Double-Real-Virtual and Double-Virtual-Real Corrections to the Three-Loop Thrust Soft Function

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Outline

Thrust observable

Definition

Thrust   Brandt et al., PL (1964), Farhi, PRL (1977)

$$T = 1 - \tau = \mathsf{Max}_{\hat{\mathbf{n}}} \frac{\sum_i |\hat{\mathbf{n}} \cdot \vec{k}_i|}{\sum_i |\vec{k}_i|}\,, \tag{1}$$

where $\vec{k}_i$ are the momenta of the final-state particles or jets, and the maximum is obtained for the Thrust axis $\hat{\mathbf{n}}$.

- One of the fundamental tools to test the theory of strong interactions(such as the asymptotic freedom of QCD)
- Can be measured very accurately
- Infrared safe, theoretically clean, and feasible to high perturbative order computations
- Suitable for the determination of the strong coupling $\alpha_s(M_Z)$ with a high precision

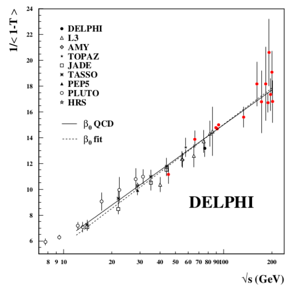

Figure: $\frac{1}{\langle 1-T \rangle}$ as a function of the cms energy.   Kluth (2006)

Near the dijet limit ($\tau \to 0$), fixed-order perturbative calculation breaks down due to the presence the large logarithms.

$$\frac{d\sigma}{d\tau} = \sum_{n=0}^{\infty} \sum_{k=1}^{2n+1} \alpha_s^n \, C_k^{(n)} \, \frac{1}{\mu} \mathcal{L}_{2n-k} \left(\frac{\tau}{\mu} \right) , \tag{2}$$

where the logarithms

$$\mathcal{L}_{2n-k}(\tau) = \left(\frac{\ln^{2n-k} \tau}{\tau} \right)_+ ,$$
$$\mathcal{L}_{-1}(\tau) = \delta(\tau) .$$

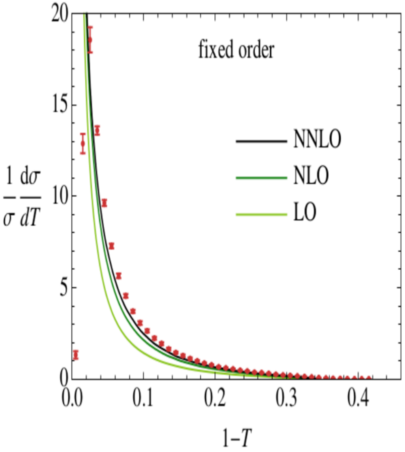

Figure: Fixed-order distribution Becher & Schwartz (2008)

Resummation

Soft-collinear effective theory (SCET) provides a systematic method to resum large logarithms. Near the dijet region, the thrust distribution is factorized in SCET: Becher & Schwartz, JHEP (2008)

$$\frac{1}{\sigma_0}\frac{d\sigma_{sing.}}{d\tau} = H(\mu)\int \prod_{i=n,\bar{n},s} d\tau_i \mathcal{J}_n(\tau_n,\mu)\mathcal{J}_{\bar{n}}(\tau_{\bar{n}},\mu)S(\tau_s)\delta(\tau-\tau_n-\tau_{\bar{n}}-\tau_s,\mu)\,,$$

where σ_0 is the leading order cross section for $e^+e^- \to q\bar{q}$. We assume that q is along the light-like direction $n=(1,0,0,1)$ while $\bar{q}$ along $\bar{n}=(1,0,0,-1)$. H is the hard function, $\mathcal{J}$ is the inclusive jet function, and S is the soft function defined as the vacuum expectation of Wilson lines.
The logarithmic terms can be resumed to all orders through the standard resummation techniques.

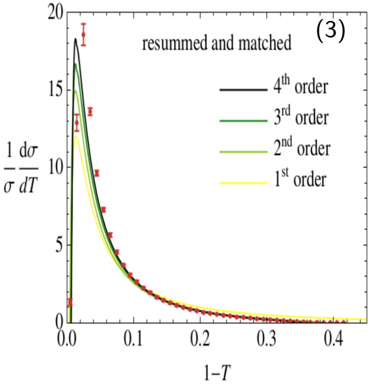

Figure: Resummed distribution Becher & Schwartz (2008)

Definition

Soft function

$$S(\tau) = \frac{1}{N_c} \sum_X \text{Tr} \left\{ \langle \Omega | Y_{\bar{n}}^\dagger Y_n | X \rangle \langle X | Y_n^\dagger Y_{\bar{n}} | \Omega \rangle \right\} \delta \left(\tau - \sum_k \min \{ k \cdot n, k \cdot \bar{n} \} \right), \quad (4)$$

where Y_n and $Y_{\bar{n}}$ are the soft Wilson lines, $|\Omega\rangle$ is the vacuum, and the summation is taken over all the final hadronic states of soft scales.

Known results:

- two-loop
  Monni et al. JHEP(2011),
  Kelley et al. PRD(2011),
  Boughezal et al. PRD(2015)
- three-loop
  triple-real:
  Baranowski et al. PRD(2022)
  double-real-virtual &
  double-virtual-real:
  Chen et al. arXiv:2206.12323

Logarithmic Order	fixed-order matching	Anomalous Dimension γ_i	Γ_{cusp}
LL	1	-	1-loop
NLL	1	1-loop	2-loop
NLL'	α_s	1-loop	2-loop
NNLL	α_s	2-loop	3-loop
NNLL'	α_s^2	2-loop	3-loop
N^{3}LL	α_s^2	3-loop	4-loop
N^{3}LL'	α_s^3	3-loop	4-loop
N^{4}LL	α_s^3	4-loop	5-loop

Table: The definitions for the logarithmic accuracy of the resummation calculation.

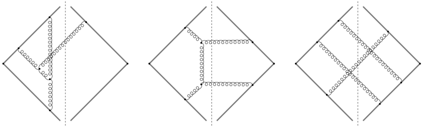

Figure: Representative diagrams for three-loop thrust soft function.

Double-real-virtual (RRV) contribution:

$$S^{(3)}_{RRV}(\tau;\epsilon) = \int \mathrm{d}\Phi_2\,\Theta(\tau;k_1,k_2)\int \frac{d^D l}{(2\pi)^D}\,\omega^{(3)}_{RRV}(l,k_1,k_2;n,\bar{n})\,, \tag{5}$$

where $\mathrm{d}\Phi_n$ denotes the n-body phase-space integration measure, $\omega^{(3)}_{RRV}$ is the squared amplitude, and

$$\Theta(\tau;k_1,k_2) = \int d\tau_1 d\tau_2 \delta(\tau-\tau_1-\tau_2)\Theta_{\tau_1}(k_1)\Theta_{\tau_2}(k_2),$$

with

$$\Theta_\tau(k) \equiv \theta(k^+-k^-)\delta(\tau-k^-)+\theta(k^--k^+)\delta(\tau-k^+)\,.$$

Difficulty: Traditional integration-by-parts (IBP) method and differential-equation (DE) technique break down due to the presence of Heaviside theta functions.

Parametrization

Solution: Integrals with theta functions can be reduced by constructing and solving linear relations in the parametric representation. Chen JHEP(2020), EPJC(2020, 2021)

$$\frac{1}{D_i^{\lambda_i+1}} \quad \Longleftrightarrow \quad \frac{e^{-\frac{\lambda_i+1}{2}i\pi}}{\Gamma(\lambda_i+1)}\int_0^\infty \mathrm{d}x_i\; e^{ix_iD_i}x_i^{\lambda_i}$$

$$\theta(D_i) \quad \Longleftrightarrow \quad -\frac{i}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}x_i\,\frac{e^{ixD_i}}{x_i+i0^+}$$

We generalize theta function and define

$$w_\lambda(u) \equiv e^{-\frac{\lambda+1}{2}i\pi}\int_{-\infty}^{\infty}\mathrm{d}x\,\frac{1}{x^{\lambda+1}}e^{ixu}. \tag{6}$$

Obviously, $w_0(u) = 2\pi\theta(u)$, $w_{-1}(u) = 2\pi\delta(u)$, and $w_{-2}(u) = 2\pi\delta'(u)$.
We consider the parametrization of a general scalar integral:

$$J(\lambda_1,\lambda_2,\ldots,\lambda_n) = \int [\mathrm{d}l_1][\mathrm{d}l_2]\cdots[\mathrm{d}l_L]\,\frac{w_{\lambda_1}(D_1)w_{\lambda_2}(D_2)\cdots w_{\lambda_m}(D_m)}{D_{m+1}^{\lambda_{m+1}+1}D_{m+2}^{\lambda_{m+2}+1}\cdots D_n^{\lambda_n+1}}, \tag{7}$$

where L is the number of loops, l_i are the loop momenta, D_i are the denominators of propagators, and we denote the integration measure as $[\mathrm{d}l_i] \equiv \frac{1}{\Gamma(1+\epsilon)}\frac{\mathrm{d}^d l_i}{\pi^{d/2}}$ with d the space-time dimension.

$$J = s_g^{-\frac{L}{2}} e^{i\pi\lambda_f} I(\lambda_0, \lambda_1, \ldots, \lambda_n) = s_g^{-\frac{L}{2}} e^{i\pi\lambda_f} \int \mathrm{d}\Pi^{(n+1)} \mathcal{I}^{(-n-1)}, \quad \text{with} \tag{8}$$

$$\mathcal{I}^{(-n-1)} = \frac{\Gamma(-\lambda_0)}{\Gamma(1+\epsilon)^L \prod_{i=m+1}^{n+1} \Gamma(\lambda_i + 1)} \mathcal{F}^{\lambda_0} \prod_{i=1}^{n+1} x_i^{\lambda_i}. \tag{9}$$

Here s_g the determinant of the d-dimensional space-time metric, $\lambda_0 = -d/2$, and $\lambda_f = \frac{1}{2}Ld - \frac{1}{2}m - \sum_{i=m+1}^{n}(\lambda_i + 1)$. The integration measure

$$\mathrm{d}\Pi^{(n+1)} \equiv \prod_{i=1}^{n+1} \mathrm{d}x_i \delta(1 - f(x)), \tag{10}$$

with the positive definite function $f(x)$ satisfying $f(\alpha x) = \alpha f(x)$. The polynomial $\mathcal{F}$ is related to the Symanzik polynomials U and F through

$$\mathcal{F}(x) \equiv F(x) + U(x) x_{n+1}, \quad \text{with} \tag{11}$$

$$U(x) \equiv \det A, \quad \text{and} \quad F(x) \equiv U(x) \left(\sum_{i,j=1}^{L} (A^{-1})_{ij} B_i \cdot B_j - C \right). \tag{12}$$

Here A, B, and C are linear in x and are determined from $\sum_{i=1}^{n} x_i D_i \equiv \sum_{i,j=1}^{L} A_{ij} l_i \cdot l_j + 2\sum_{i=1}^{L} B_i \cdot l_i + C$.

By virtue of the homogeneity of the integrands of the parametric integrals, it can be proven that

$$0 = \int \mathrm{d}\Pi^{(n+1)} \frac{\partial}{\partial x_i} \mathcal{I}^{(-n-1)}, \qquad i = 1, 2, \ldots, m, \tag{13a}$$

$$0 = \int \mathrm{d}\Pi^{(n+1)} \frac{\partial}{\partial x_i} \mathcal{I}^{(-n-1)} + \delta_{\lambda_i 0} \int \mathrm{d}\Pi^{(n)} \left. \mathcal{I}^{(-n)} \right|_{x_i=0}, \qquad i = m+1, m+2, \ldots, n+1. \tag{13b}$$

A parametric integral can be understood as a function of the indices λ_i. We define the operators $\mathcal{R}_i$, $\mathcal{D}_i$, and $\mathcal{A}_i$, such that.

$$\begin{aligned}
\hat{\mathcal{R}}_i I(\lambda_0, \ldots, \lambda_i, \ldots) =& (\lambda_i + 1) I(\lambda_0, \ldots, \lambda_i + 1, \ldots), \\
\hat{\mathcal{D}}_i I(\lambda_0, \ldots, \lambda_i, \ldots) =& I(\lambda_0, \ldots, \lambda_i - 1, \ldots), \\
\hat{\mathcal{A}}_i I(\lambda_0, \ldots, \lambda_i, \ldots) =& \lambda_i I(\lambda_0, \ldots, \lambda_i, \ldots).
\end{aligned}$$

And we define

$$\begin{aligned}
\hat{x}_i =& \mathcal{D}_i, \quad i = 1,\ 2, \ldots, m, \\
\hat{x}_i =& \mathcal{R}_i, \quad i = m+1,\ m+2, \ldots, m, \\
z_i =& -\mathcal{R}_i, \quad i = 1,\ 2, \ldots,\ m, \\
z_i =& \mathcal{D}_i, \quad i = m+1,\ m+2, \ldots,\ n, \\
\hat{a}_i =& -\mathcal{A}_i - 1, \quad i = 1,\ 2, \ldots,\ m, \\
\hat{a}_i =& \mathcal{A}_i, \quad i = m+1,\ m+2, \ldots,\ n.
\end{aligned}$$

We further introduce the operators $\hat{x}_{n+1}$ and $\hat{z}_{n+1}$ formally defined by $\hat{z}_{n+1}I = I$ and $\hat{x}^i_{n+1}I = (\hat{a}_{n+1}+1)(\hat{a}_{n+1}+2)\dots(\hat{a}_{n+1}+i)I$, with $\hat{a}_{n+1} \equiv -(L+1)\mathcal{A}_0 - \sum_{i=1}^{n}(\hat{a}_i+1)$. The eq. (13) can be written in the following form:

$$\left(\mathcal{D}_0 \frac{\partial \mathcal{F}(\hat{x})}{\partial \hat{x}_i} - \hat{z}_i\right) I = 0, \quad i = 1,\ 2, \dots, n+1. \tag{14}$$

Let y be a kinematical variable, and we assume that the polynomial $U(x)$ is free of y, then the parametric integrals satisfy the differential equation

$$\frac{\partial}{\partial y} I = -\mathcal{D}_0 \frac{\partial \mathcal{F}}{\partial y} I. \tag{15}$$

We introduce the dimensionless variables by re-scaling the momenta as $\tau_i \to \hat{\tau}_i \tau$, $k_i \to \hat{k}_i \tau$ and $l \to \hat{l}\tau$, to factorize out the τ dependence from the rest of the integral and find

$$S^{(3)}_{RRV}(\tau;\epsilon) = \tau^{-1-6\epsilon} \int d\hat{\Phi}_2 \, \frac{d^D \hat{l}}{(2\pi)^D} \, \omega^{(3)}_{RRV}(\hat{l}, \hat{k}_1, \hat{k}_2; n, \bar{n}) \, \Theta(1; \hat{k}_1, \hat{k}_2) \,,$$

Thus we see that the τ-dependence of the soft function is trivial. An extra scale can be introduced by inserting into the master integrals a trivial integral of the form $\int \mathrm{d}y \delta(K-y)$, with K a linear combination of Lorentz scalars of the loop momenta. That is

$$J_i \equiv \int [\mathrm{d}k_1][\mathrm{d}k_2][\mathrm{d}l]\mathcal{J} = \int \mathrm{d}y \int [\mathrm{d}k_1][\mathrm{d}k_2][\mathrm{d}l]\delta(K-y)\mathcal{J} \equiv \int \frac{\mathrm{d}y}{2\pi} \; J_i'(y),$$

Integrals free of theta functions: $K = k_1 \cdot k_2,\quad y = \frac{1}{2}(1-x)^2,$

Integrals containing theta functions: $K = k_1^{+/-} + k_2^{+/-},\quad y = \frac{1}{x}.$

Integrals J_i can be obtained from $J_i'(y)$ by integrating out y. For integrals with $K = k_1 \cdot k_2 - y$, the integration is straightforward. For integrals with $K = k_1^{+/-} + k_2^{+/-}$, $J_i(y)$ is singular at $y = \infty$ $(x = 0)$. The integration can be carried out by subtracting the singular part of $J(y)$ (denoted by Sing$\{J(y)\}$) and integrating the singular part in d dimensions. That is,

$$\begin{aligned} J_i &= \int_0^\infty \frac{\mathrm{d}y}{2\pi}\ J_i'(y) \\ &= \int_0^\infty \frac{\mathrm{d}y}{2\pi}\ \mathrm{Ser}\{J_i'(y) - \mathrm{Sing}\{J'(y)\}\} + \int_0^\infty \frac{\mathrm{d}y}{2\pi}\ \mathrm{Sing}\{J'(y)\}. \end{aligned} \tag{16}$$

Here we use Ser to denote the series expansion of a function with respect to ϵ.

We decompose the two-gluon-virtual contribution according to the colour structures by

$$\hat{S}^{(3)}_{\text{RRV,gg}} = (4\pi)^6 \cos(\pi\epsilon)\alpha_s^3 C_A C_F \times \left(N_f \hat{S}^{(3)}_{\text{RRV,gg},a} + C_A \hat{S}^{(3)}_{\text{RRV,gg},b} + C_F \hat{S}^{(3)}_{\text{RRV,gg},c} \right). \tag{17}$$

Here

$$\hat{S}^{(3)}_{\text{RRV,gg},a} = \frac{1}{54\epsilon^3} + \frac{31}{162\epsilon^2} + \frac{1}{\epsilon}\left(\frac{95}{81} - \frac{1}{9}\zeta_2\right) + \frac{17}{27}\zeta_2 - \frac{41}{27}\zeta_3 + \frac{3778}{729} + \epsilon\left[\frac{62}{27}\zeta_2 + \frac{43}{81}\zeta_3 - \frac{149}{18}\zeta_4 + \frac{39568}{2187}\right], \tag{18}$$

$$\begin{aligned}\hat{S}^{(3)}_{\text{RRV,gg},b} = & -\frac{5}{3\epsilon^5} - \frac{523}{144\epsilon^4} + \frac{1}{\epsilon^3}\left(\frac{34}{3}\zeta_2 - \frac{2129}{216}\right) + \frac{1}{\epsilon^2}\left(\frac{409}{24}\zeta_2 + 42\zeta_3 - \frac{76}{3}\right) \\ & + \frac{1}{\epsilon}\left(-\frac{386}{3}\zeta_2 + \frac{14399}{72}\zeta_3 + \frac{478}{3}\zeta_4 - \frac{101329}{1944}\right) - \frac{596}{3}\zeta_3\zeta_2 \\ & + \frac{6277}{54}\zeta_2 - \frac{121919}{108}\zeta_3 + \frac{8889}{8}\zeta_4 + \frac{2360}{3}\zeta_5 - 4\log(2)\zeta_2 - \frac{649801}{5832} \\ & + 1244.294408\epsilon,\end{aligned} \tag{19}$$

and

$$\hat{S}^{(3)}_{\text{RRV,gg},c} = \frac{6}{\epsilon^5} - \frac{60}{\epsilon^3}\zeta_2 - \frac{300}{\epsilon^2}\zeta_3 - \frac{810}{\epsilon}\zeta_4 + 3000\zeta_2\zeta_3 - 8100\zeta_5 + \epsilon\left[7500\zeta_3^2 - 19635\zeta_6\right]. \tag{20}$$

Similarly, for the two-ghost-virtual contribution, we have

$$\hat{S}^{(3)}_{\text{RRV,gh.gh.}} = (4\pi)^6\alpha_s^3 C_A C_F\left(N_f \hat{S}^{(3)}_{\text{RRV,gh.gh.},a} + C_A \hat{S}^{(3)}_{\text{RRV,gh.gh.},b}\right), \tag{21}$$

with

$$\hat{S}^{(3)}_{\text{RRV,gh.gh.},a} = -\frac{1}{108\epsilon^3} - \frac{13}{324\epsilon^2} + \frac{1}{\epsilon}\left(\frac{1}{18}\zeta_2 - \frac{10}{81}\right) - \frac{17}{54}\zeta_2 + \frac{41}{54}\zeta_3 - \frac{566}{729} + \epsilon\left[\frac{26}{27}\zeta_2 - \frac{457}{162}\zeta_3 + \frac{149}{36}\zeta_4 - \frac{14575}{4374}\right], \tag{22}$$

and

$$\hat{S}^{(3)}_{\mathsf{RRV,gh.gh.},b} = -\frac{5}{288\epsilon^4} - \frac{5}{144\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{5}{144}\zeta_2 - \frac{2}{81}\right) + \frac{1}{\epsilon}\left(-\frac{5}{27}\zeta_2 + \frac{11}{48}\zeta_3 - \frac{167}{3888}\right)$$
$$-\frac{55}{324}\zeta_2 - \frac{289}{216}\zeta_3 - \frac{1}{16}\zeta_4 + 2\log(2)\zeta_2 + \frac{35}{48}$$
$$+\epsilon\left[\frac{6583\zeta_2}{972} - \frac{655}{144}\zeta_2\zeta_3 - \frac{529}{162}\zeta_3 + \frac{2543}{144}\zeta_4 + \frac{425}{96}\zeta_5 + 4\log^2(2)\zeta_2\right.$$
$$\left.-\frac{59}{6}\log(2)\zeta_2 - 24\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{20513}{34992} - \log^4(2)\right]. \tag{23}$$

The double-virtual-real contribution reads

$$S^{(3)}_{V^2R}(\tau) = \frac{\alpha^3_{s,\mathsf{r}}}{8\pi^3}\frac{1}{\mu}\left(\frac{\tau}{\mu}\right)^{-1-6\epsilon}\left(C_F C_A N_f S^{(3)}_{V^2R,C_F C_A n_f} + C_F C_A^2 S^{(3)}_{V^2R,C_F C_A^2}\right), \tag{24}$$

Results

with

$$S^{(3)}_{V^2R,C_FC_An_f} = \frac{1}{9\epsilon^4} + \frac{5}{27\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{19}{81} - \frac{23}{18}\zeta_2\right) + \frac{1}{\epsilon}\left(\frac{65}{243} - \frac{115}{54}\zeta_2 - \frac{7}{3}\zeta_3\right)$$
$$+ \frac{211}{729} - \frac{545}{162}\zeta_2 - \frac{35}{9}\zeta_3 - \frac{13}{48}\zeta_4 \qquad (25)$$
$$+ \left(\frac{665}{2187} - \frac{2359\zeta_2}{486} - \frac{223\zeta_3}{27} - \frac{65\zeta_4}{144} - \frac{341\zeta_5}{15} + \frac{161\zeta_3\zeta_2}{6}\right)\epsilon\,,$$

and

$$S^{(3)}_{V^2R,C_FC_A^2} = -\frac{1}{3\epsilon^5} - \frac{11}{18\epsilon^4} - \frac{1}{\epsilon^3}\left(\frac{67}{54} + \frac{23}{6}\zeta_2\right) - \frac{1}{\epsilon^2}\left(\frac{193}{81} - \frac{253}{36}\zeta_2 - 2\zeta_3\right)$$
$$+ \frac{1}{\epsilon}\left(-\frac{1142}{243} + \frac{1541}{108}\zeta_2 + \frac{77}{6}\zeta_3 + \frac{409}{48}\zeta_4\right)$$
$$- \frac{6820}{729} + \frac{4547}{162}\zeta_2 + \frac{469}{18}\zeta_3 + \frac{143\zeta_4}{96} + \frac{52}{3}\zeta_2\,\zeta_3 - \frac{122}{15}\zeta_5\,, \qquad (26)$$
$$+ \left(-\frac{40856}{2187} + \frac{13403\zeta_2}{243} + \frac{1441\zeta_3}{27} + \frac{871\zeta_4}{288} + \frac{3751\zeta_5}{30} - \frac{1771\zeta_2\zeta_3}{12}\right.$$
$$\left.- \frac{637\zeta_6}{1152} - \frac{31}{2}\zeta_3^2\right)\epsilon\,,$$

Cross check

- Some master integrals are numerically checked by using the sector decomposition method.
- The leading pole (ϵ^{-5}) of the $C_F C_A^2$ term is validated by the strongly ordered limit calculation.
- The $C_F C_A^2 \epsilon^{-5}$ term is canceled upon summing the RRR, RRV, and VVR contributions.
- The $C_F^2 C_A$ term agrees with the nonabelian exponentiation theorem.
- Some diagrams (such as the vacuum polarization diagrams) are checked through direct calculations.

Summary

- We analytically calculate the RRV and VVR contribution to the thrust soft function.
- The application of our method to the calculation of the RRR contribution is straightforward. This is the only missing piece of the complete N^{3}LO thrust soft function, which is the key ingredient for the N^{3}LL$'$ resummation of the thrust observable in the dijet configuration.
- Our results can be used for the 0-jettiness in Drell-Yan/ggH process as well as the 1-jettiniess observable in DIS through analytic continuation. The results are also indispensable components to fulfill the N^{3}LO calculation using the N-jettiness subtraction scheme.
- The calculation demonstrates the feasibility of our reduction method to future jet and substructure precision calculations.

The End

Thanks for your attention !

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