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NOISE IN (PARTICLE) DETECTOR SYSTEMS

AN INTRODUCTION

FEBRUARY 24, 2022

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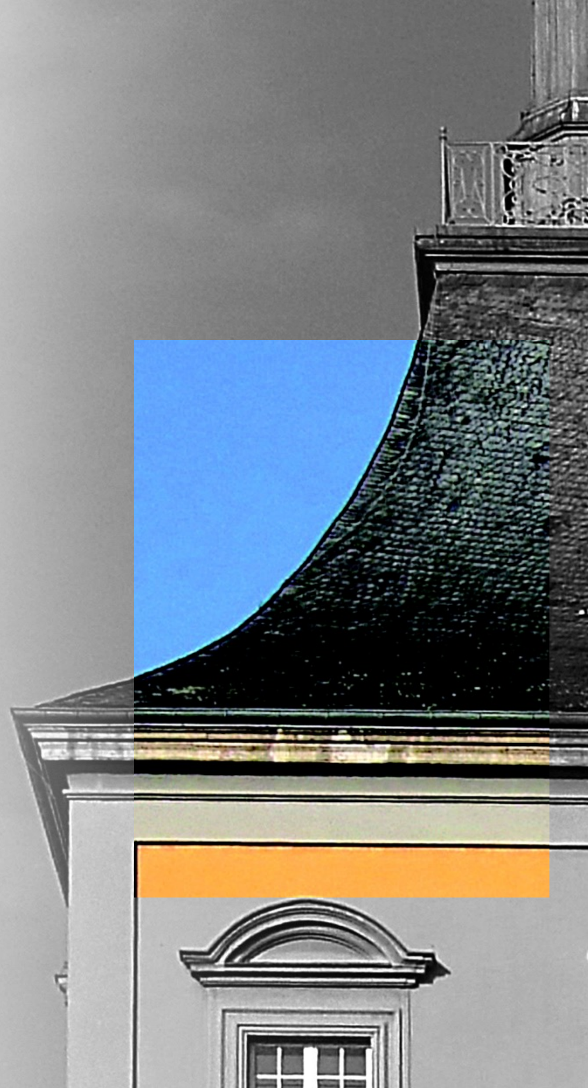

Disclaimer

Outline

- ❑ Noise – what do you mean?
- ❑ Signal fluctuations versus electronic noise
- ❑ Physical noise origins
- ❑ Noise in a typical detector readout system

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Noise ... what?

Noise terms

shot noise

Funkelrauschen

current noise

Widerstandsrauschen

burst noise

series noise

switching noise

Schrotrauschen

popcorn noise

pink noise

flicker noise

thermal noise

Nyquist noise

kT/C noise

Johnson noise

parallel noise

voltage noise

white noise

1/f noise ⟷ RTS noise

pick-up noise

common mode noise

Distinguish

- Signal noise (better: signal fluctuations)
- Electronic noise

} ▪ inherent to a system

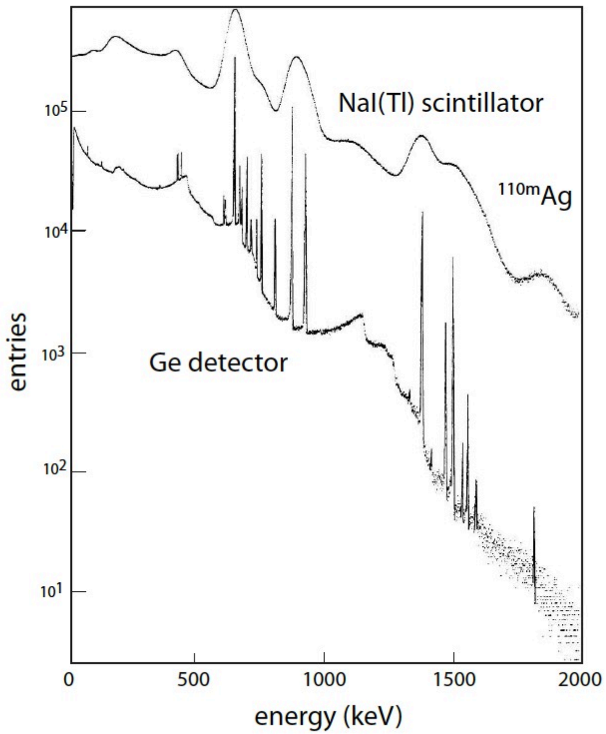
- EMI (electromagnetic interference)
  RFI (radio frequency interference)
  "pick-up" noise

often causing so-called "common-mode" noise

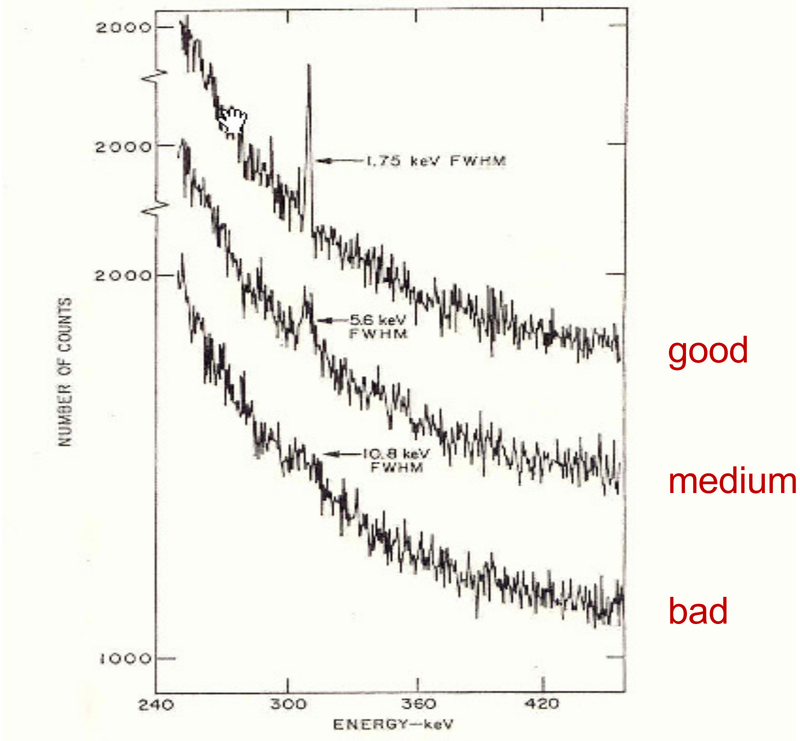
} ▪ introduced externally*
▪ different for every system
▪ => recognize and minimise

* e.g. from power supplies, digital switching, external RF signals, common grounding

Why bother?



(J.Cl. Philippot, IEEE Trans. Nucl. Sci. NS-17/3 (1970) 446)



G.A. Armantrout *et al.*, IEEE Trans. Nucl. Sci. NS-19/1 (1972) 107

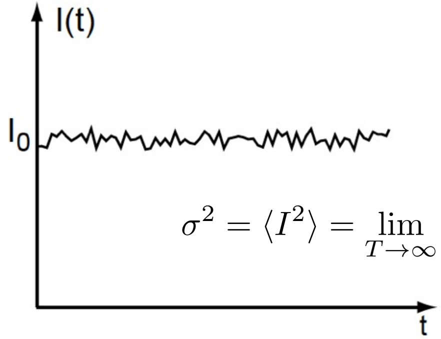
Low noise improves the resolution and the ability to distinguish (signal) structures.

Low noise improves the signal-to-noise ratio
(narrow signal counts are in fewer bins and thus compete with fewer background counts).

examples from H. Spieler, 2005

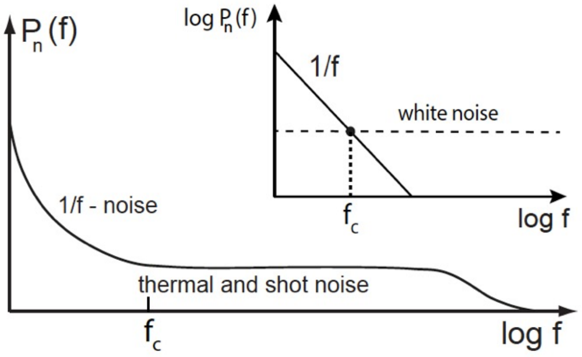
Quantifying Noise

“noise is a variation about a mean value” => quantified by the variance $\langle i^2 \rangle$ or $\langle \mathrm{v}^2 \rangle$



$$\sigma^2 = \langle I^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T (I(t) - I_0)^2 \, dt$$

(a) Current noise as a function of time.



(b) Spectral noise density (schematic) as a function of frequency.

f = frequency

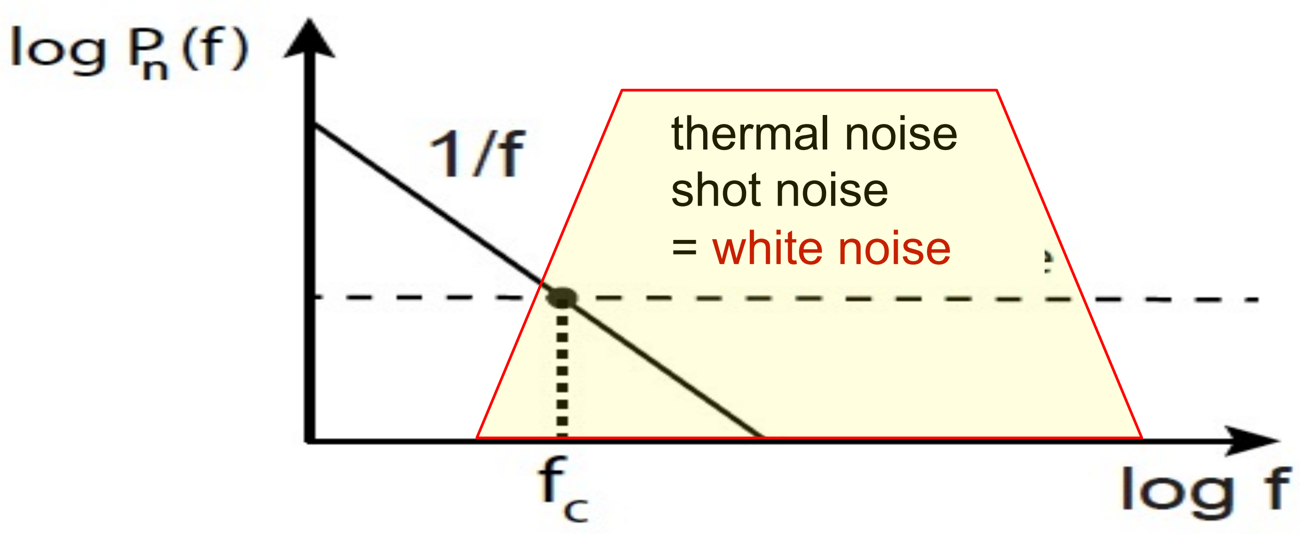
spectral noise power density

$$\frac{dP_n}{df} = \frac{1}{R}\frac{d\left\langle v^2 \right\rangle}{df} = R\frac{d\left\langle i^2 \right\rangle}{df} \quad \Rightarrow \quad P_n = \int_0^\infty \frac{dP_n}{df} df$$

unit of “voltage noise power density“ = $[\sqrt{v^2/df}] = V/\sqrt{Hz}$

unit of “current noise power density” = $[\sqrt{i^2/df}] = A/\sqrt{Hz}$

Notice …



filtering, i.e. limiting the Bandwidth by high- (CR) and low-pass (RC) filters

- reduces the noise
- but: yields a slower response

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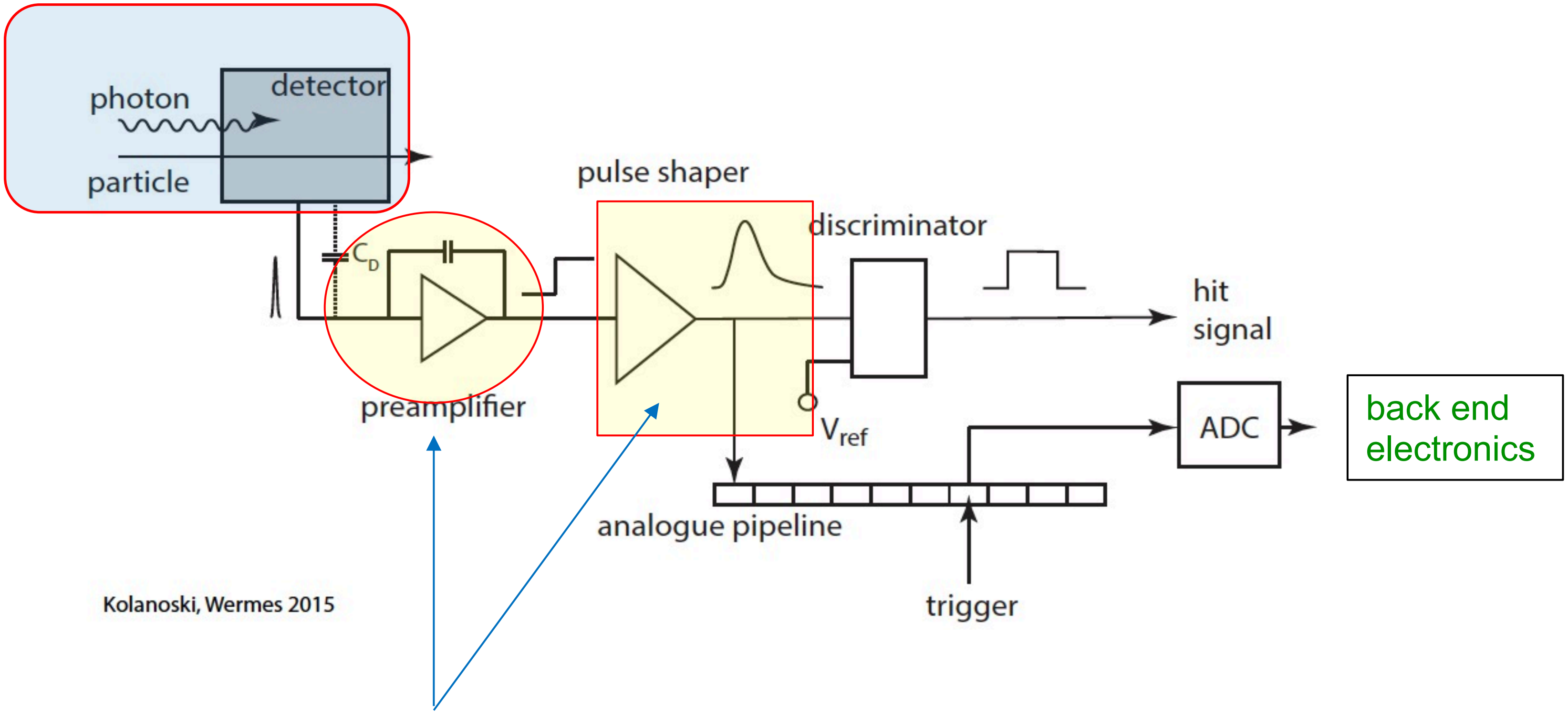
Signal fluctuations and (electronic) noise

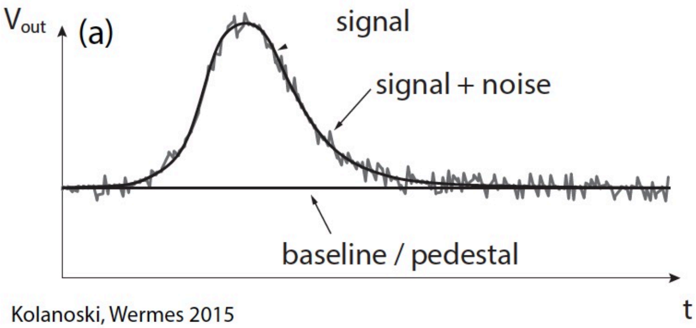
Generic detector & R/O scheme: the dominant noise components

signal fluctuations electronic noise



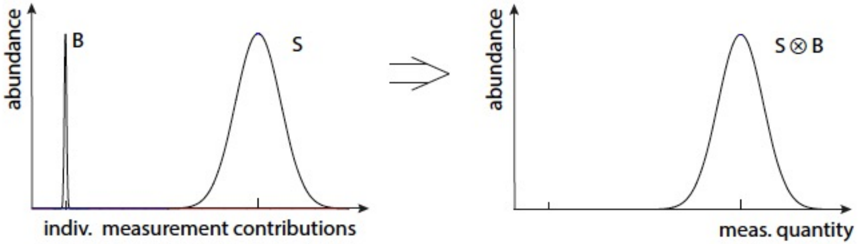
The dominant electronic noise of a system is hidden in these parts

Signal fluctuations and electronic noise



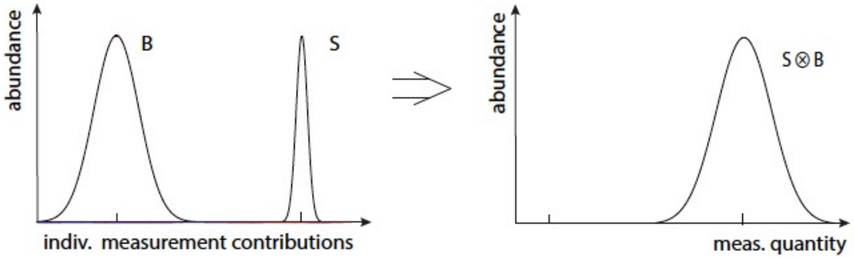
Kolanoski, Wermes 2015

case A: $\sigma_B \ll \sigma_S$



typical for NaI (Tl) crystal w/ PMT readout

case B: $\sigma_B \gg \sigma_S$



typical for silicon strip/pixel detectors

skip

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Noise origins
(… a bit tricky in parts)

Noise origins

A current $i = \frac{Nev}{d}$

can fluctuate in number

- fluctuations in carrier emission over a barrier
- fluctuations in trap/release processes

and in velocity

$$(di)^2 = \left(\frac{ev}{d}dN\right)^2 + \left(\frac{eN}{d}dv\right)^2$$

- Brownian motion (thermal)

Anticipating: noise physical origins

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$$\langle i^2 \rangle = 2q\langle i \rangle df$$

shot noise

number fluctuation

$$\langle i^2 \rangle = \frac{4kT}{R} df$$

thermal noise

velocity fluctuation

$$\langle i^2 \rangle = \text{const. } 1/\text{f}^{\alpha} df$$

1/f noise

number fluctuation

The thermal noise formula I (somewhat simplified, full … see e.g. Kolanoski, Wermes, Oxford U. Press 2020 & refs.)

origin: thermal (Brownian motion) of charge carriers

Two ways to derive from first principles

1. Thermal velocity distribution of carriers
   => time (or frequency) dependence of induced current → difficult derivation

2. Application of Planck's law for thermal radiation
   ("hides" a bit the physics behind a general result of statistical mechanics)
   => yields the spectral density of the radiated power
   i.e. the power that can be extracted in thermal equilibrium

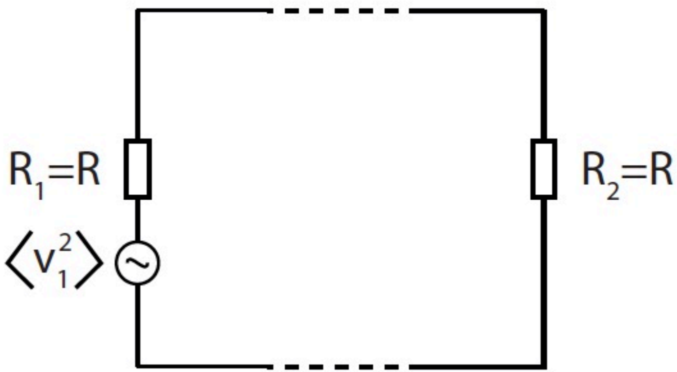
$$\frac{dP}{d\nu} = \frac{h\nu}{exp\left(\frac{h\nu}{kT}\right) - 1} \quad \rightarrow \text{(for } h\nu \ll kT\text{)} \quad = \frac{h\nu}{1 + \frac{h\nu}{kT} - 1} = kT$$

i.e. at sufficiently low frequencies (< THz)
is P independent of ν and is always the same amount in a bandwidth interval $\Delta\nu$

$$P = kT\,\Delta\nu \quad \rightarrow kT\,\Delta f \qquad (*)$$

The thermal noise formula II

To see how this relates to the noise of a resistor, consider an open resistor R_1 which generates a (quadratic) noise voltage $\langle v_1^2 \rangle$.



When both resistors are short-circuited, the noise voltage $\langle v_1^2 \rangle$ over R_1 yields a noise power in R_2, where v is the voltage over R_2 caused by $\langle v_1^2 \rangle$.
With R_1 and R_2 having equal resistances: $R_1 = R_2 = R$ we have

$$P_{1\to 2} = \frac{v^2}{R_2} = \frac{\langle v_1^2 \rangle}{R_2}\left(\frac{R_2}{R_1 + R_2}\right)^2 = \frac{\langle v_1^2 \rangle}{4R}$$

In thermal equilibrium R_2 transfers the same noise power to R_1

$$P_{1\to 2} = P_{2\to 1}$$

for every frequency portion of the noise fluctuation.

The power spectrum hence is a function of f, R, and of the temperature T.

The thermal noise formula III

with (*) P = kT df … we get

$$d\langle v_n^2\rangle = 4kTR\,df$$

and with Ohm's law relating $\langle i_n^2\rangle$ and $\langle v_n^2\rangle$

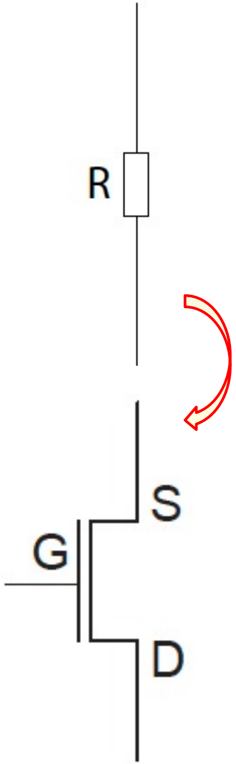
$$d\langle i_n^2\rangle = d\frac{\langle v_n^2\rangle}{R^2} = \frac{4kT}{R}df$$

Note: Thermal noise is always there (if T>0). It does not need power.

In a 1 kΩ resistor we find a current independent thermal current noise of or a voltage fluctuation over R of

$$\sqrt{\frac{d\,\langle i^2\rangle}{df}} = 4\frac{\mathrm{pA}}{\sqrt{\mathrm{Hz}}} \quad \text{or} \quad \sqrt{\frac{d\,\langle v^2\rangle}{df}} = 4\frac{\mathrm{nV}}{\sqrt{\mathrm{Hz}}}$$

Thermal noise in a MOSFET



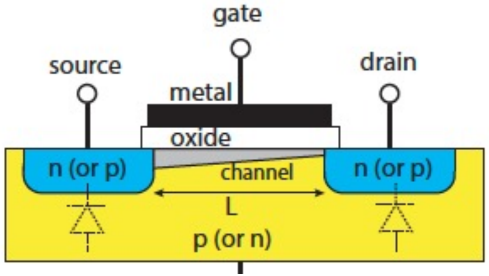
$$d\langle v_n^2\rangle = 4kTR\,df$$

$$d\left\langle i_n^2\right\rangle = \frac{4kT}{R}df$$

$$d\left\langle v_n^2\right\rangle = 4kT\gamma\frac{1}{g_m}df$$

$$d\left\langle i_n^2\right\rangle = 4kT\gamma g_m df$$

$g_m = dI_D/dV_{GS}$
transconductance



γ = adjustment factor

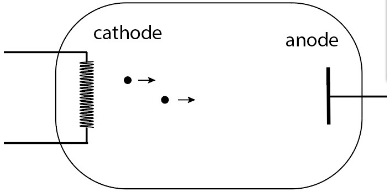
$\frac{2}{3}$ in strong inversion

$\frac{1}{2}$ in weak inversion (subthreshold operation)

Shot noise (somewhat simplified derivation, full … see e.g. Kolanoski, Wermes, Oxford U. Press 2020)

origin: excess e⁻ injection into a device when quantisation plays a role (over a barrier, i.e. NOT in a resistor)

e.g.

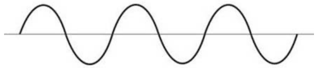


… but also in a pn boundary (detector diode)
e/h in depletion zone induce current pulses until recombination (short)

- the current pulses can be regarded as δ - functions, i.e. all frequencies contribute => white noise

$$\int_{-\infty}^{\infty} i_e(t)dt = e$$ ➔ $di_e/df = e \cdot 2$ (convention: $0 < f < \infty$ → $-\infty < f < \infty$)

- for infinitely narrow df the spectral component *k* contributing is one sine wave with mean = 0 and rms = 1/√2



$$\Rightarrow \sqrt{\frac{d\langle i_{e,k}^2\rangle}{df}} = \frac{2e}{\sqrt{2}} = \sqrt{2}e$$

- for N electrons of total average current $I = Ne/t = Ne\,\Delta f$ we get

$$\langle i^2\rangle = \sum_{k=1}^{N}\left(\frac{d_{i,k}}{df}\right)^2 (df)^2 = 2Ne^2(df)^2 = 2e\underbrace{(Nedf)}_{\langle i\rangle}df = 2e\langle i\rangle df$$

needs a current

1mA (leakage) current yields

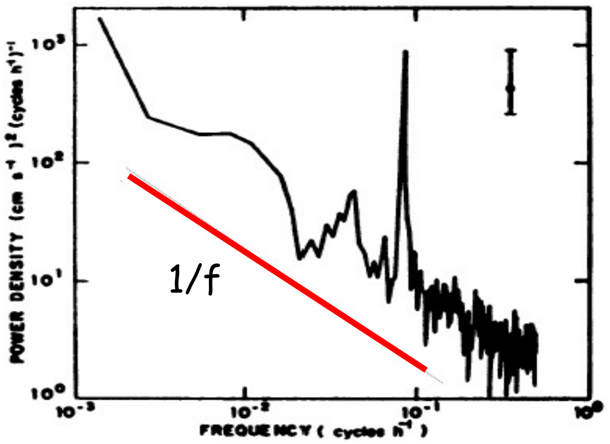
$$\sqrt{\frac{\Delta\langle i^2\rangle}{\Delta f}} = \sqrt{2eI_0} = 18\frac{\text{pA}}{\sqrt{\text{Hz}}}$$

of noise

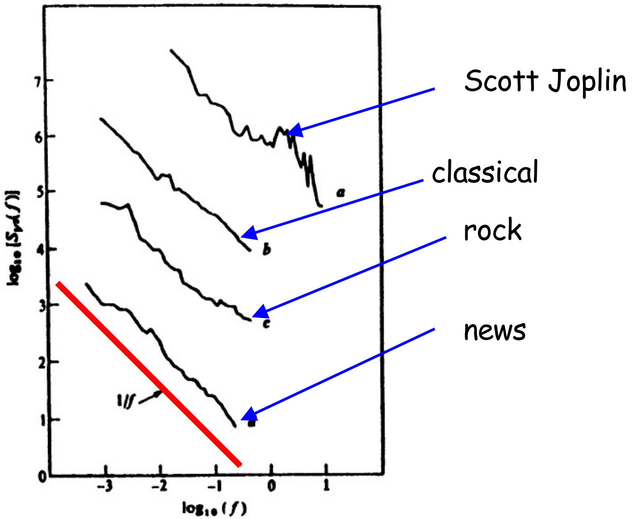
1/f noise - I

<u>origin:</u>

- <u>superposition of relaxation processes</u> with different time constants
- appears in many systems (ocean current velocity, music, broad casting, earthquake frequency spectra)
- many papers in literature (all you ever wanted to know) http://www.nslij-genetics.org/wli/1fnoise/



east-west component of ocean current velocity

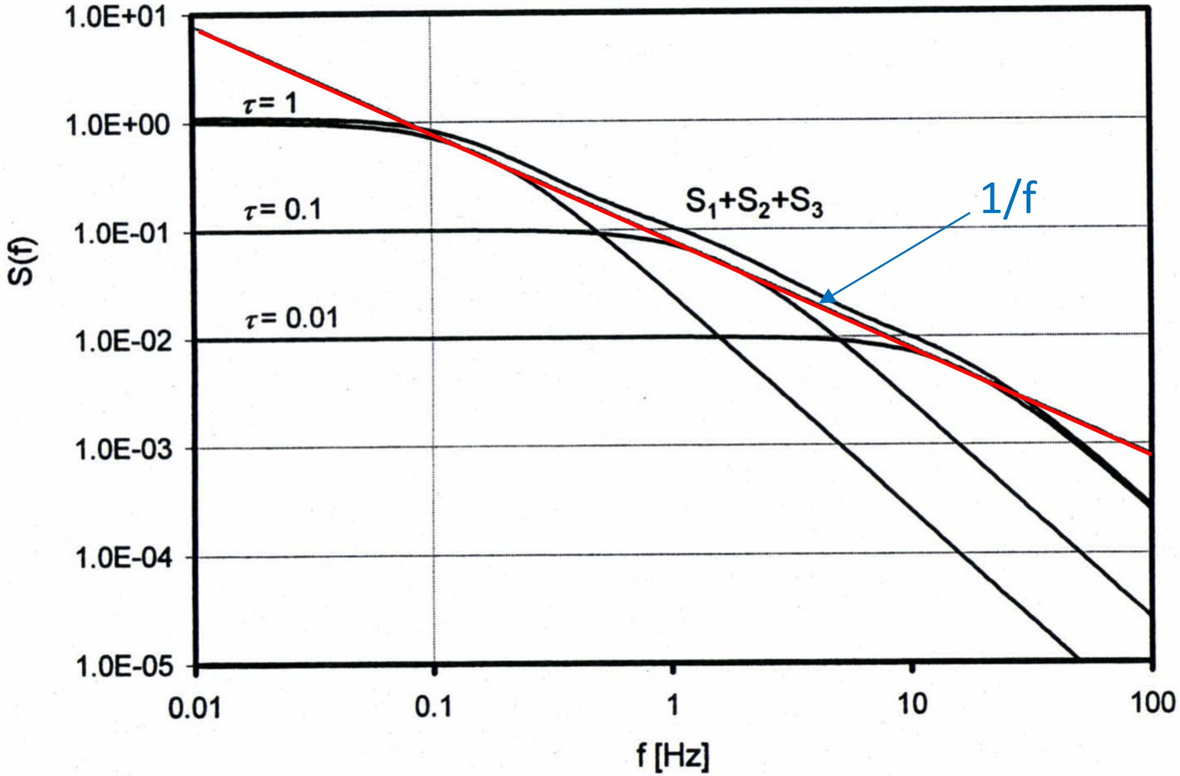


loudness fluctuations spectra of radio broadcasting

pictures from:
E. Milotti
U Udine

1/f noise - II

superposition of $1/f^2$ spectra with 3 time constants



1/f noise – III … in trap/release processes

Assume a trapping site with relaxation time constant τ which releases electrons according to

$$N(t) = N_0\, \mathrm{e}^{-t/\tau} \quad \text{for } t \geq 0\,, \qquad N(t) = 0 \ \text{ else}$$

Fourier transforming this into the frequency domain yields

$$F(\omega) = \int_{-\infty}^{\infty} N(t) \mathrm{e}^{-i\omega t}\, dt = N_0 \int_0^{\infty} \mathrm{e}^{-(1/\tau + i\omega)t}\, dt = N_0 \frac{1}{1/\tau + i\omega}$$

For a whole sequence of such relaxation processes occurring at different times t_k

$$N(t, t_k) = N_0\, \mathrm{e}^{-\frac{t - t_k}{\tau}} \quad \text{for } t \geq t_k \ , \qquad N(t, t_k) = 0 \ \text{ else}$$

…. but still with the same trapping time constant $\boldsymbol{\tau}$**,** one gets

$$F(\omega) = N_0 \sum_k \mathrm{e}^{-i\omega t_k} \int_0^{\infty} \mathrm{e}^{-(1/\tau + i\omega)t}\, dt = \frac{N_0}{1/\tau + i\omega} \sum_k \mathrm{e}^{i\omega t_k}$$

The power spectrum then is obtained as

$$P(\omega) = \lim_{T \to \infty} \frac{1}{T} \left\langle |F(\omega)|^2 \right\rangle = \frac{N_0^2}{(1/\tau)^2 + \omega^2} \lim_{T \to \infty} \frac{1}{T} \left\langle \left| \sum_k \mathrm{e}^{i\omega t_k} \right|^2 \right\rangle = \frac{N_0^2}{(1/\tau)^2 + \omega^2} n$$

where n is the average rate of trapping/relaxation processes

1/f noise - IV

If one in addition assumes that the relaxation time constants are different
i.e. $\tau \rightarrow \tau_i$ and we integrate/sum over uniformly distributed $\tau_1 < \tau_i < \tau_2$, one finds

$$P(\omega) = \frac{1}{\frac{1}{\tau_1} - \frac{1}{\tau_2}} \int_{\frac{1}{\tau_2}}^{\frac{1}{\tau_1}} \frac{N_0^2\, n}{\left(\frac{1}{\tau}\right)^2 + \omega^2}\, d\left(\frac{1}{\tau}\right) = \frac{N_0^2\, n}{\omega\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)} \left[\arctan\frac{1}{\omega\tau_1} - \arctan\frac{1}{\omega\tau_2}\right]$$

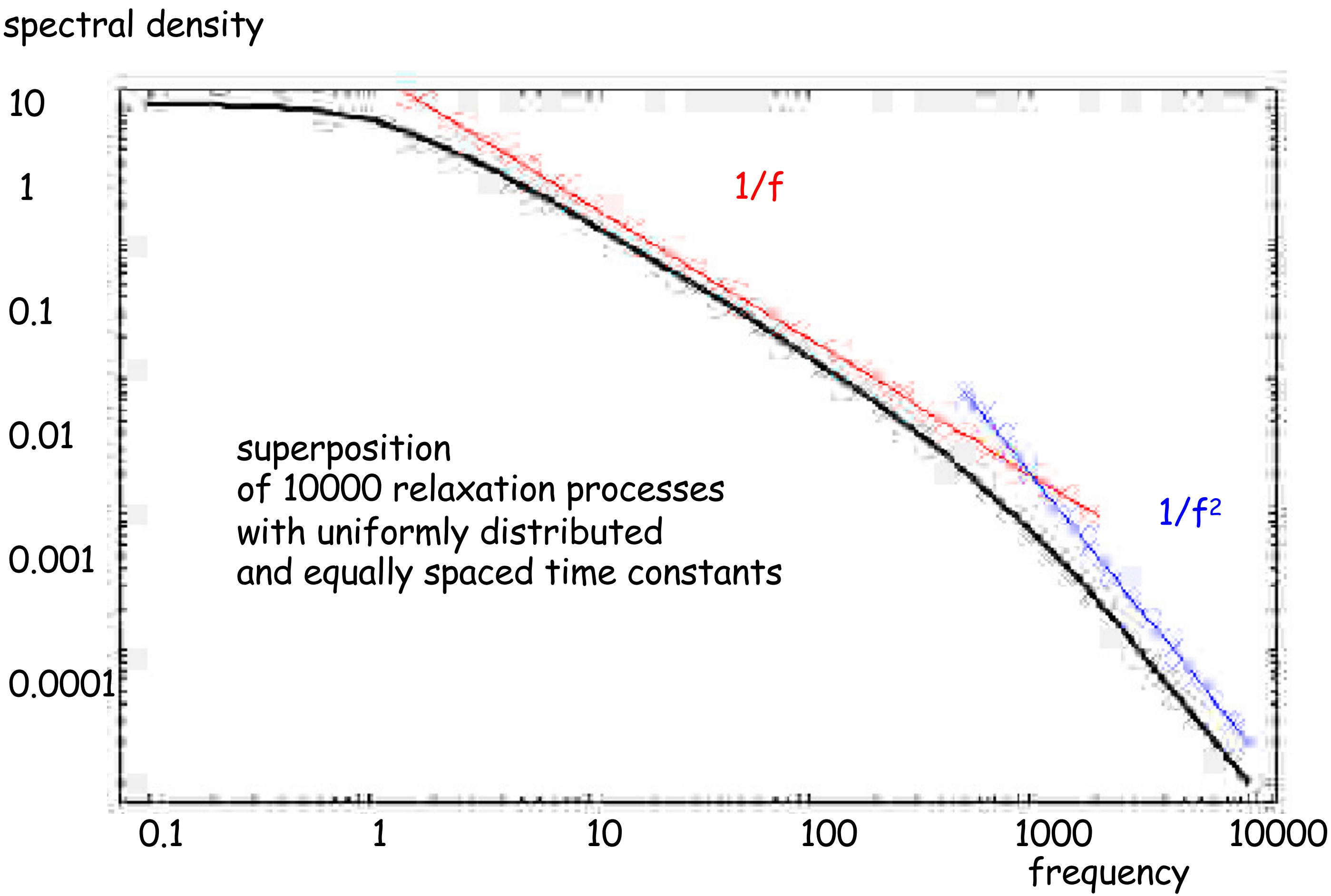
$$\approx \begin{cases} N_0^2\, n & \text{if } 0 < \omega \ll \frac{1}{\tau_1}, \frac{1}{\tau_2} \quad \rightarrow \quad \text{const.}, \\ \dfrac{N_0^2\, n\, \pi}{2\omega\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)} & \text{if } \frac{1}{\tau_2} \ll \omega \ll \frac{1}{\tau_1} \quad \rightarrow \quad \dfrac{1}{f}, \\ \dfrac{N_0^2\, n}{\omega^2} & \text{if } \frac{1}{\tau_1}, \frac{1}{\tau_2} \ll \omega \quad \rightarrow \quad \dfrac{1}{f^2}. \end{cases} \qquad (I.23)$$

const

1/f

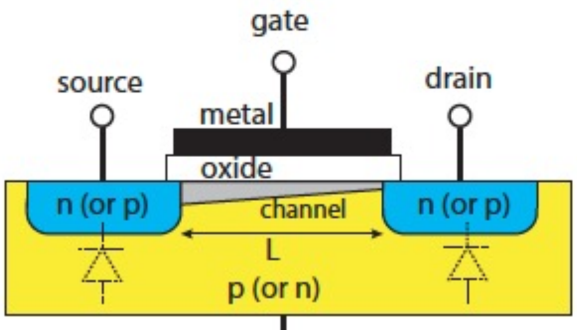
$1/f^2$

1/f noise - V



from:
E. Milotti
U Udine

1/f noise in a MOSFET



origin

- trapping and release of channel charges in gate oxide
- depends on gate area A = W × L

$$\frac{d\left\langle v_{1/f}^2\right\rangle}{df} = K_f \frac{1}{C'_{ox} W L} \frac{1}{f}$$

empirical parametrisation (e.g. PSPICE)

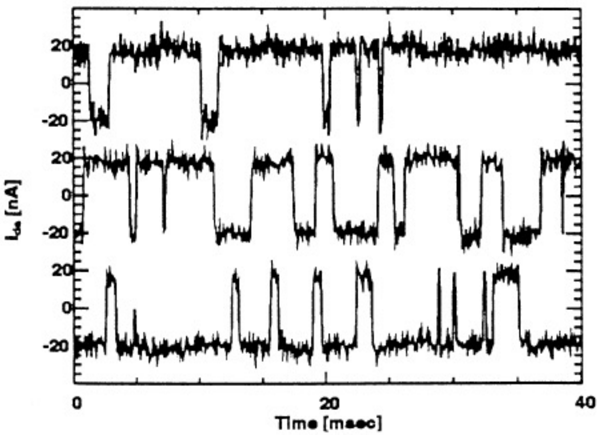
$$C'_{ox} = \frac{3}{2}\frac{C_{GS}}{WL} \approx \epsilon_0\epsilon/d$$

$K_f^{NMOS} \approx 30 \times 10^{-25}$ J, $K_f^{PMOS} \approx 0.05\text{-}0.1 \times K_f^{NMOS}$

RTS noise

RTS noise = random telegraph signal noise

also called “burst noise” or “popcorn noise”



Occurs in electronics devices usually related to trapping/detrapping processes. The popping-up nature of individual RTS bursts eventually leads to the 1/f noise spectral density when noise of several traps with (very) different trapping times are superimposed.

Given the low frequency it is <u>difficult to filter out</u> and a nuisance for very low noise devices.

Fazit: Only three physical noise origins to consider …

- $\langle i^2 \rangle = 2q\langle i\rangle df$

thermal fluctuations (Brownian motion)
velocity fluctuation

thermal noise

(in resistors, transistor channels)

- $\langle i^2 \rangle = \frac{4kT}{R} df$

fluctuations in hopping over
a barrier (shot)
number fluctuation

shot noise

(where currents due to barrier crossings
appear, e.g. in diodes, NOT in resistors)

- $\langle i^2 \rangle = \text{const. } 1/\text{f}^\alpha df$

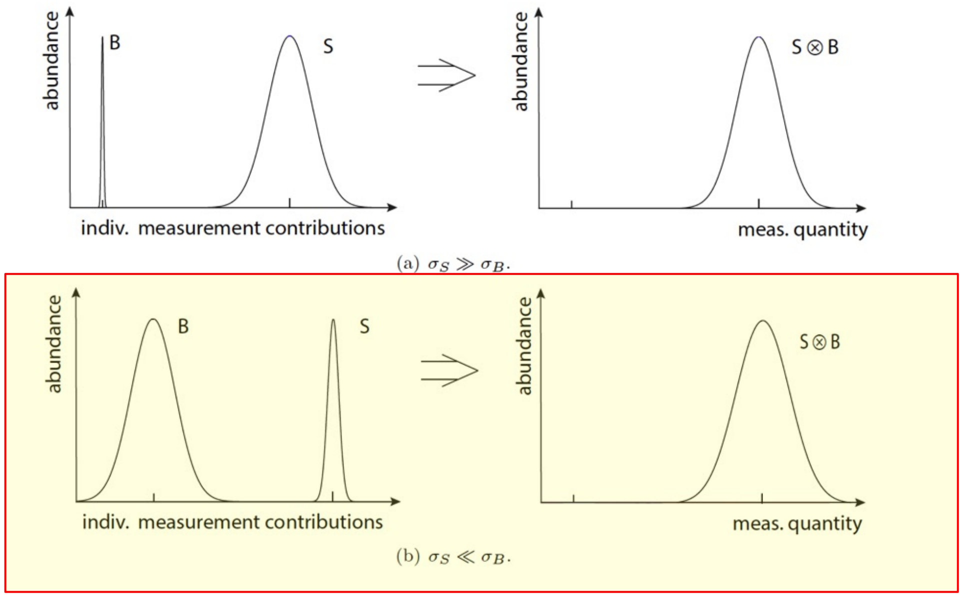
trap/release fluctuations of carriers
number fluctuation

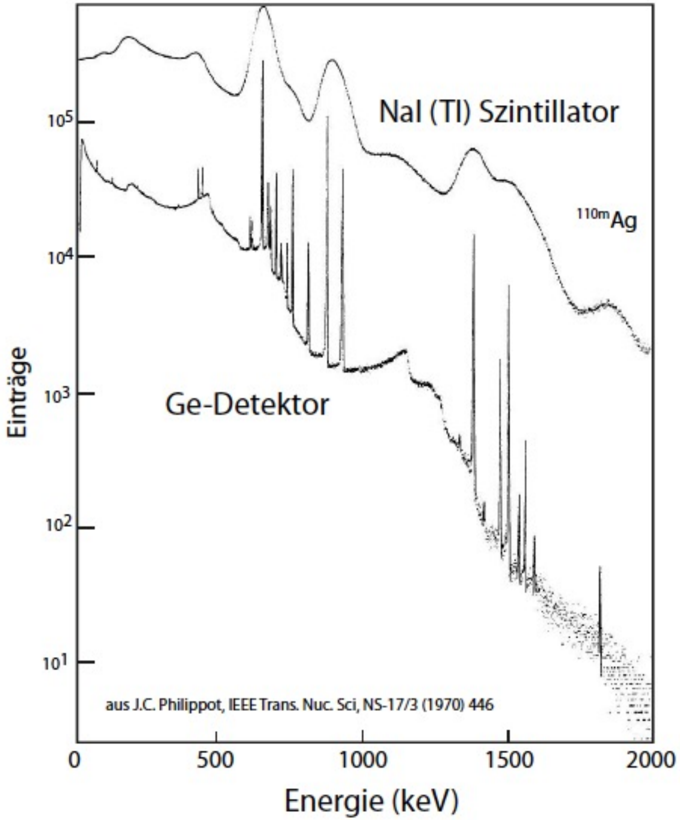
1/f noise

(whenever trapping occurs,
e.g. in (MOS) transistor channels)

Remember when to care about noise …

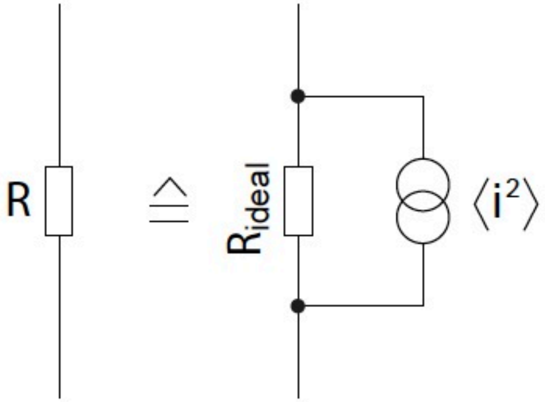
- always …
- but particularly, when the situation is like this





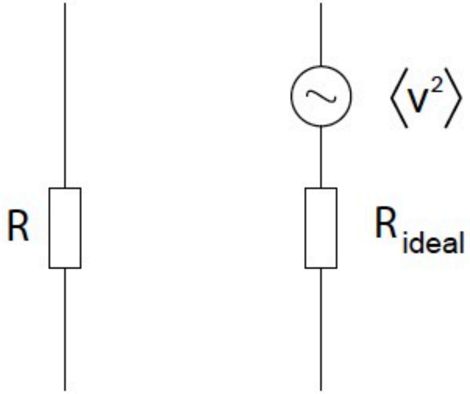
- Even if you are not interested in an energy measurement, remember … thresholds

Noisy circuit elements

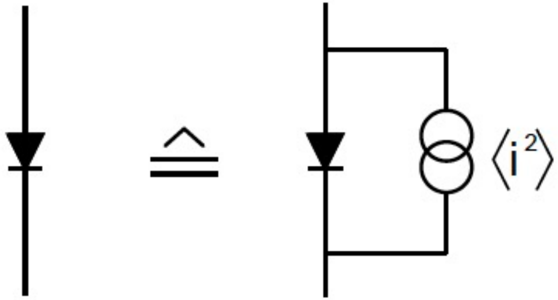


(a) Replacement circuit with parallel current noise source.

or



(b) Replacement circuit with serial voltage noise source.



real (noisy) diode

ideal diode with noise current source

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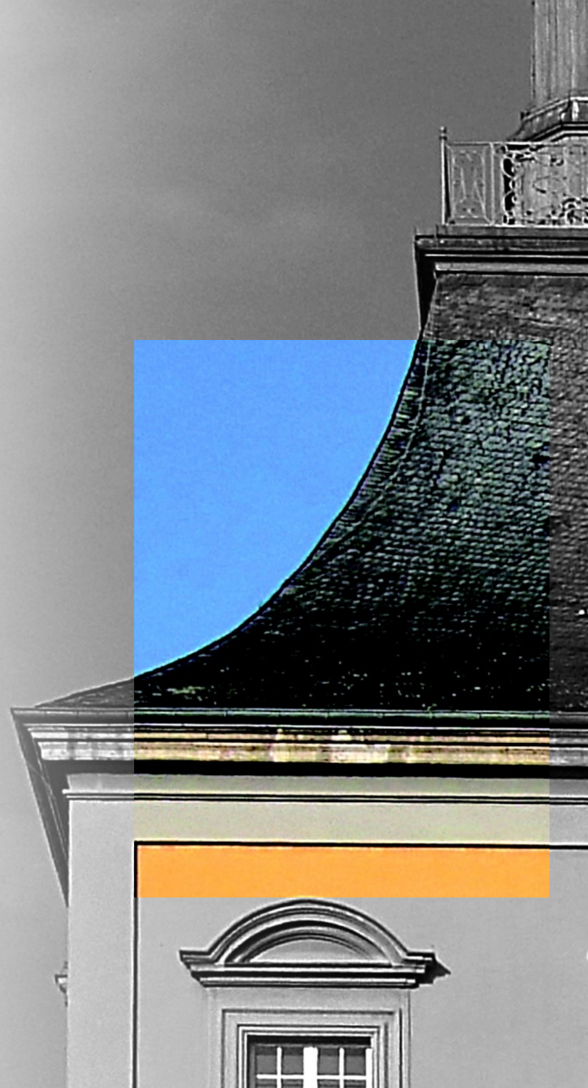
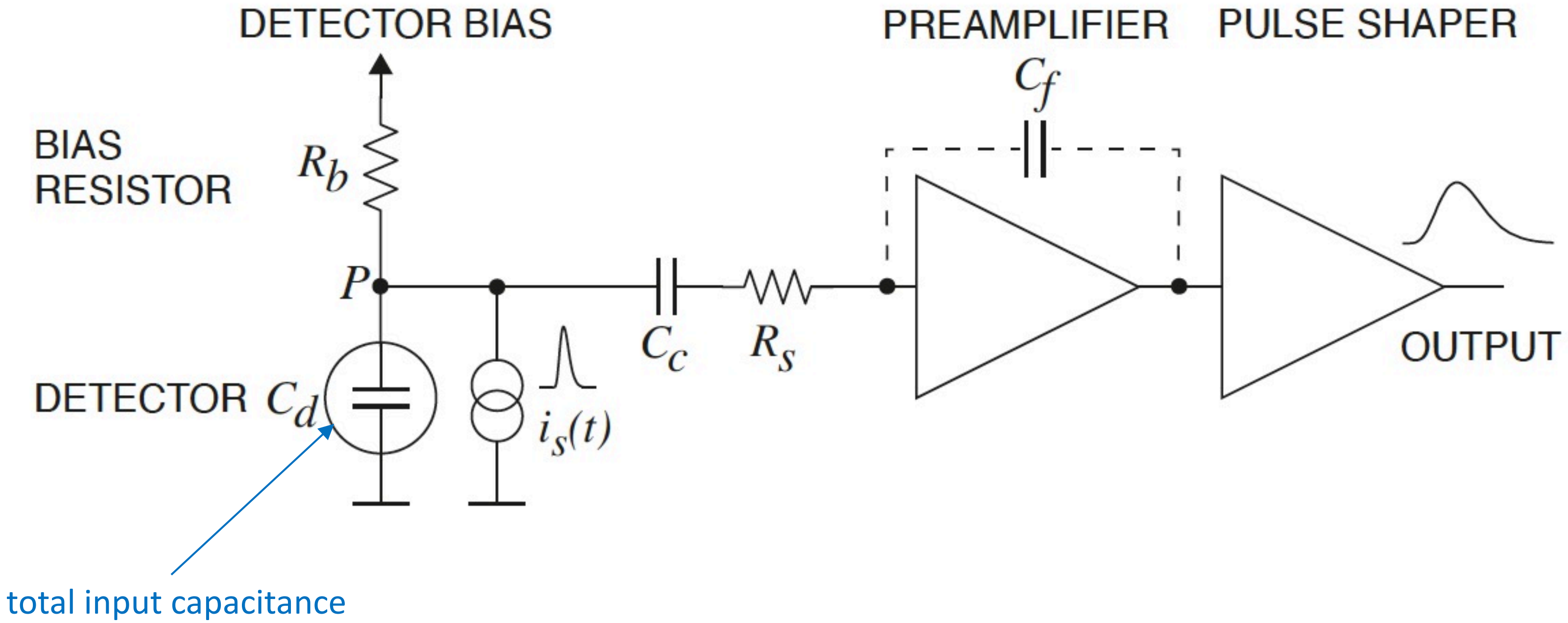
Noise in a typical detector readout system

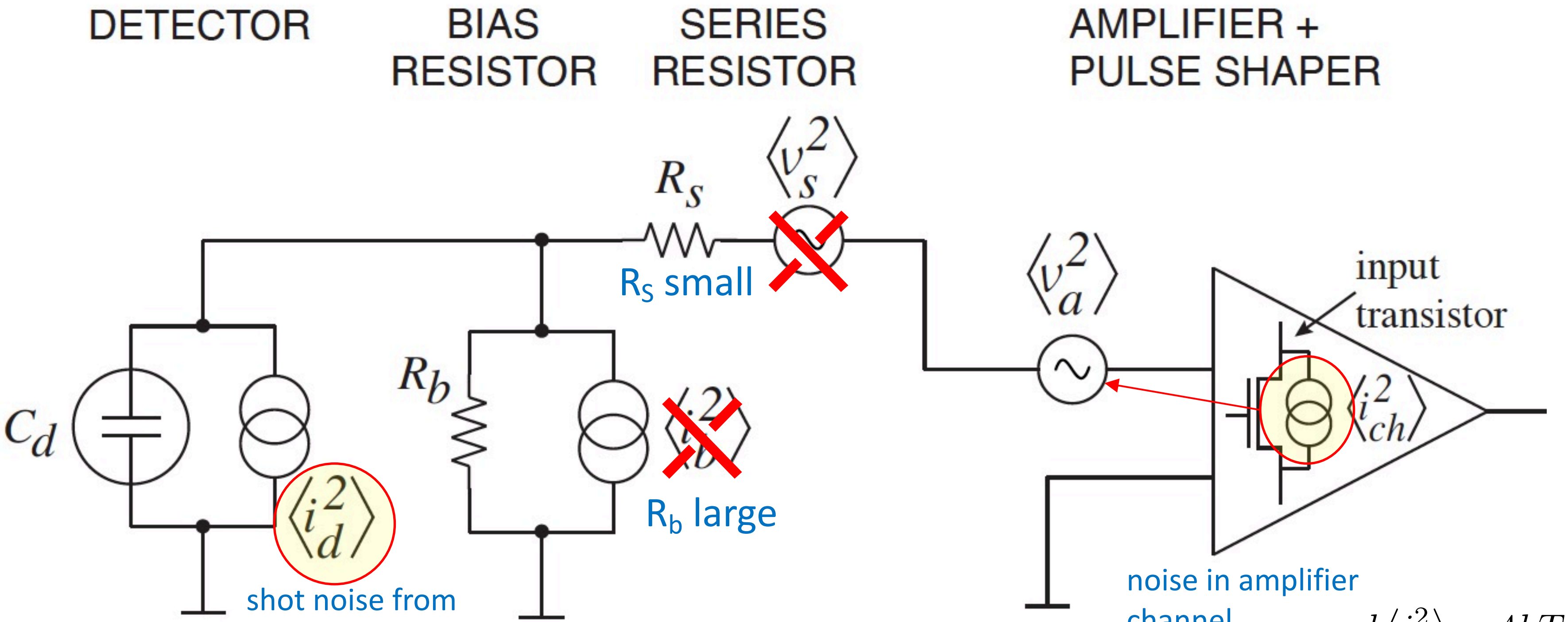


Detector system



Review on Low-noise detector readout, NW (2021), H. Spieler (2013)
in P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01
(2020) and 2021 update.

Circuit diagram for equivalent noise analysis



shot noise from detector (leakage) current

$$d\langle i_d^2 \rangle = 2eI_d\, df$$

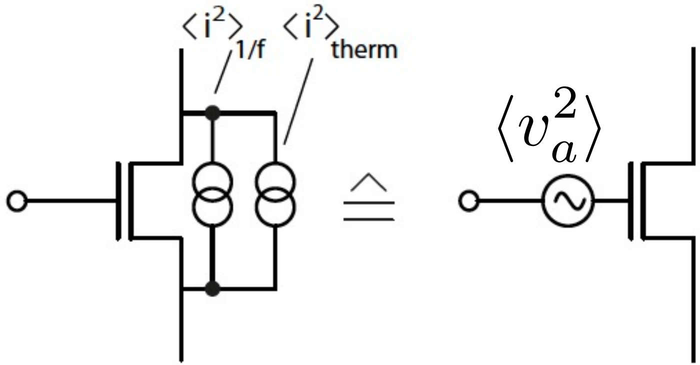
noise in amplifier channel

1. thermal $\quad d\left\langle i^2 \right\rangle = 4kT\gamma g_m df$
2. 1/f noise $\quad d\left\langle i^2 \right\rangle = K_\alpha \frac{1}{f^\alpha} df$

Review on Low-noise detector readout, NW (2021), H. Spieler (2013) in P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020) and 2021 update.

Noise in a MOSFET (current or voltage noise?)

"parallel current noise can also be described by serial voltage noise"



via

$$\langle i_{ch}^2 \rangle = \langle (g_m v_a)^2 \rangle$$

1/f noise
parametrized by C'_{ox} and K_f

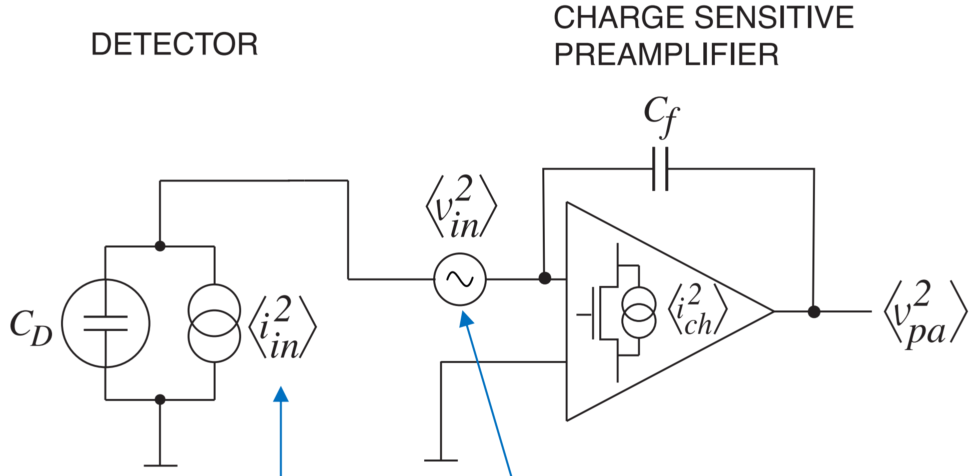
thermal noise

$$\frac{d\langle v_a^2 \rangle}{df} = 4kT\gamma \frac{1}{g_m} + K_f \frac{1}{C'_{ox} W L} \frac{1}{f}$$

* contributions assumed uncorrelated, adding in quadrature

Preamplifier output noise

let's now use a CSA and compute the noise output voltage …



The noise current, flowing through the feedback capacitance C_f , as well as the noise voltage at the preamplifier input, generate a noise voltage behind the preamplifier.

$$\langle v_{pa}^2 \rangle = \langle v_{in}^2 \rangle \left(\frac{\omega C_D}{\omega C_f} \right)^2$$

$$\langle v_{pa}^2 \rangle = \langle i_{in}^2 \rangle \left(\frac{1}{\omega C_f} \right)^2$$

shot noise (det. leak.)

$$\frac{d \langle i^2 \rangle_{shot}}{df} = 2eI_d$$

thermal noise
1/f noise
in transistor channel

$$\frac{d\langle v_a^2 \rangle}{df} = 4kT\gamma \frac{1}{g_m} + K_f \frac{1}{C'_{ox} W L} \frac{1}{f}$$

$$\omega = 2\pi f$$

$$C_d \rightarrow C_D = C_{in}^{tot}$$

We then get …

current noise input

$$\frac{d\left\langle i^2\right\rangle_{shot}}{df} = 2eI_d$$

voltage noise input

$$\frac{d\langle v_a^2\rangle}{df} = 4kT\gamma\frac{1}{g_m} + K_f\frac{1}{C'_{ox}WL}\frac{1}{f}$$

$$\frac{d\left\langle v_{\mathrm{pa}}^2\right\rangle}{d\omega} = \underbrace{\frac{eI_d}{\pi\omega^2C_f^2}}_{shot} + \underbrace{K_f\frac{1}{C'_{ox}WL}\frac{C_D^2}{C_f^2}\frac{1}{\omega}}_{1/f} + \underbrace{\frac{2\gamma}{\pi}\frac{kT}{g_m}\frac{C_D^2}{C_f^2}}_{thermal}$$

$$= \sum_{k=-2}^{0} c_K\omega^k$$

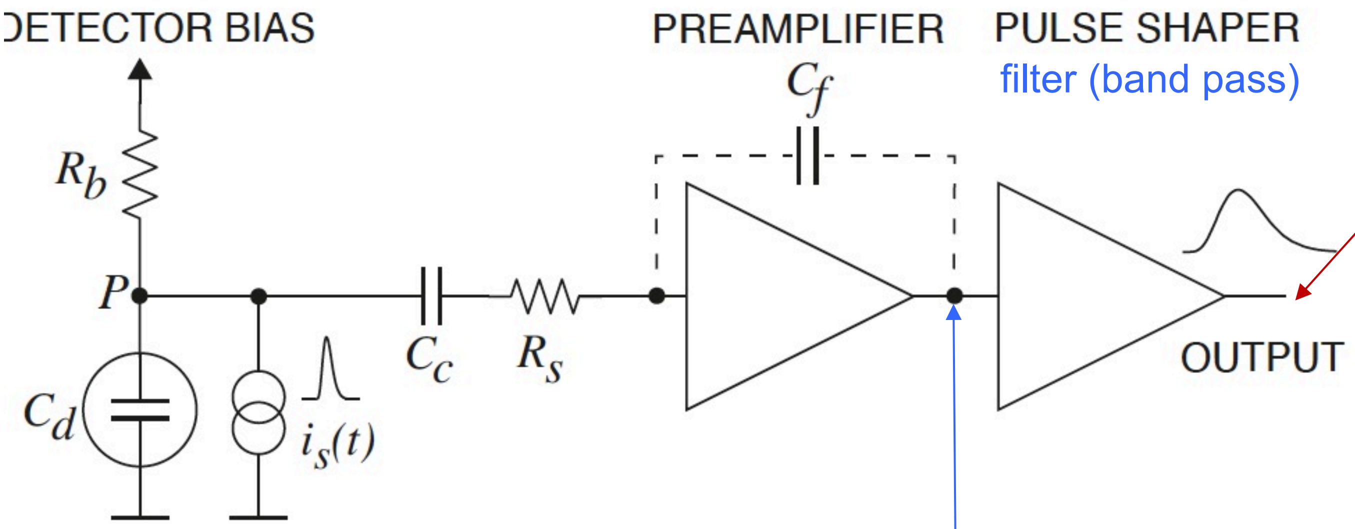
voltage noise output behind the CSA

with coefficients

$$c_{-2} = \frac{e}{\pi}I_d\frac{1}{C_f^2}, \qquad c_{-1} = K_f\frac{1}{C'_{ox}WL}\frac{C_D^2}{C_f^2}, \qquad c_0 = \frac{2\gamma}{\pi}kT\frac{1}{g_m}\frac{C_D^2}{C_f^2}$$

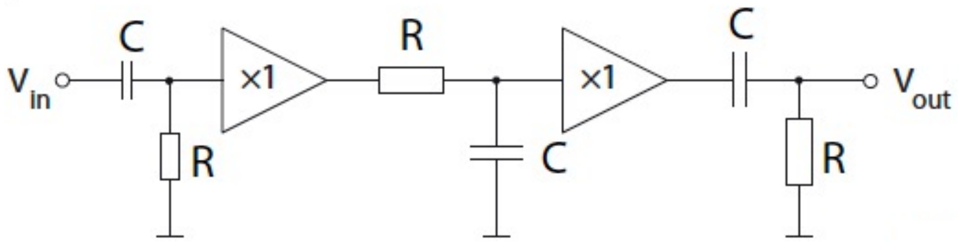
Noise in a preamp + shaper system

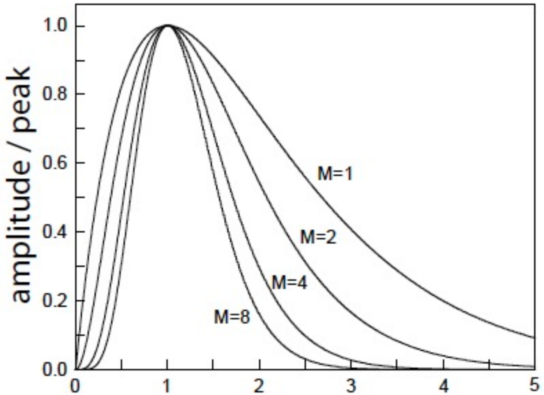
now $\langle v_{sh}^2 \rangle$?



so far: $\langle v_{pa}^2 \rangle$

shaper = high pass plus M low passes

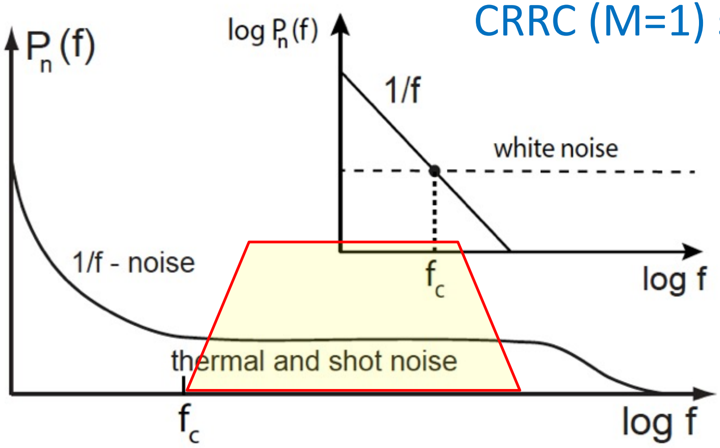




$$f^{(1,M)}(t) = \frac{1}{M!}\left(\frac{t}{\tau}\right)^M e^{-t/\tau}$$

easiest and often realised

CRRC (M=1) shaper



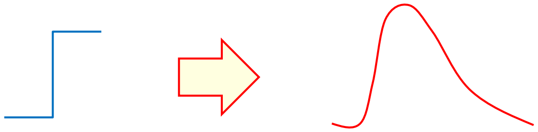
consequences for noise

BW limitation => lower noise

at the expense of loosing speed

Shaper transfer function (in → out)

in the time domain



$$v_{sh}(t) = A\frac{t}{\tau}\,\mathrm{e}^{-\frac{t}{\tau}}$$

A = amplitude

peaks at τ with peak height A / 2.71

in the frequency domain $1/s$

$$H(s) = \frac{s\tau}{(1+s\tau)^2} \rightarrow |H(\omega)|^2 = A^2\left(\frac{\omega\tau}{1+\omega^2\tau^2}\right)^2$$

$$(\text{with } s \rightarrow i\omega)$$

for noise
need square

$$\langle v_{\mathrm{sh}}^2\rangle = \int_0^\infty \frac{d\langle v_{\mathrm{pa}}^2\rangle}{d\omega}\,|H(\omega)|^2\,d\omega$$

$$\langle v_{\mathrm{sh}}^2\rangle = \sum_{k=-2}^{0}\int_0^\infty c_k\omega^k\,|H(\omega)|^2\,d\omega$$

$$= A^2\frac{1}{2}\sum_{k=-2}^{0} c_k\tau^{-k-1}\,\Gamma\left(1+\frac{k+1}{2}\right)\Gamma\left(1-\frac{k+1}{2}\right)$$

$$\Gamma(x+1) = x\Gamma(x), \qquad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}, \qquad \Gamma(1) = 1$$

Shaper transfer function (in → out)

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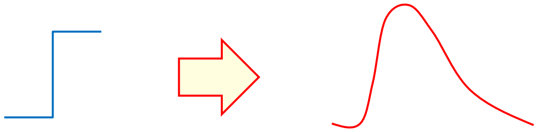
in the time domain



$$v_{sh}(t) = A\frac{t}{\tau}\,\mathrm{e}^{-\frac{t}{\tau}}$$

A = amplitude

peaks at τ with peak height A / 2.71

in the frequency domain $1/s$

$$H(s) = \frac{s\tau}{(1+s\tau)^2} \rightarrow |H(\omega)|^2 = A^2\left(\frac{\omega\tau}{1+\omega^2\tau^2}\right)^2$$

$$(\text{with } s \rightarrow i\omega)$$

executing the sum yields

$$\langle v_{\mathrm{sh}}^2 \rangle = \frac{\pi}{4}A^2\left(c_{-2}\,\tau + \frac{2}{\pi}c_{-1} + c_0\frac{1}{\tau}\right)$$

with

$$c_{-2} = \frac{e}{\pi}I_d\frac{1}{C_f^2}\,, \qquad c_{-1} = K_f\frac{1}{C'_{ox}WL}\frac{C_D^2}{C_f^2}\,, \qquad c_0 = \frac{2\gamma}{\pi}kT\frac{1}{g_m}\frac{C_D^2}{C_f^2}$$

Equivalent Noise Charge (ENC)

… want to express the noise in units of the signal at the input, i.e. "how many electrons would produce the noise voltage output behind the shaper that I see?"

$$\mathrm{ENC} = \frac{\text{noise output voltage (V)}}{\text{output voltage of a signal of } 1\,\mathrm{e}^- \ (\mathrm{V}/\mathrm{e}^-)}$$

$$\mathrm{ENC}^2 = \frac{\langle v_{\mathrm{sh}}^2 \rangle}{v_{\mathrm{sig}}^2}$$

Equivalent noise charge

for 1e at the input we get $$v_{\text{sig}} = \frac{A}{2.71}\frac{e}{C_f}$$

peak of shaper pulse

$$\text{ENC}^2\,(e^{-2}) = \frac{\langle v_{\text{sh}}^2 \rangle}{v^2_{\text{signal}}\,(1e^-)} = \frac{(2.71)^2}{4e^2}\left(eI_d\tau + 2C_D^2 K_f \frac{1}{C'_{ox}WL} + \gamma\frac{2kT}{g_m}\frac{C_D^2}{\tau}\right)$$

$$= a_{\text{shot}}\,\tau + a_{1/\text{f}}C_D^2 + a_{\text{therm}}\frac{C_D^2}{\tau}$$

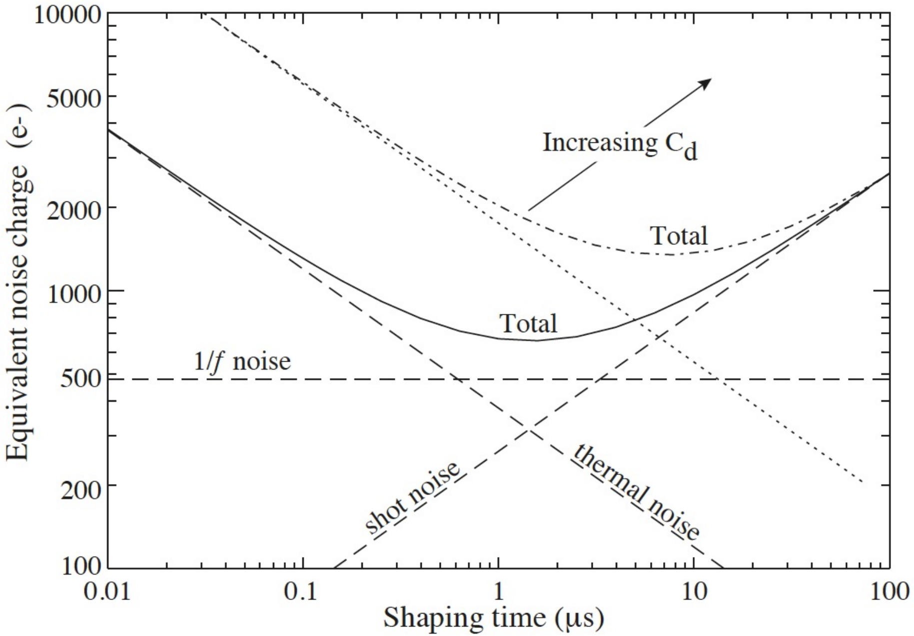
using $\gamma = 2/3$ and $C'_{ox} = 6\text{fF}/\mu\text{m}^2, K_f = 33\times 10^{-25}J$

$$\text{ENC}^2\,(e^{-2}) = 11\,\frac{I_d}{\text{nA}}\frac{\tau}{\text{ns}} + 740\,\frac{1}{WL/(\mu\text{m}^2)}\frac{C_D^2}{(100\,\text{fF})^2} + 4000\,\frac{1}{g_m/\text{mS}}\frac{C_D^2/(100\,\text{fF})^2}{\tau/\text{ns}}$$

Optimal filter time

$$\mathrm{ENC}^2 = a_{\mathrm{shot}}\,\tau + a_{1/\mathrm{f}}\,C_D^2 + a_{\mathrm{therm}}\,\frac{C_D^2}{\tau}$$

$$\tau_{\mathrm{opt}} = \left(\frac{a_{\mathrm{therm}}}{a_{\mathrm{shot}}}\,C_D^2\right)^{1/2} = \left(\frac{4kT}{3\,eI_0 g_m}\,C_D^2\right)^{1/2}$$



Review on Low-noise detector readout, NW (2021), H. Spieler (2013) in P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020) and 2021 update.

Examples

Pixel detector. As an example featuring small electrodes and correspondingly small input capacitances we choose a silicon pixel detector (section 8.7) with parameters $C_D = 200\,\mathrm{fF}$, $I_0 = 1\,\mathrm{nA}$, $\tau = 50\,\mathrm{ns}$, $W = 20\,\mu\mathrm{m}$, $L = 0.5\,\mu\mathrm{m}$, $g_m = 0.5\,\mathrm{mS}$, where we assumed a typical leakage current before the detector received substantial radiation damage. With (17.110) an equivalent noise charge of

$$\mathrm{ENC}^2 \approx (24\,e^-)^2(\mathrm{shot}) + (17\,e^-)^2(1/\mathrm{f}) + (25\,e^-)^2(\mathrm{therm}) \approx (40\,e^-)^2$$

Strip detector. For a typical silicon microstrip detector (see section 8.6.2) after radiation damage one obtains with $C_D = 20\,\mathrm{pF}$, $I_0 = 1\,\mu\mathrm{A}$, $\tau = 50\,\mathrm{ns}$, $W = 2000\,\mu\mathrm{m}$, $L = 0.4\,\mu\mathrm{m}$, $g_m = 5\,\mathrm{mS}$:

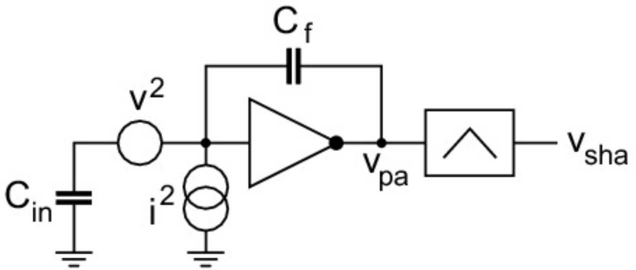
$$\mathrm{ENC}^2 \approx (750\,e^-)^2(\mathrm{shot}) + (200\,e^-)^2(1/\mathrm{f}) + (800\,e^-)^2(\mathrm{therm}) = (1100\,e^-)^2.$$

Liquid argon calorimeter. As an example of a detector with a large electrode capacitance we take a liquid argon calorimeter cell with typical values as given by the ATLAS electromagnetic calorimeter (see section 15.5.3.2 on page 597) in the central region. With the parameters $C_D = 1.5\,\mathrm{nF}$, $I_0 = <2\,\mu\mathrm{A}$, $\tau = 50\,\mathrm{ns}$, $W = 3000\,\mu\mathrm{m}$, $L = 0.25\,\mu\mathrm{m}$, $g_m = 100\,\mathrm{mS}$, i.e. assuming only a small (negligible) parallel shot noise (leakage current), one obtains:

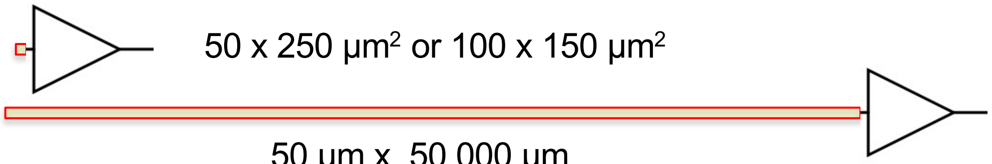
$$\mathrm{ENC}^2 \approx (1000\,e^-)^2(\mathrm{shot}) + (15000\,e^-)^2(1/\mathrm{f}) + (13500\,e^-)^2(\mathrm{therm}) \approx (20200\,e^-)^2.$$

Examples: Noise in pixel/strip/liq.Ar detector (ionisation detector)

… with CSA preamplifier & shaper



comparing pixels
and strips



	C_D	I_0	τ	W	L	g_m	ENC therm	ENC 1/f	ENC shot	ENC tot
pixel	200 fF	1 nA	50 ns	20 μm	0.5 μm	0.5 mS	25 e⁻	17 e⁻	24 e⁻	**40 e⁻**
strip	20 pF	1 μA	50 ns	2000 μm	0.4 μm	5 mS	800 e⁻	200 e⁻	750 e⁻	**1100 e⁻**
liq. Ar	1.5 nF	2 μA	50 ns	3000 μm	0.25 μm	100 mS	13 500 e⁻	15 000 e⁻	1000 e⁻	**20 200 e⁻**

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Thank you for your attention

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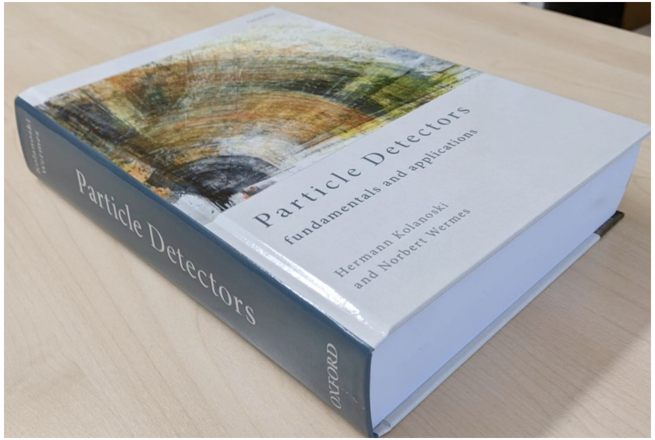
Content of lecture based on …

- Particle Data Group Review (2021)

35.9 Low-noise detector readout

Revised November 2021 by N. Wermes (Bonn U.), revised November 2013 by H. Spieler (LBNL).



- Kolanoski, H. und Wermes, N.
  Teilchendetektoren – Grundlagen und Anwendungen (Springer/Spektrum 2016)

- Kolanoski, H. and Wermes, N.
  Particle Detectors – fundamentals and applications (Oxford University Press 2020)