QCD and Monte Carlo techniques – Exercise 3

Divergencies again...

- collinear divergencies factored into renormalized parton distributions
- what about soft divergencies ?

  treated with "plus" prescription

with

$$\frac{1}{1-z} \to \frac{1}{1-z}_+ \qquad \int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}$$

- soft divergency treated with Sudakov form factor:

$$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z)\right]$$

Exercise 2

6. Calculate the Sudakov form factor for the scales $t_2 = 10, 100, 500$ GeV^2 as a function of t_1 and plot it as a function of t_1. Use q as the argument for α_s, and check the differences. For the z integral use $z_{min} = 0.01$ and $z_{max} = 0.99$.

$$\log \Delta_S = -\int_{t_1}^{t_2} \frac{dt}{t} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z)$$

Use the gluon and also the quark splitting functions :

$$P_{gg} = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right)$$

and

$$P_{qq} = \frac{4}{3}\frac{1+z^2}{1-z}$$

Exercise 2

7. write a program to evolve a parton density $g(x) = 3(1-x)^5/x$ from a starting scale $t_0 = 1$ GeV^2 to and higher scale $t = 100$ GeV^2. Do the evolution only with fixed $\alpha_s = 0.1$ and an approximate gluon splitting function $P_{gg} = 6(\frac{1}{z} + \frac{1}{1-z})$. To avoid the divergent regions use $z_{min} = \epsilon$ and $z_{max} = 1 - \epsilon$ with $\epsilon = 0.1$. Calculate the Sudakov form factor for evolving from t_1 to t_2 using only the $\frac{1}{(1-z)}$ part of the splitting function. Generate z according to P_{gg}. Repeat the branching until you reach the scale t. Plot the $xg(x)$ as a function of x for the starting distribution and for the evolved distribution. Repeat the same exercise but with $P_{qq} = \frac{4}{3}\frac{1+z^2}{1-z}$.

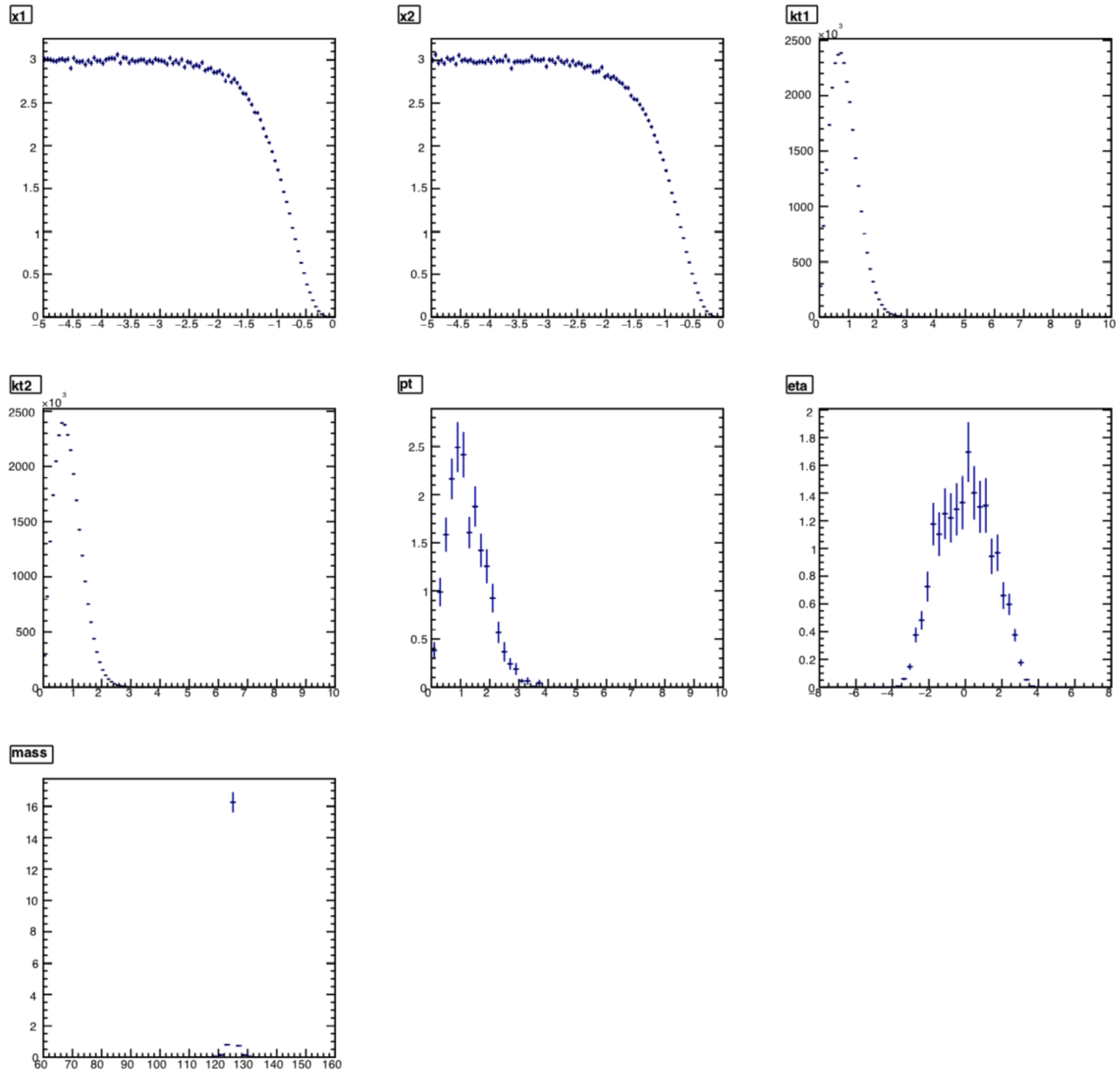
   Calculate and plot the transverse momentum of the parton after the evolution. At the starting scale the partons can have a intrinsic k_t, which is generated by a gauss distribution with $\mu = 0$ and $\sigma = 0.7$ (use generating a gauss distribution from Exercise 1).

   Compare the k_t distribution using P_{gg} and P_{qq}. What is different ?

Exercise 3 – Example 8

8. Calculate $\sigma(p+p \to h)$ (Higgs production via gluon fusion) in lowest order. Take $\sqrt{s} = 7000$ GeV. Calculate the total cross section, and plot x_1, x_2 and y_h. Require $120 < m_h < 130$ GeV. Plot the transverse momenta of the incoming partons. Use for simplicity parton density of the form $xg(x) = 3(1-x)^5$.
   The Higgs cross section is:
   $$\sigma(g+g \to h) = \alpha_s^2 \frac{\sqrt{2}}{\pi} \frac{G_F}{576}$$
   with $G_F = 1.166 \cdot 10^{-5}$ GeV^{-2} and $\alpha_s = 0.1$.
   Use a Breit-Wigner form for the Higgs:
   $$P(m) = \frac{1}{2\pi} \frac{\Gamma_h}{(m-m_h)^2 + \Gamma_h^2/4}$$
   with $m_h = 125$ GeV and $\Gamma_h = 0.4$ GeV. Calculate the cross section.
   Include in the calculation a small intrinsic transverse momentum from both of the incoming partons. Assume $h(k_t) = \exp(-bk_t^2)$. Using $b = 1$ corresponds to a gauss distribution with $\mu = 0$ and $\sigma \sim 0.7$. Plot the transverse momentum k_t and the transverse momentum squared k_t^2 of both incoming partons and the resulting h.
   Write the code in a modular way, such that it can be used for the last exercise.

Example 8 - result



Exercise 3 – Example 9

9. Use the evolved pdf (from previous exercise) to calculate higgs production from above. Set the scale $t = 10000$ GeV^2. Use for simplicity the a gluon density $xg(x) = 3(1-x)^5$ as a starting distribution and use P_{gg}. Calculate the transverse momentum of the incoming partons and calculate the transverse momentum of the Higgs. Plot the x-values of the incoming partons and the transverse momenta.

Example 9 - result