Primordial black holes in an early matter era and stochastic inflation[1]

[1]Based on [1912.01638], [2001.08220], [2006.14597], and two more works in preparation

Introduction

- Primordial Black Holes are dark matter candidates. They are interesting because they do not require physics beyond inflation. Their astrophysical signatures (gravitational waves, lensing, etc.) could be probed within the next decade[2].
- PBHs can account for the totality of the (un)observed dark matter if[3]

$$10^{-16}\, M_{\odot} \lesssim M_{\mathrm{PBH}} \lesssim 10^{-11} M_{\odot}.$$

- We want to answer the following questions:

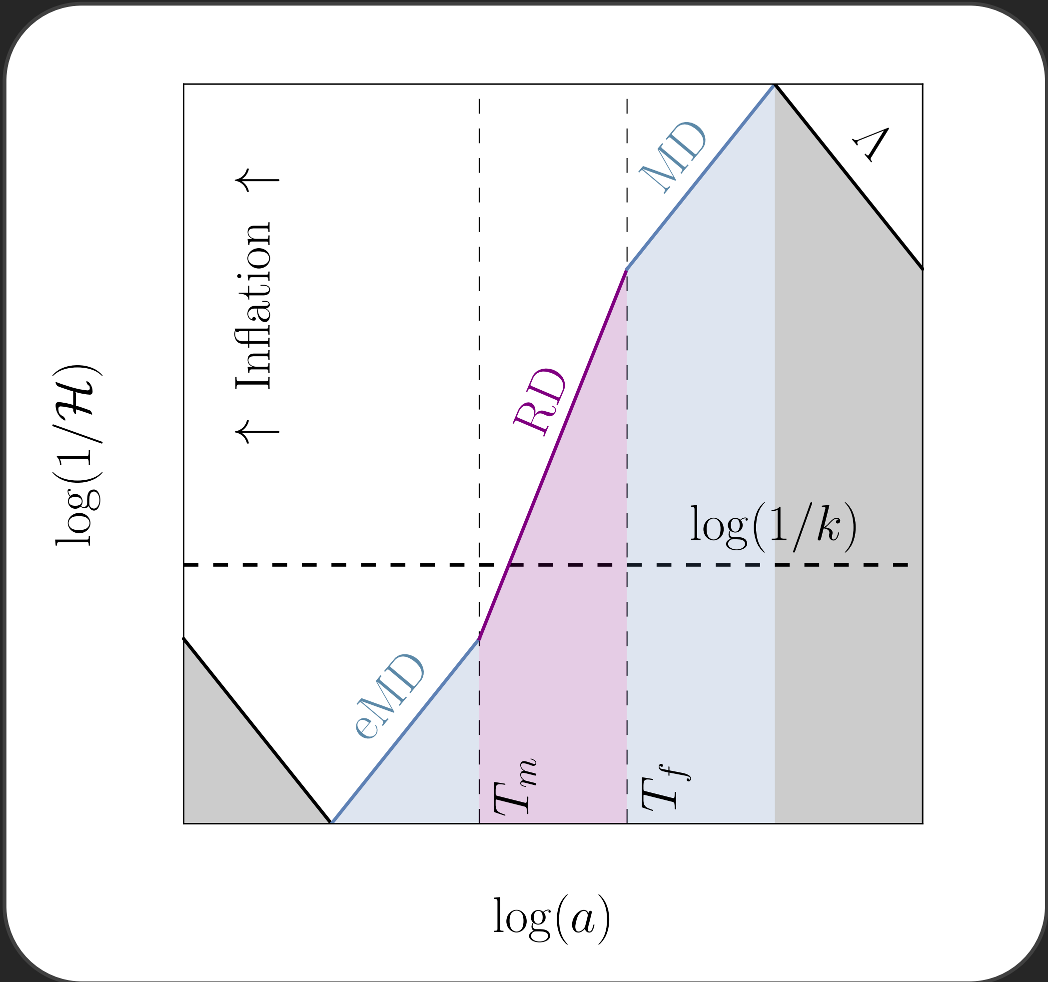
  1. How many different ways are there to obtain PBHs from inflation?
  2. Is PBH formation more likely during an early matter era?
  3. Does quantum backreaction in inflation affect the formation probability?
  4. How can we probe each one of these scenarios?

[2] M. Sasaki et al. [1801.05235]

[3] B. Carr, et al. [0912.5297], A. Arbey et al. [1906.04750], H. Niikura et al. [1701.02151], A. Katz et al. [1807.11495]

Primordial black holes

PBHs are formed in the early universe by mechanisms different to stellar collapse. There are many possibilities, from the collision of vacuum bubbles to inflation.



- For PBHs to form, we need large density fluctuations $\delta = \delta\rho/\rho$.
- These are produced during inflation. They leave the horizon and induce collapse upon re-entry.
- We assume transitions are instantaneous.

Collapse in the radiation era

The mass of the PBHs that form is some fraction of the total energy in a Hubble patch, and thus depends on the scale of the fluctuation[4],

$$M_{\mathrm{RD}} = \gamma \frac{4}{3}\pi\rho H^{-3} \propto \gamma k^{-2}, \qquad f_{\mathrm{RD}} \propto \beta_{\mathrm{RD}} M_{\mathrm{RD}}^{-1/2}.$$

where $\gamma \lesssim 1$ because of causality, f_{RD} is the fraction of DM in the form of PBHs, and

$$\beta_{\mathrm{RD}}(k) \propto \frac{1}{\sqrt{\mathcal{P}_\delta}} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\mathcal{P}_\delta}\right) d\delta$$

is the fraction of energy that collapses (beware, only for Gaussian fluctuations).

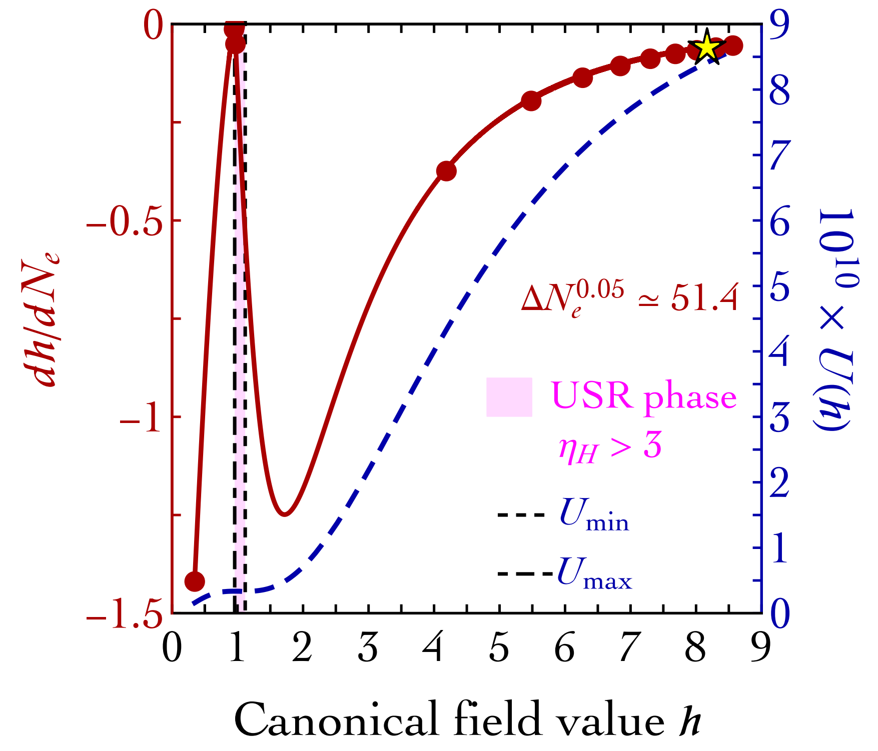
- The power spectrum $\mathcal{P}_\delta(k)$ tells us how fluctuations in energy density δ are distributed across different scales, and is what CMB experiments measure.
- Fluctuations at CMB scales do not produce enough PBHs to explain all DM. We need to enhance the power spectrum at small scales.

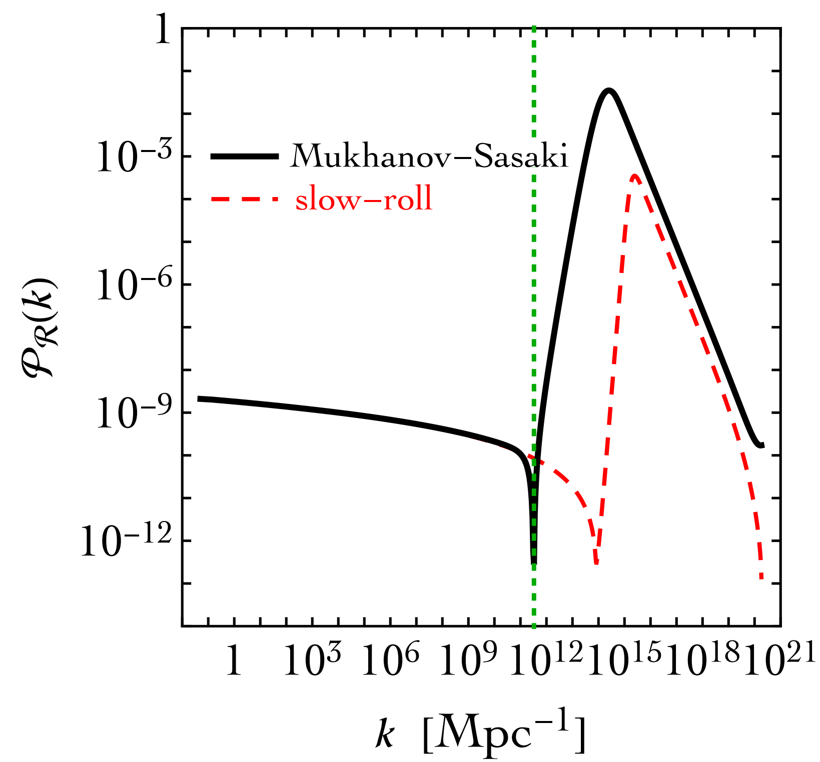
[4] B. Carr [10.1086/153853]

Black holes from inflation

Roughly speaking (only in slow roll),

$$\mathcal{P}_\delta(k) \simeq \frac{H^4}{\dot{\phi}^2},$$

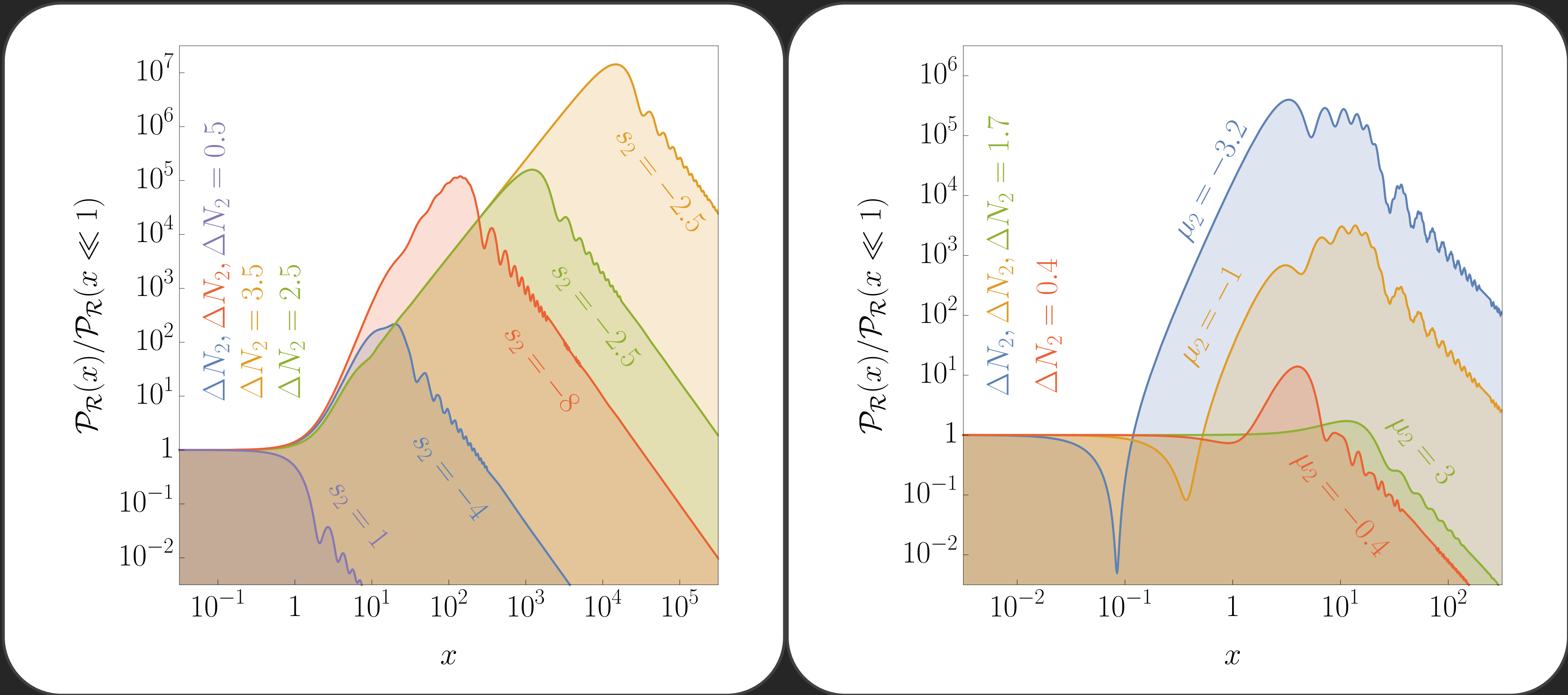




Other mechanisms (work in progress!)

Consider a generic action for curvature perturbations $\mathcal{R} \simeq \delta$

$$\mathcal{S} = \int d^4x \, M^2 \frac{a^3 \epsilon}{c_s^2} \left[\dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2} (\nabla \mathcal{R})^2 \right] \qquad \rightarrow \qquad \mathcal{P}_\delta \simeq \frac{H^4 M_p^2}{c_s \dot{\phi}^2 M^2}.$$



The simplest model

Consider a scalar field coupled to gravity in the Jordan frame[5]

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(M_p^2+\xi\phi^2)R+\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi)\right].$$

We can get rid of the coupling to R by redefining the fields,

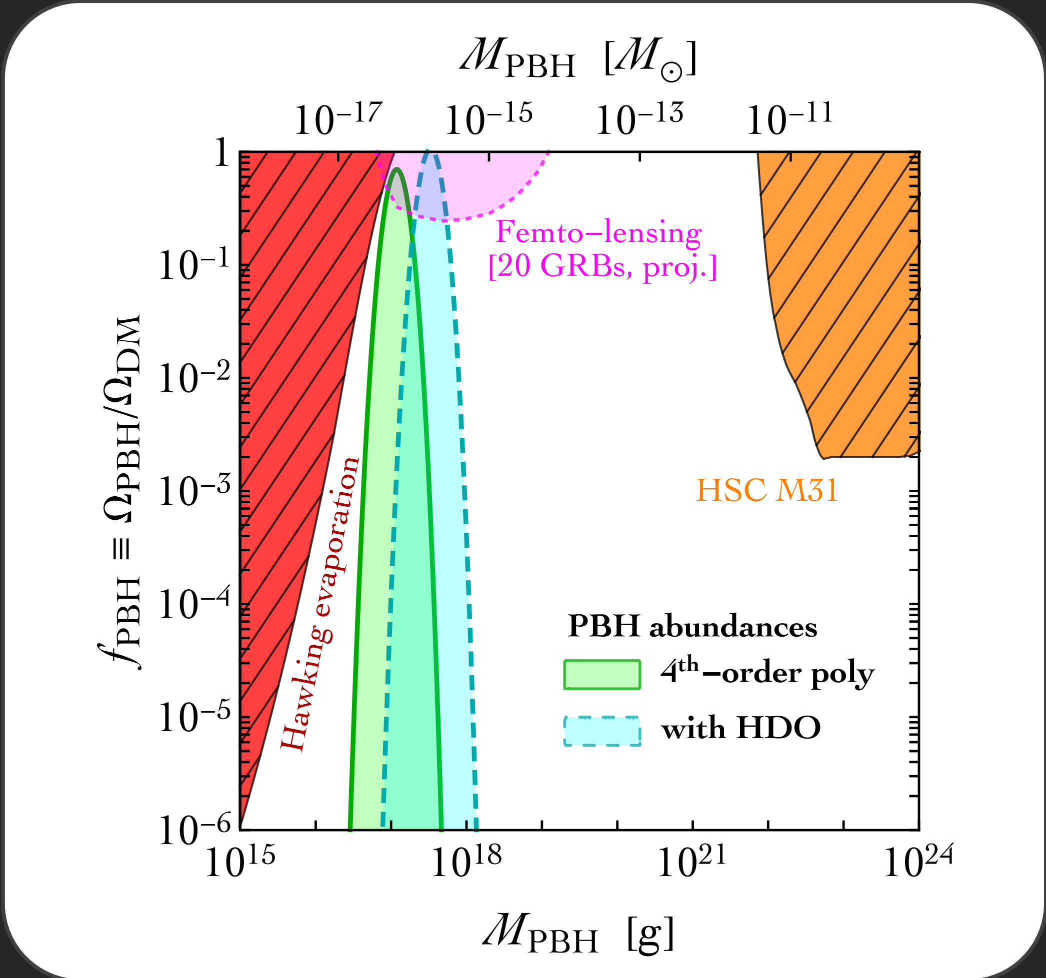
$$\begin{aligned} \Omega^2 &\equiv 1+\xi\phi^2/M_p^2, \\ g_{\mu\nu} &\to \Omega^2[\phi]g_{\mu\nu}, \\ \Omega^2\frac{dh}{d\phi} &= \left[\Omega^2+\frac{3}{2}M_p^2\left(\frac{d\Omega^2}{d\phi}\right)^2\right]^{1/2}, \end{aligned}$$

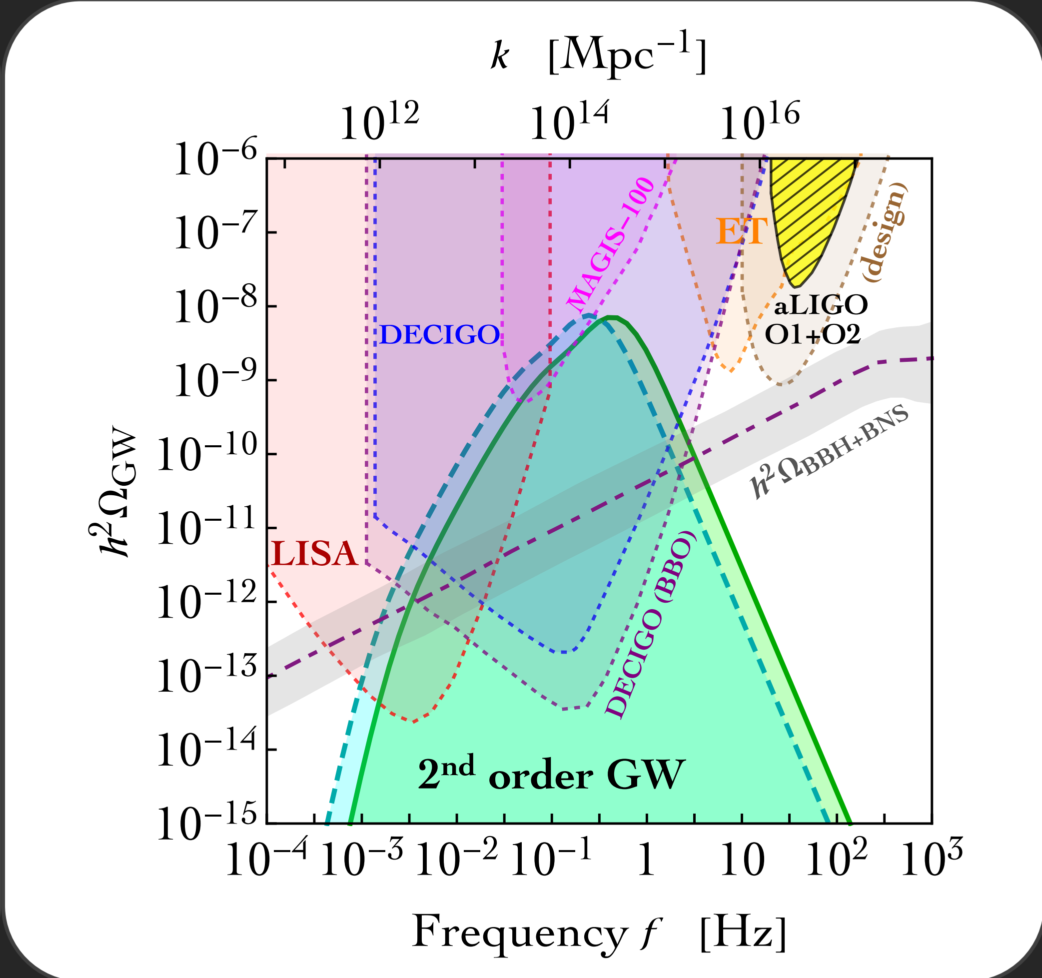
where h is such that the kinetic term is canonically normalized. The resulting potential is (where the denominator helps fit the CMB data)

$$U(h) \equiv \frac{V}{\Omega^4} = \left.\frac{a_2\phi^2+a_3\phi^3+a_4\phi^4}{(1+\xi\phi^2/M_p^2)^2}\right|_{\phi=\phi(h)}.$$

[5]G. Ballesteros and M. Taoso [1709.05565], G. Ballesteros et al. [2001.08220]

By adjusting the tilt of $\mathcal{P}_\mathcal{R}$ at CMB scales we run into problems with evaporation bounds, $n_s^{\rm poly} \simeq 0.949$ but $n_s^{\Lambda\rm CDM} = 0.9649 \pm 0.0042$. To fix this we can either extend ΛCDM, or add higher-dimensional operators to the potential.





Quantum backreaction

In stochastic inflation, quantum fluctuations backreact on the classical trajectory of the inflaton, modifying its background evolution[6],

$$\frac{d\bar{\phi}}{dN} = -\frac{\partial_\phi V}{3H^2} + \frac{H}{2\pi}\xi_\phi \quad \rightarrow \quad \mathcal{P}_\mathcal{R} \ll 1 \ \ \text{(slow roll)}$$

The field is split into a coarse-grained part and a perturbation,

$$\phi(t,\boldsymbol{x}) = \bar{\phi}(t) + \underbrace{\int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}} W[k - k_\sigma(t)] \Big[a_{\boldsymbol{k}} \phi_{\boldsymbol{k}}(t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \text{hc.} \Big]}_{\hat{\phi}_Q(t,\boldsymbol{x})},$$

where $k_\sigma = \sigma a H$ is a cutoff that separates classical, superhorizon modes from quantum, subhorizon modes. The perturbation is a stochastic variable.

Defining ”classicality” is tricky, but it turns out to be equivalent to the freeze-out time of perturbations once they leave the horizon.

[6] G. Ballesteros, and M. Taoso [1709.05565], A. Starobinsky [10.1007/3-540-16452-9-6], M. Biagetti et al. [1804.07124], J. M. Ezquiaga and J. García-Bellido [1805.06731]

The quantity of interest now is the probability distribution for the coarse-grained variables $\Phi = (\bar{\phi}, \bar{\pi})$, $P(\Phi, N)$. This can be found by solving the Fokker-Planck equation,

$$\frac{d}{dN} P(\Phi, N) = \mathcal{L}_{\rm FP}(\partial_\Phi) P(\Phi, N).$$

If we are only interested in the power spectrum, we can derive equations for the statistical moments by using $\bar{\phi} = \phi_{\rm cl} + \delta\phi_{\rm st}$ and

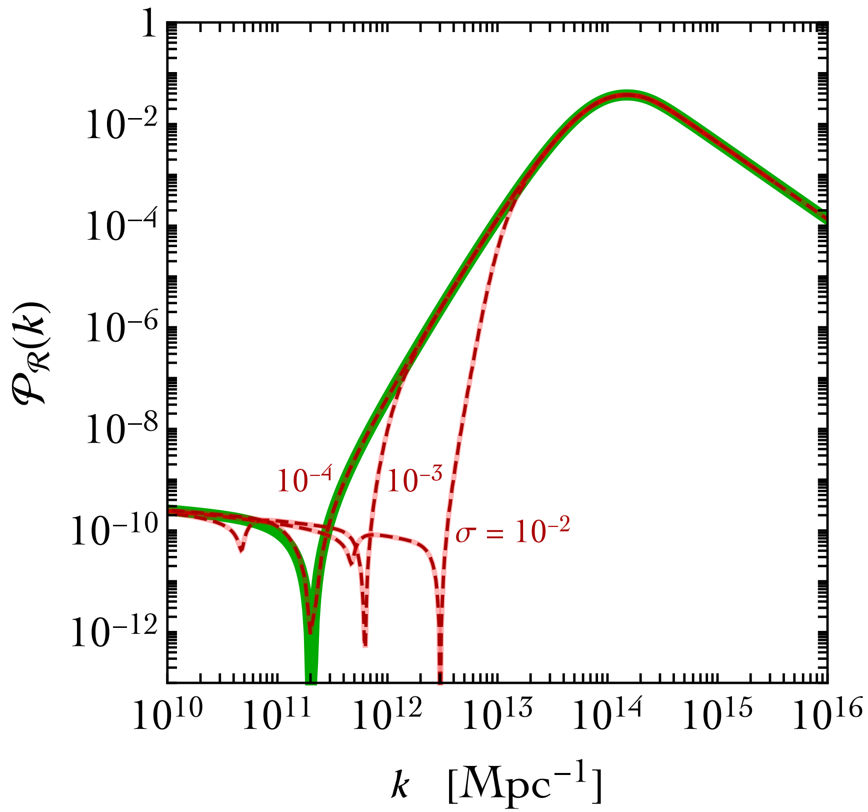
$$\langle \delta\phi_{\rm st}^n \delta\pi_{\rm st}^m \rangle(N) = \int d\Phi \, P(\Phi, N)(\bar{\phi} - \phi_{\rm cl})^n (\bar{\pi} - \pi_{\rm cl})^m.$$

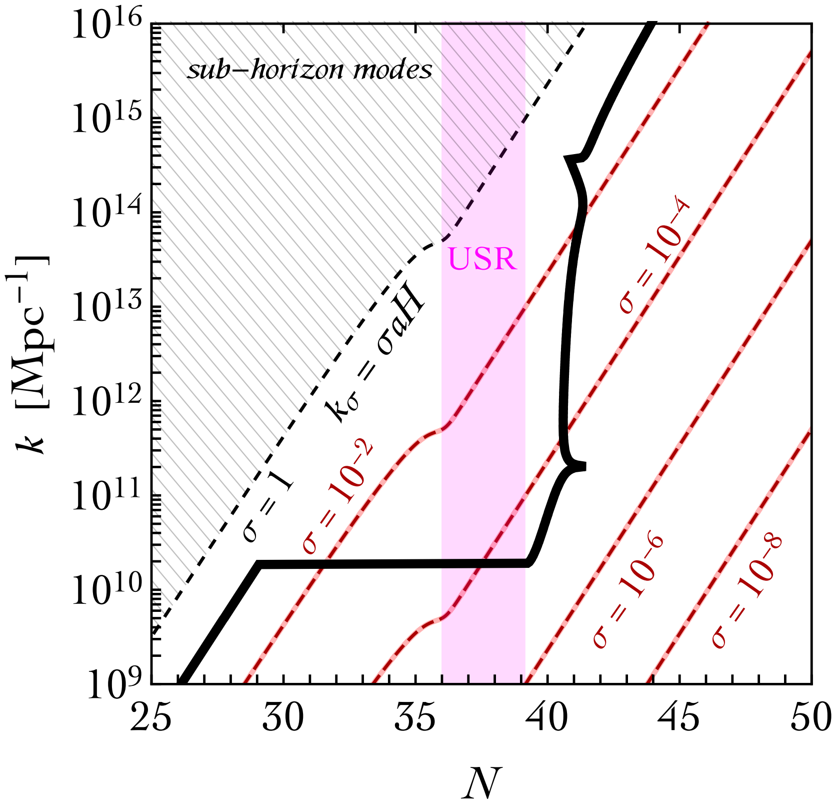
The power spectrum is given by

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{2\epsilon_{\rm cl}} \Big[D_{\phi\phi} + 2\langle \delta\phi_{\rm st}\delta\pi_{\rm st}\rangle - 2(\epsilon_{\rm cl} - \eta_{\rm cl})\langle \delta\phi_{\rm st}^2 \rangle \Big],$$

where $D_{\phi\phi}$ is a noise correlator that depends on the window function we chose earlier. The equations of motion for the statistical moments can be solved analytically by considering a series of phases with constant η to show $2\langle \delta\phi_{\rm st}\delta\pi_{\rm st}\rangle - 2(\epsilon_{\rm cl} - \eta_{\rm cl})\langle \delta\phi_{\rm st}^2 \rangle = 0$, as long as the cutoff is chosen correctly.

We show that the power spectrum coincides, at the linear level, with the perturbative result in the $\sigma \to 0$ limit, even in the presence of an ultra slow roll phase.





We focused on the power spectrum, but in stochastic inflation it is possible to compute the full PDF. Recent analyses in perfect USR suggest an exponential tail [7].

[7] D. Figueroa et al. [2012.06551], J. M. Ezquiaga et al. [1912.05399], C. Pattison et al. [2101.05741], M. Biagetti et al. [2105.07810]

Collapse in an early matter era

If collapse occurs during an early matter-dominated era, the abundance is

$$f_{\mathrm{RD}} \propto \beta_{\mathrm{RD}} M_{\mathrm{RD}}^{-1/2} \qquad f_{\mathrm{MD}} \propto \beta_{\mathrm{MD}} T_m$$

The β function represents the fraction of energy density that collapses. This function has very different forms in MD and RD [8],

$$\beta_{\mathrm{RD}}(k) \propto \frac{1}{\sqrt{\mathcal{P}_\delta}} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\mathcal{P}_\delta}\right) d\delta,$$
$$\beta_{\mathrm{MD}}(k) \propto \mathcal{I}^6 \mathcal{P}_\delta \exp\left[-\alpha \left(\frac{\mathcal{I}^4}{\mathcal{P}_\delta}\right)^{1/3}\right].$$

The latter takes into account the non-sphericity and angular momentum (related to $\mathcal{I}$) of the collapsing cloud.

Intuitively, collapse should be easier during an eMD era because of the lack of radiation pressure (roughly speaking).

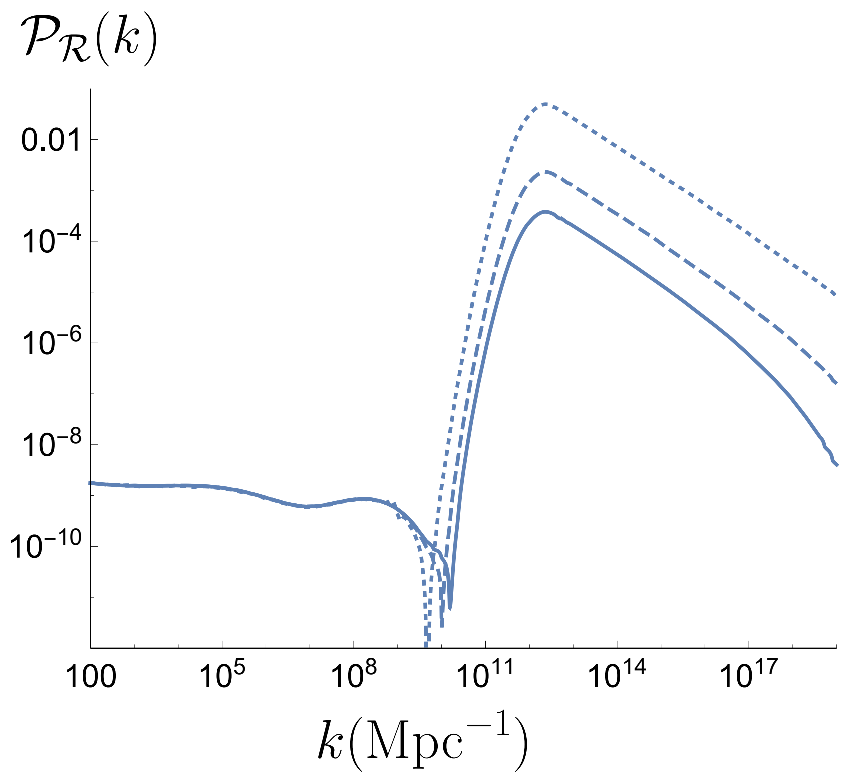
[8] T. Harada et al. [1609.01588], T. Harada et al. [1707.03595]

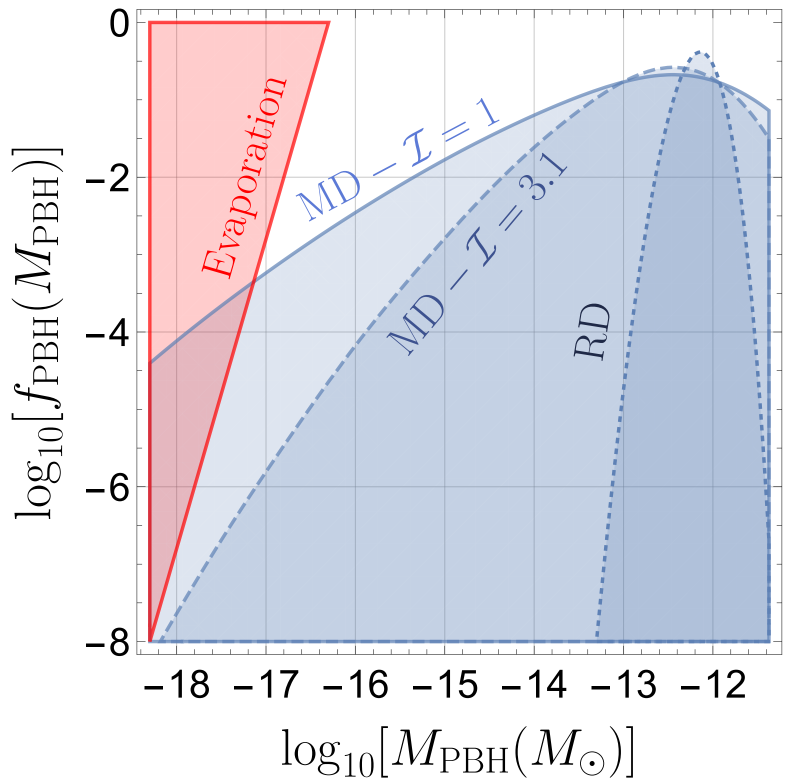
Advantages and disadvantages

Collapse during matter-domination has two advantages,

1. The power spectrum required to get a significant PBH abundance is much smaller in MD than in RD, $\mathcal{P}_{\rm RD} \sim 10^{-2}$ vs. $\mathcal{P}_{\rm MD} \sim 10^{-4}$ (at $T_m \sim 10^5$GeV).

2. The abundance is **much** less sensitive to small changes in $\mathcal{P}_{\mathcal{R}}$, since β is different.



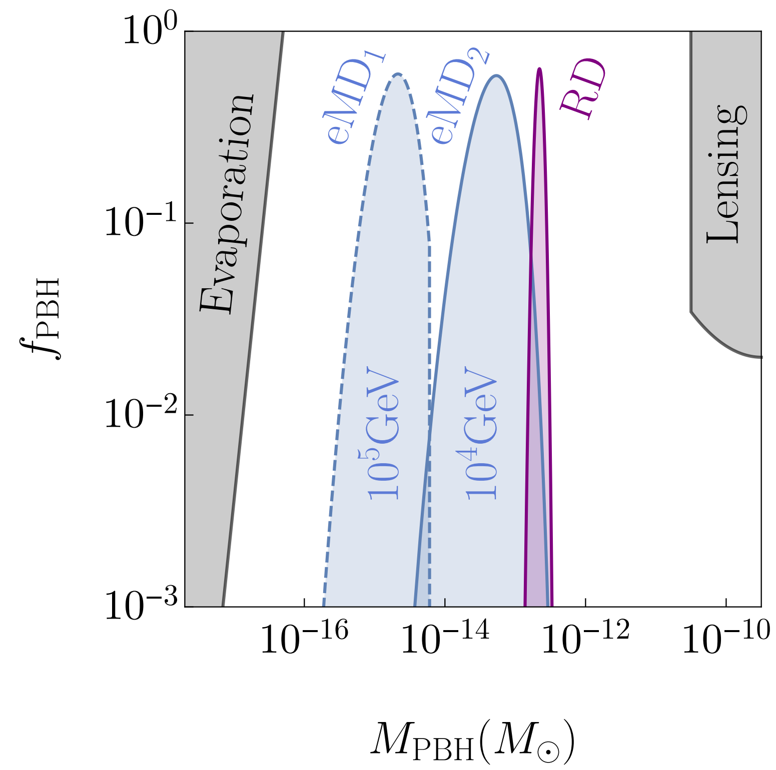


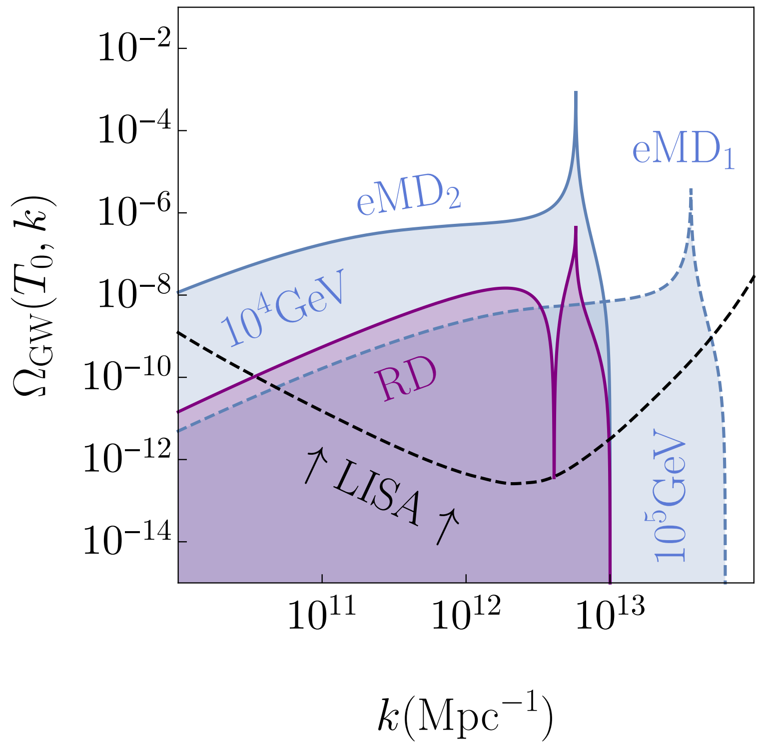
Gravitational wave signal (work in progress!)

The gravitational wave signal arises at second order in Einstein's equations,

$$\boldsymbol{h}_{ij}^{\mathrm{TT}''} + 2\mathcal{H}\boldsymbol{h}_{ij}^{\mathrm{TT}'} - \nabla^2\boldsymbol{h}_{ij}^{\mathrm{TT}} = -4\mathcal{T}_{ij}{}^{lm}s_{lm},$$

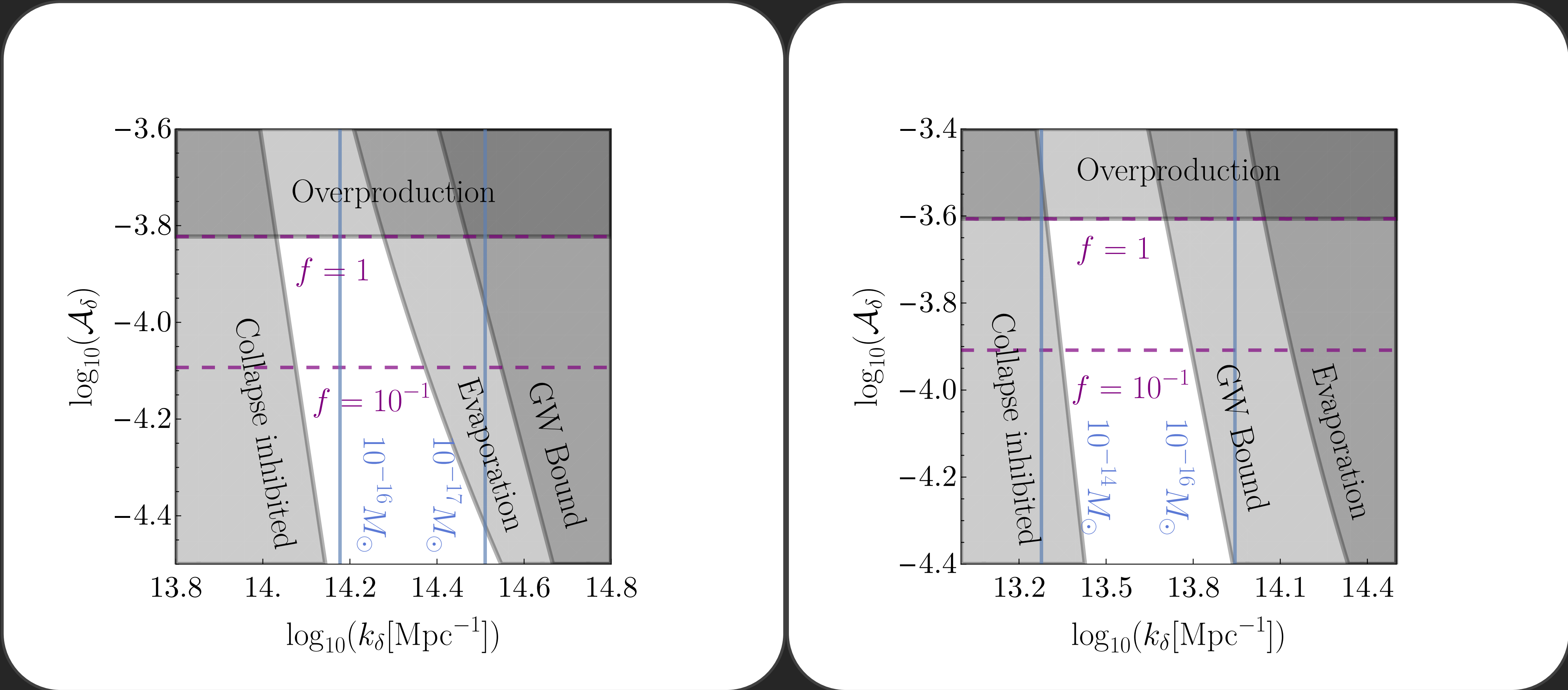
where the source is quadratic in first-order scalars.





There is a bound on $\Omega_{\rm GW}$ arising from both the CMB and the abundance of light elements produced during Big-Bang Nucleosynthesis[9] ($T_m = 10^{5.7}$GeV vs $T_m = 10^5$GeV),

$$\Omega_{\rm GW} h^2 < 1.8 \times 10^{-6}.$$

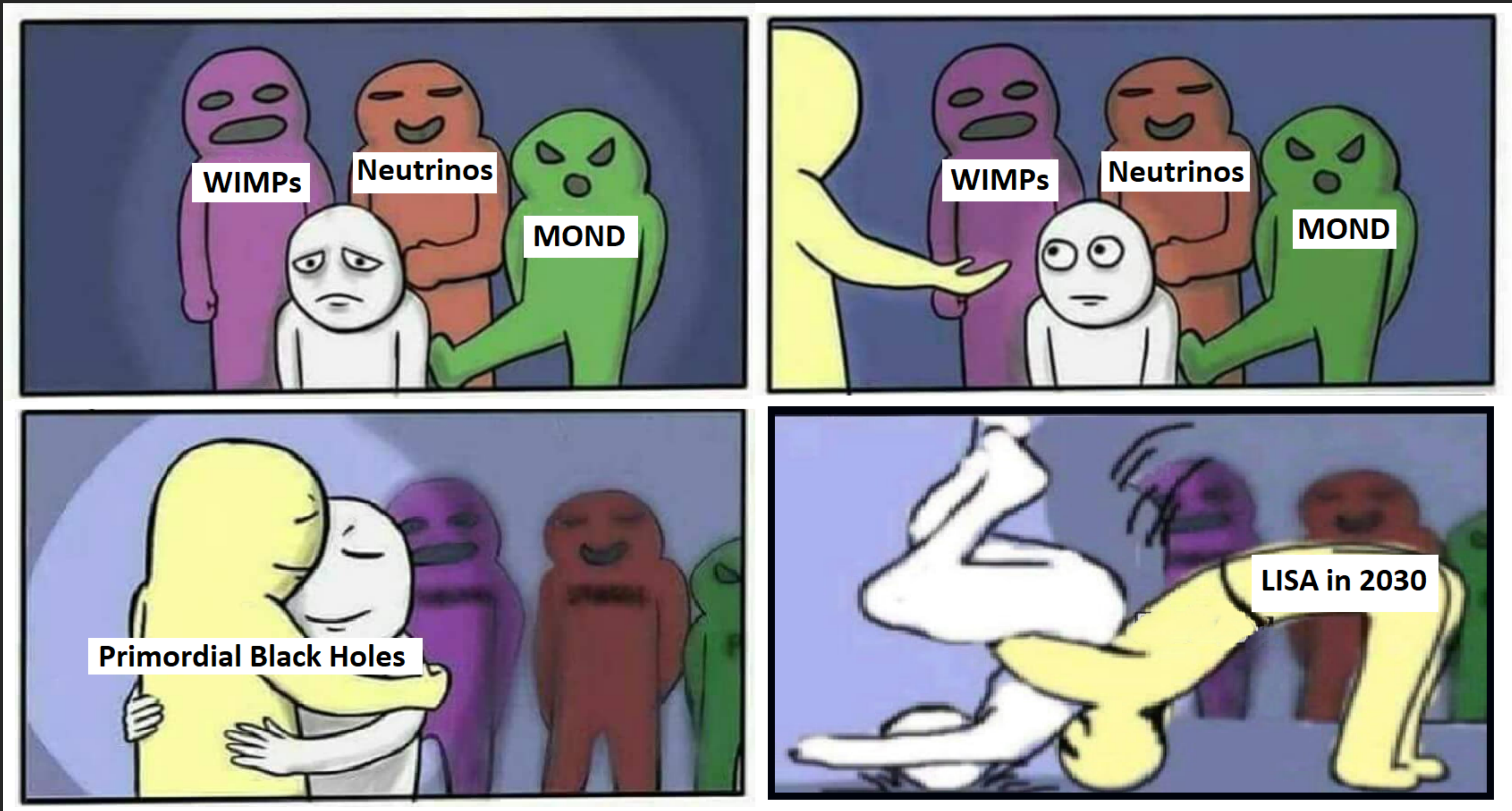


[9] L. Pagano et al. [1508.02393]

Conclusions and remarks

- The simplest potential that can produce PBHs is viable, provided ΛCDM is extended, or higher-dimensional operators are considered.
- If dark matter is in the form of PBHs, the corresponding GW signal should be observable by LISA and DECIGO. The MD case requires care and leads to bounds on the abundance.
- We have shown that, at leading order, stochastic inflation does not affect the power spectrum, even in the presence of a USR phase. The full probability distribution is another story.
- PBH formation in an early matter-dominated era has significant advantages, namely, that a smaller enhancement of the power spectrum is required, and the potential parameters are less tuned. It is more difficult to evade evaporation bounds.
- There are many sources of uncertainty in the MD formulas. A more thorough analytical description is necessary, together with numerical simulations.

Thank you!


WIMPs
Neutrinos
MOND
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Primordial Black Holes
LISA in 2030