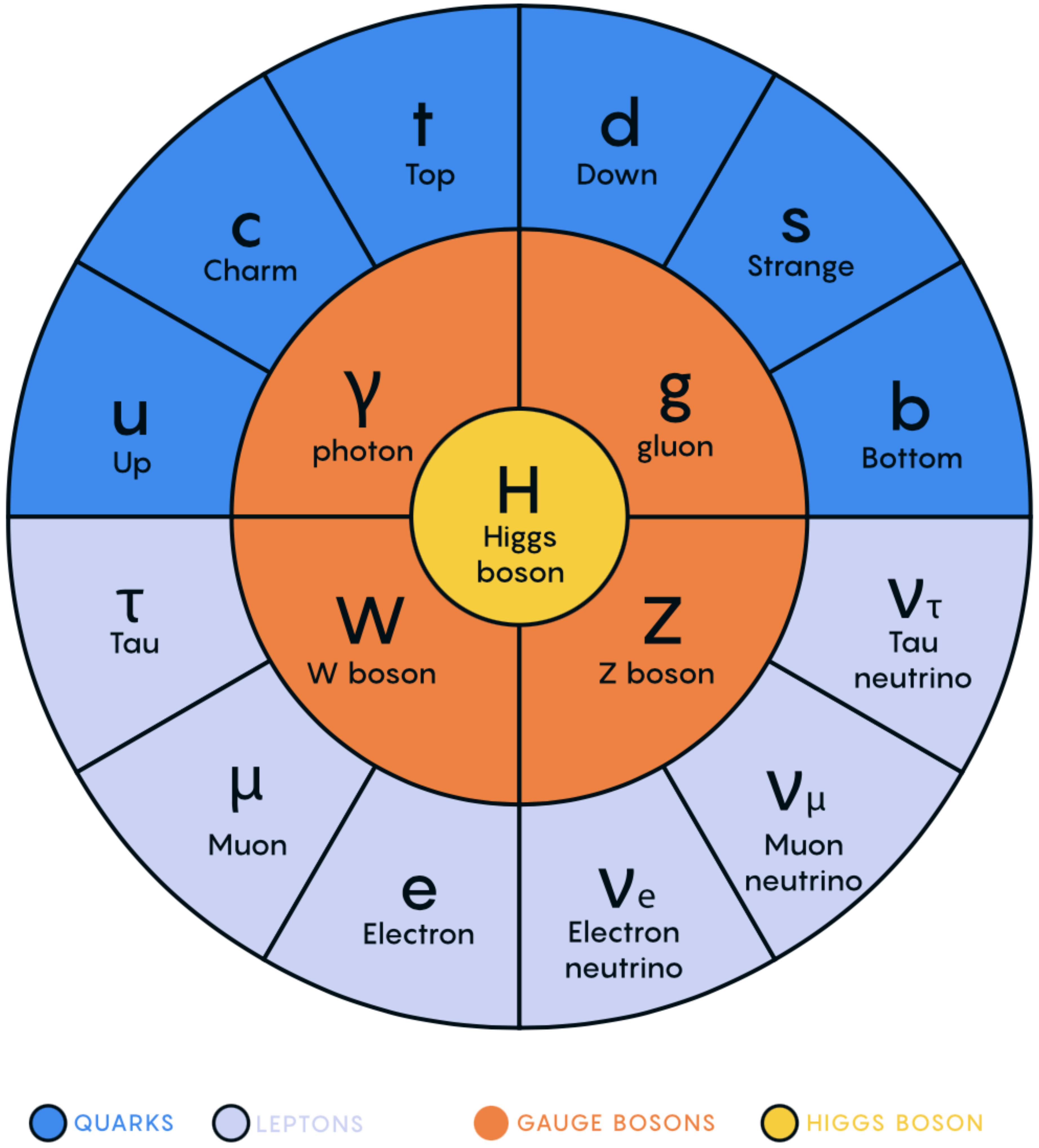
  ex.

  Fermi theory is the low energy approximation of the Standard Model



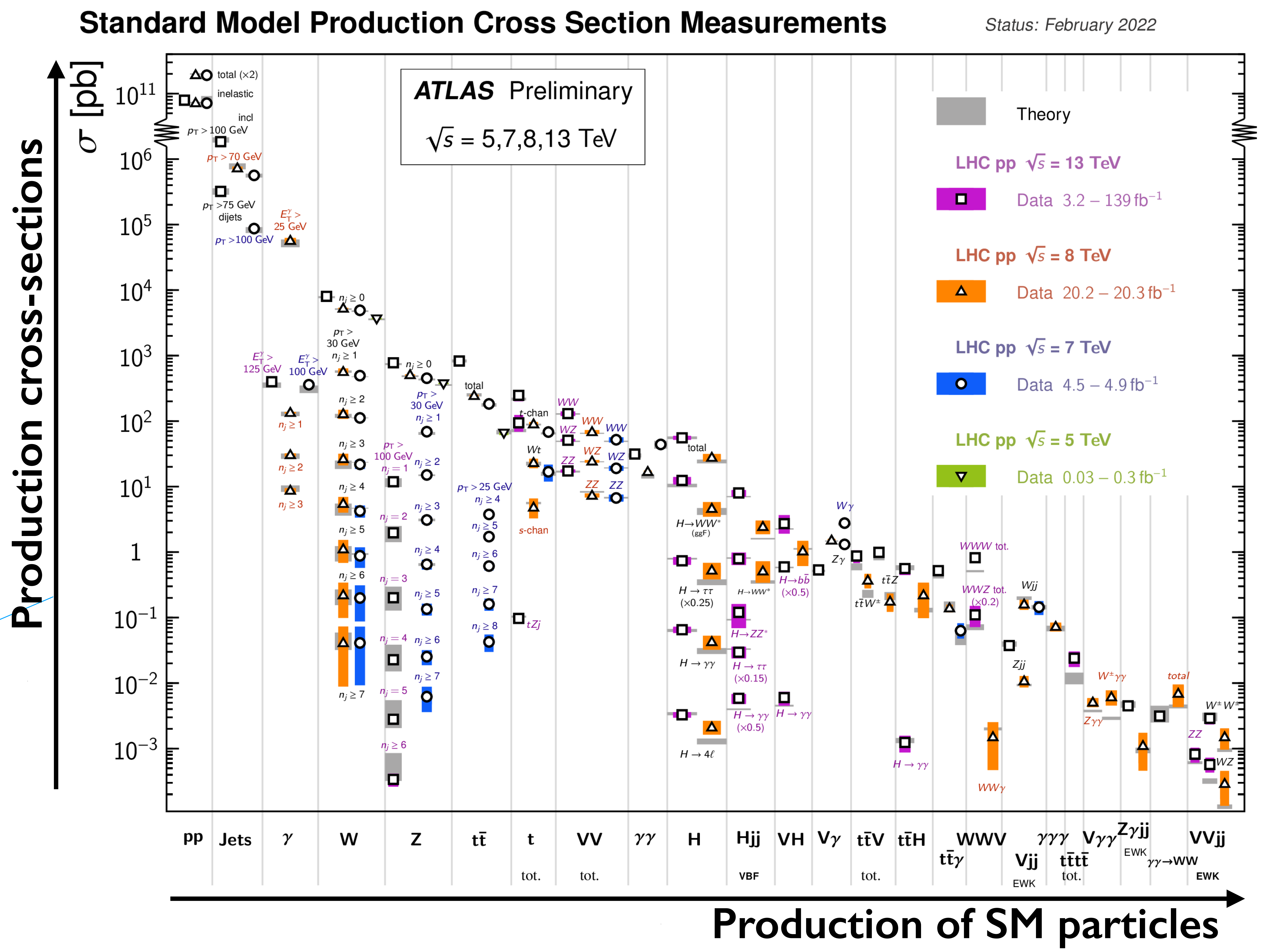
Standard Model

- The Standard Model (SM) is a remarkably successful theory
- It is being studied extensively at the LHC and other experiments around the world
- No direct evidence for new members of the family
- However lacks explanation for many phenomena !
  dark matter, matter-antimatter asymmetry, etc..



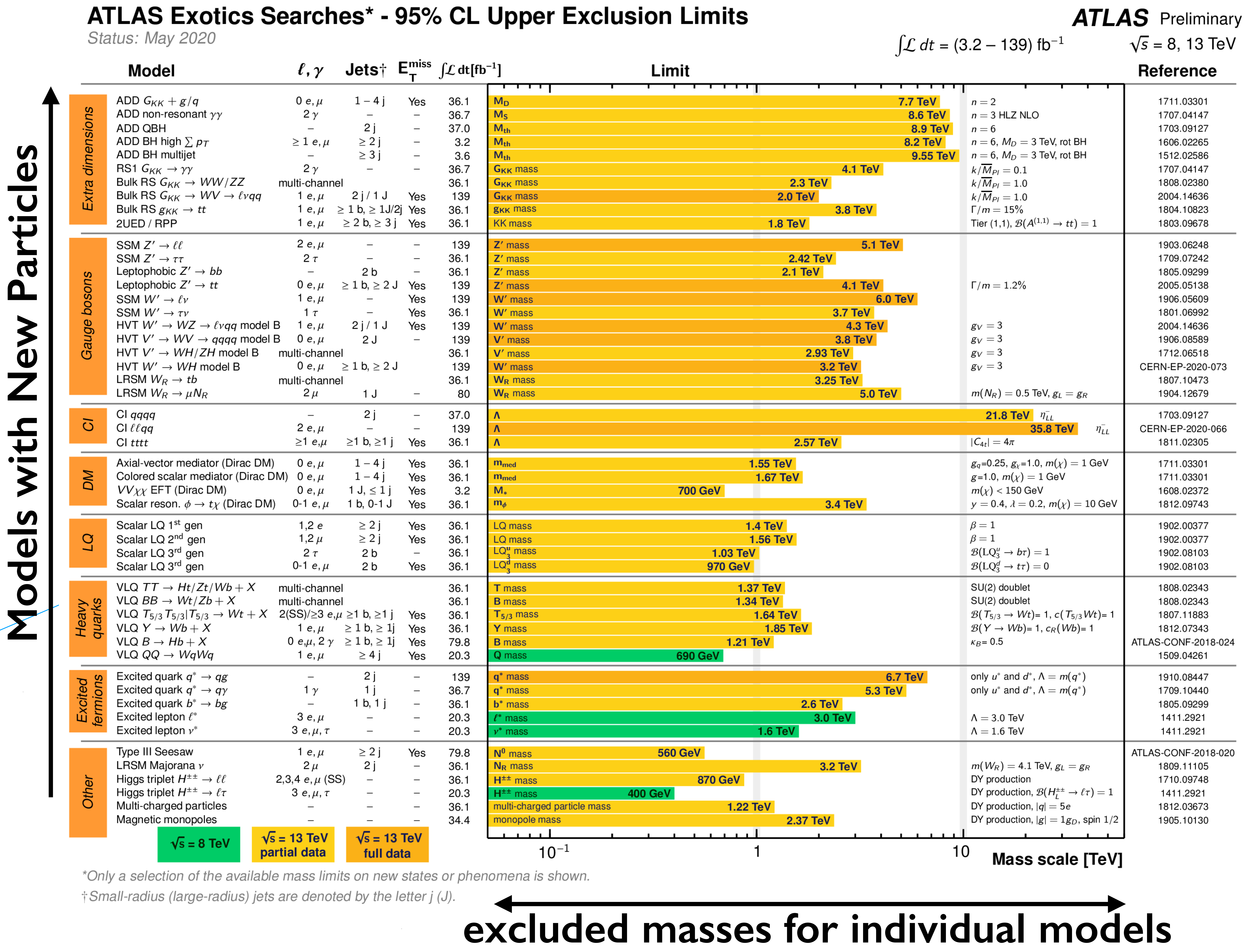
Standard Model - highly predictive !

- The Standard Model predictions agree well with measurements at the LHC
- The energy scale probed by these measurements typically around vev (246 GeV)
- Important to measure differential distributions to separate out regions with higher energy reach !

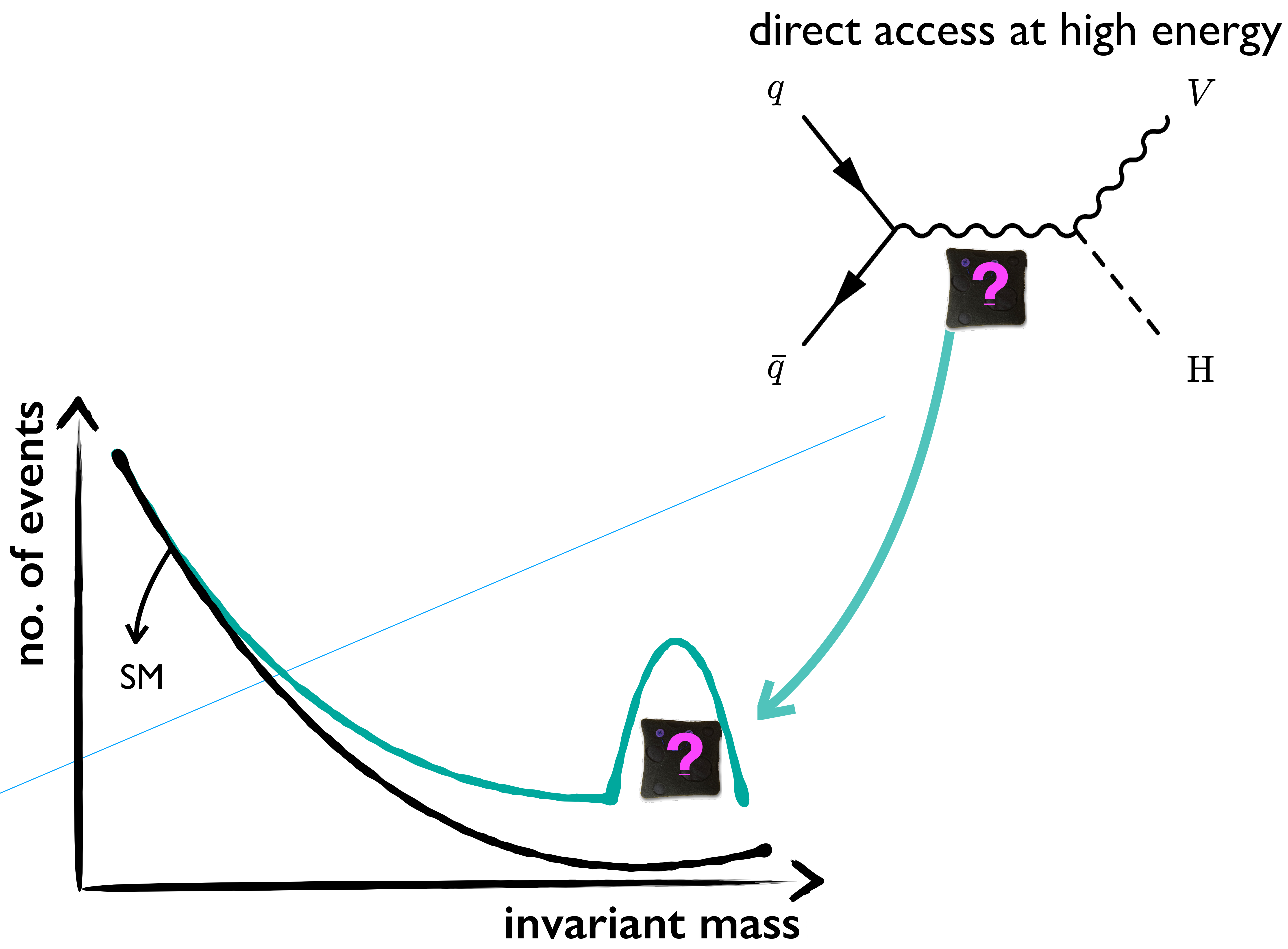


Direct searches for new particles

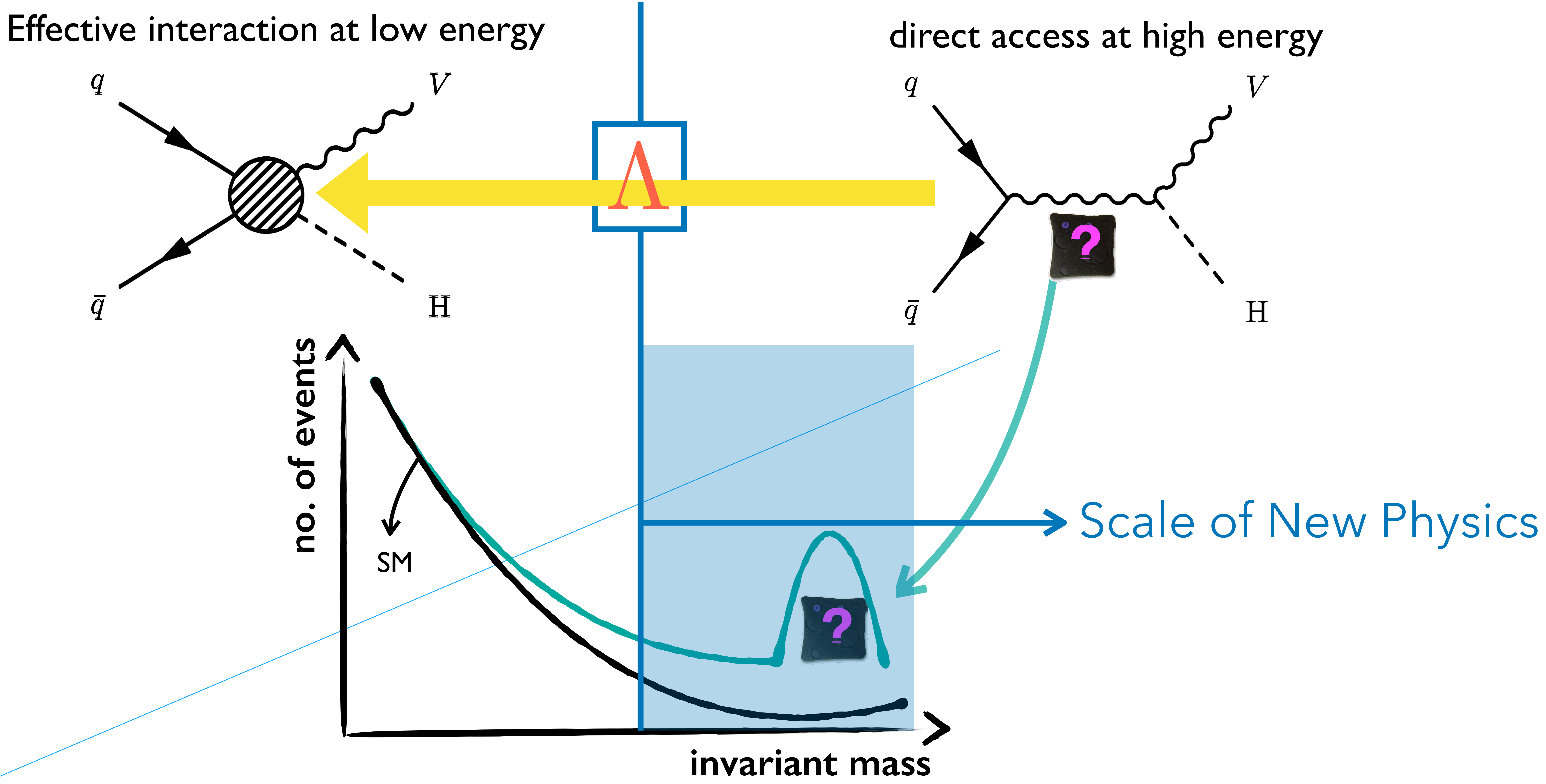
- Direct searches probe resonant production or non-resonant effects direct production of new particles
- Usually based on a particular models with unique signature(s)
- Mass reach saturated at 1-10 TeV



Looking for direct signatures



Looking for indirect signatures



- SMEFT allows to probe BSM physics at energies much higher than direct energy reach

EFT Lagrangian

- EFT Lagrangian written out as an expansion in terms of $1/\Lambda$ where $\Lambda \gg$ E, vev

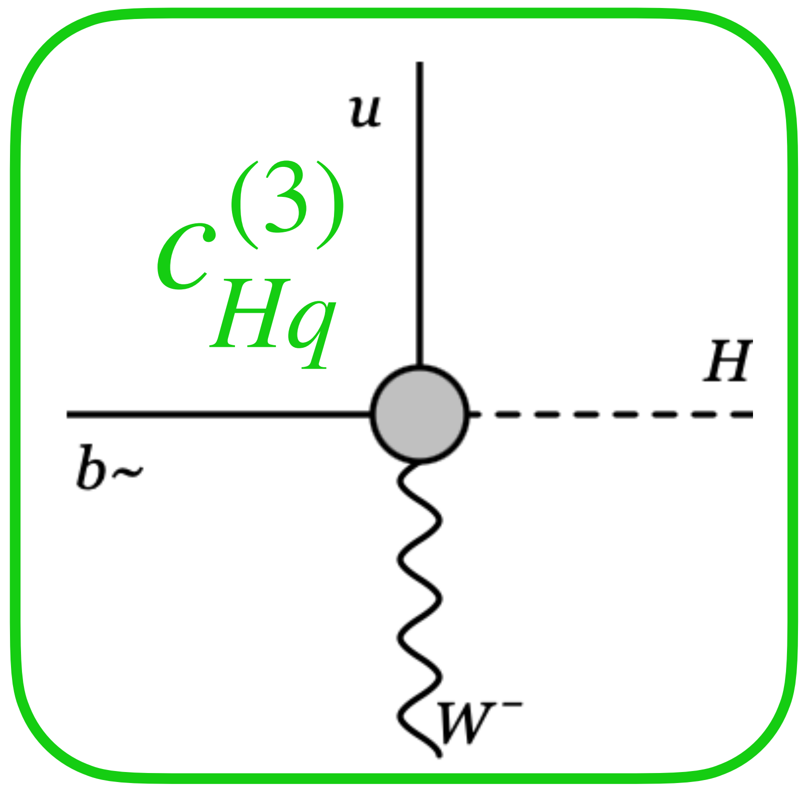
$$\boxed{\mathscr{L} = \mathscr{L}_{SM} + \frac{1}{\Lambda}\mathscr{L}^{(d=5)} + \frac{1}{\Lambda^2}\mathscr{L}^{(d=6)} + \cdots + \frac{1}{\Lambda^{D-4}}\mathscr{L}^{(d=D)} + \cdots}$$

- $\mathscr{L}_{SM}$ contains no information of Λ, accessible by higher order terms which are given as sum of all possible operators $\{\mathcal{O}_i^d\}$ where,

$$\boxed{\mathscr{L}^{(d)} = \sum_i c_i \mathcal{O}_i}$$

- $\{c_i\}$ are wilson coefficients are correspond to free parameters of the model
- $\mathcal{O}_i$ all possible terms with known fields that respect allowed symmetries
  → Lorentz invariance, Gauge symmetry
- $\{\mathcal{O}_i\}$ acts as a basis to describe all allowed deformation to the Standard Model

$$\mathcal{O}^{(3)}_{Hq} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$$



Observables in EFT

- We want to model the dependence of observable distribution on the parameters of the model $\{c_i\}$, ex.

$$\frac{d\sigma}{dp_T}$$

- Distribution predictions can be generated at choice of parameters values using monte-carlo simulations

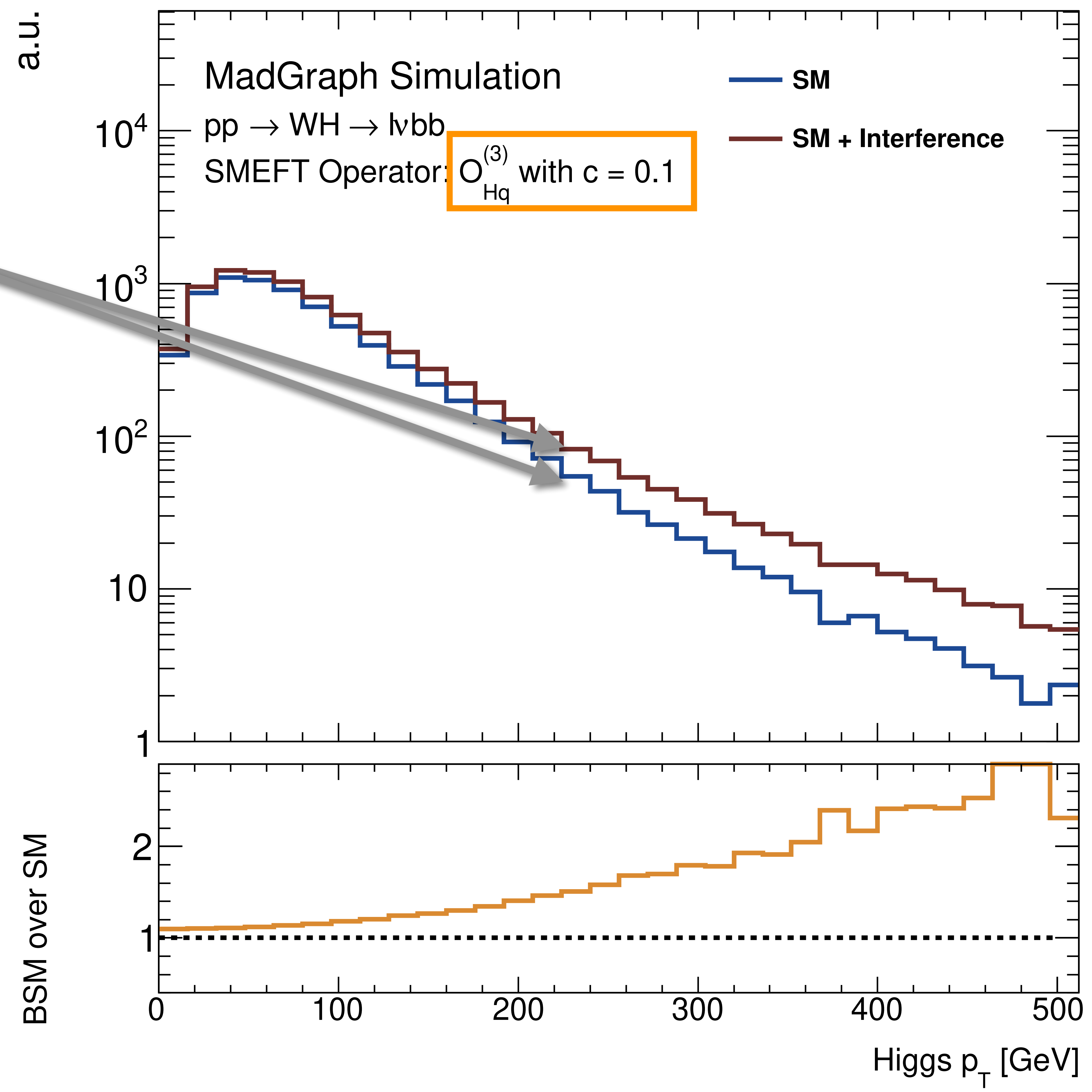
- However expensive to construct a fine grid of simulations, grid simulations scales exponential with number exponentially with number of parameters involved
  → Prohibitive !

- Crucial to construct likelihood function in terms of EFT parameters,

L(data| $\vec{c}$, Nuisance parameters)

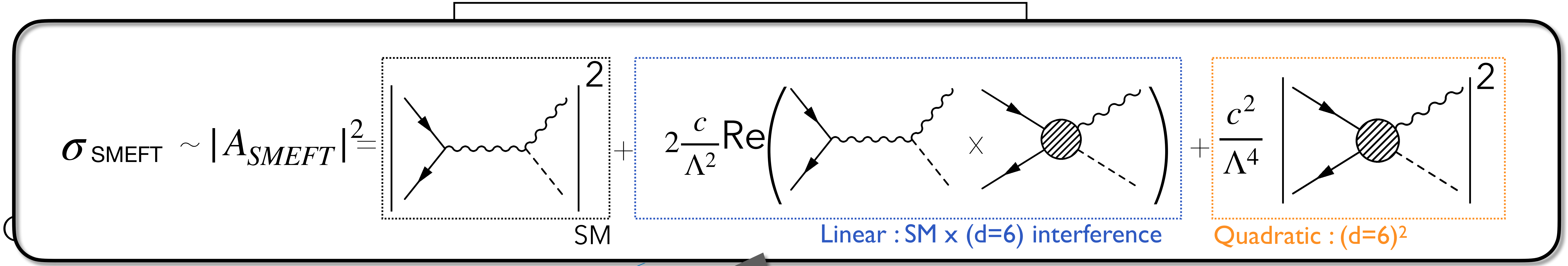
[See Glen Cowan's talk on likelihood based combinations]

B. Moser's thesis



From Lagrangian to observables

- Given the Lagrangian, can write down transition amplitude for process as sum of amplitudes,
- Let's consider a one operator at d=6 case,



$$|A_{SMEFT}(c)|^2 = |\mathcal{O}_{SM}|^2 + 2.\frac{c}{\Lambda^2}\Re(\mathcal{O}^*_{SM}\mathcal{O}_{EFT}) + \frac{c^2}{\Lambda^4}|\ \mathcal{O}_{EFT}|^2$$

Distributions estimated from MC simulations

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Mapping distributions to operators

- Distributions map to contributions of all the different operators,

$$\boxed{|A_{SMEFT}(c)|^2 = |\mathcal{O}_{SM}|^2 + 2.\frac{c}{\Lambda^2}\Re(\mathcal{O}^*_{SM}\mathcal{O}_{EFT}) + \frac{c^2}{\Lambda^4}|\ \mathcal{O}_{EFT}|^2}$$

- For one operator, templates generated at three values of the grid should suffice to span the whole parameter space,

$$\sigma(c=0) \propto |\mathcal{O}_{SM}|^2$$

$$\sigma(c=1) \propto |\mathcal{O}_{SM}|^2 + 2.\frac{c}{\Lambda^2}\Re(\mathcal{O}^*_{SM}\mathcal{O}_{EFT}) + \frac{c^2}{\Lambda^4}|\ \mathcal{O}_{EFT}|^2$$

$$\sigma(c=-1) \propto |\mathcal{O}_{SM}|^2 - 2.\frac{c}{\Lambda^2}\Re(\mathcal{O}^*_{SM}\mathcal{O}_{EFT}) + \frac{c^2}{\Lambda^4}|\ \mathcal{O}_{EFT}|^2$$

Distribution in the full parameter space

- Three samples can be used to generate the continuous description, can fix $\Lambda = 1$ TeV for reference,

σ

- In this example we saw a one parameter case, however this principle can be extended to arbitrary number of parameters which can be sampled at any set of independent points in the parameter space

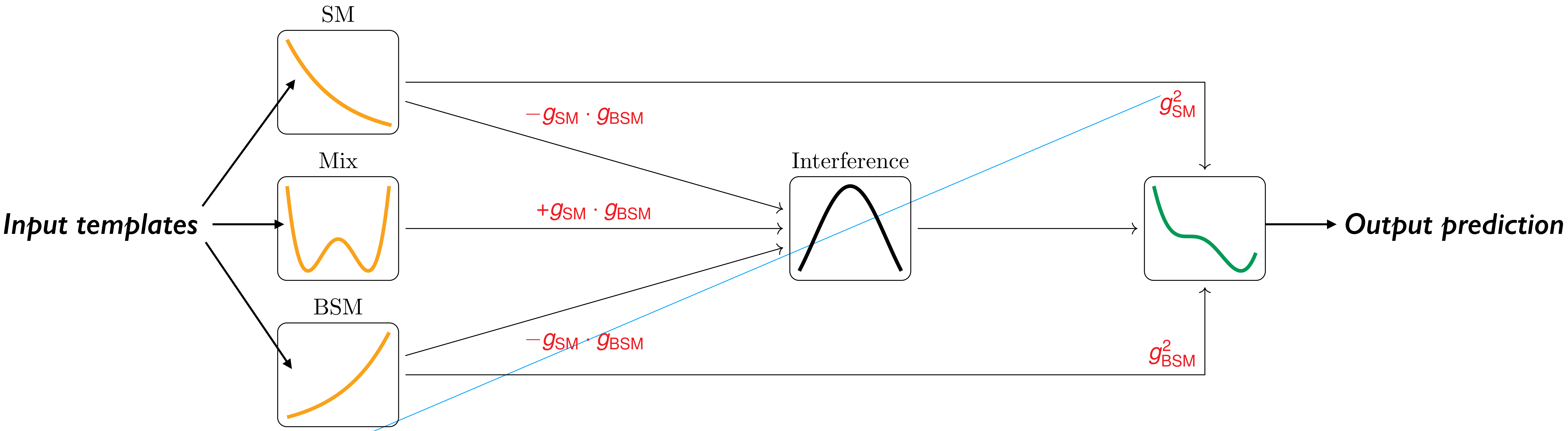
- Morphing → procedure to turn a collection of probability models for individual points in parameter space to a continuous description

{L(data| c=0, NPs), L(data| c= -1, NPs), L(data| c=+1, NPs)} → L(data| c, NPs)

Morphing principle

- The morphing principle is based on linear algebra, very robust ! Only relies on independent sample to have well defined weight matrix

$$(g_{\mathrm{SM}}^2 - g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}})\,\sigma_{\mathrm{SM}} + (g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}})\,\sigma_{\mathrm{Mix}} + (g_{\mathrm{BSM}}^2 - g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}})\,\sigma_{\mathrm{BSM}} = \sigma_{\mathrm{out}}(g_{\mathrm{SM}}, g_{\mathrm{BSM}})$$



	g_{SM}	g_{BSM}
SM	1	0
Mix	1	1
BSM	0	1

Parameter Card

$$\xrightarrow{\text{polynomials}} \quad \begin{array}{c c} & \begin{array}{ccc} g_{\mathrm{SM}}^2 & g_{\mathrm{SM}}g_{\mathrm{BSM}} & g_{\mathrm{BSM}}^2 \end{array} \\ \begin{array}{c} \mathrm{SM} \\ \mathrm{Mix} \\ \mathrm{BSM} \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \quad \xrightarrow{\text{inversion}} \quad \begin{array}{c c} & \begin{array}{ccc} \mathrm{SM} & \mathrm{Mix} & \mathrm{BSM} \end{array} \\ \begin{array}{c} g_{\mathrm{SM}}^2 \\ g_{\mathrm{SM}}g_{\mathrm{BSM}} \\ g_{\mathrm{BSM}}^2 \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

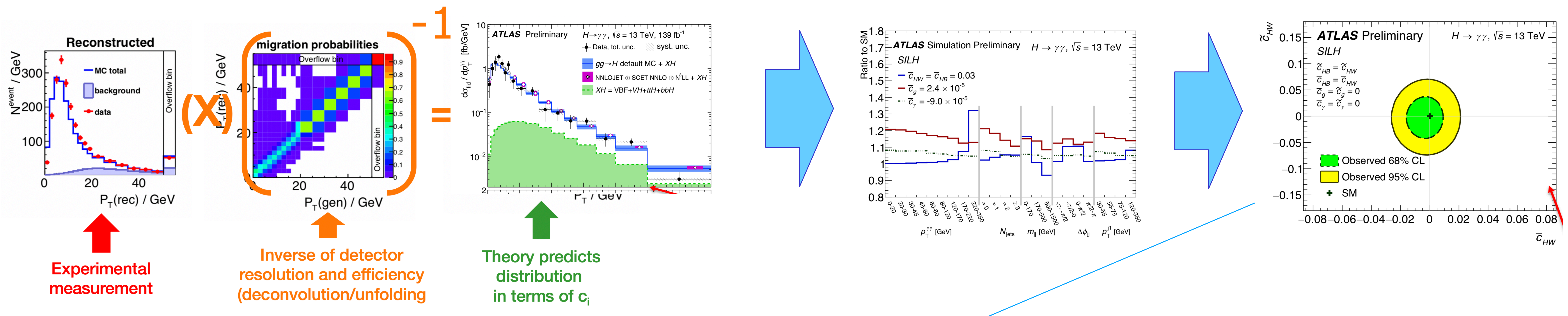
Weight Matrix **Morphing Matrix**

interface between data and theory - unfolded distributions

- Three possible approaches for the interface between data and theory for experimentalist

I. Publishing theory-level distributions

1) **Full unfolding** (very complicated) → SM(EFT) parametrization (easy) → Measurement



- Unfolding is a numerically very difficult problem that requires 'regularization' to make deconvolution step numerically stable

- Many algorithms on the market – with variable sensitivity to assumptions, biases, etc

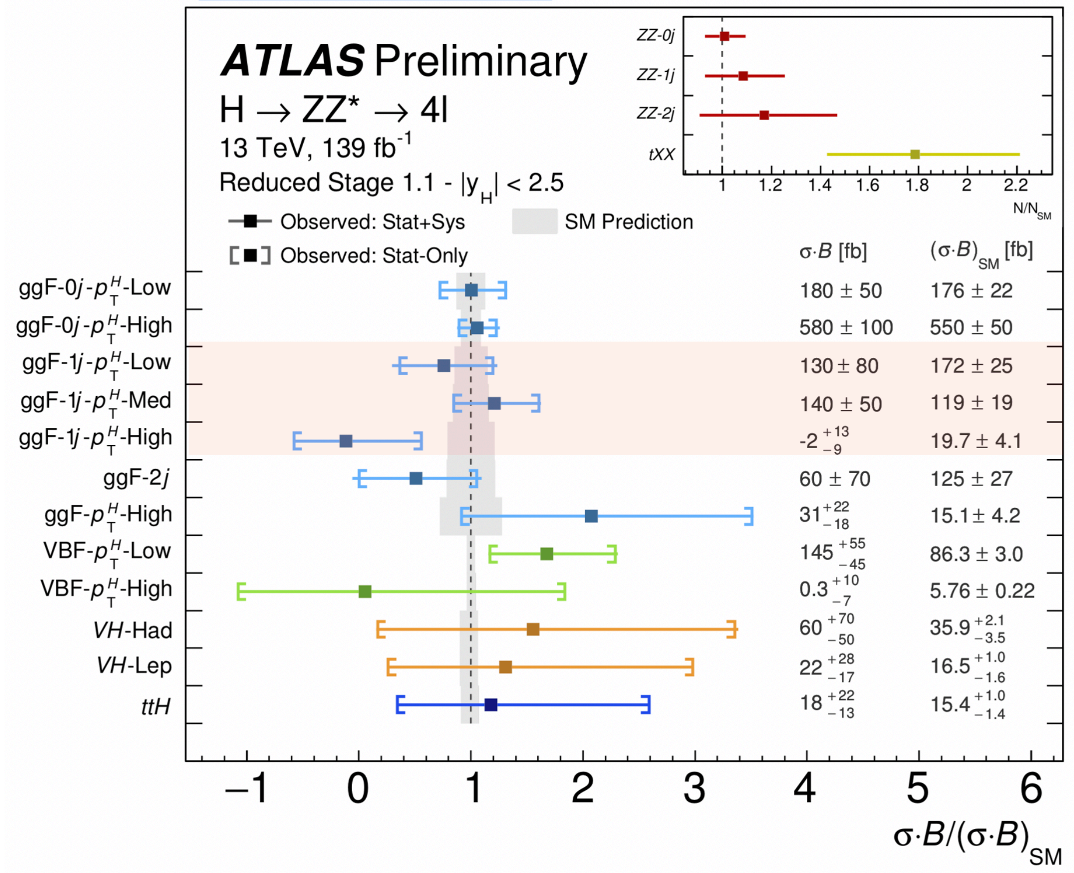
[See Carsten Burgard' talk on unfolding]

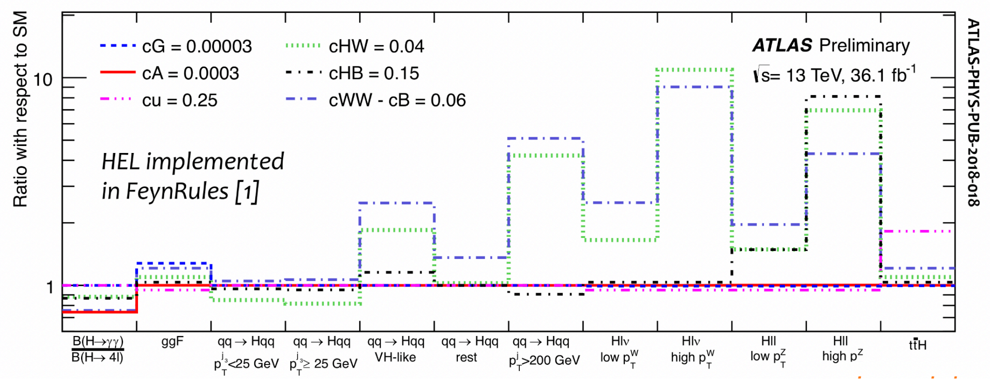
- Unfolded physics distributions are extremely time and resource intensive for collaborations to produce!

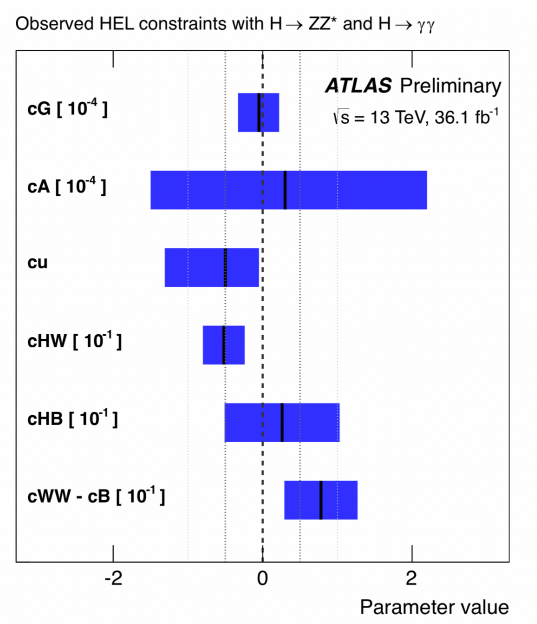
interface between data and theory - template cross-sections

II. Publishing template distribution close to reconstruction level

2) **Template cross-sections** (med. hard) → SM(EFT) param. (med. hard) →Measurement







- Avoid unfolding by publishing template cross-sections in regions close to analysis reconstruction region
- Typically perform for Higgs cross-sections, may not necessarily extend for all use cases
- Additional work involved in mapping each measurement σ to the expression in c_i

interface between data and theory - template cross-sections

II. Publishing template distribution close to reconstruction level

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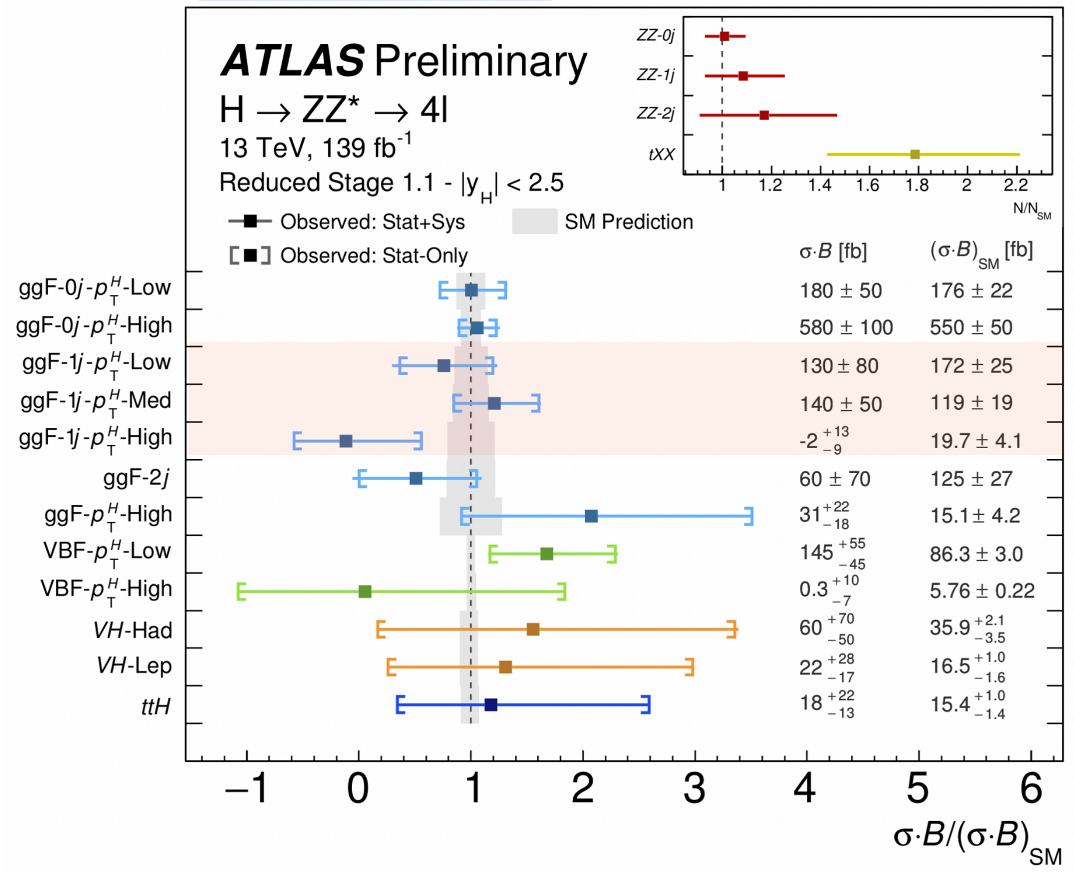
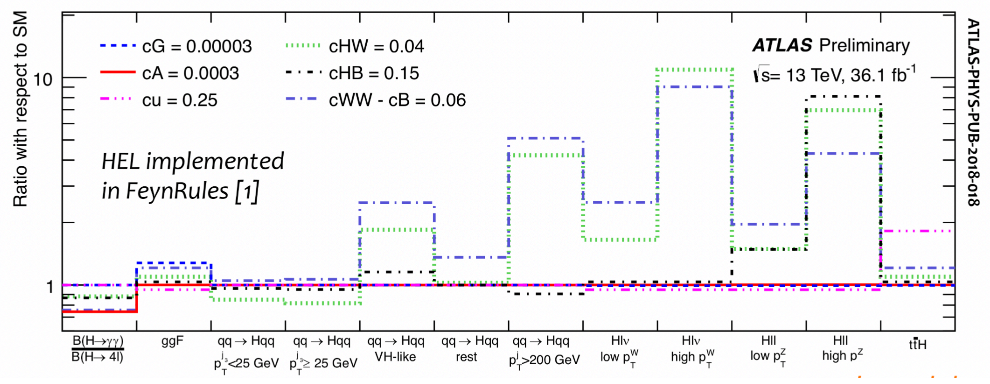
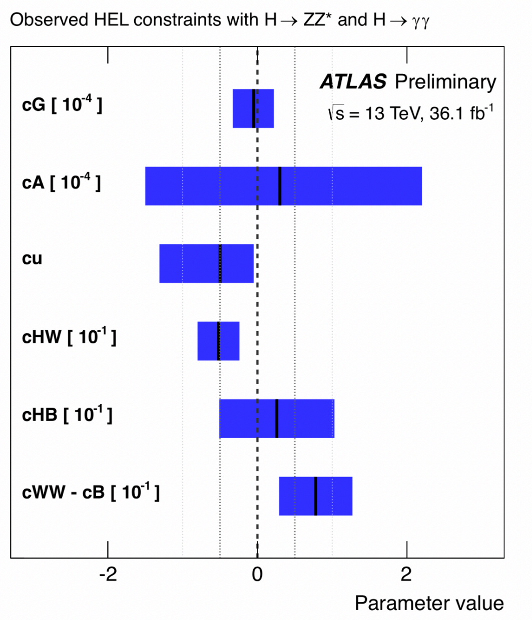


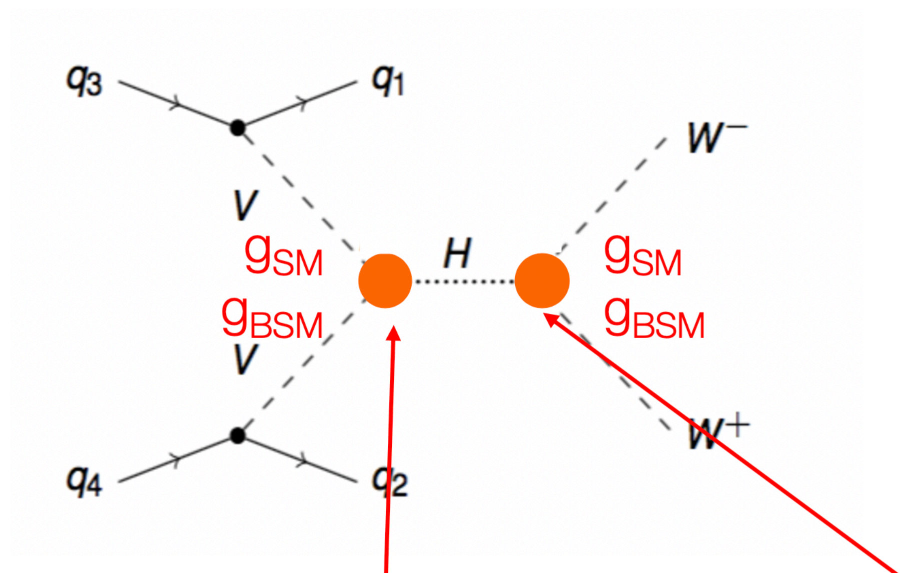




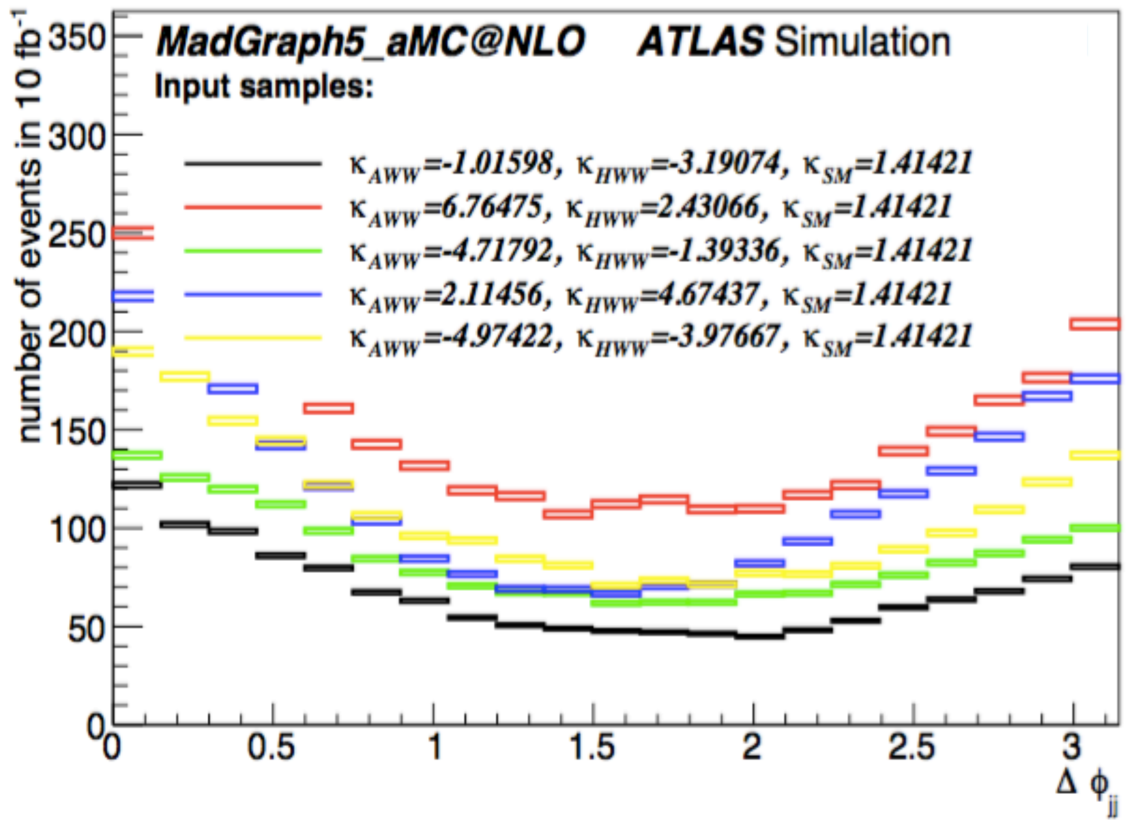
- Avoid unfolding by publishing template cross-sections in regions close to analysis reconstruction region
- Typically perform for Higgs cross-sections, may not necessarily extend for all use cases
- Additional work involved in mapping each measurement σ to the expression in c_i

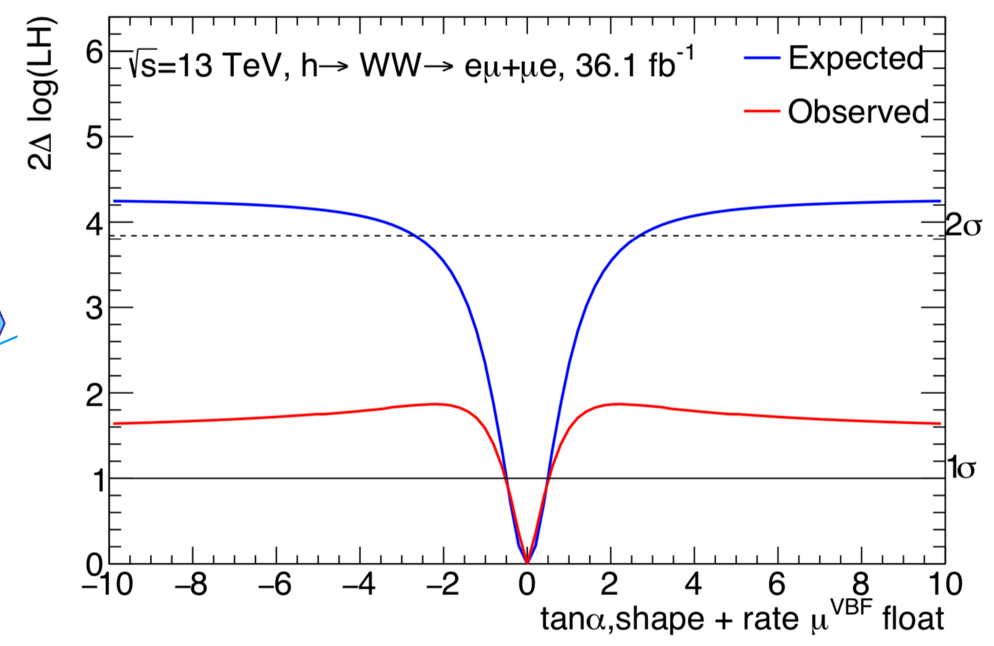
II. Publishing template distribution close to reconstruction level

3) **Template morphing** with SM(EFT) parametrization (medium) → Measurement



$$\mathcal{M}(g_{SM}, g_{BSM}) = \left(g_{SM} \cdot O_{SM,p} + g_{BSM} \cdot O_{BSM,p}\right) \cdot \left(g_{SM} \cdot O_{SM,d} + g_{BSM} \cdot O_{BSM,d}\right).$$





- ✦ Publishing directly results on the parameters of the effective lagrangian
- ✦ The lagrangian provides a natural description to measure physics parameter across different process !
- ✦ Full power of EFT lies in global combinations !

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Describing model within RooFit

Statistical Models with RooFit

- RooFit : statistical modelling toolkit based on C++ to create and perform inference on statistical models of arbitrary complexity
- Design principle : Mathematical functions map directly to C++ classes

Mathematical concept		RooFit class
variable	x	RooRealVar
function	$f(x)$	RooAbsReal
PDF	$f(x)$	RooAbsPdf
space point	$\vec{x}$	RooArgSet
integral	$\int_{x_{min}}^{x_{max}} f(x)dx$	RooRealIntegral
list of space points		RooAbsData

Data Modelling

- Modelling composite objects based on clear connection between mathematical functions and class objects

Math

$$gauss\,(x, m, \sqrt{s})$$

RooFit diagram

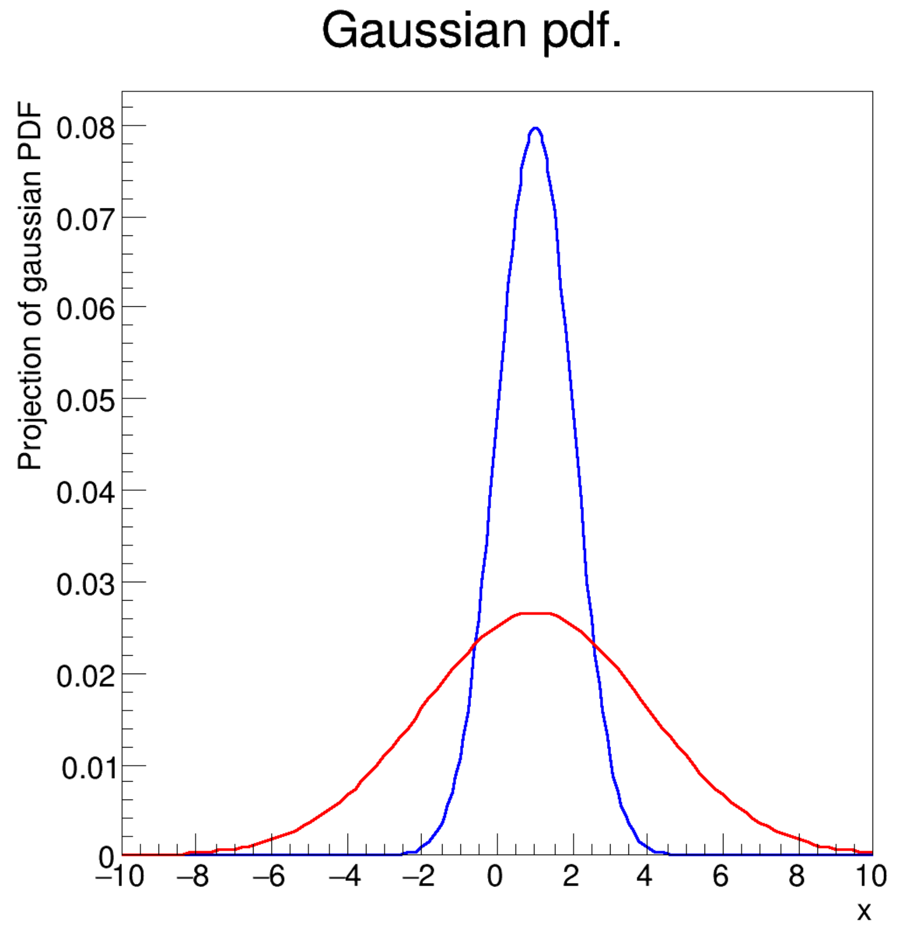
⑤ RooGaussian g

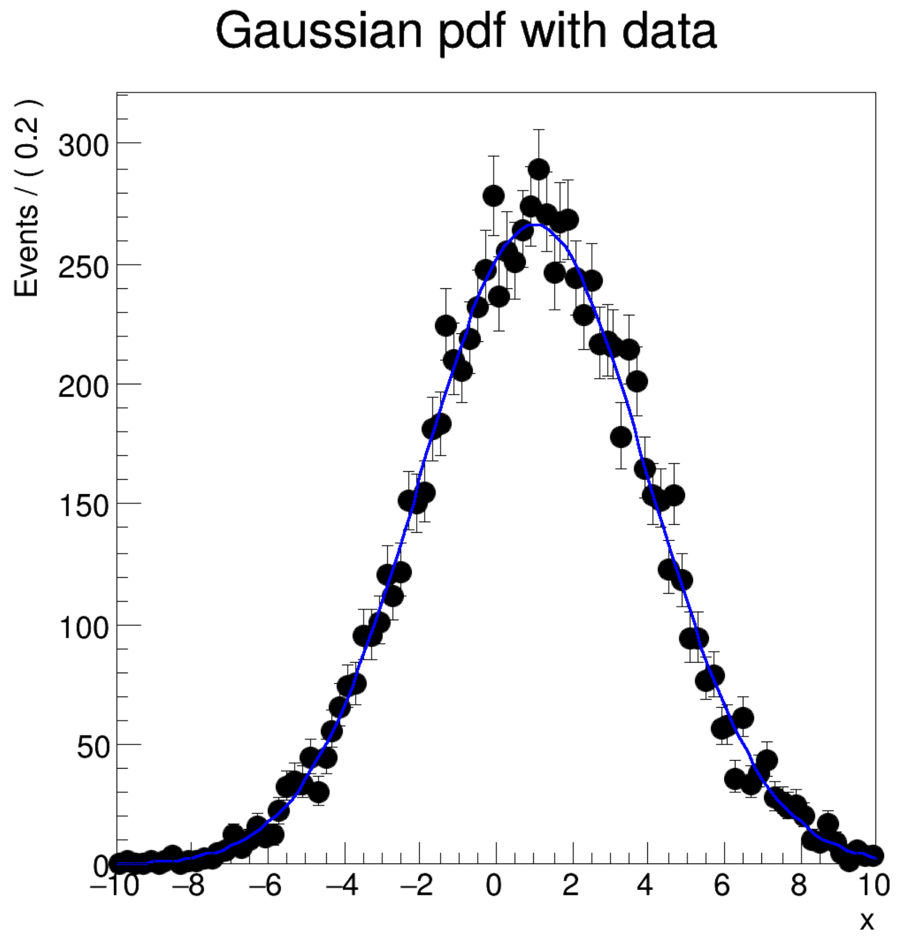
① RooRealVar x — ② RooRealVar m — ④ RooFormulaVar sqrts

③ RooRealVar s

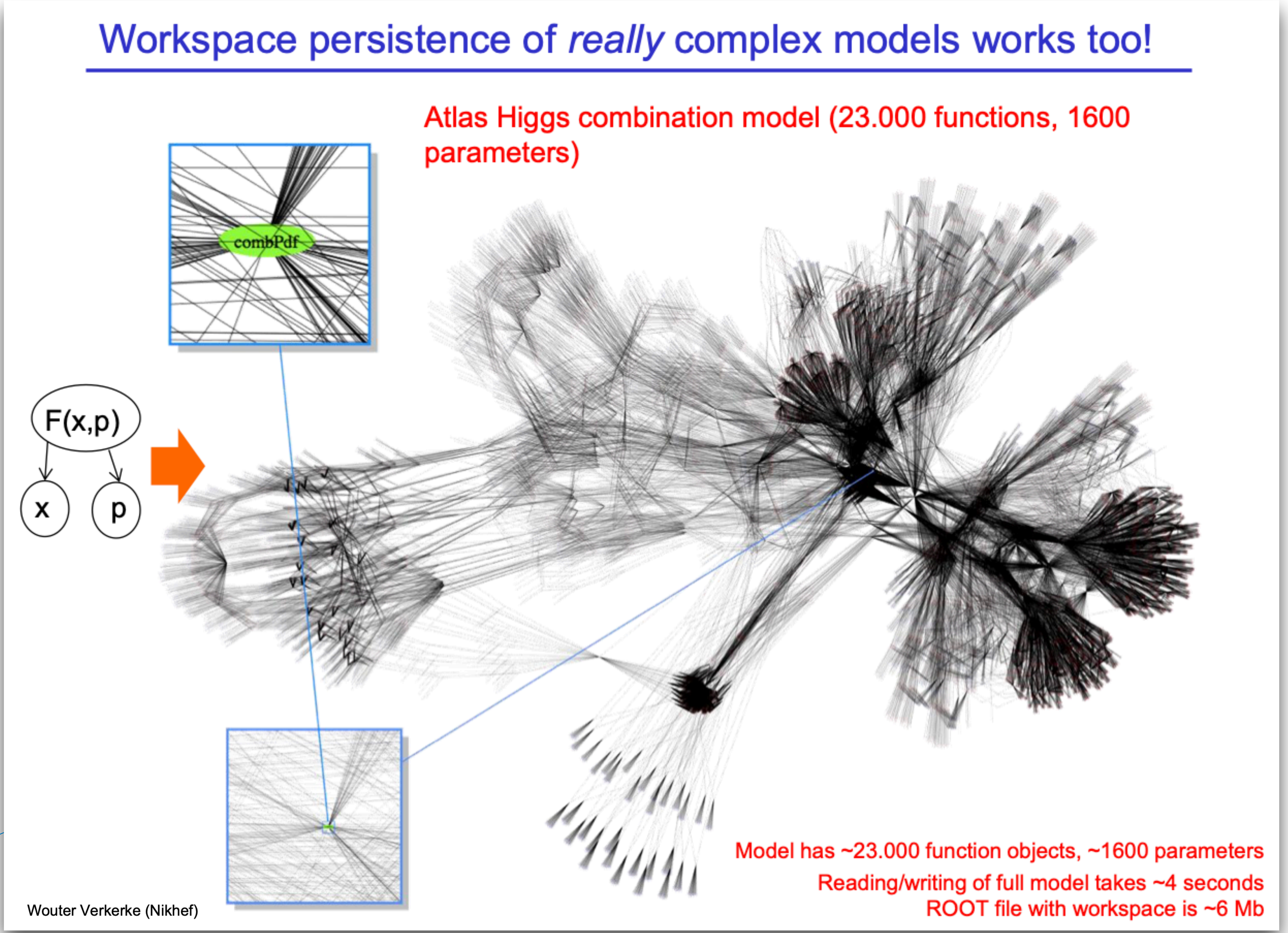
RooFit code

```
① RooRealVar x("x","x",-10,10) ;
② RooRealVar m("m","mean",0) ;
③ RooRealVar s("s","sigma",2,0,10) ;
④ RooFormulaVar sqrts("sqrts","sqrt(s)",s) ;
⑤ RooGaussian g("g","gauss",x,m,sqrts) ;
```





Modular design performs well with complex models



RooLagrangianMorphFunc

- RooLagrangianMorphFunc : RooFit class implementing the lagrangian morphing

$$\overbrace{\overbrace{(g_{\mathrm{SM}}^2 - g_{\mathrm{SM}}\cdot g_{\mathrm{BSM}})\,\sigma_{\mathrm{SM}} + (g_{\mathrm{SM}}\cdot g_{\mathrm{BSM}})\,\sigma_{\mathrm{Mix}} + (g_{\mathrm{BSM}}^2 - g_{\mathrm{SM}}\cdot g_{\mathrm{BSM}})\,\sigma_{\mathrm{BSM}}}^{\texttt{RooRealSumFunc}}}^{\texttt{RooLagrangianMorphFunc}} = \sigma_{\mathrm{out}}(g_{\mathrm{SM}}, g_{\mathrm{BSM}})$$

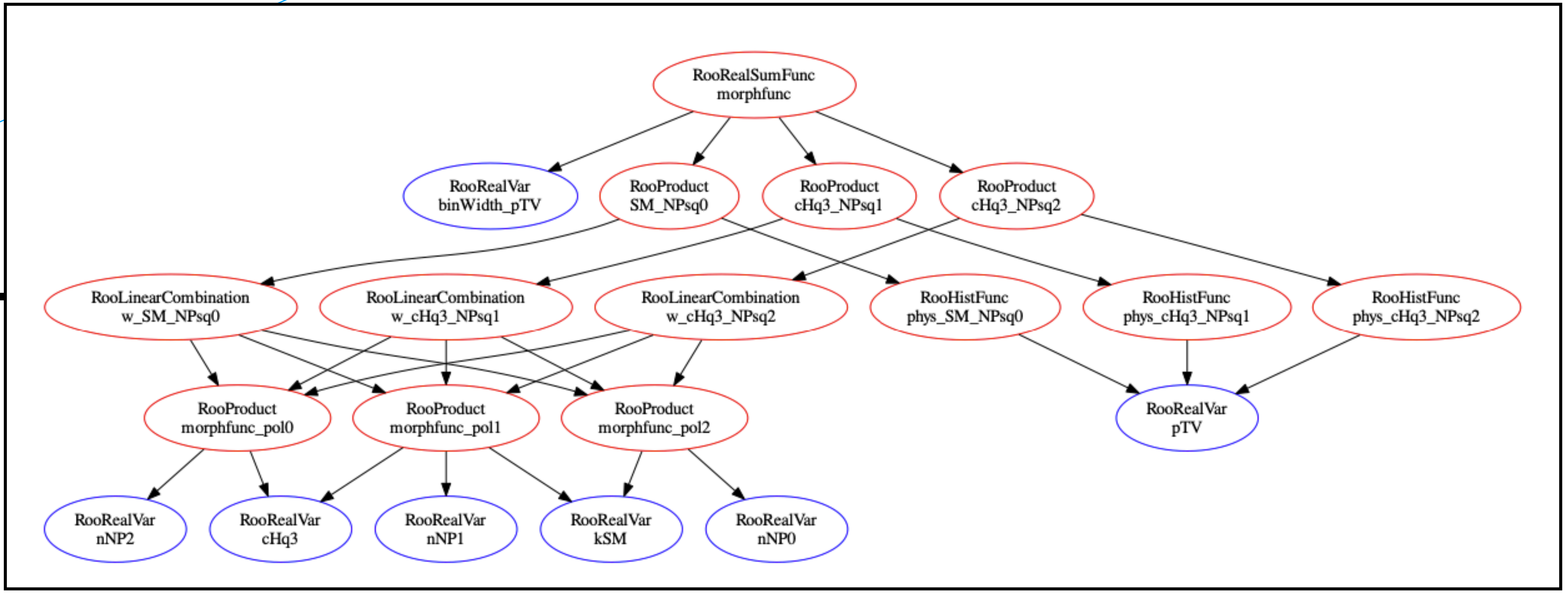
RooLinearCombination

RooProduct

Underlying objects : **RooRealVar**, **RooDataHist**

Morphing object built as a composite object of other underlying RooFit objects

Computational dependency graph for an example object of RooLagrangianMorphFunc



One parameter use case

ROOT file containing input dists.

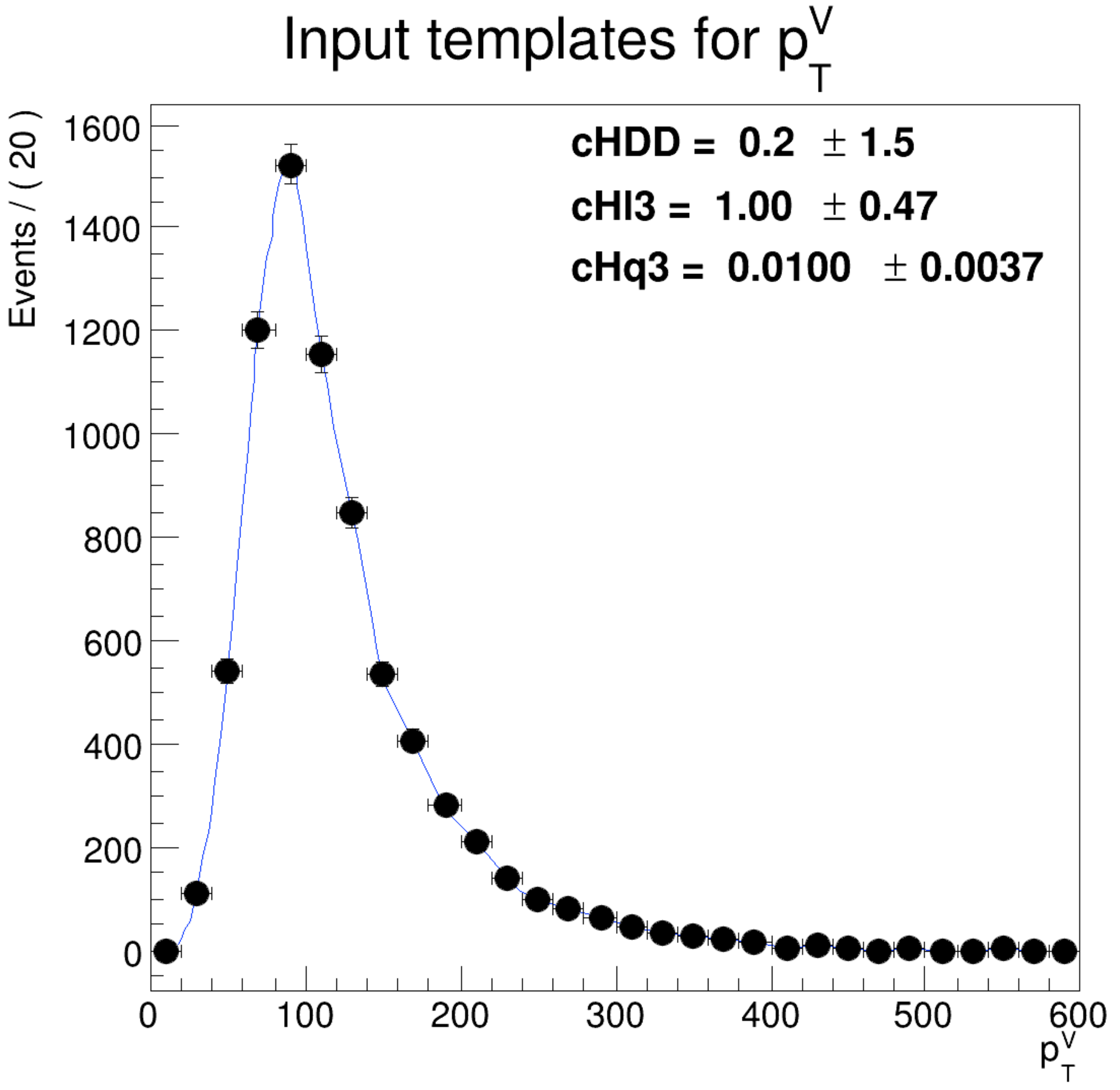
observable name

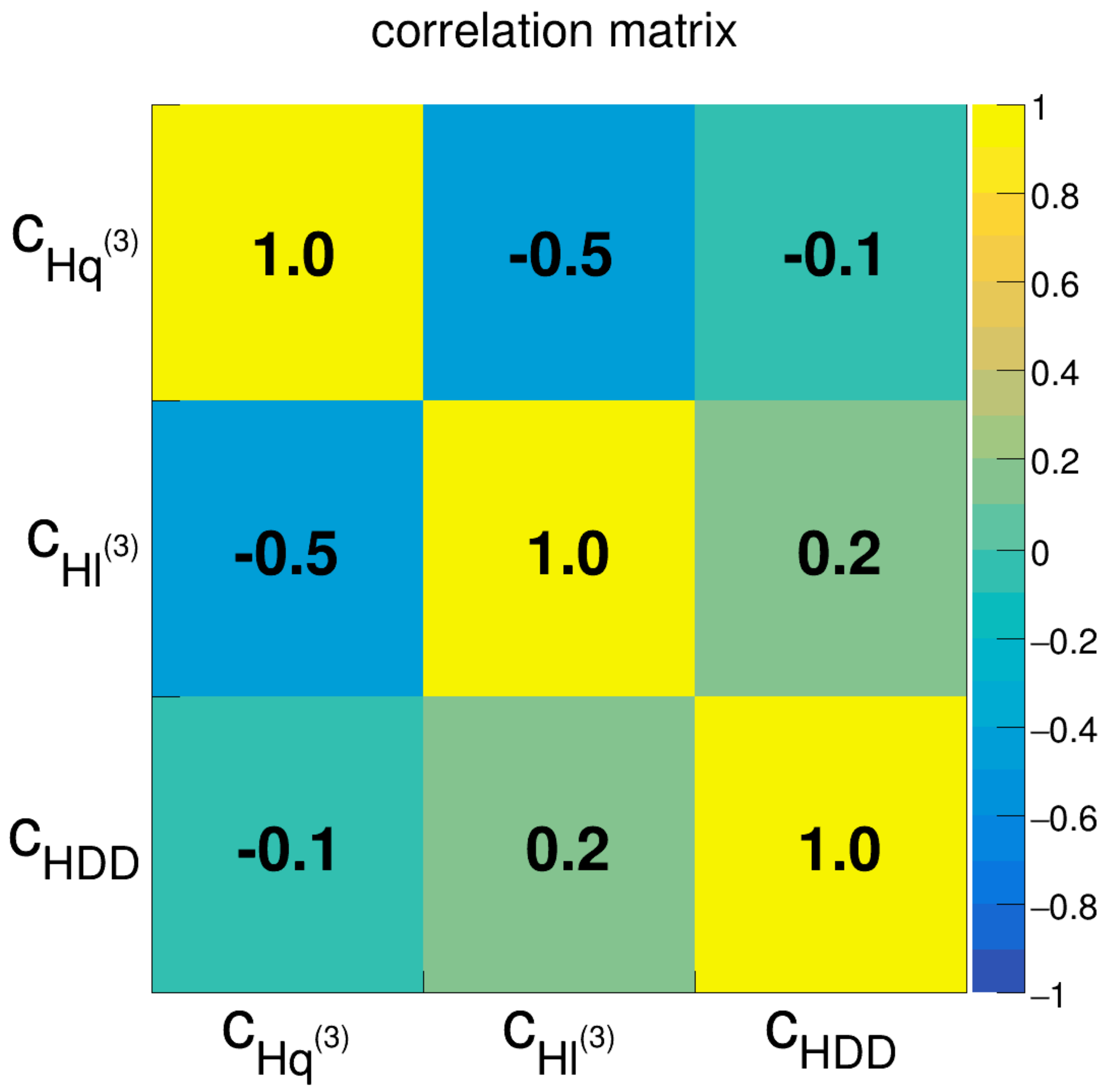
sample folders

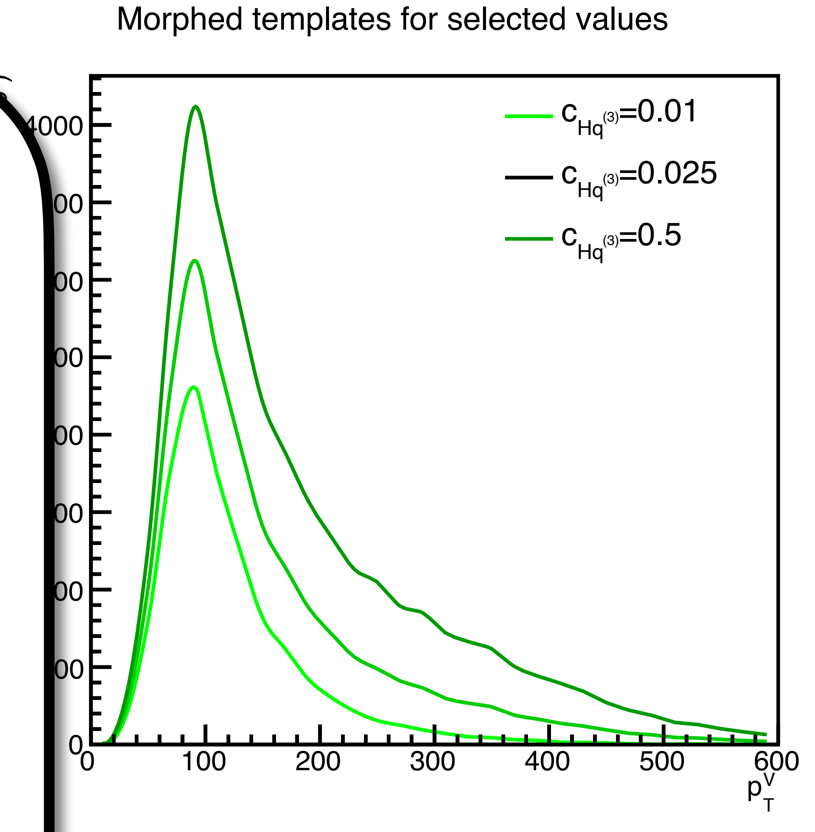
```
// usage of RooLagrangianMorphFunc for simple case
// prepa

// nece
std::st
std::st
std::ve

// crea
RooReal
RooReal
// Set
cHq3.se

// Setu
RooLagr
config
config
config
config
config

// Setu
RooLagr
```

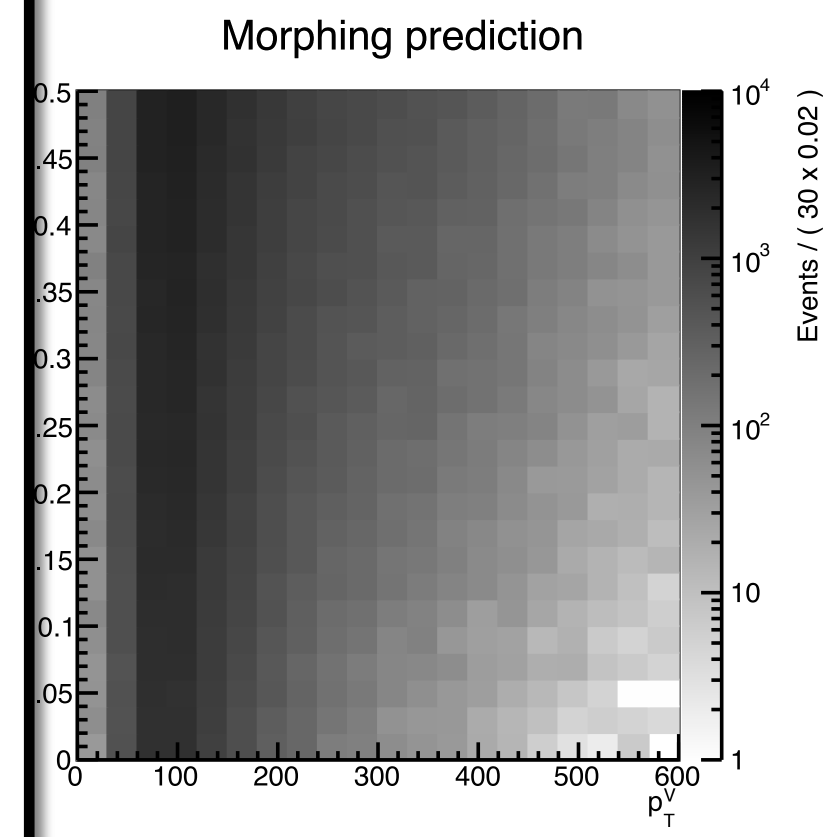
Easy to extend to arbitrary number of parameters,

fit to pseudo data of distribution affected by three operators and correspond Pearson correlation coefficients










```
ory interface
.root'),
),
$NewPhysics(cHq3=1),
$folders({'SM_NPsq0','cHq3_NPsq1','cHq3_NPsq2'})");
```

Summary

SMEFT becoming the standard interpretation framework for measurements at the LHC to look for indirect signs of physics beyond the Standard Model

The morphing technique provide a powerful way to model the EFT distributions in combined likelihoods, now available with ROOT release v6.26.00

SMEFT is global, provides an unifying framework to interpret measurements consistently across different sectors and experiments

Thanks for your attention !