Resolving negative NLO cross sections problem in quarkonium production via matching with High-Energy Factorization [1]

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[1] Based on JHEP **05** (2022) 083 and ongoing work

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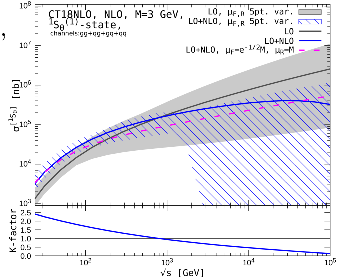
Perturbative instability of quarkonium total cross sections

Inclusive η_c-hadroproduction (CSM):

$$p+p \to c\bar{c}\left[{}^1S_0^{[1]}\right]+X,\ \text{LO: } g(p_1)+g(p_2) \to c\bar{c}\left[{}^1S_0^{[1]}\right],$$

$$\sigma(\sqrt{s_{pp}}) = f_i(x_1,\mu_F) \otimes f_j(x_2,\mu_F) \otimes \hat{\sigma}(z),$$

where $z = \frac{M^2}{\hat{s}}$ with $\hat{s} = (p_1+p_2)^2$.

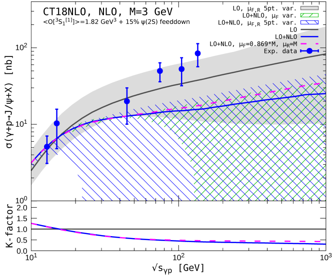


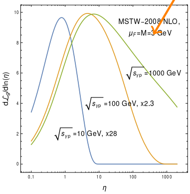
Inclusive J/ψ-photoproduction (CSM):

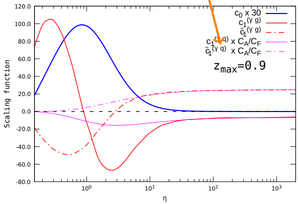
$$\gamma+p \to c\bar{c}\left[{}^3S_1^{[1]}\right]+X,\ \text{LO: } \gamma(q)+g(p_1) \to c\bar{c}\left[{}^3S_1^{[1]}\right]+g,$$

$$\sigma(\sqrt{s_{\gamma p}}) = f_i(x_1,\mu_F) \otimes \hat{\sigma}(\eta),$$

where $\eta = \frac{\hat{s}-M^2}{M^2}$ with $\hat{s} = (q+p_1)^2$.







Scale-fixing solution

Studied in [Lansberg, Ozcelik, 20'], [Lansberg *et.al*, 21']. For J/ψ **photoproduction**:

$$\frac{d\sigma_{\gamma p}^{(\mathrm{LO+NLO})}}{d\ln\mu_F^2} \propto \left(\frac{\alpha_s}{2\pi}\right)^2 \int\limits_0^{\eta_{\max}} d\eta \left\{ \ln(1+\eta)\left[c_1(\eta\to\infty) + \bar{c}_1(\eta\to\infty)\ln\frac{M^2}{\mu_F^2}\right] \right.$$

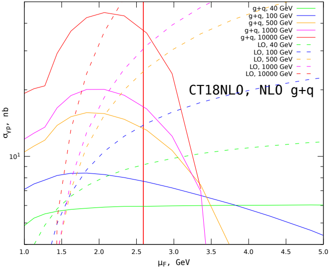
$$\left. \times \left(f_g(x_\eta,\mu_F^2) + \frac{C_F}{C_A} f_q(x_\eta,\mu_F^2)\right) + \text{non-singular terms at } \eta \gg 1 \right\}$$

"principle of minimal scale-sensitivity" $\Rightarrow$ for J/ψ **photoproduction**:

$$\hat{\mu}_F = M \exp\left[\frac{\bar{c}_1(\eta\to\infty)}{2\bar{c}_1(\eta\to\infty)}\right] \simeq 0.87M,$$

for η_c-hadroproduction:

$$\hat{\mu}_F = M \exp\left[\frac{A_1}{2}\right] = \frac{M}{\sqrt{e}} \simeq 0.61M.$$

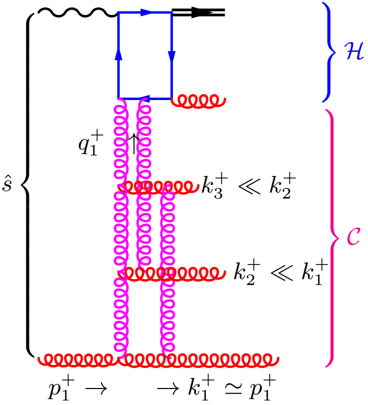


The $\hat{\mu}_F$-scale removes corrections $\propto \alpha_s^n \ln^{n-1}(1+\eta)$ from $\hat{\sigma}_i(\eta)$ and resums them into PDFs. But is such resummation complete?

High-Energy Factorization

The **LLA** ($\sum_n \alpha_s^n \ln^{n-1}(1+\eta)$) formalism is due to [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']

Physical picture in the **LLA** for photoproduction:



Glauber exchanges($k_+ k_- \ll \mathbf{k}_T^2$) form the **Reggeized gluon** in the t-channel.

$$\hat{\sigma}_{\mathrm{HEF}}(\eta) \propto \int_0^{1+\eta} \frac{dy}{y} \int_0^{\infty} d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \times \mathcal{H}(y, \mathbf{q}_{T1}^2) + \mathrm{NLLA} + O(1/\eta).$$

- The resummation factor $\mathcal{C}$ is the solution of the LL **BFKL** equation with collinear divergences subtracted,
- The coefficient function $\mathcal{H}$ can be calculated at LO[Kniehl, Vasin, Saleev, 06'] and NLO (needed for **NLLA**),
- For consistency with fixed-order **DGLAP** evolution the anomalous dimension γ_{gg} in $\mathcal{C}$ should be truncated:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA}} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \dots}_{\text{LLA}}$$

- Expansion of $\hat{\sigma}_{\mathrm{HEF}}(\eta)$ in α_s **correctly reproduces** $\hat{\sigma}_{\mathrm{NLO}}(\eta \gg 1)$ and predicts the $\hat{\sigma}_{\mathrm{NNLO}}(\eta \gg 1)$.

LLA evolution w.r.t. $\ln 1/z$

In the LL($\ln 1/z$)-approximation, the $Y = \ln 1/z$-evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$-factor has the form:

$$\tilde{\mathcal{C}}(x, \mathbf{q}_T) = \delta(1-x)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int_x^1 \frac{dz}{z} \int d^{2-2\epsilon}\mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2)\tilde{\mathcal{C}}\left(\frac{x}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

$$K(\mathbf{k}_T^2, \mathbf{p}_T^2) = \delta^{(2-2\epsilon)}(\mathbf{k}_T)\ \frac{(\mathbf{p}_T^2)^{-\epsilon}}{\epsilon}\ \frac{(4\pi)^\epsilon \Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{1}{\pi(2\pi)^{-2\epsilon}\mathbf{k}_T^2}.$$

It is convenient to go from $(z, \mathbf{q}_T)$-space to $(N, \mathbf{x}_T)$-space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon}\mathbf{q}_T\ e^{i\mathbf{x}_T\mathbf{q}_T} \int_0^1 dx\ x^{N-1}\ \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- Mellin convolutions over z turn into products: $\int \frac{dz}{z} \to \frac{1}{N}$
- Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^{k} \frac{1}{z} \to \frac{\alpha_s^{k+1}}{N^{k+1}}}$
- All *collinear divergences* are contained inside $\mathcal{C}$ in $\mathbf{x}_T$-space.

Exact LL solution

In $(N, \mathbf{q}_T)$-space, subtracted $\mathcal{C}$, which resums all terms $\propto (\hat{\alpha}_s/N)^n$ (complete LLA) has the form:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}(N, \alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, 82']:

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma),$$

where $\psi(\gamma) = d \ln \Gamma(\gamma)/d\gamma$ – Euler's ψ-function. The first few terms:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA}} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \ldots}_{\text{LLA}}$$

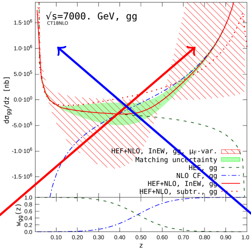
The function $R(\gamma)$ is

$$R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).$$

Matching with NLO

The HEF is valid in the **leading-power** in $M^2/\hat{s}$, so for $\hat{s} \sim M^2$ we match it with NLO CF by the *Inverse-Error Weighting Method* [Echevarria *et.al.*, 18'].

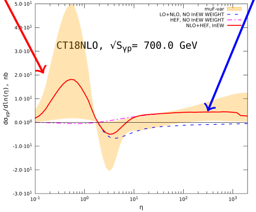
η_c**-hadroproduction**, $z = M^2/\hat{s}$:

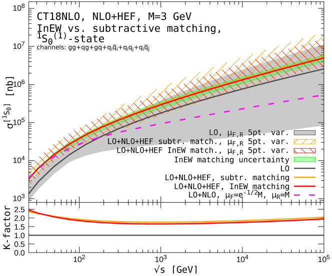


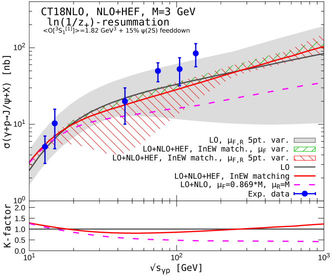
NLO

HEF

J/ψ**-photoproduction**, $\eta = (\hat{s} - M^2)/M^2$:







Inverse Error Weighting (InEW) matching

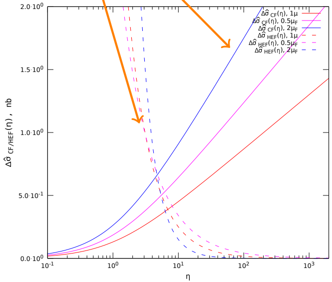
Development of an idea from [Echevarria *et al.*, 18'] :

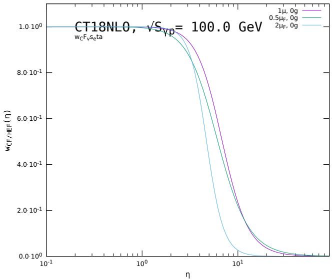
$$\hat{\sigma}(\eta) = w_{\rm CF}(\eta)\hat{\sigma}_{\rm CF}(\eta) + (1 - w_{\rm CF}(\eta))\hat{\sigma}_{\rm HEF}(\eta),$$

the weights are determined through the estimates of “errors”:

$$w_{\rm CF}(\eta) = \frac{\Delta\hat{\sigma}^{-2}_{\rm CF}(\eta)}{\Delta\hat{\sigma}^{-2}_{\rm CF}(\eta) + \Delta\hat{\sigma}^{-2}_{\rm HEF}(\eta)}, \quad w_{\rm HEF}(\eta) = 1 - w_{\rm CF}(\eta).$$

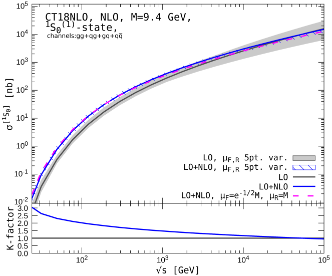
- $\Delta\hat{\sigma}_{\rm CF}(\eta)$ is due to **missing higher orders and large logarithms,** it can be estimated from the α_s expansion of $\hat{\sigma}_{\rm HEF}(\eta)$:
  $$\Delta\hat{\sigma}_{\rm CF}(\eta) = \hat{\alpha}_s^2 \ln(1+\eta) \left(f_2 + f_1 \ln \frac{M^2}{\mu_F^2} + \frac{\bar{f}_1}{2} \ln^2 \frac{M^2}{\mu_F^2} \right)$$
- $\Delta\hat{\sigma}_{\rm HEF}(\eta)$ is due to **missing power corrections in** $1/\eta$:
  $\Delta\hat{\sigma}_{\rm HEF}(\eta) = A\eta^{-\alpha_{\rm HEF}}$. We determine A and $\alpha_{\rm HEF}$ from behaviour of $\hat{\sigma}_{\rm CF}(\eta) - \hat{\sigma}_{\rm CF}(\infty)$ at $\eta \gg 1$.

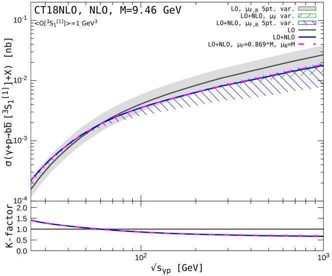




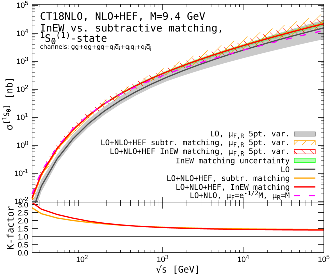
Results for bottomonia

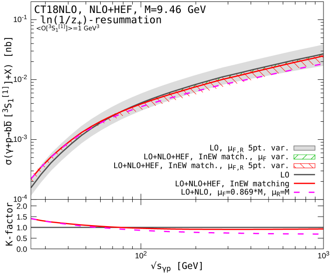
NLO:





NLO+HEF:





Towards NLL: the "Monster logs" at small $\mathbf{q}_T$ are not scary

$$\hat{\sigma}_{\text{HEF}}(\eta) \propto \int_0^{1+\eta} \frac{dy}{y} \int_0^{\infty} d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(y, \mathbf{q}_{T1}^2).$$

At NLO for $\mathcal{H}$ one typically encounters corrections $\propto \alpha_s \ln^n \frac{M^2}{\mathbf{q}_T^2}$ at $\mathbf{q}_T^2 \ll M^2$ with $n = 1, 2$. Let's study their effect in N-space (note that $\gamma_N = \hat{\alpha}_s/N$):

$$\int_0^{\mu_F^2} d\mathbf{q}_T^2 \; \mathcal{C}_{\text{DLA}}(N, \mathbf{q}_T^2, \mu_F^2) \times \hat{\alpha}_s \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2} = \hat{\alpha}_s \gamma_N \int_0^{\mu_F^2} \frac{d\mathbf{q}_T^2}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^{\gamma_N} \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2}$$

$$= \hat{\alpha}_s \frac{(-1)^n n!}{\gamma_N^n} = \begin{cases} -N & \text{for } n = 1 \\ \frac{2N^2}{\hat{\alpha}_s} & \text{for } n = 2 \end{cases} \xrightarrow[\text{Mellin transform}]{} \begin{cases} -\delta'(\eta) & \text{for } n = 1 \\ \frac{2}{\hat{\alpha}_s}\delta''(\eta) & \text{for } n = 2 \end{cases}$$

So these contributions *do not belong to NLA in* $\eta = (\hat{s} - M^2)/M^2 \gg 1$ and *will be removed by the matching!*

Conclusions and outlook

- The perturbative instability of p_T-integrated quarkonium production cross sections at NLO comes from the region $\hat{s} \gg M^2$
- The problem can be solved via matching of NLO calculation at $\hat{s} \sim M^2$ and LLA HEF calculation at $\hat{s} \gg M^2$
- The *Inverse-Error Weighting(InEW) method* is an efficient matching prescription without free parameters. The uncertainties due to matching are smaller than residual scale uncertainties
- The LLA HEF has to be truncated down to DLA for resummation factors, to be consistent with NLO DGLAP evolution
- The inclusive η_c hadroproduction and J/ψ photoproduction have been considered as examples
- Calculations for rapidity-dependent cross sections as well as $\chi_{c0,2}$-meson production cross sections are in progress
- The *next-to-DLA* calculation is needed to further reduce scale-uncertainties. *The logarithms* $\ln M^2/\mathbf{q}_T^2$ *for* $\mathbf{q}_T^2 \ll M^2$ *in the NLO HEF coefficient function* ($\mathcal{H}$) *are not a problem for the matching calculation!*

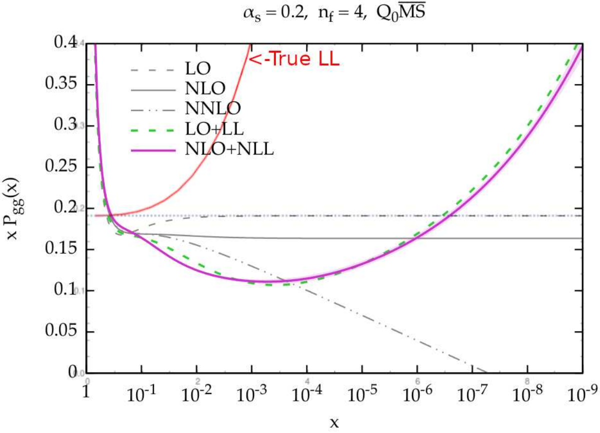
Thank you for your attention!

Backup: DGLAP P_{gg} at small z

Plot from hep-ph/1607.02153 with my curve (in red) for the **strict LLA**

$\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$; LO:

$P_{gg}(z) = \frac{2C_A}{z} + \ldots \Leftrightarrow \gamma_N = \frac{\hat{\alpha}_s}{N}$



The “LO+LL” and “NLO+NLL” curves represent a form of matching between DGLAP and BFKL expansions, in a scheme by Altarelli, Ball and Forte, more complicated than **strict LL or NLL approximation**.