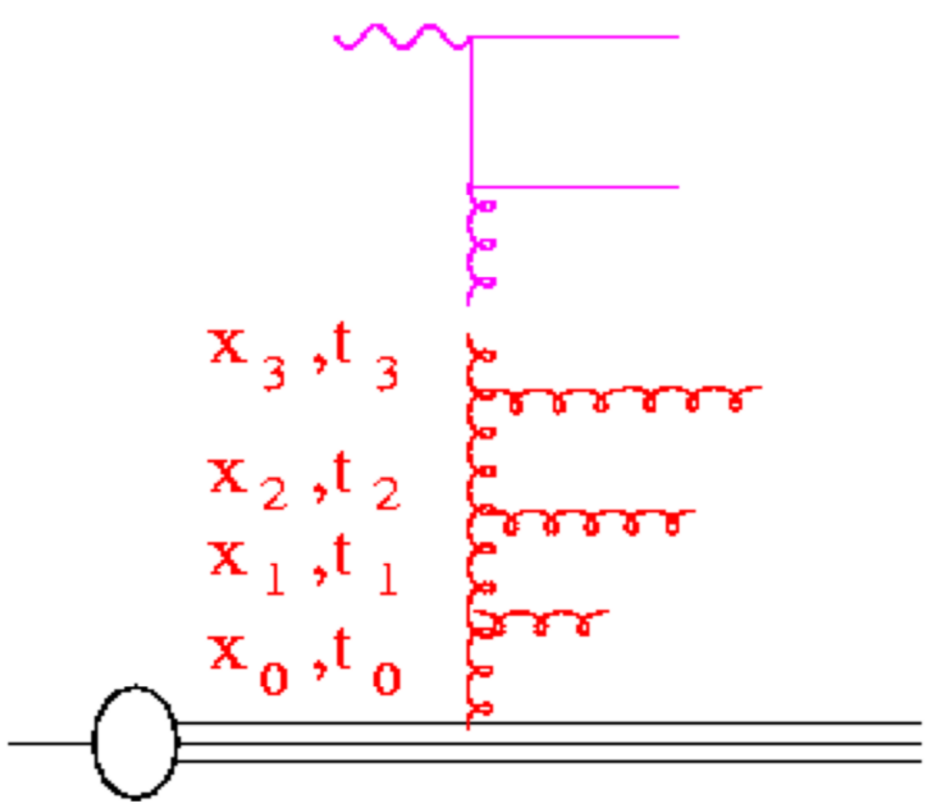
QCD and Monte Carlo techniques

- Hope that you are all ok !

Including kinematic effects into evolution ?

Approximations so far ....

- Only inclusive quantities were considered:
  - nothing was said about “real” emissions or gluons or quarks although implicitly assumed....
  - in deriving DGLAP splitting functions we assumed: $\hat{t} \ll \hat{s}$
  - and also in the small *t* limit: $\hat{t} \sim \frac{-{k_T}^2}{1-z}$



- neglect *t* in previous branchings

$$t_0 \ll t_1 \ll t_2 \ll t_3 \cdots \ll \mu^2$$

  - strong ordering condition
  - strong ordering: neglect all kinematics of previous branchings...

- ordering in *x*

$$x_0 > x_1 > x_2 > x_3$$

Better treatment including Transverse Momenta

- start from integral equation:

$$f(x,q) = f(x,Q_0)\Delta_s(q) + \int \frac{dz}{z}\int \frac{d^2q'}{\pi q'^2}\cdot\frac{\Delta_s(q)}{\Delta_s(q')}\tilde{P}(z) f\left(\frac{x}{z},q'\right)$$

- use TMD (Transverse Momentum Dependent, un-integrated pdfs):

$$x\mathcal{A}(x,k_T,q) = x\mathcal{A}_0(x,k_T)\Delta_s(q) + \int dz \int \frac{d^2q'}{\pi q'^2}$$

$$\cdot\Delta_s(q,q')\tilde{P}(z,q',k_T)\Theta(\mathcal{O})\frac{x}{z}\mathcal{A}\left(\frac{x}{z},k_T{}',q'\right)$$

because of phi integration:

$$\frac{dt}{t} \to \frac{dq^2}{q^2} \to \frac{d^2q}{\pi q^2}$$

define updf (TMD):

$$xg(x,Q) = \int \frac{d^2k_T}{\pi} x\mathcal{A}(x,k_T,Q)\Theta(Q-k_T)$$

- same as before.... but included explicitly dependence on transverse momentum k_t in addition to evolution scale q

- what are the ordering constraints $\Theta(\mathcal{O})$?

Insert: Light-cone variables

$$V = (V^0, V^1, V^2, V^3) = (V^0, V_\perp, V^3)$$

$$V^+ = \frac{1}{\sqrt{2}}(V^0 + V^3) \Longrightarrow V_0 = \frac{1}{\sqrt{2}}(V^+ + V^-)$$

$$V^- = \frac{1}{\sqrt{2}}(V^0 - V^3) \qquad V^3 = \qquad -$$

$$A.B = A^0 B^0 - \vec{A}\vec{B}$$

$$V.W = V^+ V^- + V^- W^+ - V_\perp W_\perp$$

$$V.V = 2 V^+ V^- - V_\perp^2$$

$$\text{Boost} \quad V'^0 = \frac{V^0 + v V^3}{\sqrt{1-v^2}}, \quad V'^3 = \frac{v V^0 + V^3}{\sqrt{1-v^2}}$$

$$V'_1 = V_1, \quad V'_2 = V_2$$

Insert: Light-cone variables

$$V^{+\prime} = \frac{1}{\sqrt{2}}\left(V'^{0} + V'^{3}\right) = \frac{1}{\sqrt{2}} \frac{V^0 + vV^3 + vV^3 + V^0}{\sqrt{1-v^2}}$$

$$= \frac{1}{\sqrt{2}} \sqrt{\frac{1+v}{1-v}} \left(V^0 + V^3\right)$$

$$V^{-\prime} = \frac{1}{2} \sqrt{\frac{1-v}{1+v}} \left(V^0 - V^3\right)$$

$$\psi = \frac{1}{2} \ln \frac{1+v}{1-v}$$

$$e^{\psi} = \sqrt{\frac{1+v}{1-v}}$$

$$V^{+\prime} = V^{+} e^{\psi}$$

$$V^{-\prime} = V^{-} e^{\psi}$$

$$p^{rest} = \left(\frac{m}{\sqrt{2}}, \frac{m}{\sqrt{2}}, 0\right)$$

$$p' = \left(p^{+\prime}, p^{-\prime}, 0\right) = \left(\frac{m}{\sqrt{2}} e^{\psi}, \frac{m}{\sqrt{2}} e^{-\psi}, 0\right)$$

$$\frac{p^{+\prime}}{p^{-\prime}} = e^{2\psi} \Rightarrow \ln \frac{p^{+\prime}}{p^{-\prime}} = 2\psi$$

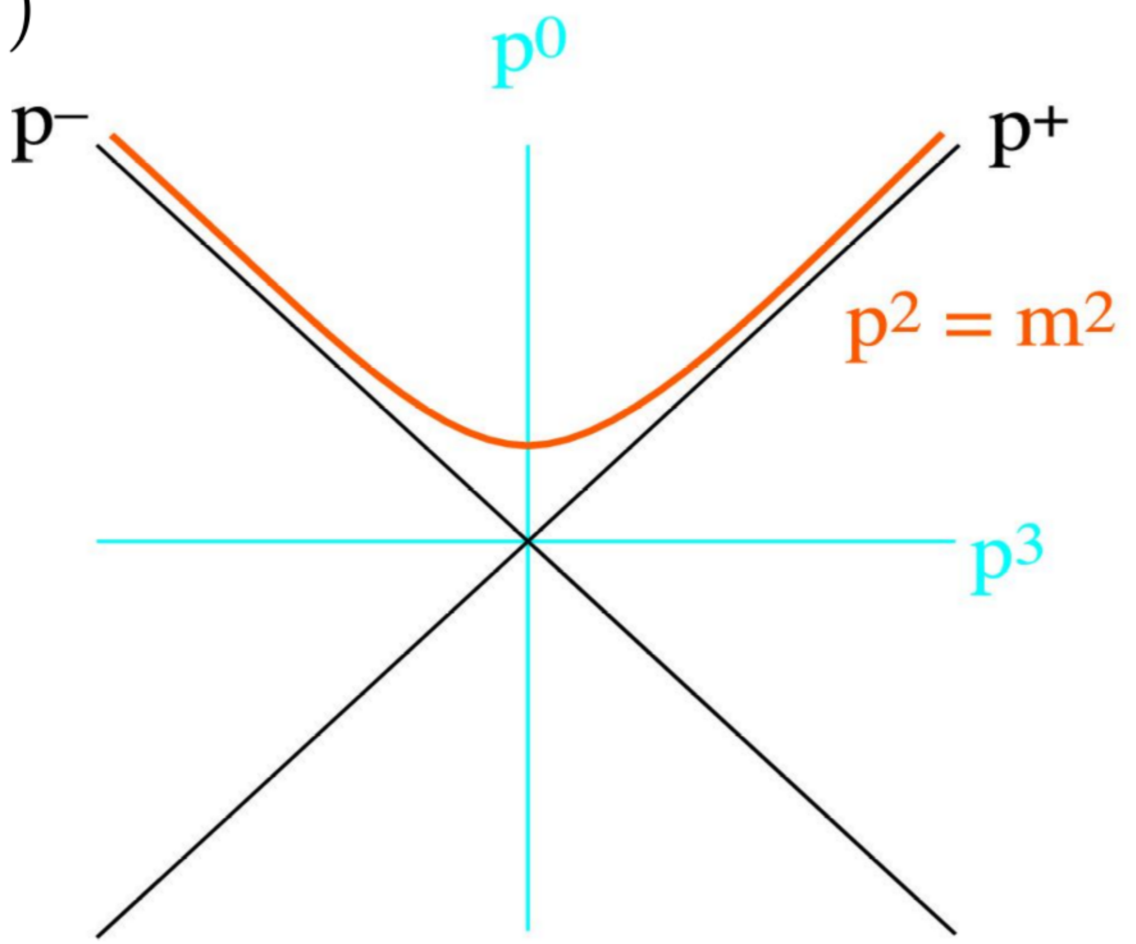
$$\Rightarrow \quad y = \psi = \frac{1}{2} \ln \frac{p^{+}}{p^{-}}$$

Light-cone variables

- Light Cone variables:

J. Collins hep-ph/9705393
D Soper, CTEQ 2001

$$V = (V^0, V^1, V^2, V^3) = (V^0, \mathrm{V}_t, V^3)$$
$$V^+ = \frac{1}{\sqrt{2}}(V^0 + V^3)$$
$$V^- = \frac{1}{\sqrt{2}}(V^0 - V^3)$$
$$V = (V^+, V^-, \mathrm{V}_t)$$
$$V.W = V^+W^- + V^-W^+ - \mathrm{V}_t\mathrm{W}_t$$
$$V^2 = 2V^+V^- - V_t^2$$

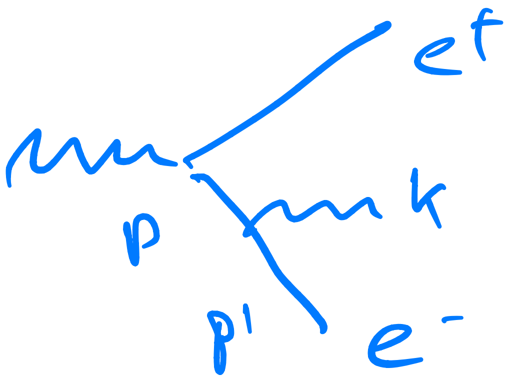


- lorentz boosts:

$$V'^0 = \frac{V^0 + vV^3}{\sqrt{1-v^2}}$$
$$V'^3 = \frac{vV^0 + V^3}{\sqrt{1-v^2}}$$
$$V'^+ = V^+e^{\psi}$$
$$V'^- = V^-e^{-\psi}$$
$$\psi = \frac{1}{2}\ln\frac{1+v}{1-v}$$

Reconsider ordering conditions

Angular ordering in QED



$$\Delta t = \frac{1}{\Delta E}$$

$$p = (p^+, p^-, 0) = (p^+, 0, 0)$$

$$p' = \left((1-z)p^+, p'^-, -k_\perp\right) = \left((1-z)p^+, \frac{k_\perp^2}{2(1-z)p^+}, -k_\perp\right)$$

$$p'^2 = 2p^+p^- - k_t^2$$

$$p^- = \frac{-k_\perp^2}{2p^+}$$

$$p' + k = \left((1-z)p^+ + zp^+, \frac{k_\perp^2}{2(1-z)p^+} + \frac{k_\perp^2}{\dots}, 0\right) = \left(p^+, \frac{k_\perp^2}{(1-z)2p^+}, 0\right)$$

$$\Delta E = (p' + k - p)$$

$$= \frac{1}{\sqrt{2}}\left(p^+ + \frac{k_\perp^2}{(1-z)2p^+} - p^+\right)$$

$$= \frac{1}{\sqrt{2}} \frac{k_\perp^2}{(1-z)\cdot 2p^+}$$

$$z \longrightarrow 0$$

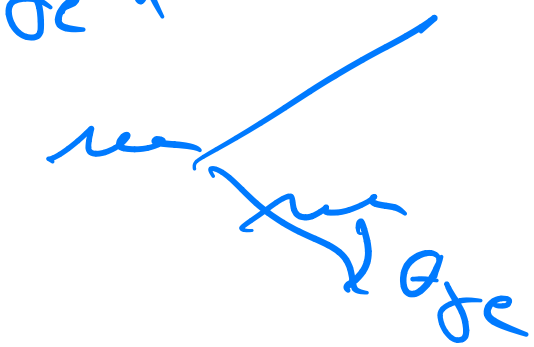
$$k_t \sim z p^+ \Theta$$

$$\Delta E = \frac{1}{\sqrt{2}} \frac{k_t^2}{zp^+} \sim \frac{1}{\sqrt{2}} z p^+ \Theta^2$$

Angular ordering in QED

$$\Delta t = \frac{1}{\Delta E} = \frac{1}{2p^+\Theta^2} = \frac{1}{k\Theta^2} = \frac{\lambda_\perp}{\Theta_{\gamma e}}$$

$$\lambda_\perp^{-1} = k_\perp = \Theta_{\gamma e}\, k$$



during Δt $\quad \varrho_\perp^{e^+e^-} = \Delta\,\Delta t = \Theta_{ee}\,\Delta t = \Theta_{ee}\,\frac{\lambda_\perp}{\Theta_{e\gamma}}$

For $\Theta_{e\gamma} \gg \Theta_{ee}$ $\quad \varrho_\perp \ll \lambda_\perp$

$\Longrightarrow$ γ sees only e^+e^-, charge $= 0$ $\rightarrow$ x-sect $\rightarrow 0$

$\Theta_{e\gamma} \ll \Theta_{ee}$ $\Longrightarrow$ γ sees electrons x-sect $\neq 0$

$\Longrightarrow$ Cosmic ray radiation "Chudakow effect" (1955)

Angular ordering in QED

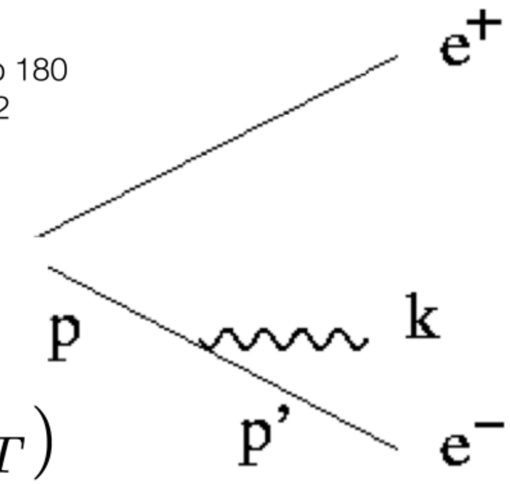
- assume QED
- use light-cone vectors:

$$p = (p^+, p^-, 0) = (p^+, 0, 0)$$

$$p' = ((1-z)p^+, p'^-, -k_T) = ((1-z)p^+, \frac{{k_T}^2}{(1-z)p^+}, -k_T)$$

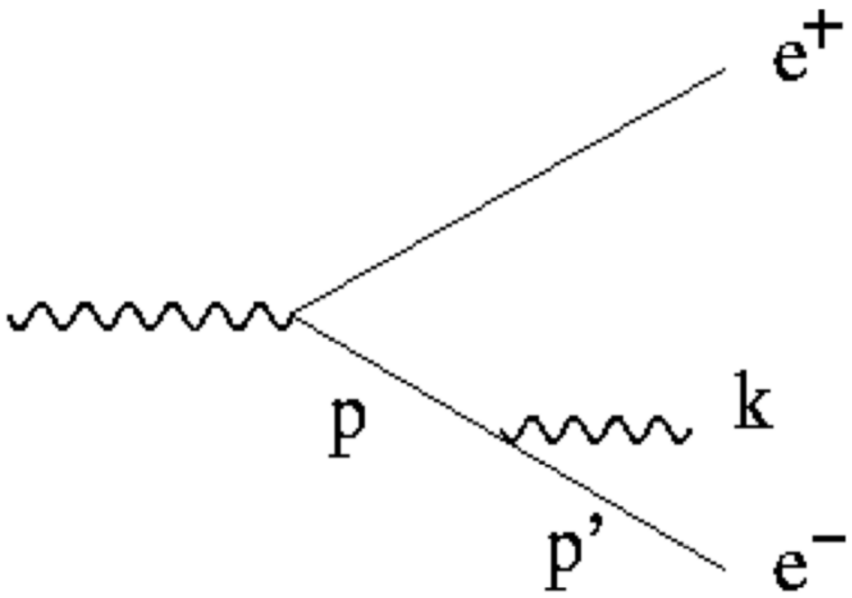
$$k = (zp^+, k^-, k_T) = (zp^+, \frac{{k_T}^2}{zp^+}, k_T)$$

Ellis,Webber,Stirling, p 180
Dokshitzer,Khoze p 92



- use energy imbalance: $\Delta E \sim \frac{{k_T}^2}{zp^+} = zp^+ \Theta^2_{e\gamma}$
- define transverse wavelength: $k_T = k\Theta_{e\gamma} = \lambda_\perp^{-1}$
- from uncertainty principle: $\Delta t = \frac{\lambda_\perp}{\Theta_{e\gamma}}$
- during Δt e^+e^- pair has travelled a distance: $\rho_t^{e^+e^-} = \Delta x \Delta t \sim \Theta_{e^+e^-} \frac{\lambda_\perp}{\Theta_{e\gamma}}$

Angular ordering in QED



Ellis,Webber,Stirling, p 180
Dokshitzer,Khoze p 92

- photon emissions allowed for:

  $$\text{for } \Theta_{\gamma,e} < \Theta_{e^+,e^-}$$

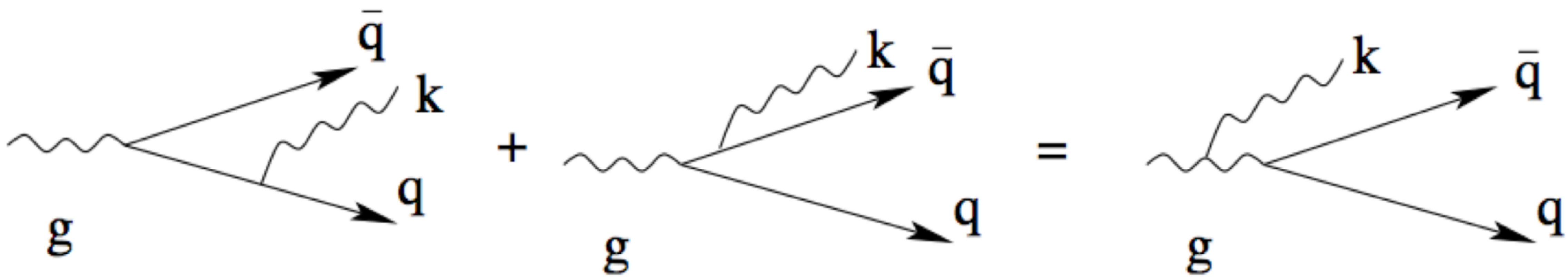
- radiation strongly suppressed for:

  $$\text{for } \Theta_{\gamma,e} > \Theta_{e^+,e^-}$$

➔ since photon cannot resolve any structure of e^+e^- pair

Angular ordering and color coherence

Dokshitzer,Khoze p 92

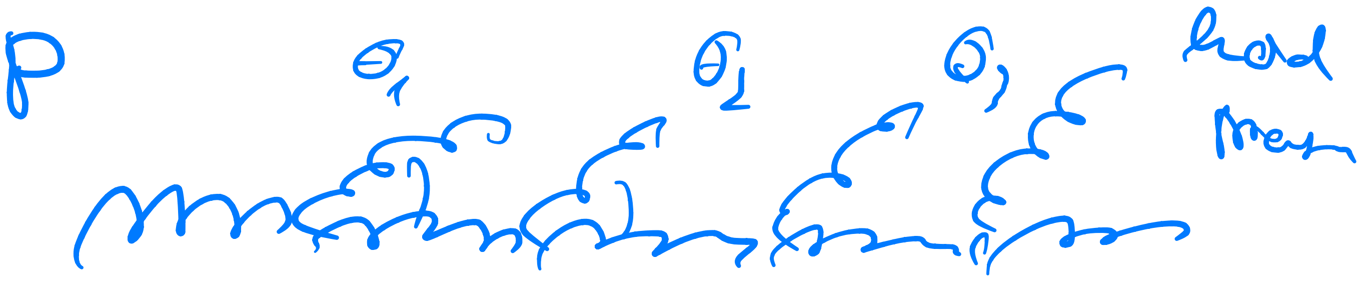


- gluon emissions are allowed

$$\begin{array}{ll} \text{off } q & \text{for } \Theta_{kq} < \Theta_{q\bar{q}} \\ \text{off } \bar{q} & \text{for } \Theta_{k\bar{q}} < \Theta_{q\bar{q}} \\ \text{off parent } g & \text{for } \Theta_{kg} > \Theta_{q\bar{q}} \end{array}$$

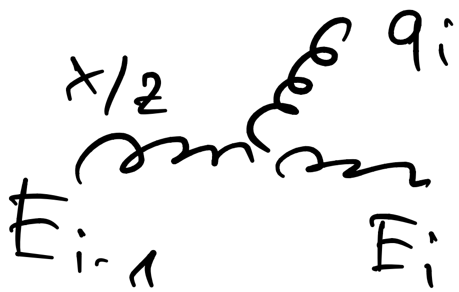
- calculations done explicitly in Ellis,Stirling & Webber

Parton Branching evolution with ang. ord.



$$p_z = |p|\cos\Theta$$
$$p_\perp = |p|\cdot\sin\Theta$$

$$\Theta_1 < \Theta_2 < \Theta_3 \ldots$$



$$\frac{E_i}{E_{i-1}} = z$$

$$E_i = x P$$
$$E_{i-1} = x/z \, P$$

$$E_{i-1} = E_i + E_{q_i} \Rightarrow E_{q_i} = E_{i-1}(1-z) \; ; \quad q_{ti} = E_{q_i}\sin\Theta_i = E_{i-1}(1-z_i)\sin\Theta_i$$

$$q_i' = \frac{q_{ti}}{1-z} = E_{i-1}\cdot\sin\Theta_i \sim E_{i-1}\cdot\Theta_i$$

$$\frac{q_i'}{E_{i-1}} > \frac{q_{i-1}'}{E_{i-2}} \Rightarrow q_i' > \frac{E_{i-1}}{E_{i-2}} q_{i-1}' = z_{i-1} q_{i-1}'$$

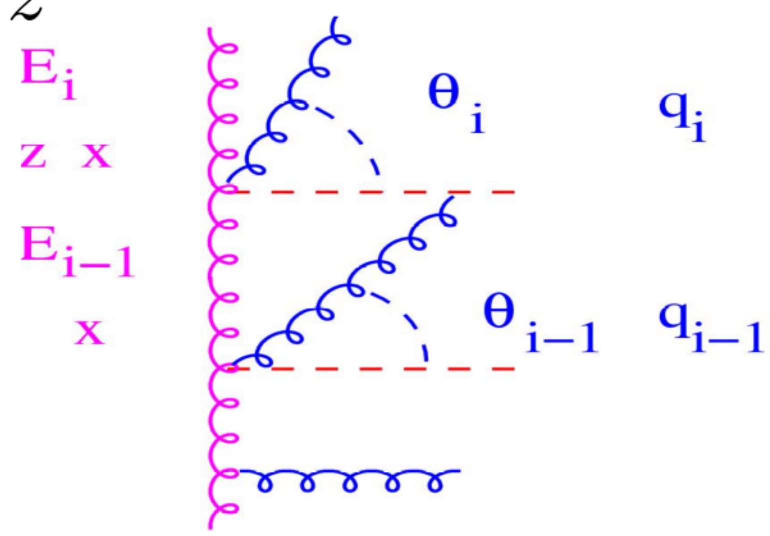
$$q_i' > q_{i-1}'$$

Parton Branching evolution with ang. ord.

$$
\begin{aligned}
p_{ti} &= |q_i^0| \sin\Theta_i \\
z &= \frac{E_i}{E_{i-1}}
\end{aligned}
$$

$$
\begin{aligned}
\text{with: } q_i &= \frac{p_{ti}}{1-z_i} \\
\to \Theta_i &= \frac{q_i}{E_{i-1}} \\
\Theta_{i+1} &= \frac{q_{i+1}}{E_i}
\end{aligned}
$$

$$
\begin{aligned}
E_{i-1} &= E_i + q_i^0 = zE_{i-1} + q_i^0, \\
\to q_i^0 &= (1-z)E_{i-1} \\
p_{ti} &= q_i^0 \sin\Theta_i \simeq (1-z)E_{i-1}\Theta_i \\
\frac{p_{ti}}{1-z} &\simeq E_{i-1}\Theta_i
\end{aligned}
$$



- Apply color coherence in form of angular ordering

$$\mu > z_n q_n, q_n > z_{n-1} q_{n-1}, \ldots, q_1 > Q_0$$

- true angular ordering (in terms of rescaled momentum):

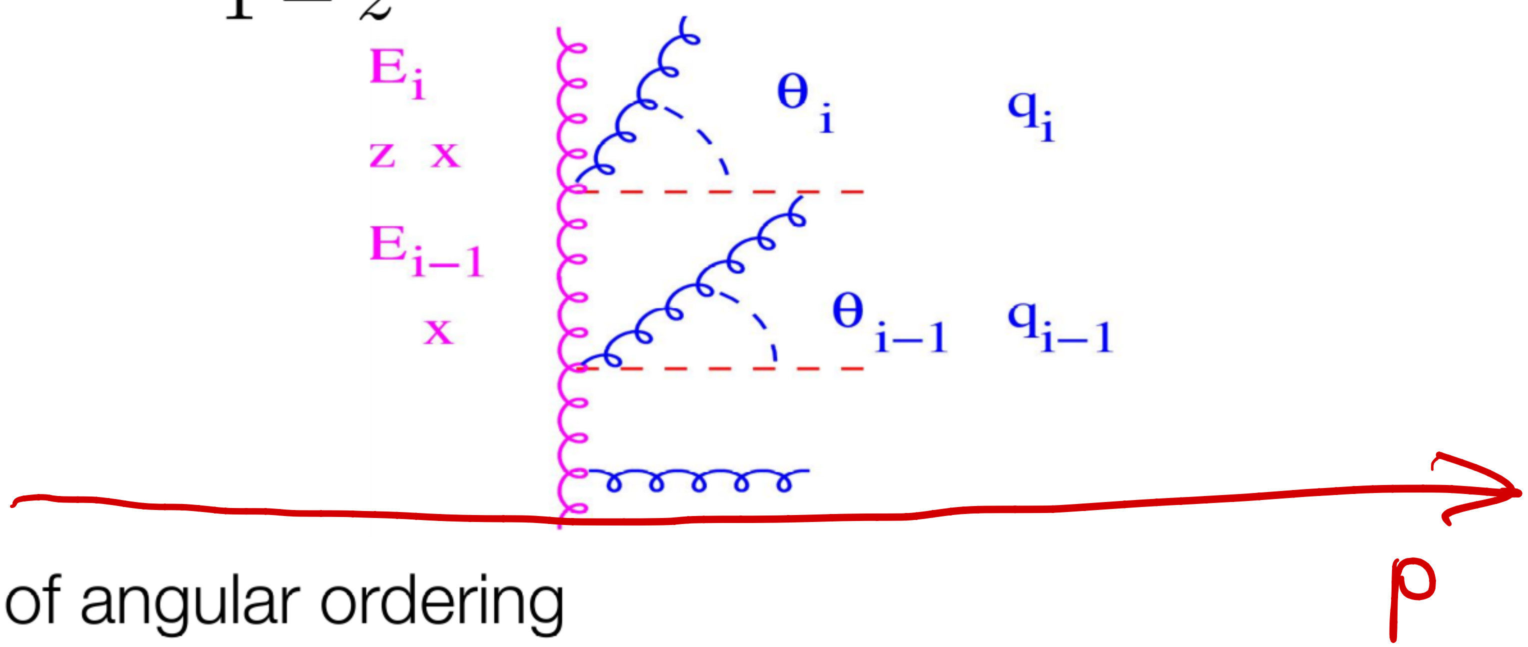
$$q_i > z_{i-1} q_{i-1}$$

Parton Branching evolution with ang. ord.

$$p_{ti} = |q_i^0| \sin\Theta_i$$
$$z = \frac{E_i}{E_{i-1}}$$

$$\text{with: } q_i = \frac{p_{ti}}{1-z_i}$$
$$\rightarrow \Theta_i = \frac{q_i}{E_{i-1}}$$
$$\Theta_{i+1} = \frac{q_{i+1}}{E_i}$$

$$E_{i-1} = E_i + q_i^0 = zE_{i-1} + q_i^0,$$
$$\rightarrow q_i^0 = (1-z)E_{i-1}$$
$$p_{ti} = q_i^0 \sin\Theta_i \simeq (1-z)E_{i-1}\Theta_i$$
$$\frac{p_{ti}}{1-z} \simeq E_{i-1}\Theta_i$$



- Apply color coherence in form of angular ordering

$$\mu > z_n q_n, q_n > z_{n-1} q_{n-1}, \ldots, q_1 > Q_0$$

- true angular ordering (in terms of rescaled momentum):

$q_i > z_{i-1} q_{i-1}$ but “HERWIG and PB ” use: $q_i > q_{i-1}$

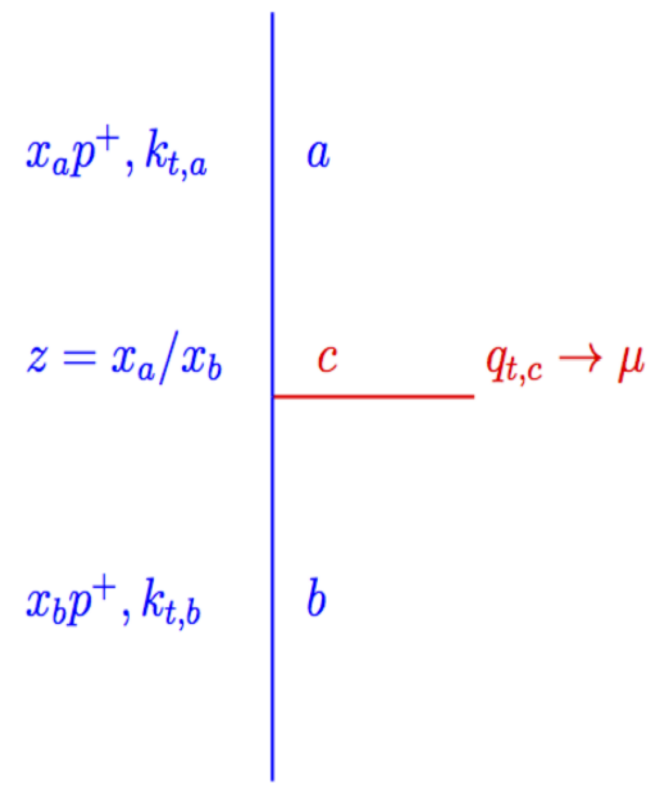
Transverse Momentum Dependence

- Parton Branching evolution generates every single branching:
  - kinematics can be calculated at every step
- Give physics interpretation of evolution scale:
  - in high energy limit: p_T -ordering:

$$\mu = q_T$$

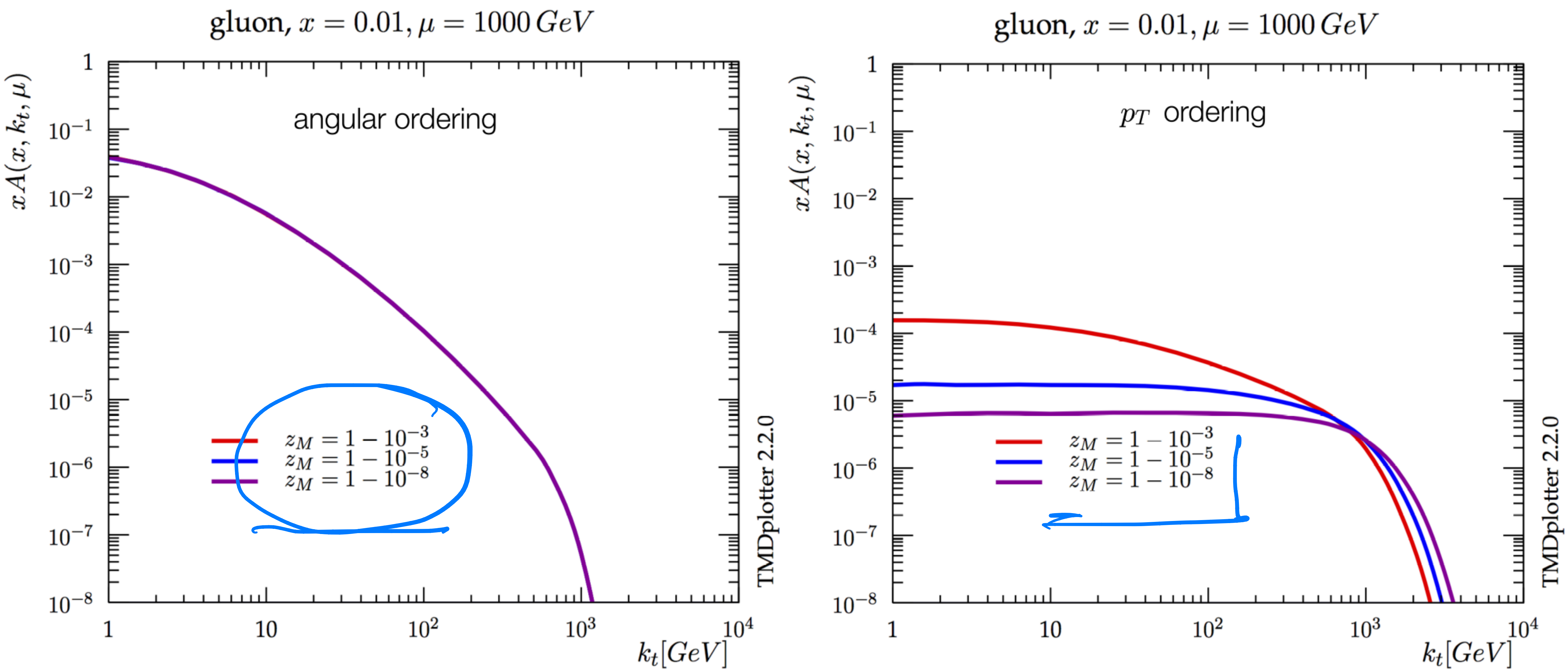
  - angular ordering:

$$\mu = q_T/(1-z)$$



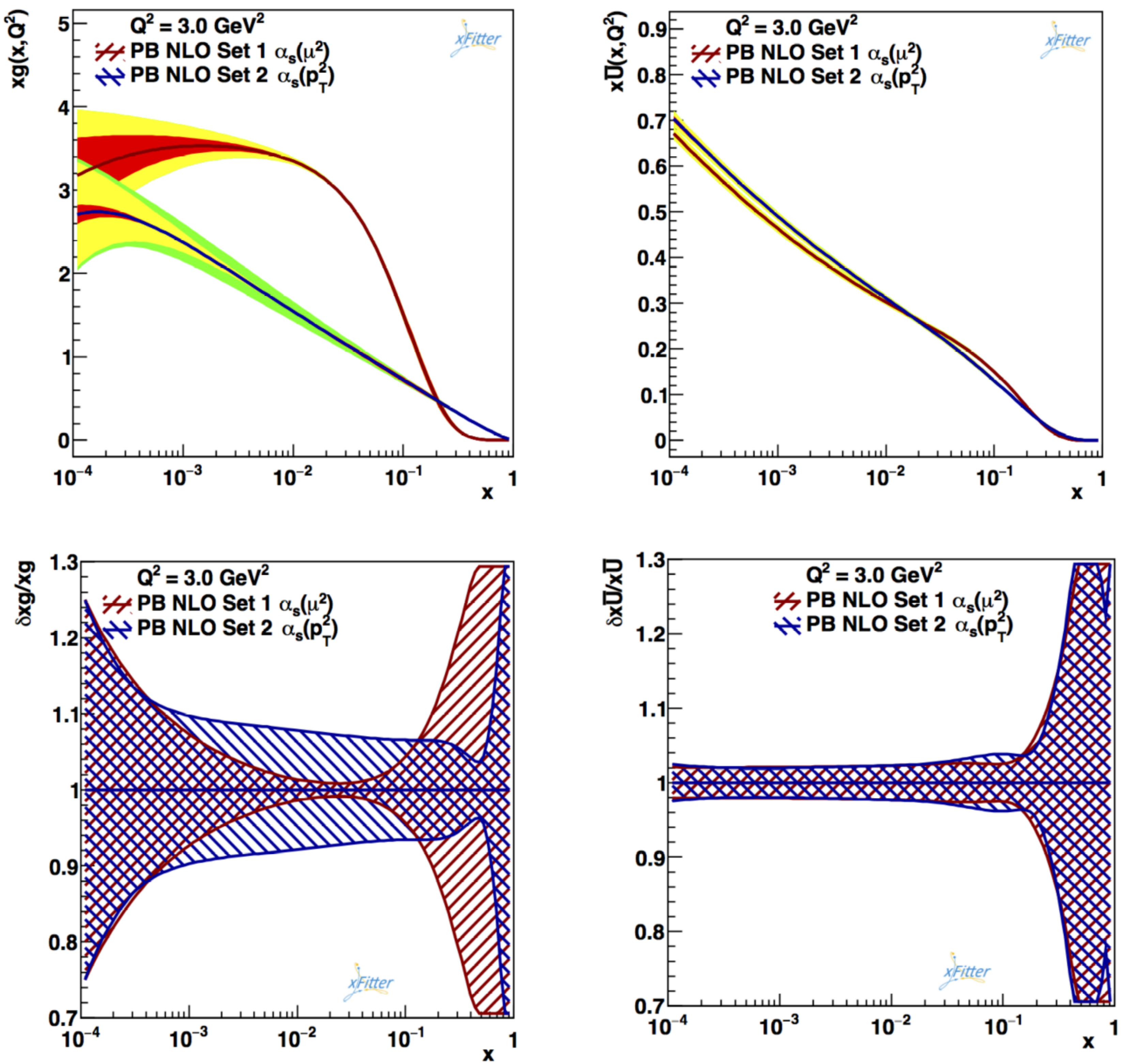
$$\Delta_s = \exp\left\{ - \int \frac{dt}{t} \int^{2\pi} dz \, P(z) \right\}$$

Transverse Momentum: dependence on z_M



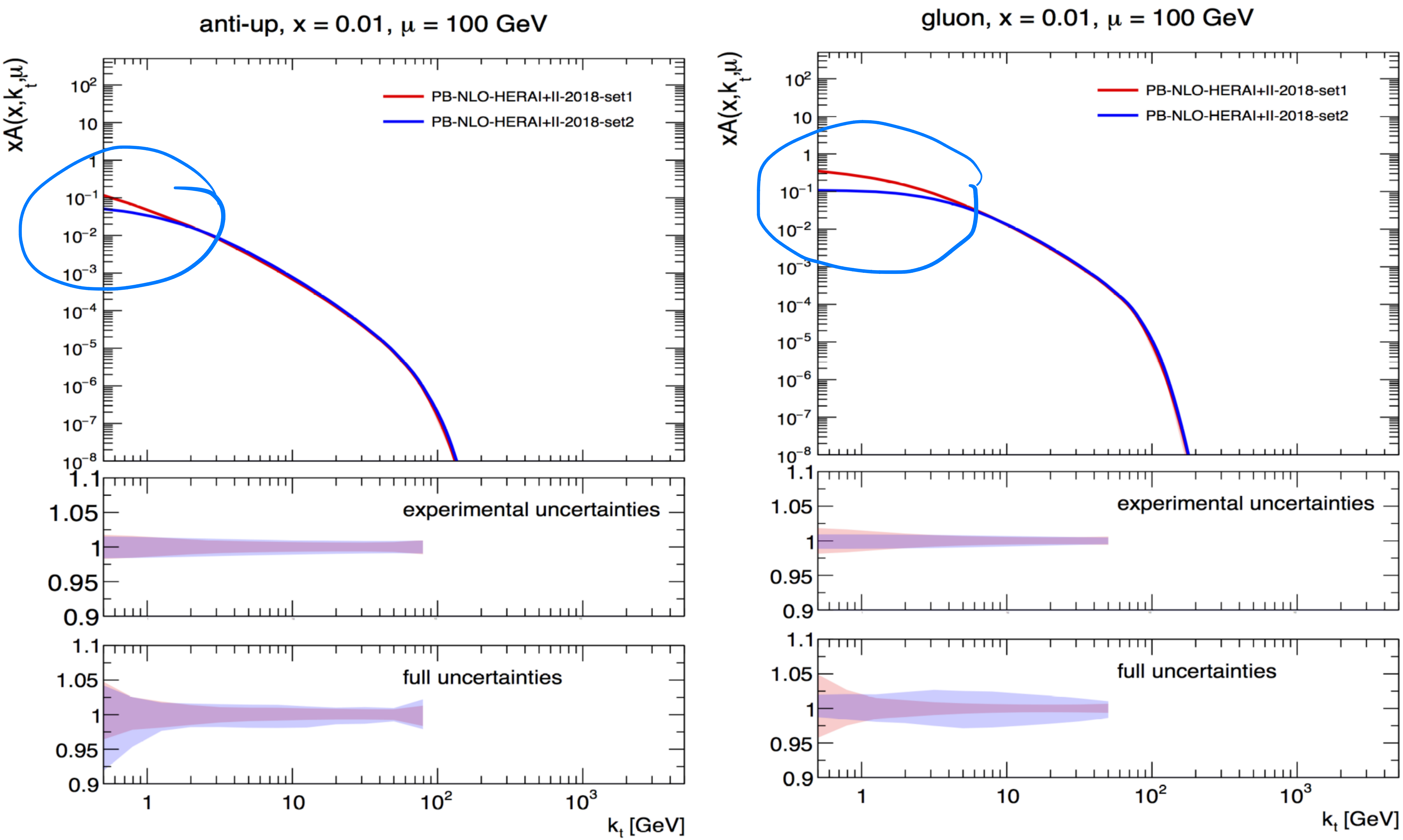
- p_T – ordering ($\mu = q_T$) shows significant dependence on z_M: unstable result because of soft gluon contribution
- angular ordering ($\mu = q_T/(1-z)$) is independent of z_M: stable results since soft gluons are suppressed (angular ordering)

Fit with changed $\alpha_s(p_T)$: at small Q^2



- fit 1 with $\alpha_s(q)$
  - as good as HERAPDF2.0 $\chi^2/ndf = 1.2$
- fit 2 with $\alpha_s(q(1-z))$
  - $\chi^2/ndf = 1.21$
- very different gluon distribution obtained at small Q^2

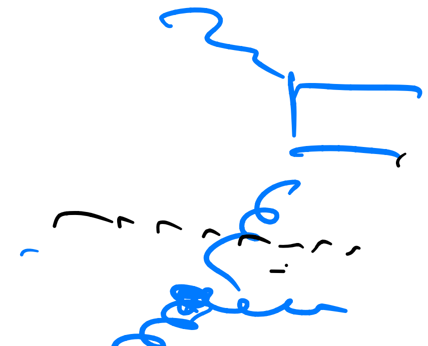
TMD distributions



- model dependence larger than experimental uncertainties

How to calculate x-sections then ?

k_t - factorization



- use high energy (kt -) factorization:
  (Catani, Ciafaloni, Hautmann NPB 366 (1991) 135,
  Gribov, Levin, Ryskin, Phys. Rep.100 ,(1983),1,
  Collins, Ellis, NPB 360 ,(1991) ,3)

- with $\sigma(\mathrm{ep} \to \mathrm{e}'\mathrm{q}\bar{\mathrm{q}}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \int d^2k_t \hat{\sigma}(\hat{s}, k_t,) x_g \mathcal{A}(x_g, k_t,)$

$$\int^{Q^2} d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$$

- t-channel gluon with virtuality $k^2 = -k_t^2$ dominates the process in the high energy limit $s \gg \hat{s}$
- collinear limit obtained by: $\hat{\sigma}(\hat{s}, 0, Q) \cdot \Theta(Q - k_T)$

- BUT k_t-factorization is proven only for small x ....

Why off-shell matrix elements ?

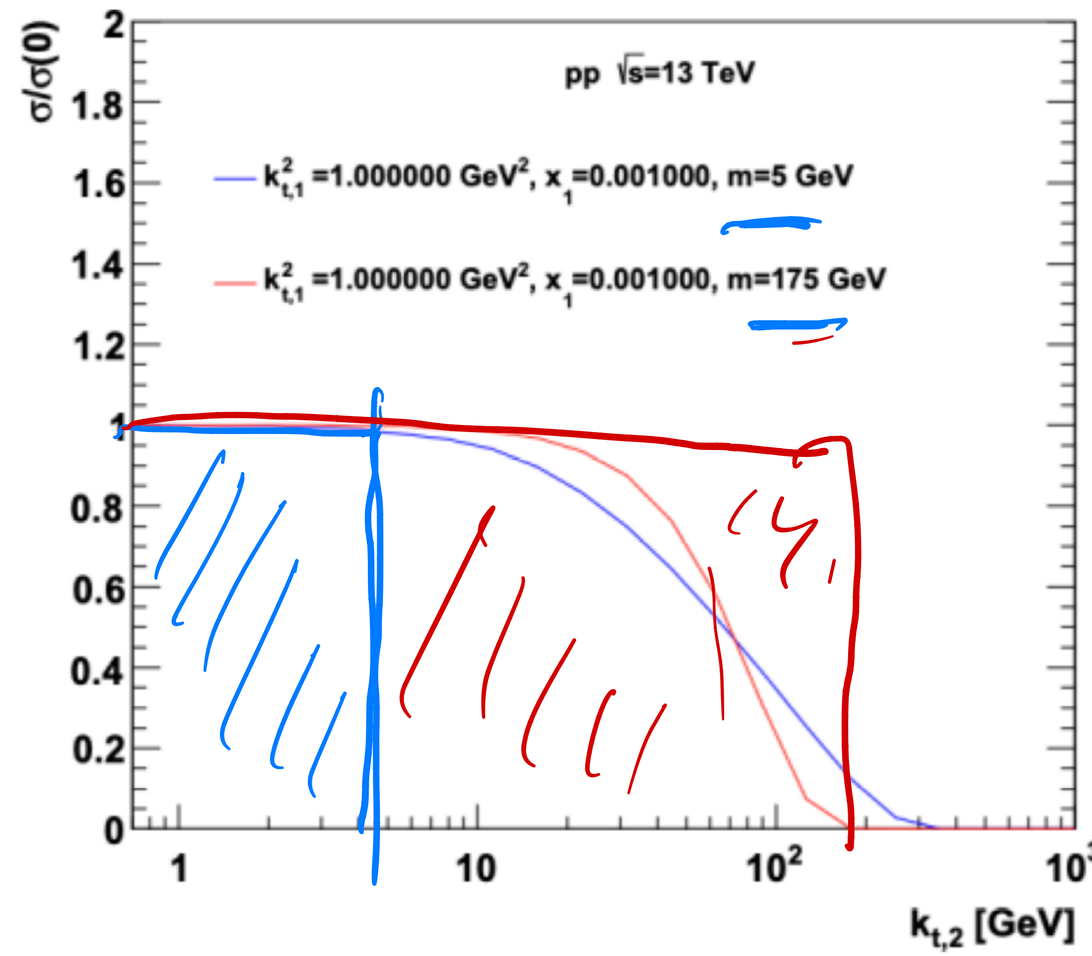
- Behavior of ME as function of k_t:

Baranov, S. and others CASCADE3 A Monte Carlo event generator based on TMDs, Eur. Phys. J. C, 81(2021), 425

  - for small k_t converges to collinear result
  - for large k_t has suppression
  - → suppression appears at "standard factorization scale": $Q^2 + 4\,m^2$
  - collinear factorization:

    $\mu^2 \sim Q^2 + 4\,m^2$

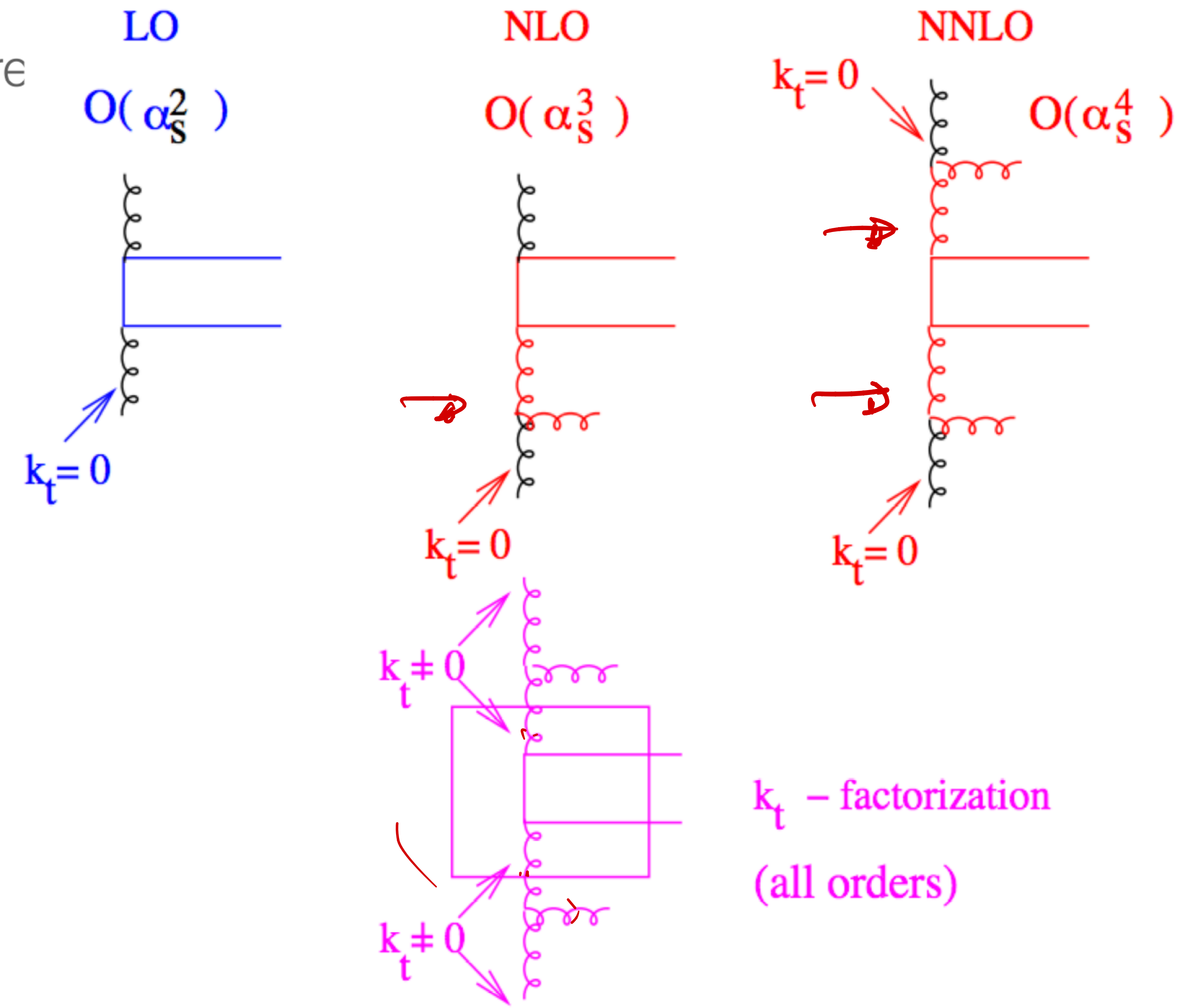
$$\int_0^{\mu^2} d\hat{\sigma}(, \ldots)$$



- k_t-fact proven smaller
- apply also medium x, without prove!

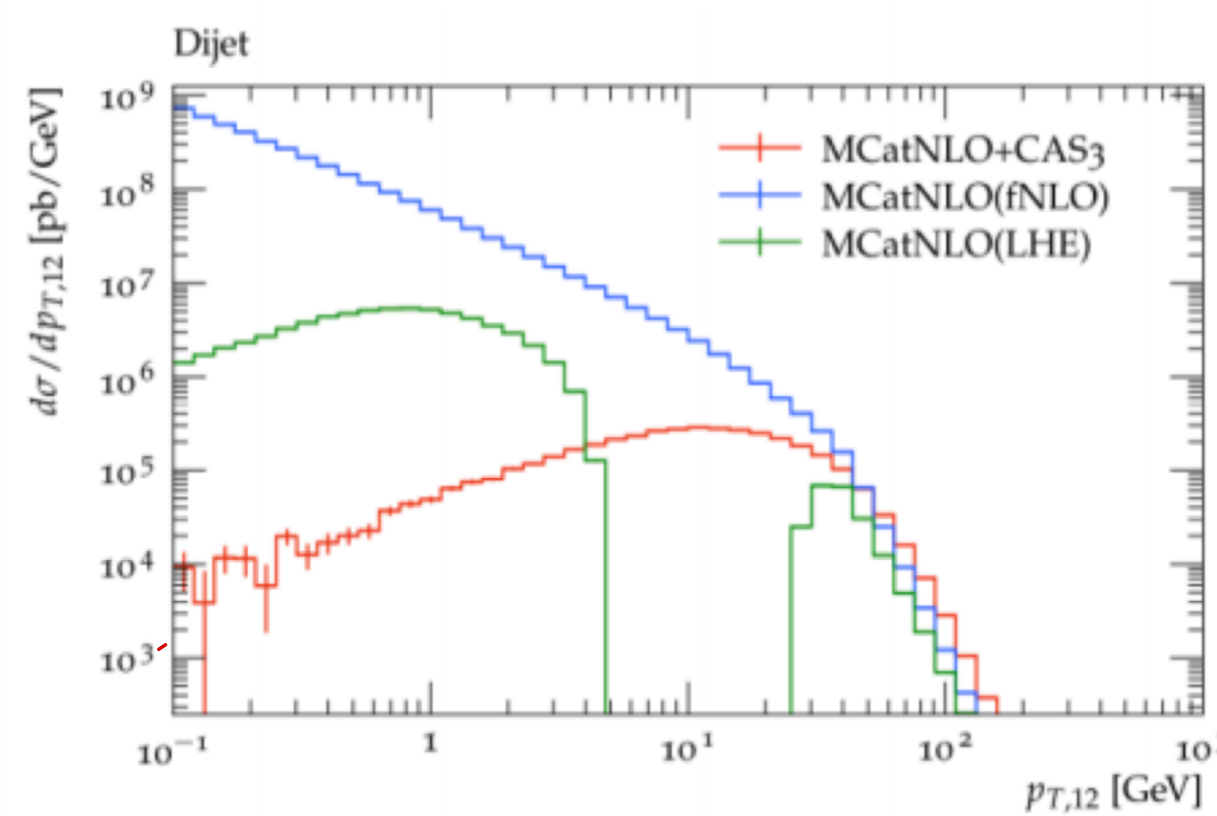
kt-factorization and coll. NLO calcs

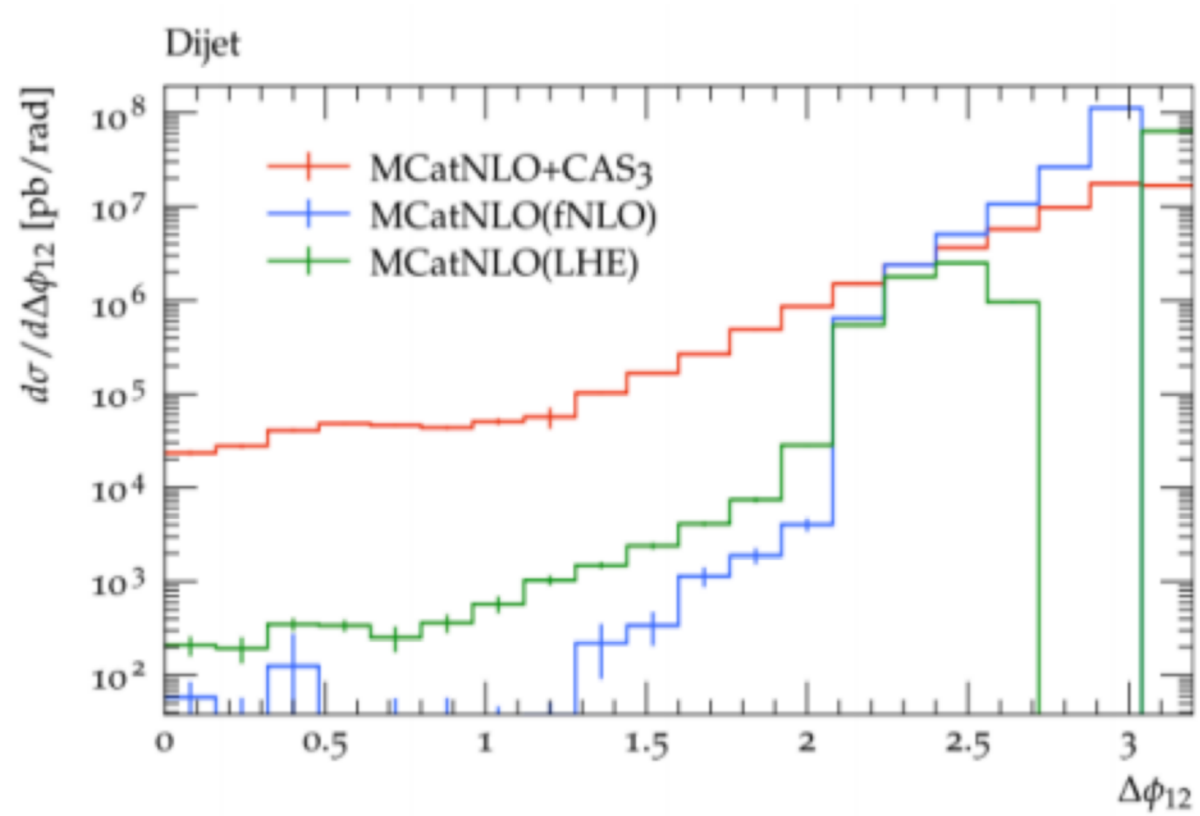
- fit of uPDF to inclusive structure functions /x-sections used to determine normalization
  - ➔ includes “all-orders” !!!!
- off-shell matrix element simulates part of real NLO corrections

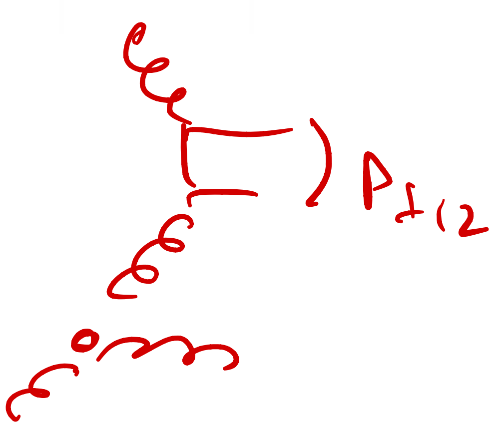


Calculations using NLO matrix elements

- Applying a NLO calculation, where the divergent parts are subtracted, and then added by TMDs and Parton shower: **MCatNLO** method (*Frixione, S. and Webber, B. R. Matching NLO QCD computations and parton shower simulations, JHEP, 06(2002), 029*)

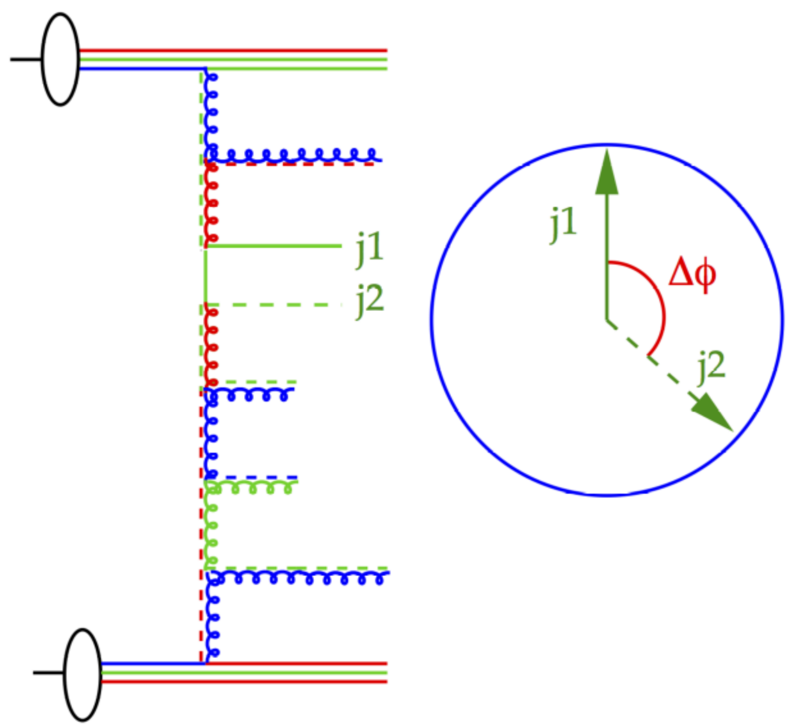


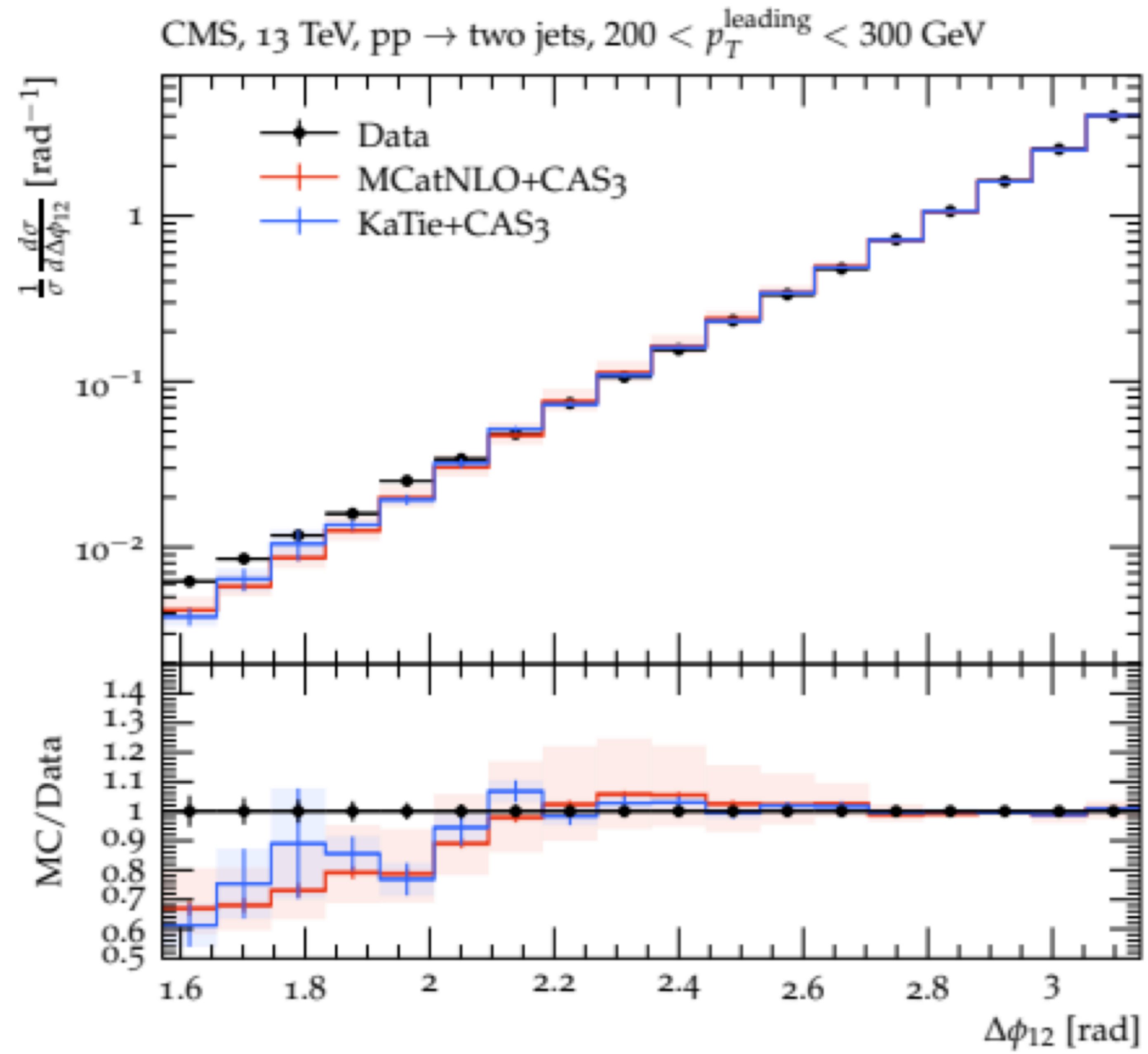




Application to high p_T dijets in pp

- Dijet production at in pp, a test for TMDs and PS :





- NLO+TMD calculations very similar to calculations using off-shell ME (KaTie) at LO