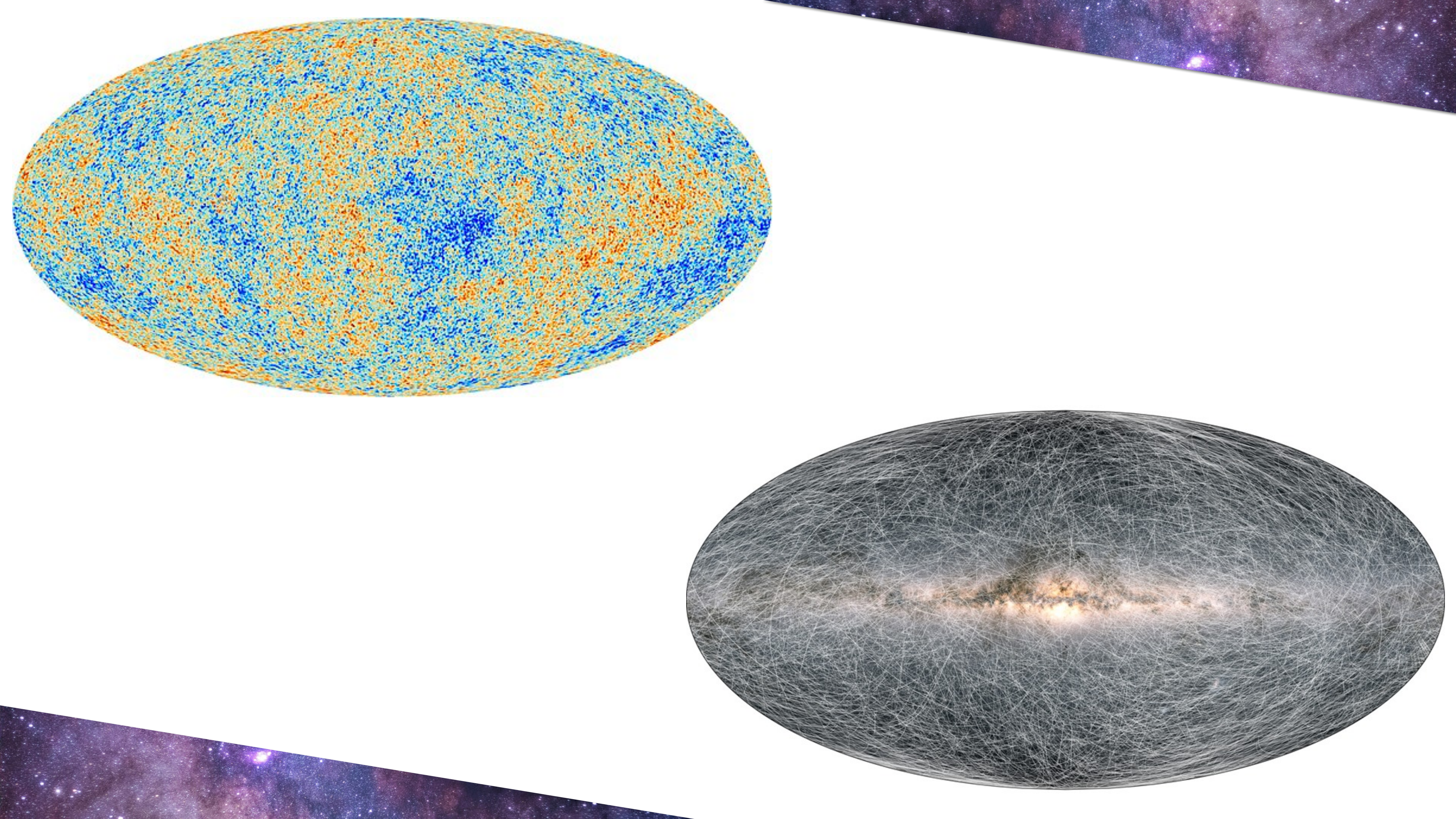


A NEW CHAPTER IN THE QUEST FOR DARK MATTER: THE ULTRALIGHT REGIME



Raffaele Tito D'Agnolo - IPhT Saclay

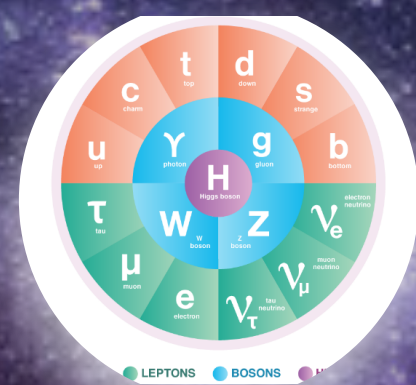


DARK MATTER MASS

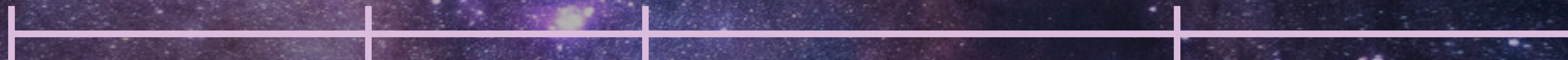
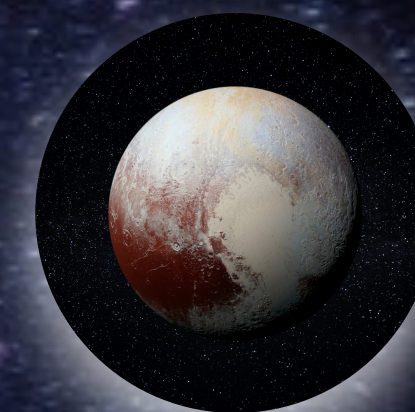


one order of magnitude

DARK MATTER MASS



Person



Neutrino

Higgs



Pluto

Self-coupling

100 O.M.

Mass

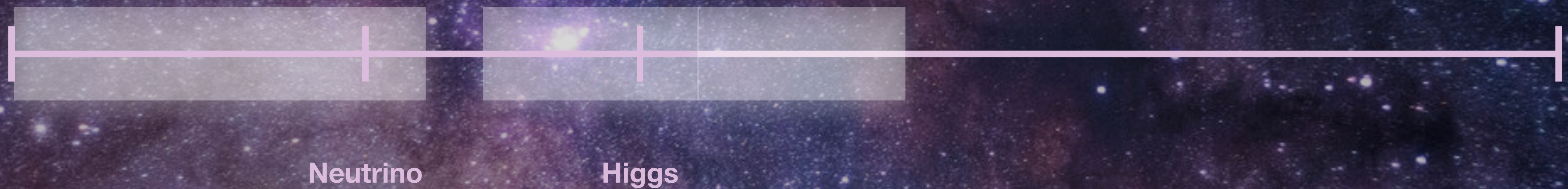
50 O.M.

80 O.M.

Couplings to ordinary matter



DARK MATTER MASS



Theory Spotlight

Natural Units

$$\hbar = c = 1$$

$$[m] = \text{eV}$$

$$\text{eV} \simeq 10^{-36} \text{kg} \simeq 10^{-66} M_{\odot}$$



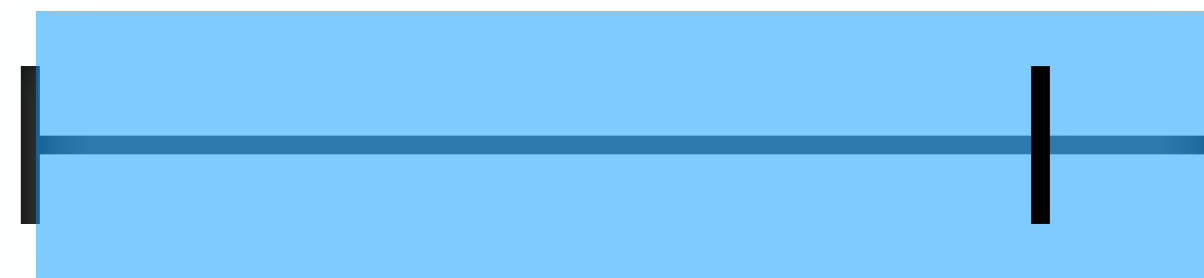
ULTRALIGHT DARK MATTER



Neutrino



**Bosonic
Dark Matter**



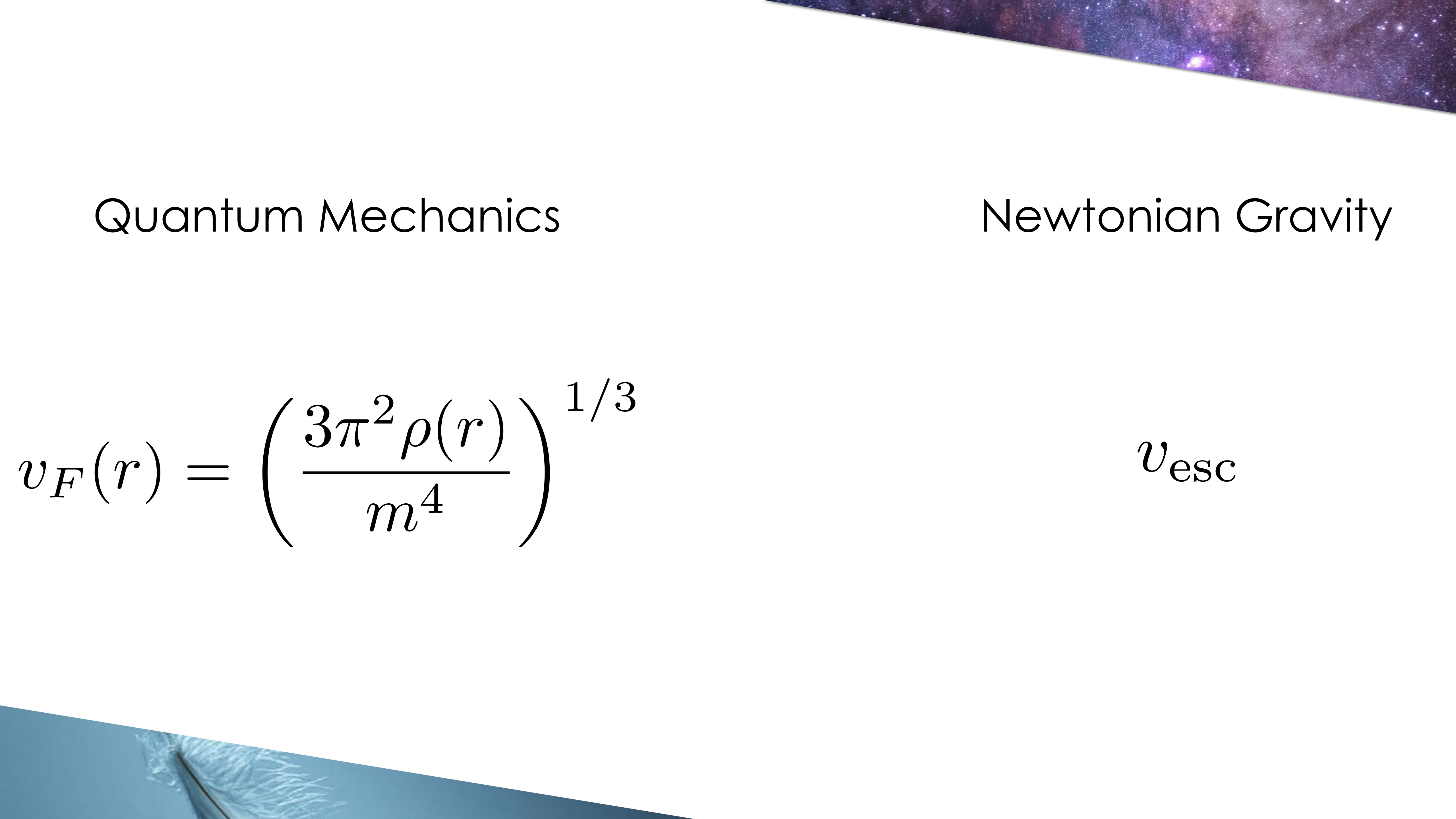
Neutrino

**Bosonic
Dark Matter**



Quantum Mechanics

$$v_F(r) = \left(\frac{3\pi^2 \rho(r)}{m^4} \right)^{1/3}$$

A decorative background image featuring a cosmic scene with a purple and blue nebula at the top right and a blue, textured surface at the bottom left.

Quantum Mechanics

$$v_F(r) = \left(\frac{3\pi^2 \rho(r)}{m^4} \right)^{1/3}$$

Newtonian Gravity

$$v_{\text{esc}}$$



Quantum Mechanics

Newtonian Gravity

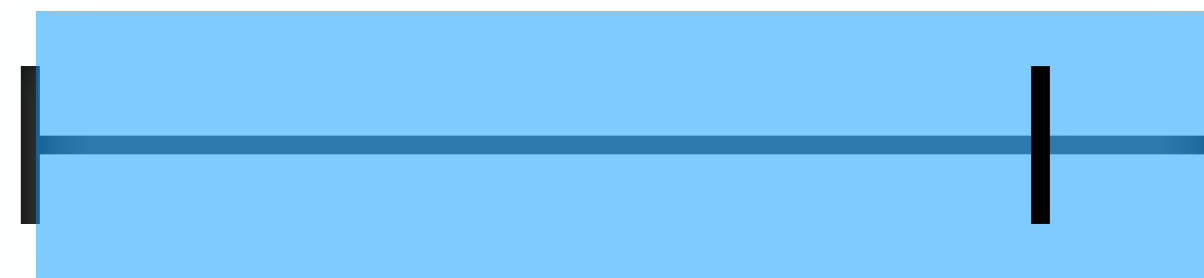
$$v_F(r) = \left(\frac{3\pi^2 \rho(r)}{m^4} \right)^{1/3} <$$

v_{esc}





**Bosonic
Dark Matter**



Neutrino

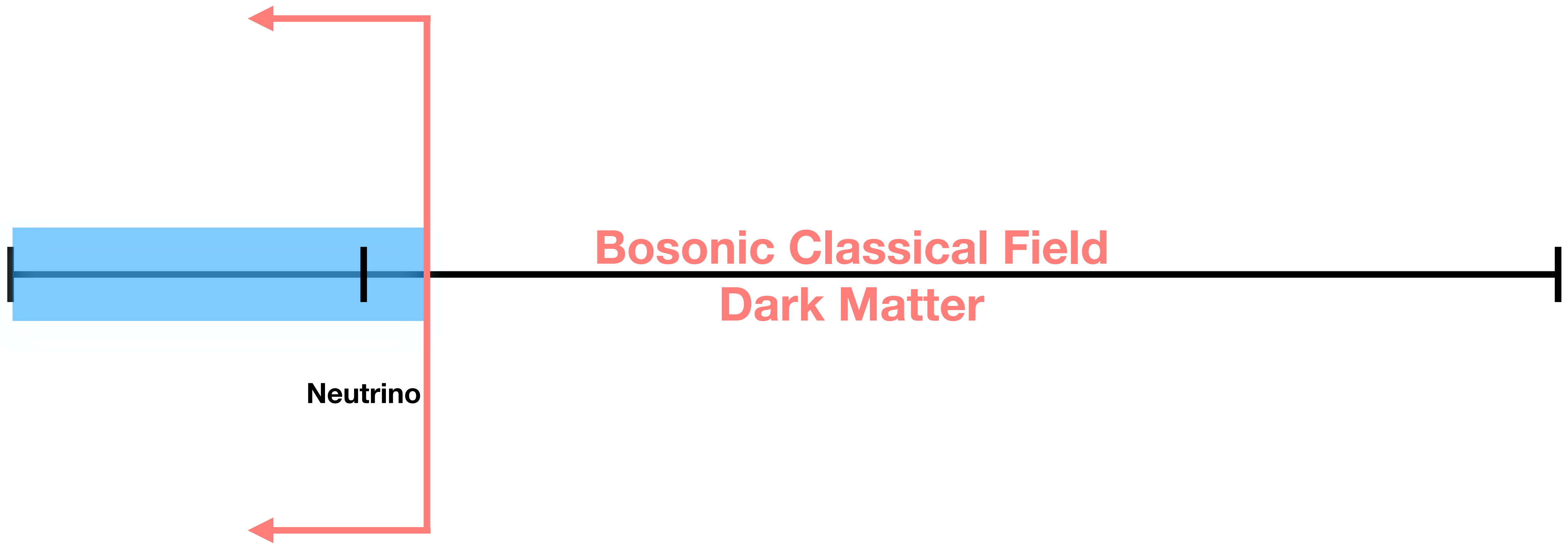
**Bosonic
Dark Matter**



Dark Matter Particles in a de Broglie Volume **Today**

Galaxy: $N_{\text{DM}} \simeq 10^3 \left(\frac{\text{eV}}{m_{\text{DM}}} \right)$

Universe: $N_{\text{DM}} \simeq 10^{-3} \left(\frac{\text{eV}}{m_{\text{DM}}} \right)$





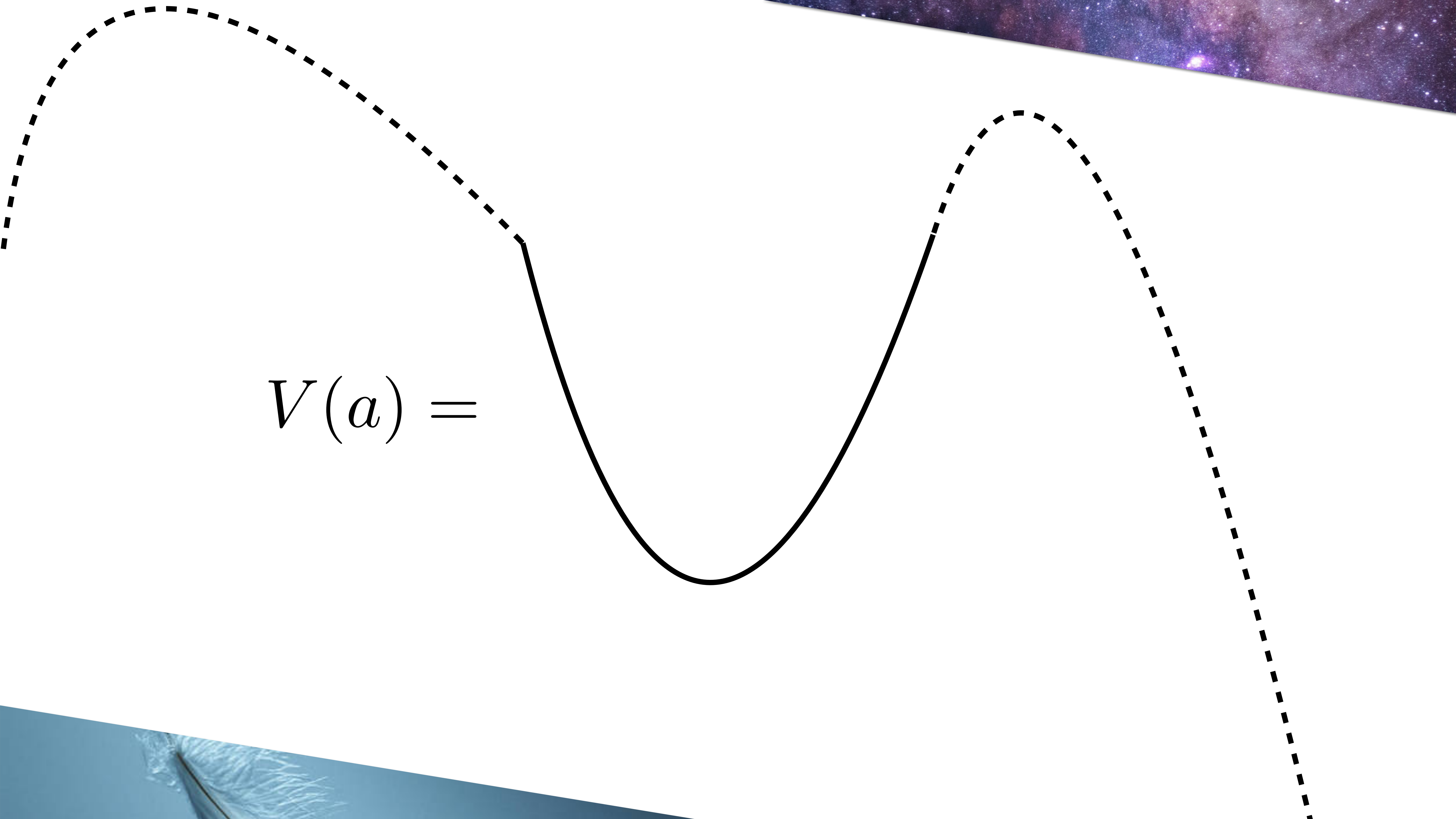
ULTRALIGHT DARK MATTER

- Minimal Extension of what we know to exist
 - Simple and predictive cosmology
- 

A decorative background image featuring a cosmic scene with a purple and blue nebula in the upper right corner and a blue, textured surface resembling water or a planet's horizon in the lower left corner.

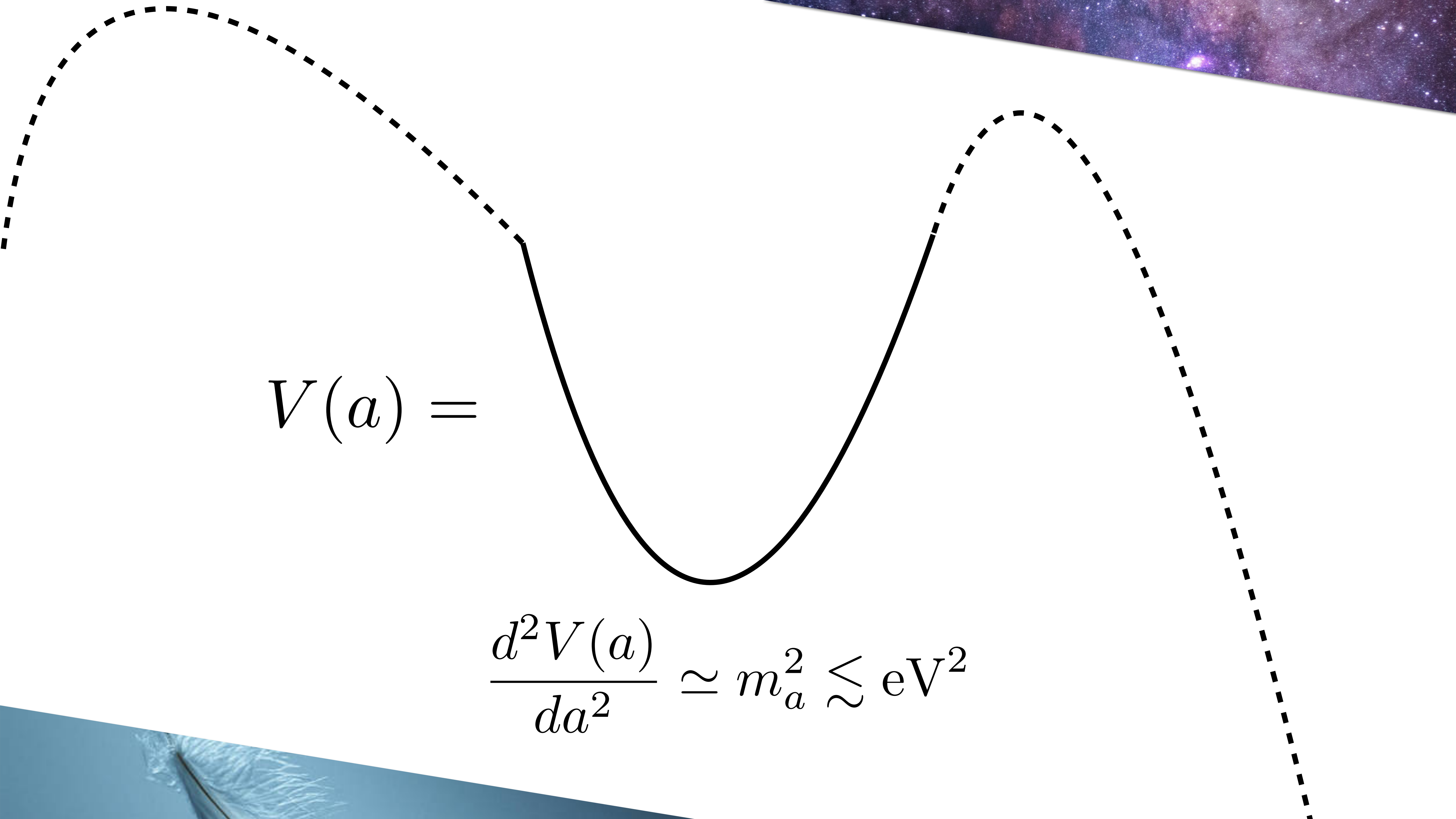
ULTRALIGHT DARK MATTER

- Minimal Extension of what we know to exist
- Simple and predictive cosmology
- Solve another big problem in particle physics



The image features a graph of a potential function $V(a)$. The central part of the graph is a solid black curve that starts at a local maximum on the left, descends to a global minimum in the center, and then ascends to a local maximum on the right. From the first local maximum, a dashed line extends upwards and to the left. From the second local maximum, a dashed line extends upwards and to the right, eventually curving downwards. The background is split: the top right shows a purple and blue galaxy, while the bottom left shows a blue quill pen.

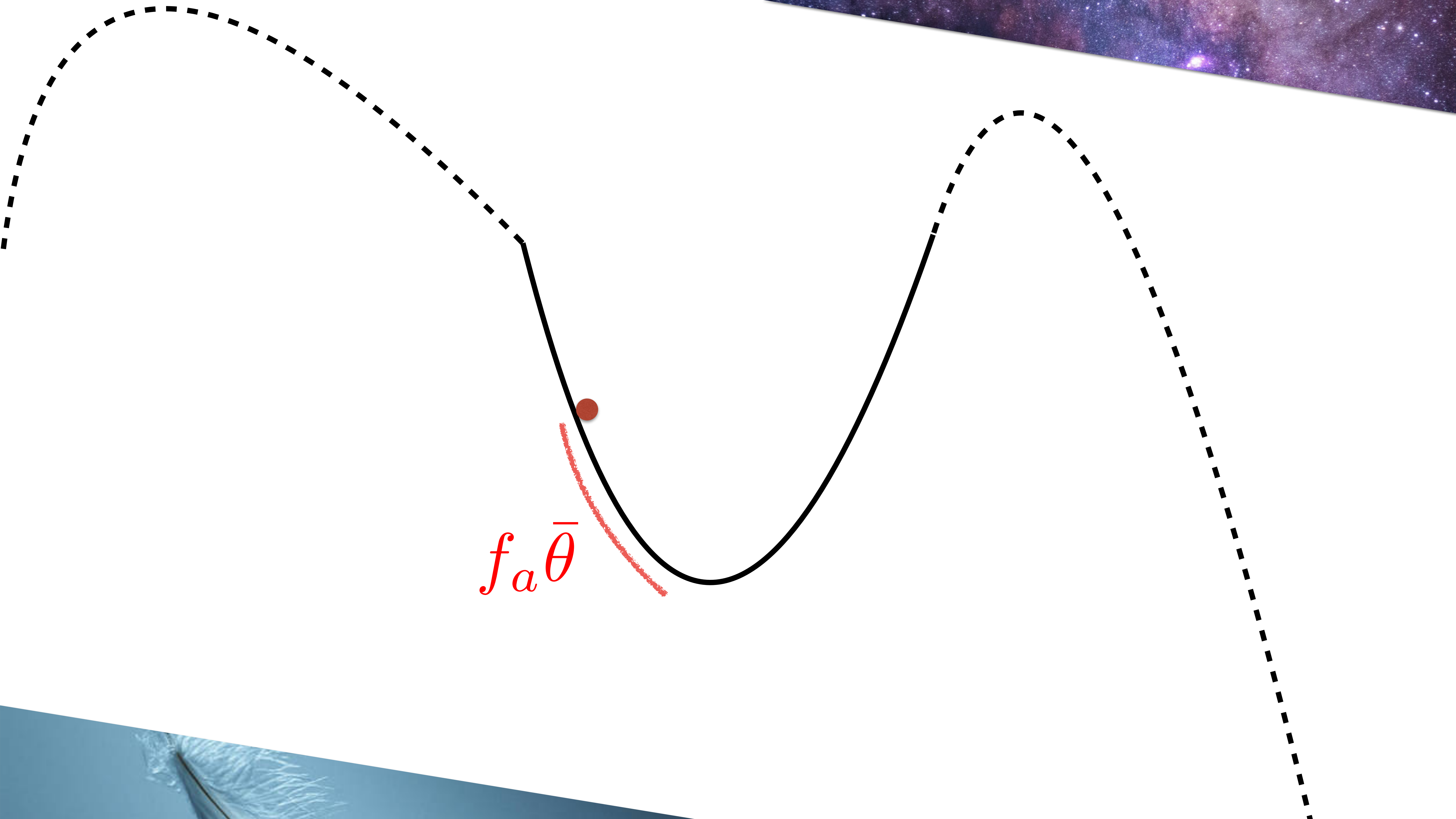
$$V(a) =$$

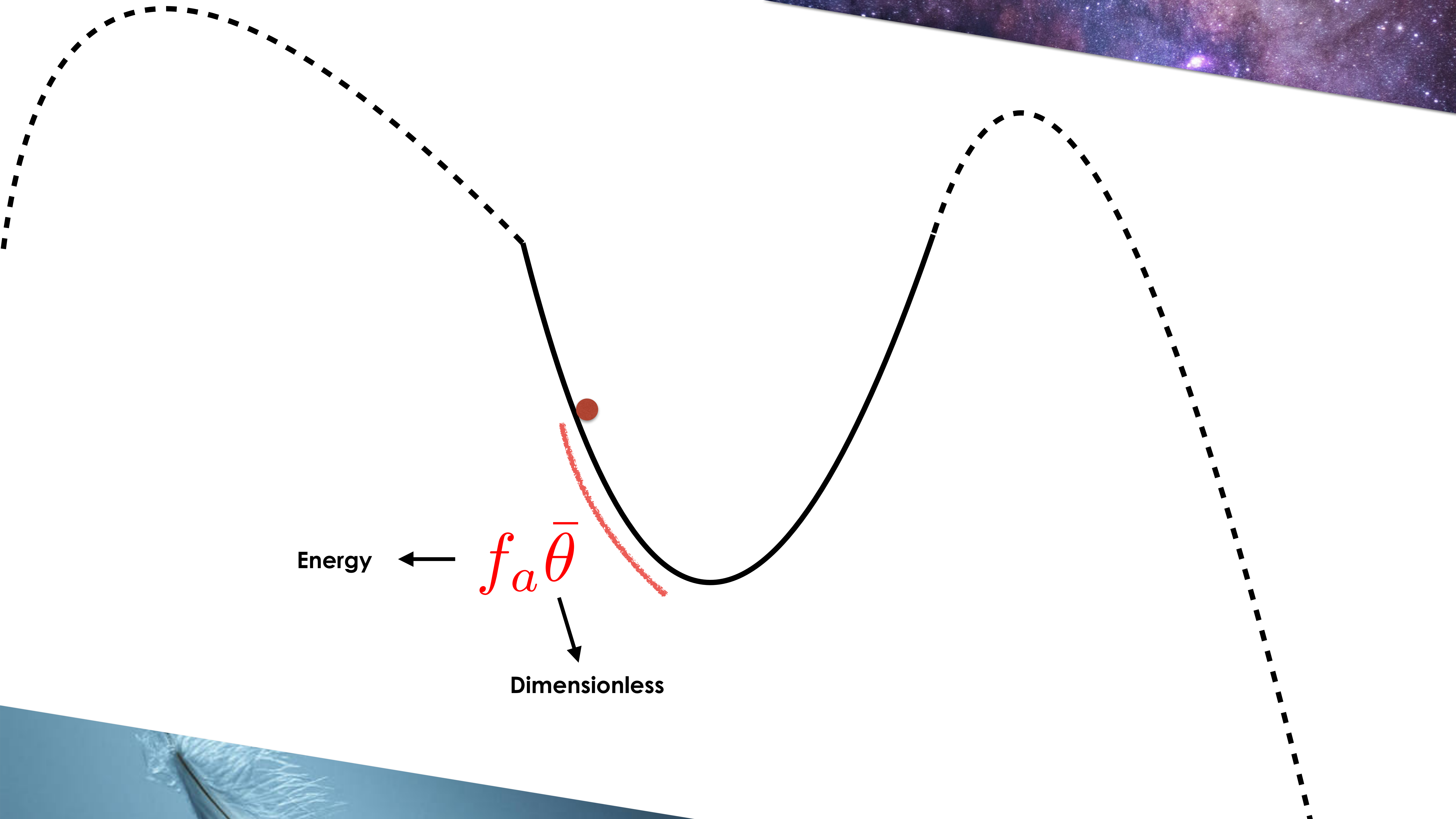


The image shows a potential energy curve $V(a)$ plotted against a . The curve consists of a solid black parabolic segment with a minimum, and two dashed black segments that extend from the ends of the solid segment. The background features a purple and blue nebula at the top and a blue textured surface at the bottom.

$$V(a) =$$

$$\frac{d^2 V(a)}{da^2} \simeq m_a^2 \lesssim \text{eV}^2$$

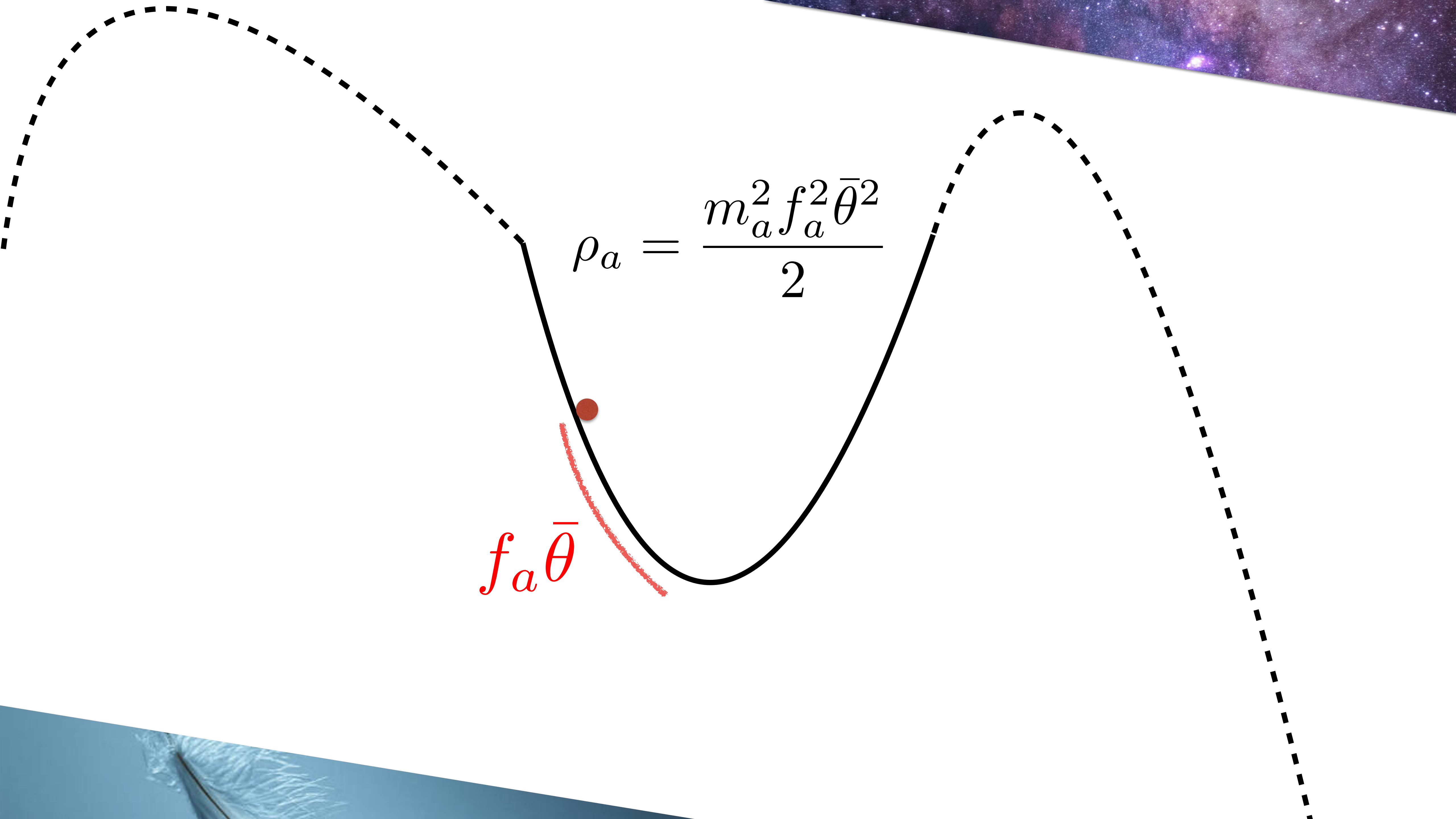




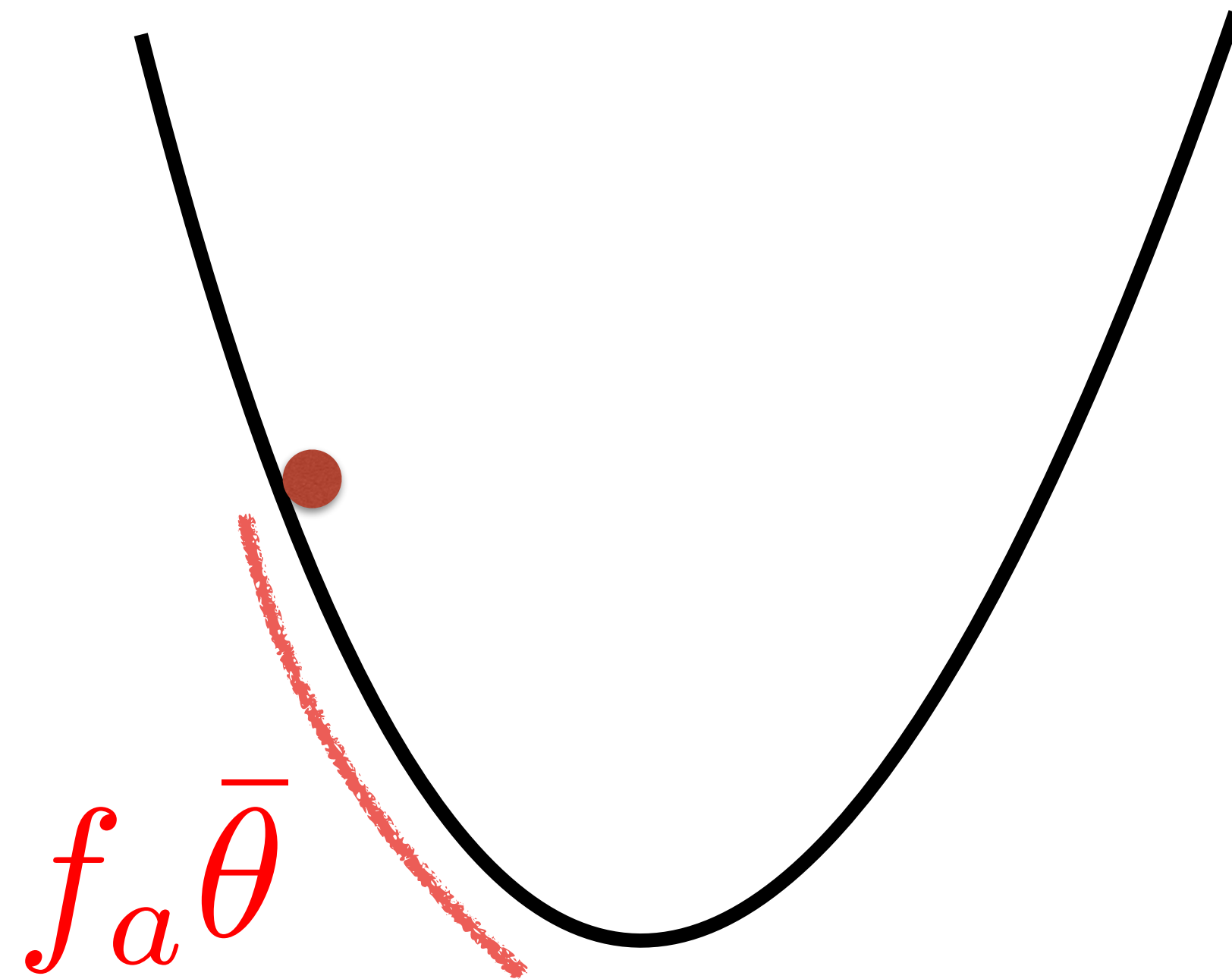
Energy

$$f_a \bar{\theta}$$

Dimensionless

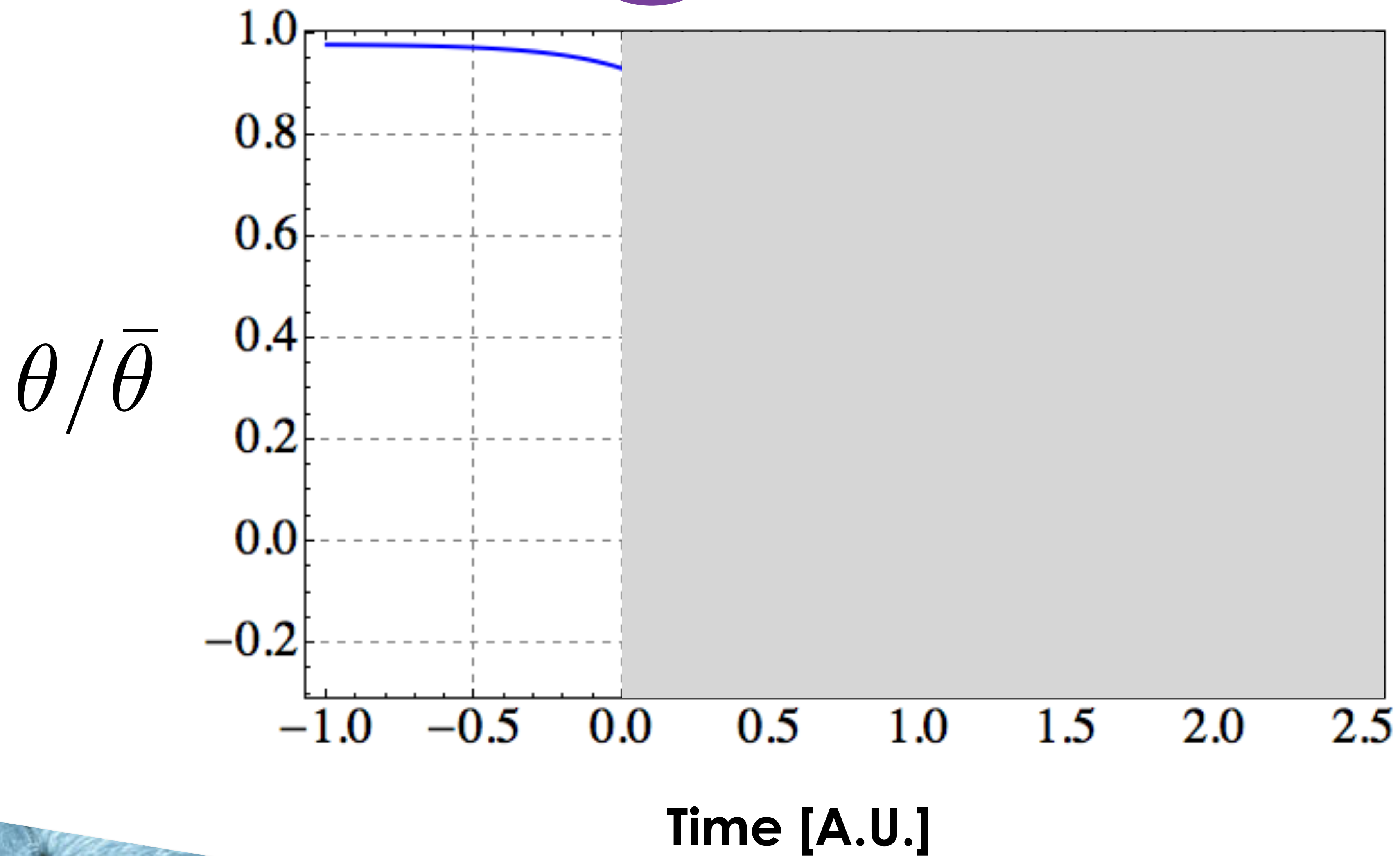


Classical Equation of Motion

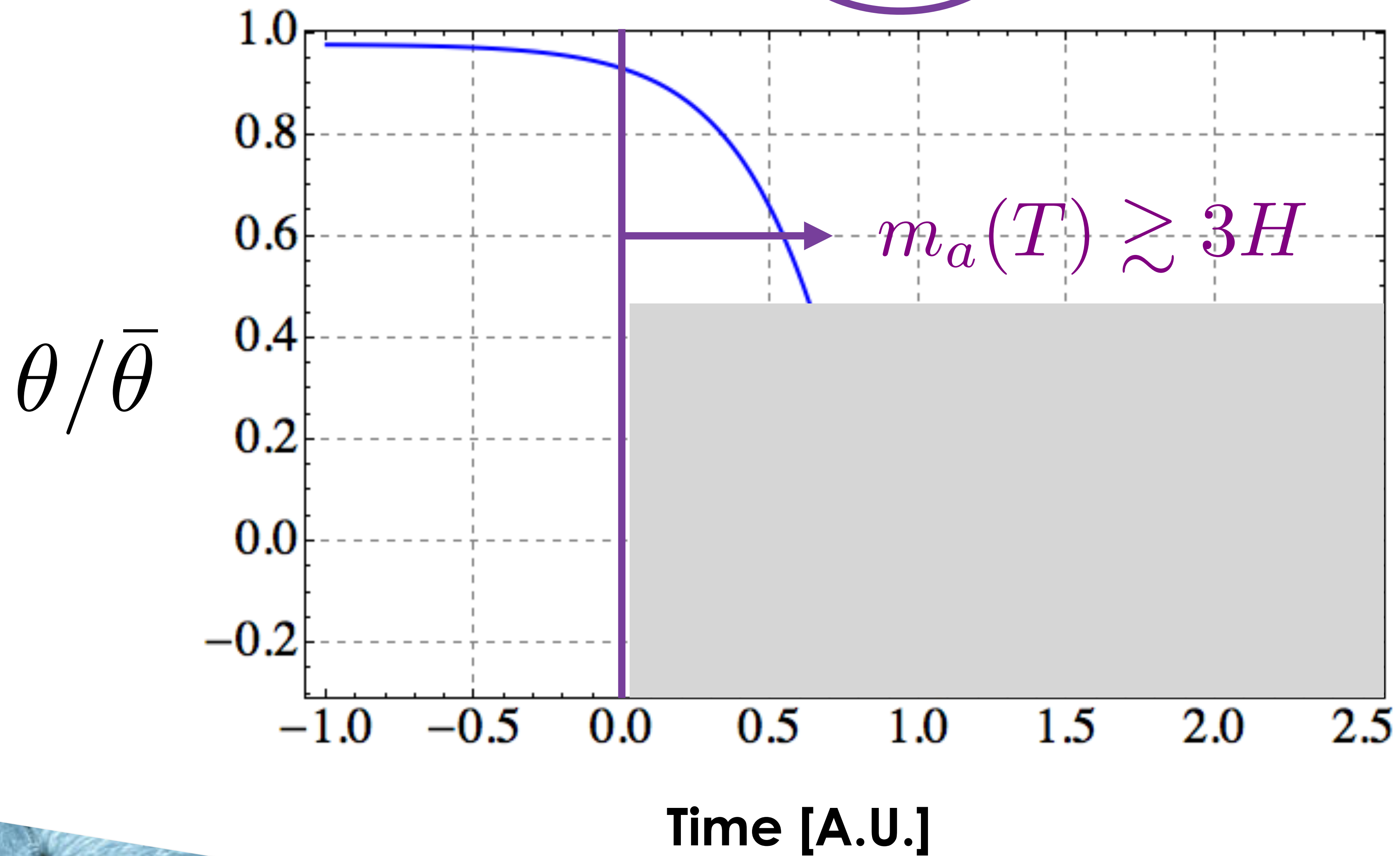


$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T)\theta = 0$$

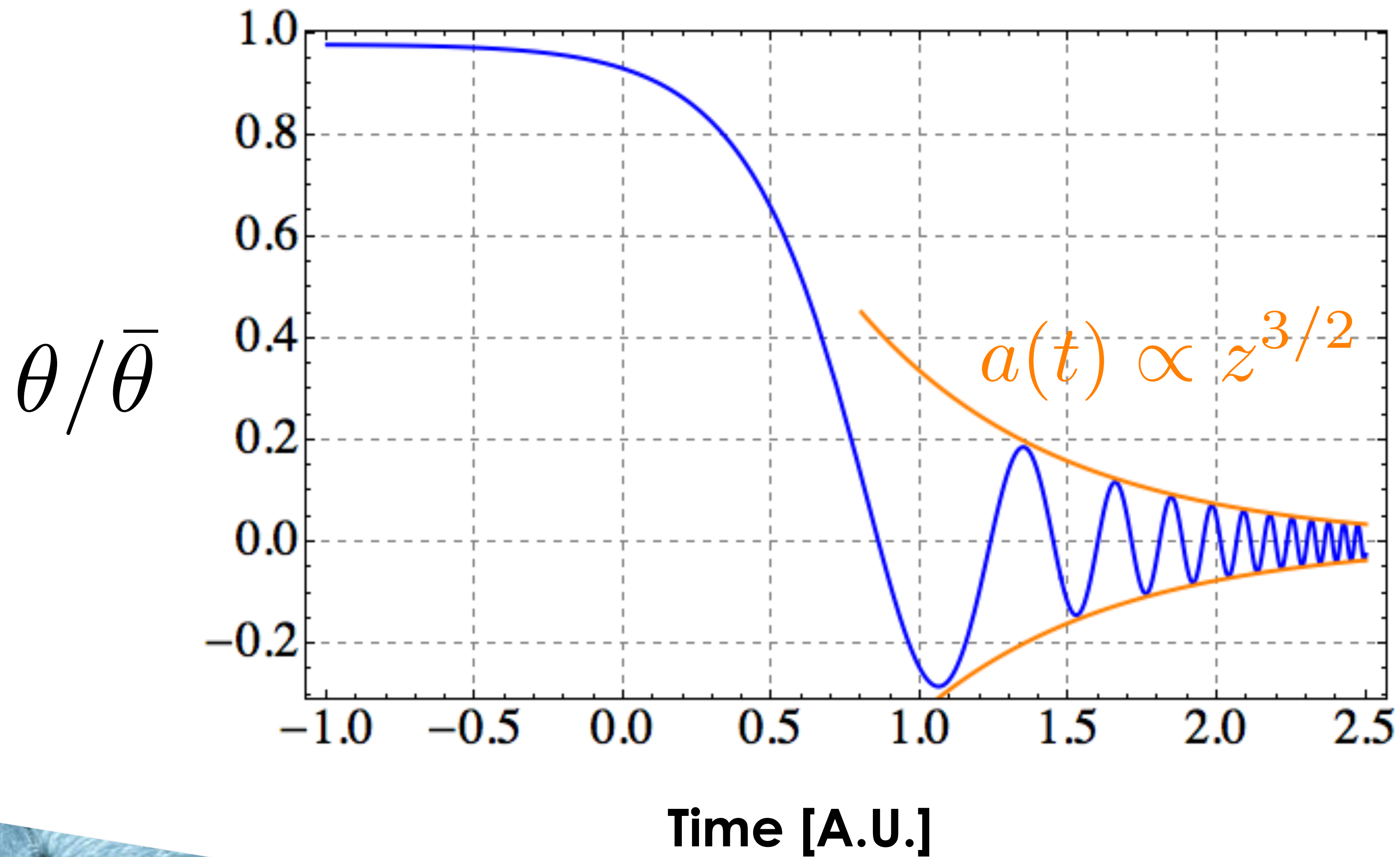
$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T)\theta = 0$$



$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T)\theta = 0$$



$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T)\theta = 0$$



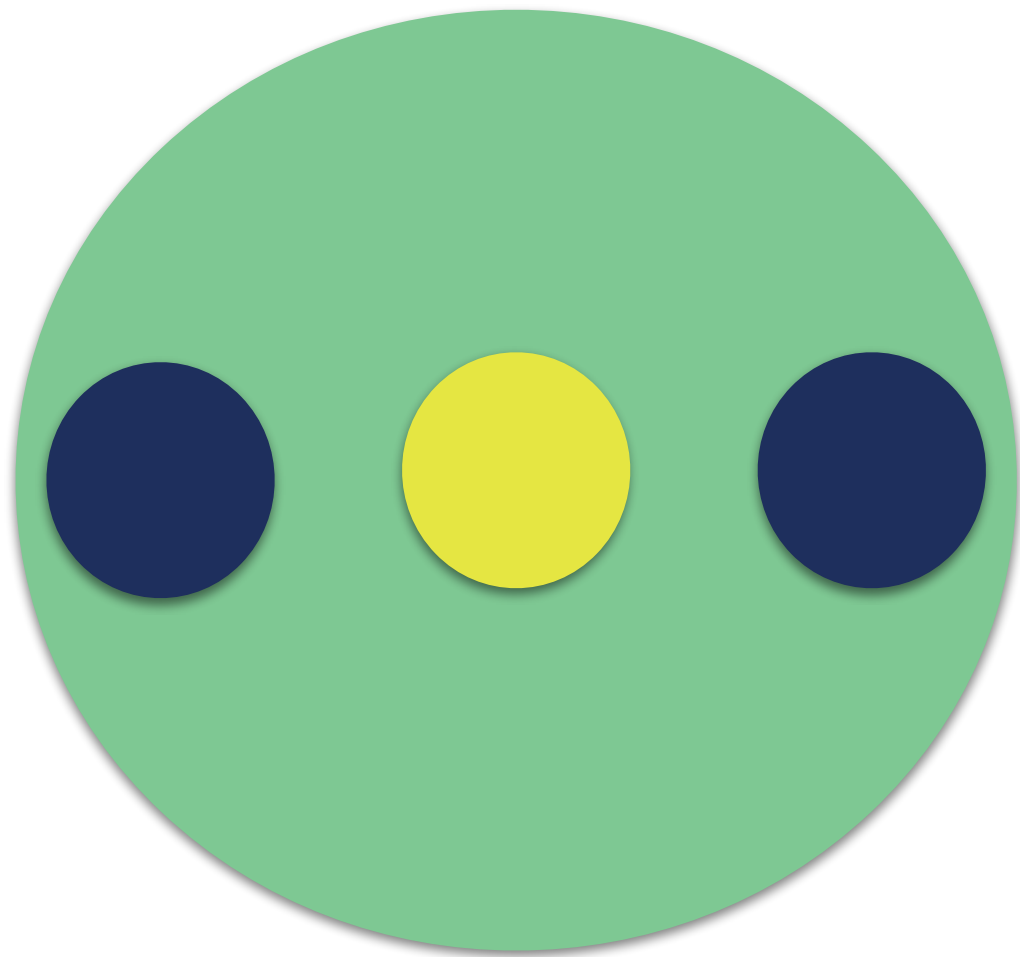
A decorative background image featuring a cosmic scene with a purple and blue nebula in the top right corner and a blue, textured surface resembling water or a planet's horizon in the bottom left corner.

ULTRALIGHT DARK MATTER

- Minimal Extension of what we know to exist
- Simple and predictive cosmology

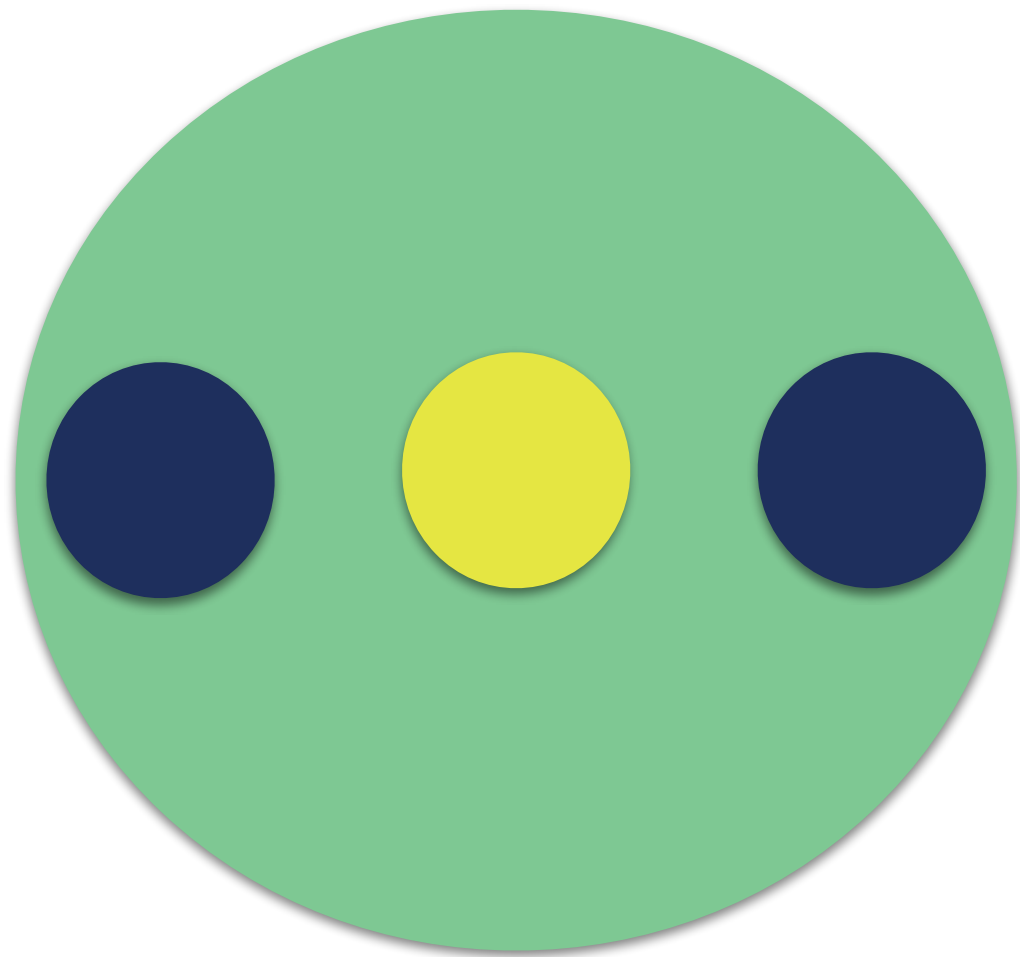
$$\theta G \tilde{G}$$

Neutron $\theta = 0$

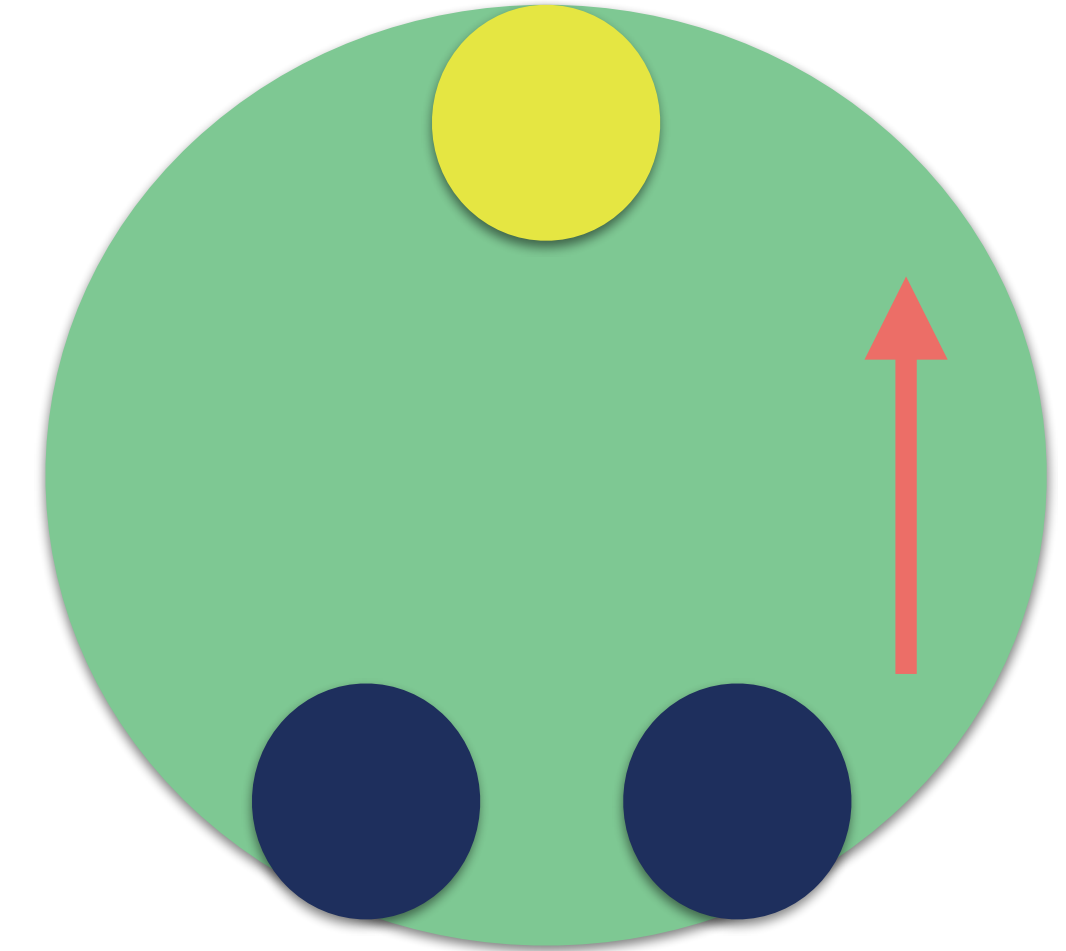


$$\theta G \tilde{G}$$

Neutron $\theta = 0$



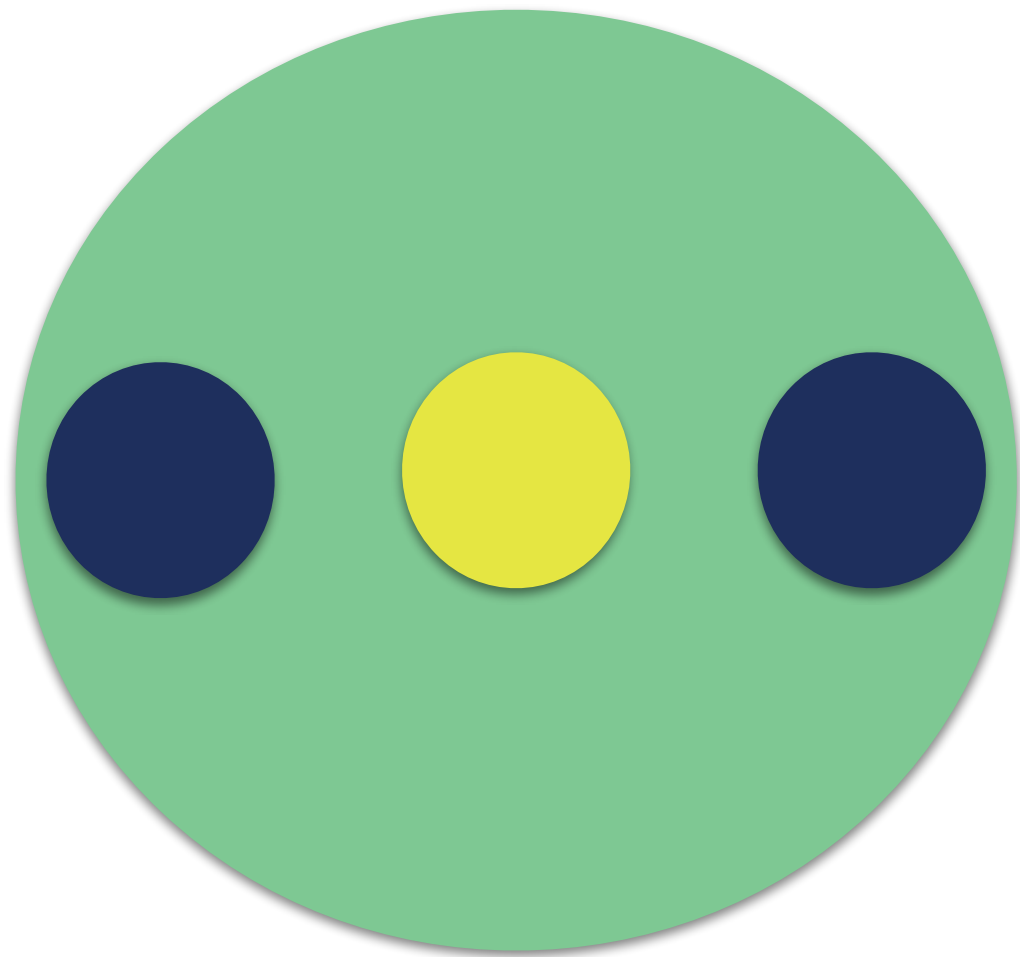
Neutron $\theta \neq 0$



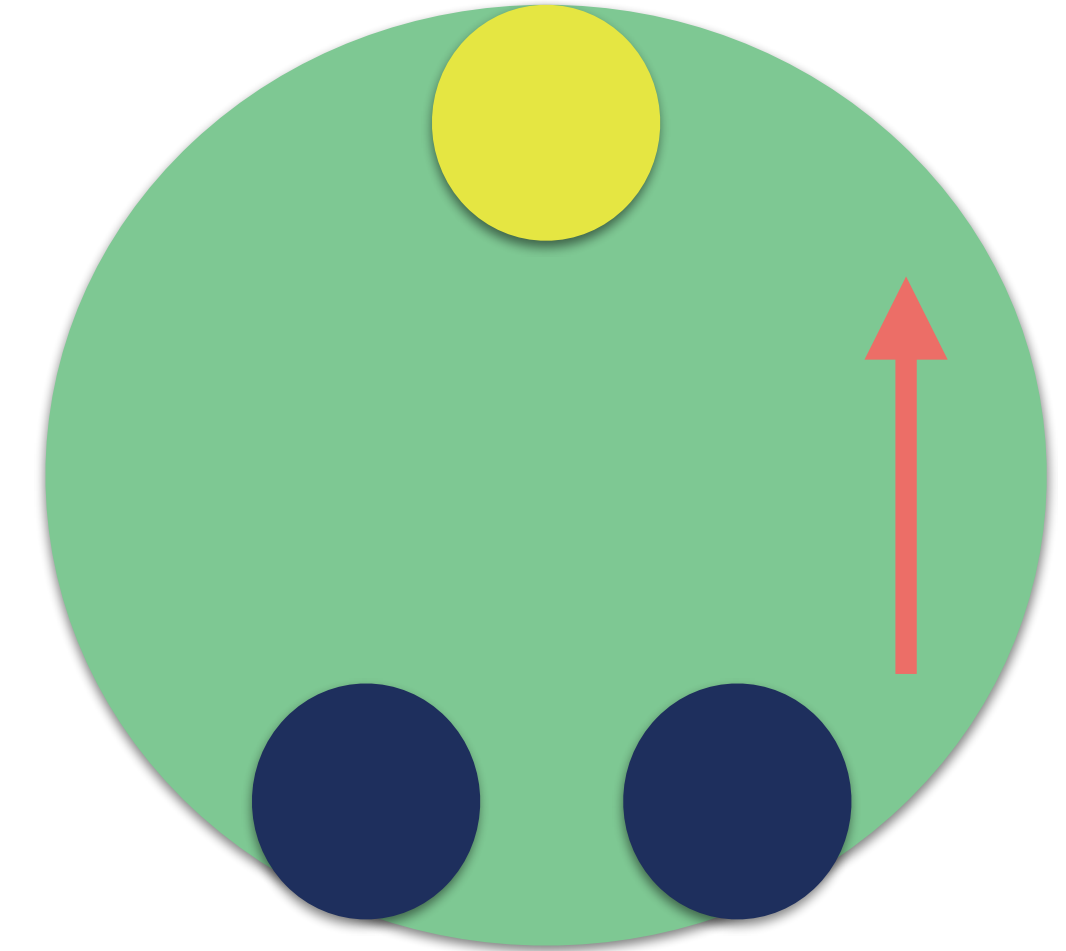
Electric
Dipole

$$\theta G \tilde{G}$$

Neutron $\theta = 0$



Neutron $\theta \neq 0$



Electric
Dipole

$$|\theta| \lesssim 10^{-10} \quad \text{Experimentally}$$

Introduce a new **global symmetry** at f_a

$$\theta G \tilde{G} \quad \longrightarrow \quad \left(\theta + \frac{a}{f_a} \right) G \tilde{G}$$

At the minimum

$$\langle a \rangle = -\theta f_a$$



AXION-LIKE DARK MATTER

- Minimal Extension of what we know to exist
 - Simple and predictive cosmology
 - Solve another big problem in particle physics
- 

Does anything change
as we vary the Higgs mass?

LOCAL

$$\text{Tr}[G \wedge G] \equiv G \tilde{G}$$

NON-LOCAL

On-shell N-point
functions of massive SM
particles



Does anything change
as we vary the Higgs mass?

$$\text{Tr}[G \wedge G] \equiv G\tilde{G}$$

Present in many
explanations of the small
value of the Higgs mass

[Dvali, Vilenkin '03], [Dvali '04], [Geller,
Hochberg, Kuflik, '18], [RTD, Teresi,
'21], [Arkani-Hamed, RTD, Kim, '20],

...





AXION-LIKE DARK MATTER

- Minimal Extension of what we know to exist
 - Simple and predictive cosmology
 - Potentially solves **two big problems** in particle physics
- 

AXION

$$\frac{a}{f_a} G \tilde{G}$$

QCD Phase Transition



$$\frac{a}{f_a} \frac{\pi}{f_\pi} + \dots$$

AXION

QCD Phase Transition

$$\frac{a}{f_a} G \tilde{G}$$



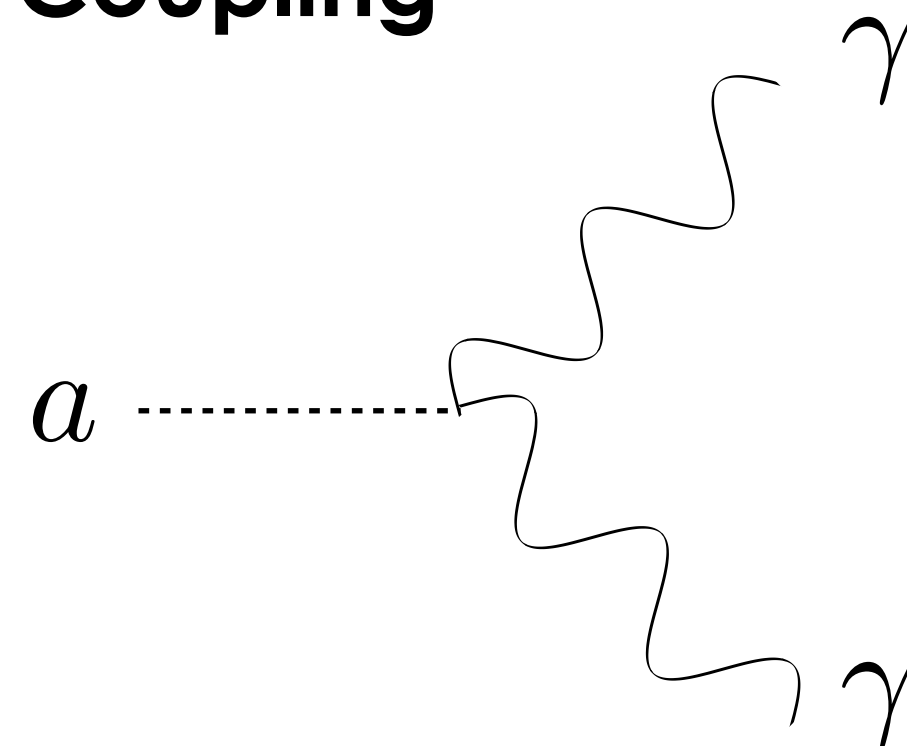
$$\frac{a}{f_a} \frac{\pi}{f_\pi} + \dots$$

Mass

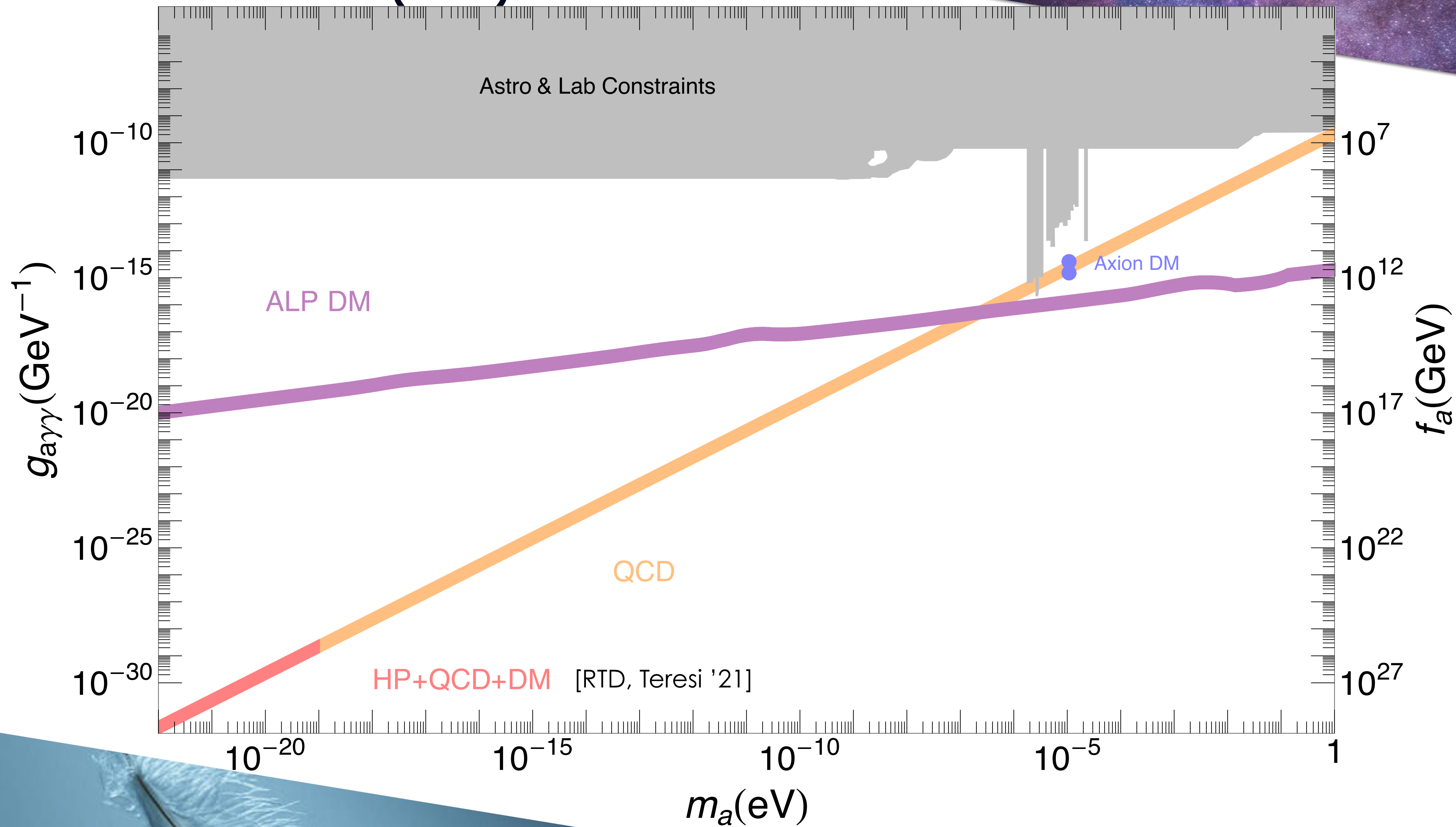
$$m_a \sim \frac{m_\pi^2}{f_a} \sim 10^{-2} \text{ eV} \frac{10^9 \text{ GeV}}{f_a}$$

Relevant Coupling

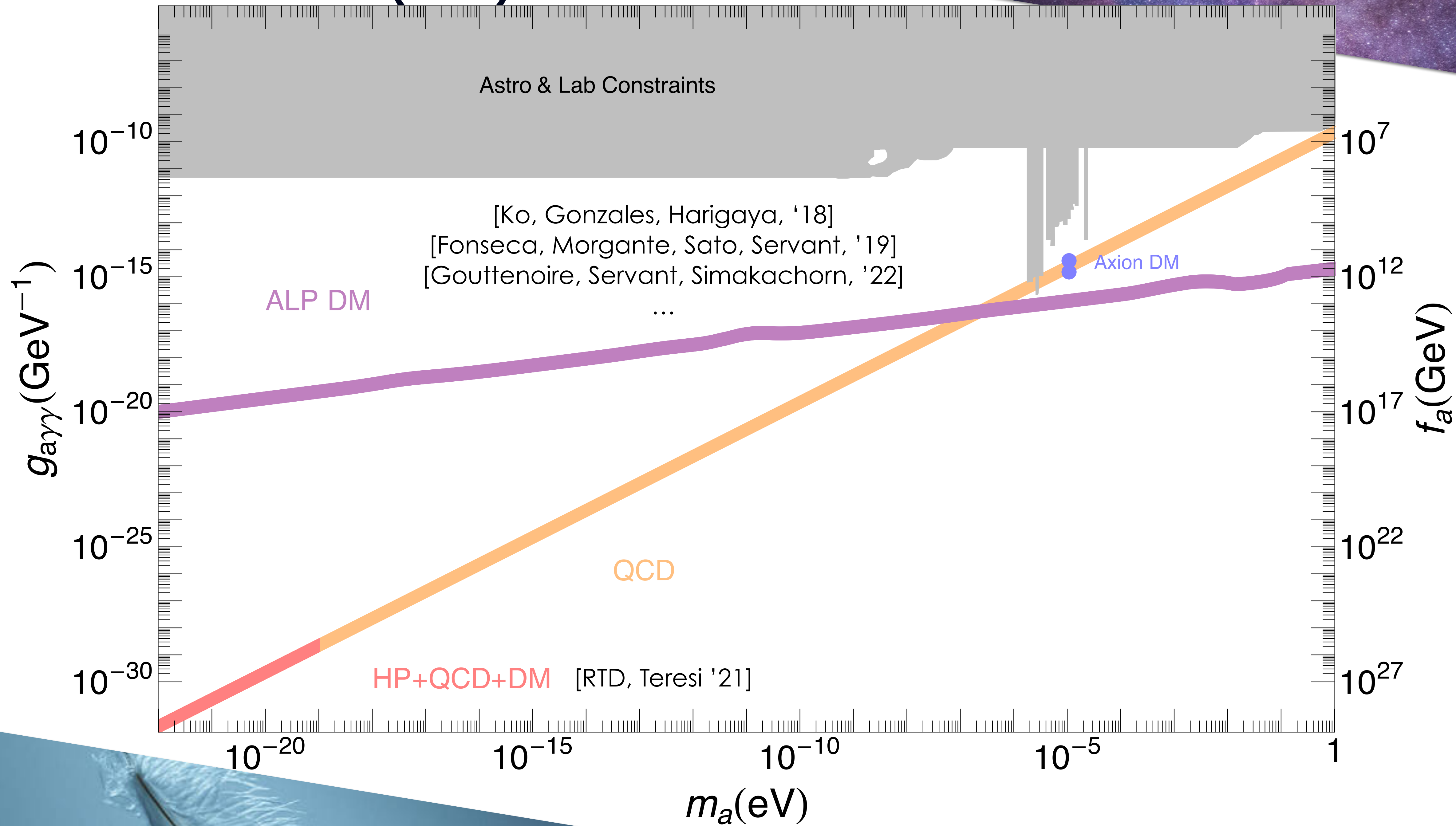
$$\frac{a}{f_a} \mathbf{E} \cdot \mathbf{B}$$



AXION LIKE PARTICLES (ALPs)



AXION LIKE PARTICLES (ALPs)



DESY and Hamburg University are at the
forefront of this effort:
ALPS II, BabyIAXO, ...

The background of the slide is a composite image. The left side shows a close-up of the ALPS (Advanced Large Photon Spectrometer) detector, featuring several large, polished, metallic hemispherical components arranged in a row. The right side of the image transitions into a deep space view of the Milky Way galaxy, showing a dense field of stars and the characteristic reddish-pink dust lanes of the galactic core.

ALPS DETECTION

Produced Colder than the SM (even if not via misalignment)

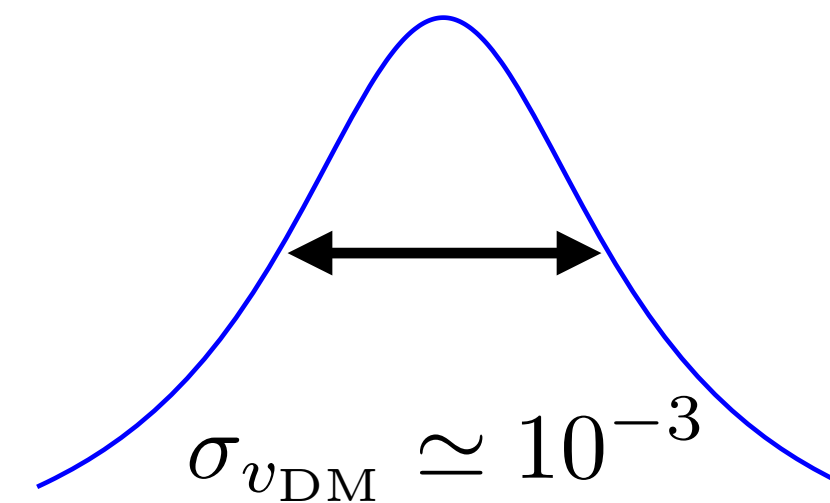
$$E_a \approx m_a$$

Produced Colder than the SM (even if not via misalignment)

$$E_a \approx m_a$$

It acquires a **small velocity dispersion** from virialization **in the Milky Way**

$$E_a \simeq m_a \left(1 + \frac{v_{\text{DM}}^2}{2} \right)$$

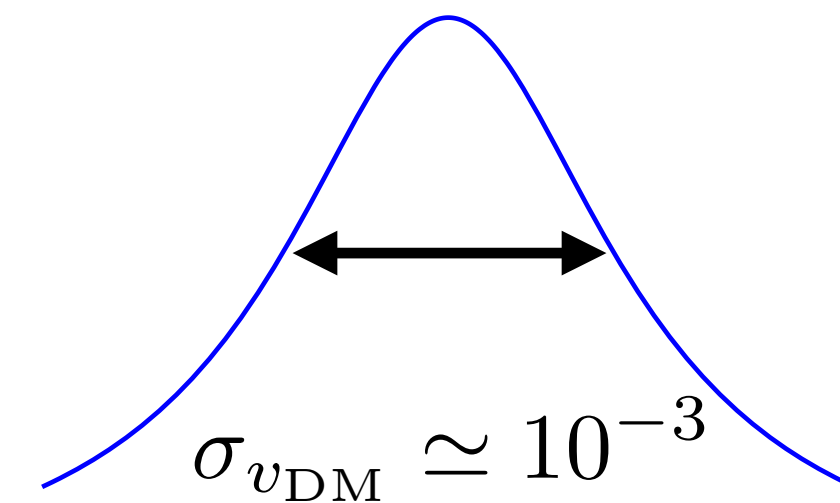


Produced Colder than the SM (even if not via misalignment)

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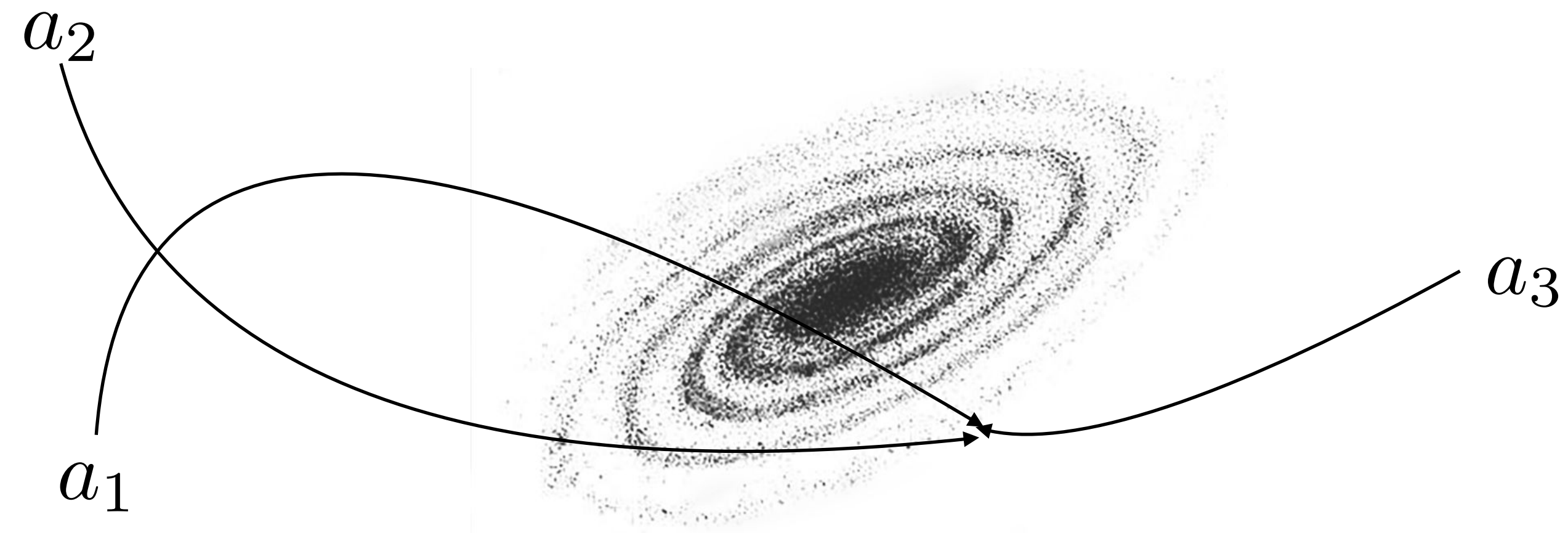
$$E_a \simeq m_a \left(1 + \frac{v_{\text{DM}}^2}{2} \right)$$



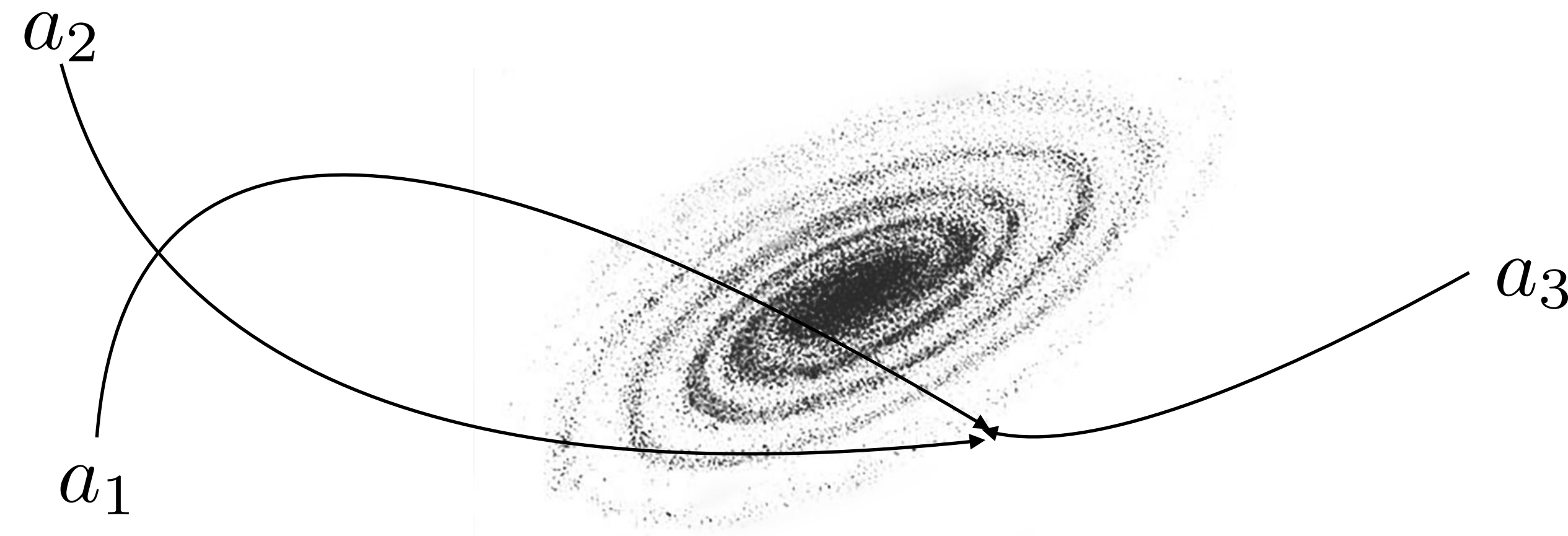
Lots of axions in each frequency bin that we can resolve
(even more in a De Broglie volume):

$$\Delta N_a \simeq \frac{\rho_{\text{DM}} V}{m_a^2 t_{\text{int}} v_{\text{DM}}^2} \simeq 10^{24} \left(\frac{10^{-14} \text{ eV}}{m_a} \right)^2 \left(\frac{\text{year}}{t_{\text{int}}} \right) \left(\frac{V}{\text{m}^3} \right)$$

In each experimental bin we are **summing over a multitude of plane waves** with different phases



In each experimental bin we are **summing over a multitude of plane waves** with different phases



$$a(t) = a_0 \left[\cos \left(m_a \left(1 + \frac{v_1^2}{2} \right) t + \phi_1 \right) + \cos \left(m_a \left(1 + \frac{v_2^2}{2} \right) t + \phi_2 \right) + \dots \right]$$

$$\simeq a_0 \cos(m_a t + \phi) [\cos(\delta\omega_a t + \phi') + \dots]$$

$$\delta\omega_a \simeq \frac{1}{m_a \langle v_{\text{DM}}^2 \rangle} \simeq \frac{10^6}{m_a}$$

Effectively: very **slow modulation** of an approximately **monochromatic field**

$$a(t) \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \cos(\omega_a t + \phi)$$

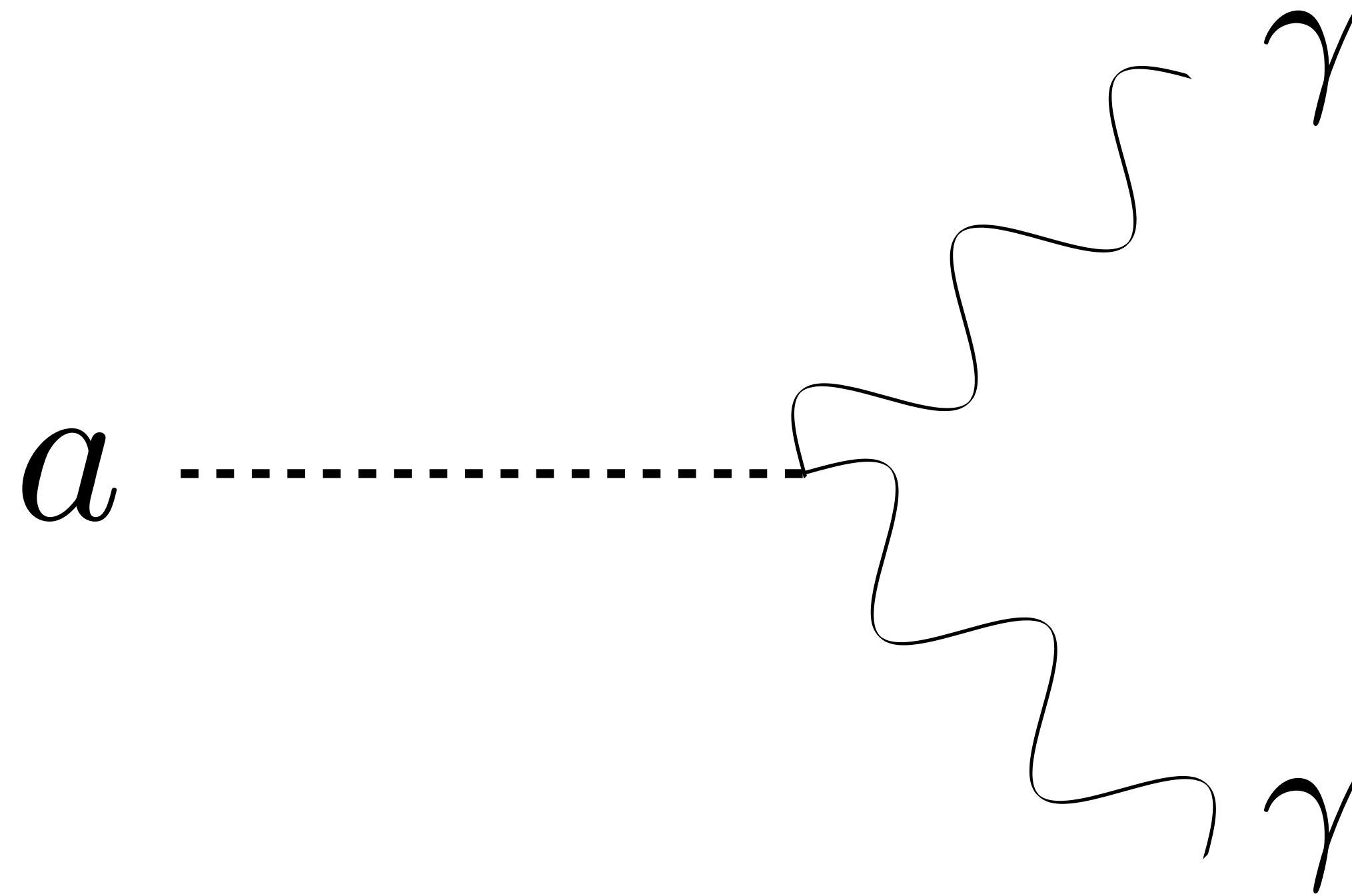
$$a(t) \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \cos(\omega_a t + \phi)$$

Frequency: $\omega_a \simeq \text{GHz} \frac{m_a}{10^{-6} \text{ eV}}$

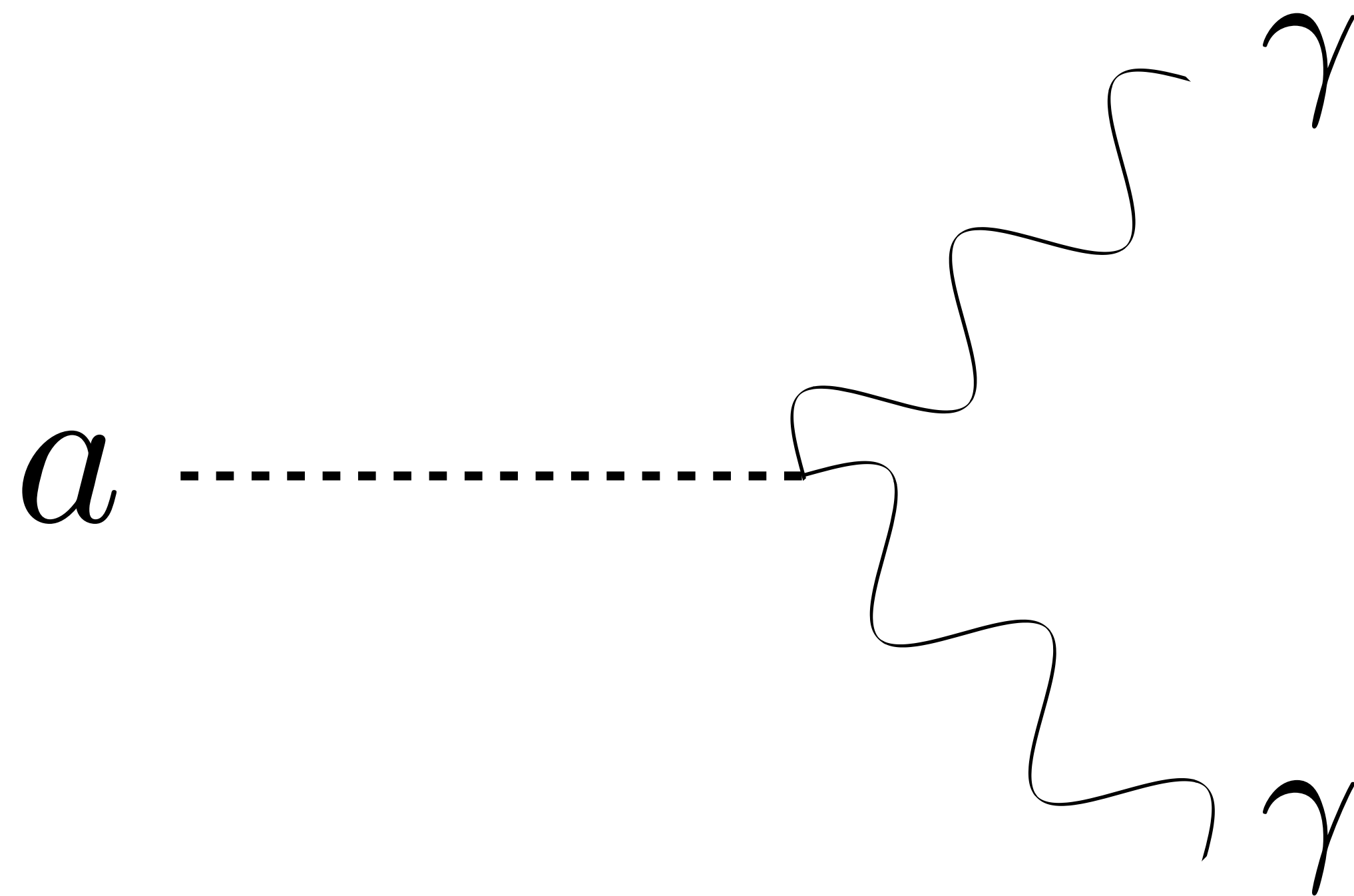
Coherence: $\tau_a \simeq \text{ms} \frac{10^{-6} \text{ eV}}{m_a}$

Max Exp. Size: $\lambda_a \simeq 200 \text{ m} \frac{10^{-6} \text{ eV}}{m_a}$

ALP DARK MATTER DETECTION



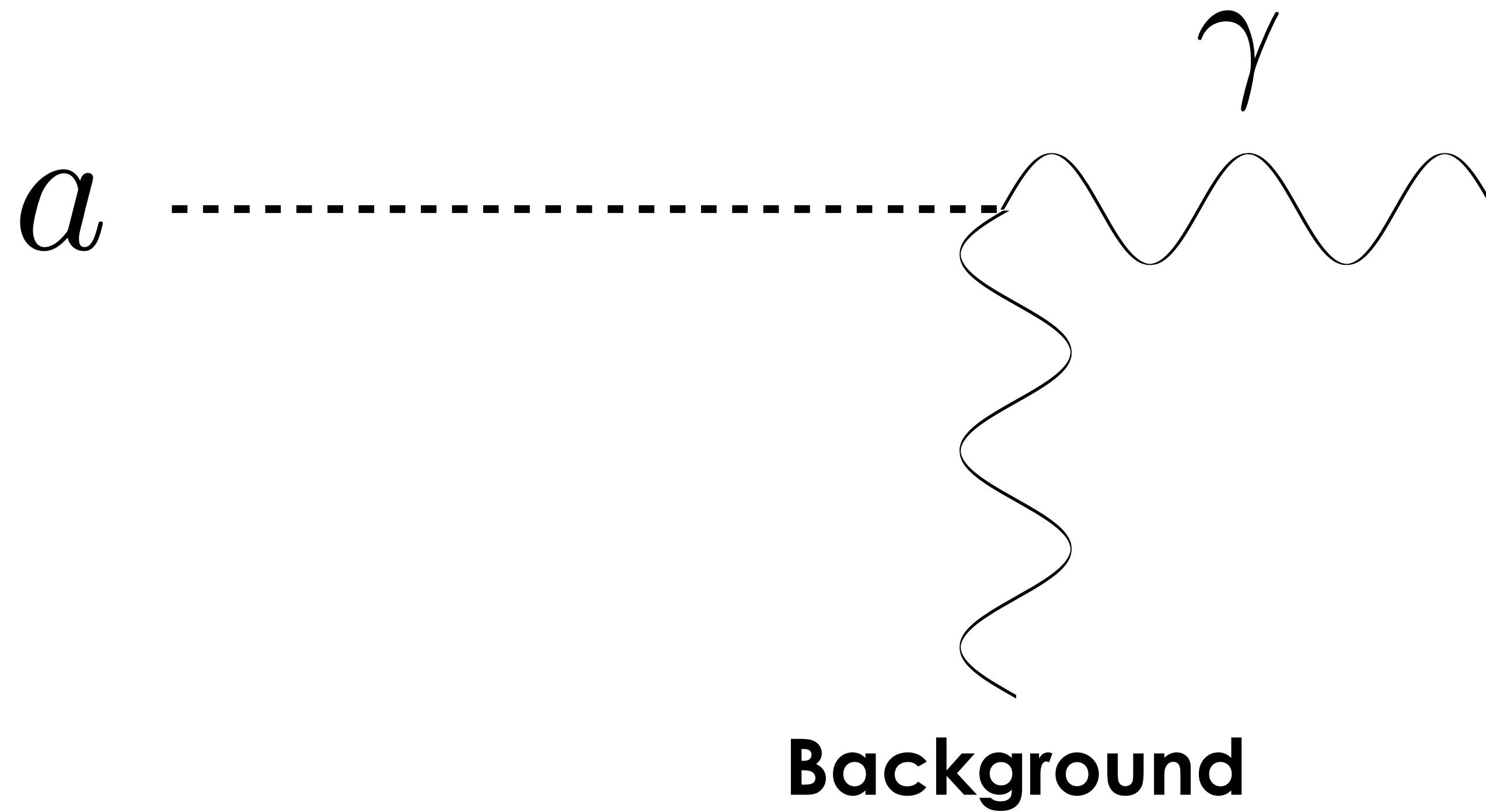
$$\nabla \times \mathbf{B} \simeq \partial_t \mathbf{E} + \mathbf{J} + \underline{g_{a\gamma\gamma} \mathbf{B} \partial_t a}$$



$$\nabla \times \mathbf{B} \simeq \partial_t \mathbf{E} + \mathbf{J} + \underline{g_{a\gamma\gamma} \mathbf{B} \partial_t a}$$

$$J_{\text{eff}}(t) \sim g_{a\gamma\gamma} B_0(t) \sqrt{\rho_{\text{DM}}} \cos m_a t$$

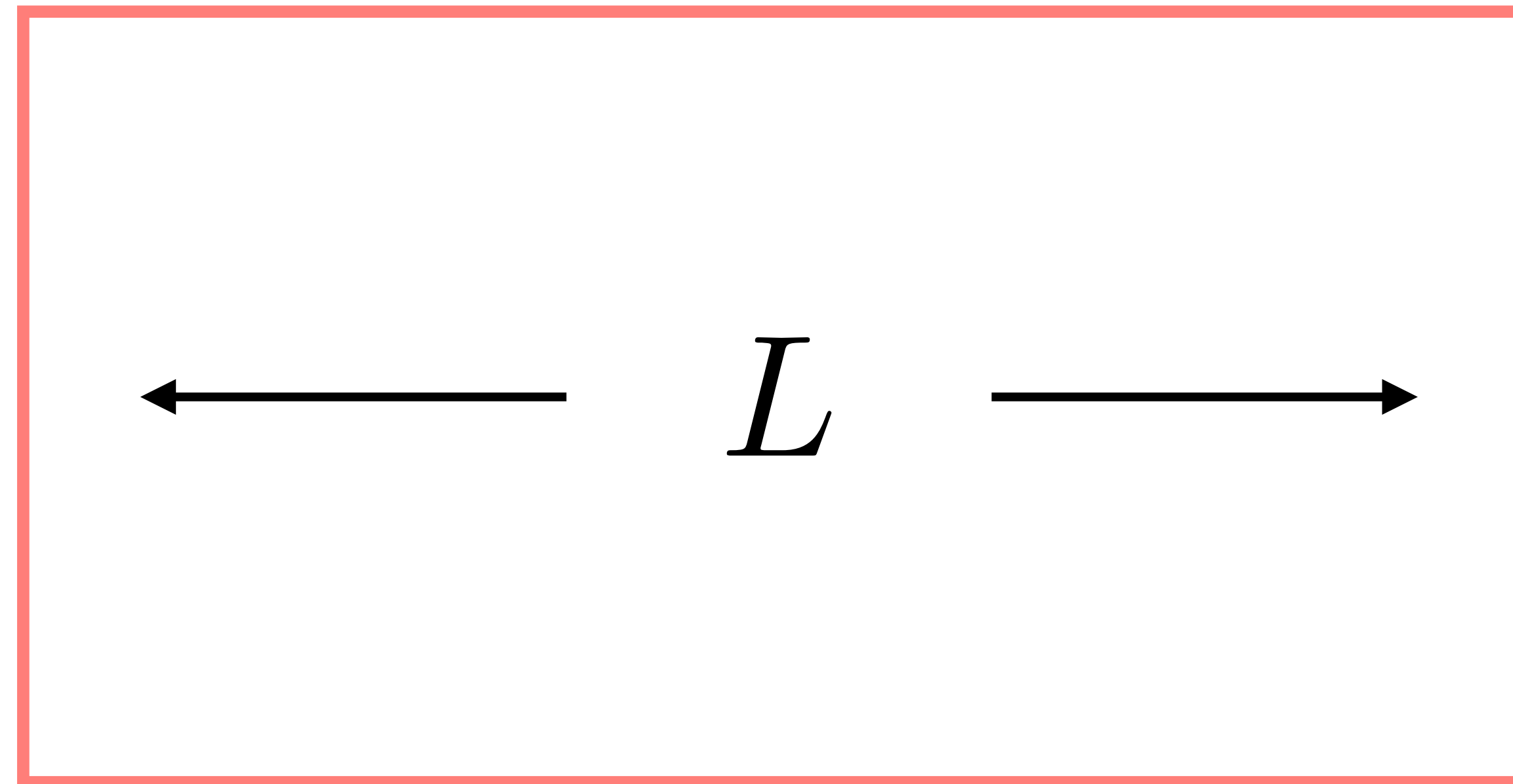
ALP DARK MATTER DETECTION



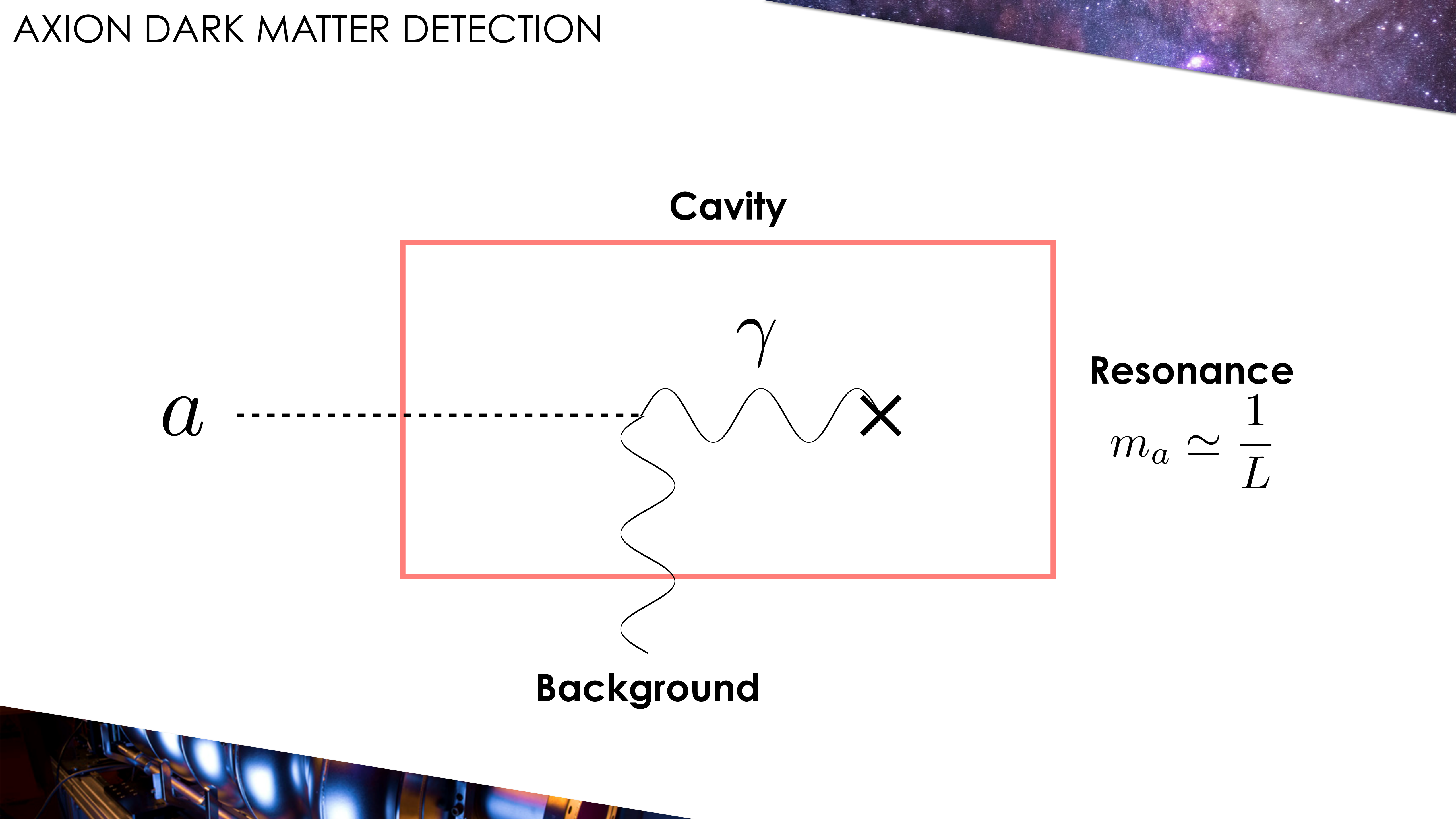
$$\sim \frac{a}{f_a} E_{\text{bkg}} \simeq 10^{-21} E_{\text{bkg}}$$

AXION DARK MATTER DETECTION

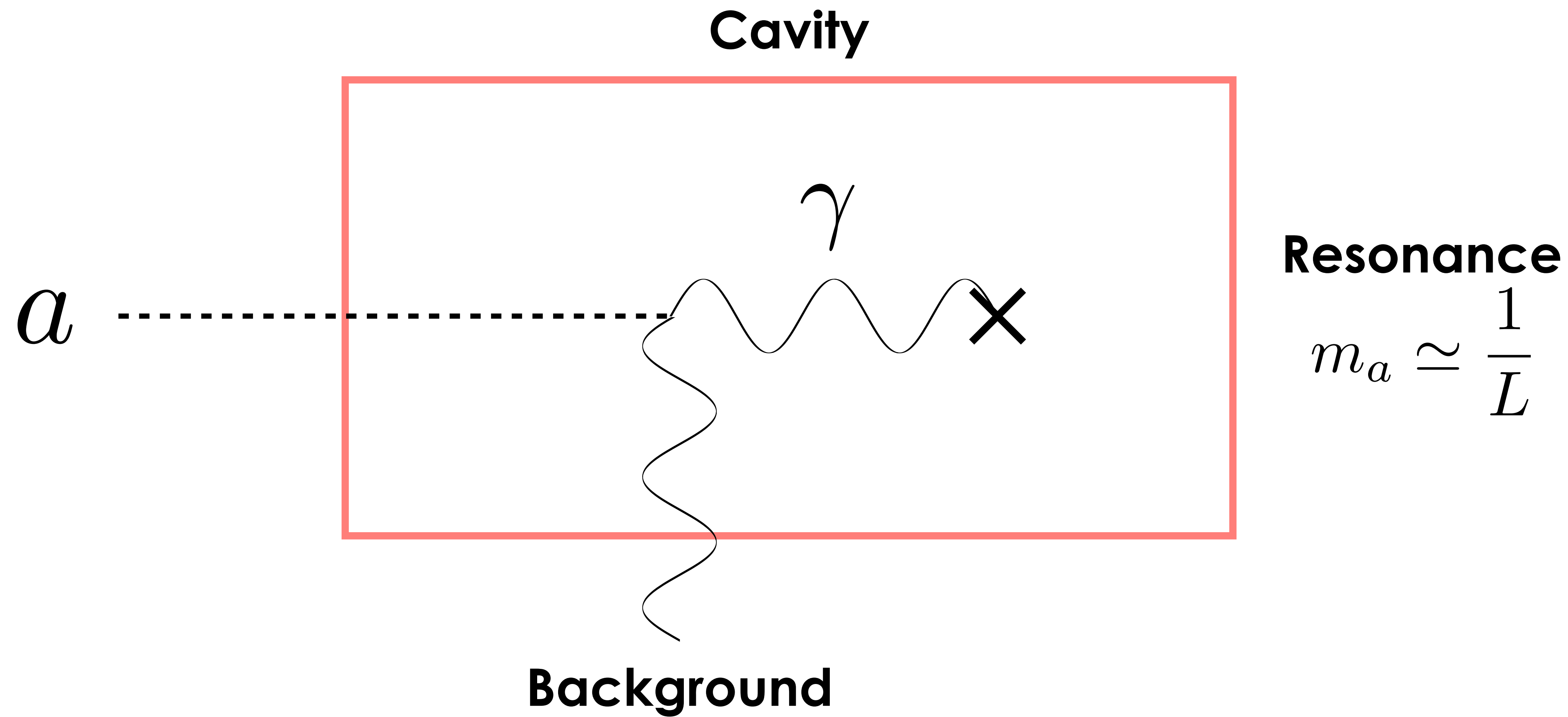
Cavity

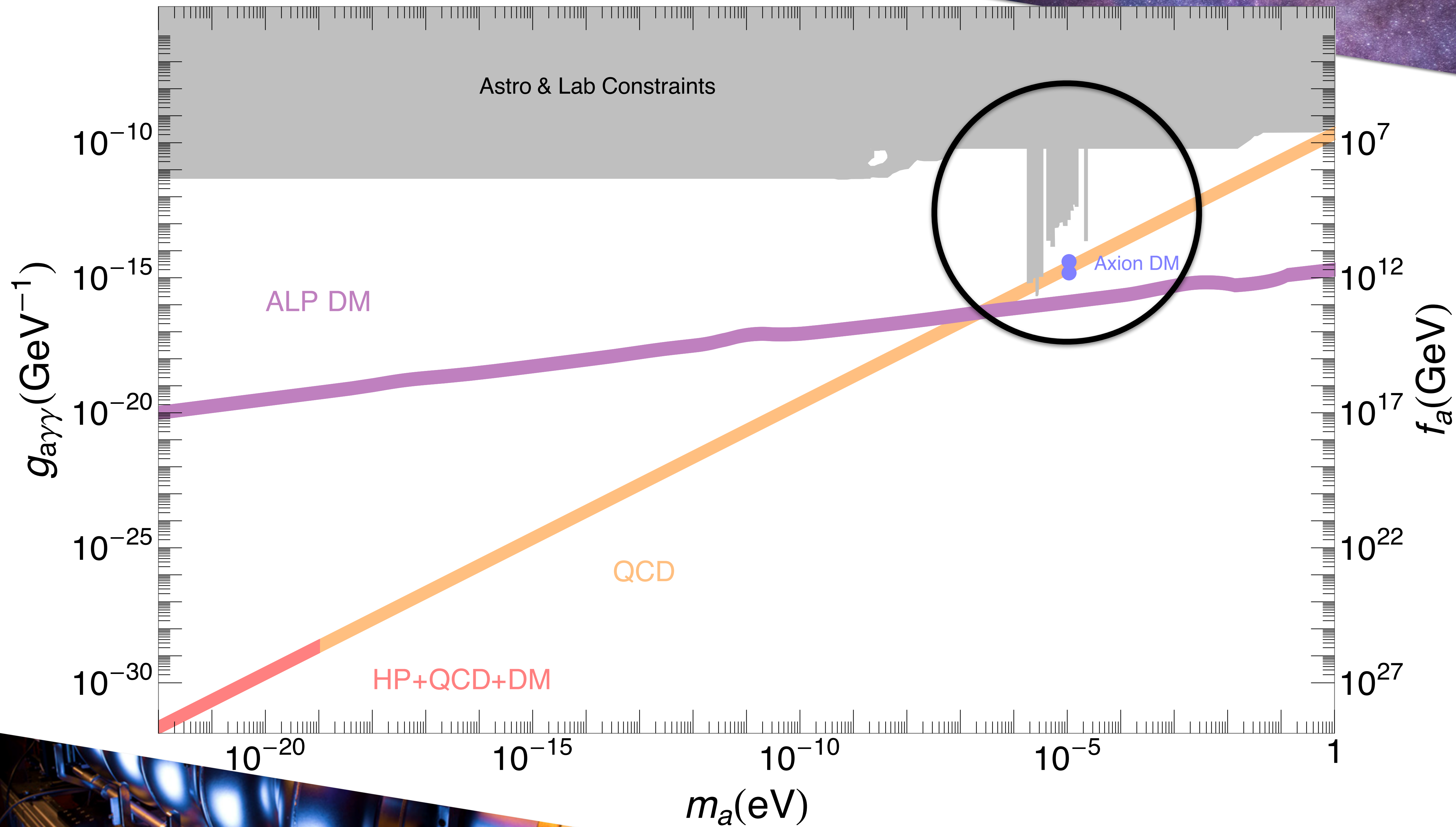


$$m_\gamma \simeq \frac{1}{L}$$



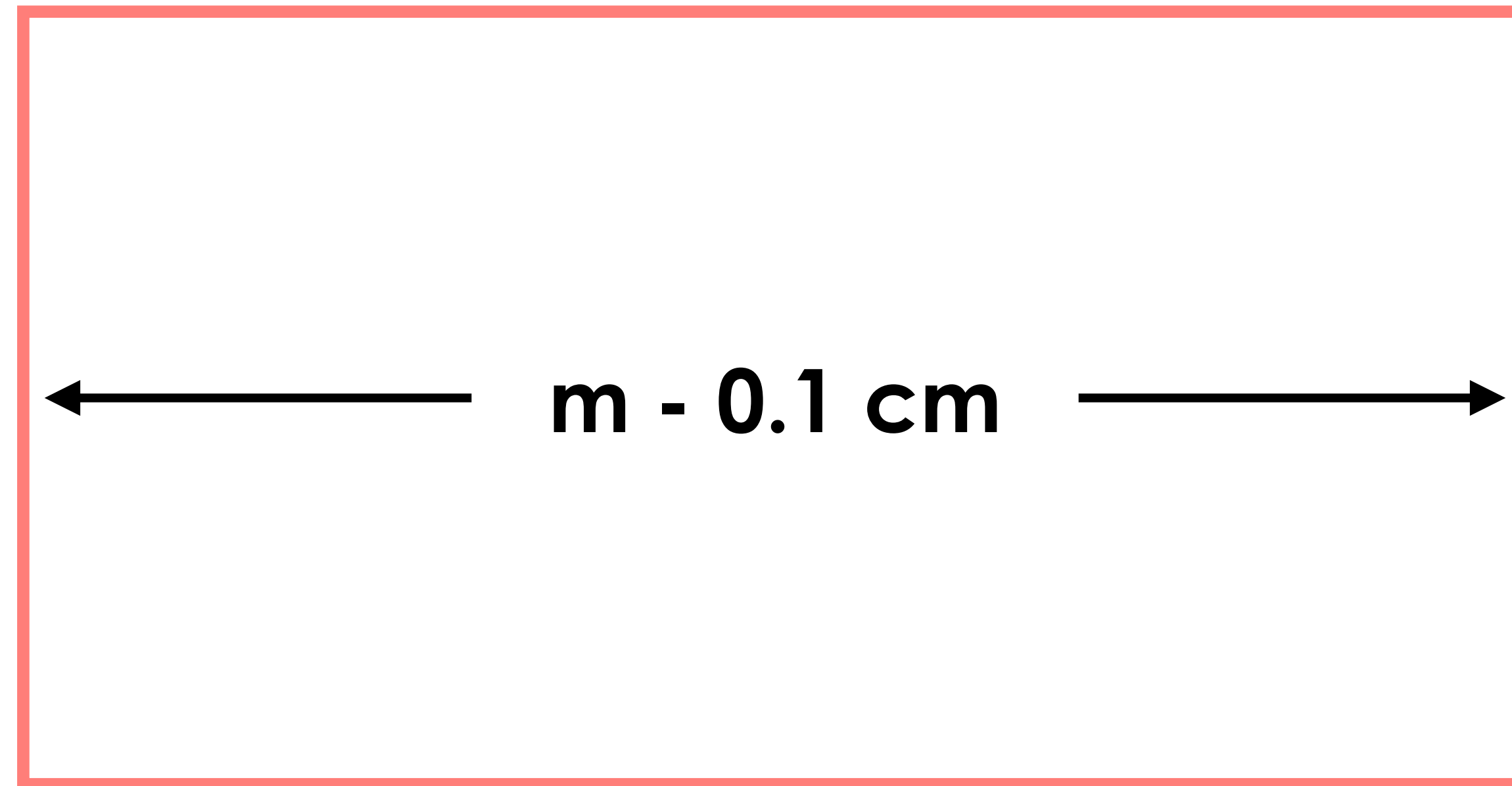
AXION DARK MATTER DETECTION



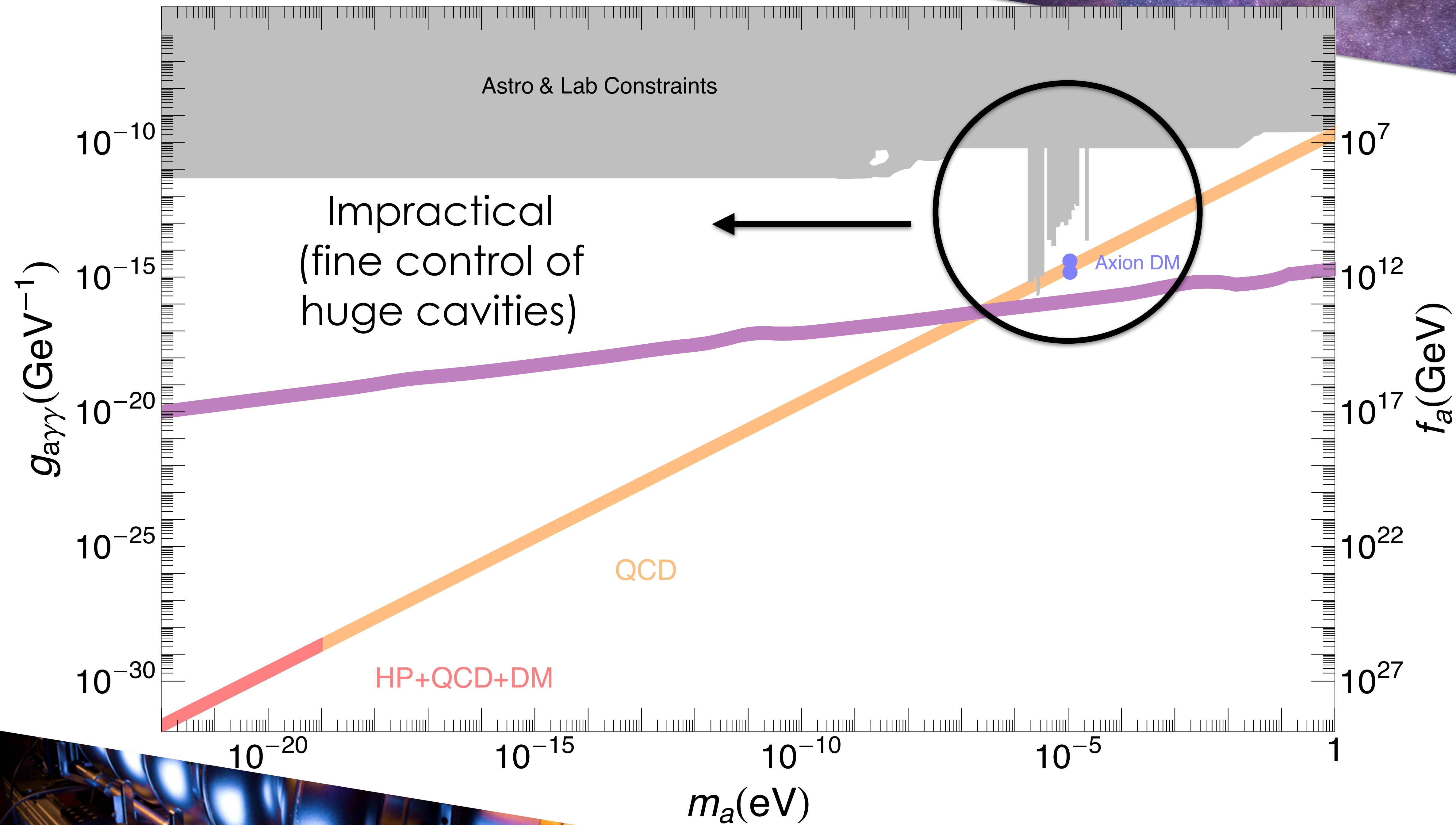


AXION DETECTION

Cavity



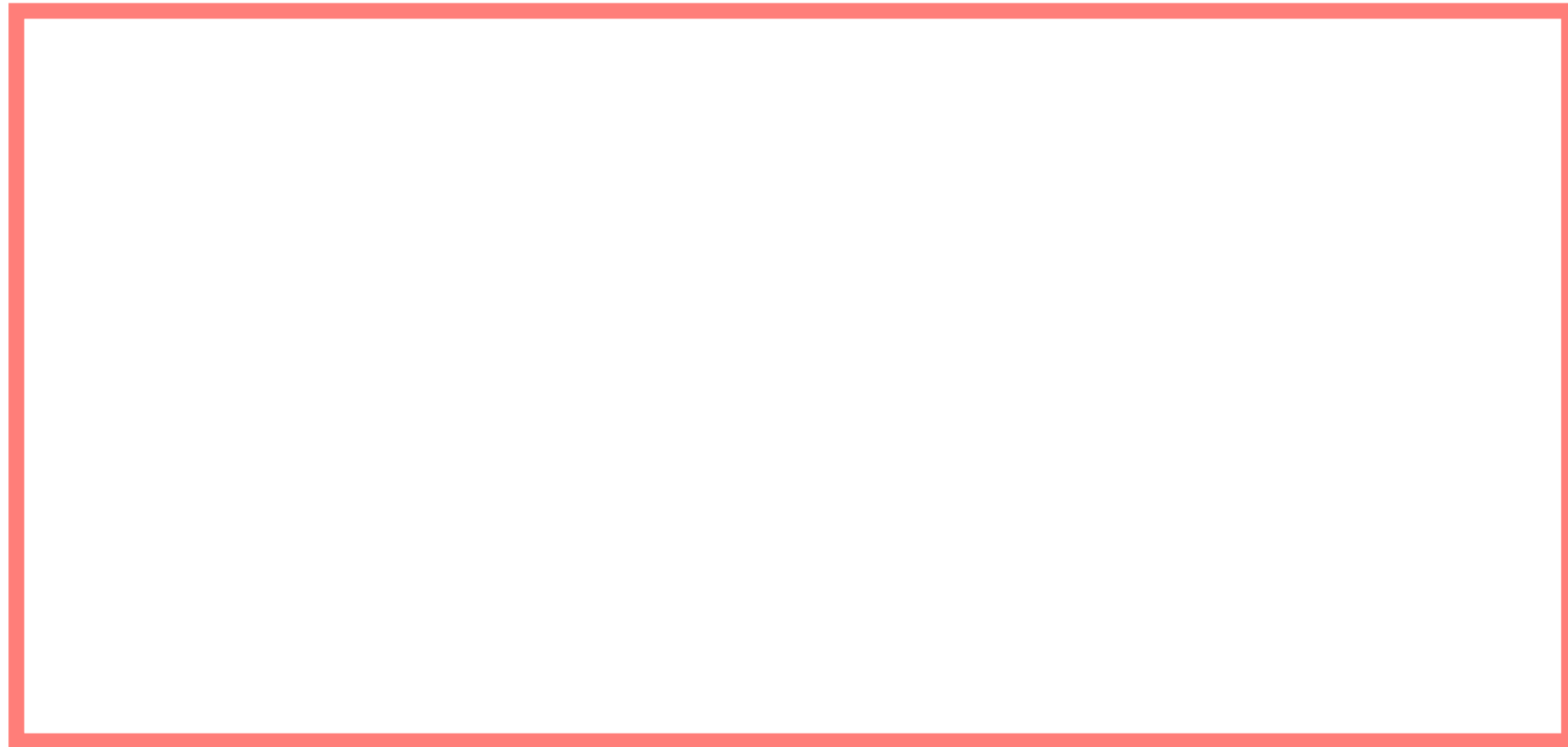
Low noise electronics
High quality factors



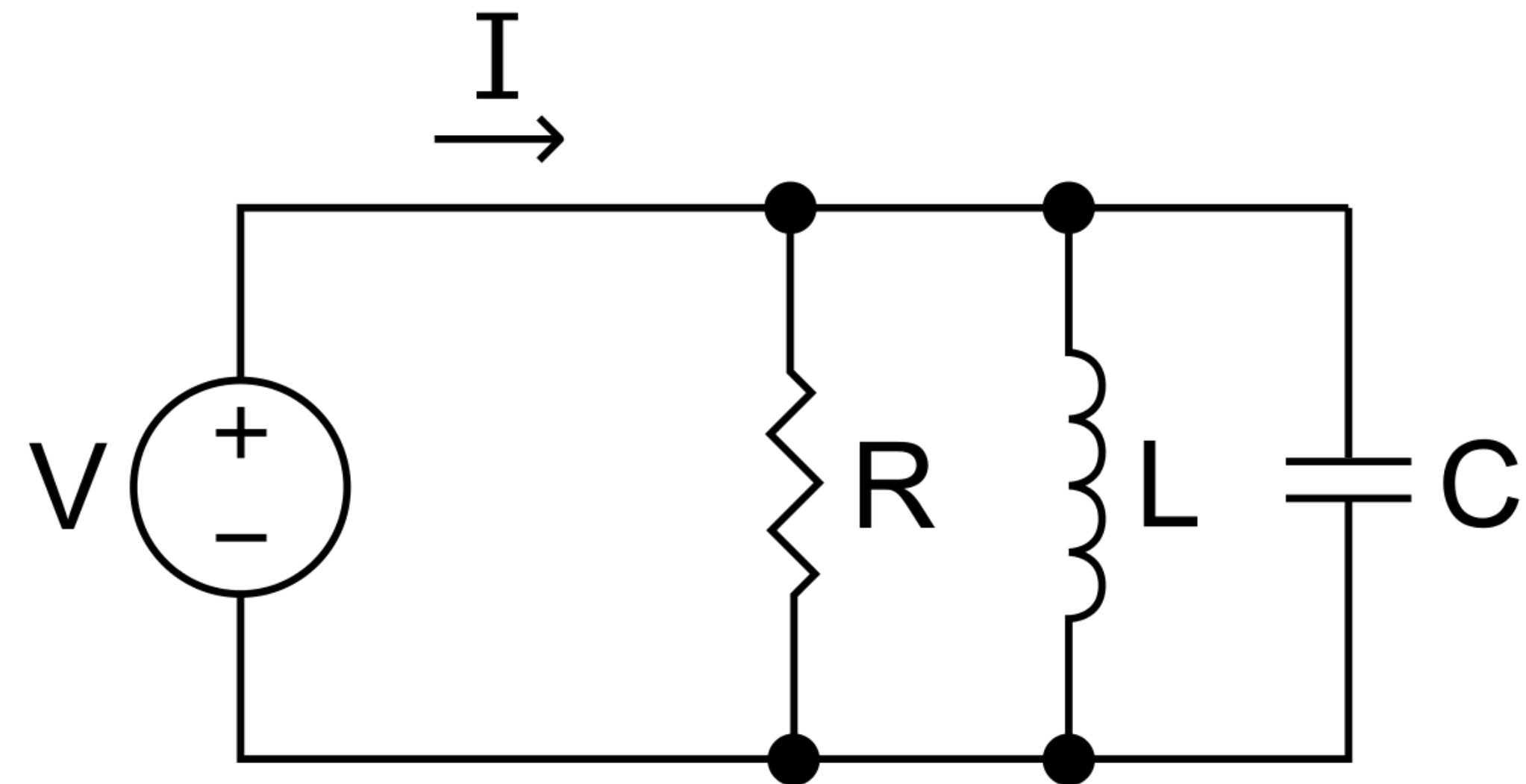
DM RADIO AND ABRACADABRA

[Kahn, Safdi, Tahler '16][Chauduri, Irwin, Graham, Mardon '18-'19]

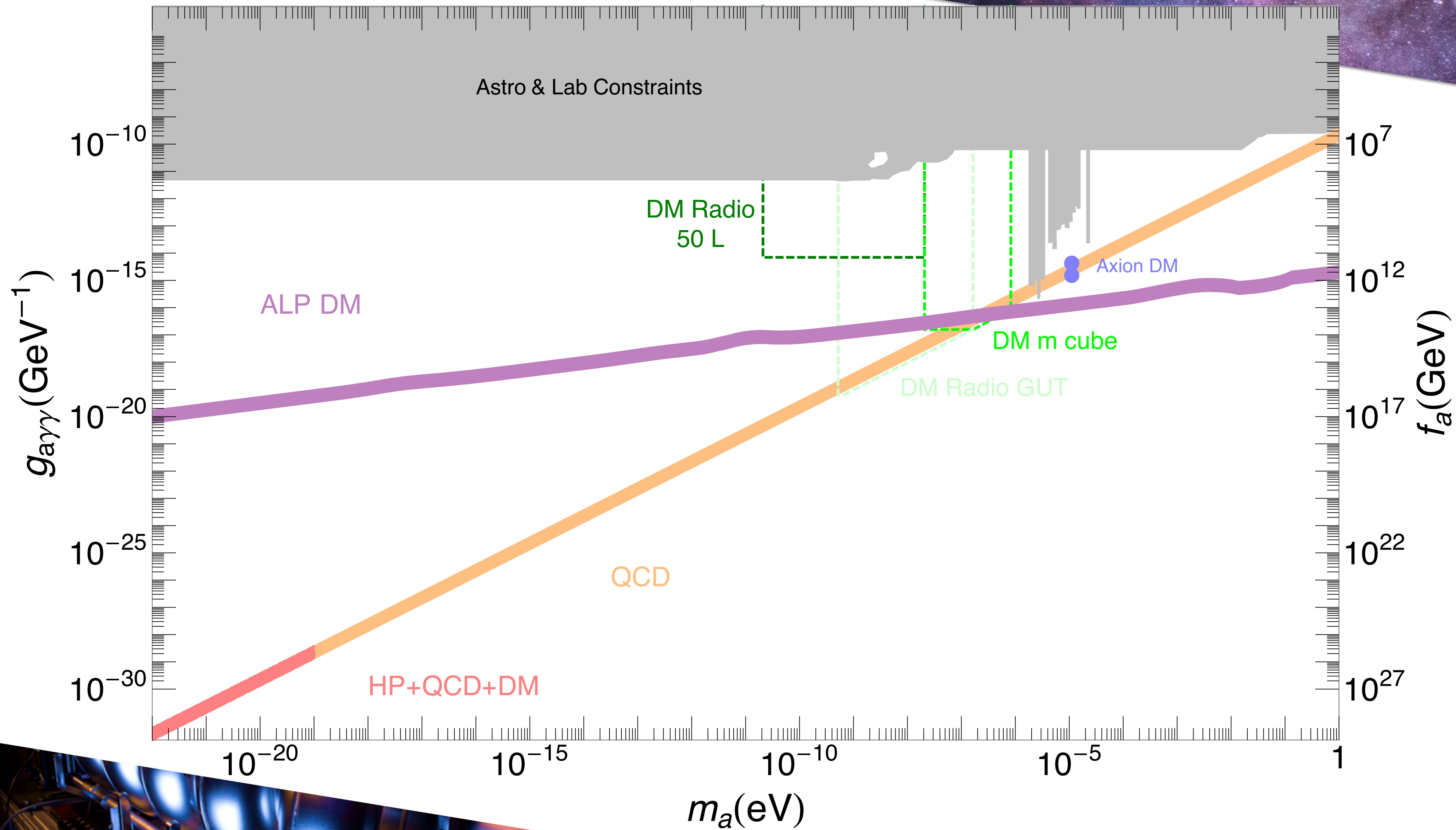
Cavity



RLC Circuit



$$\omega = \frac{1}{\sqrt{LC}}$$



Cavity:

$$\sum_n \left(\partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) \mathbf{E}_n = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a)$$

Cavity:

$$\sum_n \left(\partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) \mathbf{E}_n = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a)$$

$$\omega_1 \simeq m_a \qquad \partial_t(\mathbf{B}) \simeq 0$$

Cavity:

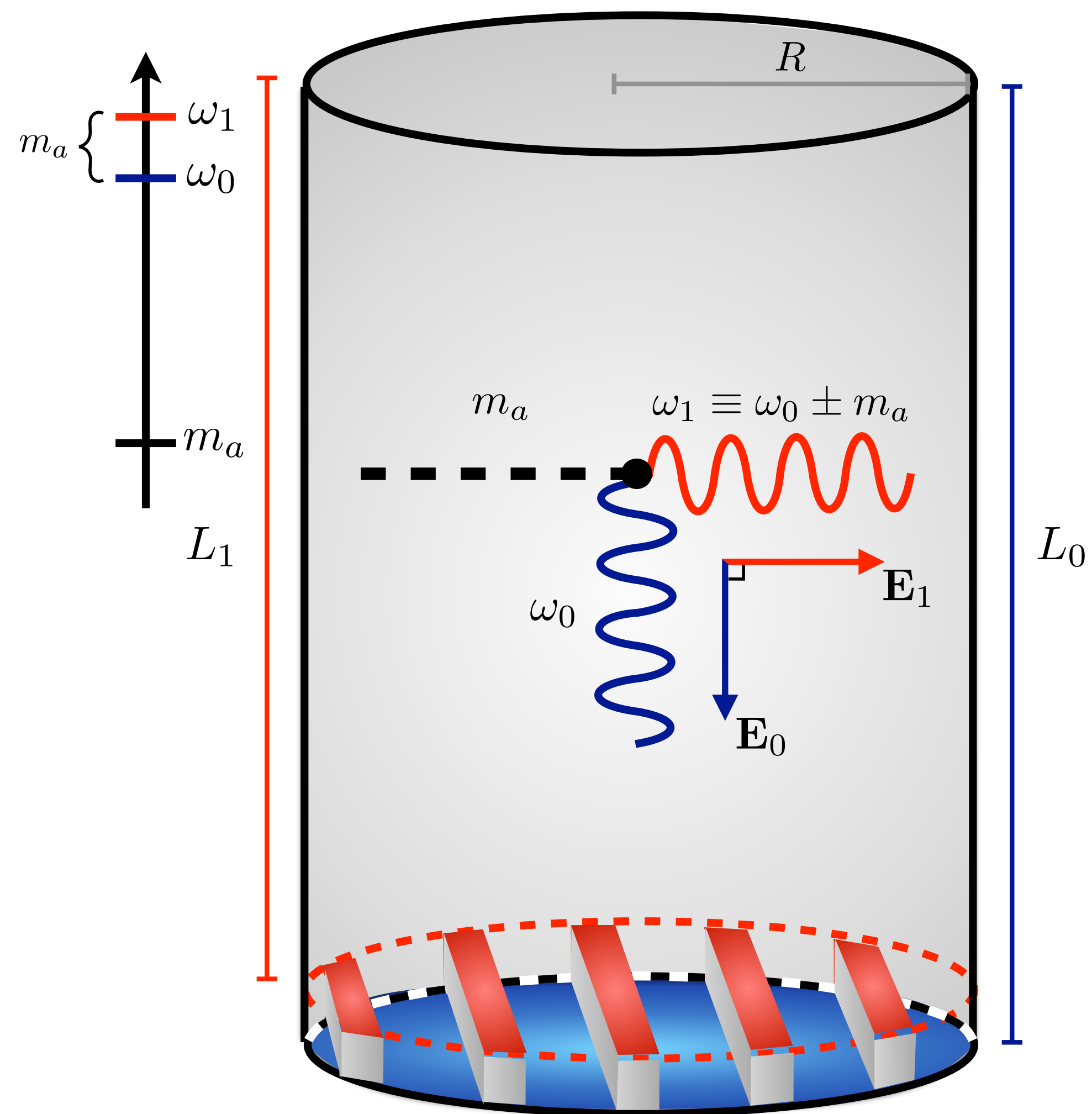
$$\sum_n \left(\partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) \mathbf{E}_n = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a)$$

$$\omega_1 \simeq m_a \quad \partial_t (\mathbf{B}) \simeq 0$$

$$\left(\partial_t^2 + \frac{m_a}{Q_1} \partial_t + m_a^2 \right) \mathbf{E}_1 = g_{a\gamma\gamma} \mathbf{B} \sqrt{\rho_{\text{DM}}} m_a \cos m_a t$$

HETERODYNE DETECTION

[Berlin, RTD, S. Ellis, C. Nantista, J. Nielson, P. Schuster, S. Tantawi, N. Toro, K. Zhou '19]



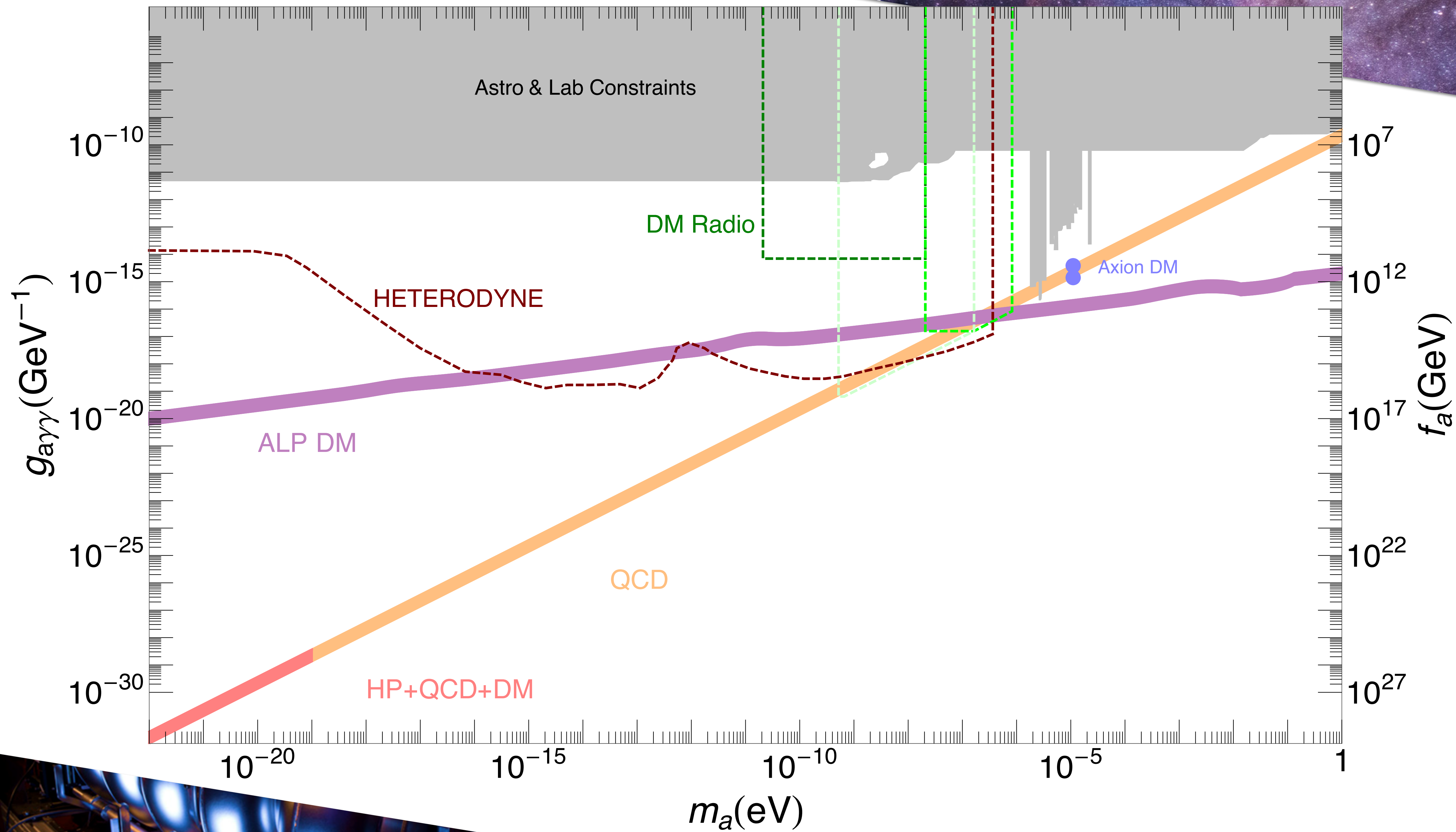
HETERODYNE DETECTION

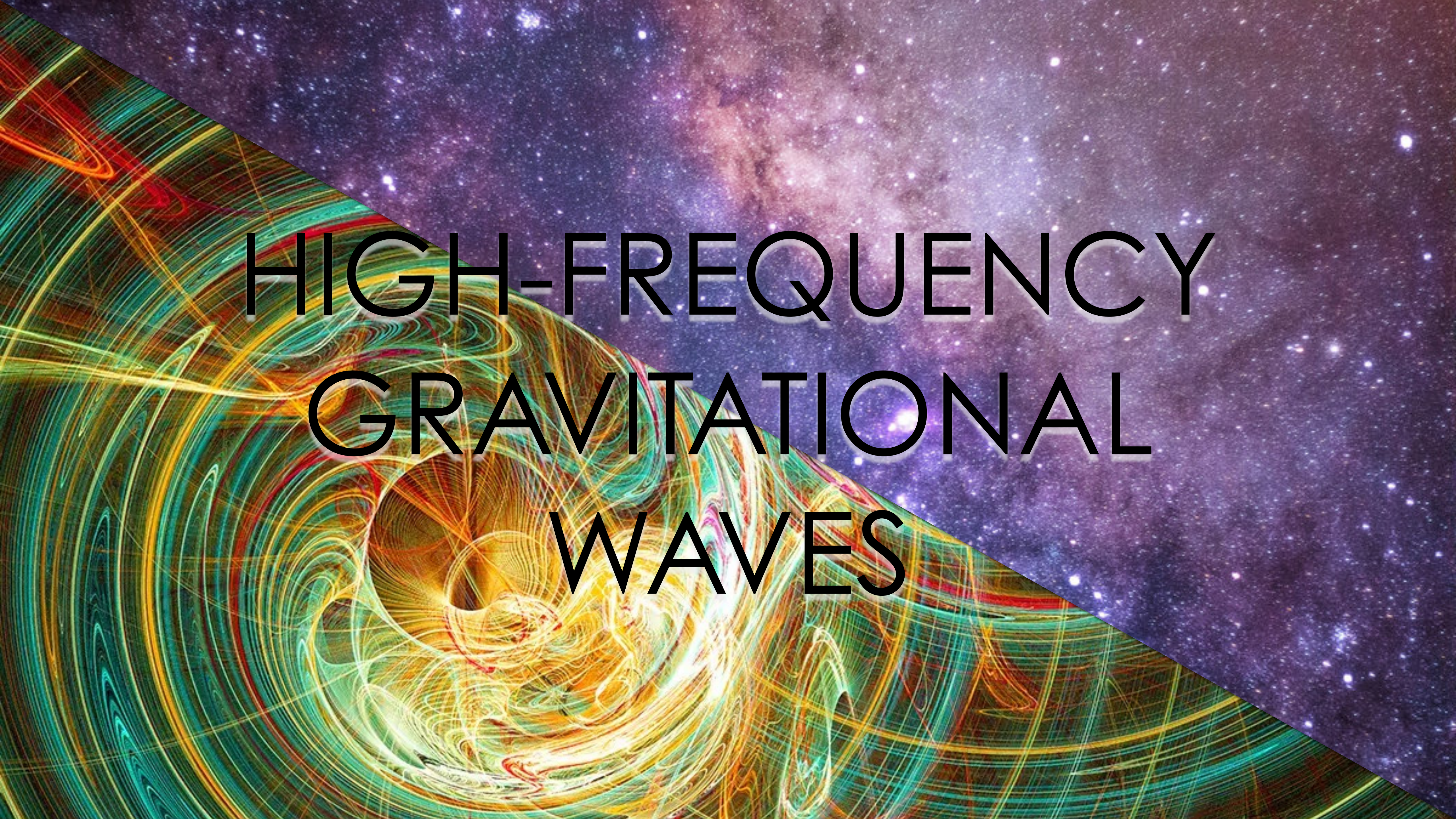
[Berlin, RTD, S. Ellis, C. Nantista, J. Nielson, P. Schuster, S. Tantawi, N. Toro, K. Zhou'19]

$$\sum_n \left(\partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) \mathbf{E}_n = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a)$$

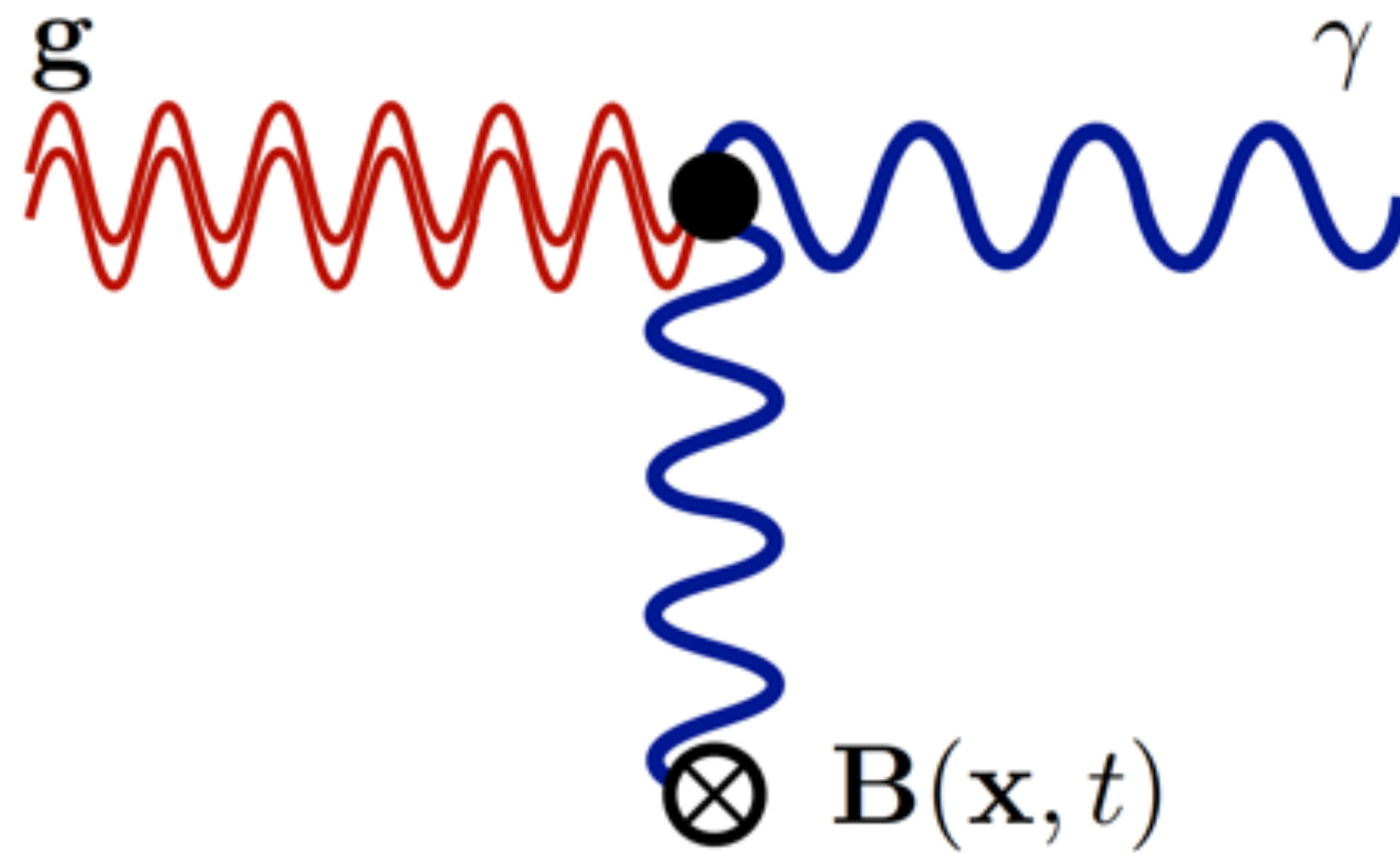
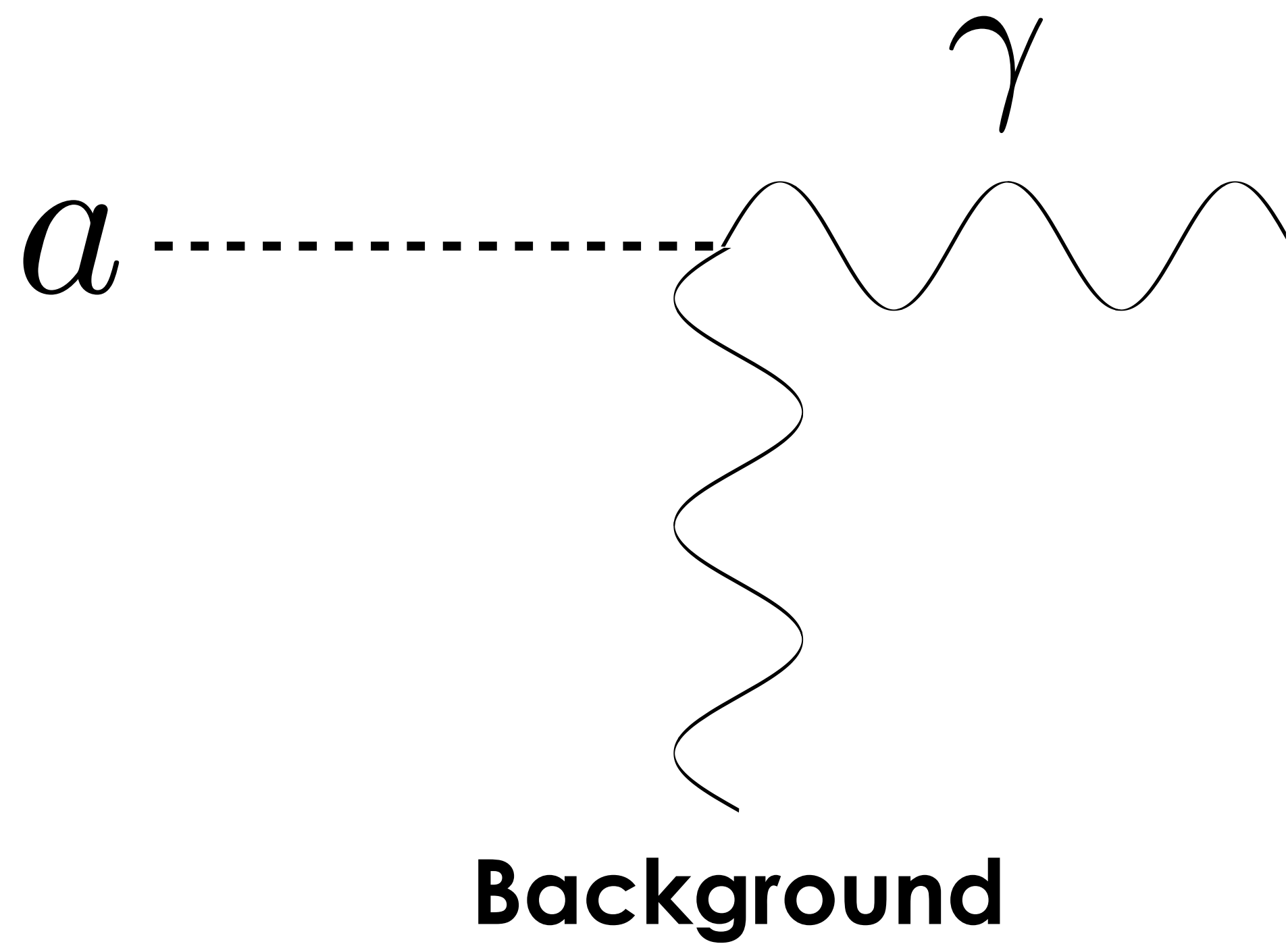
$$\partial_t (\mathbf{B}) \simeq i\omega_0 \mathbf{B} \quad \omega_1 \simeq \omega_0 + m_a$$

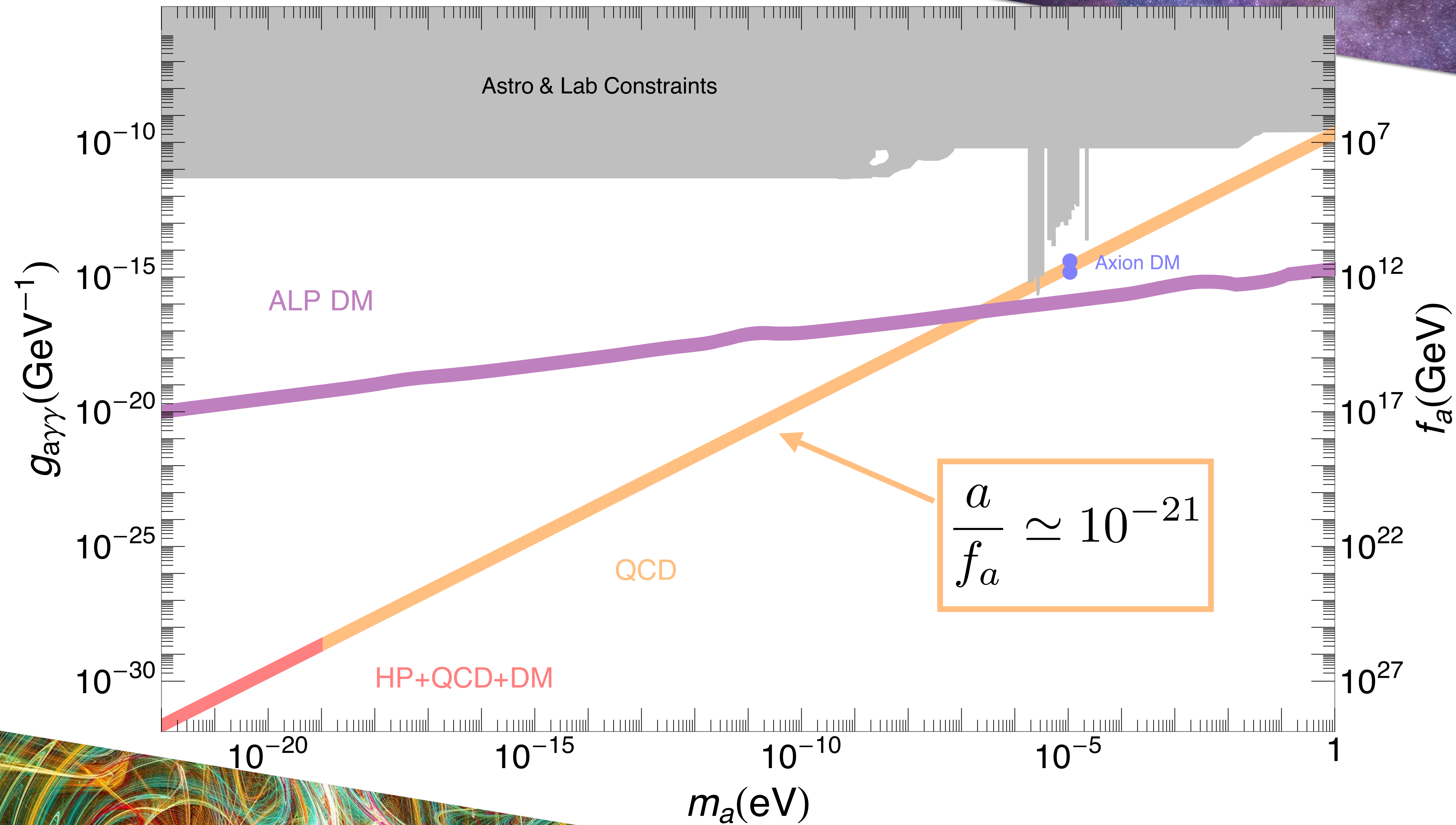
$$\partial_t J_{\text{eff}} = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a) \propto \omega_0 m_a \gg m_a^2$$





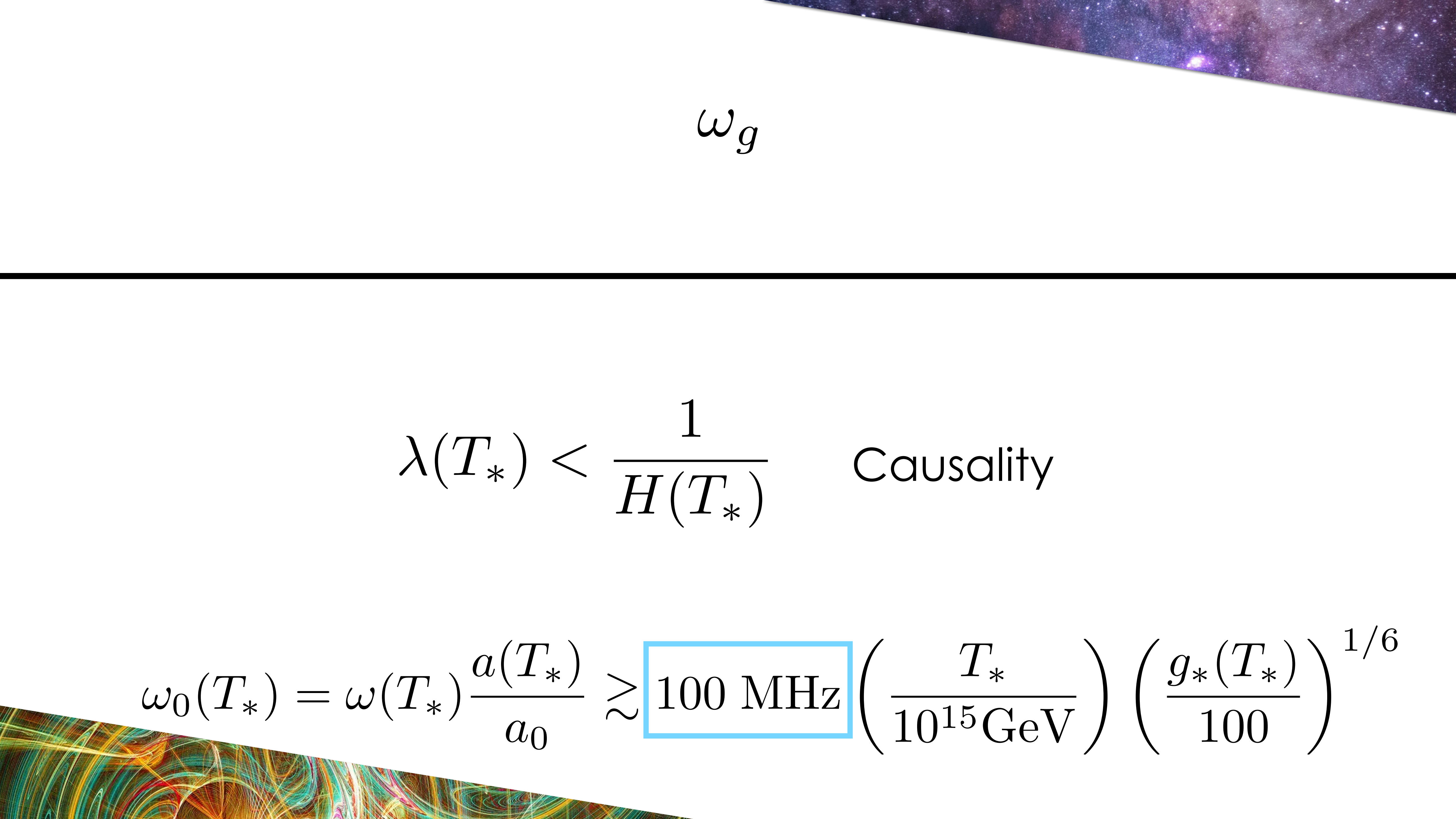
HIGH-FREQUENCY GRAVITATIONAL WAVES



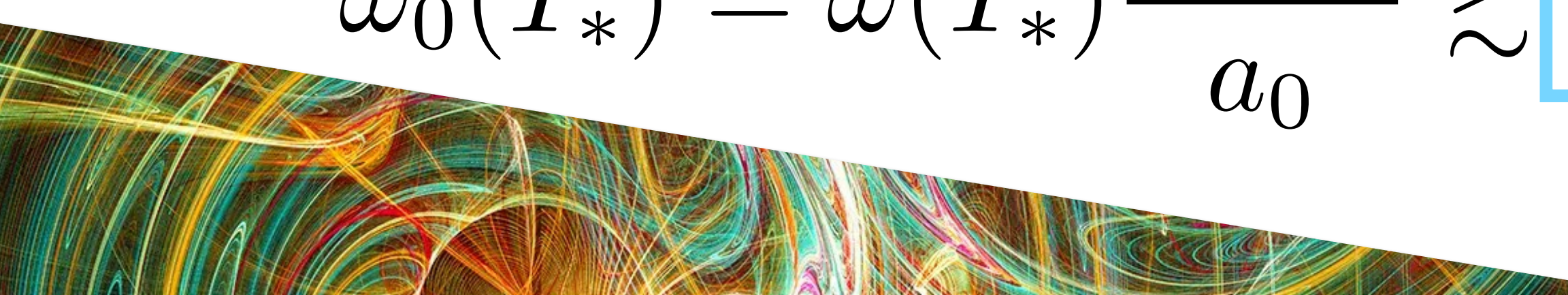


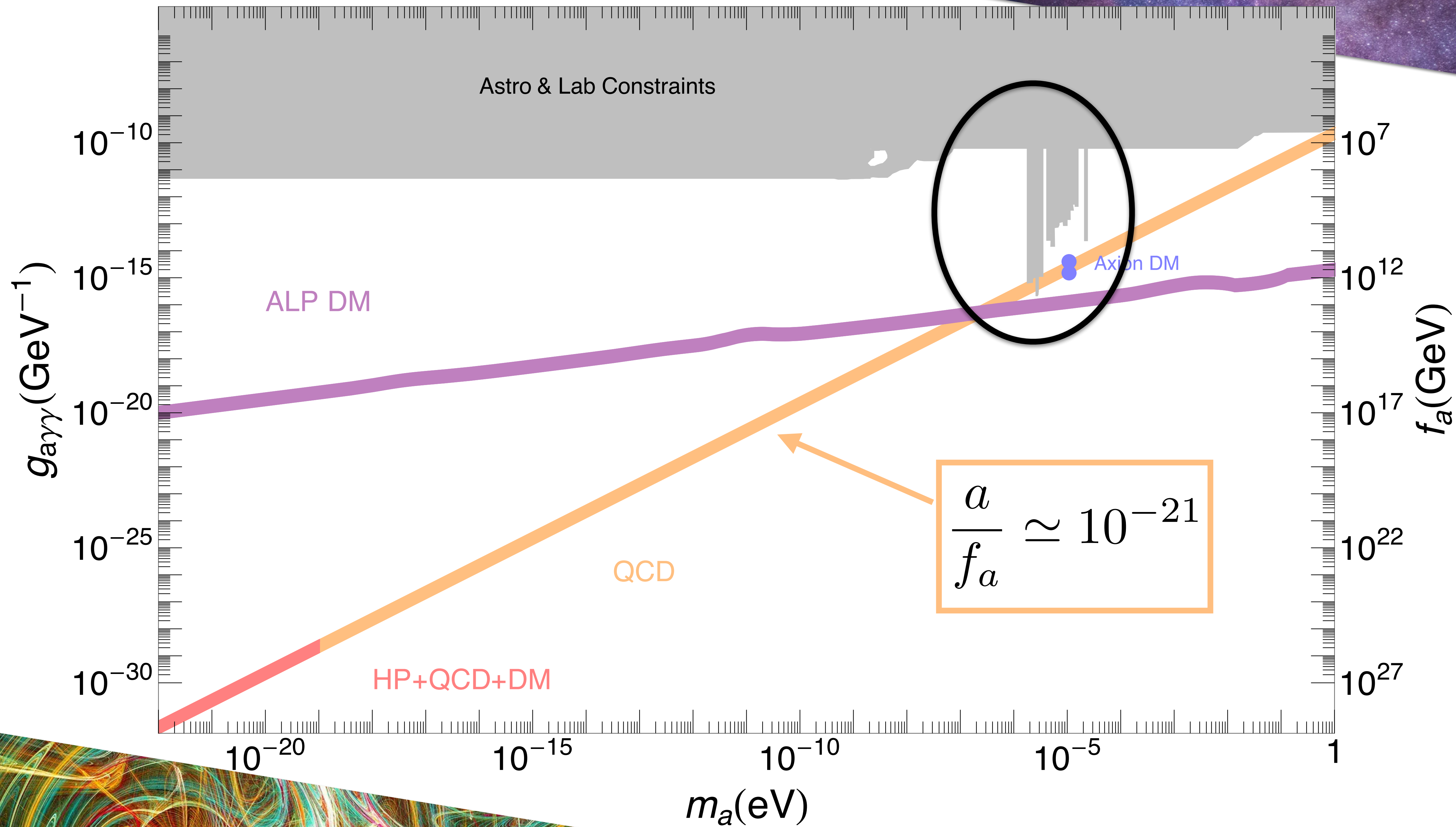

$$\omega_g$$

$$\lambda(T_*) < \frac{1}{H(T_*)} \quad \text{Causality}$$


A decorative header image showing a deep space scene with a purple and blue nebula and numerous stars.
$$\omega_g$$

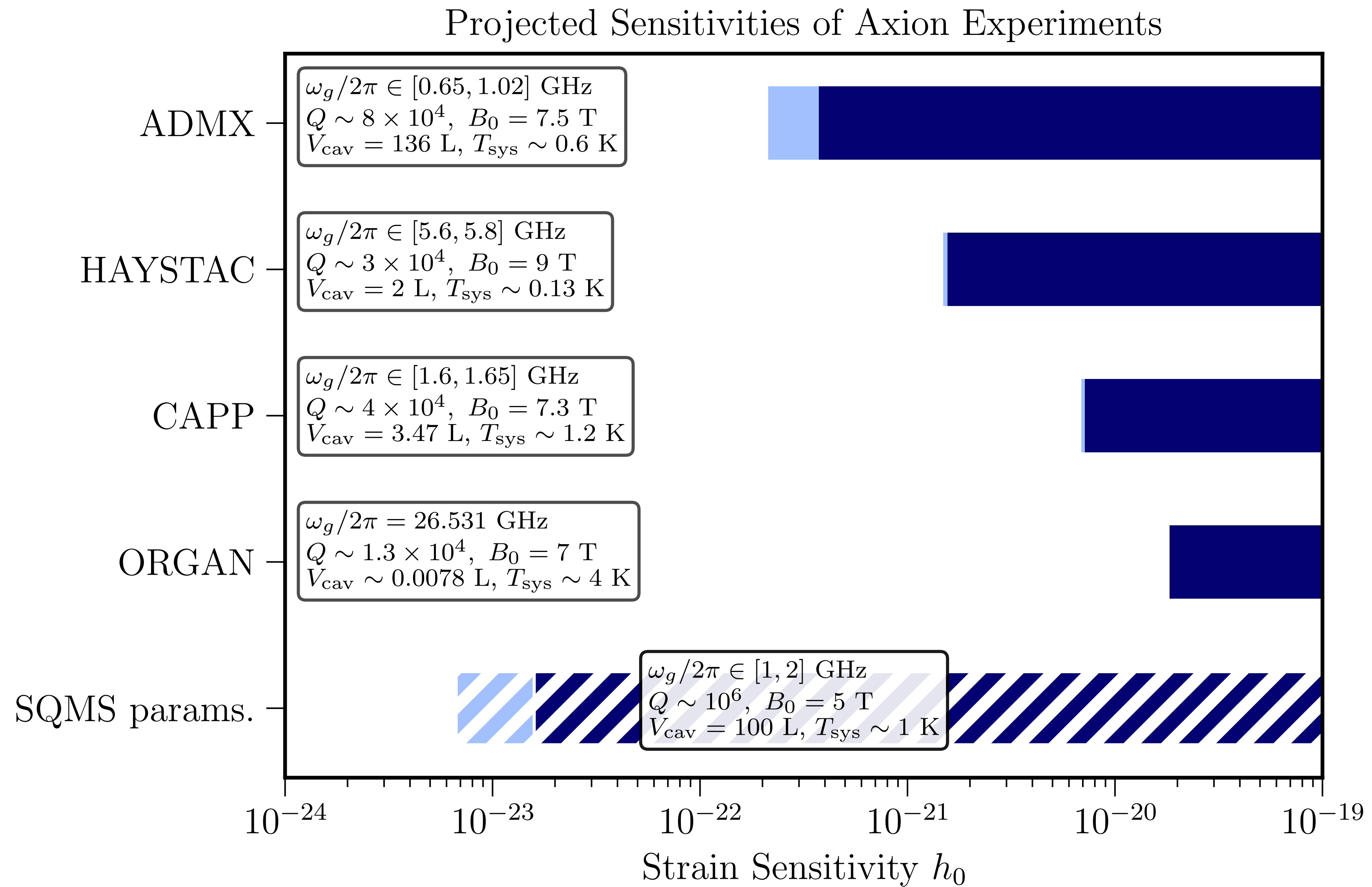
$$\lambda(T_*) < \frac{1}{H(T_*)} \quad \text{Causality}$$

$$\omega_0(T_*) = \omega(T_*) \frac{a(T_*)}{a_0} \gtrsim \boxed{100 \text{ MHz}} \left(\frac{T_*}{10^{15} \text{ GeV}} \right) \left(\frac{g_*(T_*)}{100} \right)^{1/6}$$
A decorative footer image featuring a complex, colorful, swirling pattern of lines in shades of green, yellow, and red.





GRAVITATIONAL WAVE DETECTION

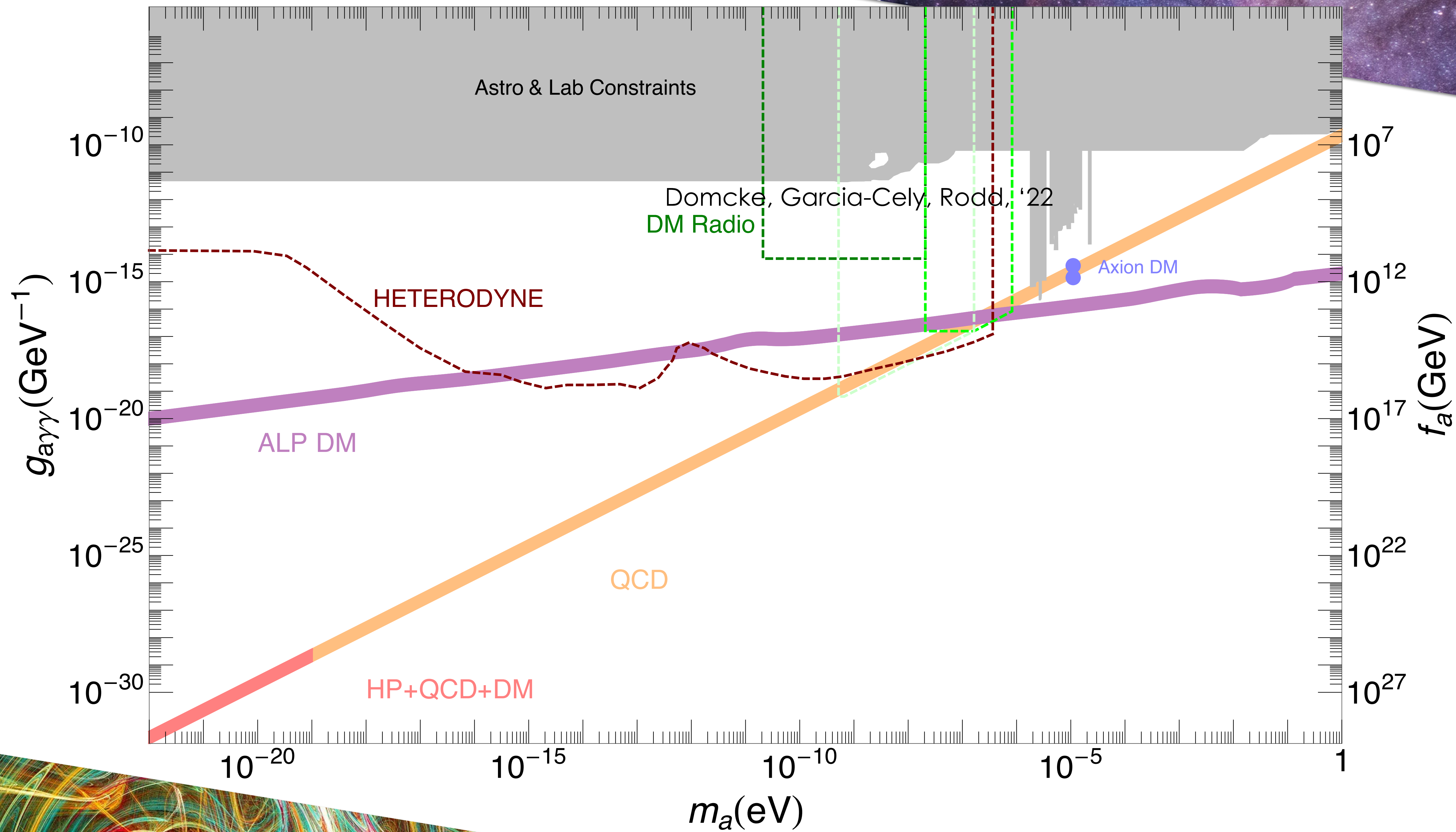


SUPERRADIANCE

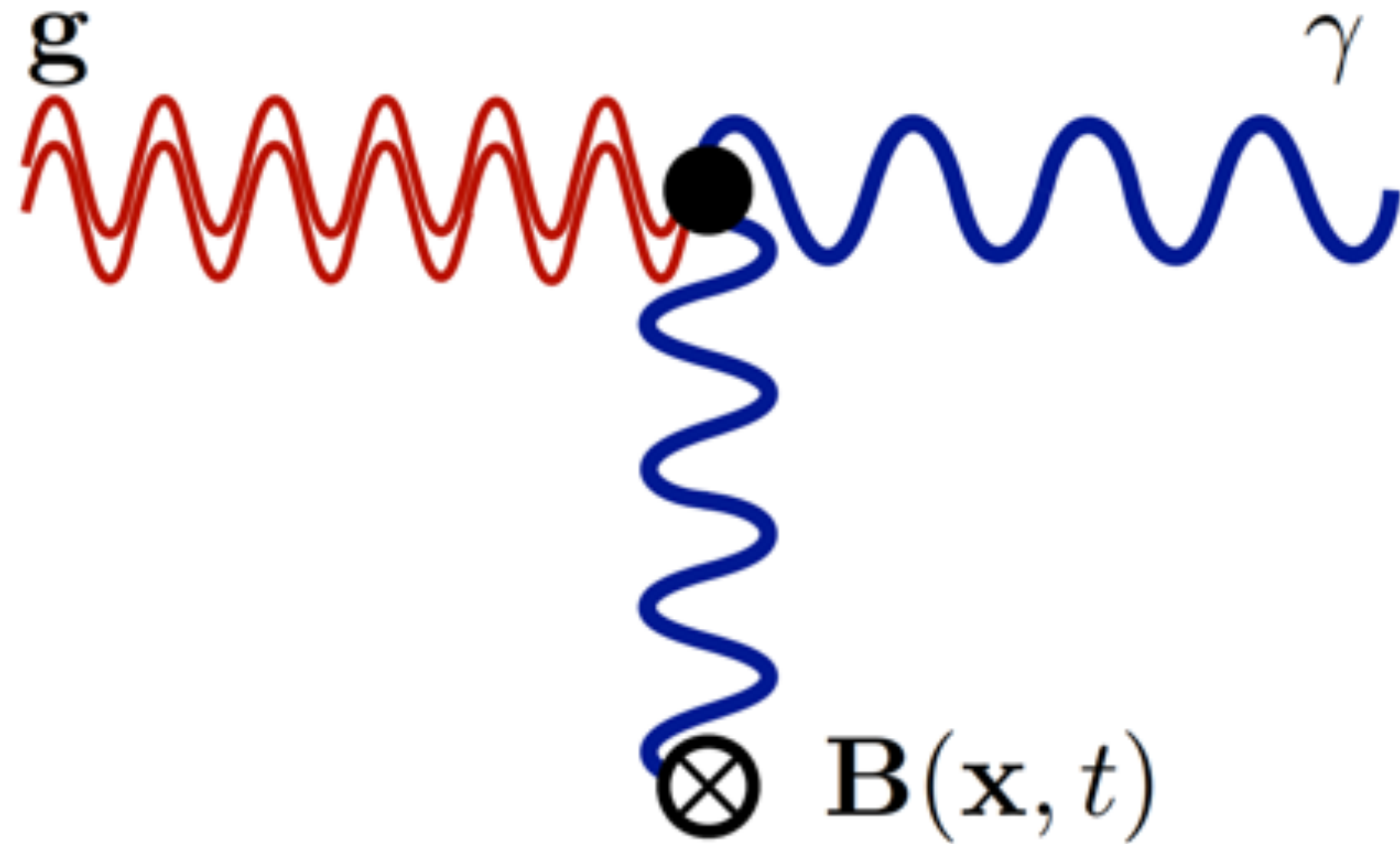
$$\omega_g \simeq 2m_a \simeq \text{GHz} \left(\frac{10^{-4} M_\odot}{M_{\text{BH}}} \right)$$

$$h \simeq 10^{-23} \left(\frac{1 \text{ pc}}{D} \right) \frac{\text{GHz}}{\omega_g}$$

Signal from:[Arvanitaki, Geraci '12]

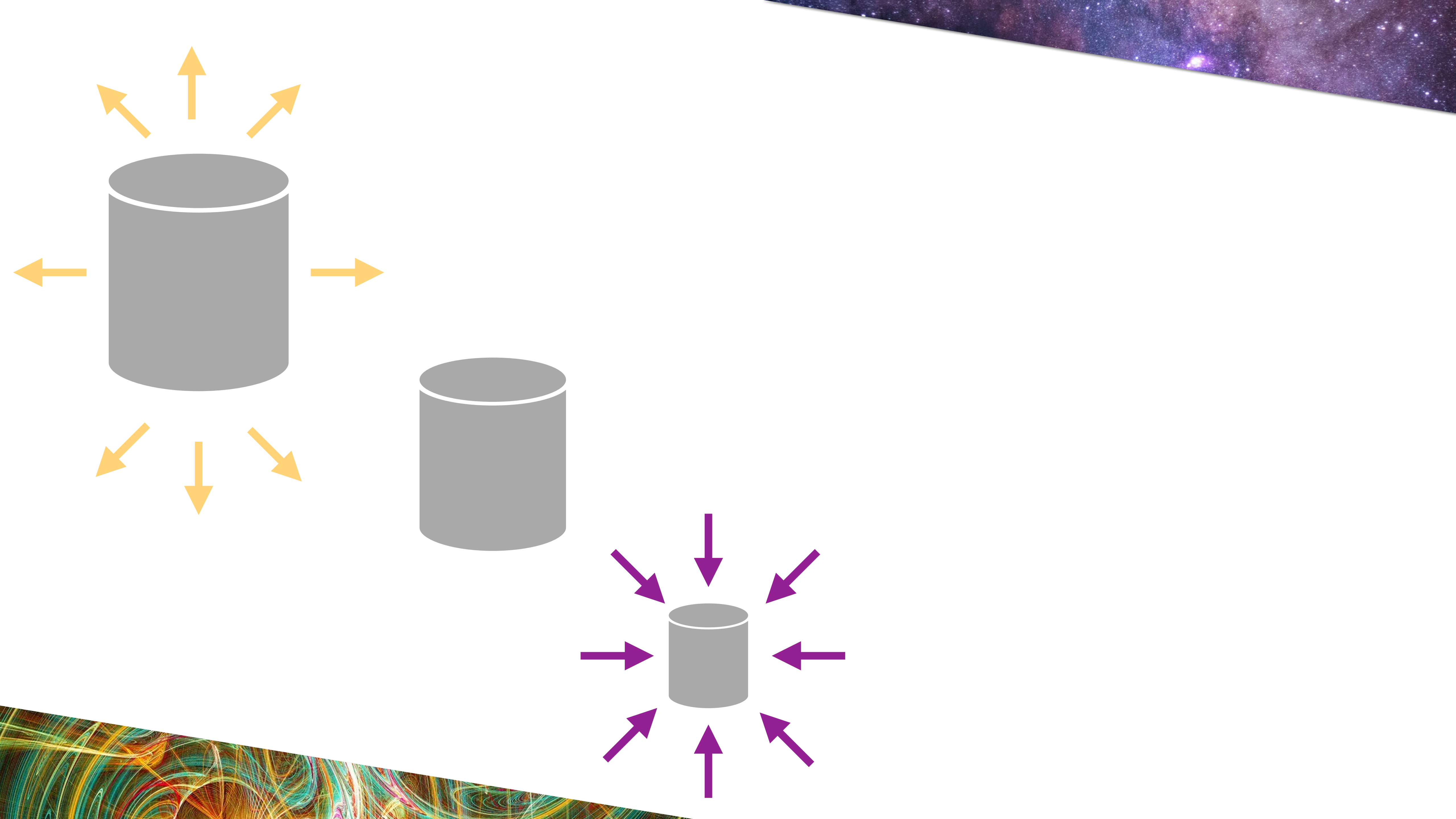


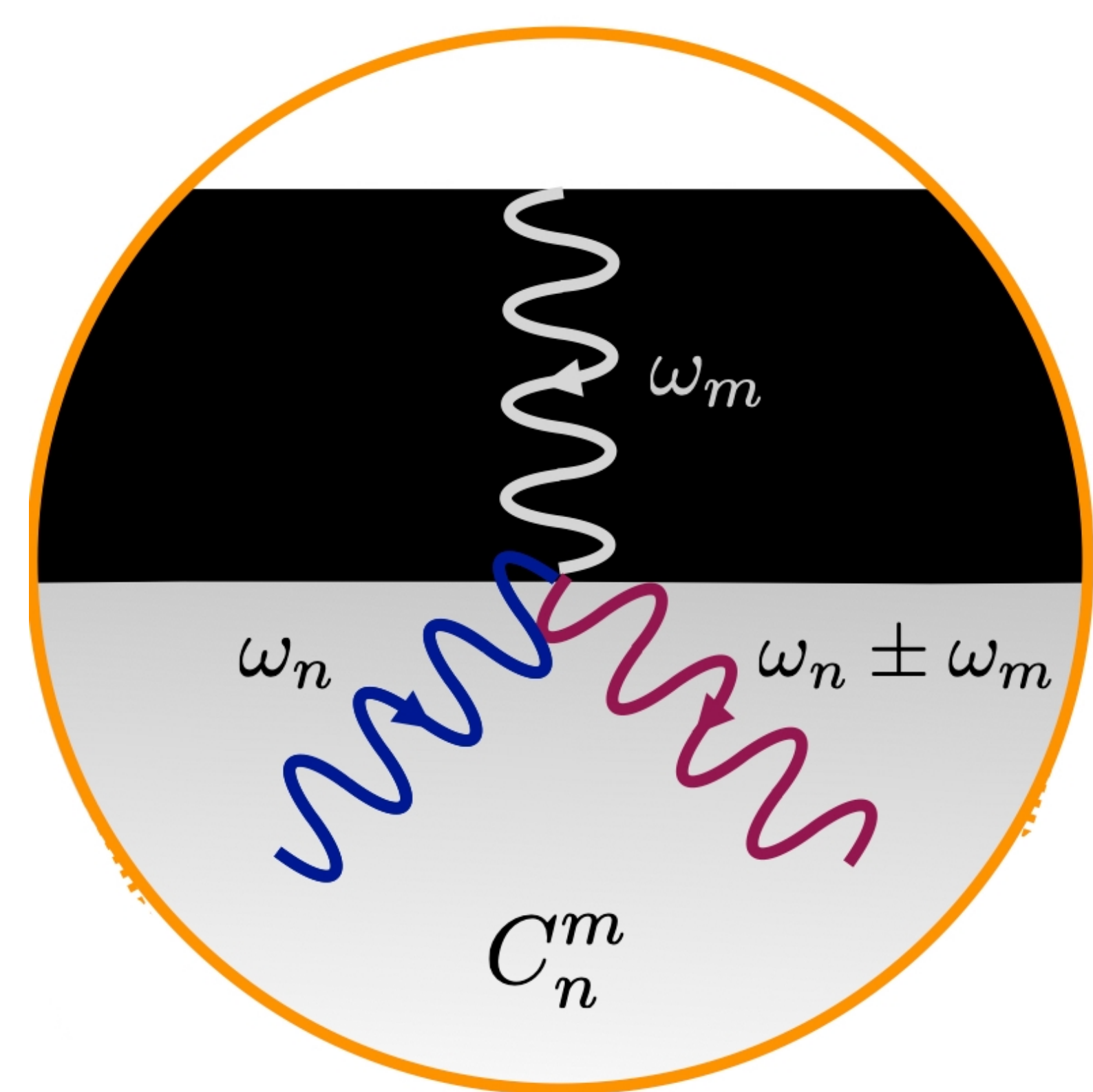
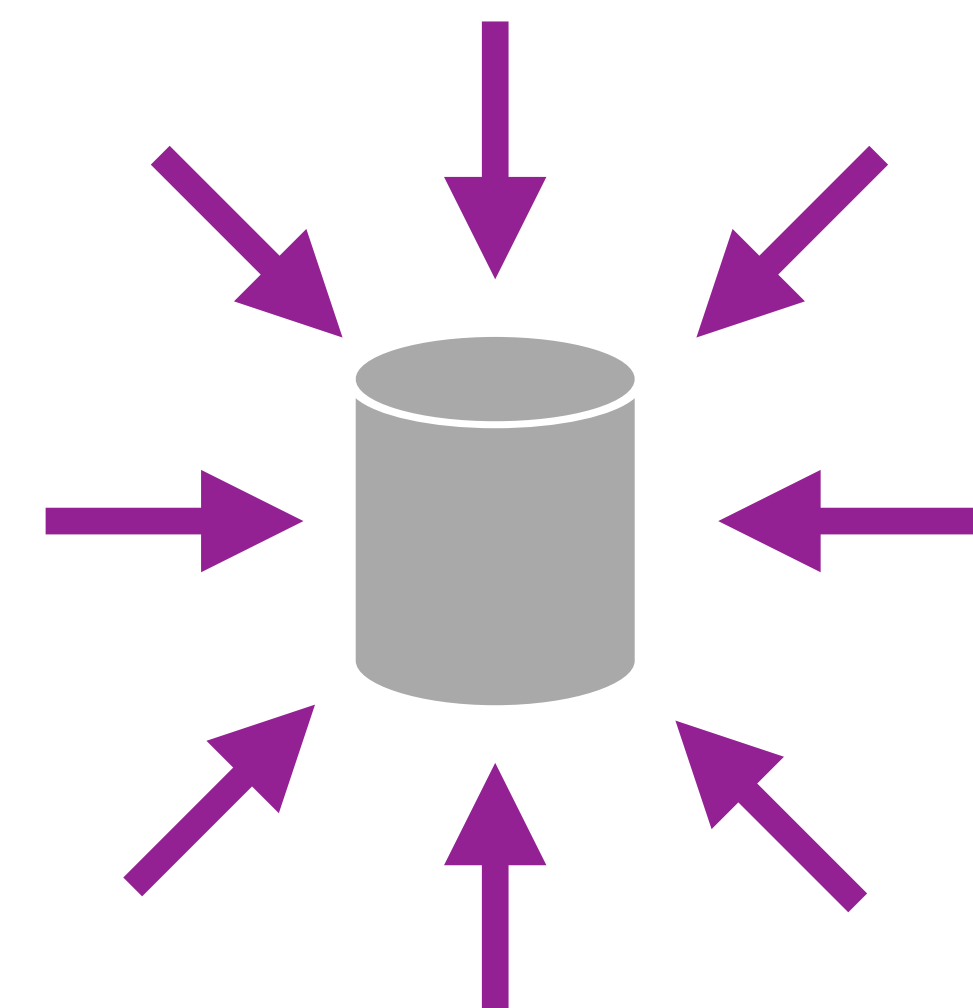
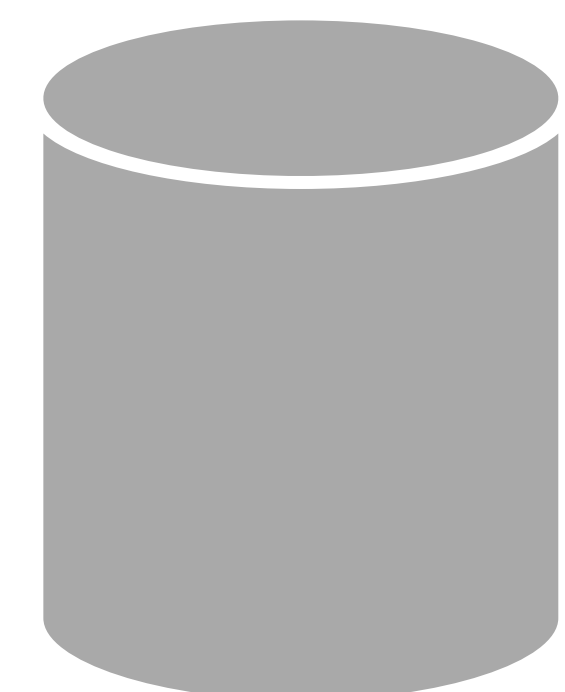
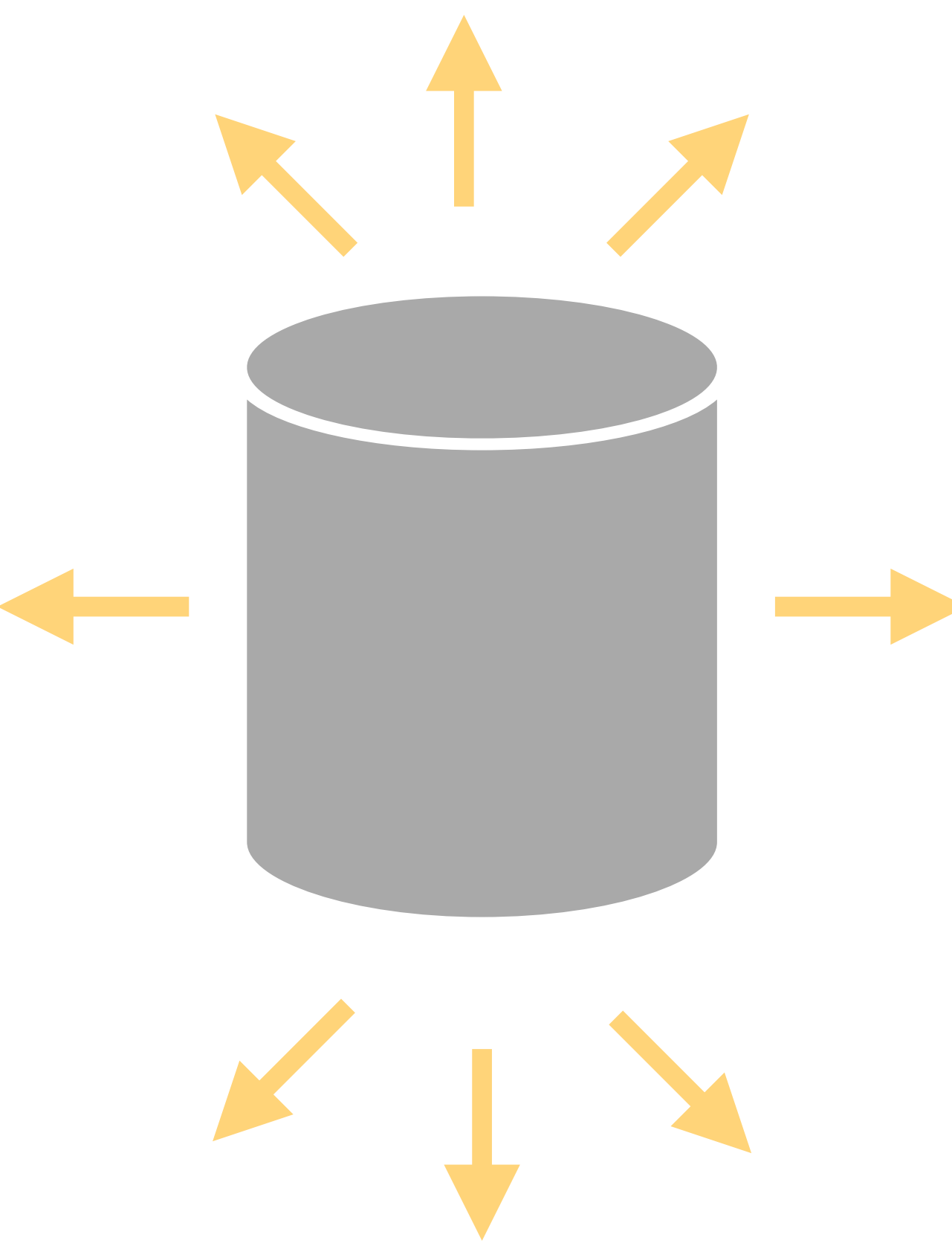
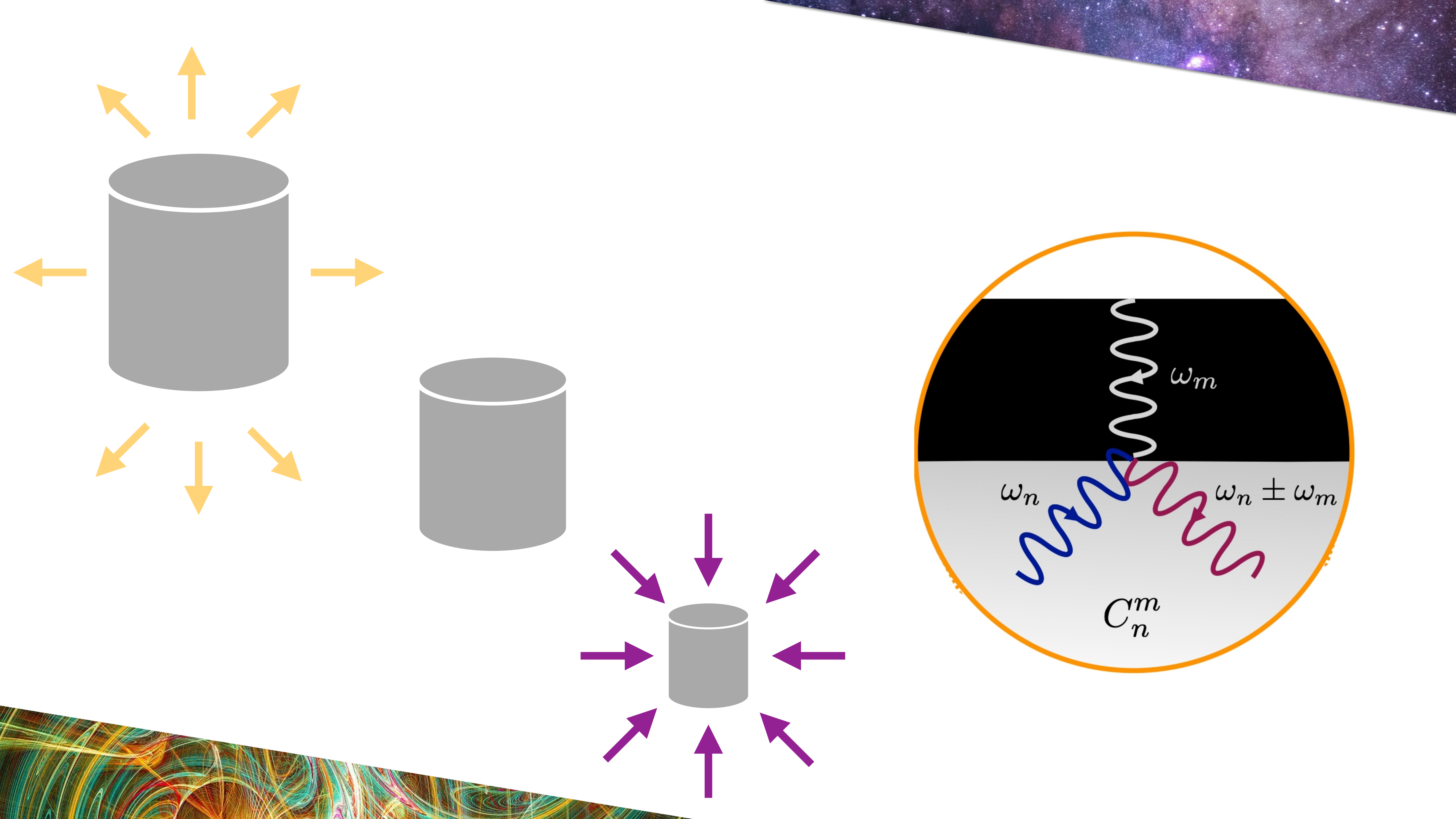
ELECTROMAGNETIC



$$E_{\text{sig}} \sim Q h E_0 (\omega_g L_{\text{cav}})^2$$

$$(\omega_g L_{\text{cav}}) \ll 1$$





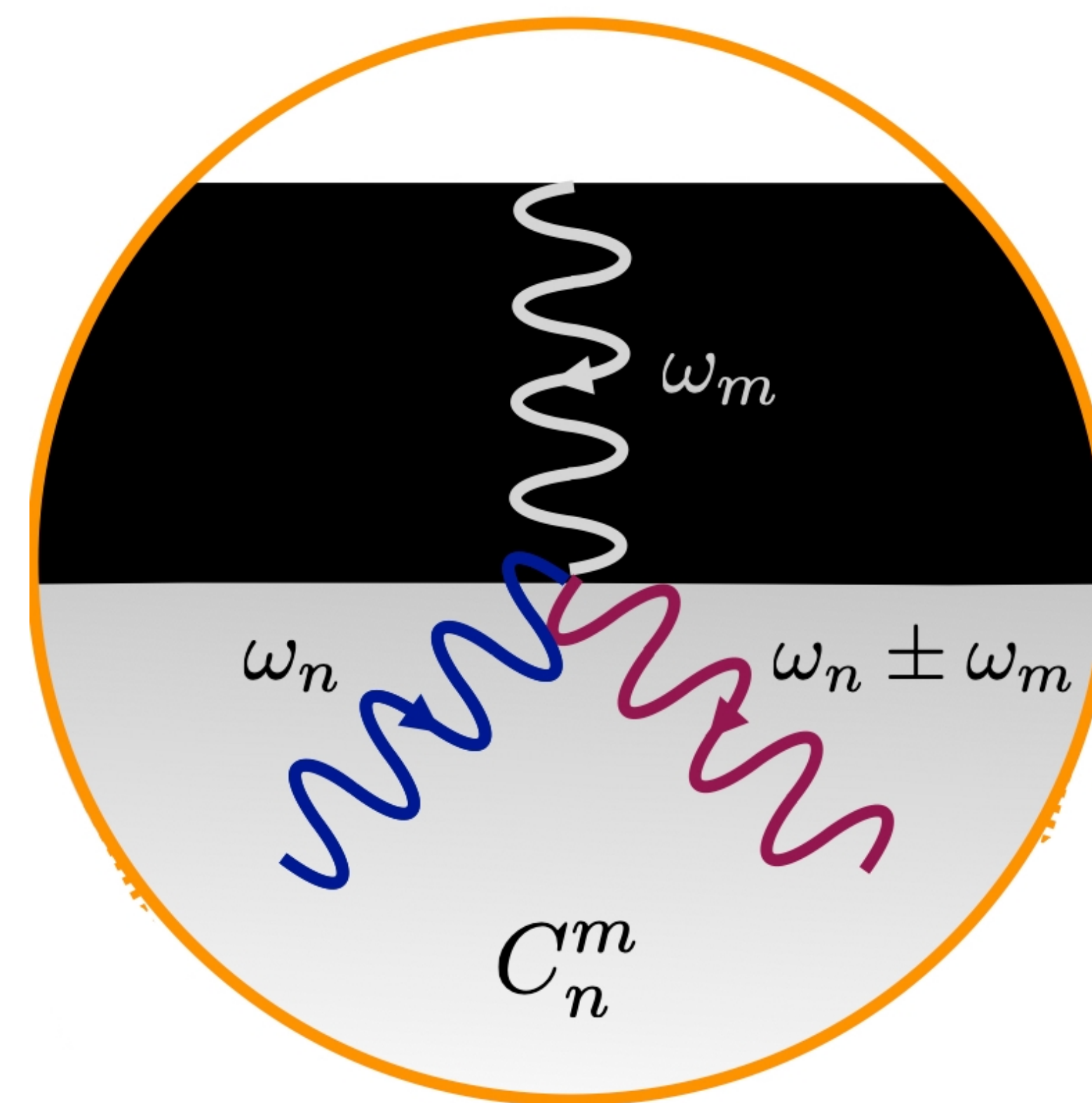
On resonance

$$E_{\text{sig}} \sim Q_m Q(hB_0)$$

Below resonance

$$E_{\text{sig}} \sim Q(hB_0) \frac{\omega_g^2}{(\omega_m^{\text{min}})^2}$$

MECHANICAL



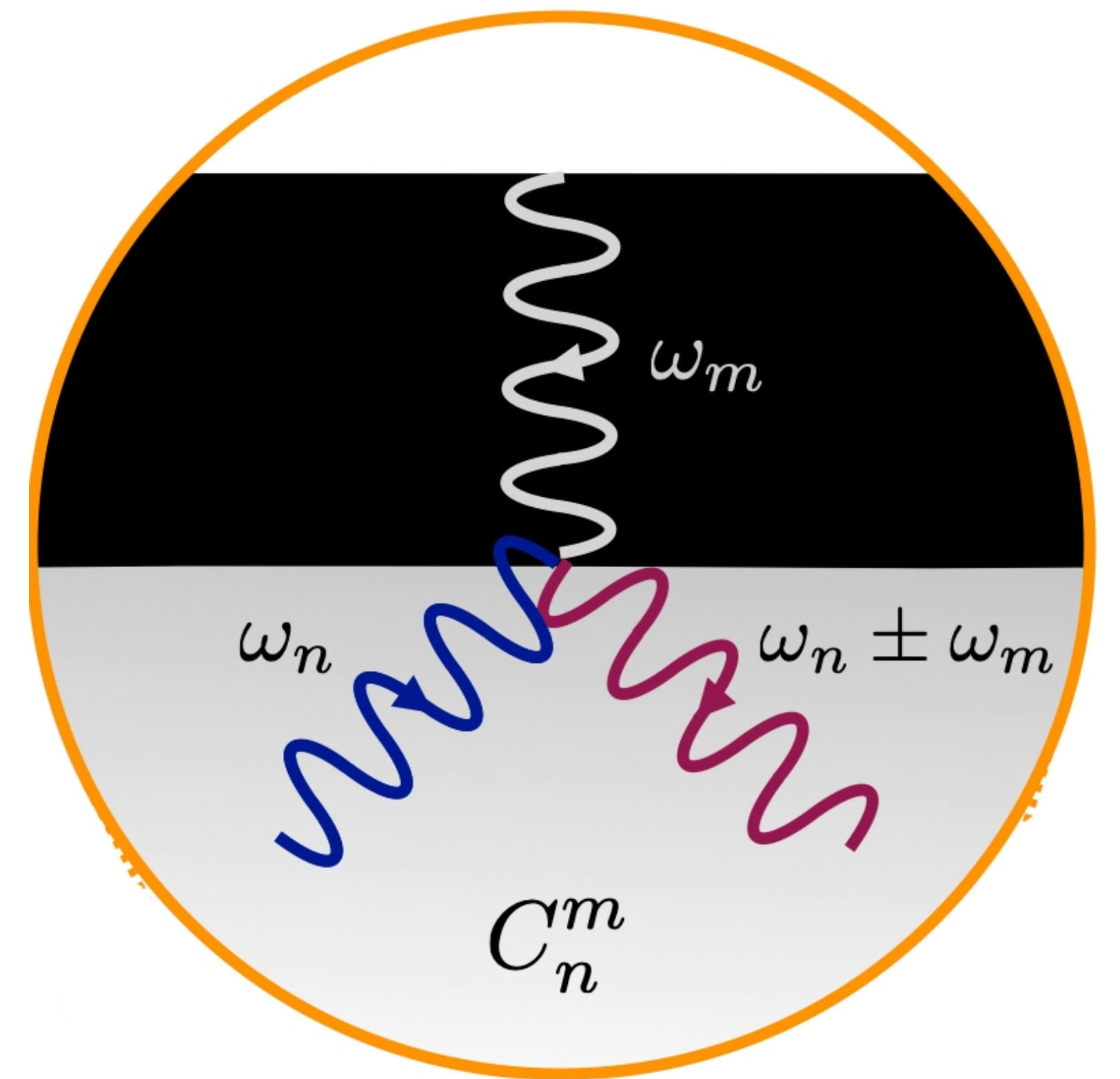
Mechanical Mode

$$\omega_m \sim \frac{c_s}{L_{\text{cav}}} < 1/L$$

$$\omega_m^{\text{min}} \simeq \text{kHz}$$

Niobium ~0.5 m cavity

MECHANICAL



[Caves '78], [Radicati, Pegoraro, Picasso '78]

CONCLUSION

- We have motivated dark matter candidates in a large range of masses
- We have probed the heaviest particle candidates for decades
- Ultralight dark matter is just as motivated, but experimentally we have only just started to scratch the surface!
- There is a fascinating interplay between ultralight dark matter and gravitational waves searches

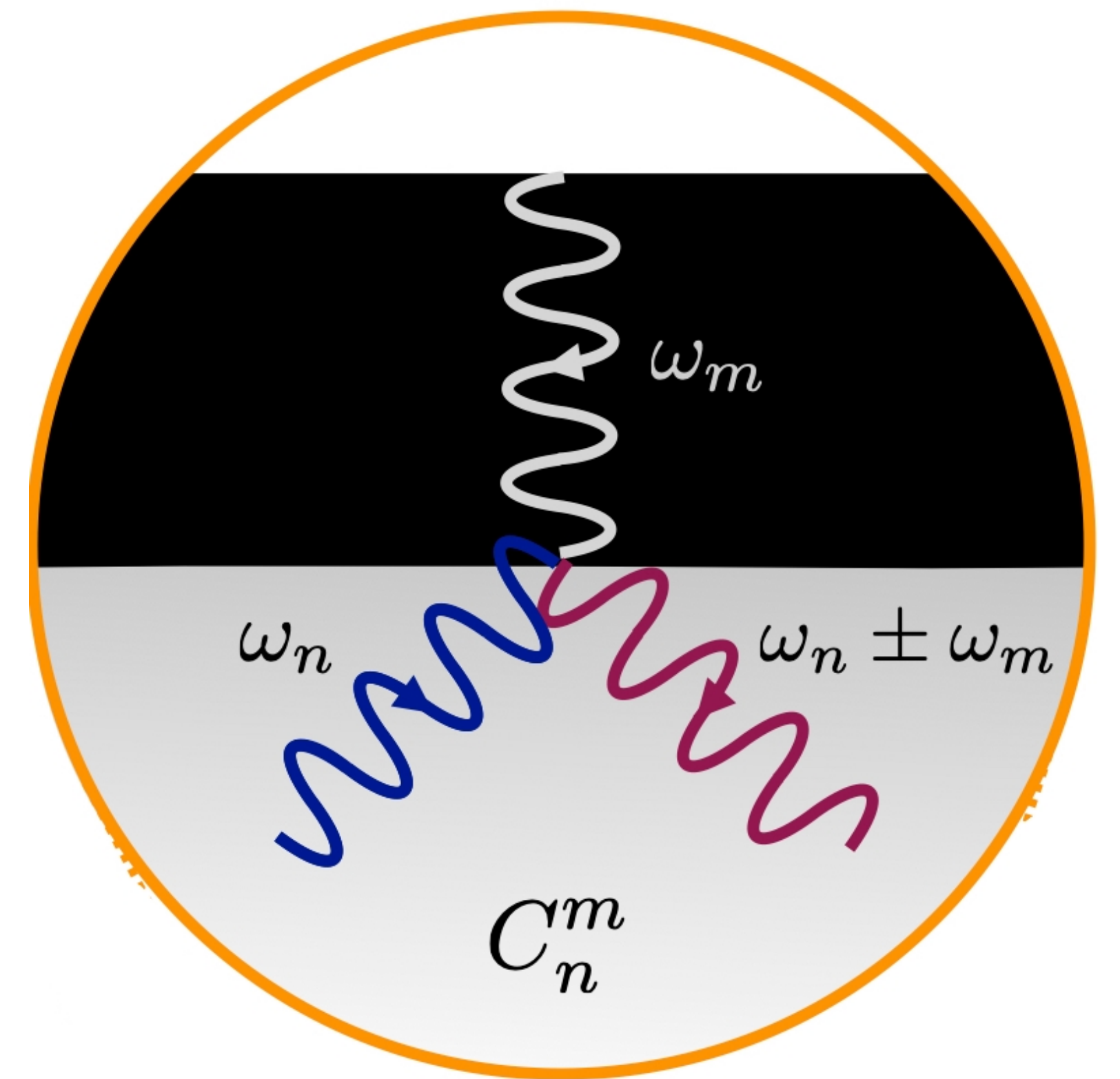
BACKUP

Mechanical Mode

$$\left(\partial_t^2 + \frac{\omega_m}{Q} \partial_t + \omega_m^2 \right) \underline{u_m} = \frac{F_m}{M_{\text{cav}}}$$

$F_m \sim$ Riemann

MECHANICAL



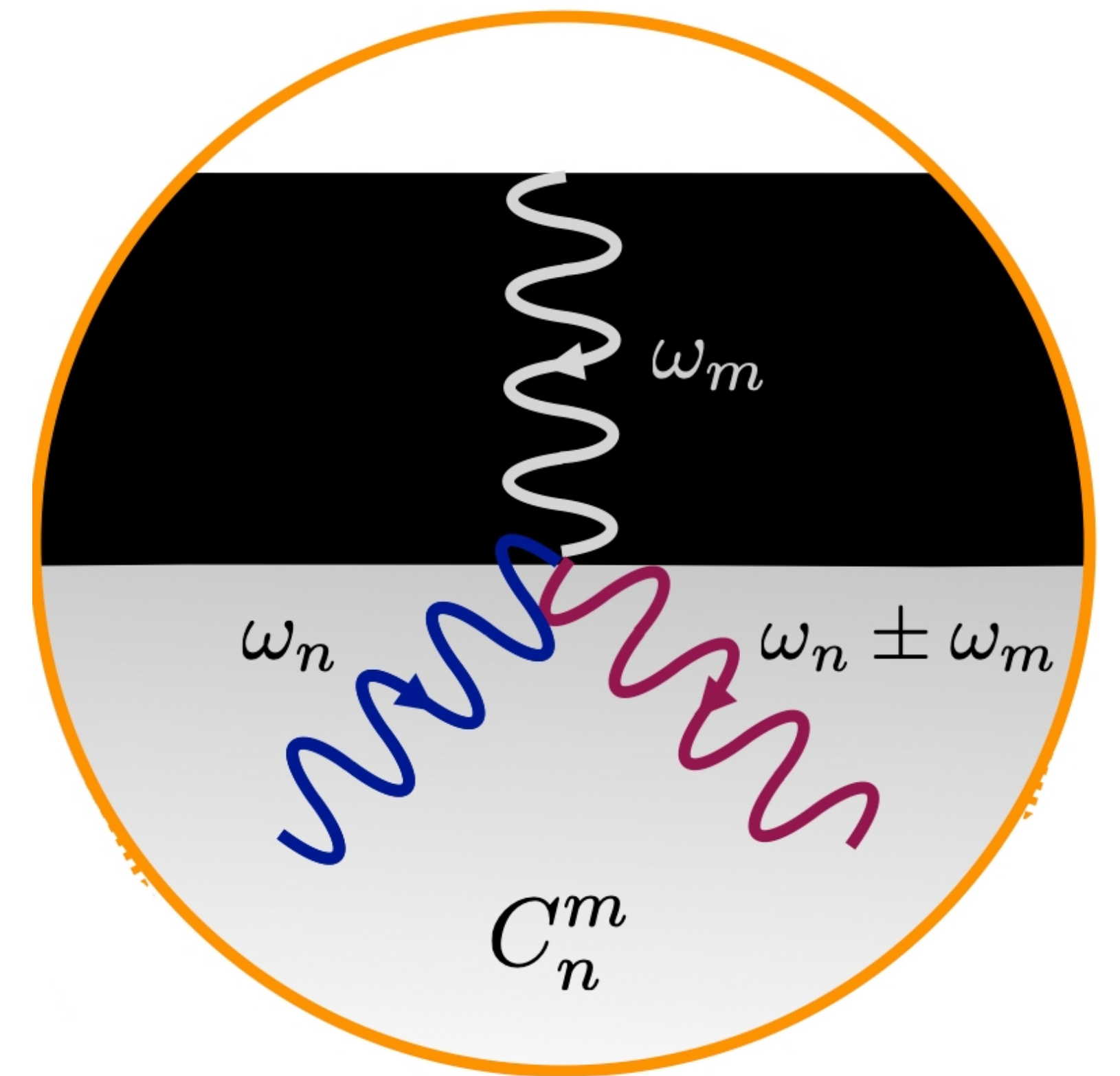
Mechanical Mode

$$\left(\partial_t^2 + \frac{\omega_m}{Q} \partial_t + \omega_m^2 \right) \underline{u_m} = \frac{F_m}{M_{\text{cav}}}$$

$$F_m \sim \text{Riemann}$$

$$u_m \sim Q \frac{\omega_g^2}{\omega_m^2} (h L_{\text{cav}})$$

MECHANICAL



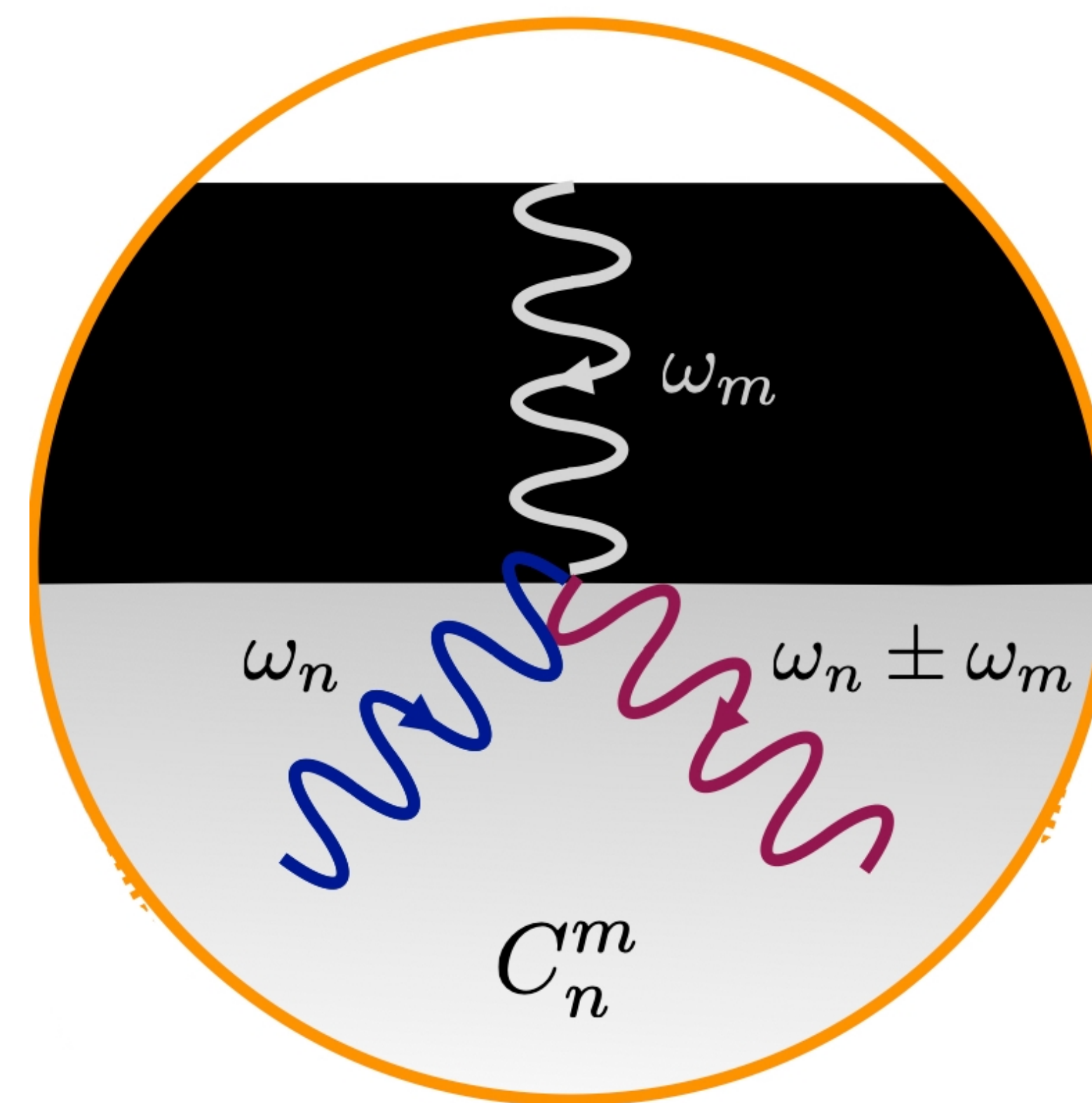
On resonance

$$E_{\text{sig}} \sim Q_m Q_1 (hB_0)$$

Below resonance

$$E_{\text{sig}} \sim Q_1 (hB_0) \frac{\omega_g^2}{(\omega_m^{\text{min}})^2}$$

MECHANICAL



MADMAX and LAMPOST

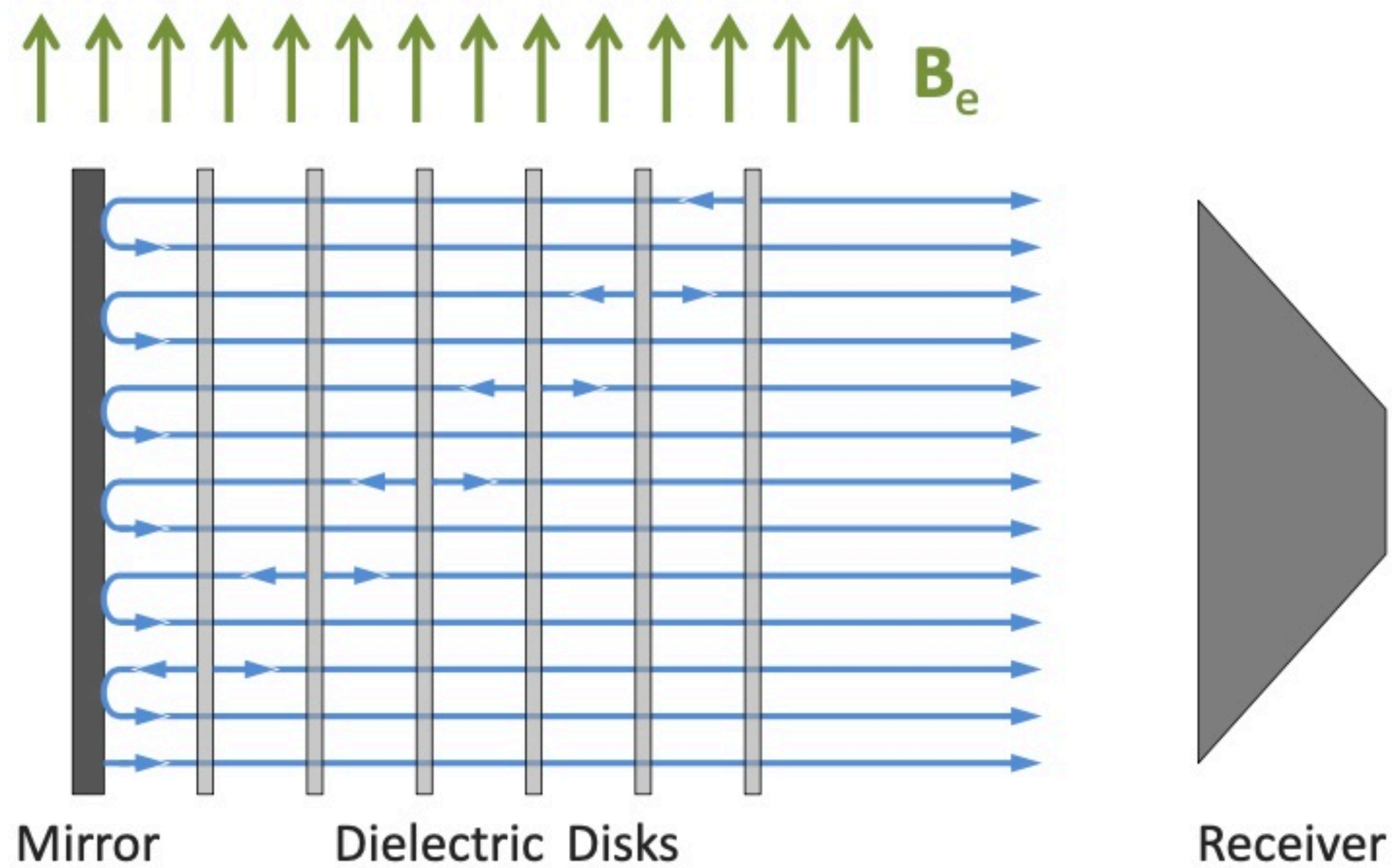
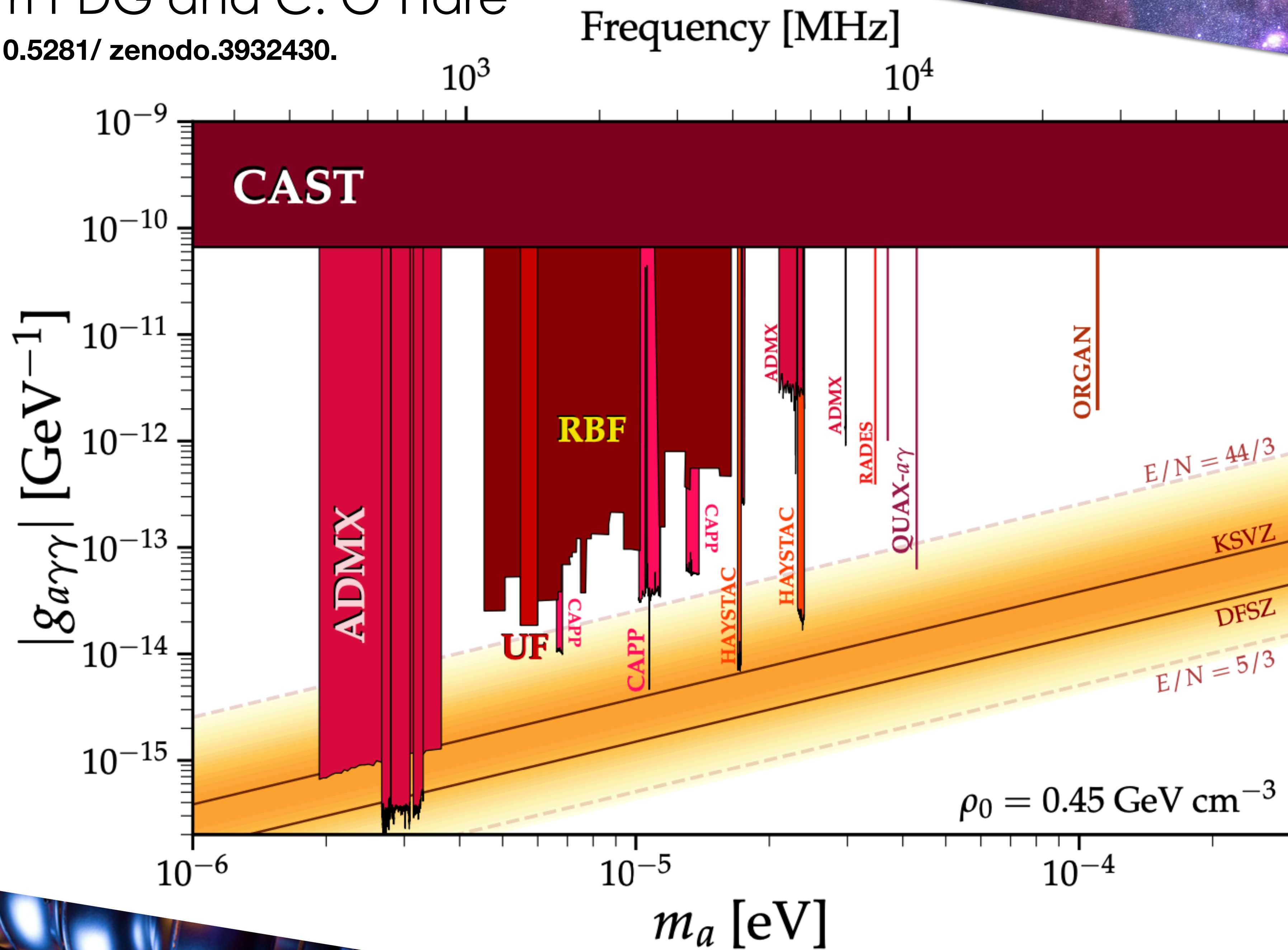


Figure from PDG and C. O'Hare
<https://doi.org/10.5281/zenodo.3932430>.



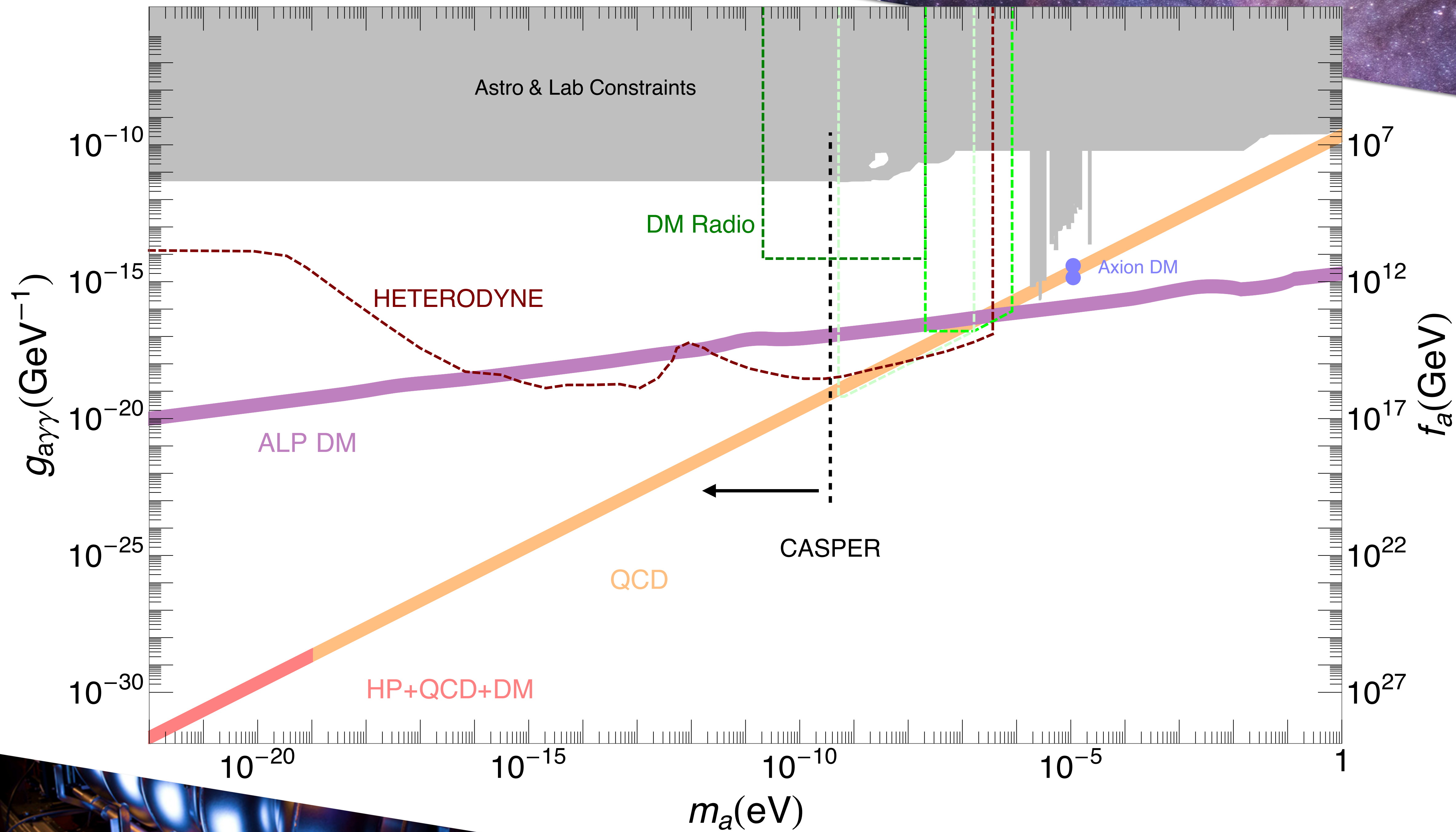
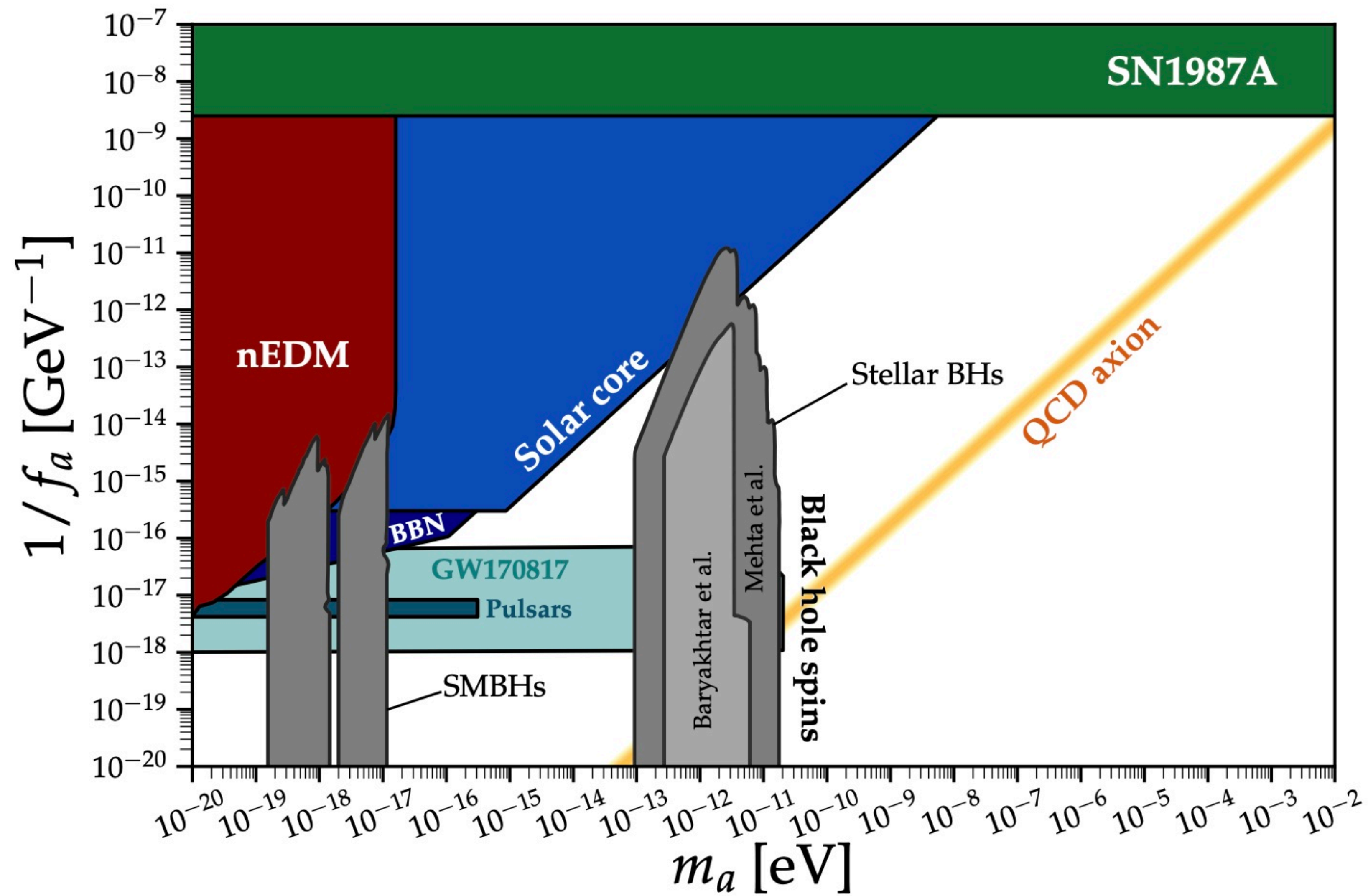


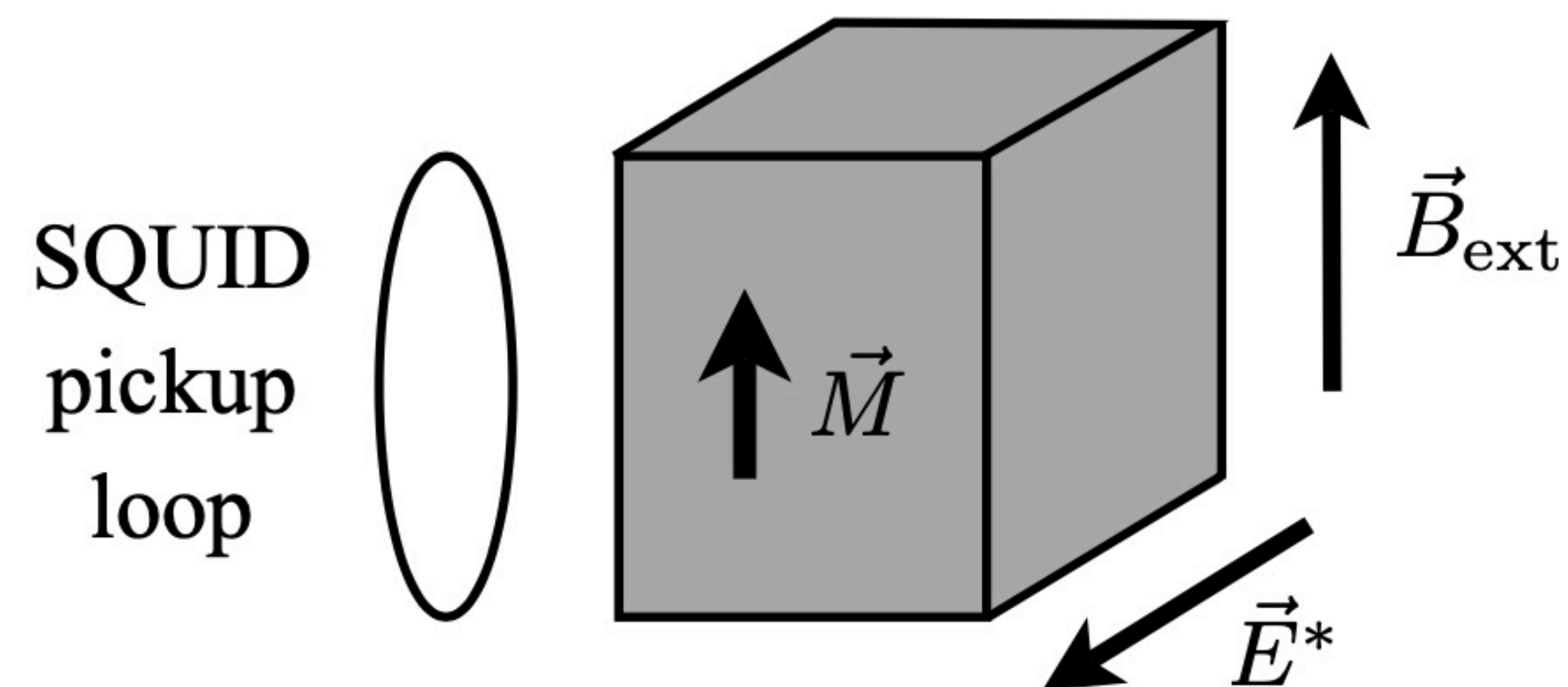
Figure from PDG and C. O'Hare

<https://doi.org/10.5281/zenodo.3932430>.



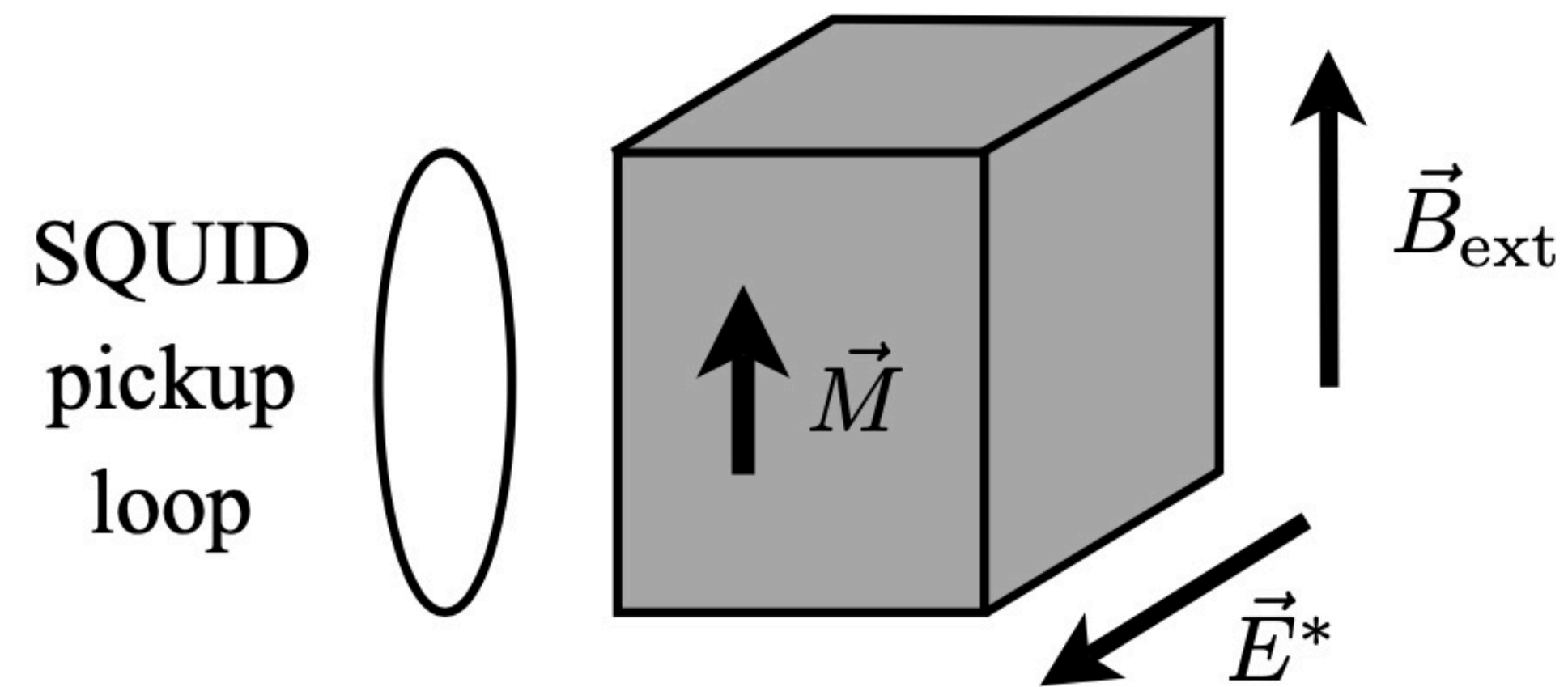
Valid for QCD Axion!

$$H \simeq \frac{a}{m_N f_a} \vec{\sigma} \cdot \vec{E}$$

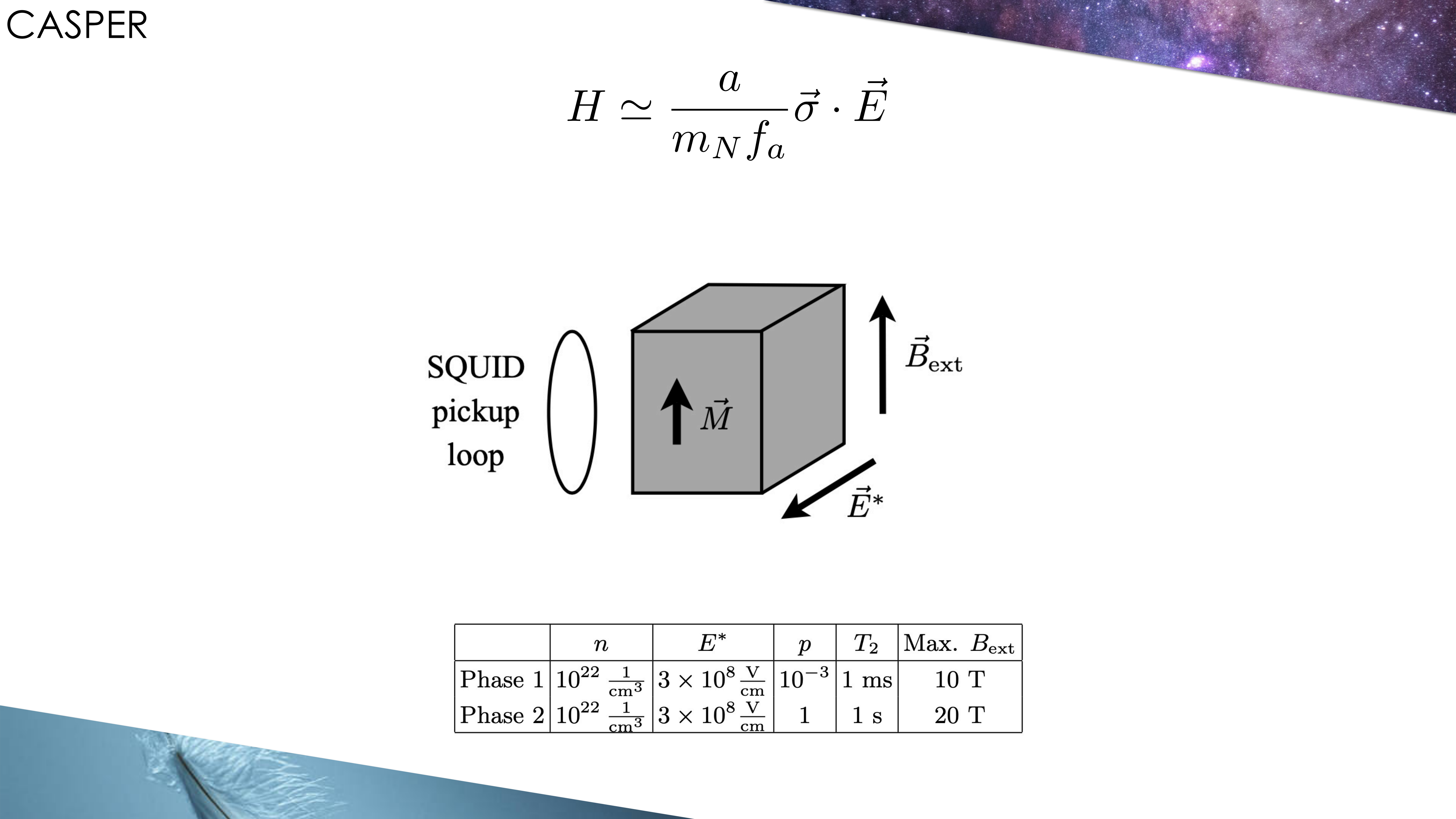


Without the axion you have a magnetisation component precessing around B (Larmor frequency)

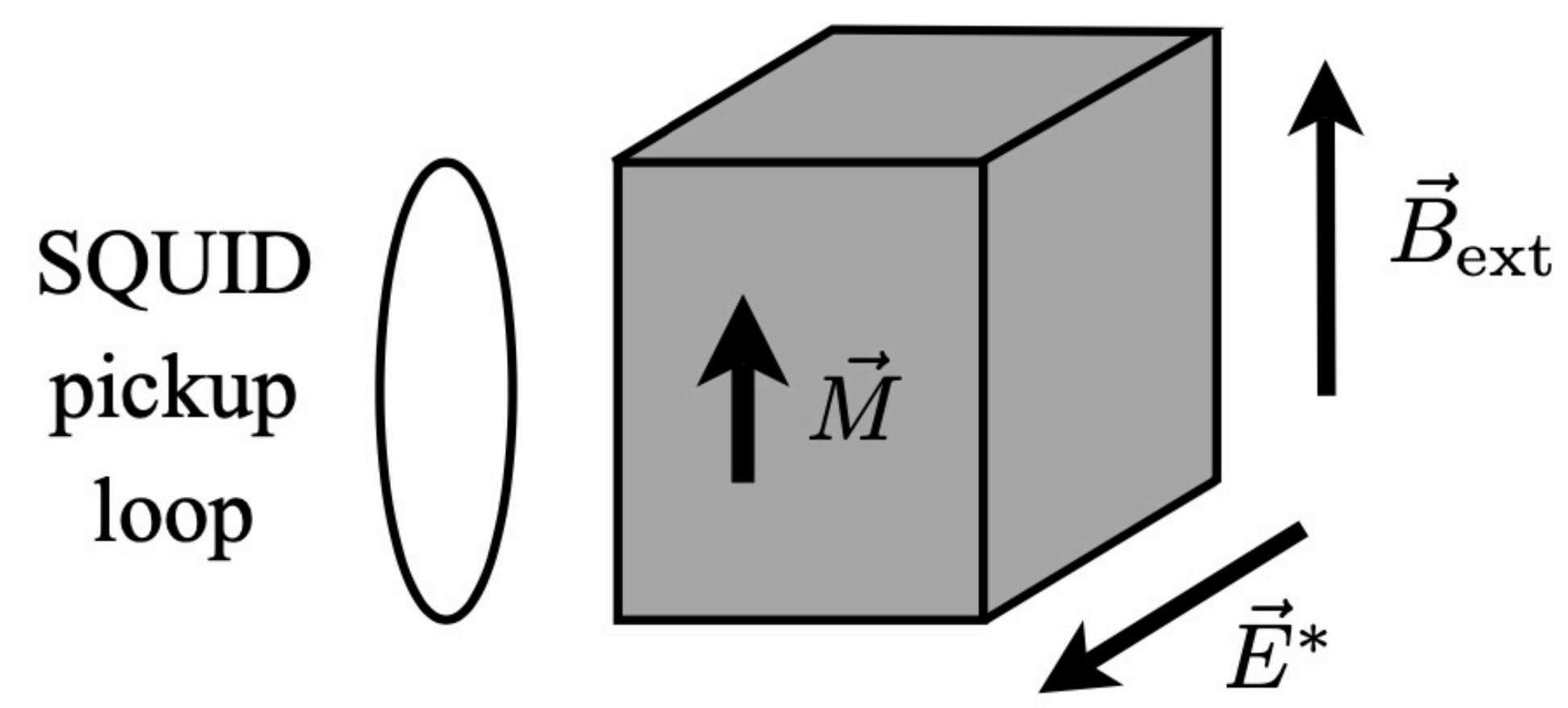
$$H \simeq \frac{a}{m_N f_a} \vec{\sigma} \cdot \vec{E}$$



$$M(t) \approx np\mu E^* \epsilon_S d_n \frac{\sin \left[\left(\frac{2\mu B_{\text{ext}} - m_a c^2}{\hbar} \right) t \right]}{\frac{2\mu B_{\text{ext}} - m_a c^2}{\hbar}} \sin (2\mu B_{\text{ext}} t)$$



$$H \simeq \frac{a}{m_N f_a} \vec{\sigma} \cdot \vec{E}$$

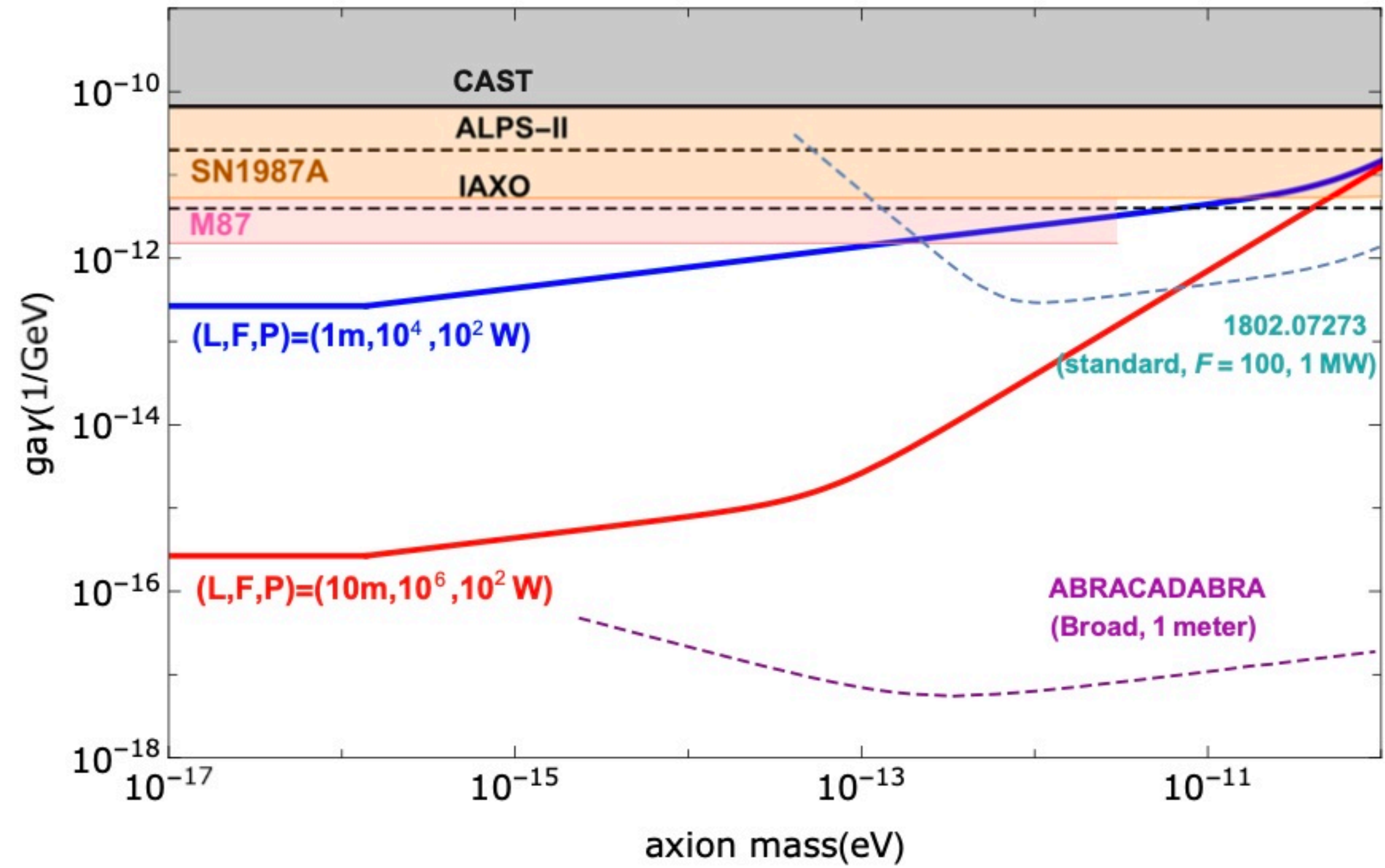


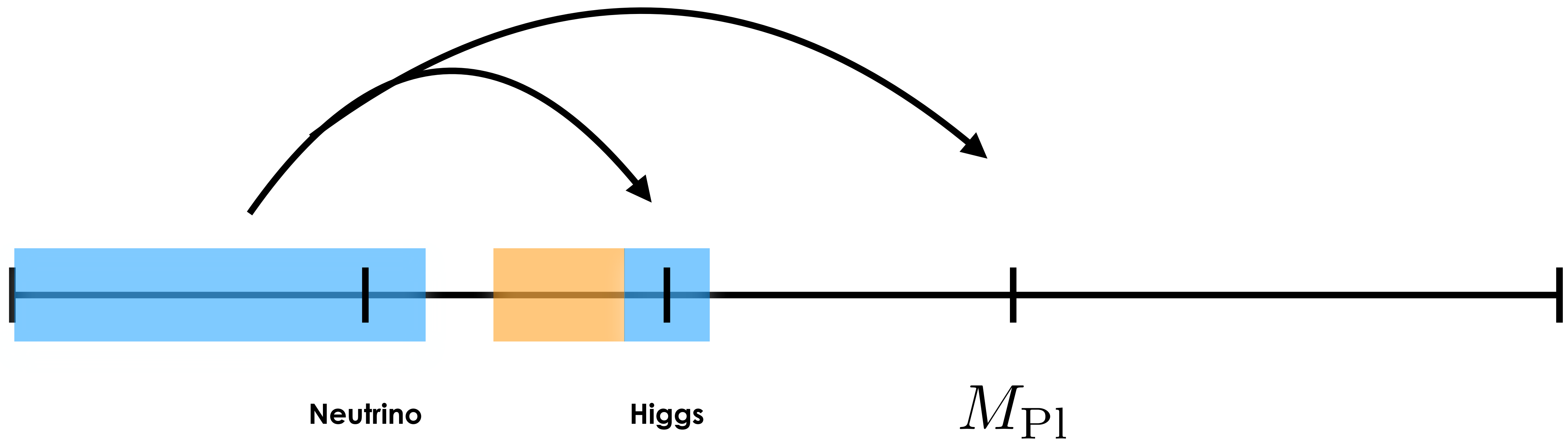
	n	E^*	p	T_2	Max. B_{ext}
Phase 1	$10^{22} \frac{1}{\text{cm}^3}$	$3 \times 10^8 \frac{\text{V}}{\text{cm}}$	10^{-3}	1 ms	10 T
Phase 2	$10^{22} \frac{1}{\text{cm}^3}$	$3 \times 10^8 \frac{\text{V}}{\text{cm}}$	1	1 s	20 T



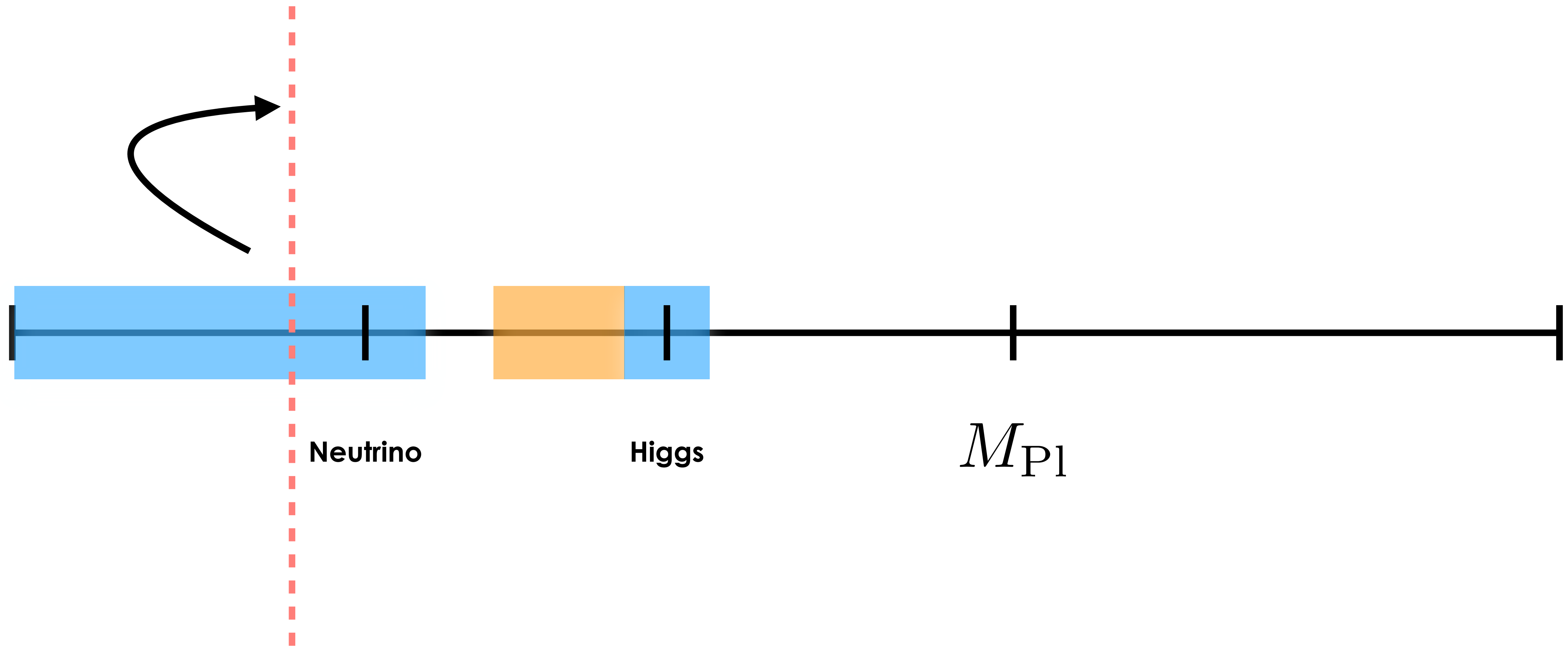
DANCE

Obata, Fujita, Michimura, '18





New Symmetry



AXION

AXION

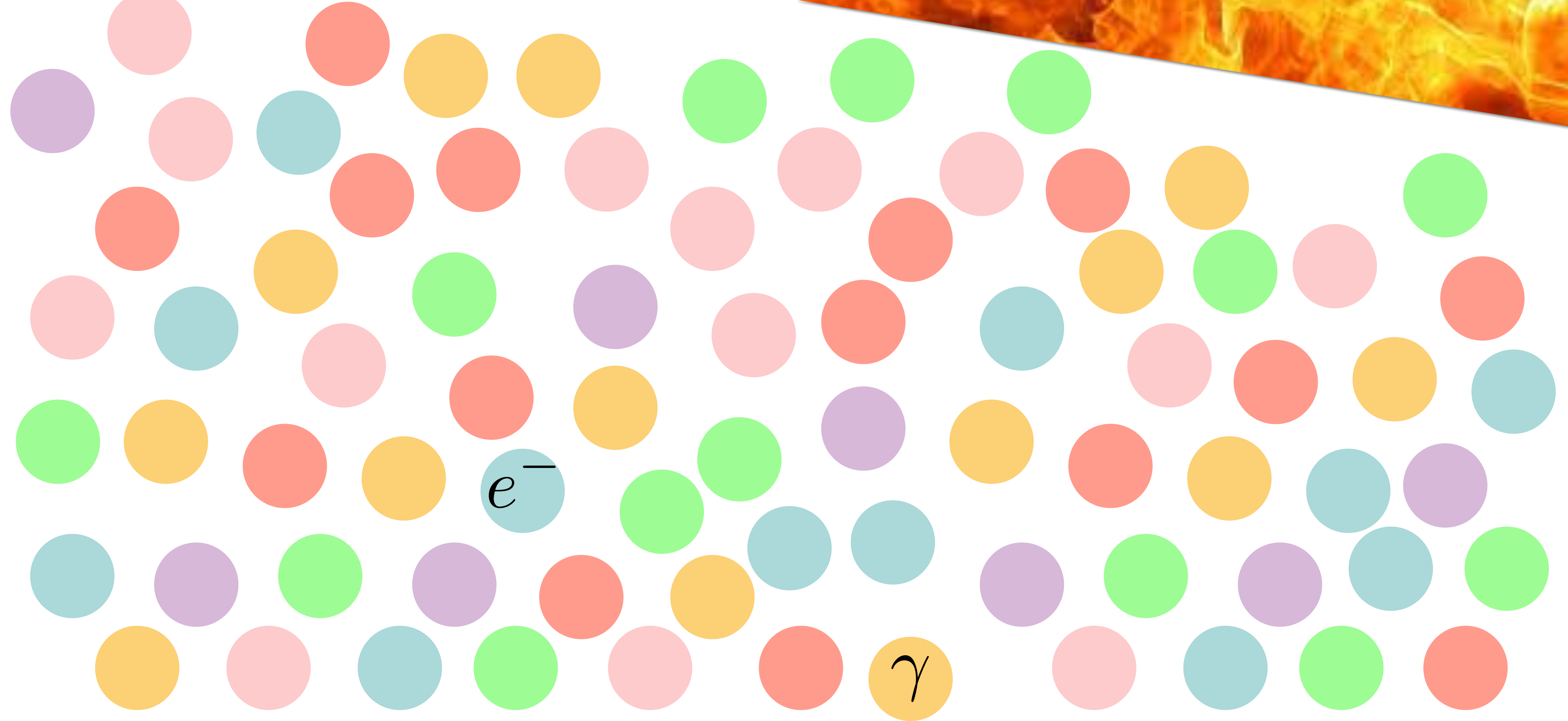
- **Minimal Extension of what we know to exist**
- **Naturally light**
- Simple and predictive cosmology
- Solve two other big problems in particle physics



Higgs

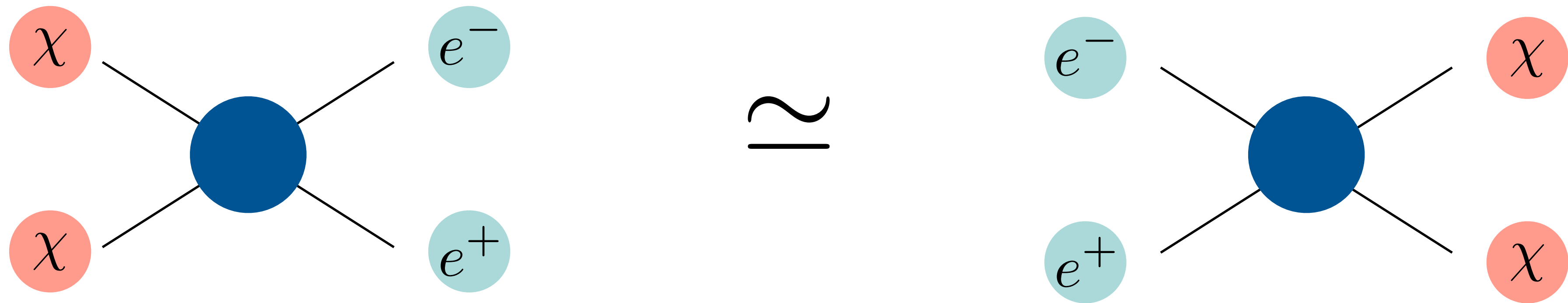
The background is a diagonal split. The top-right half features a close-up of bright orange and yellow flames, representing high temperature. The bottom-left half features a close-up of light blue ice with sharp, jagged edges, representing low temperature. A solid light blue triangle is in the top-left corner.

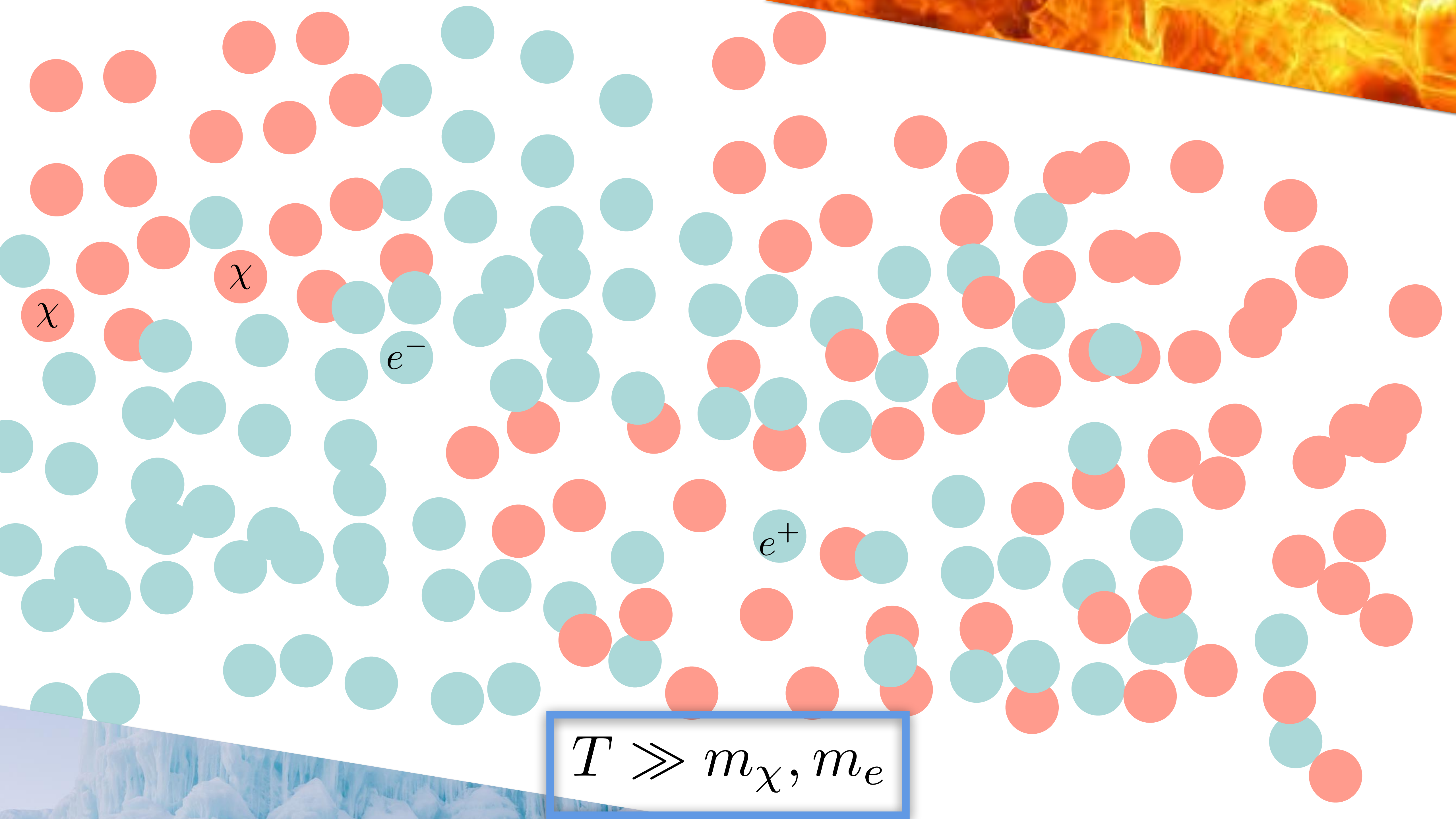
THERMAL DARK MATTER



Standard Model Thermal Bath

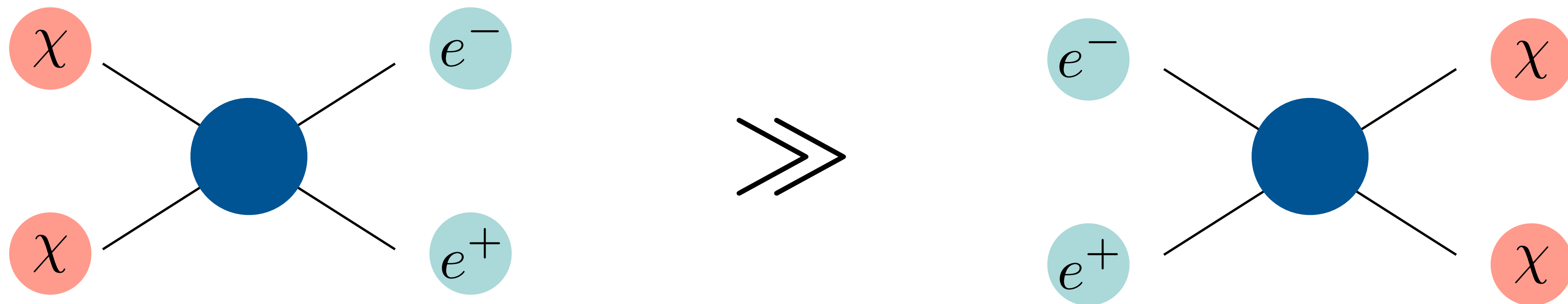
$$T \gg m_\chi, m_e$$





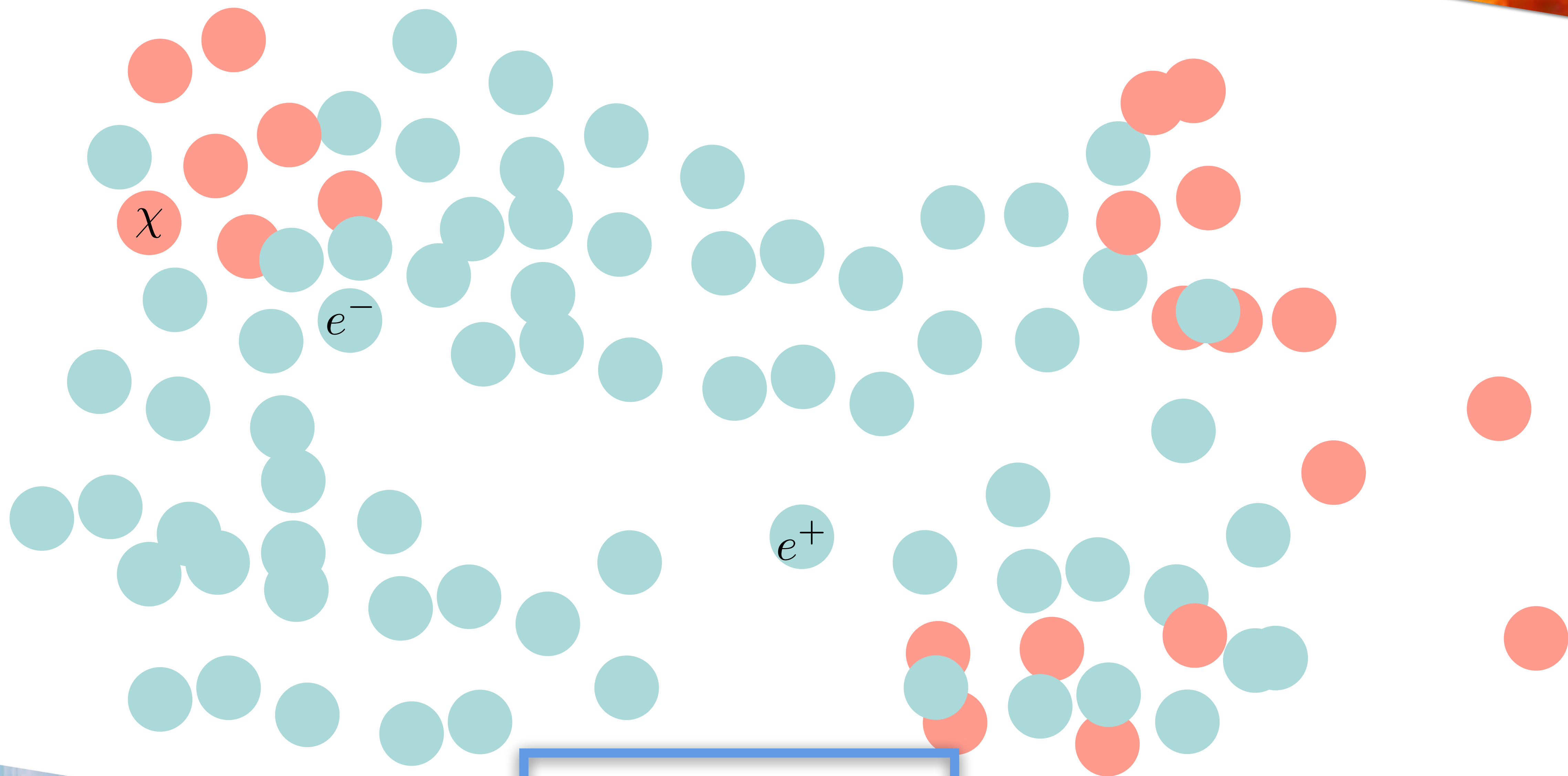
$$T \gg m_\chi, m_e$$


$$m_e \ll T \lesssim m_\chi$$



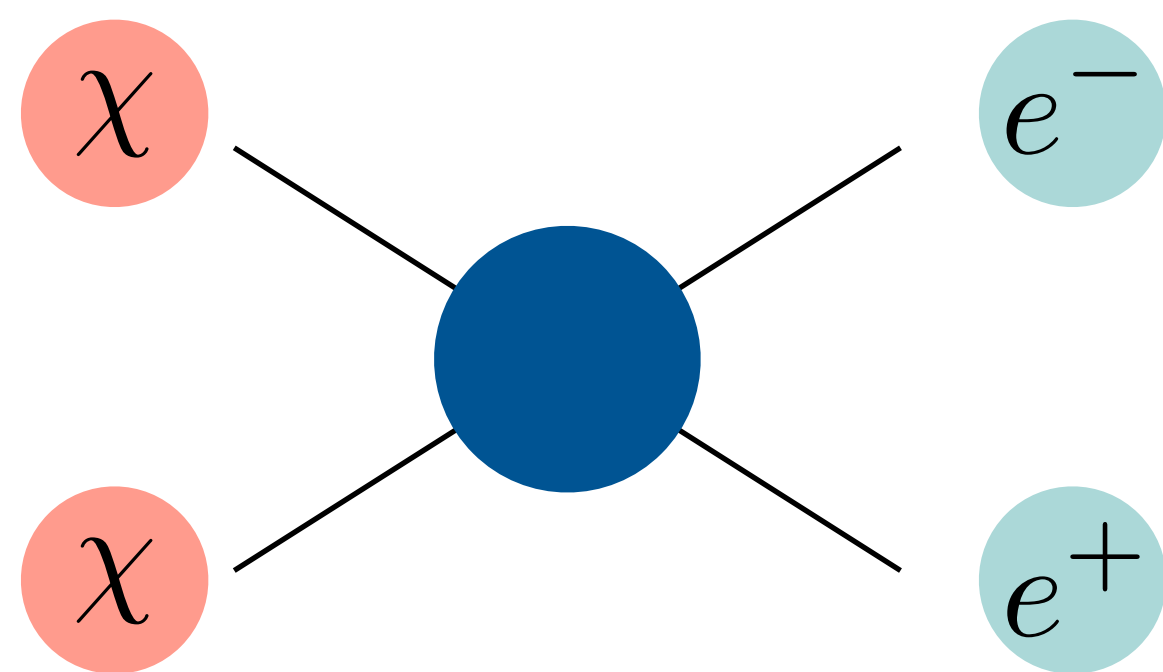
Not enough Kinetic Energy to Produce DM



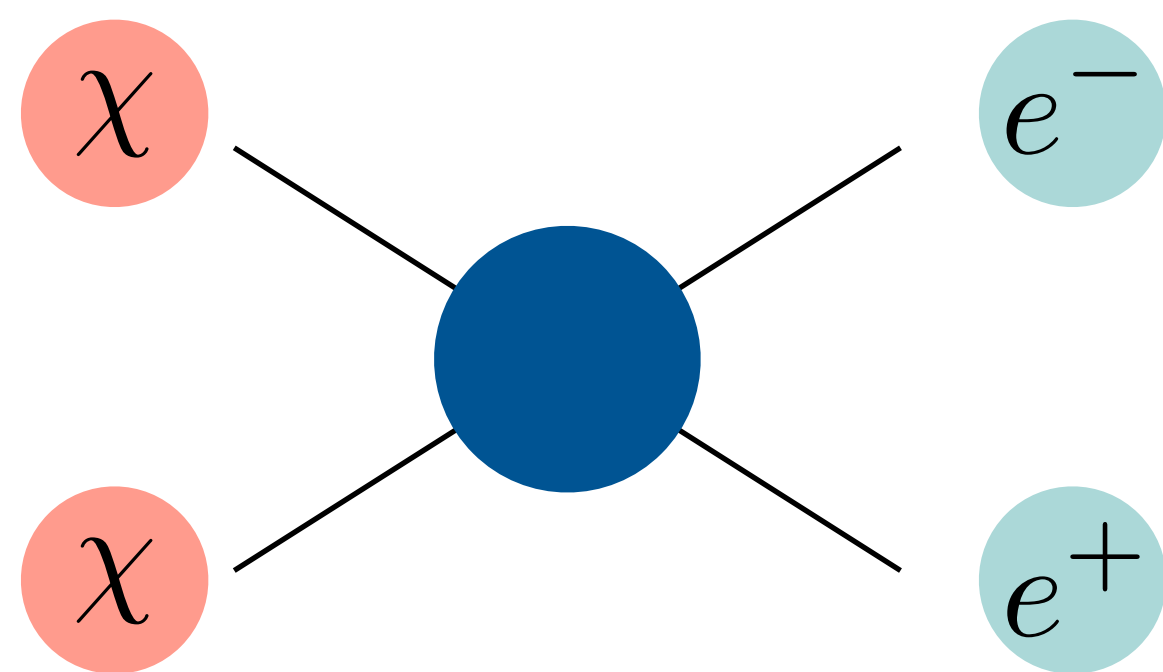


$$m_e \ll T \lesssim m_\chi$$

$$\Gamma_{\chi \rightarrow \text{SM}} = n_{\chi} \langle \sigma v \rangle$$




$$\Gamma_{\chi \rightarrow \text{SM}} = n_{\chi} \langle \sigma v \rangle$$



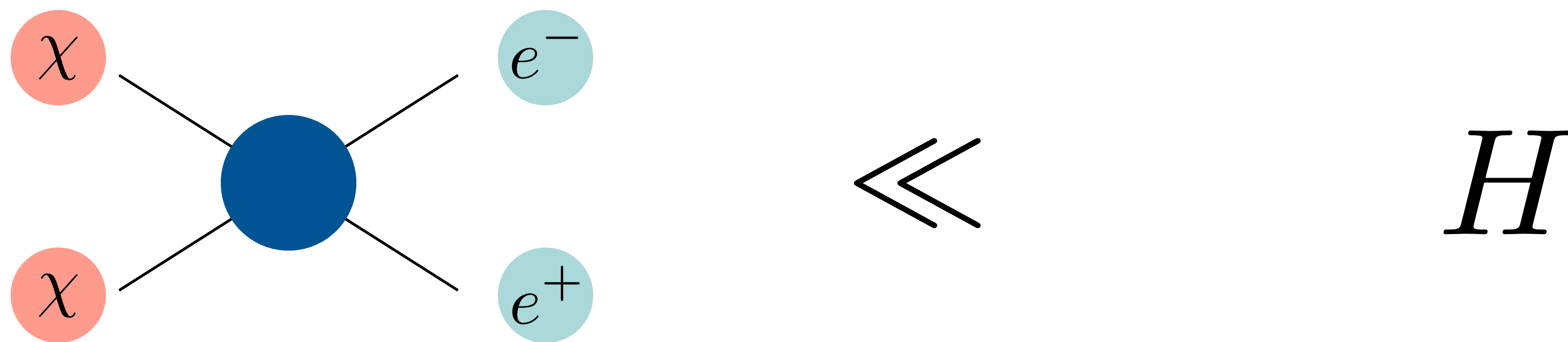
$\ll$

H

Expansion Rate
of the Universe

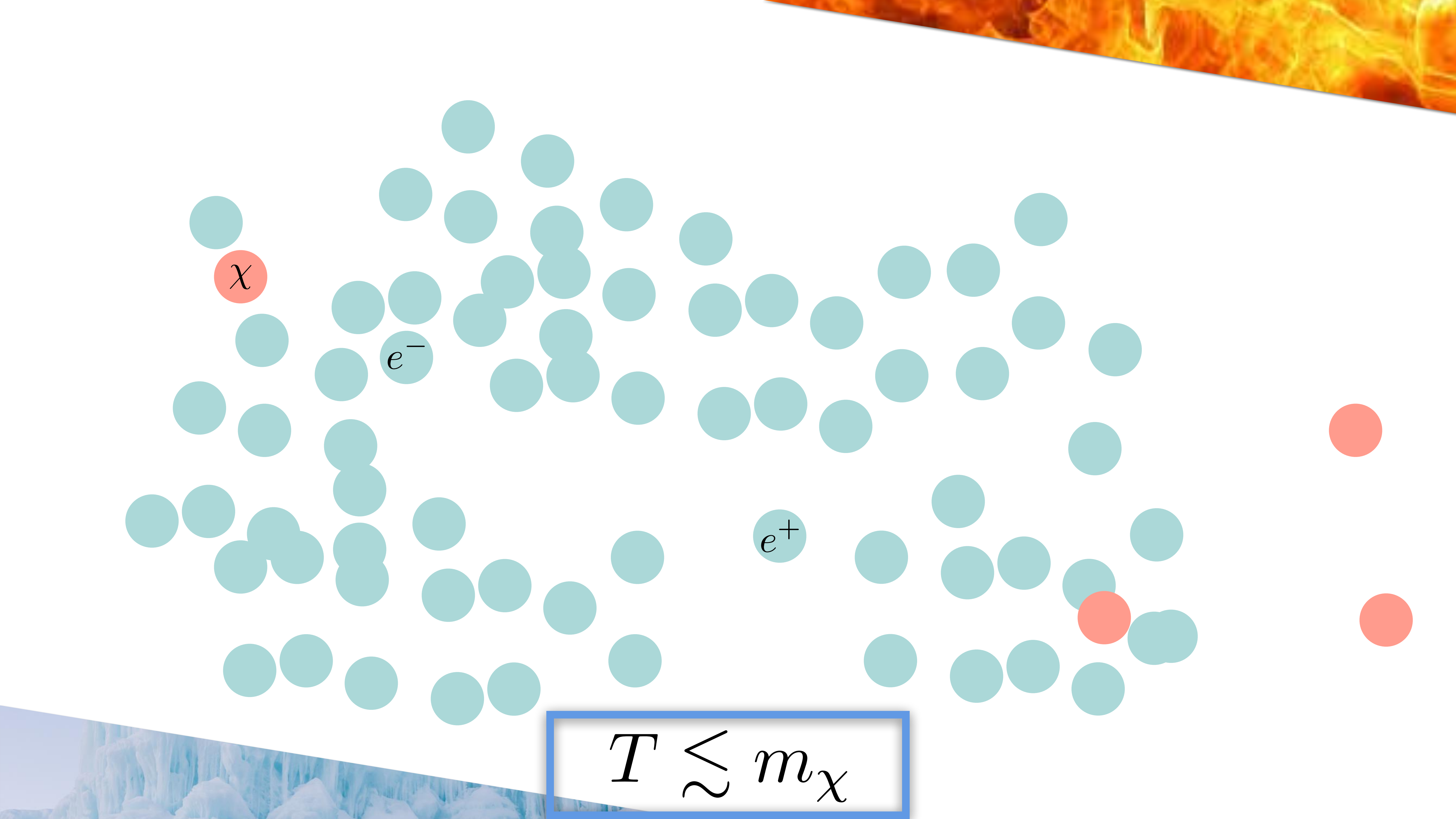



$$\Gamma_{\chi \rightarrow \text{SM}} = n_{\chi} \langle \sigma v \rangle$$



DM particles (almost) never meet: Number of DM particles is frozen

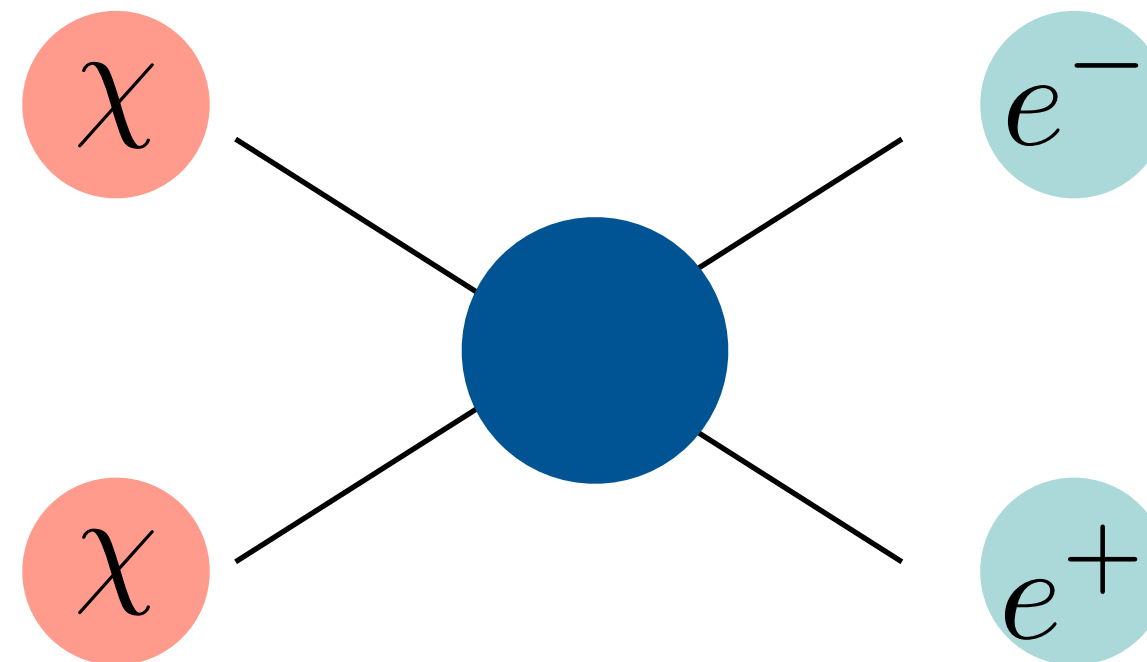

$$T \lesssim m_{\chi}/20$$

 χ e^- e^+

$$T \lesssim m_\chi$$

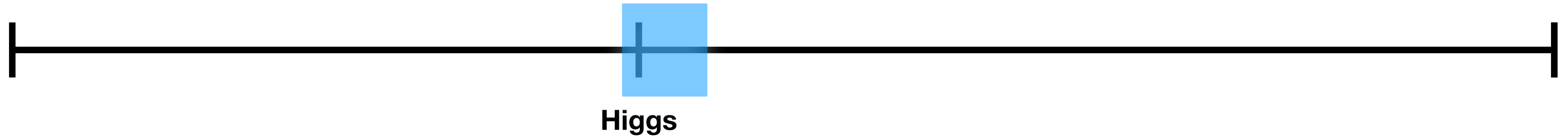
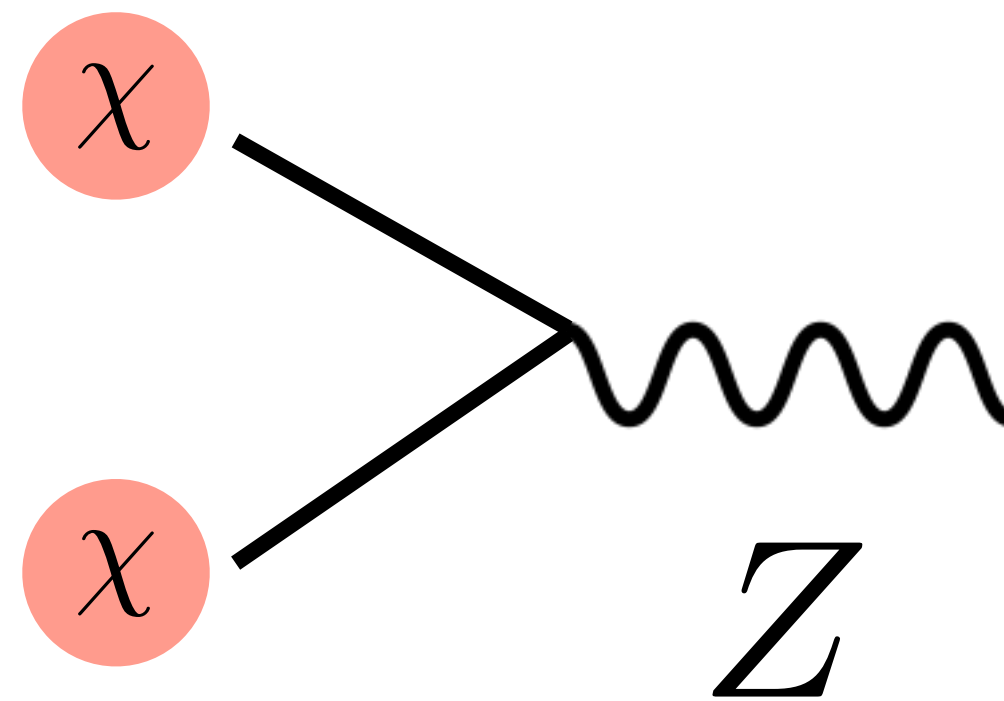
The amount of DM today depends on its interactions

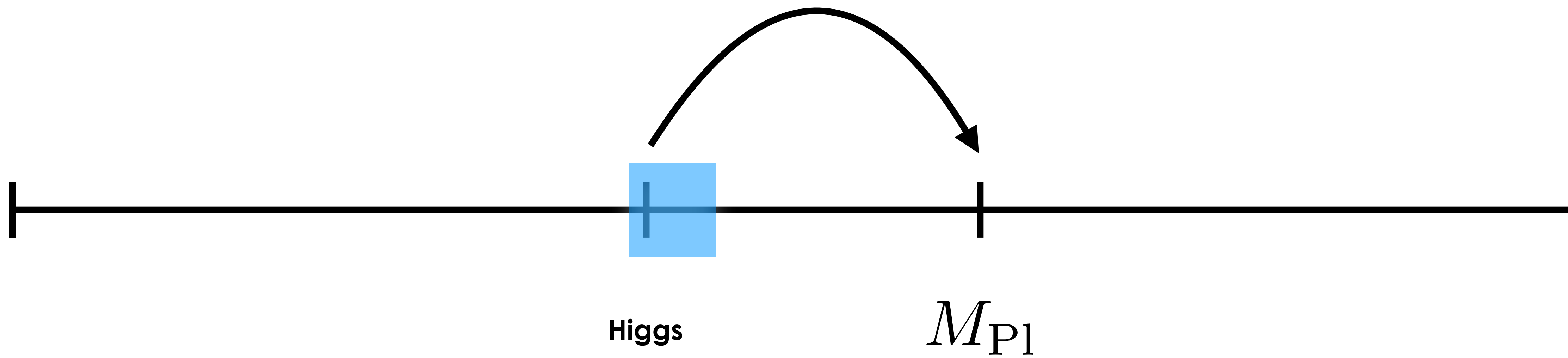
$$\Gamma_{\chi \rightarrow \text{SM}} = n_{\chi} \langle \sigma v \rangle \simeq H$$



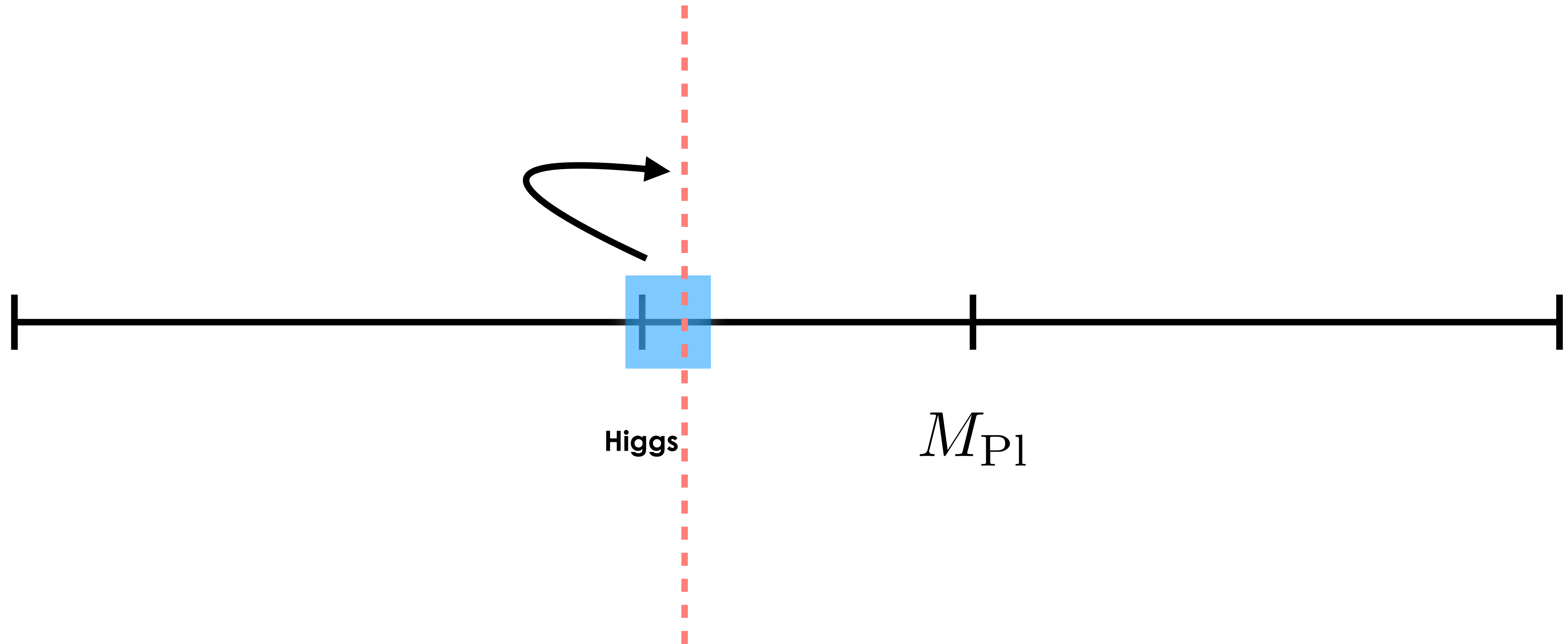
$$n_{\chi}^{\text{today}} \sim \frac{1}{\langle \sigma v \rangle} \sim \frac{m_{\chi}^2}{\alpha_{\chi}^2}$$

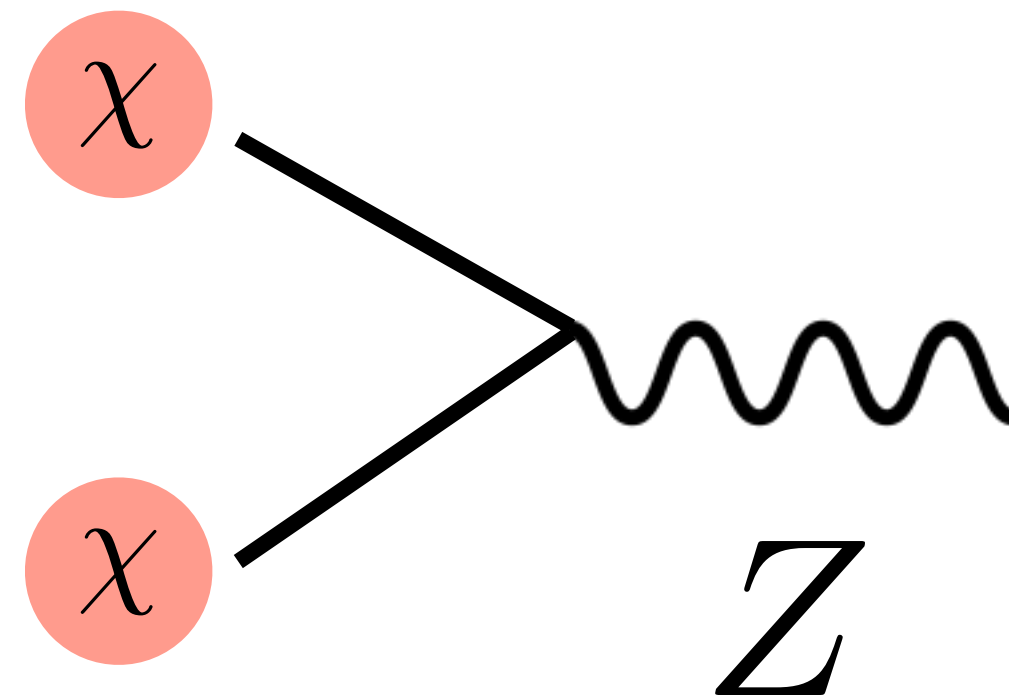
Perfect match!





New Symmetry



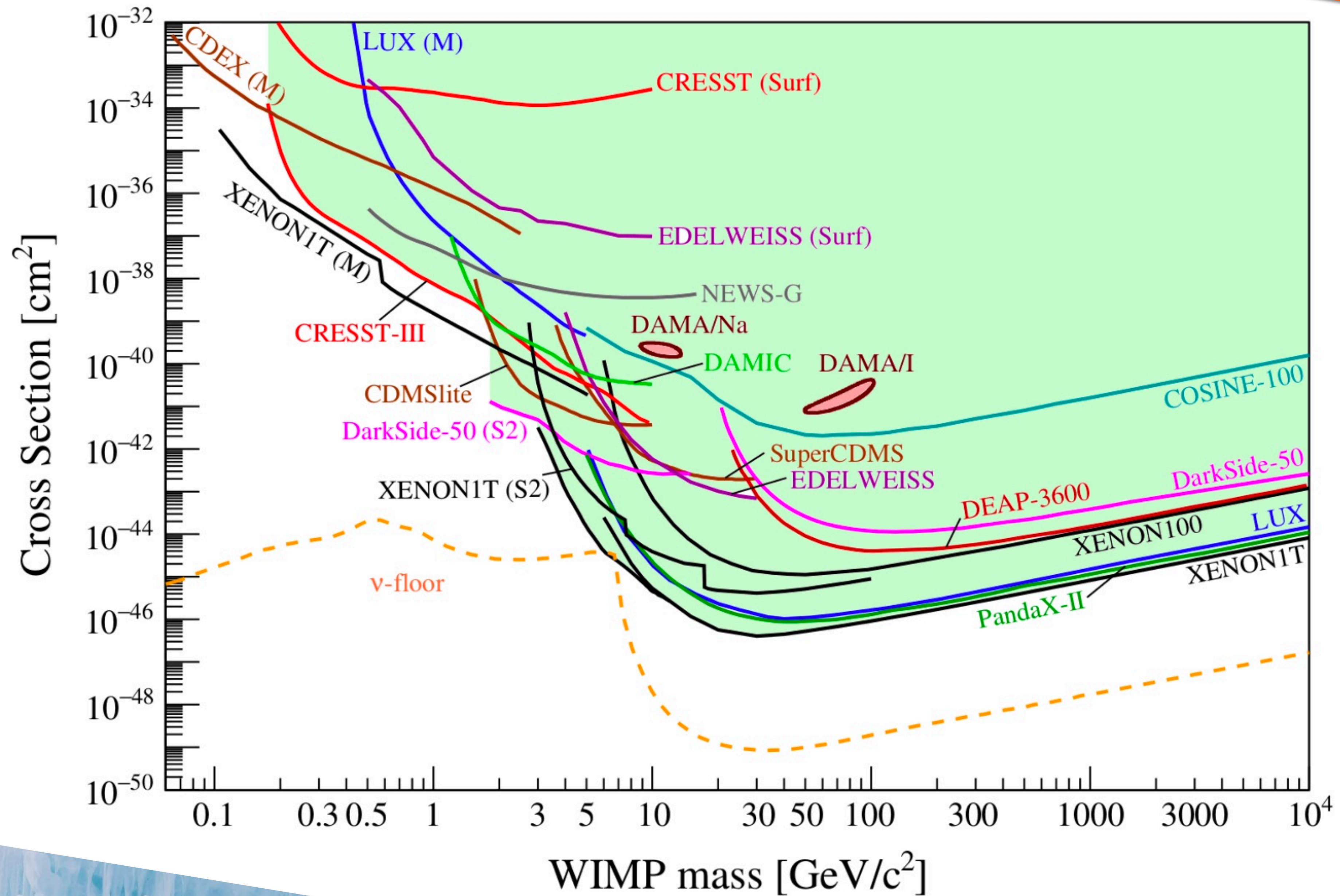


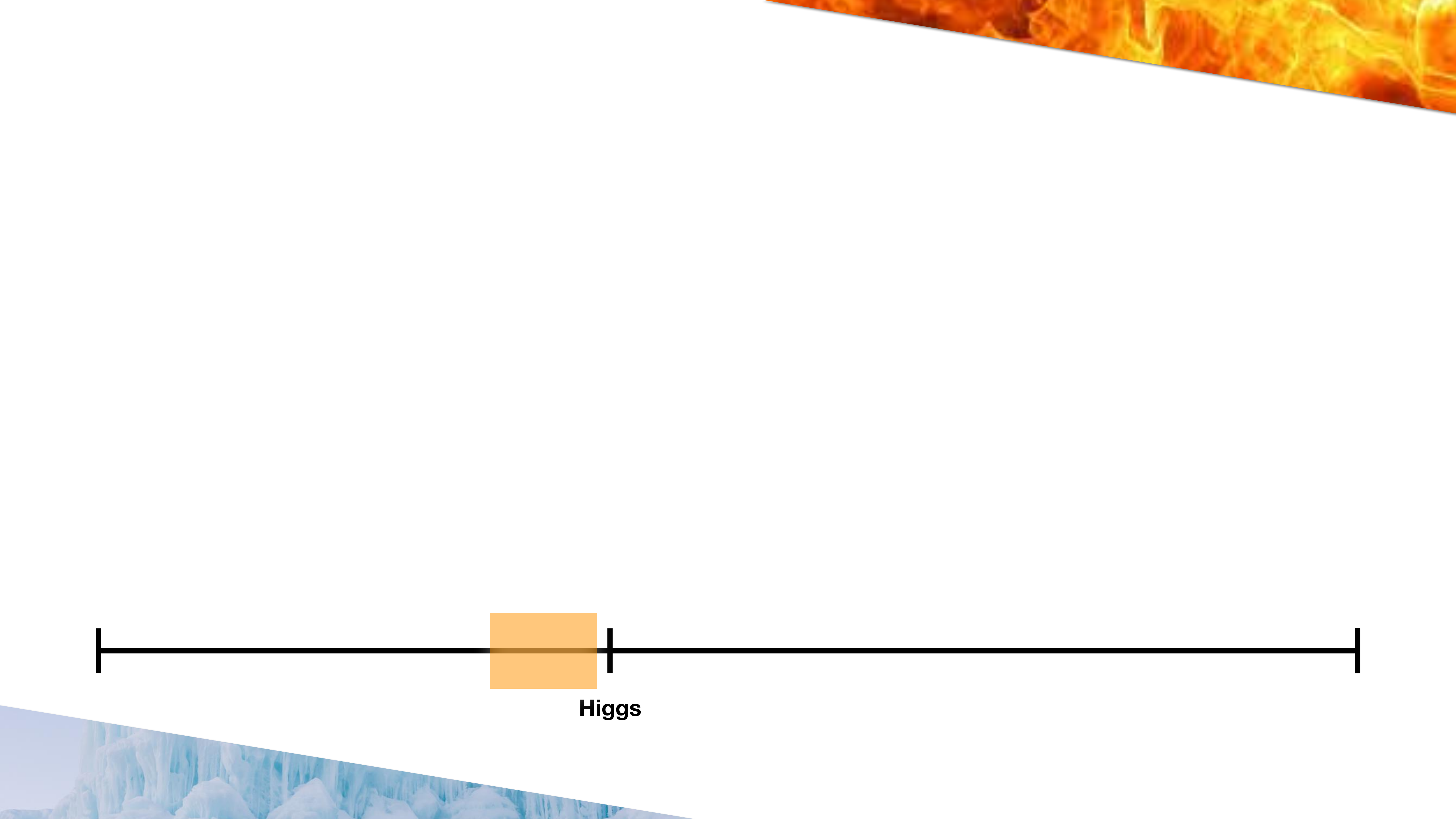
- Minimal Extension of what we know to exist
- Simple and predictive cosmology
- Solves another big problem in particle physics



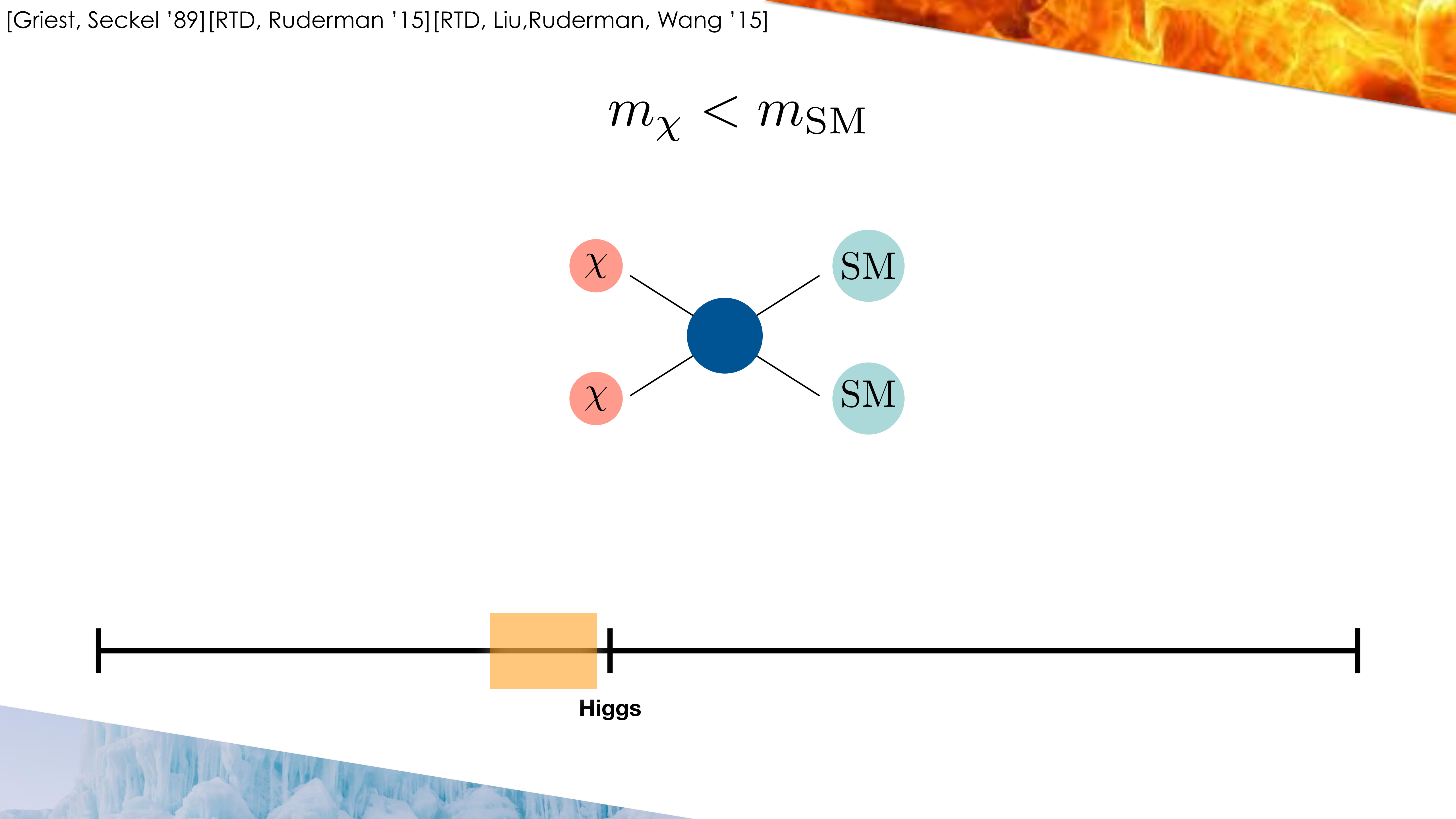
DIRECT DETECTION OF THERMAL DM

[APPEC '21, 2104.07634]

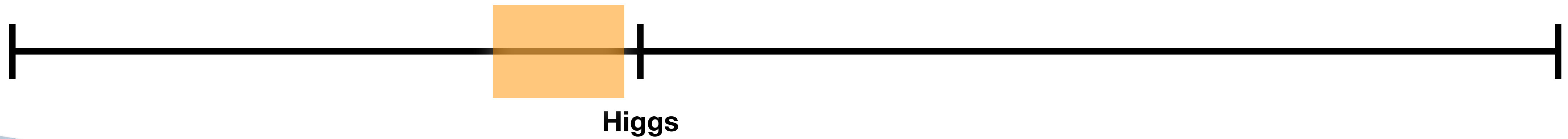
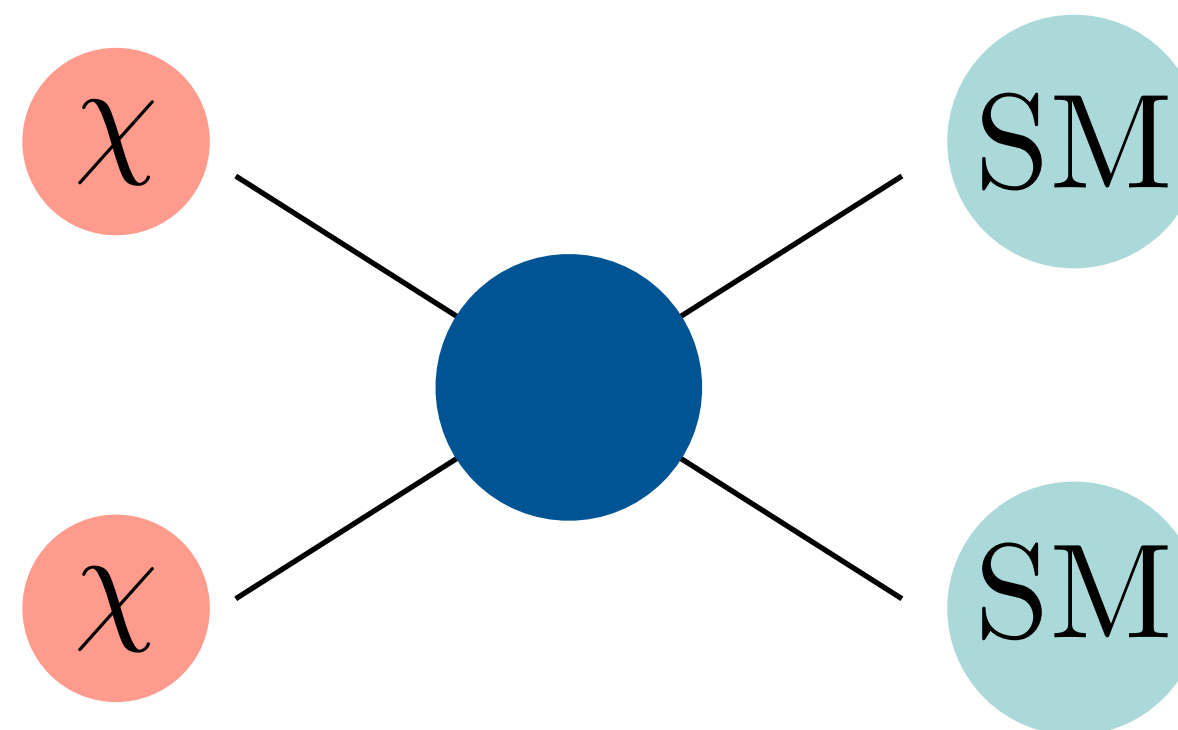




Higgs



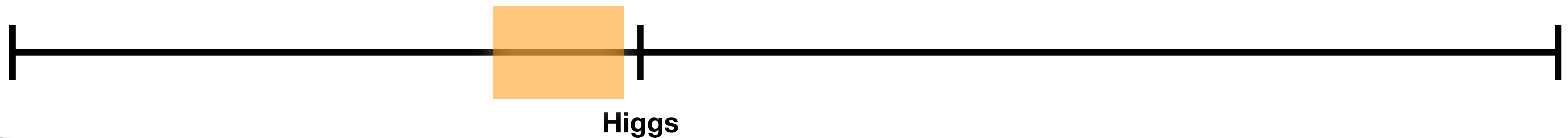
$$m_\chi < m_{\text{SM}}$$



$$m_\chi < m_{\text{SM}}$$

$$n_\chi^{\text{today}} \sim \frac{1}{\langle \sigma v \rangle} \sim \frac{m_\chi^2}{\alpha_\chi^2} e^{\Delta \frac{m_\chi}{T_f}}$$

The **same amount of DM** requires a **smaller mass**





$$S \supset -\frac{1}{2} \int d^4x \, j_{\text{eff}}^\mu A_\mu$$

GW

Axion

$$\underline{j_{\text{eff}}^\mu} = \partial_\nu \left(\frac{1}{2} h F^{\mu\nu} + h^\nu{}_\alpha F^{\alpha\mu} - h^\mu{}_\alpha F^{\alpha\nu} \right)$$

$$\underline{j_{\text{eff}}^\mu} = \epsilon^{\mu\nu\rho\sigma} \partial_\nu (a F_{\rho\sigma})$$

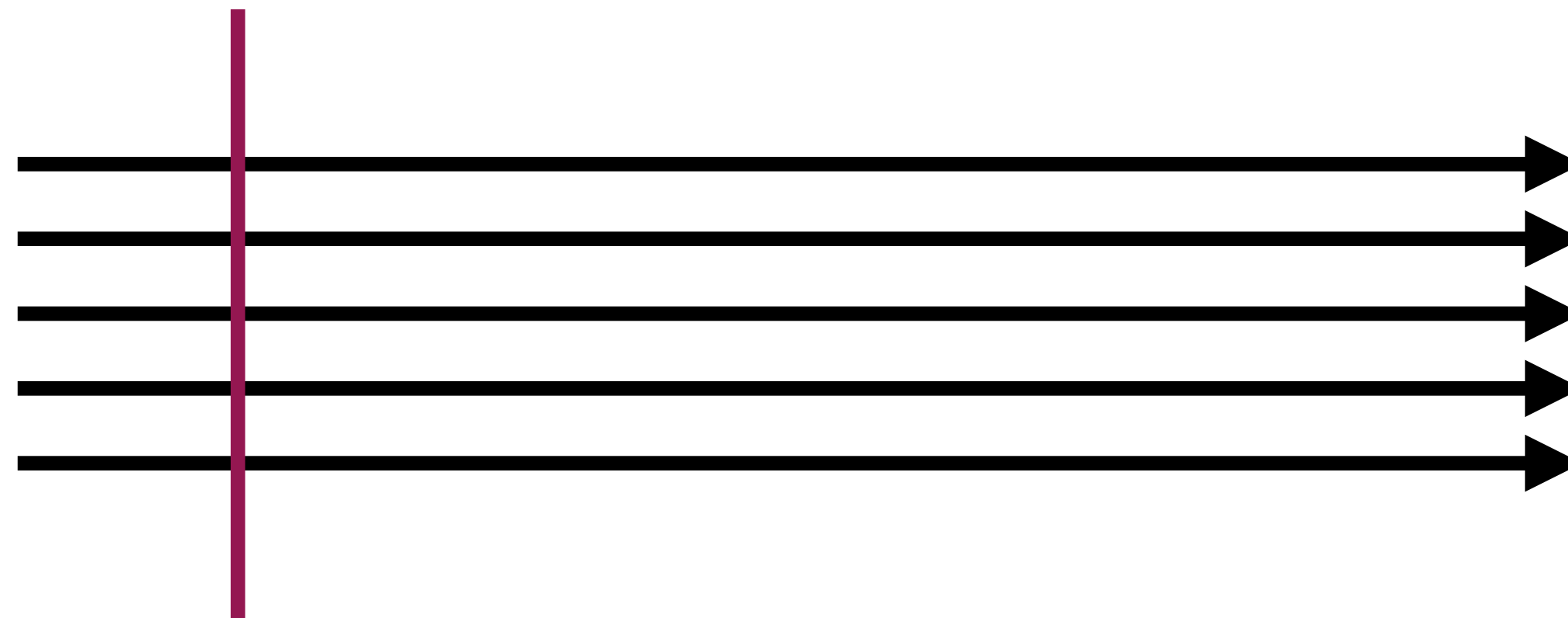
Not a covariant vector



Proper Detector Frame (PDF) Vs Transverse Traceless (TT) Frame
(Fermi Normal Coordinates)

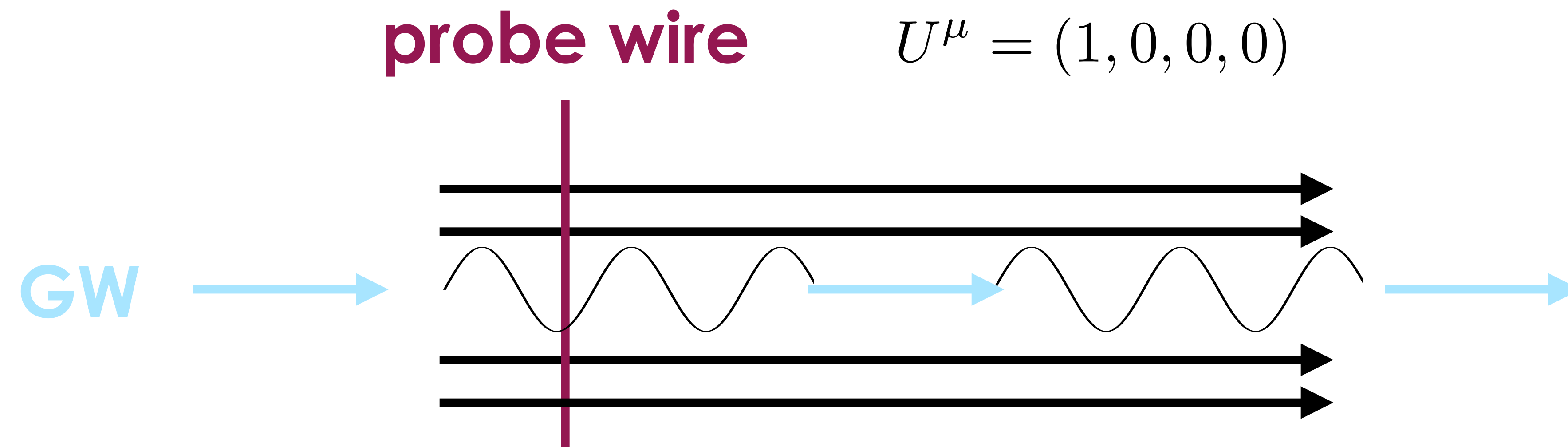
proper detector frame = laboratory
 (t, x, y, z)

probe wire $U^\mu = (1, 0, 0, 0)$



$$\mathbf{B} = B_0 \hat{z}$$

proper detector frame = laboratory
 (t, x, y, z)

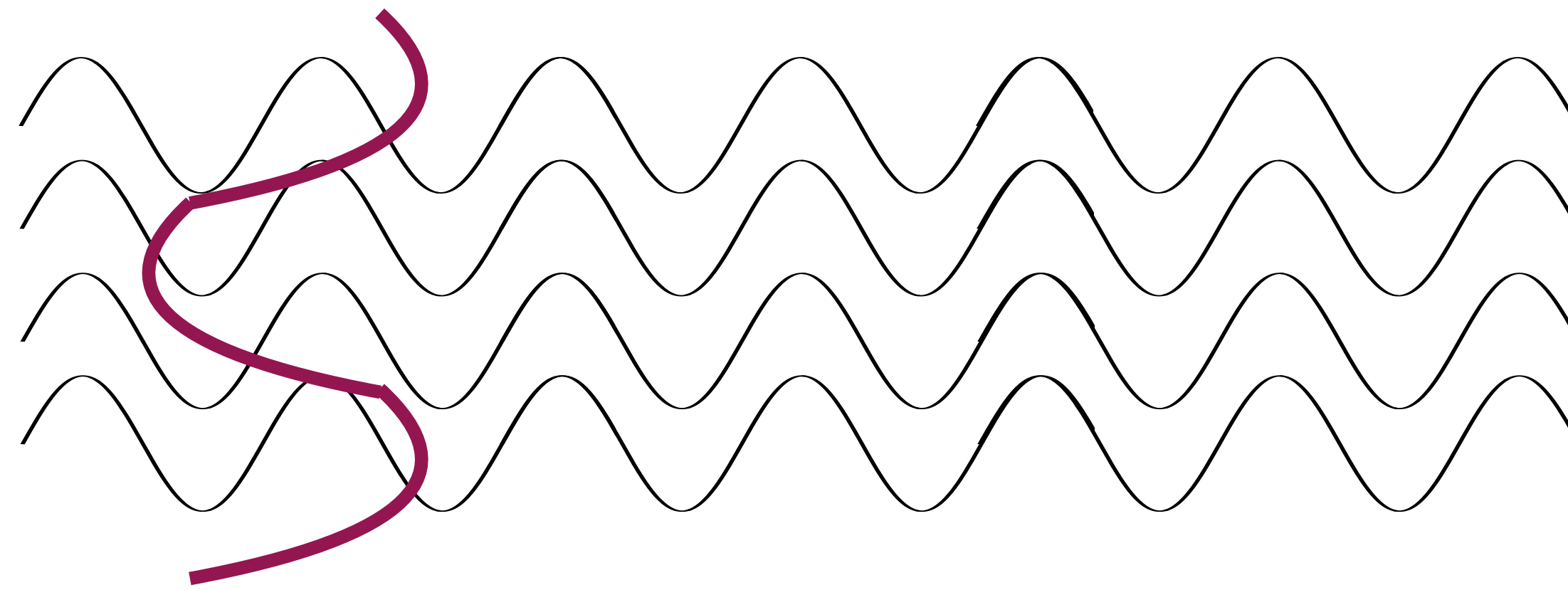


TT gauge = comoving with the wave
 $(t_{\text{TT}}, x_{\text{TT}}, y_{\text{TT}}, z_{\text{TT}})$

$$\begin{aligned} t_{\text{TT}} &\simeq t - \frac{i}{4} \omega_g (x^2 - y^2) h_+ e^{i\omega_g t}, & x_{\text{TT}} &\simeq x - \frac{1}{2} x (1 - i\omega_g z) h_+ e^{i\omega_g t} \\ y_{\text{TT}} &\simeq y + \frac{1}{2} y (1 - i\omega_g z) h_+ e^{i\omega_g t}, & z_{\text{TT}} &\simeq z - \frac{i}{4} \omega_g (x^2 - y^2) h_+ e^{i\omega_g t} \end{aligned}$$

TT gauge = comoving with the wave
 $(t_{\text{TT}}, x_{\text{TT}}, y_{\text{TT}}, z_{\text{TT}})$

probe wire



$$\mathbf{B} = B_0 \hat{z} + \frac{i}{2} (h_+ B_0) e^{i\omega_g t} (\omega_g x, -\omega_g y, 0) + \mathcal{O}(h^2)$$

TT gauge = comoving with the wave
 $(t_{\text{TT}}, x_{\text{TT}}, y_{\text{TT}}, z_{\text{TT}})$

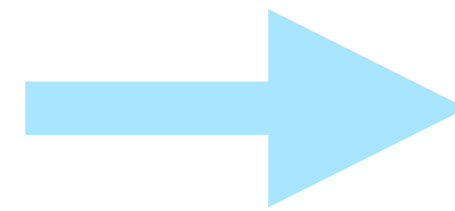


$$\mathbf{B} = B_0 \hat{z}$$

Theorem: $j_{\text{eff,TT}}^{\mu} = 0$

TT gauge = comoving with the wave
 $(t_{\text{TT}}, x_{\text{TT}}, y_{\text{TT}}, z_{\text{TT}})$

Theorem: $j_{\text{eff}, \text{TT}}^\mu = 0$



Wrong Conclusion

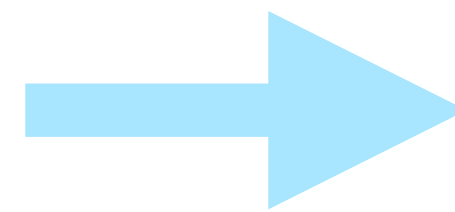
No signal

Surprisingly common mistake in the literature
(confusion between TT frame and laboratory frame)

TT gauge = comoving with the wave
 $(t_{\text{TT}}, x_{\text{TT}}, y_{\text{TT}}, z_{\text{TT}})$

Wrong Conclusion

Theorem: $j_{\text{eff}, \text{TT}}^{\mu} = 0$

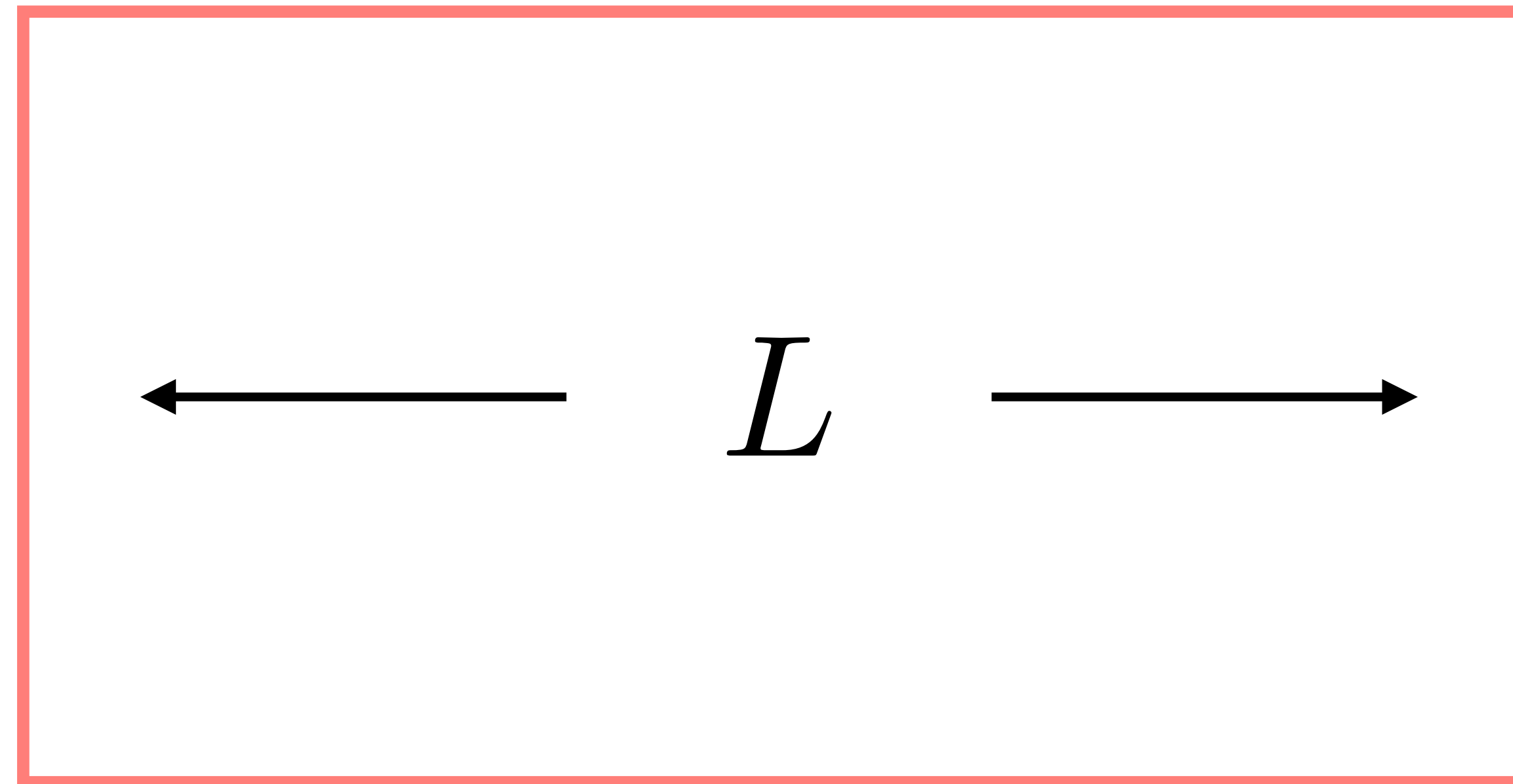


No signal

Doubly Wrong:

1. Impossible to prepare a uniform B-field in the TT frame
2. Even if you could do it, there would still be a signal (wire moving)

Cavity



$$\lambda_g \simeq L$$

ADMX, HAYSTAC, ...

$$\mathbf{j}_\times \cdot \hat{z} \propto \sin \phi \sin \beta \left(-1 + \frac{2i}{3} \omega_g (z \cos \beta - r \sin \phi \sin \beta) + \mathcal{O}(\omega_g^2 L_{\text{det}}^2) \right)$$

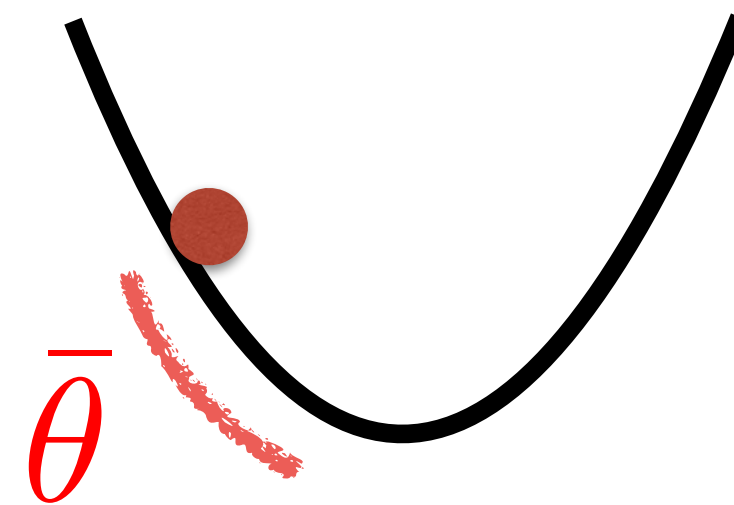
The volume integral of the leading
order piece vanishes

The Universe cools below the QCD scale



$$m_a(T) \approx 0.1 m_a \left(\frac{\Lambda_{\text{QCD}}}{T} \right)^4$$

AXION



$$\rho_a = \frac{m_a^2 f_a^2 \bar{\theta}^2}{2}$$

ALP DARK MATTER DETECTION

