# Sudakov resummation and gluon saturation at NLO

Resummation, Evolution, Factorization
November 3rd, 2022

Farid Salazar

Based on

(1) 2108.06347 [JHEP 11 (2021) 222]
(2) 2208.13872 [Submitted to JHEP]

In collaboration with

P. Caucal

B. Schenke

R. Venugopalan

# Outline

- Azimuthal correlations a window to gluon saturation

- Inclusive dijet production in the CGC at NLO

  P. Caucal, FS, R. Venugopalan. *2108.06347* *[JHEP 11 (2021) 222]*

  For inclusive dihadron see Jamal Jalilian Marian's talk on Tuesday

  For diffractive dihadron see Emilie Li's talk on Thursday

- Back-to-back limit: gluon saturation and Sudakov

  P. Caucal, FS, B. Schenke ,R. Venugopalan. *2208.13872* *[Submitted to JHEP]*

  For back-to-back dijet in photoproduction see Pieter Taels' talk on Wednesday

- Outlook

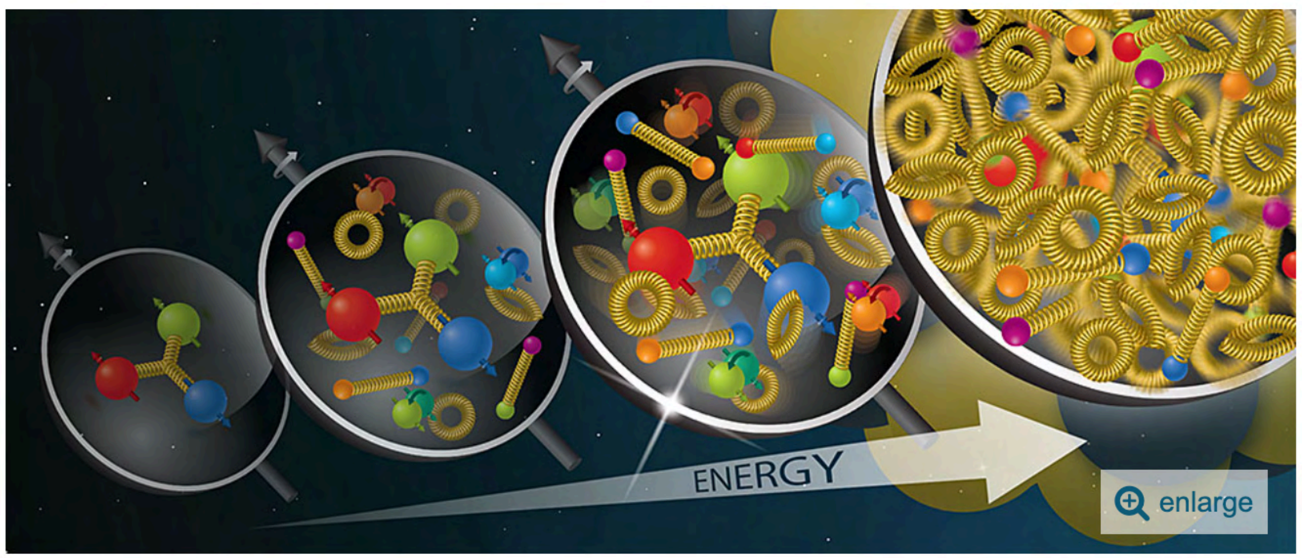
# Azimuthal correlation a window to gluon saturation

## Forward dihadron azimuthal correlations at RHIC

### Signs of Saturation Emerge from Particle Collisions at RHIC

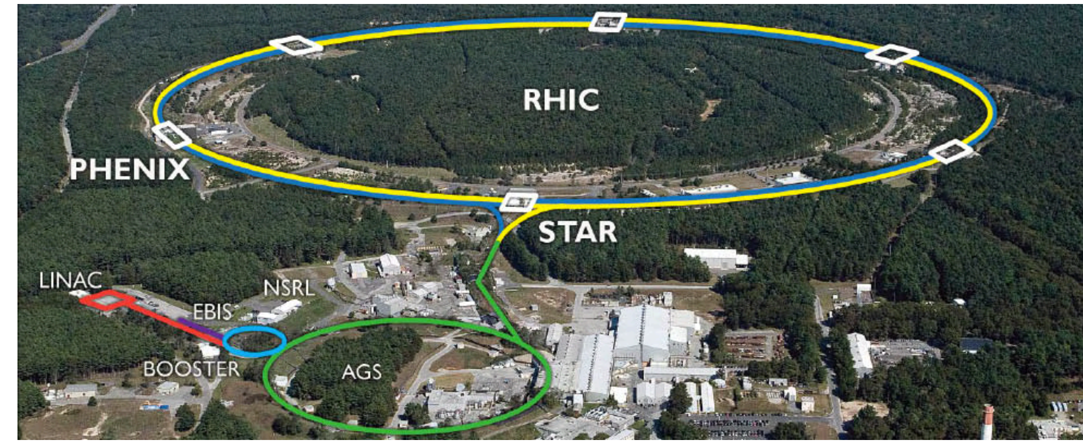
**Suppression of a telltale sign of quark-gluon interactions presented as evidence of multiple scatterings and gluon recombination in dense walls of gluons**

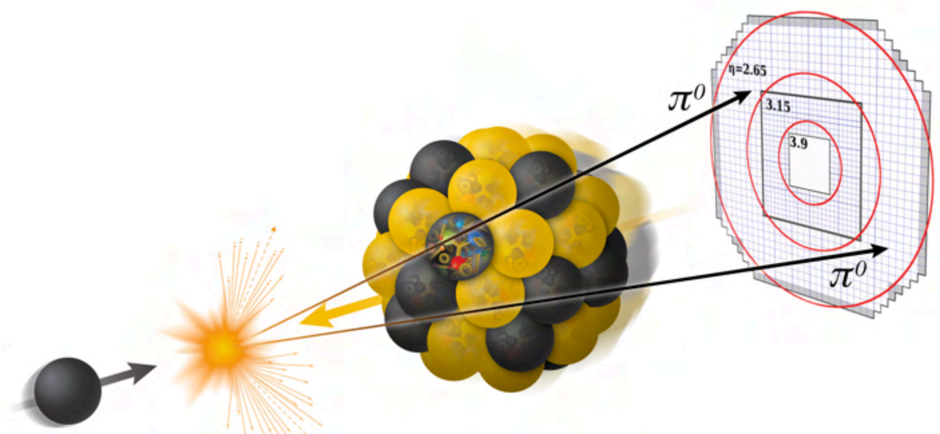
August 31, 2022




Members of the STAR collaboration report new data that indicate nuclei accelerated to very high energies at the Relativistic Heavy Ion Collider (RHIC) may be reaching a state where gluons are starting to saturate.

"We varied the species of the colliding ion beam because theorists predicted that this sign of saturation would be easier to observe in heavier nuclei," explained Brookhaven Lab physicist Xiaoxuan Chu, a member of the STAR collaboration who led the analysis. "The good thing is RHIC, the world's most flexible collider, can accelerate different species of ion beams. In our analysis, we used collisions of protons with other protons, aluminum, and gold."



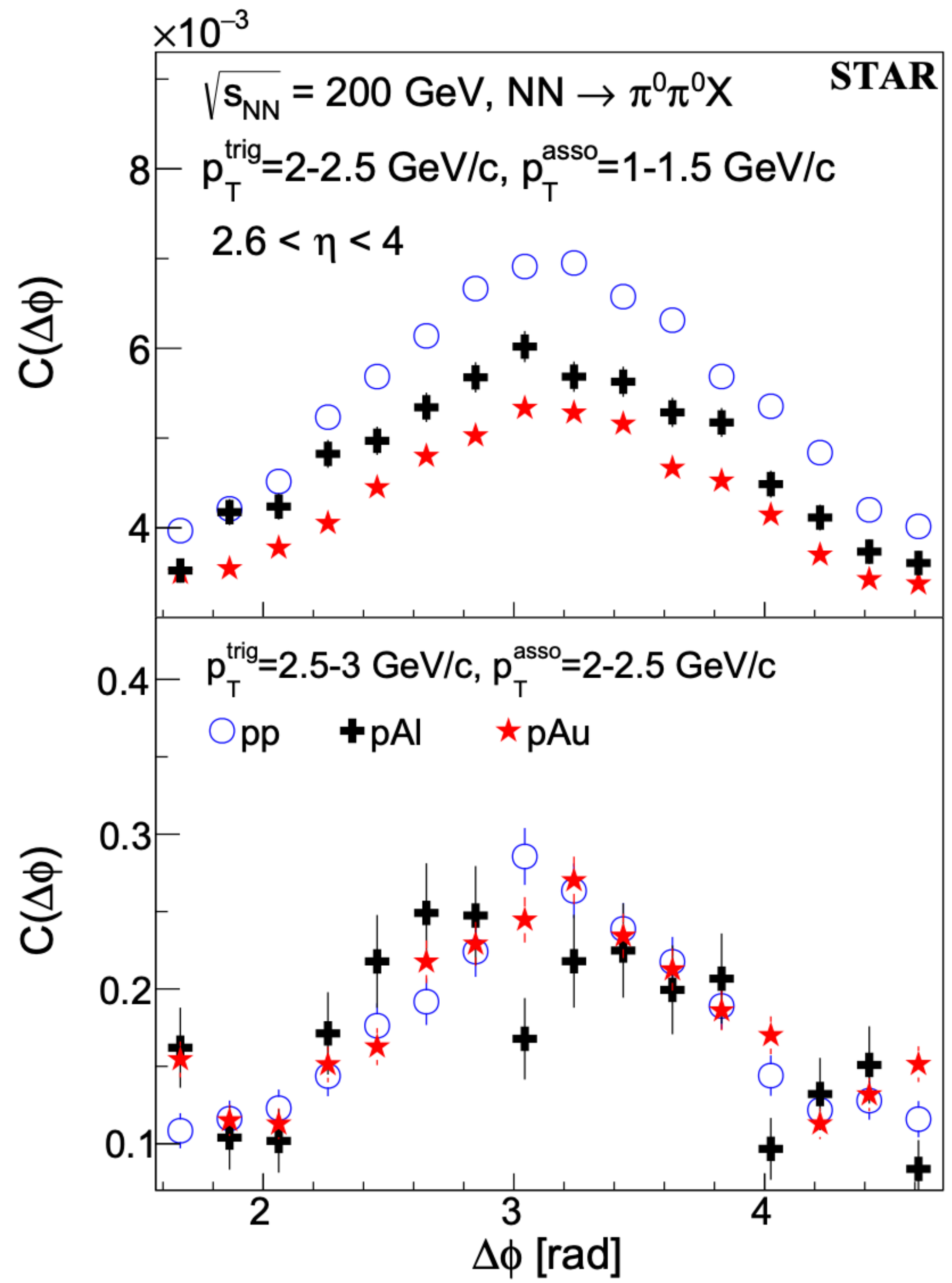

Xiaoxuan Chu and Elke Aschenauer

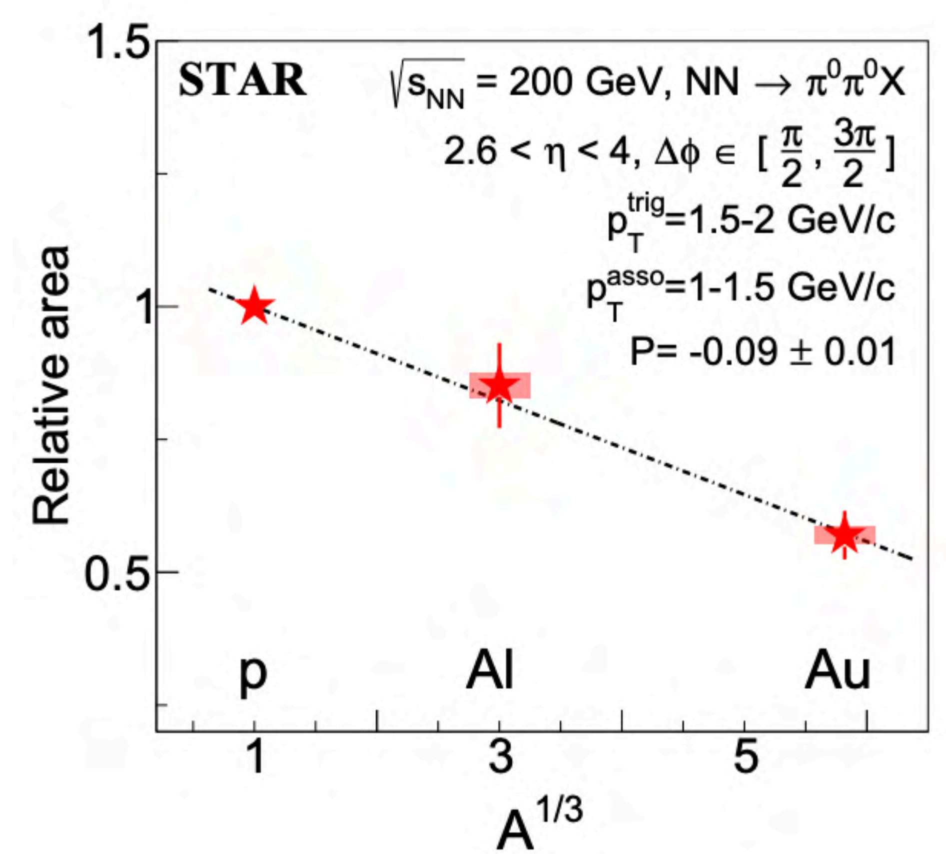
Source: BNL Newsroom

# Azimuthal correlation a window to gluon saturation

## Forward dihadron azimuthal correlations at RHIC

Evidence for Nonlinear Gluon Effects in QCD and Their Mass Number Dependence at STAR



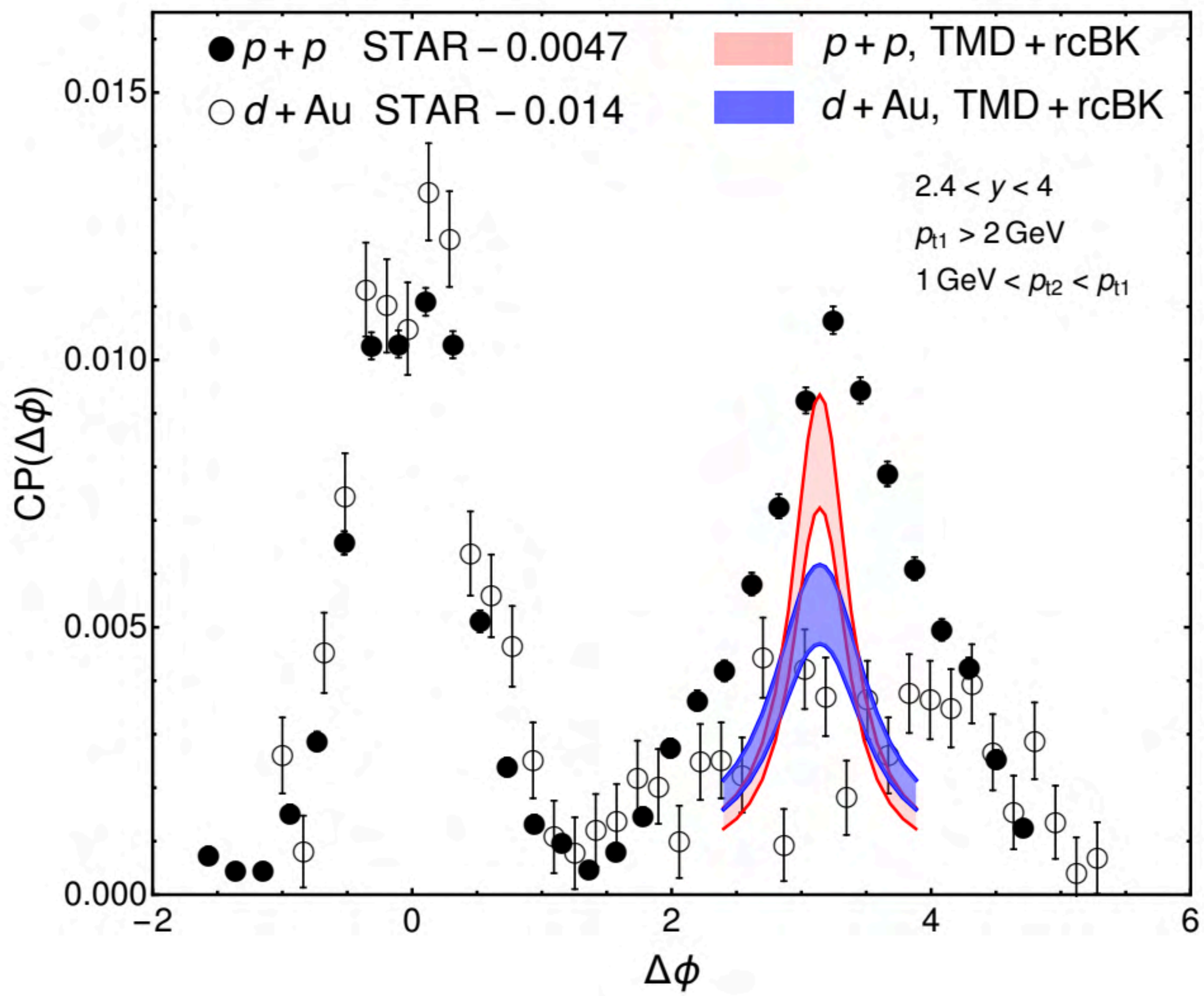

$A^{1/3}$ suppression supports saturation picture

STAR Collaboration
*Phys. Rev. Lett. 129, 092501 (2022)*

For dijets at LHC see Piotr Kotko's talk on Thursday

# Azimuthal correlation a window to gluon saturation

Gluon saturation without Sudakov




**Experimental data:** E. Braidot [STAR Collaboration] *arXiv:1005.2378*

**Theory curves:** J. Albacete, G. Giacalone, C. Marquet, M. Matas. *Phys.Rev.D 99 (2019) 1, 014002*

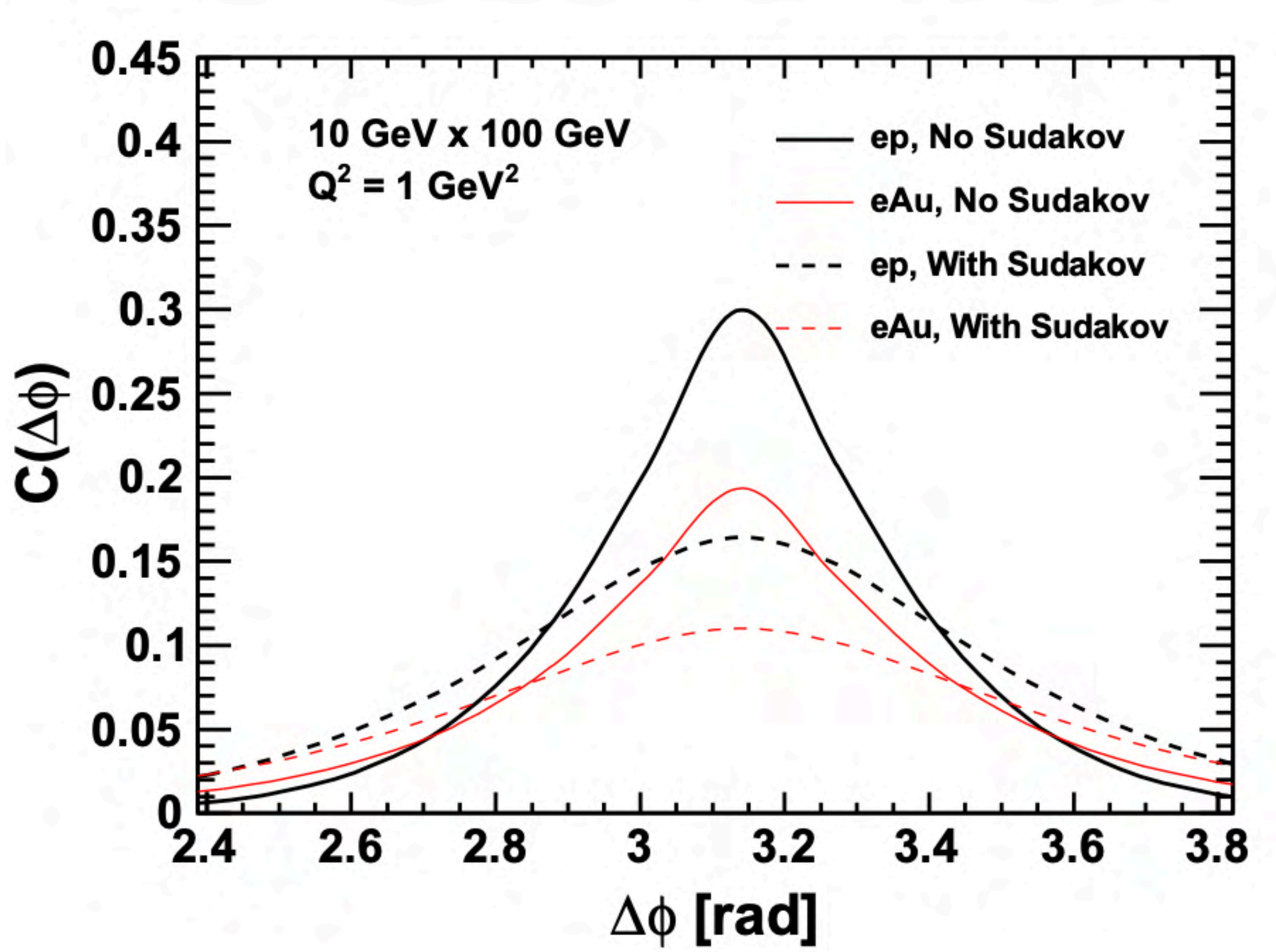
Gluon saturation alone cannot describe data

For recent phenomenology of Sudakov + Gluon Saturation see talks at DIS2022 by Cyrille Marquet and Sanjin Benić

See also Anton Perkov's talk on Thursday

# Azimuthal correlation a window to gluon saturation

Forward dihadron azimuthal correlations at the EIC



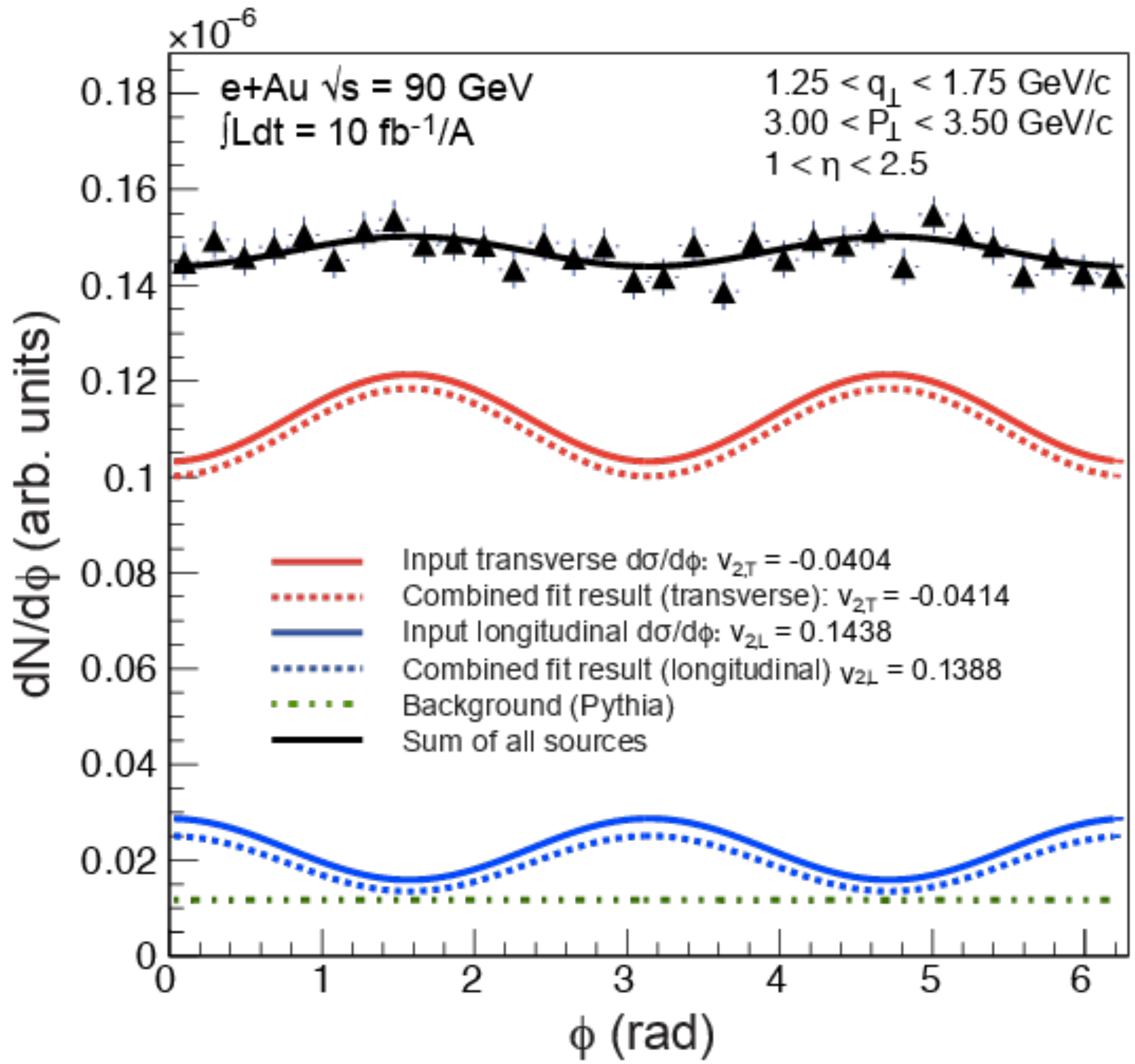

L. Zheng, E.C. Aschenauer, J.H. Lee, B.W. Xiao. *Phys.Rev.D 89 (2014) 7, 074037*

Advantages of EIC over RHIC: better control over kinematics, no pedestal, one-channel (depends only on WW distribution)

For more on EIC program see Elke Aschenauer's talk

# Azimuthal correlation a window to gluon saturation

Dihadron azimuthal correlations at the EIC

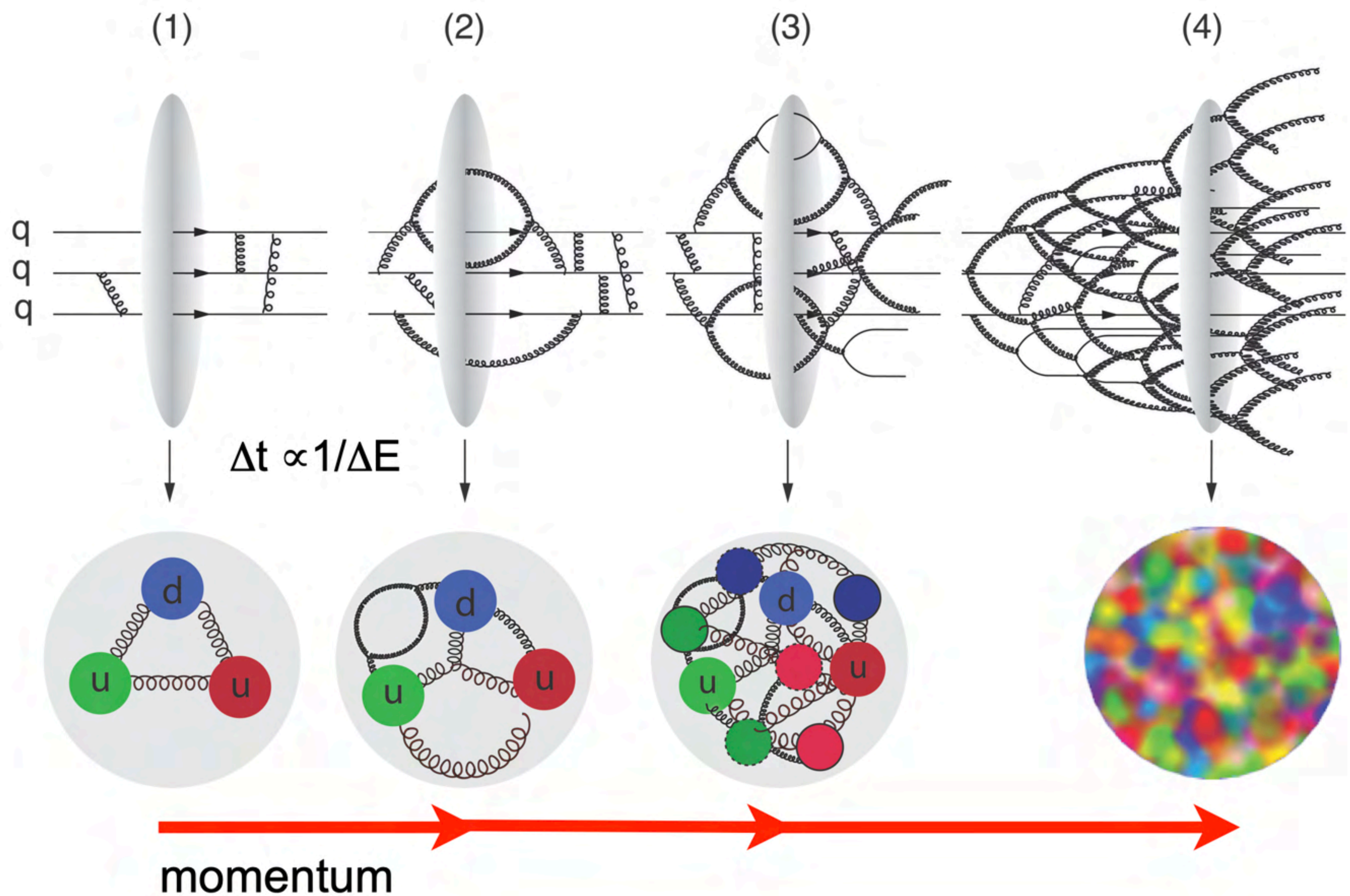



A. Dumitru, V. Skokov, T. Ullrich. *Phys.Rev.C* 99 (2019) 1, 015204

See also A. Dumitru, T. Lappi, V. Skokov. Phys. Rev. Lett. 115, 252301 (2015)

Momentum imbalance azimuthal distribution (relative to jet azimuthal angle) sensitive to linearly polarized gluons

# Anatomy of nuclear matter in the high-energy limit




Artwork: T. Ullrich

Emergence of an energy and nuclear specie dependent momentum scale

Multiple scattering (higher twist effects)

Non-linear evolution equations (BK/JIMWLK)

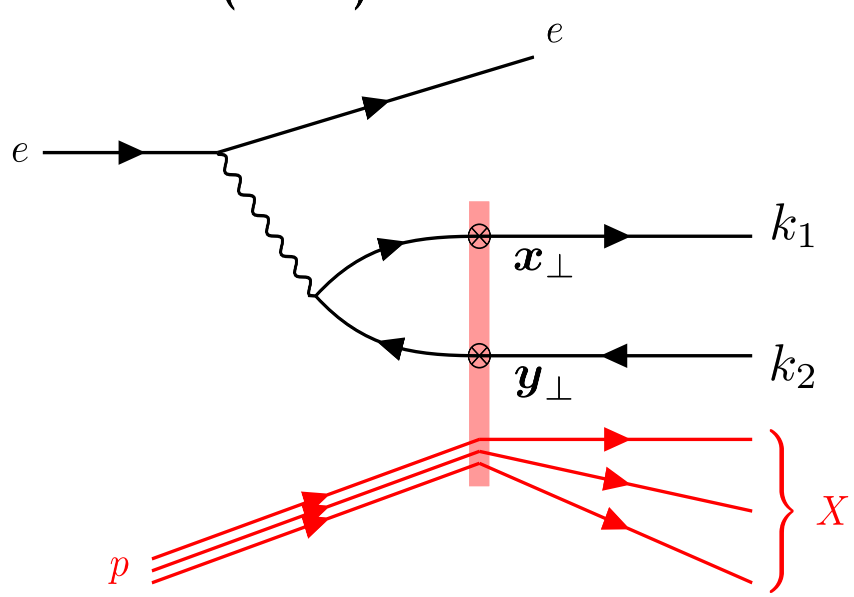
$$Q_s^2 \propto A^{1/3} x^{-\lambda}$$

For a recent review see:
Mining Gluon Saturation at Colliders. A. Morreale, FS. *Universe 2021, 7(8), 312*

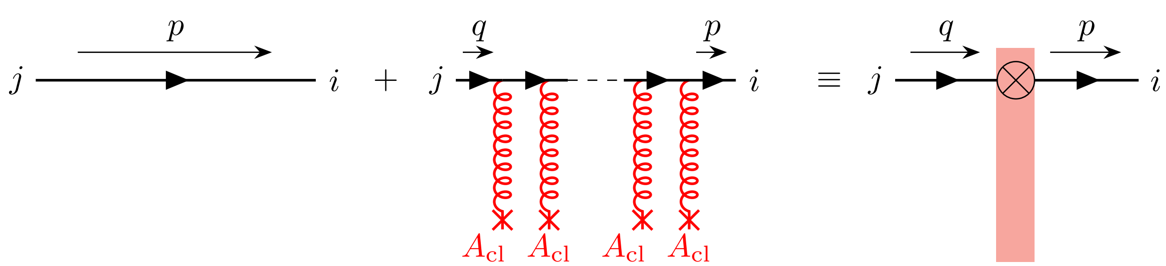
# Dijet production the CGC

## Leading order result

F. Dominguez, C. Marquet, B.W. Xiao, F. Yuan. *Phys.Rev.D* 83 (2011) 105005




Dense gluon field $A_{\rm cl} \sim 1/g$ **needs <u>resummation</u> of multiple gluon interactions**




$$V_{ij}(\boldsymbol{x}) = P\exp\left\{ig\int dx^- A^{+,a}_{cl}(\boldsymbol{x},x^-)t^a\right\}$$

Unpolarized differential cross-section:

$$\frac{\mathrm{d}\sigma^{\gamma^*_\lambda+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} \propto \int \mathrm{d}^8\boldsymbol{X}_\perp\; e^{-i\boldsymbol{k}_{1\perp}\cdot(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)}e^{-i\boldsymbol{k}_{2\perp}\cdot(\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}$$
$$\times \langle\Xi_{\rm LO}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp\boldsymbol{x}'_\perp)\rangle_Y \mathcal{R}^\lambda(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp-\boldsymbol{y}'_\perp)$$

$q\bar{q}$ interaction with nucleus — $\gamma^*$ splitting to $q\bar{q}$

$$\Xi_{\rm LO}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp\boldsymbol{y}'_\perp) = 1 - S^{(2)}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) - S^{(2)}(\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp) + S^{(4)}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)$$

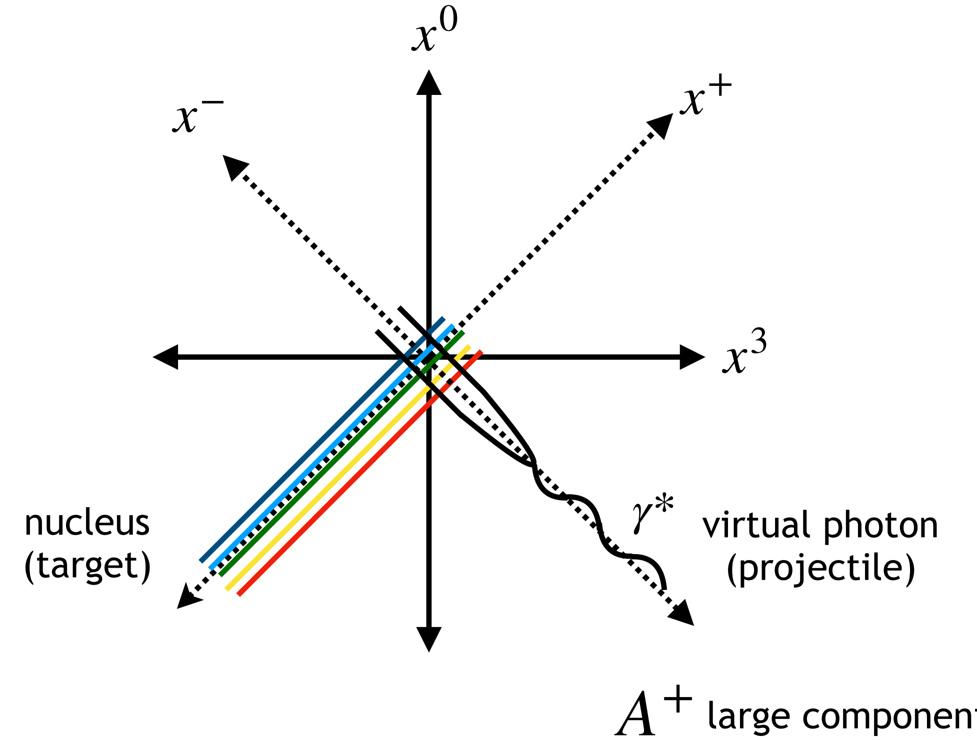
dipoles — quadrupole

$$S^{(2)}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = \frac{1}{N_c}\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]$$

# Dijet production the CGC

## Next-to-leading order results

P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*




- Divergences: UV, soft and collinear

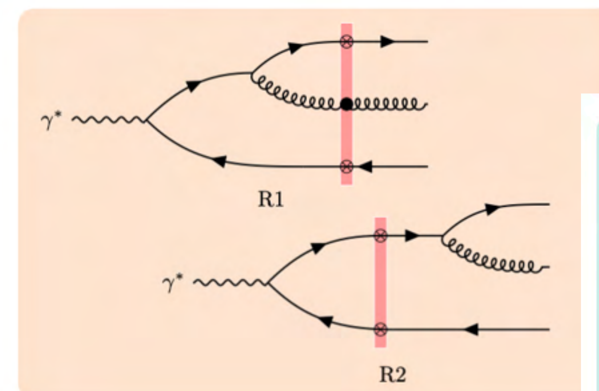
  **Dimensional regularization + longitudinal momentum cut-off + small-R cone algorithm**

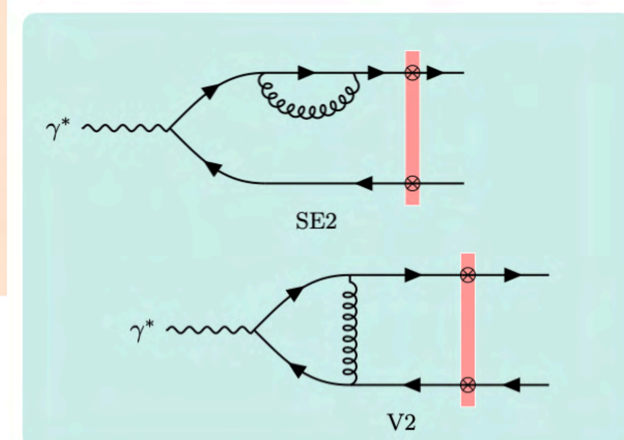
$$\int_{\Lambda^-} \frac{\mathrm{d}k_g^-}{k_g^-}\mu^\varepsilon \int \frac{\mathrm{d}^{2-\varepsilon}\boldsymbol{k_{g\perp}}}{(2\pi)^{2-\varepsilon}} f_{\Lambda^-}(k_g^-, \boldsymbol{k_{g\perp}})$$

- Large rapidity (high-energy) logs

  **Resummed via JIMWLK renormalization**

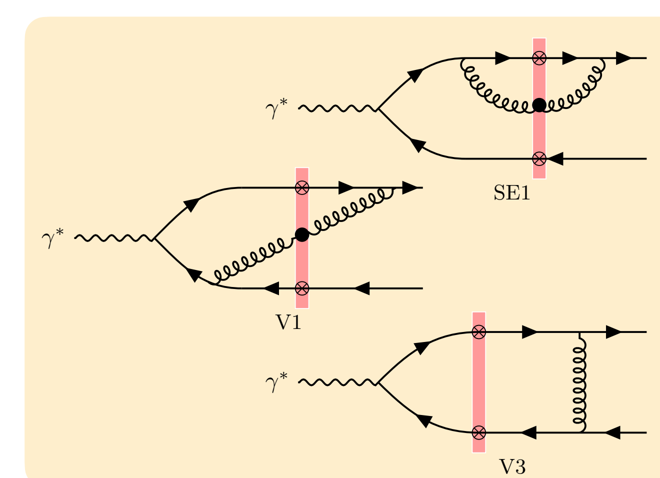
$$\left[1 + \mathcal{H}_{\mathrm{LL}} \ln\left(\Lambda_f^- / \Lambda^-\right) + \ldots\right] \langle \mathrm{d}\sigma \rangle_{\Lambda^-} = \langle \mathrm{d}\sigma \rangle_{\Lambda_f^-}$$




- Impact factor

  **Finite piece (free of large rapidity logs), but might contain other (potentially) large logs!**




- We showed cancellation of UV, soft and collinear divergences
- Absorbed large energy/rapidity logs into JIMWLK resummation
- Isolated genuine $\mathcal{O}(\alpha_s)$ contributions (aka impact factor)

# Dijet production the CGC

## Next-to-leading order results: rapidity factorization

P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \mathrm{d}\widetilde{\sigma}_0 \ln\left(\frac{z_f}{z_0}\right) + \int_0^{z} \frac{\mathrm{d}z_g}{z_g}\left[\mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}} - \mathrm{d}\widetilde{\sigma}_0 \Theta(z_f - z_g)\right] + \mathcal{O}(z_0)$$

Leading Logarithmic in x (LLx)

impact factor

$$\left.\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{NLO}}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_2}\right|_{\mathrm{LLx}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^6}\delta(1 - z_q - z_{\bar{q}}) \int \mathrm{d}\boldsymbol{X}_\perp \mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) \ln\left(\Lambda_f^-/\Lambda^-\right)$$

$$\mathcal{H}_{\mathrm{LL}}\left\langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp \boldsymbol{y}'_\perp)\right\rangle_Y$$

$$\begin{aligned}
\times \frac{\alpha_s N_c}{4\pi^2}\Bigg\langle \int \mathrm{d}^2\boldsymbol{z}_\perp \Bigg\{ &\frac{\boldsymbol{r}^2_{xy}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy}}(2D_{xy} - 2D_{xz}D_{zy} + D_{zy}Q_{y'x',xz} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\
&+ \frac{\boldsymbol{r}^2_{x'y'}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy'}}(2D_{y'x'} - 2D_{y'z}D_{zx'} + D_{zx'}Q_{xy,y'z} + D_{y'z}Q_{xy,zx'} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\
&+ \frac{\boldsymbol{r}^2_{xx'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zx'}}(D_{zx'}Q_{xy,y'z} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\
&+ \frac{\boldsymbol{r}^2_{yy'}}{\boldsymbol{r}^2_{zy}\boldsymbol{r}^2_{zy'}}(D_{y'z}Q_{xy,zx'} + D_{zy}Q_{y'x',xz} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\
&+ \frac{\boldsymbol{r}^2_{xy'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy'}}(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{zx'}Q_{xy,y'z} - D_{zy}Q_{y'x',xz}) \\
&+ \frac{\boldsymbol{r}^2_{x'y}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy}}(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{y'z}Q_{xy,zx'} - D_{xz}Q_{y'x',zy}) \Bigg\} \Bigg\rangle_Y
\end{aligned}$$

Small-x evolution of dipole and quadrupole!

Evolution of quadrupole can be found in Dominguez, Mueller, Munier, Xiao. Phys.Lett.B 705 (2011) 106-111

JIMWLK LL Hamiltonian acting on LO color structure → Renormalization of Wilson line operators

# Dijet production the CGC

## Next-to-leading order results: impact factor

P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times C_F \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{\alpha_s}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}}\right]\ln\left(\frac{\boldsymbol{k_{1\perp}}^2\boldsymbol{r}_{xx'}^2R^2\xi^2}{c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}'_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times \Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{(-\alpha_s)}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy'}}\right]\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy'}^2\xi^2}{z_2^2c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{sud1}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\times\frac{\alpha_s}{\pi} \times\left\{C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_f}{z_1}\right)\ln\left(\frac{\boldsymbol{r}_{xx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_f}{z_2}\right)\ln\left(\frac{\boldsymbol{r}_{yy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]+\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_1}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{xy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_2}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{yx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right\}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{\alpha_sC_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{\boldsymbol{k_{1\perp}}^2\boldsymbol{k_{2\perp}}^2\boldsymbol{r}_{xy}^2\boldsymbol{r}_{x'y'}^2}{c_0^4}\right)-3\ln(R)+\frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right)+\frac{11}{2}+3\ln(2)-\frac{\pi^2}{2}\right\}$$

$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}(4\alpha_sC_F)\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{e^{-i\boldsymbol{k_{g\perp}}\cdot\boldsymbol{r}_{xx'}}}{(\boldsymbol{k_{g\perp}}-\frac{z_g}{z_1}\boldsymbol{k_{1\perp}})^2}\left\{8z_1z_2^3(1-z_2)^2Q^2\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2}}r_{x'y'})\delta_z^{(3)} -\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta(z_1-z_g)\delta_z^{(2)}\right\}+(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^3\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'}) \times\frac{\alpha_s}{\pi}\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}\left\{K_0(\bar{Q}_{\mathrm{V3}}r_{xy})\left[\left(1-\frac{z_g}{z_1}\right)^2\left(1+\frac{z_g}{z_2}\right)(1+z_g)e^{i(\boldsymbol{P}_\perp+z_g\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V3}}r_{xy}) -\left(1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right)e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_\odot\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\right] +K_0(\bar{Q}r_{xy})\ln\left(\frac{z_gP_\perp r_{xy}}{c_0z_1z_2}\right)+(1\leftrightarrow 2)\right\}\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)+c.c.$$

$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8}\int \mathrm{d}^2\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}(-4\alpha_s)\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy'}}}{\boldsymbol{l}_\perp^2}\left\{8z_1^2z_2^2(1-z_2)(1-z_1)Q^2K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2'}}r_{x'y'})\left[1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right] \times e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_{xy'}}\frac{\boldsymbol{l}_\perp\cdot(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)}{(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)^2}\delta_z^{(3)}-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta\left(\frac{c_0^2}{\boldsymbol{r}_{xy'}^2}\geq\boldsymbol{l}_\perp^2\geq\boldsymbol{K}_\perp^2\right)\Theta(z_1-z_g)\delta_z^{(2)}\right\} +(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,other}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g} \times\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\left\{\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1} -\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})-\Theta(z_f-z_g)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})\right]C_F\Xi_{\mathrm{LO}} -\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}\left[\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left(1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}K_0(QX_V) -\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1}+(1\leftrightarrow 2)\right\}+c.c..$$

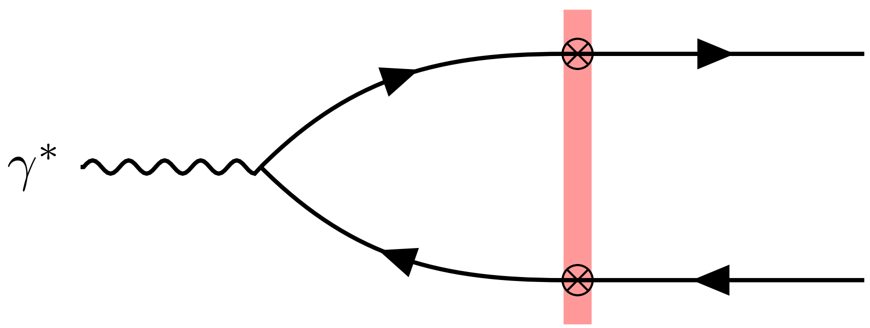
$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,other}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\frac{\mathrm{d}^2\boldsymbol{z}'_\perp}{\pi}e^{-i\boldsymbol{k_{g\perp}}\cdot\boldsymbol{r}_{zz'}} \alpha_s\left\{-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2}}r_{w'y'})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp) +\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2'}}r_{w'y'})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp) +\frac{1}{2}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp) -\frac{1}{2}\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp) +(1\leftrightarrow 2)+c.c.\right\}-\frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8}\alpha_s\Theta(z_f-z_g)\times\text{“slow”}$$

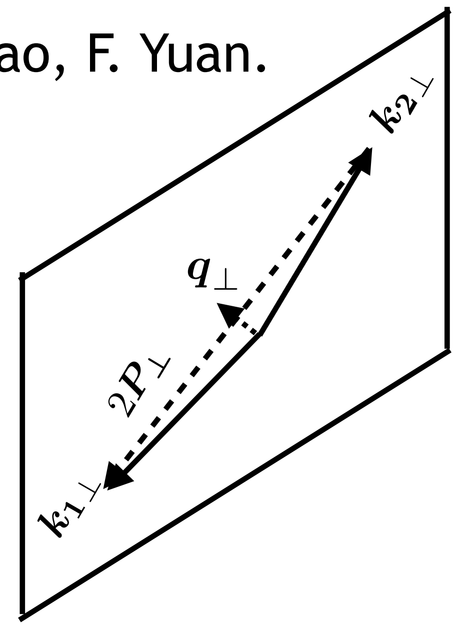
Similar expressions (but more lengthy) for transversely polarized photon

# Back-to-back limit: gluon saturation and Sudakov

Leading order results:

F. Dominguez, C. Marquet, B.W. Xiao, F. Yuan.
*Phys.Rev.D* 83 (2011) 105005




TMD valid $q_\perp, Q_s \ll k_{1\perp}, k_{2\perp}$

back-to-back hadrons/jets
and transverse momenta larger
than sat scale

In the back-to-back (or more precisely in the correlation limit) the CGC cross-section factorizes

$$\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X} \sim \mathcal{H}^{ij}(\boldsymbol{P}_\perp) \; \alpha_s G_Y^{ij}(\boldsymbol{q}_\perp)$$

Perturbatively calculable

WW gluon TMD

$$G_Y^{ij}(\boldsymbol{q}_\perp) = \frac{1}{2}\delta^{ij} G_Y^0(\boldsymbol{q}_\perp) + \frac{1}{2}\left(\frac{2\boldsymbol{q}_\perp^i \boldsymbol{q}_\perp^j}{\boldsymbol{q}_\perp^2} - \delta^{ij}\right) h_Y^0(\boldsymbol{q}_\perp)$$

Unpolarized — Linearly polarized

Azimuthal anisotropy

$$\langle \cos(2\phi) \rangle \sim h_Y^0(\boldsymbol{q}_\perp)/G_Y^0(\boldsymbol{q}_\perp)$$

$$\phi \equiv \phi_{\boldsymbol{q}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

F. Dominguez, J.W. Qiu, B.W. Xiao, F. Yuan.
*Phys.Rev.D 85 (2012) 045003*

For an explicit quantitative comparison
between TMD and CGC see

R. Boussarie, H. Mäntysaari, FS, B. Schenke.
*JHEP* 09 (2021) 178

# Back-to-back limit: gluon saturation and Sudakov

## Sudakov resummation and saturation

A.H. Mueller, Bo-Wen Xiao, Feng Yuan. *Phys.Rev.D* 88 (2013) 11, 114010

*“Large logarithms [Sudakov] can be generated due to incomplete real and virtual graph cancellation when the phase space for additional gluon emission is limited.”*

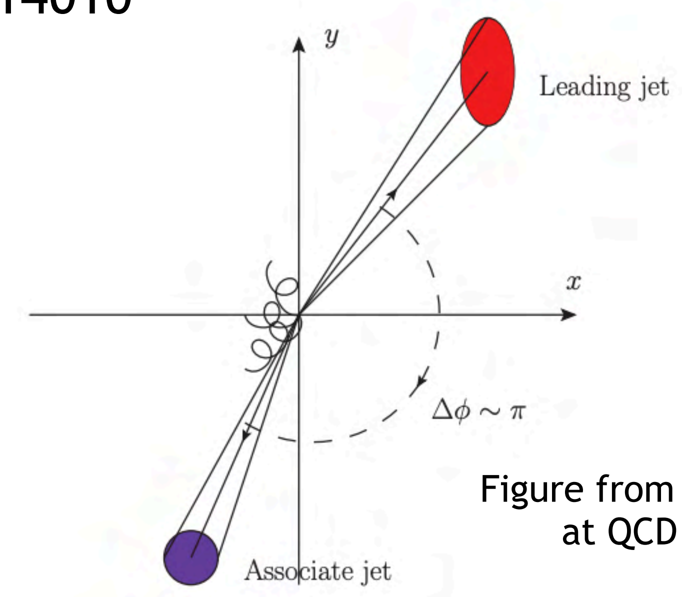



Figure from B.W. Xiao lecture at QCD master class

$$d\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X} \propto \mathcal{H}(\boldsymbol{P}_\perp) \int \frac{d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp} \alpha_s \widetilde{G}^0_Y(\boldsymbol{b}_\perp) e^{-S_{\rm Sud}(\boldsymbol{b}_\perp, \boldsymbol{P}_\perp)}$$

Perturbative Sudakov factor:

$$S_{\rm Sud}(\boldsymbol{b}_\perp, Q) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[ A \log\left(\frac{Q^2}{\bar{\mu}^2}\right) + B \right]$$

Mueller, Xiao, Yuan (2013) conclusion:

Sudakov resummation as in collinear factorization. TMD needs to include BFKL/BK/JIMWLK evolution

*Joint Sudakov+small-x resummation*

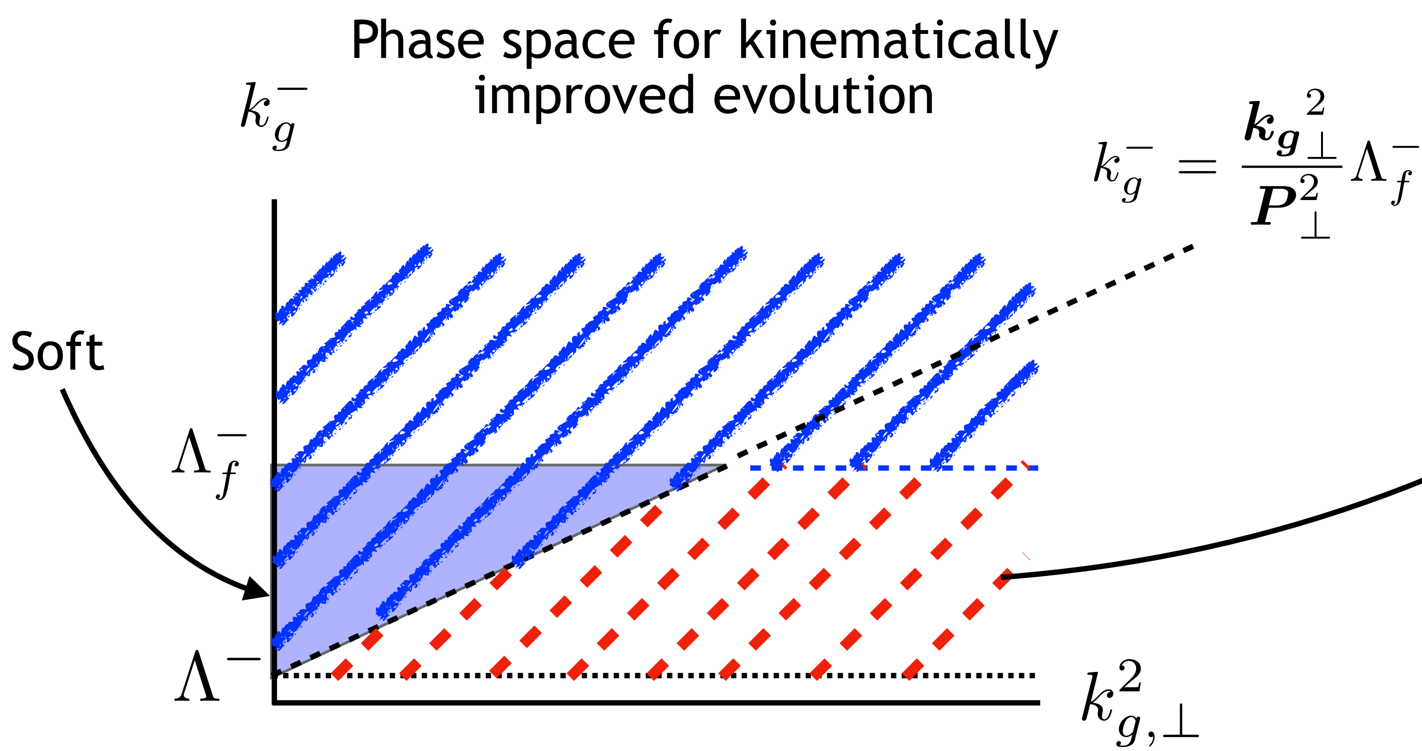
MXY assume TMD factorization in the first place

**Can we derive these results more rigorously? Can we obtain finite NLO pieces?**

# Back-to-back limit: gluon saturation and Sudakov

## Sudakov resummation and kinematically improved small-x evolution

P. Caucal, FS, B. Schenke, and R. Venugopalan. *2208.13872*




Kinematic constraint: ordering in + and - momenta

$$k_g^+ \geq \Lambda_f^+ \quad \& \quad k_g^- \leq \Lambda_f^-$$

For photo-production see
*Taels, Altinoluk, Marquet, Beuf. 2204.11650*

$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} \propto \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{b}_\perp e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$

$$\left[1 - \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{b}_\perp^2}{c_0^2}\right) + \cdots + \alpha_s \ln\left(\Lambda_f^-/\Lambda^-\right) \mathcal{K}_{LL} \otimes\right] \alpha_s \widetilde{G}_Y(\boldsymbol{b}_\perp)$$

Correct Sudakov double log

Kinematically improved small-x evolution

P. Caucal, FS, B. Schenke, and R. Venugopalan. *2208.13872*

For hadron production in pA see Shu-yi Wei's talk on Wednesday

# Back-to-back limit: gluon saturation and Sudakov

## Sudakov resummation at single log accuracy

P. Caucal, FS, B. Schenke, and R. Venugopalan. *2208.13872*

$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} \propto \mathcal{H}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2 \boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp} \alpha_s \widetilde{G}_Y^0(\boldsymbol{b}_\perp) e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, \boldsymbol{P}_\perp)}$$

$$S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, Q) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[ A \log\left(\frac{Q^2}{\bar{\mu}^2}\right) + B \right]$$

$$A = \frac{\alpha(\mu^2) N_c}{2\pi} \qquad B = \frac{\alpha(\mu^2) C_F}{2\pi} \left[ \ln\left(\frac{2(1+\cosh(\Delta Y_{12}))}{R^2}\right) + \ln\left(\frac{x_{Bj}}{z_1 z_2 c_0^2 x_f}\right) \right]$$

- Terms in blue are in agreement with Mueller, Xiao, Yuan (2013) and Y. Hatta, B.W. Xiao, F. Yuan, J. Zhou. *Phys.Rev.D 104 (2021) 5, 054037*

- Term in red is rapidity factorization scale dependent single log is new. Suggest a natural choice for the factorization scale

$$x_f = \frac{x_{\mathrm{Bj}}}{z_1 z_2 c_0^2}$$

# Back-to-back limit: gluon saturation and Sudakov

## Genuine $\mathcal{O}(\alpha_s)$ pieces (impact factor)

P. Caucal, FS, B. Schenke, and R. Venugopalan. *2208.13872*

Azimuthally averaged cross-section

$$\begin{aligned}
\mathrm{d}\sigma^{(0),\lambda=\mathrm{L}} &= \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{L}}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
&\times \left\{1 + \frac{\alpha_s C_F}{\pi}\left[\frac{3}{2}\ln(c_0^2) - 3\ln(R) + \frac{1}{2}\ln^2\left(\frac{z_1^2}{z_2^2}\right) + \frac{11}{2} + 3\ln(2) - \frac{\pi^2}{2}\right]\right. \\
&\left. + \frac{\alpha_s N_c}{2\pi}\left[\ln\left(\frac{z_f^2}{z_1 z_2}\right)\ln(c_0^2) - \ln^2\left(\frac{Q_f^2 c_0^2}{\boldsymbol{P}_\perp^2}\right)\right]\right\} \\
&+ \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{L}}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
&\times \frac{\alpha_s N_c}{2\pi}\left\{1 + \frac{2C_F}{N_c}\ln(R^2) - \frac{1}{N_c^2}\ln(z_1 z_2)\right\} \\
&+ \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \left[\frac{1}{2}\mathcal{H}_{\mathrm{NLO},1}^{\lambda=\mathrm{L},ii}(\boldsymbol{P}_\perp)\right] \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
&\times \frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}\ln\left(\frac{z_1 z_2}{z_f^2}\right) - \frac{3C_F}{2N_c}\right] \\
&+ \left\{\alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \left[\frac{1}{2}\mathcal{H}_{\mathrm{NLO},2}^{\lambda=\mathrm{L},ii}(\boldsymbol{P}_\perp)\right] \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2\right. \\
&\left.\times \left(\frac{-\alpha_s}{2\pi N_c}\right) + c.c.\right\} + \mathrm{d}\sigma_{\mathrm{other}}^{(0),\lambda=\mathrm{L}}.
\end{aligned}$$

Elliptic anisotropy

$$\begin{aligned}
\mathrm{d}\sigma^{(2),\lambda=\mathrm{L}} &= \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{L}}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \frac{\cos(2\theta)}{2} \hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
&\times \left\{1 + \frac{\alpha_s C_F}{\pi}\left[\frac{3}{2}\ln(c_0^2) - 4\ln(R) + \frac{1}{2}\ln^2\left(\frac{z_1^2}{z_2^2}\right) + \frac{11}{2} + 3\ln(2) - \frac{\pi^2}{2}\right]\right. \\
&\left. + \frac{\alpha_s N_c}{2\pi}\left[-\frac{5}{4} + \ln\left(\frac{z_f^2}{z_1 z_2}\right)\ln(c_0^2) - \ln^2\left(\frac{Q_f^2 c_0^2}{\boldsymbol{P}_\perp^2}\right)\right] + \frac{\alpha_s}{4\pi N_c}\ln(z_1 z_2)\right\} \\
&+ \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{L}}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \frac{\cos(2\theta)}{2} \hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
&\times \frac{\alpha_s N_c}{\pi}\left\{1 + \frac{2C_F}{N_c}\ln(R^2) - \frac{1}{N_c^2}\ln(z_1 z_2)\right\} \\
&+ \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \left[\frac{1}{2}\mathcal{H}_{\mathrm{NLO},1}^{\lambda=\mathrm{L},ii}(\boldsymbol{P}_\perp)\right] \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \frac{\cos(2\theta)}{2} \hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
&\times \frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}\ln\left(\frac{z_1 z_2}{z_f^2}\right) - \frac{3C_F}{2N_c}\right] \\
&+ \left\{\alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \left[\frac{1}{2}\mathcal{H}_{\mathrm{NLO},2}^{\lambda=\mathrm{L},ii}(\boldsymbol{P}_\perp)\right] \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \frac{\cos(2\theta)}{2} \hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)\right. \\
&\left.\times \left(\frac{-\alpha_s}{2\pi N_c}\right) + c.c.\right\} + \mathrm{d}\sigma_{\mathrm{other}}^{(2),\lambda=\mathrm{L}}.
\end{aligned}$$

- Only blue terms had been computed before by Hatta, Xiao, Yuan, Zhou (2021)
- Red terms break TMD factorization at NLO and grow with $Q_s^2$

Bonus: we computed all even (2n) harmonics!

# Back-to-back limit: gluon saturation and Sudakov

## TMD factorization breaking at NLO: correlators beyond WW

P. Caucal, FS, B. Schenke, and R. Venugopalan. *2208.13872*

- Color correlators at NLO

| | |
|---|---|
| $\Xi_{\rm LO}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$ | $\langle 1 - D_{xy} - D_{y'x'} + Q_{xy,y'x'}\rangle$ |
| $\Xi_{\rm NLO,1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$ | $\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{zy,y'x'}D_{xz} - D_{xz}D_{zy}\rangle - \frac{1}{2N_c}\Xi_{\rm LO}$ |
| $\Xi_{\rm NLO,2}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$ | $\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{xz,y'x'}D_{zy} - D_{xz}D_{zy}\rangle - \frac{1}{2N_c}\Xi_{\rm LO}$ |
| $\Xi_{\rm NLO,3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$ | $\frac{N_c}{2}\langle 1 - D_{xy} - D_{y'x'} + D_{xy}D_{y'x'}\rangle - \frac{1}{2N_c}\Xi_{\rm LO}$ |
| $\Xi_{\rm NLO,4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$ | $\frac{N_c}{2}\langle 1 - D_{xz}D_{zy} - D_{y'z}D_{zx'} + Q_{xz,z'x'}Q_{y'z',zy}\rangle - \frac{1}{2N_c}\Xi_{\rm LO}$ |

- Blue correlators collapse to the WW gluon TMD, red correlators result in other TMDs. e.g.

$$\Xi_{\rm LO} \approx \boldsymbol{u}^i_\perp \boldsymbol{u}'^j_\perp \frac{1}{2N_c}(-2)\underbrace{\left\langle {\rm Tr}\left[\left(V(\boldsymbol{b}_\perp)\partial^i V^\dagger(\boldsymbol{b}_\perp)\right)\left(V(\boldsymbol{0}_\perp)\partial^j V^\dagger(\boldsymbol{0}_\perp)\right)\right]\right\rangle_Y}_{\alpha_s \widetilde{G}^{ij}_Y(\boldsymbol{b}_\perp)}$$

$$\Xi_{\rm NLO,1} \approx -\boldsymbol{u}'^j_\perp \frac{1}{2N_c}(-2)\underbrace{\left\langle {\rm Tr}\left[V(\boldsymbol{b}_\perp)V^\dagger(\boldsymbol{z}_\perp)\right]{\rm Tr}\left[V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{b}_\perp)\partial^j V(\boldsymbol{0}_\perp)V^\dagger(\boldsymbol{0}_\perp)\right]\right\rangle_Y}_{\alpha_s \widetilde{G}^{j}_{Y,{\rm NLO},1}(\boldsymbol{b}_\perp,\boldsymbol{z}_\perp)}$$

breaks TMD factorization!

# Back-to-back limit: gluon saturation and Sudakov

## TMD factorization breaking at NLO: evolution and impact factor

P. Caucal, FS, B. Schenke, and R. Venugopalan. *2208.13872*

$$\alpha_s \widetilde{G}^j_{Y,\mathrm{NLO},1}(\boldsymbol{b}_\perp, \boldsymbol{z}_\perp) \qquad \alpha_s \widetilde{G}^j_{Y,\mathrm{NLO},2}(\boldsymbol{b}_\perp, \boldsymbol{z}_\perp) \qquad \alpha_s \widetilde{G}_{Y,\mathrm{NLO},4}(\boldsymbol{b}_\perp, \boldsymbol{z}_\perp)$$

Featured in

- the JIMWLK evolution of the WW gluon TMD, not closed even at large Nc

Dominguez, Mueller, Munier, Xiao (2011)

- Impact factor

see also T*aels, Altinoluk, Marquet, Beuf. 2204.11650*

Break TMD factorization

In the dilute limit (small $Q_s^2$), "new" correlators collapse to WW TMD $\alpha_s \widetilde{G}^{ij}_Y(\boldsymbol{b}_\perp)$

- JIMWLK -> BFKL (closed equation)
- Impact factor only depends on WW TMD

Recover TMD factorization (BFKL+Sudakov)

- Gluon saturation introduces factorization-breaking terms at NLO. Expected to be enhanced for nuclei!

# Outlook

- Phenomenological study of dijets at NLO. Get numbers!

  Kinematically constrained evolution, Sudakov single and double logs, genuine NLO corrections, TMD factorization

- Consequence of of our results to other processes

  Dihadrons in DIS (exclusive and inclusive), 2 particle production processes in proton-nucleus collisions. Energy correlators?

- Distinguish soft/rapidity divergences using SCET

  A systematic treatment of soft and collinear regions, and rapidity divergences. Factorization ala SCET al small-x.

*work in progress with Z. Kang and X. Liu*

# Summary

Motivation:

2-particle azimuthal correlations → powerful observables to search for saturation

Results:

small-x and soft gluon resummation at finite Nc at single logarithmic accuracy

full NLO calculation → computed all even azimuthal harmonics

Identified TMD factorization breaking terms