Prospects for α_s measurement at the LHC using soft drop jet mass

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Motivation

Fine structure constant: $\alpha = 7.297\ 352\ 5693(11) \times 10^{-3}$

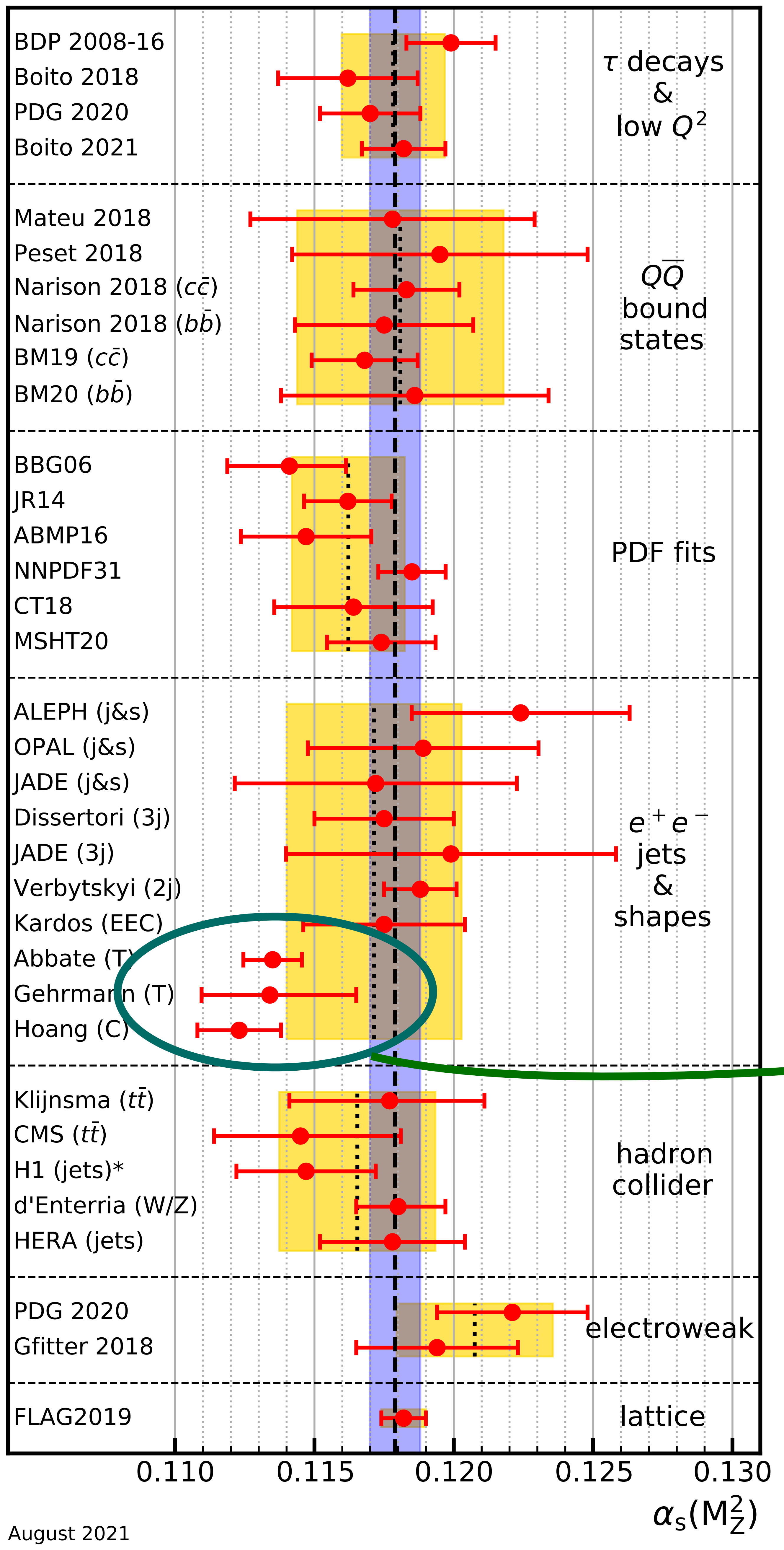
PDG

Motivation

Strong coupling constant: $\alpha_s(m_Z) = 0.1179 \pm 0.0010$

PDG, World average

PDG



Motivation

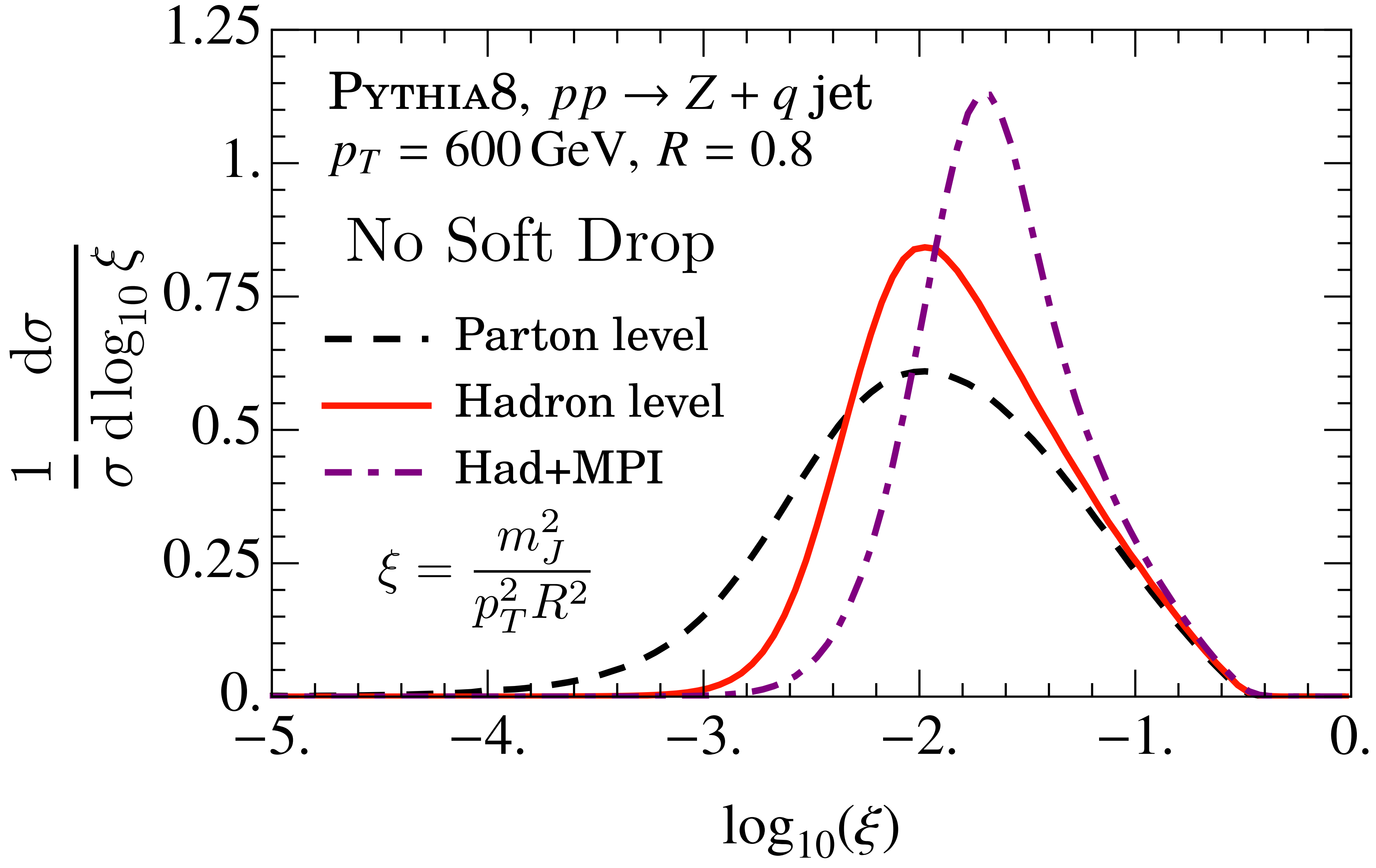
Uncertainties in α_s propagate into almost all measurements at the LHC

α_s world average: $\alpha_s(m_Z) = 0.1179 \pm 0.0010$

LEP measurements in tension with world average

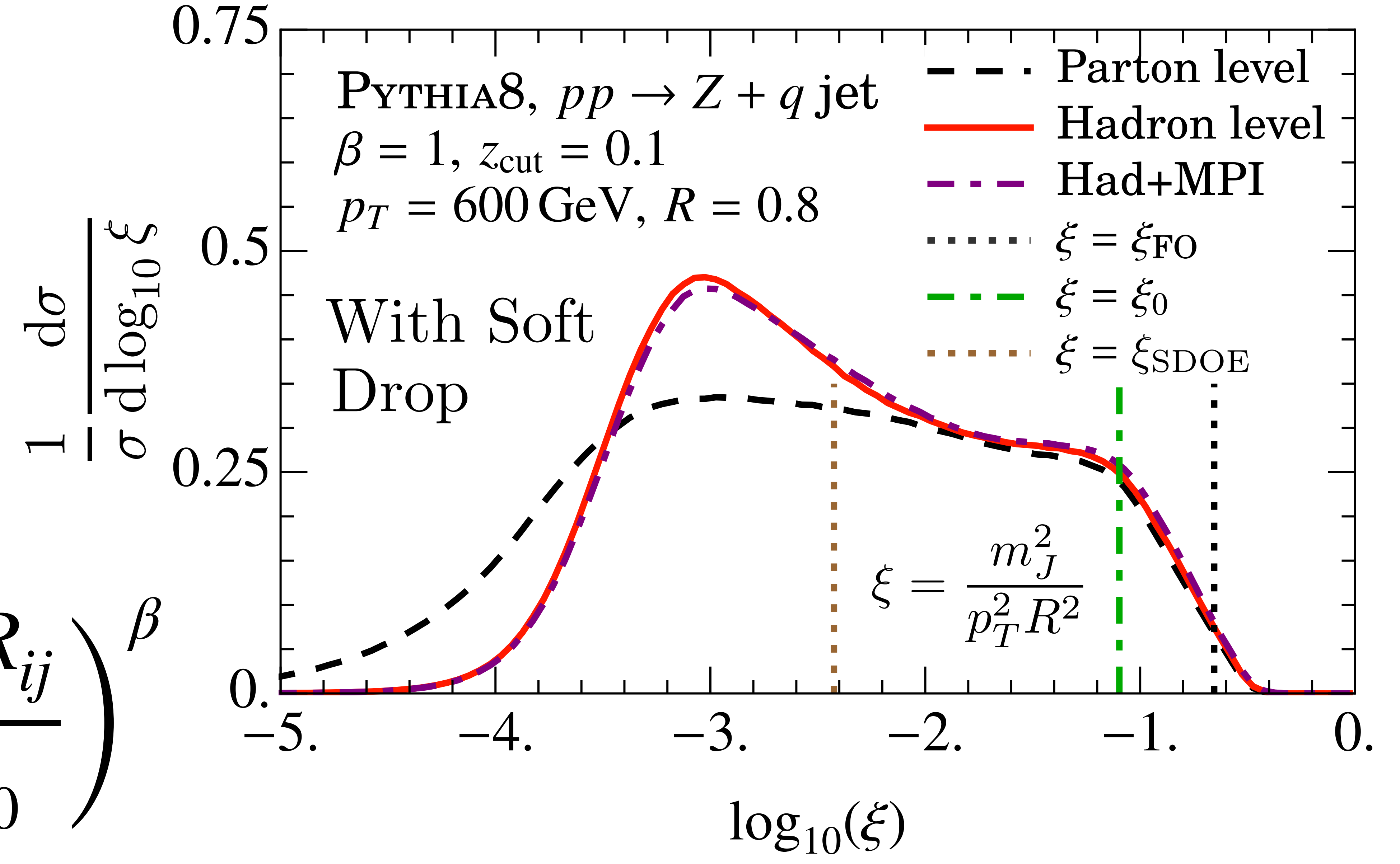
Can we get new measurements of α_s at the LHC using soft drop jet mass?

Motivation for soft drop jet mass



Apply soft drop

$$\frac{\min\{p_{T_i}, p_{T_j}\}}{p_{T_i} + p_{T_j}} > z_{\text{cut}} \left(\frac{\Delta R_{ij}}{R_0}\right)^{\beta}$$



[Larkoski, Marzani, Soyez, Thaler 2014]

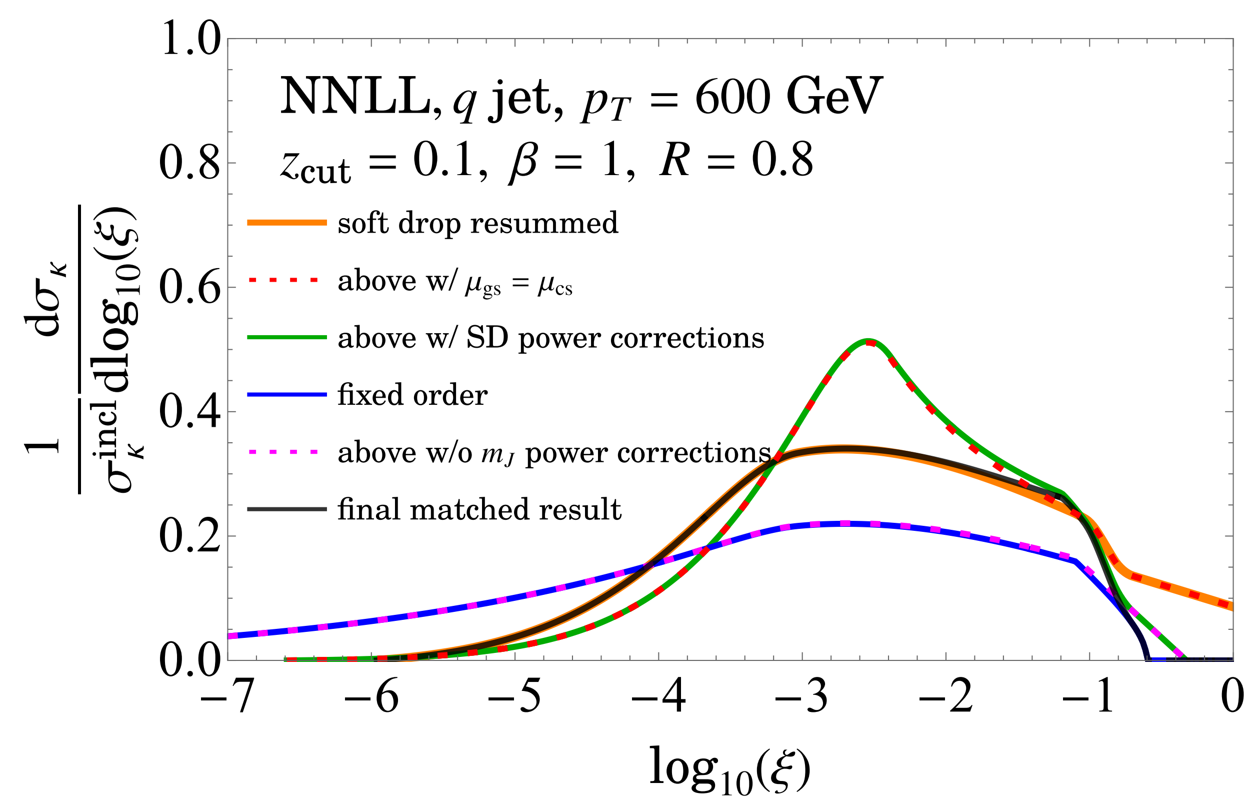
- Reduces sensitivity to the underlying event and hadronization effects
- **Measurable** and **calculable** observable for hadron colliders

[Les Houches 2017]

Outline

1. Quark-gluon fraction and PDF dependence

2. NNLL resummed cross section



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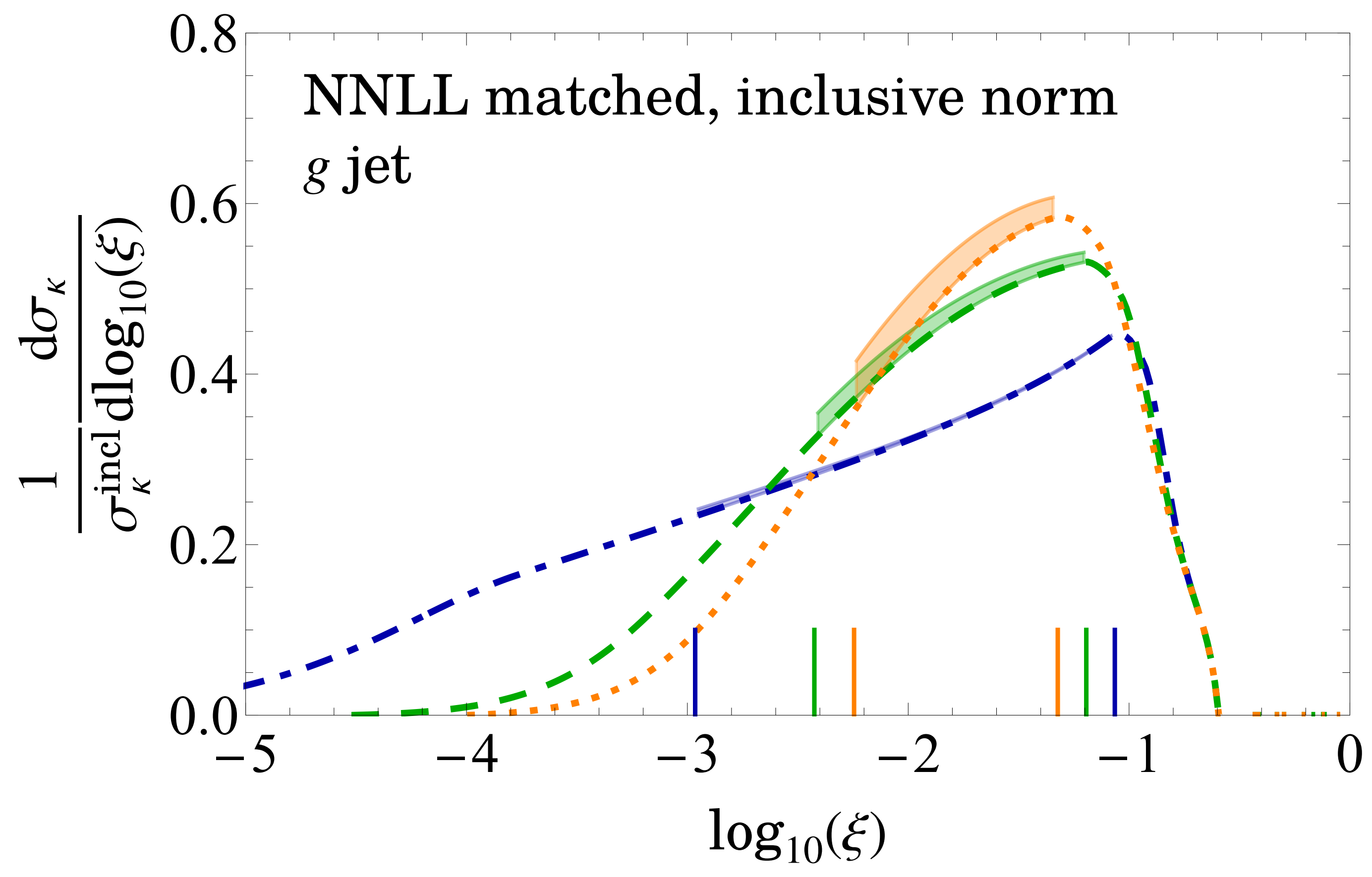
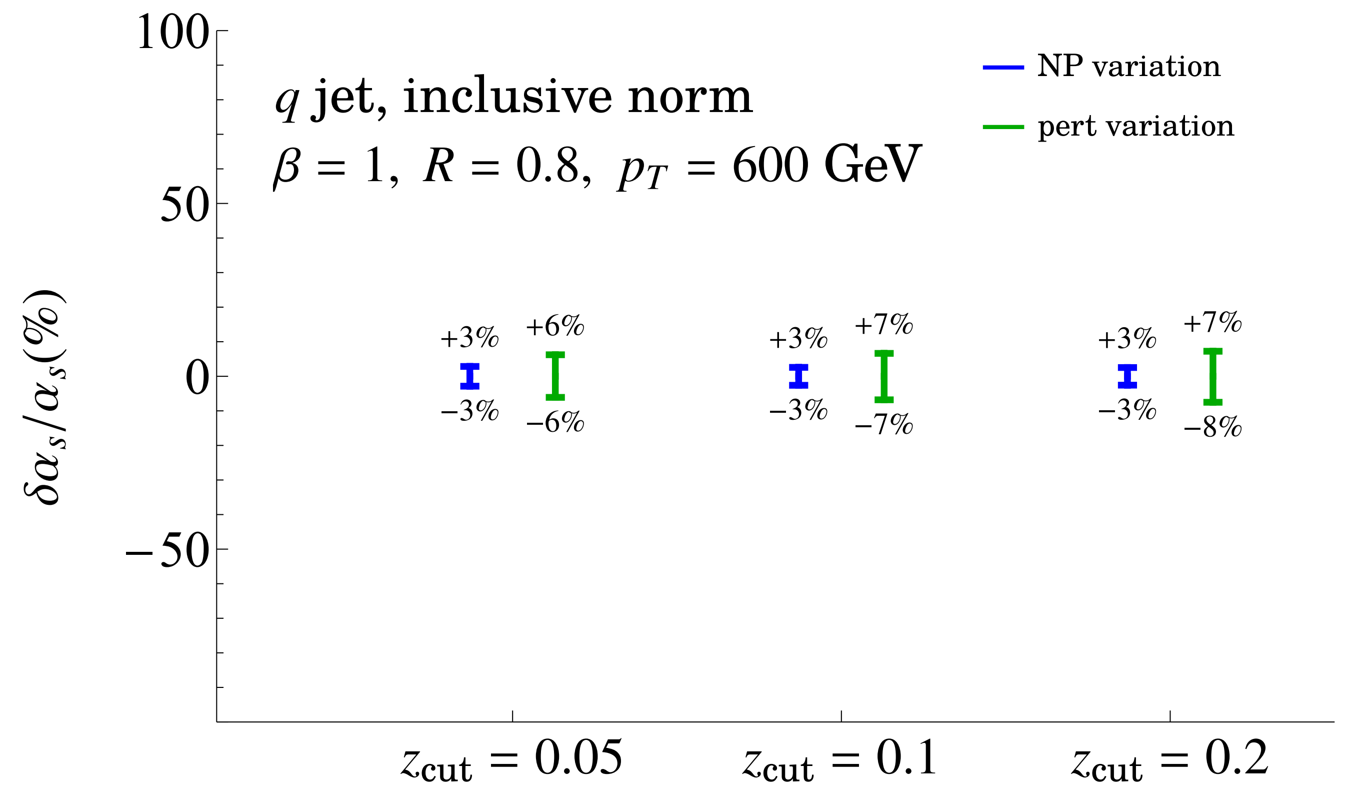


4. Results



Soft drop jet mass in inclusive jets

$$\xi = \frac{m_J^2}{p_T^2 R^2}$$

Hard function for $a + b \to c + X$

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}\xi} = \sum_{abc} \int \frac{\mathrm{d}x_a \mathrm{d}x_b \mathrm{d}z}{x_a x_b z} f_a(x_a,\mu) f_b(x_b,\mu) H_{ab}^c\left(x_a, x_b, \eta, \frac{p_T}{z}, \mu\right) \mathcal{G}_c(z, \xi, p_T, R, \mu)\,,$$

Parton distribution functions

Inclusive jet function

Hard collinear factorization for $\xi \ll 1$:

A single jet initiating parton i

$$\mathcal{G}_c\left(z, \xi, p_T, R, \mu\right) = \sum_i \mathcal{H}_{c\to i}\left(z, p_T R, \mu\right)\, \mathcal{J}_i(\xi, p_T, \eta, R, \mu)\,, \qquad \xi \ll 1$$

$$\mathcal{H}_{i\to j}\left(z, p_T R, \mu\right) = J_{ij}\left(z, p_T R, \mu\right) N_{\mathrm{incl}}^j\left(p_T R, \mu\right)$$

[Kang, Lee, Liu, Ringer 2018],
[Cal, Lee, Ringer, Waalewijn 2020]
[Hannesdottir, AP, Schwartz, Stewart 2022]

$$\mathcal{G}_c = \sum_{ij} J_{ij}(z, p_T R, \mu) \tilde{\mathcal{G}}_c(\xi, p_T, \eta, R, \mu)$$

Focus on this piece

Quark-gluon fraction

q/g fraction is **internal** to our calculation

(not an external input, not taken from experiments)

We can nevertheless "pull out" the q/g fractions and study their dependence on PDFs:

Normalize to inclusive cross section:

$$\frac{1}{\sigma_{\rm incl}(p_T,\eta)}\frac{{\rm d}^3\sigma}{{\rm d}p_T{\rm d}\eta{\rm d}\xi} = x_q\,\tilde{\mathcal{G}}_q(\xi,p_TR,\mu) + x_g\,\tilde{\mathcal{G}}_g(\xi,p_TR,\mu)$$

$$\sigma_{\rm incl}(p_T,\eta) \equiv \frac{{\rm d}^2\sigma}{{\rm d}p_T{\rm d}\eta} = \sum_{a,b,c,d} f_a\otimes f_b\otimes H_{ab}^c\otimes J_{cd}\,, \qquad x_\kappa(p_TR,\eta,\mu) \equiv \frac{\sum_{a,b,c} f_a\otimes f_b\otimes H_{ab}^c\otimes J_{c\kappa}}{\sigma_{\rm incl}(p_T,\eta)}$$

$$\tilde{\mathcal{G}}_\kappa(\xi,p_TR,\mu) \equiv \frac{1}{\sigma_\kappa^{\rm incl}}\frac{{\rm d}\sigma_\kappa}{{\rm d}\xi}(p_T,\eta) = N_{\rm incl}^\kappa(p_TR,\mu)\mathcal{J}_\kappa(\xi,p_T,\eta,R,\mu)$$

Quark-gluon fraction

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(not an external input, not taken from experiments)

We can nevertheless "pull out" the q/g fractions and study their dependence on PDFs:

PDF	α_s used	x_q	% change
NNPDF 23 LO	0.119	0.479	-6.0
NNPDF 23 NLO	0.119	0.517	1.3
NNPDF 23 NNLO	0.119	0.523	2.5
NNPDF 23 NNLO	0.120	0.514	0.84
CT18NLO_as_0119	0.119	0.514	0.87
CT18NNLO_as_0119	0.119	0.507	-0.49
MSTW2008nlo68cl	0.120	0.510	0.063
MSTW2008nlo68cl	0.117	0.514	0.87
mean	–	0.510	1.6

Using various PDFs combined with hard functions:

$$f_{\mathrm{LO}} = 0.507, \qquad f_{\mathrm{NLO}} = 0.530, \qquad f_{\mathrm{NNLO}} = 0.538$$

Subdominant effect on α_s uncertainty for normalized cross section

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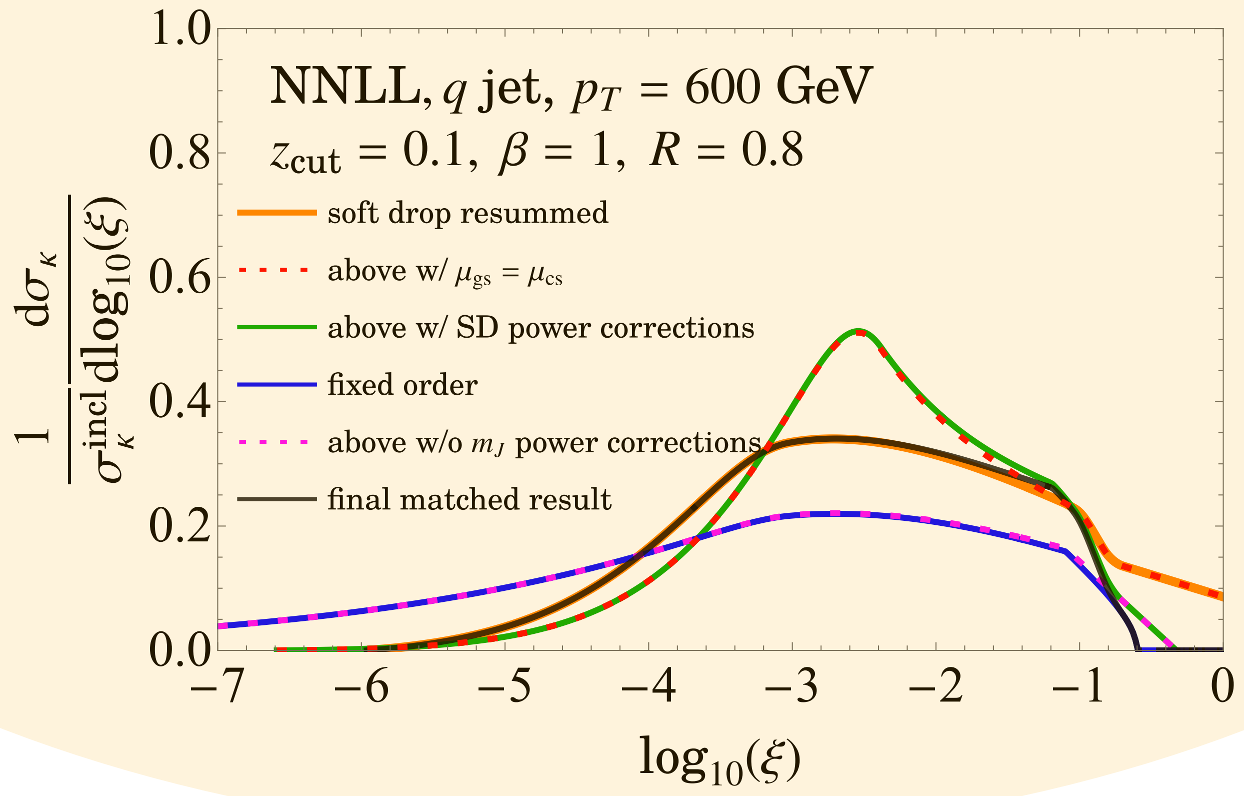

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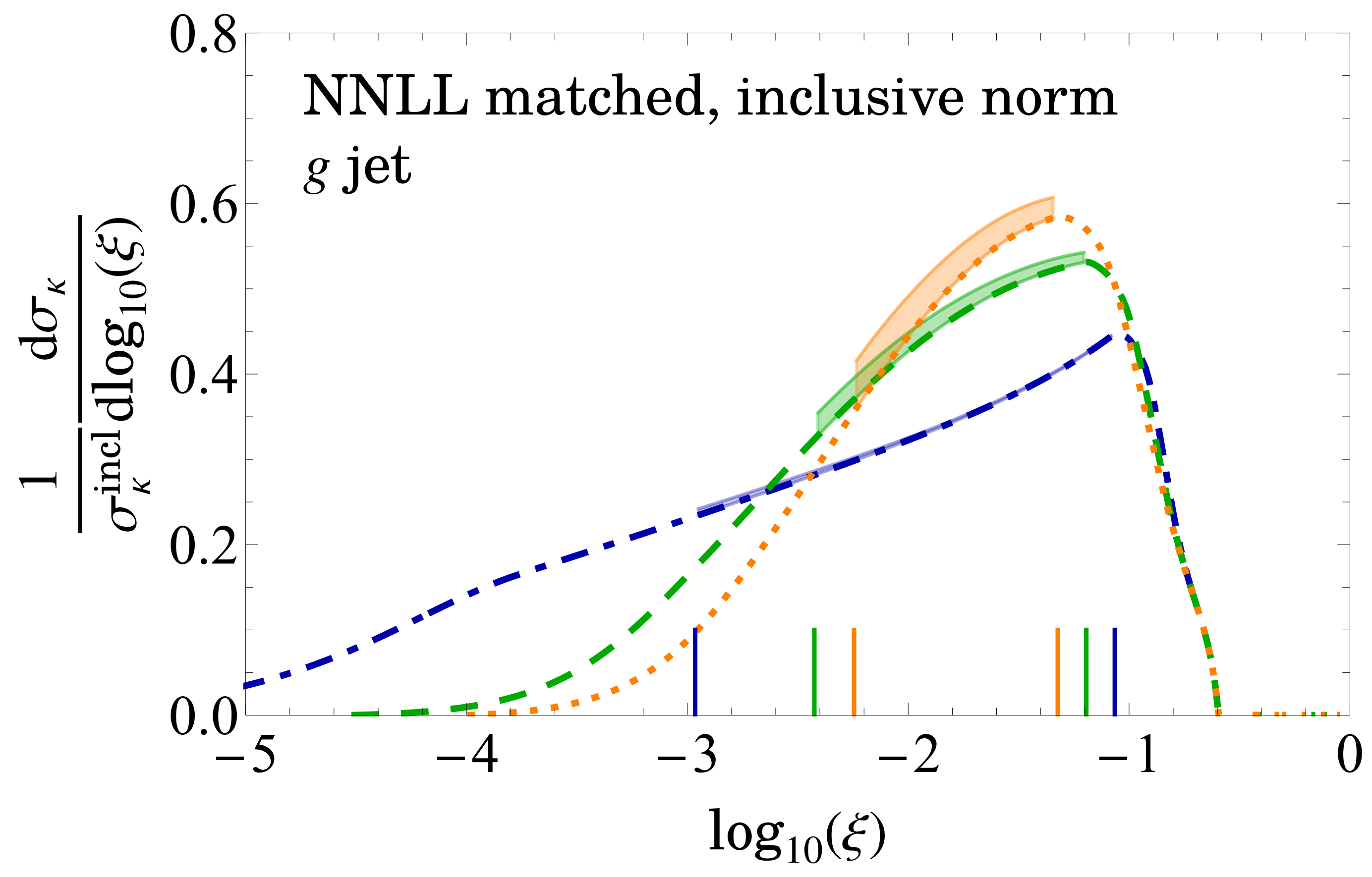

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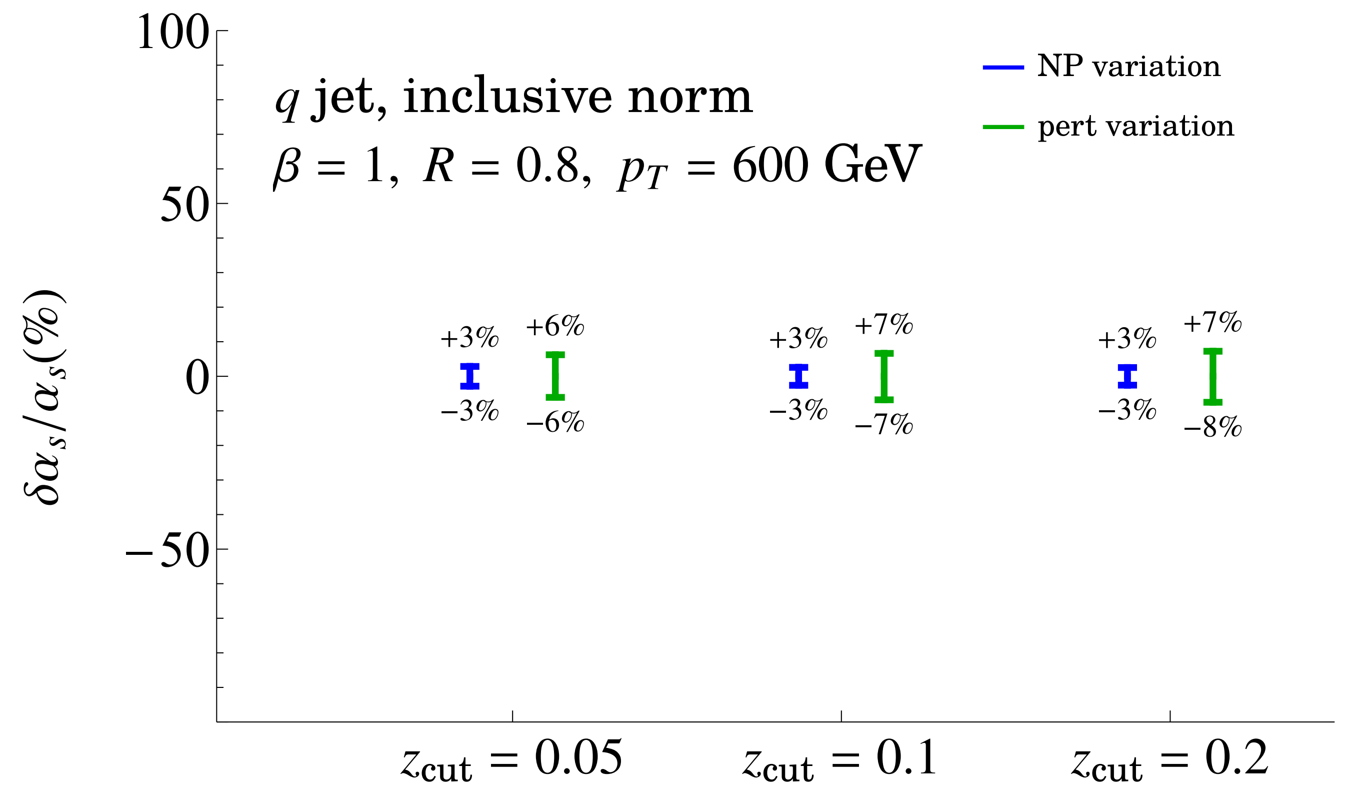
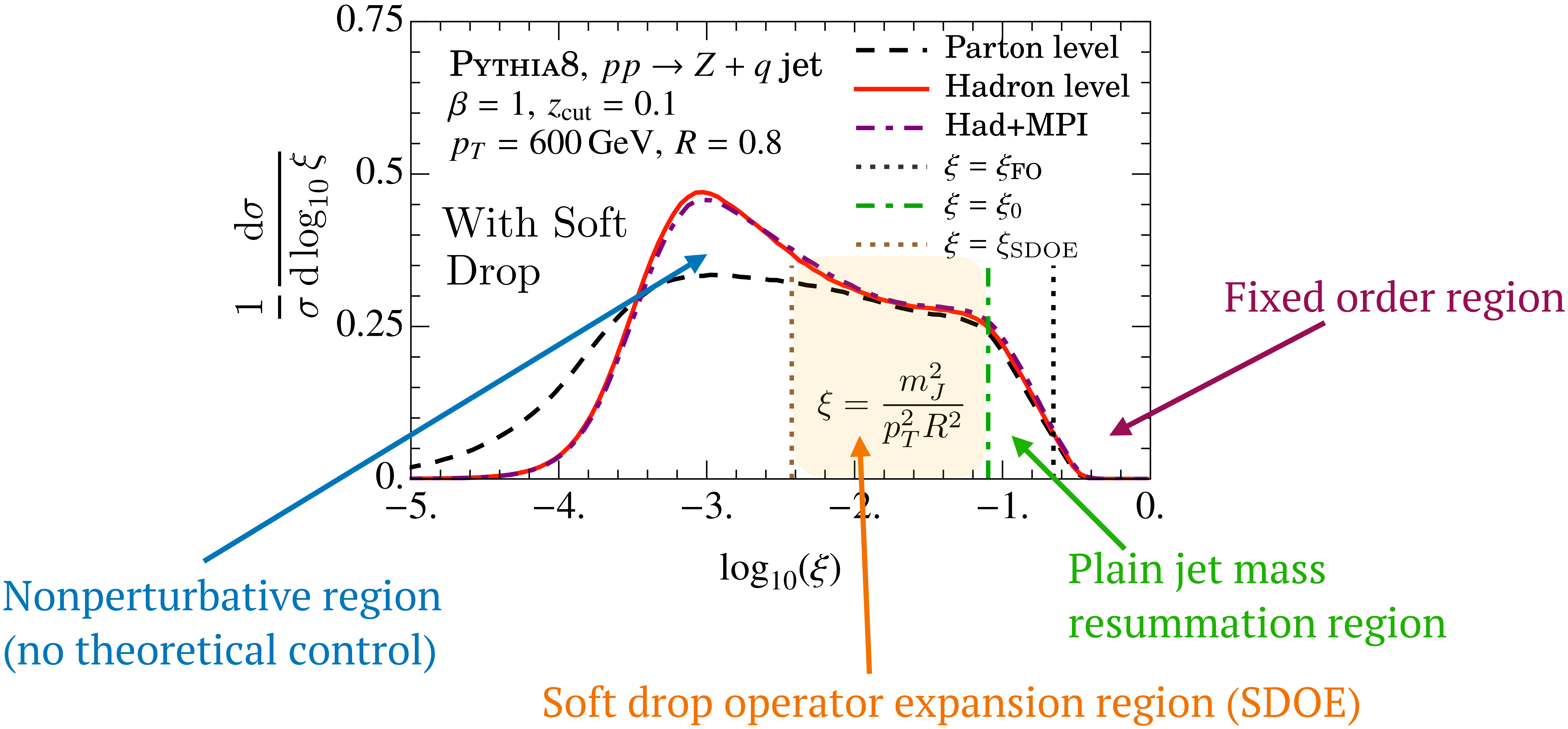


Region for fitting to α_s

$$\xi = \frac{m_J^2}{p_T^2 R^2}$$



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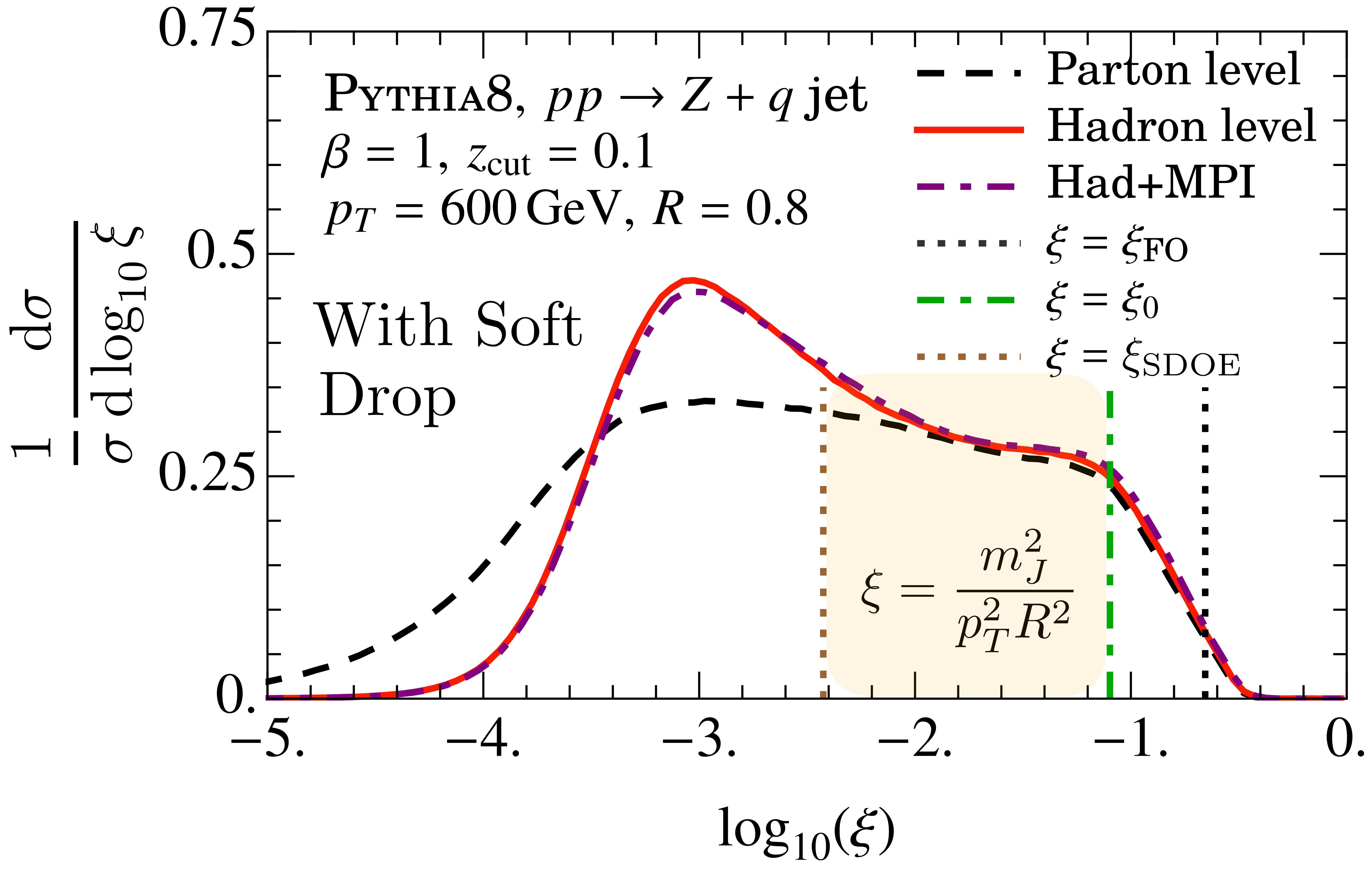
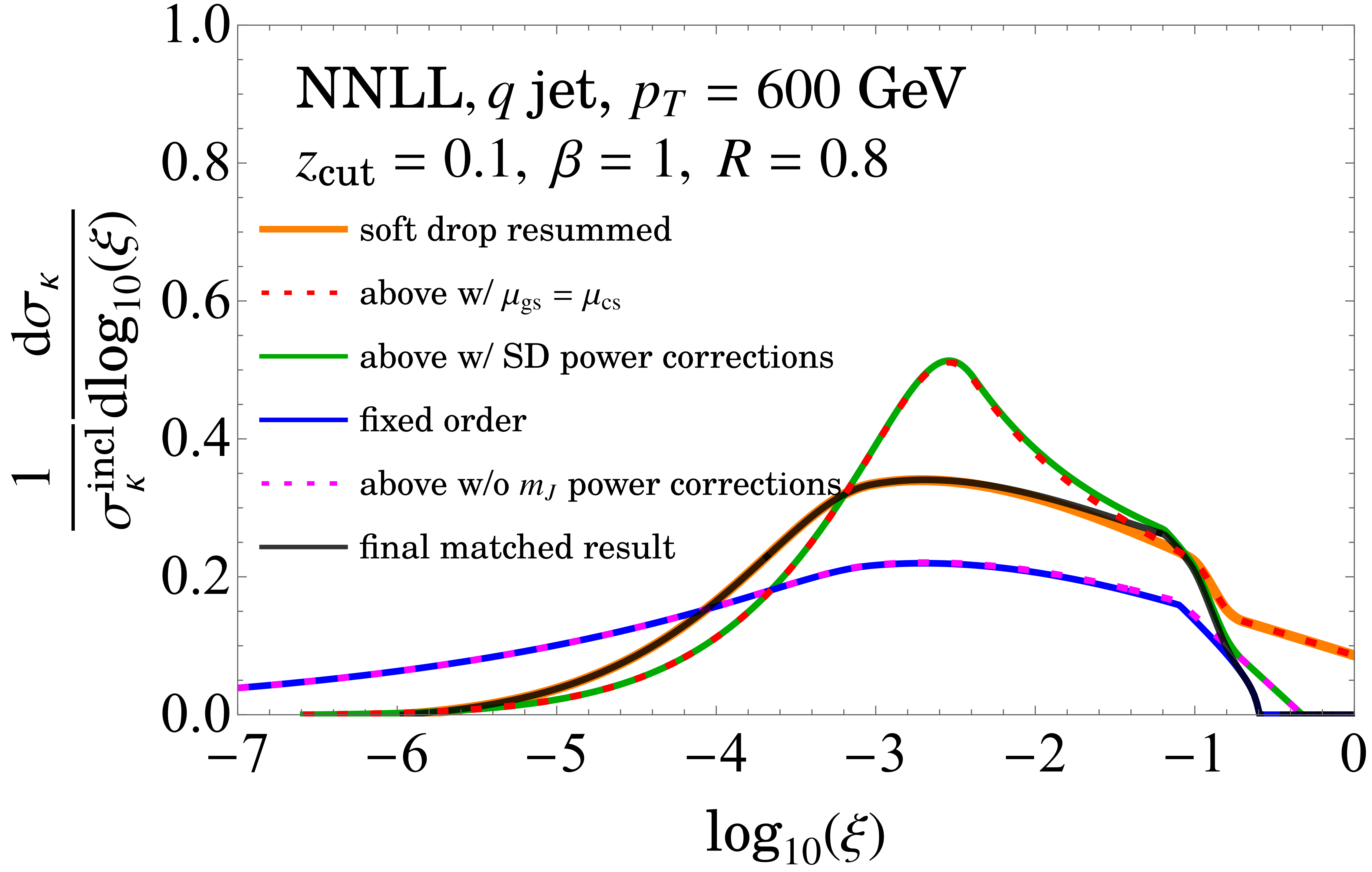


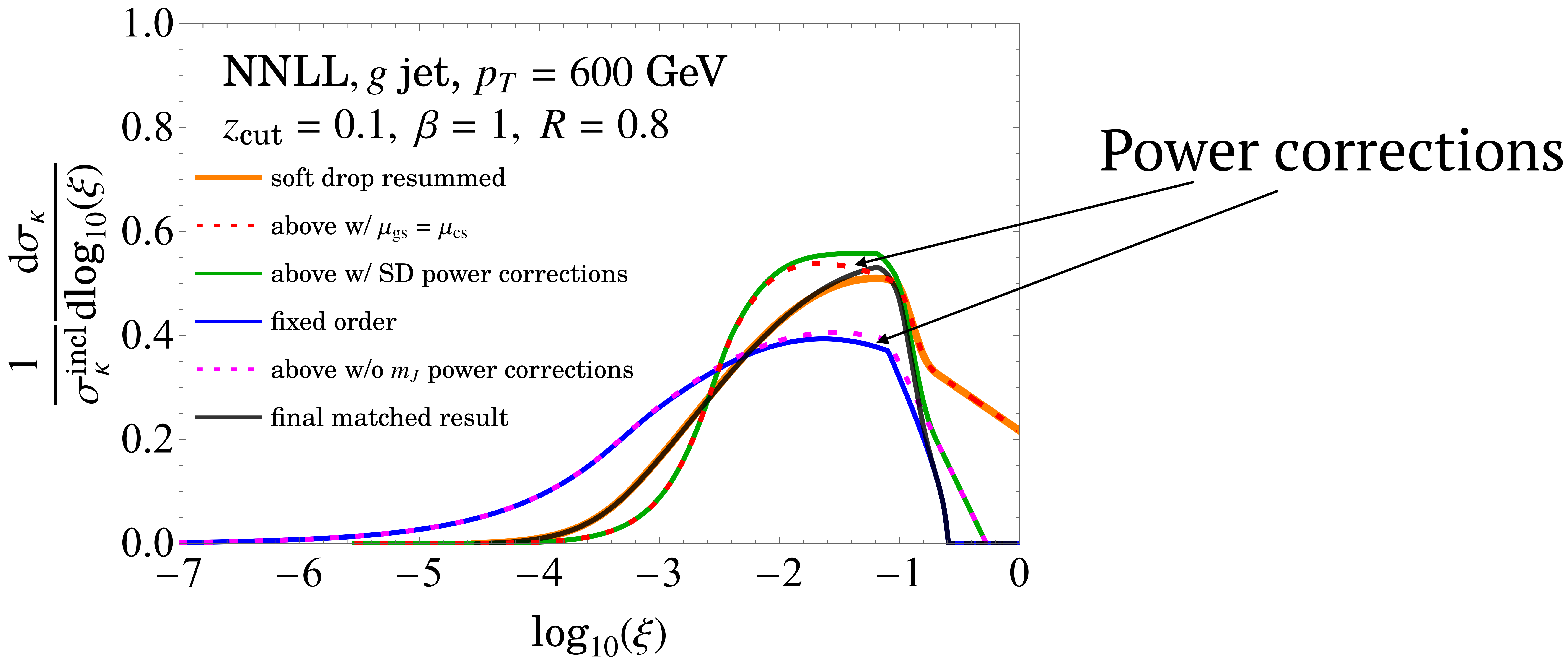
SDOE region only accessible for LHC Kinematics!

Matched cross section (for quark jets)



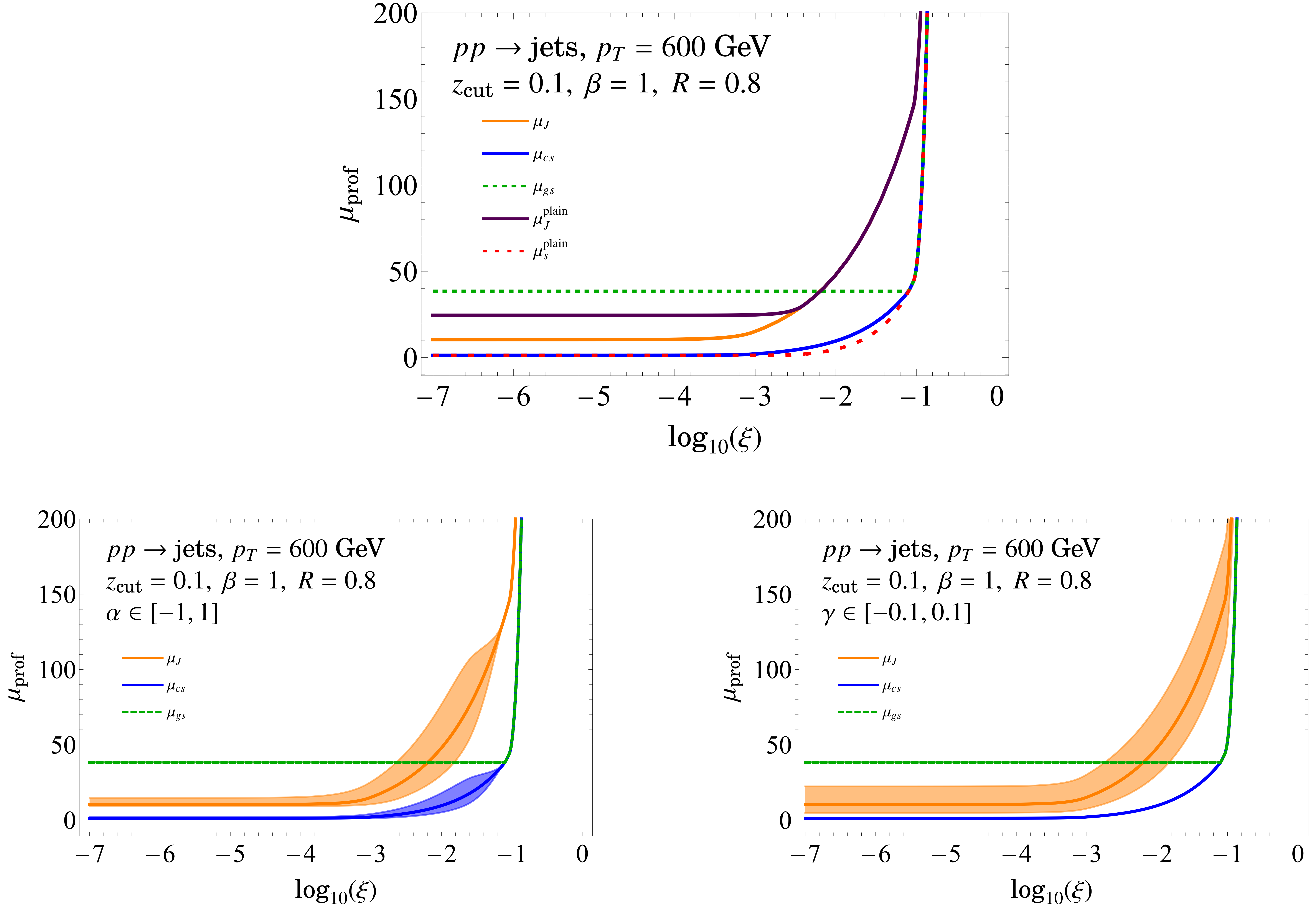
$$\tilde{\mathcal{G}}^{\rm matched}_{\kappa,\rm sd}(\xi) \equiv \tilde{\mathcal{G}}^{\rm resum}_{\kappa,\rm sd}\big(\xi, \mu_{\rm sd\to plain}\big) + \tilde{\mathcal{G}}^{\rm plain}_{\kappa,\rm sd}\big(\xi, \mu_{\rm plain}\big) - \tilde{\mathcal{G}}^{\rm resum}_{\rm sd}\big(\xi, \mu_{\rm plain}\big)$$
$$+ \tilde{\mathcal{G}}^{\rm min}_{\kappa,\rm sd}\big(\xi, r_g^{\rm max}(\xi), \mu_{\rm min\to plain}\big) - \tilde{\mathcal{G}}^{\rm int}_{\rm sd}\big(\xi, r_g^{\rm max}(\xi), \mu_{\rm min\to plain}\big)$$

Matched cross section (for gluon jets)



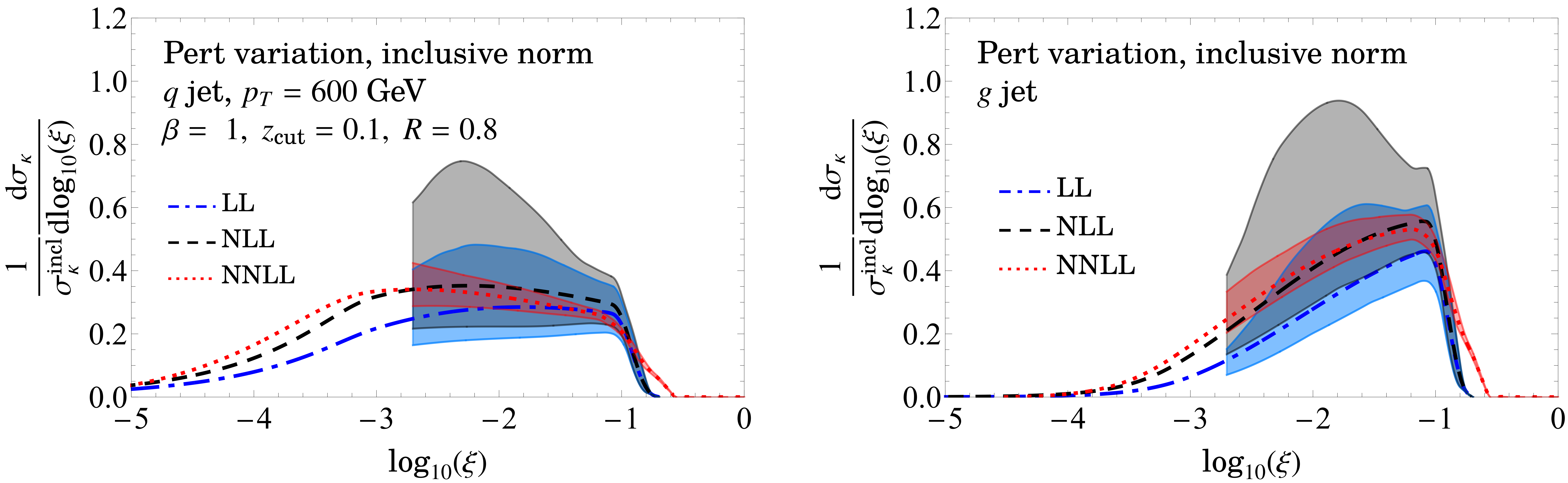
$$\tilde{\mathcal{G}}_{\kappa,\mathrm{sd}}^{\mathrm{matched}}(\xi) \equiv \tilde{\mathcal{G}}_{\kappa,\mathrm{sd}}^{\mathrm{resum}}\big(\xi, \mu_{\mathrm{sd}\to\mathrm{plain}}\big) + \tilde{\mathcal{G}}_{\kappa,\mathrm{sd}}^{\mathrm{plain}}\big(\xi, \mu_{\mathrm{plain}}\big) - \tilde{\mathcal{G}}_{\mathrm{sd}}^{\mathrm{resum}}\big(\xi, \mu_{\mathrm{plain}}\big)$$
$$+ \tilde{\mathcal{G}}_{\kappa,\mathrm{sd}}^{\mathrm{min}}\big(\xi, r_g^{\max}(\xi), \mu_{\mathrm{min}\to\mathrm{plain}}\big) - \tilde{\mathcal{G}}_{\mathrm{sd}}^{\mathrm{int}}\big(\xi, r_g^{\max}(\xi), \mu_{\mathrm{min}\to\mathrm{plain}}\big)$$

Scale variations



Scale variations

Convergence from NLL to NNLL:



Only displaying scale variations in the SDOE region

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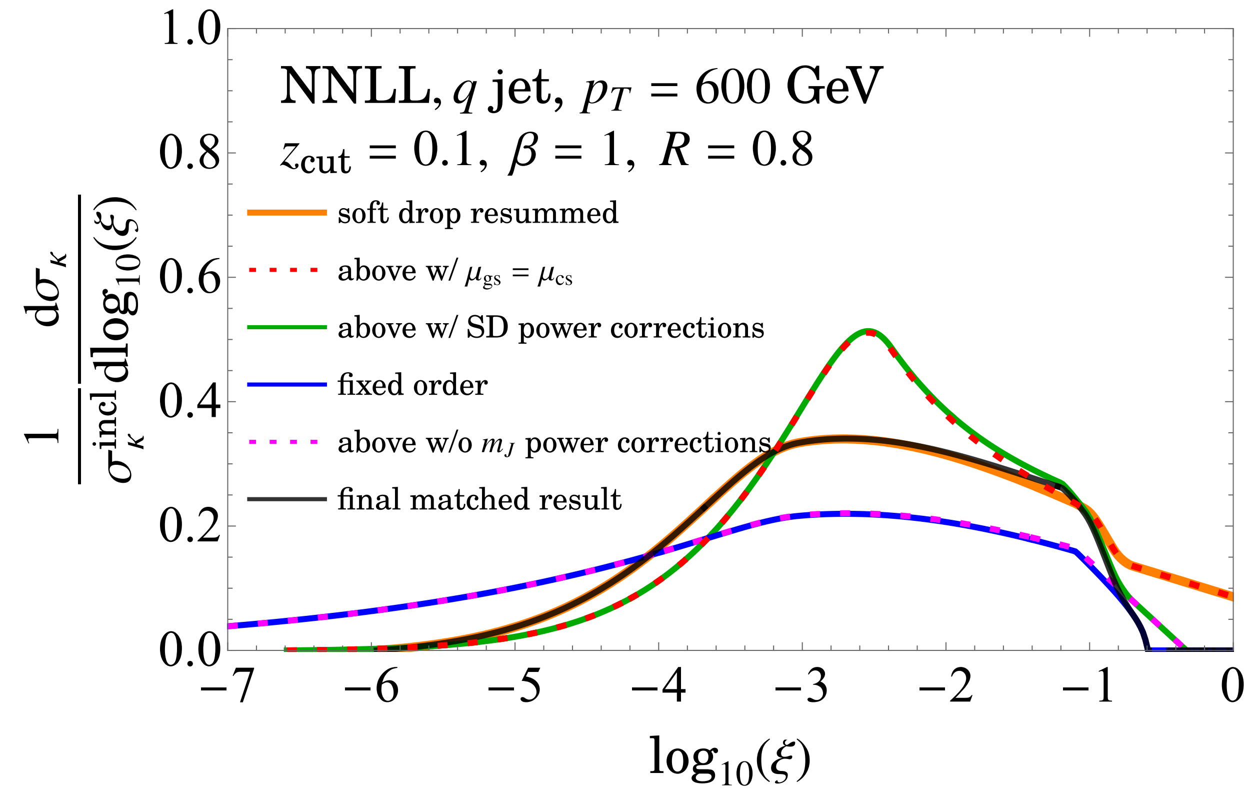

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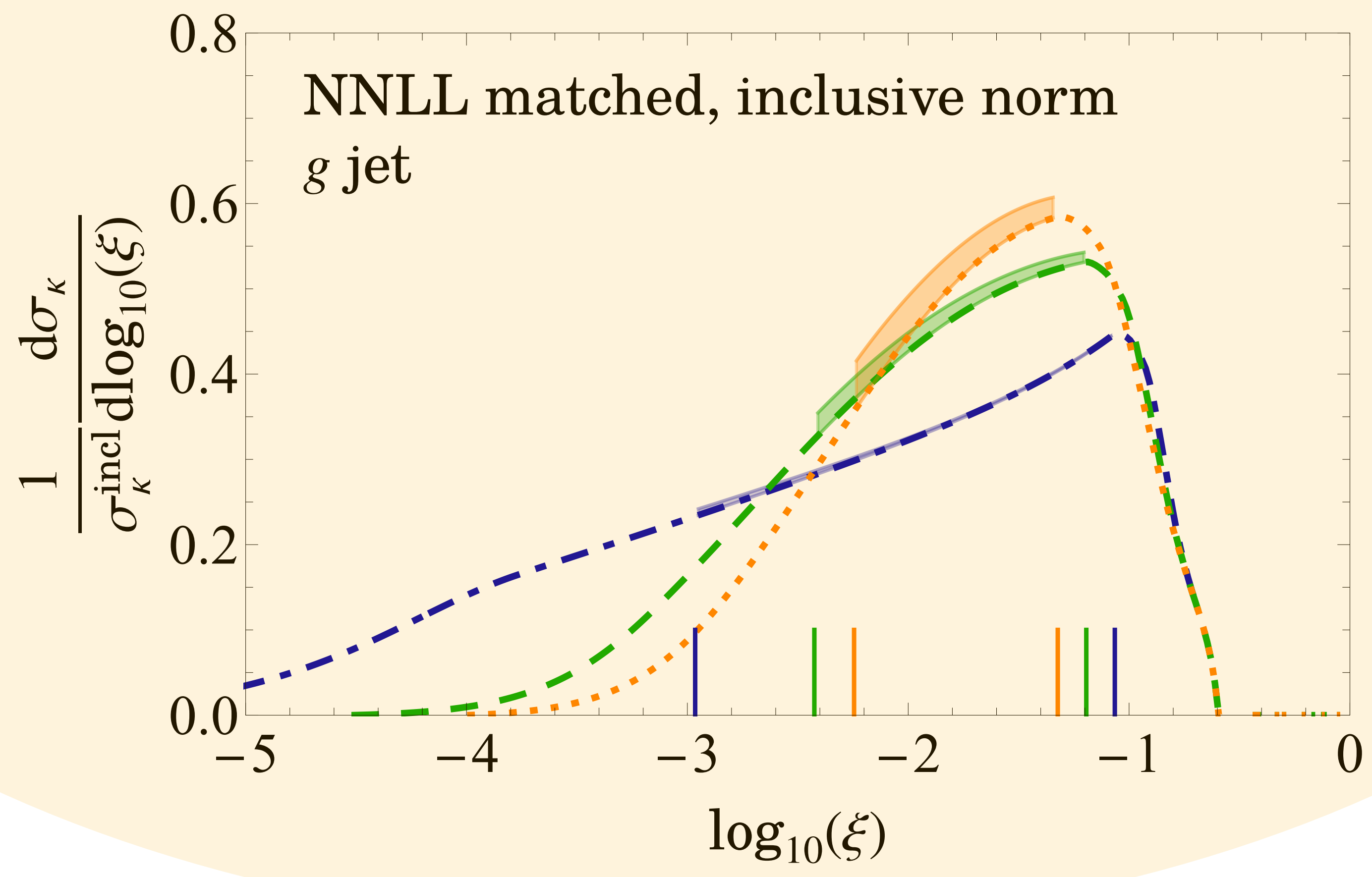

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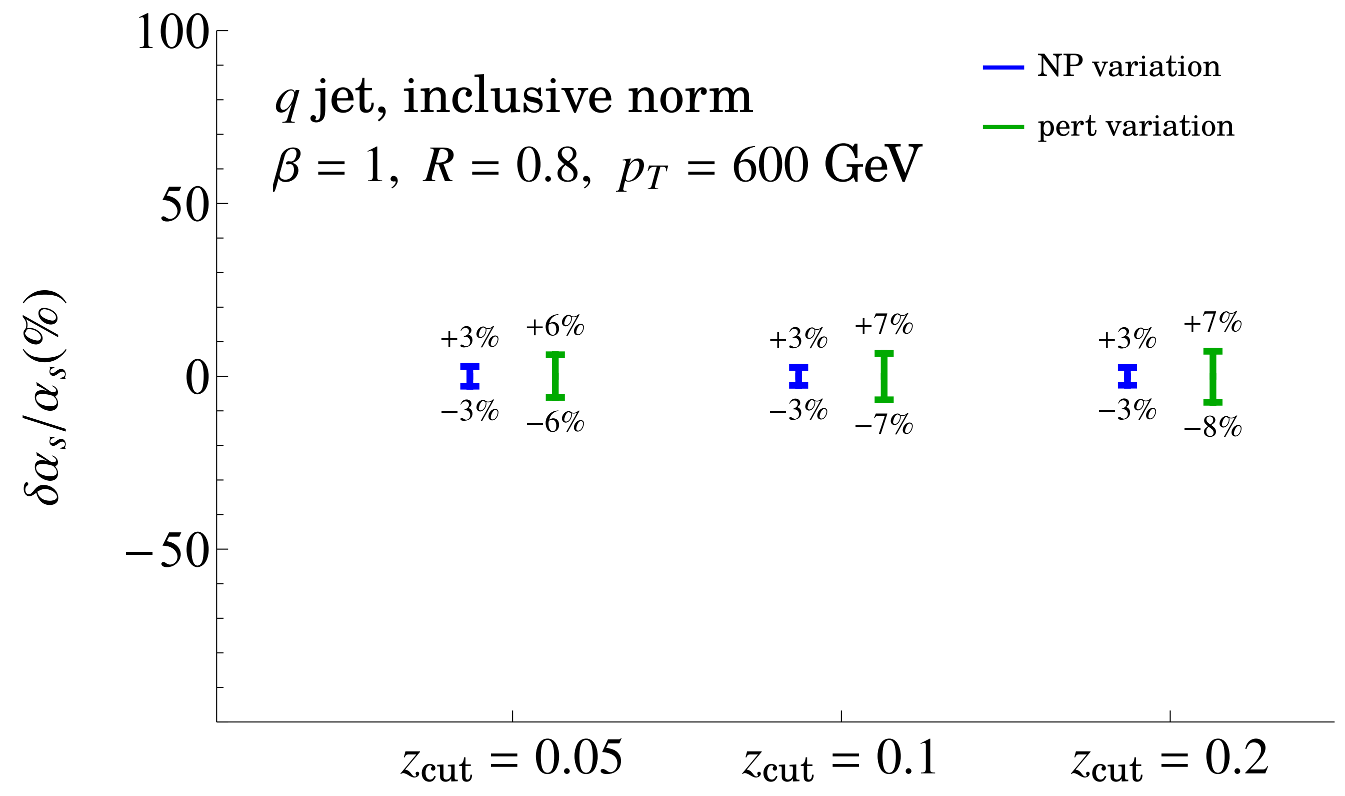
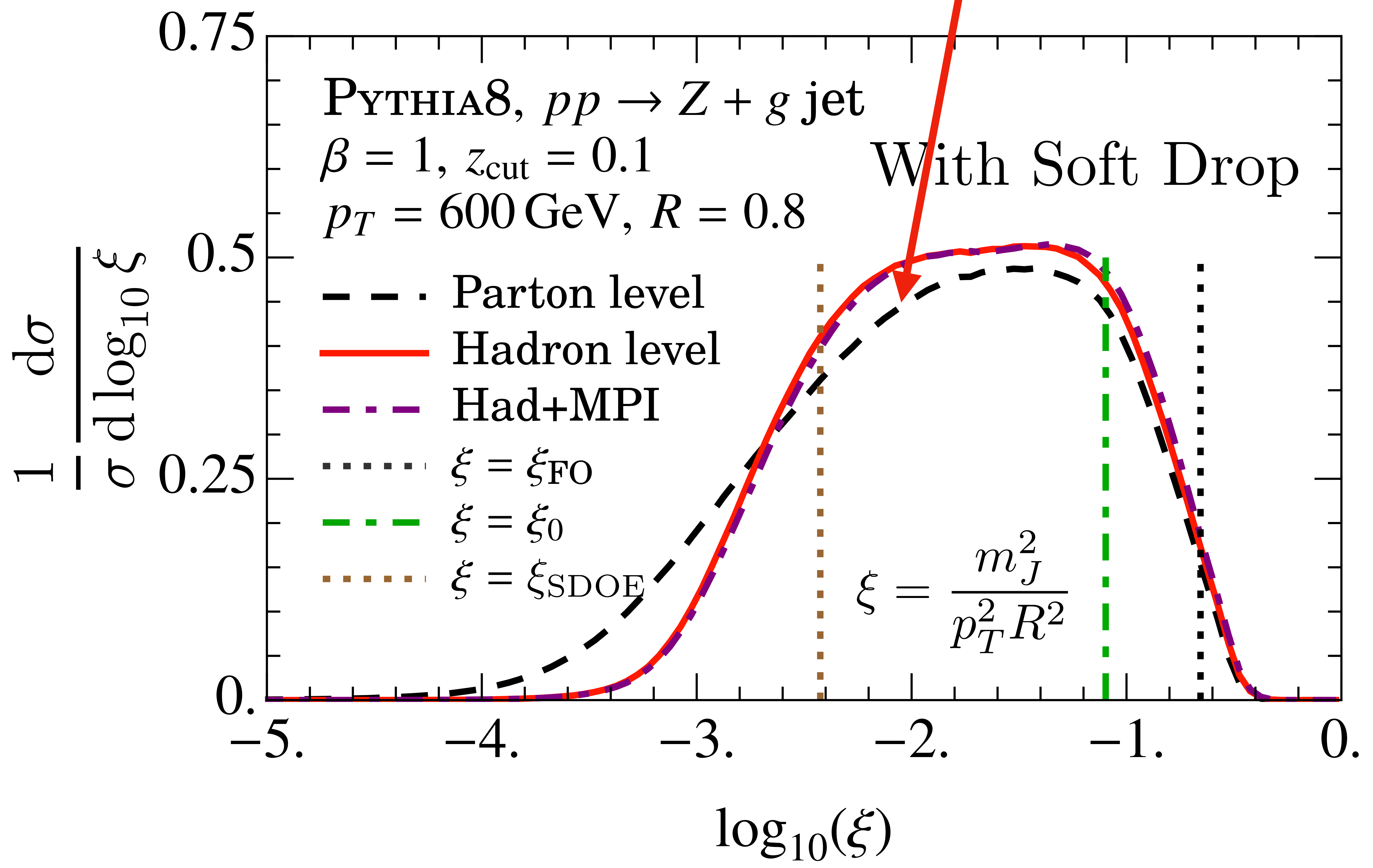


Hadronization corrections

Substantial nonperturbative effects in the α_s fit region



How can we assess impact of hadronization on α_s
in a **model-independent** way?

Hadronization corrections

Model-independent statement on hadronization power corrections:

[Hoang, Mantry, AP, Stewart 2019]

Perturbatively calculable

$$\frac{1}{\sigma_\kappa}\frac{\mathrm{d}\sigma_\kappa}{\mathrm{d}m_J^2} = \frac{1}{\hat\sigma_\kappa}\frac{\mathrm{d}\hat\sigma_\kappa}{\mathrm{d}m_J^2} - Q\Omega_{1\kappa}^{\circledcirc}\frac{\mathrm{d}}{\mathrm{d}m_J^2}\left(\frac{1}{\hat\sigma_\kappa}\frac{\mathrm{d}\hat\sigma_\kappa^{\circledcirc}}{\mathrm{d}m_J^2}\right) + \frac{\Upsilon_{1,0\kappa}^{\circledcirc} + \beta\Upsilon_{1,1\kappa}^{\circledcirc}}{Q}\frac{1}{\hat\sigma_\kappa}\frac{\mathrm{d}\hat\sigma_\kappa^{\circledcirc}}{\mathrm{d}m_J^2},$$

Constant $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ nonperturbative parameters

$$\frac{1}{\hat\sigma_\kappa}\frac{\mathrm{d}\hat\sigma_\kappa^{\circledcirc}}{\mathrm{d}m_J^2} \equiv \int \mathrm{d}r_g\, r_g \frac{1}{\hat\sigma_\kappa}\frac{\mathrm{d}^2\hat\sigma_\kappa}{\mathrm{d}m_J^2\mathrm{d}r_g}, \qquad \frac{1}{\hat\sigma_\kappa}\frac{\mathrm{d}\hat\sigma_\kappa^{\circledcirc}}{\mathrm{d}m_J^2} \equiv \int \frac{\mathrm{d}r_g\mathrm{d}z_g\,\delta\big(z_g - z_{\mathrm{cut}}r_g^\beta\big)}{r_g}\frac{1}{\hat\sigma_\kappa}\frac{\mathrm{d}^3\hat\sigma_\kappa}{\mathrm{d}m_J^2\mathrm{d}r_g\mathrm{d}z_g}.$$

Groomed jet radius

- At NLL′ in [AP, Stewart, Vaidya, Zoppi 2020],
- Improved to NNLL + matching to ungroomed region in [AP (to appear)]

What to do about hadronization?

Ideally, fit for 7 parameters to stay model-independent. Challenging?

$$(\alpha_s, \Omega_{1q}^{\infty}, \Omega_{1g}^{\infty}, \Upsilon_{1,0q}^{\circledcirc}, \Upsilon_{1,0g}^{\circledcirc}, \Upsilon_{1,1q}^{\circledcirc}, \Upsilon_{1,1g}^{\circledcirc})$$

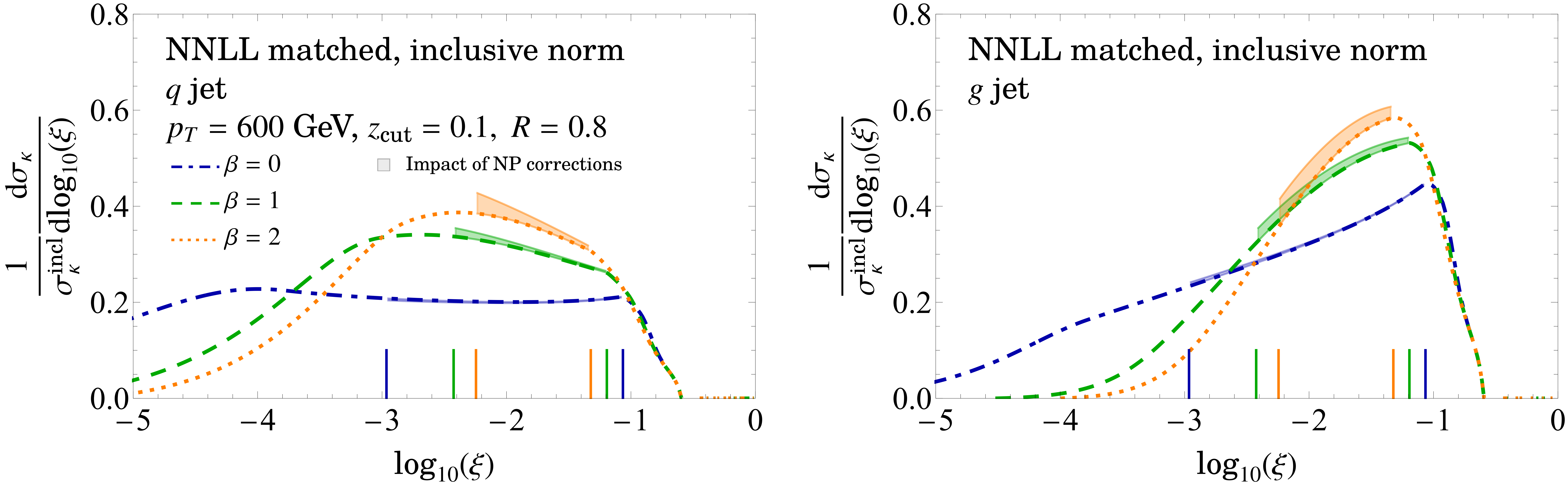
Instead, treat them as nuisance parameters

Values obtained from fit to Pythia8: [Ferdinand, Lee, AP (to appear)]

$$\Omega_{1,q}^{\infty} = 0.55\,\text{GeV}\,, \quad \Upsilon_{1,0q}^{\circledcirc} = -0.73\,\text{GeV}\,, \quad \Upsilon_{1,1q}^{\circledcirc} = 0.90\,\text{GeV}\,, \quad \text{for quarks,}$$
$$\Omega_{1g}^{\infty} = 0.91\,\text{GeV}\,, \quad \Upsilon_{1,0g}^{\circledcirc} = -0.24\,\text{GeV}\,, \quad \Upsilon_{1,0g}^{\circledcirc} = 0.90\,\text{GeV}\,, \quad \text{for gluons.}$$

Good for uncertainty estimate, exact values not important!

Impact of hadronization corrections



Take uncertainty as difference between parton and hadron level

Comparison with previous work

[Frye, Larkoski, Schwartz, Yan 2016] [Marzani, Schunk, Sodes 2017]
[Anderle, Dasgupta, El-Menoufi, Helliwell, Guzzi 2020]
[Kang, Lee, Liu, Ringer 2018] [Larkoski, 2020] [Benkendorfer, Larkoski 2021]

- **Resummation and matching**: state-of-the-art NNLL resummation, fully analytical treatment of perturbative power corrections, included finite-z_{cut} non-singular corrections (found negligible).
- **Perturbative uncertainty**: Comprehensive variation of profile parameters that break a specific canonical-relation by randomly chosen points in a well-studied variation range.
- **Transition into ungroomed region**: Consistent analytical matching to ungroomed region at NNLL at the soft-wide angle transition point.
- **Nonperturbative effects**: Incorporated using model-independent field theory based formalism to assess impact on precision of α_s-determination.
- **Resummation at soft drop cusp**: 20% shift in the cusp location suggested by [Benkendorfer, Larkoski 2021] (too large?) is only a 5% modification to the range we used for our α_s-sensitivity analysis.

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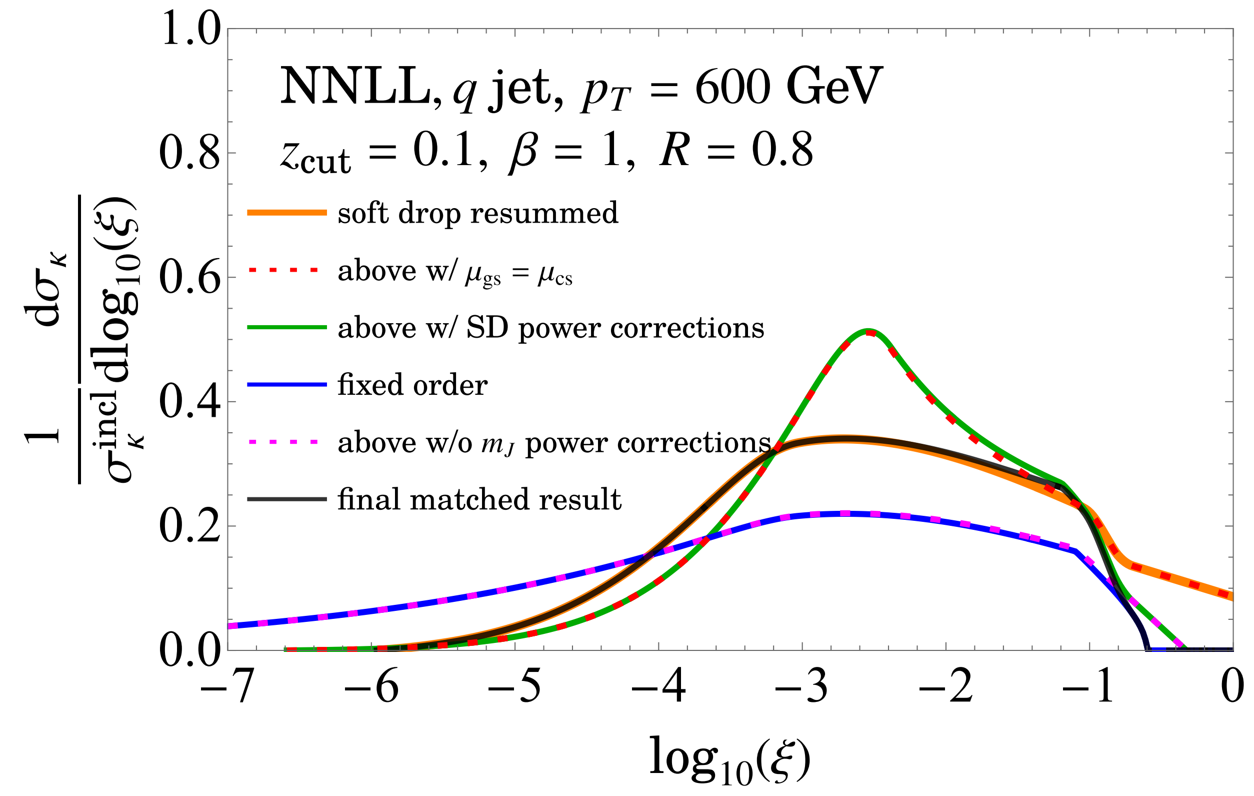

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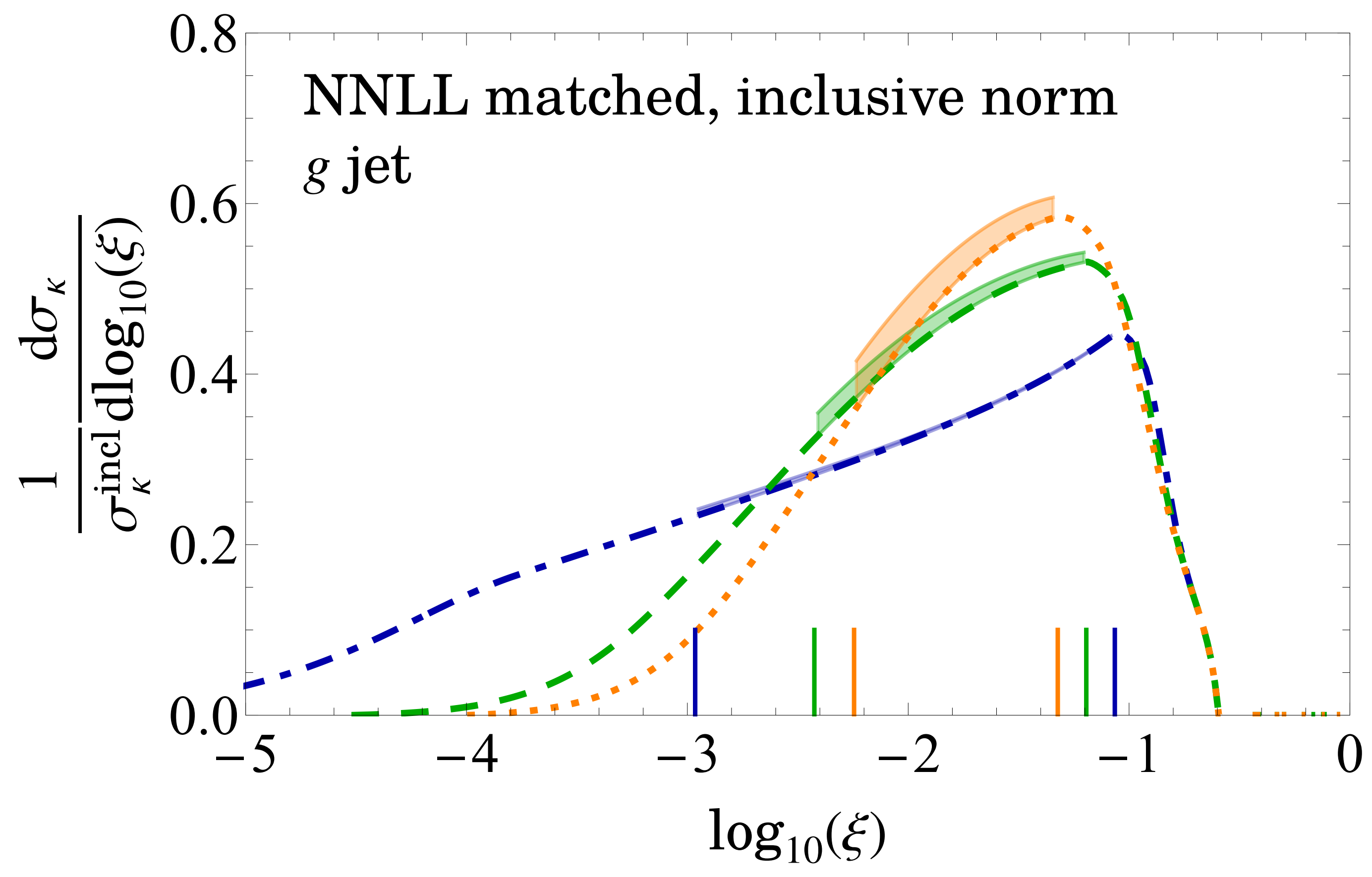

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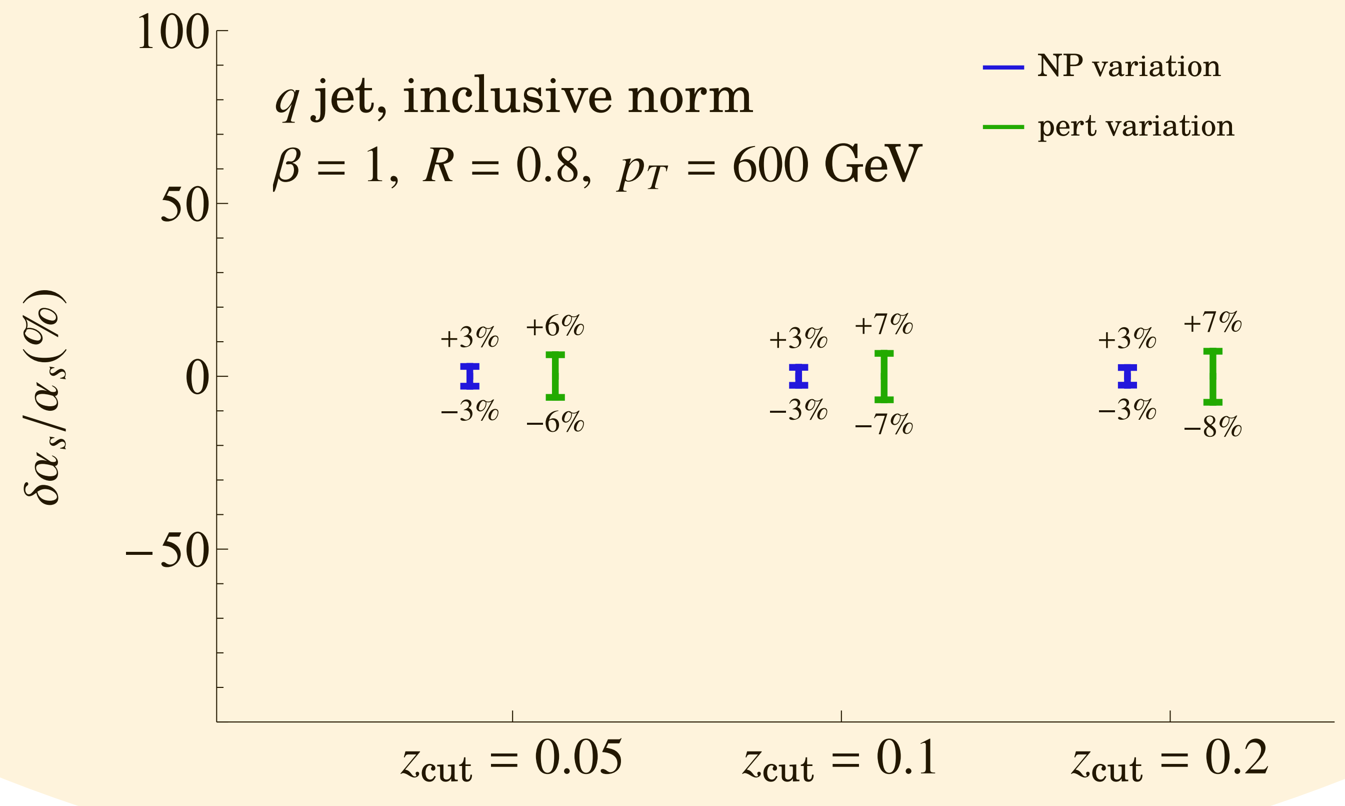


Dependence on slope

Coefficient independent of α_s, ξ

According to **leading-logarithmic** estimate:

$$\frac{\mathrm{d}\sigma^{\kappa}_{\mathrm{resum}}}{\mathrm{d}\log_{10}(\xi)} \propto \exp\Big[-\alpha_s(\mu)a_k\log_{10}(\xi)\Big] \approx 1 - \underline{\alpha_s(\mu)a_k}\log_{10}(\xi)$$

Slope depends linearly on α_s

[Les Houches 2017]

Two ways to normalize

- Normalize to inclusive cross section in the p_T-η bin: $\dfrac{1}{\sigma_{\text{incl}}}\dfrac{\mathrm{d}^3\sigma}{\mathrm{d}p_T\mathrm{d}\eta\mathrm{d}\xi}$,

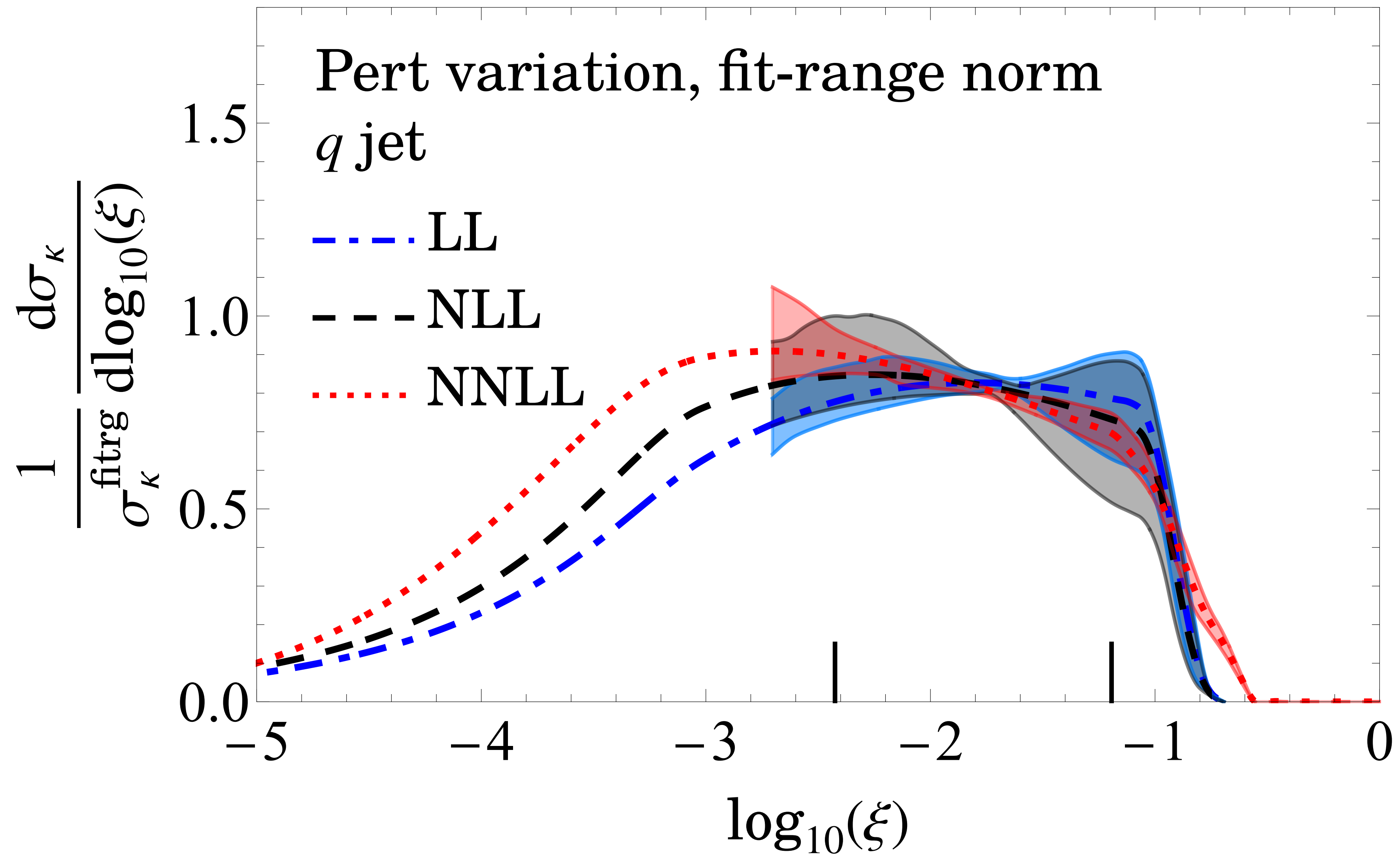
  Proposed in [Kang, Lee, Liu, Ringer 2018]

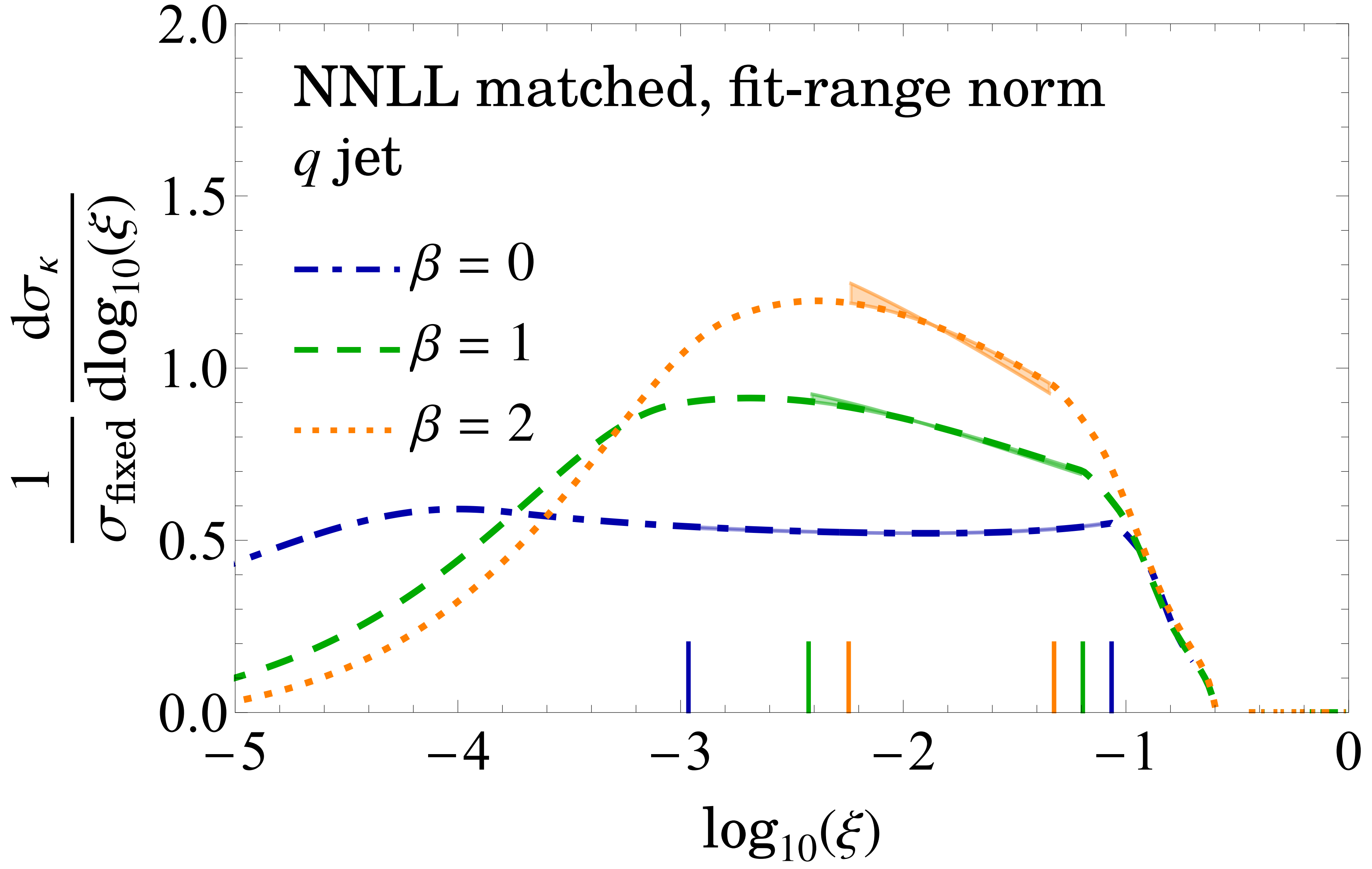
- Normalize to cross section in range: $\dfrac{1}{\sigma_{\text{fitrg}}}\dfrac{\mathrm{d}^3\sigma}{\mathrm{d}p_T\mathrm{d}\eta\mathrm{d}\xi}$

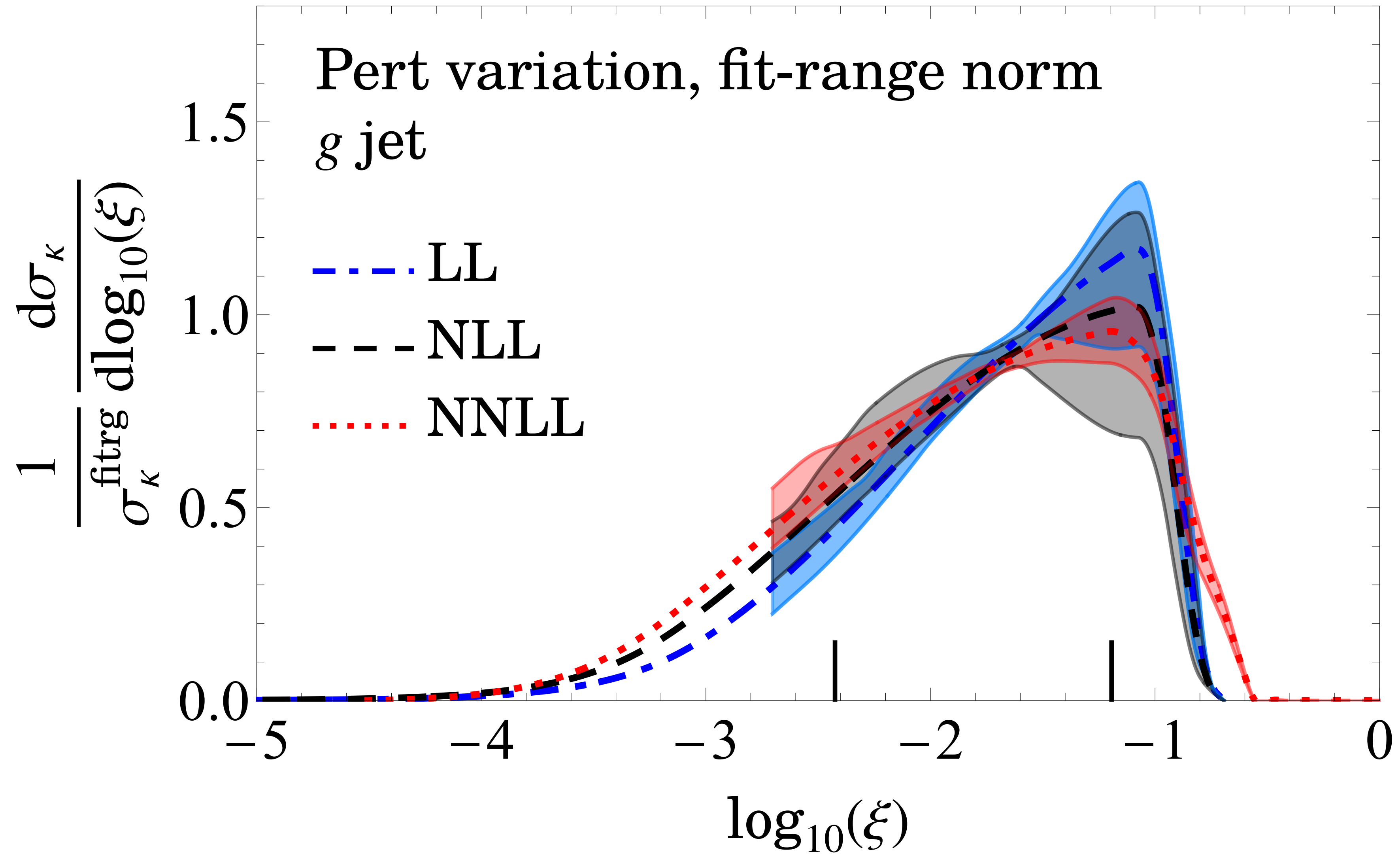
  Pursued by [ATLAS 1711.08341, 1912.09837]

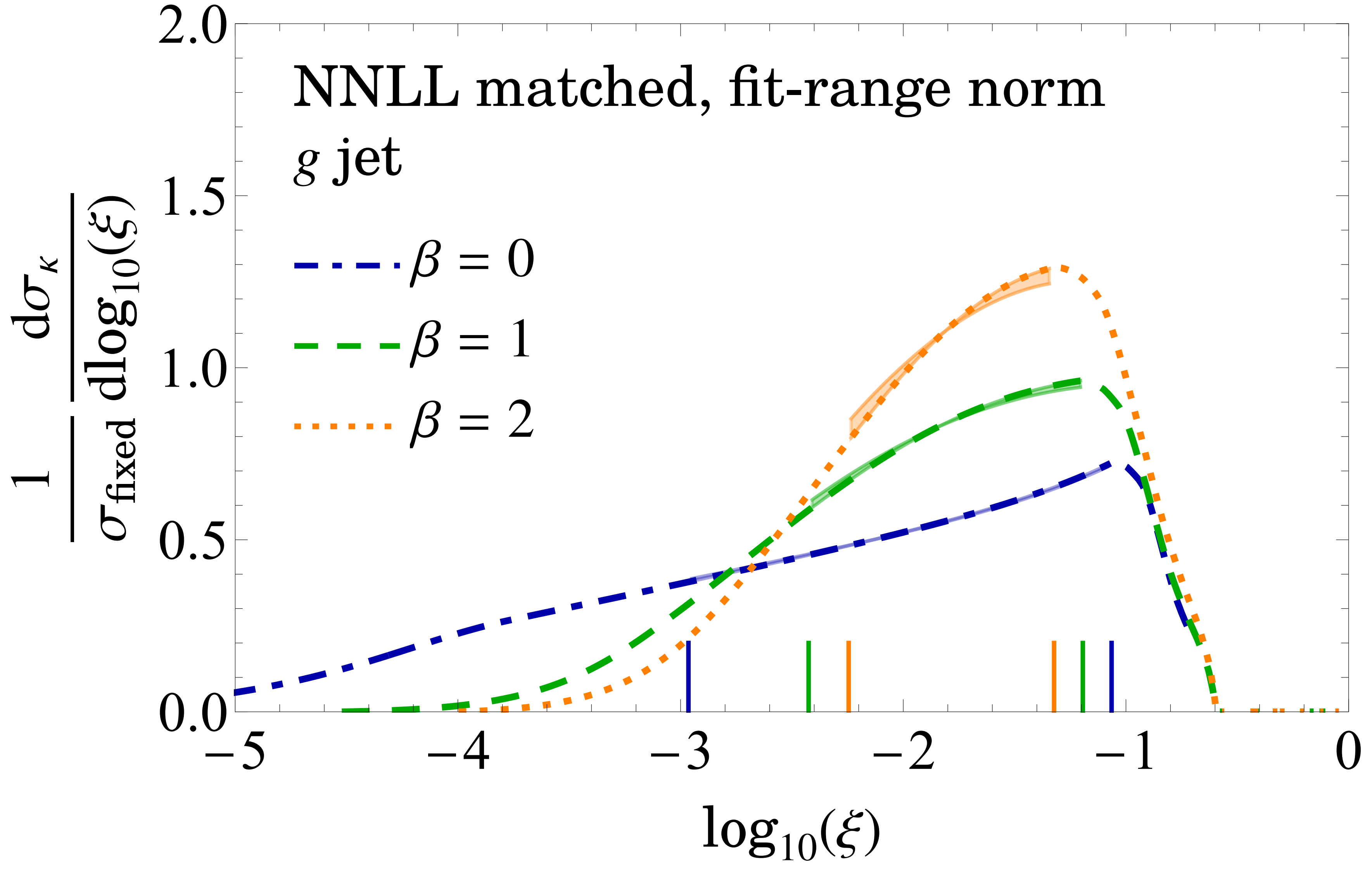
$$\sigma_{\text{incl}} = \frac{\mathrm{d}^2\sigma}{\mathrm{d}p_T\mathrm{d}\eta}, \qquad \sigma_{\text{fitrg}} = \int_{\xi_{\text{SDOE}}}^{\xi_0'} \mathrm{d}\xi \, \frac{\mathrm{d}^3\sigma}{\mathrm{d}p_T\mathrm{d}\eta\mathrm{d}\xi}$$

Uncertainties in fit-range normalized spectrum

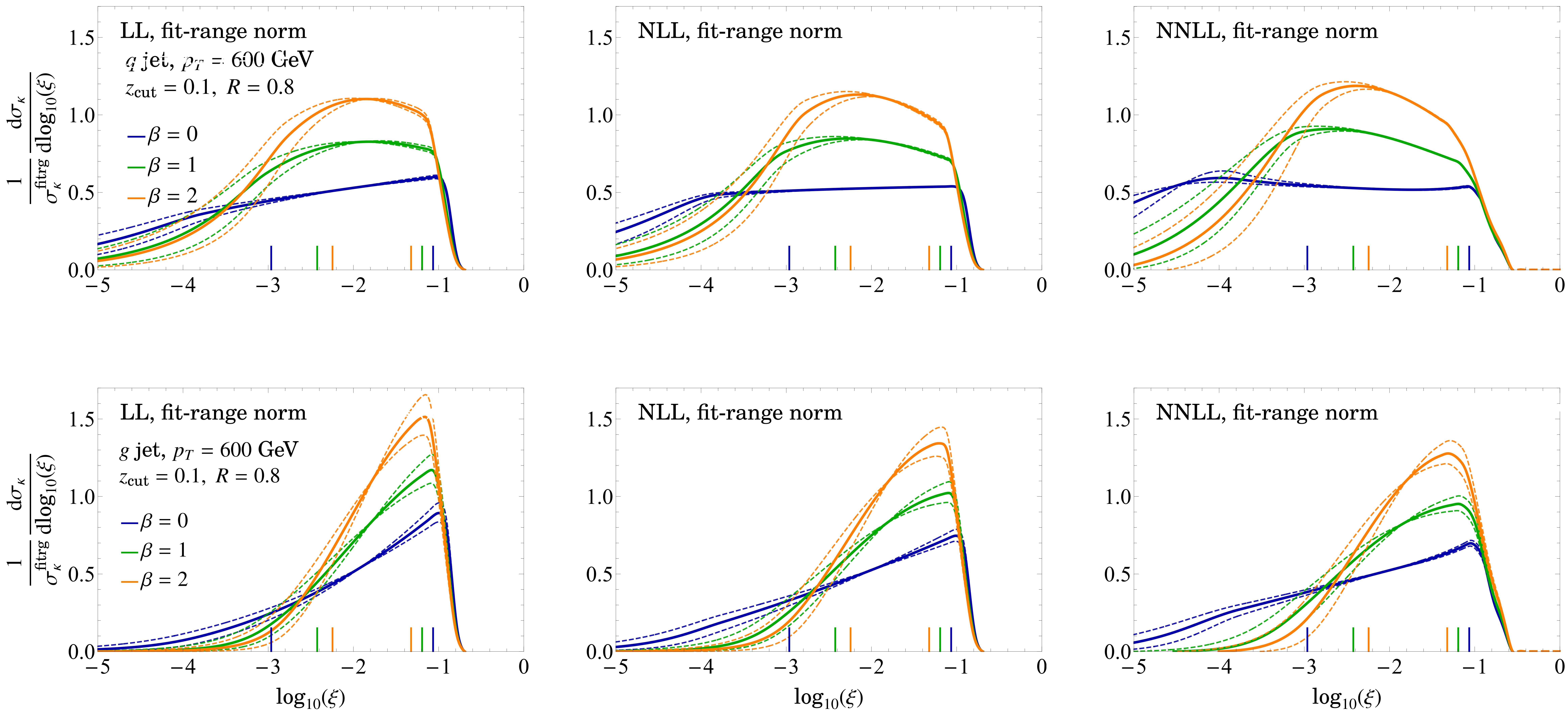








Effects of higher order resummation on slope



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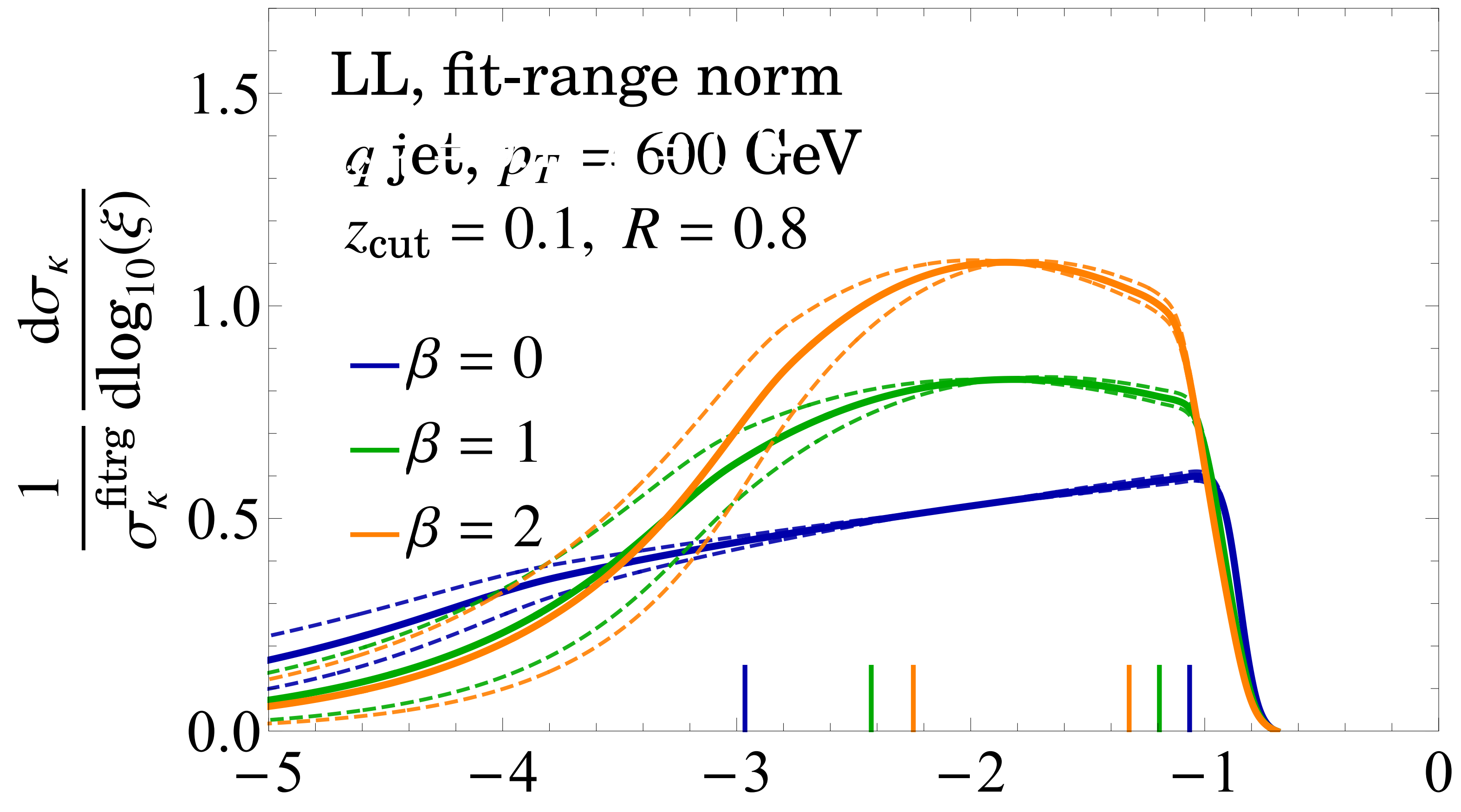
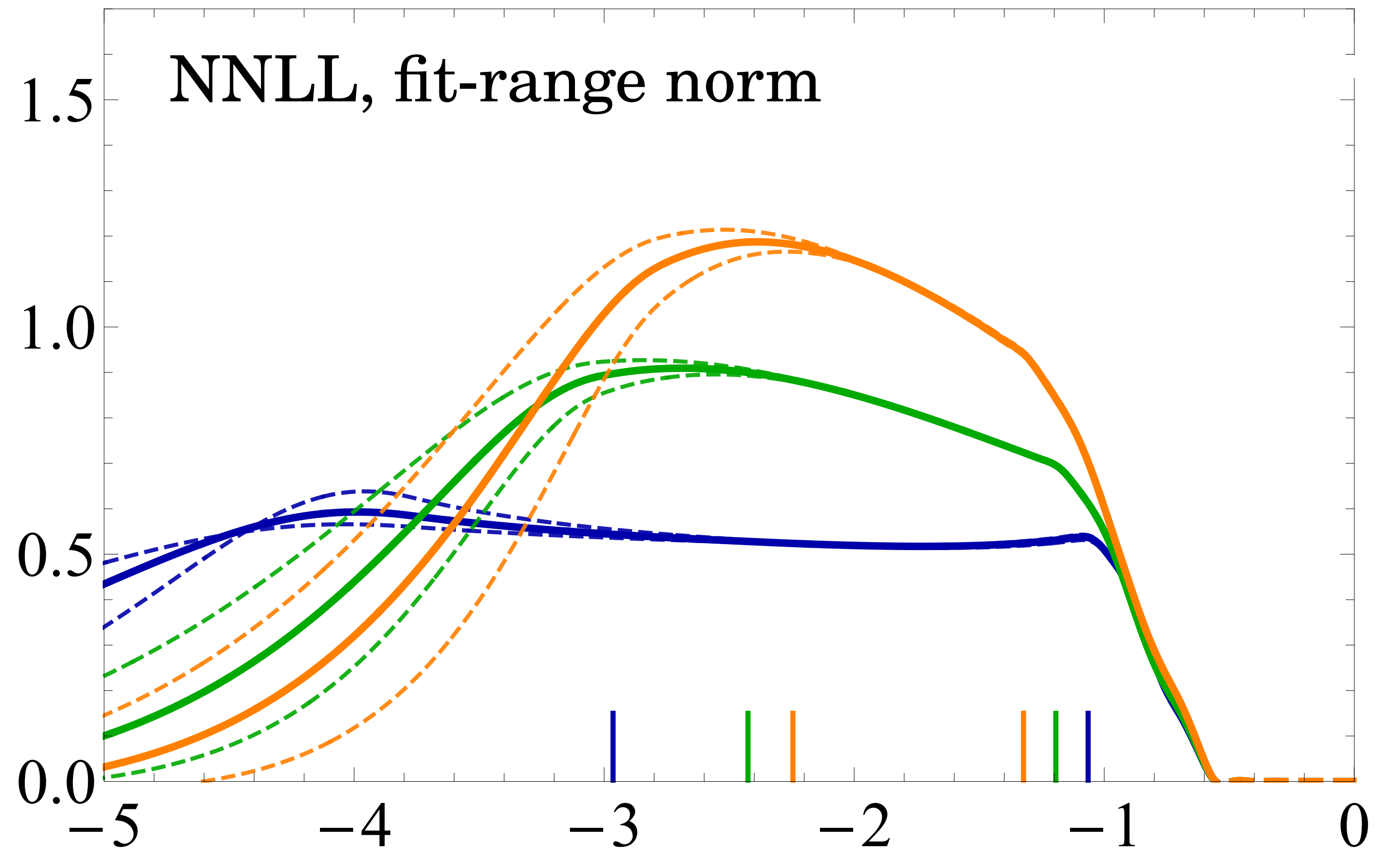
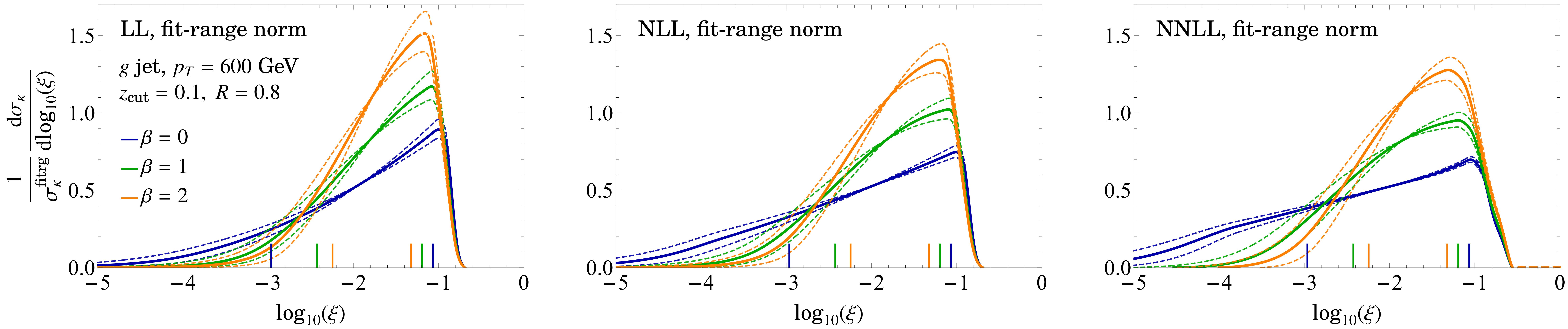


Quark jets: slope flattens





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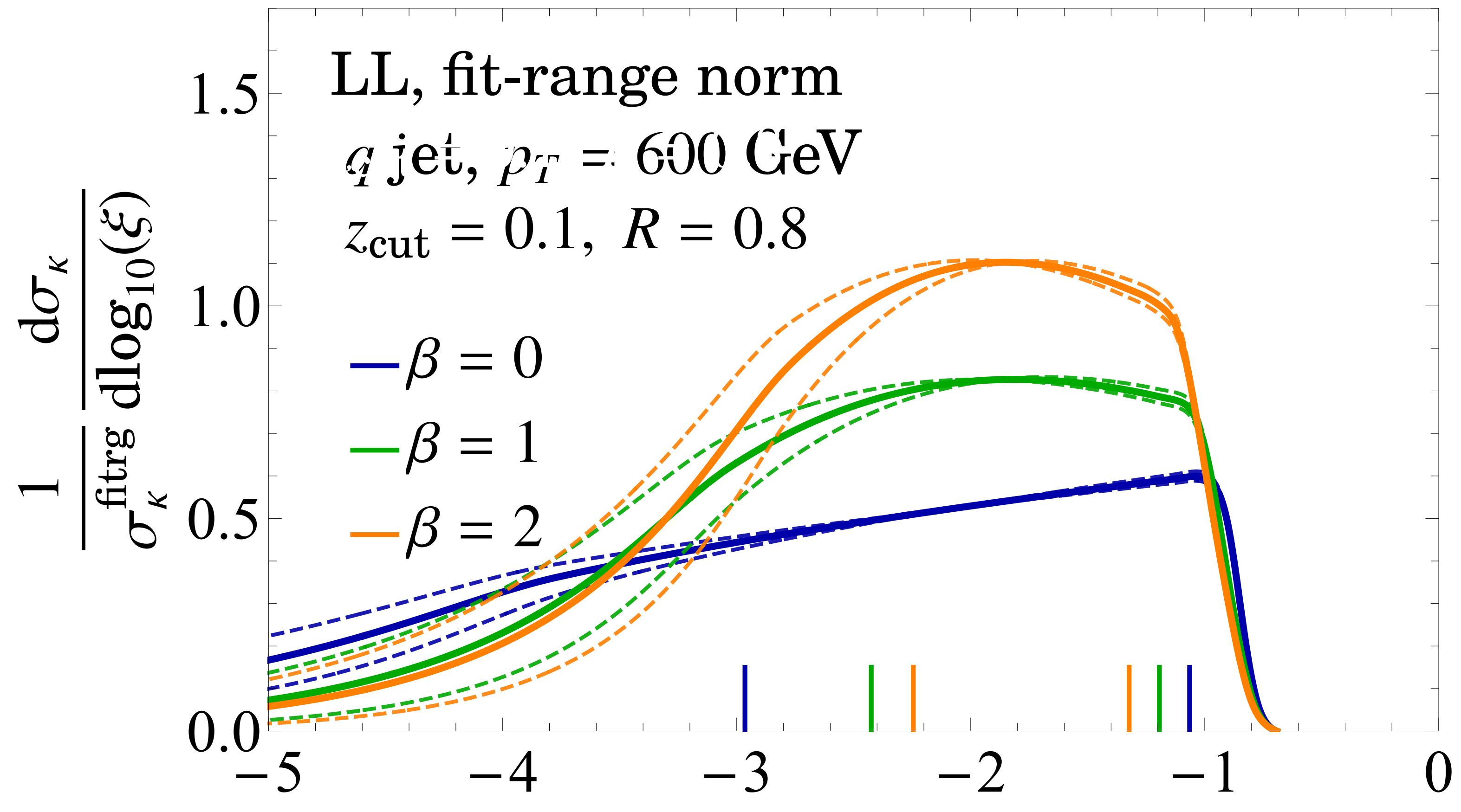
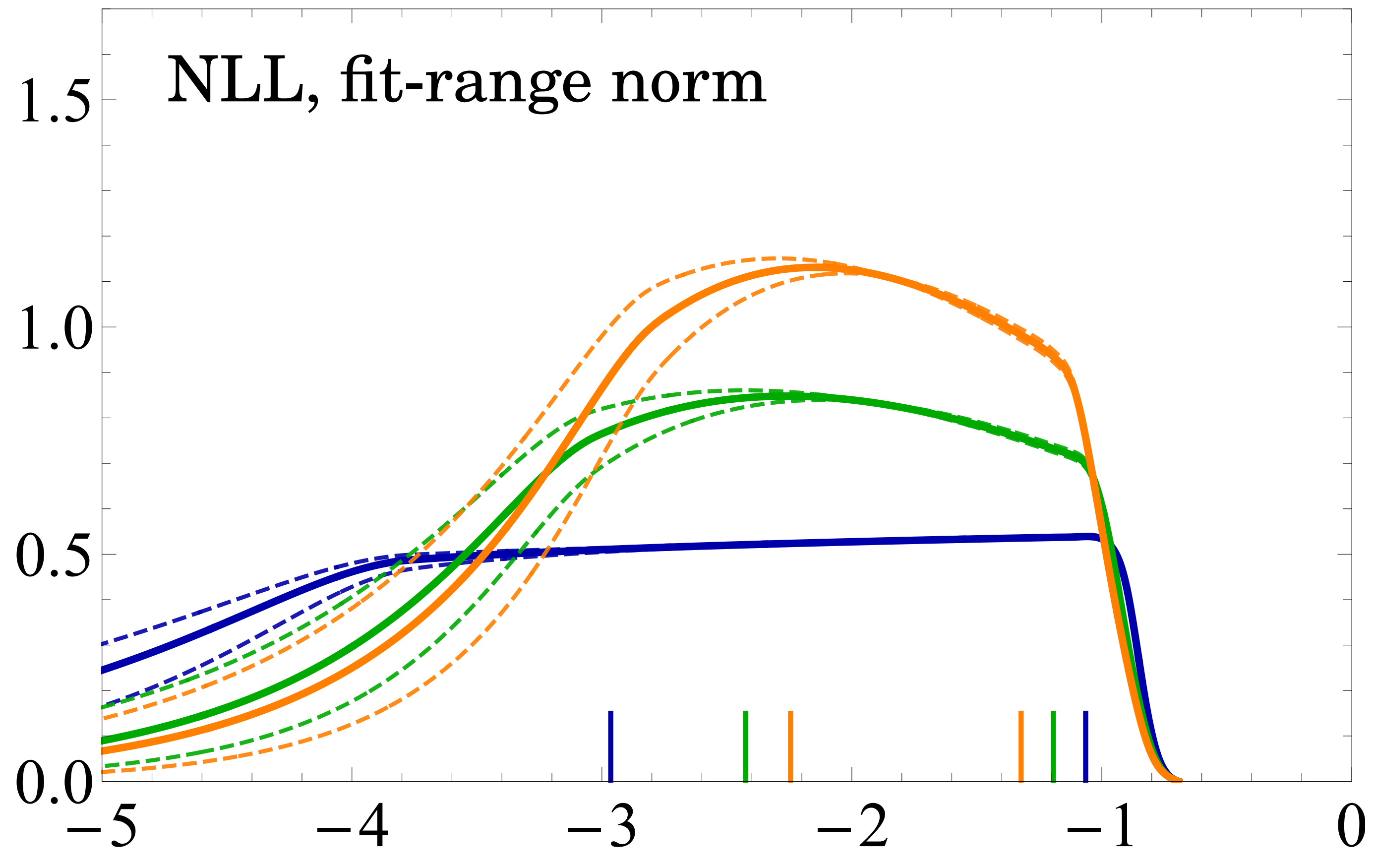
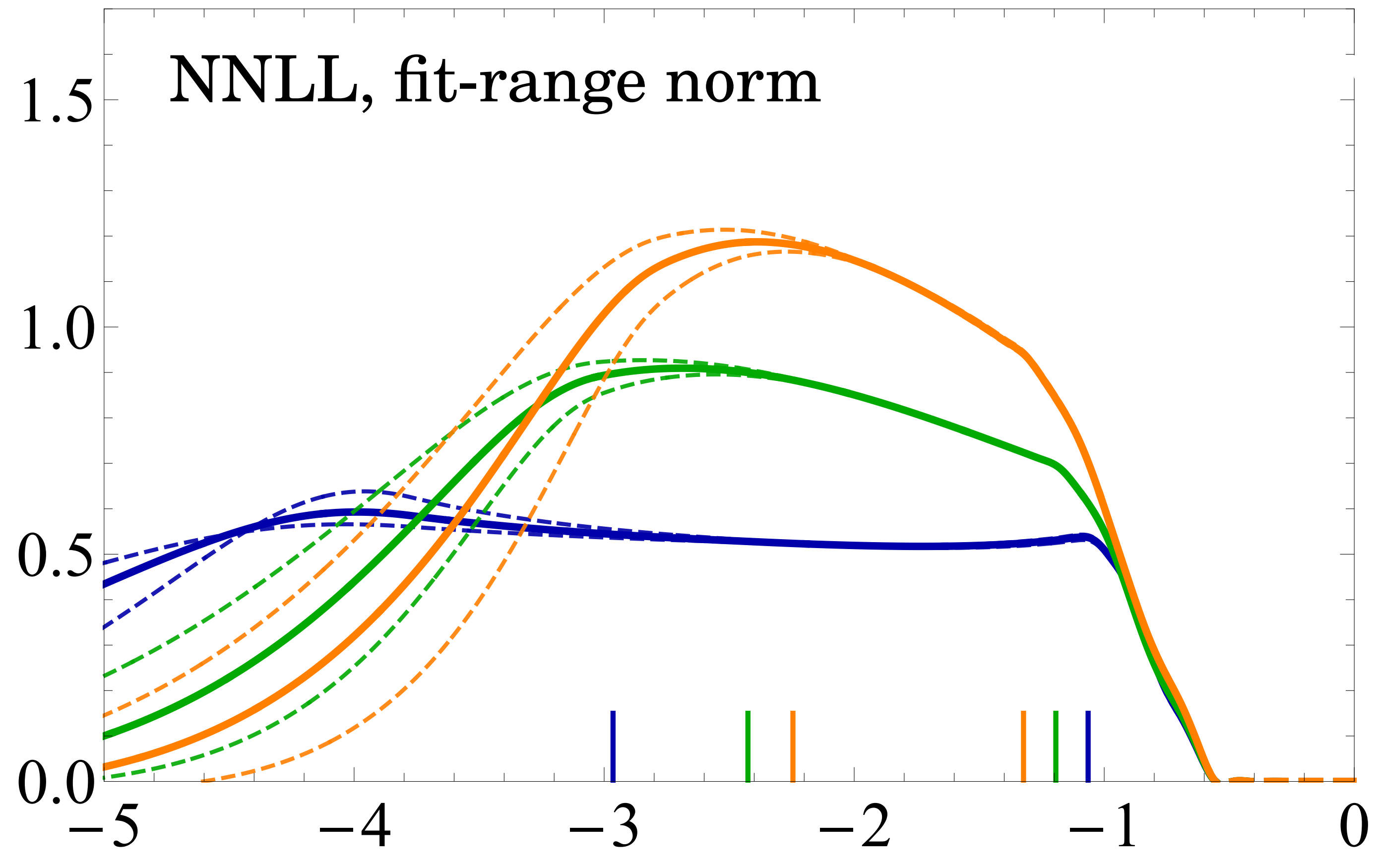
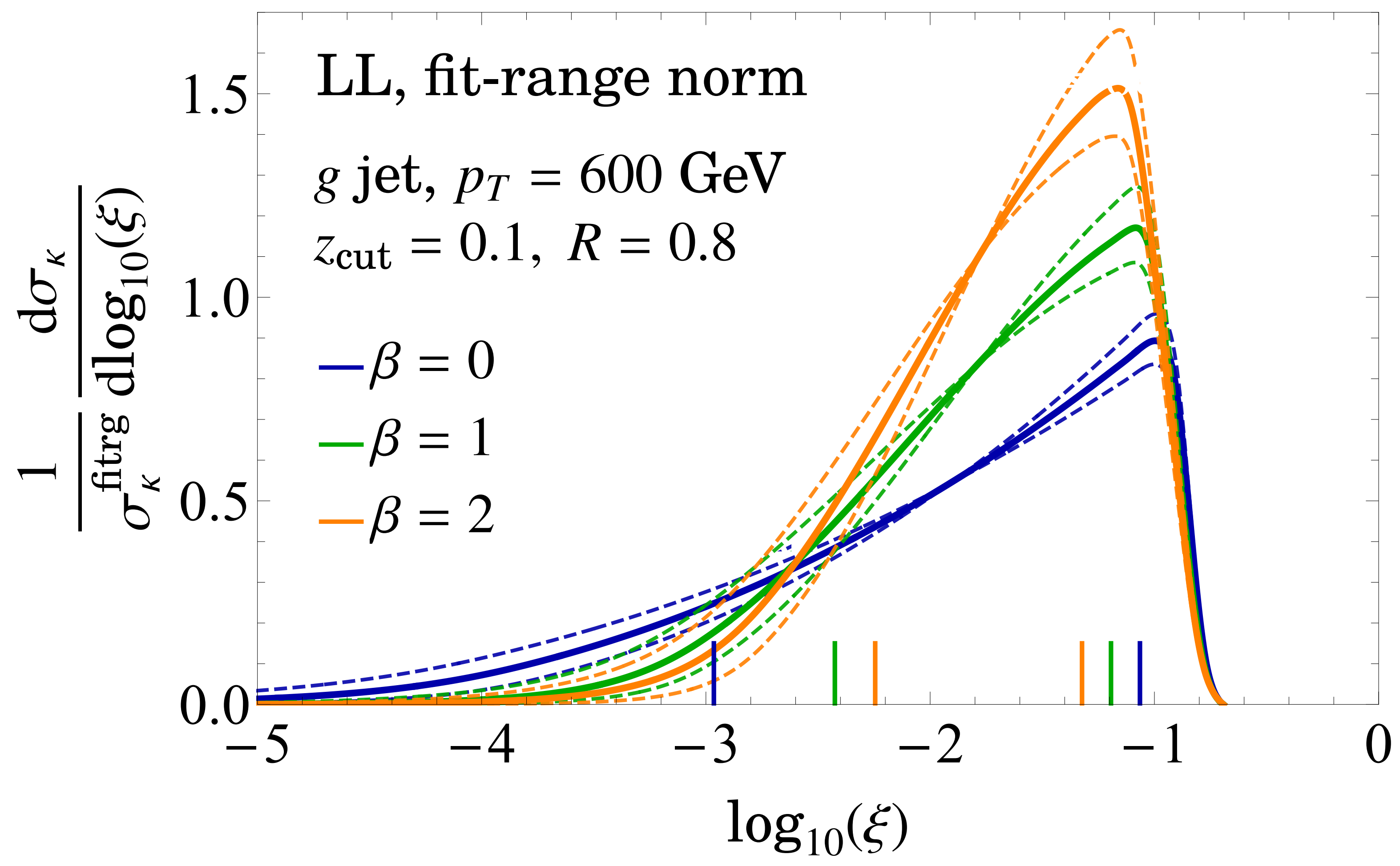
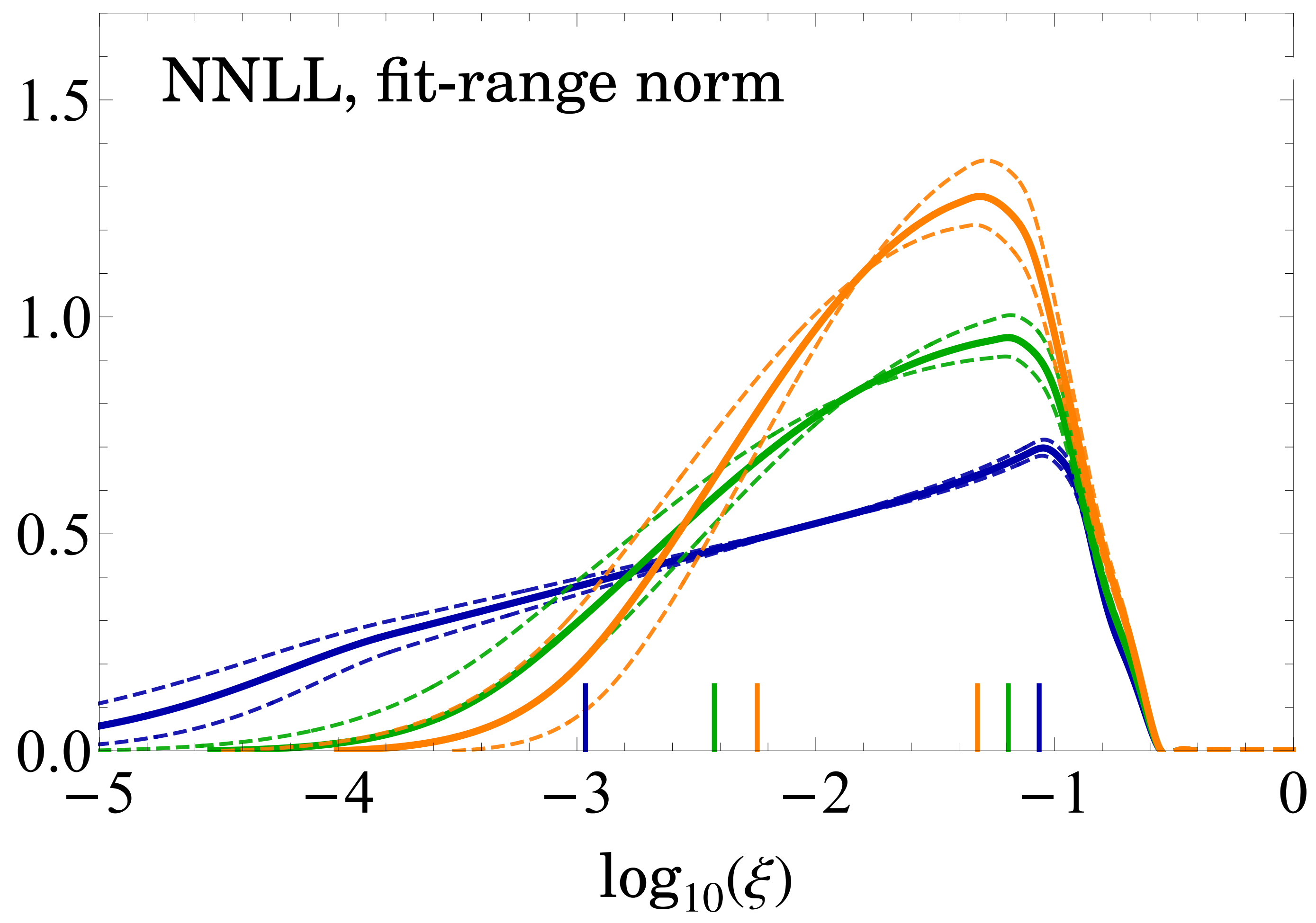








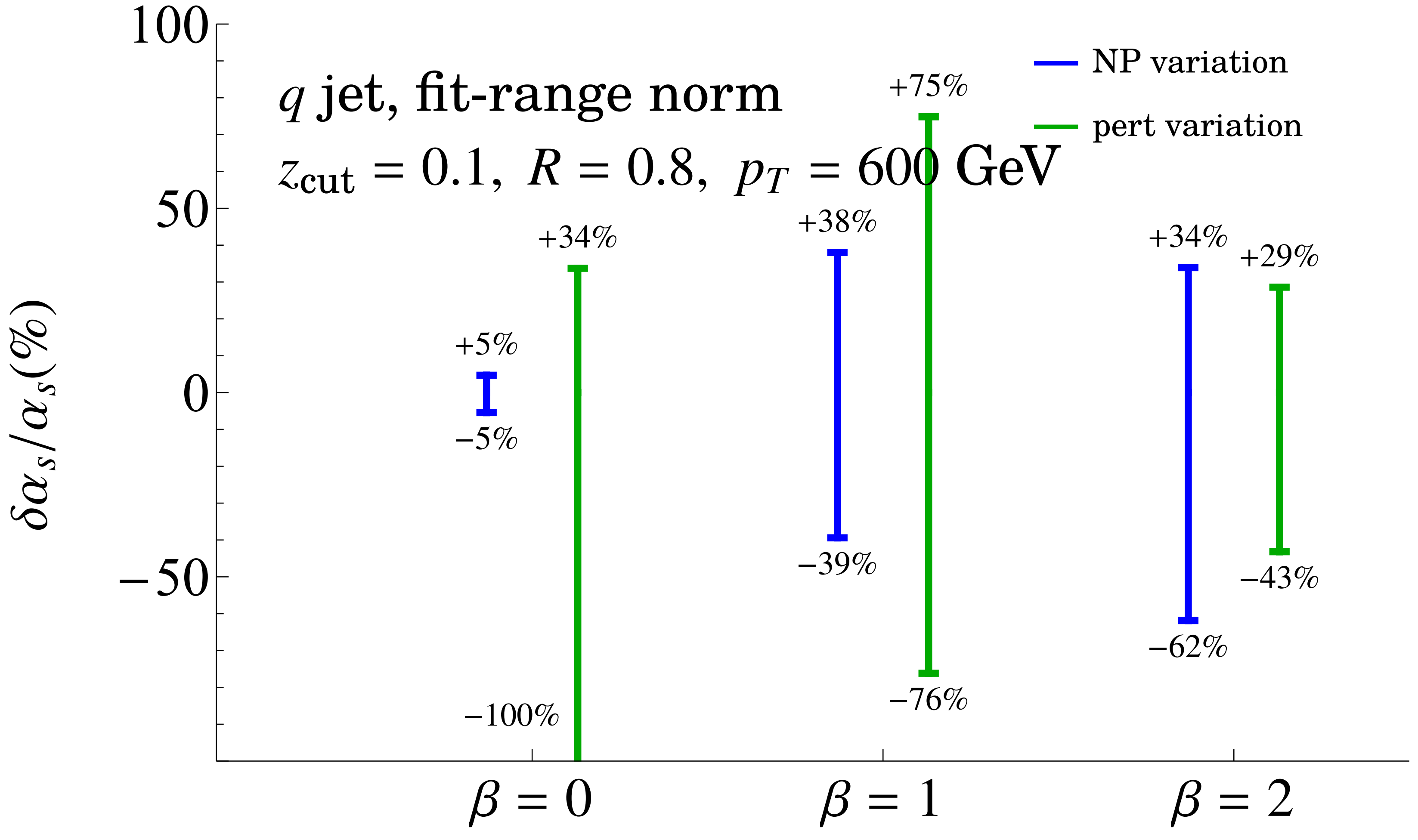
Gluon jets: retain shape

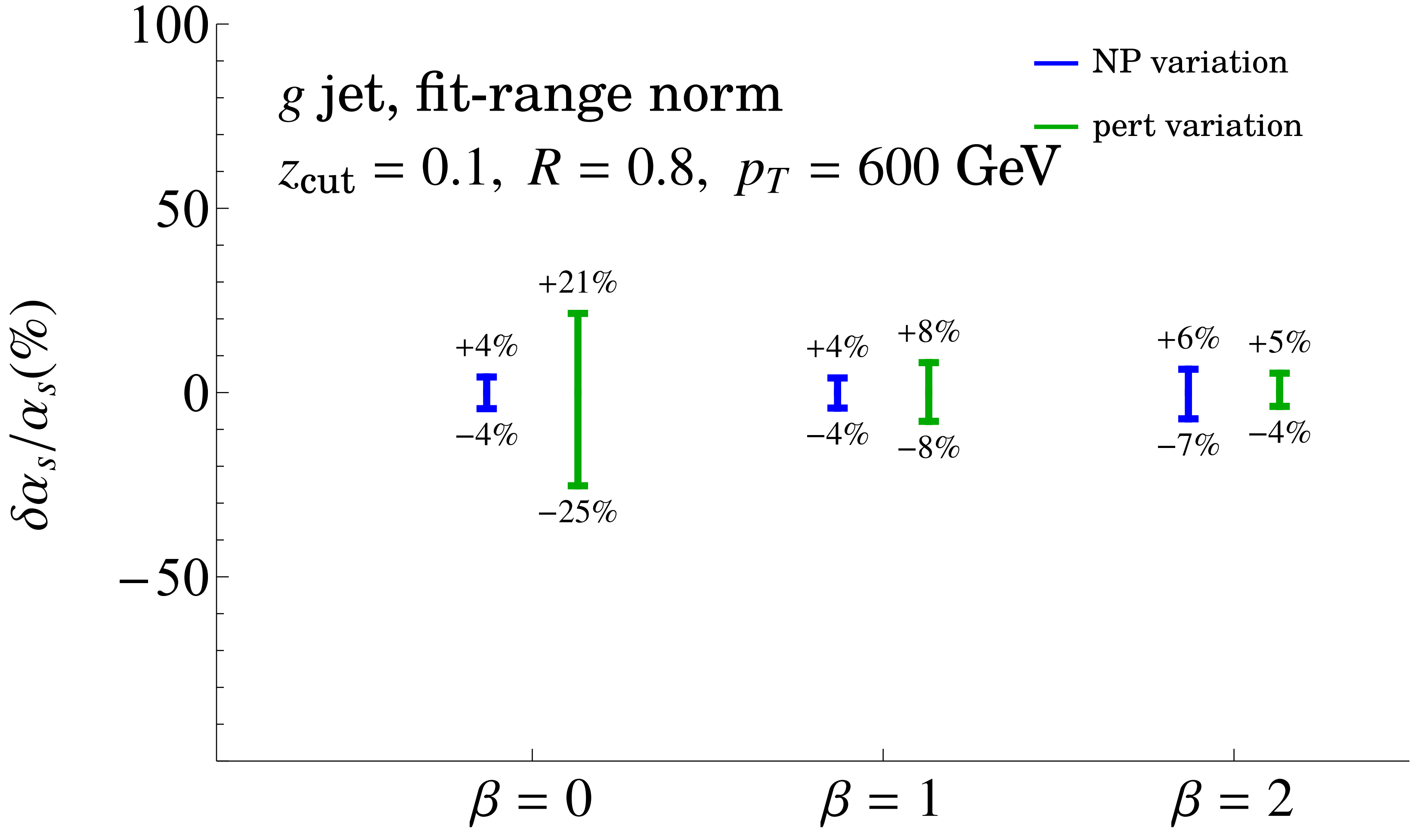


Results for fit-range normalization

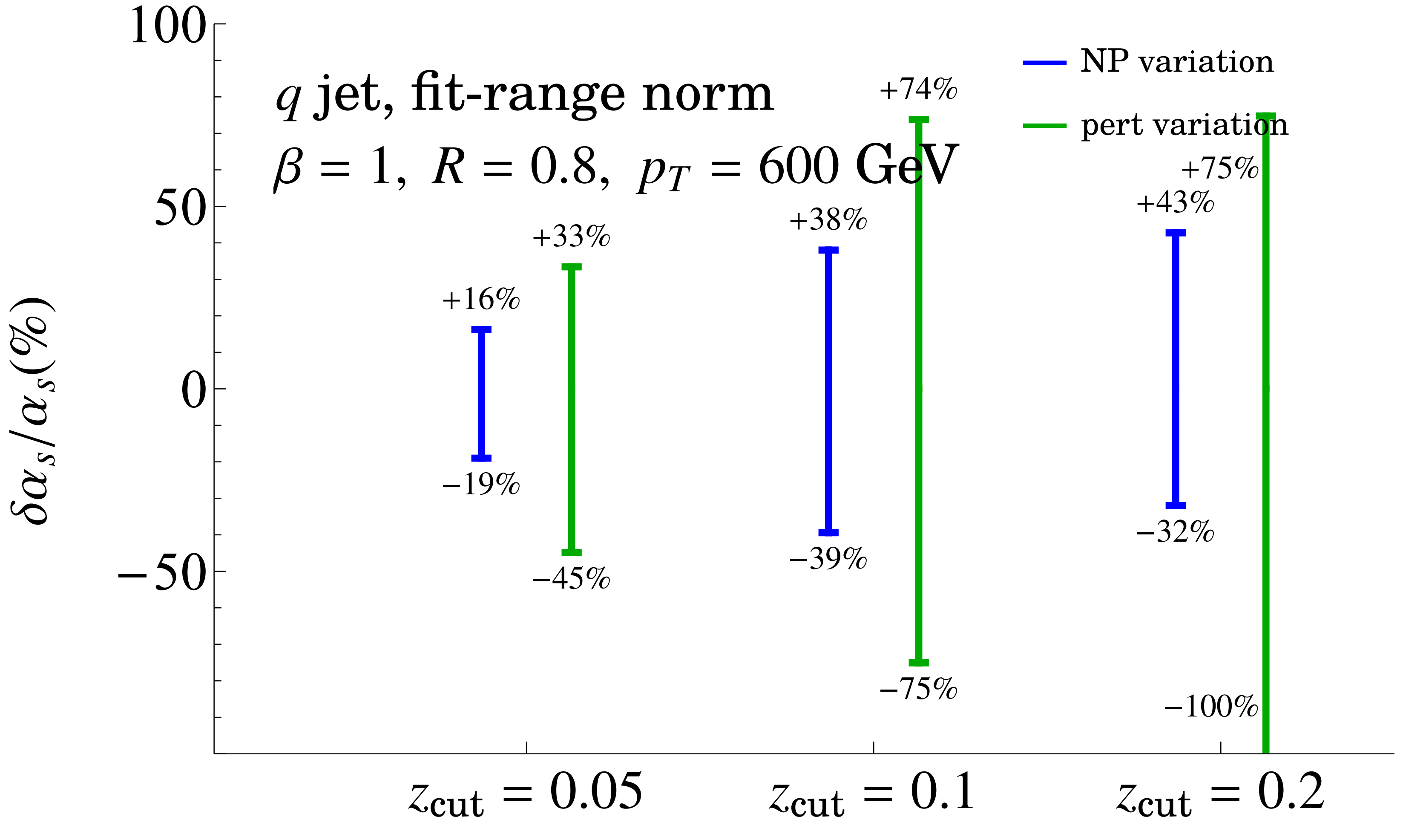
Quark jets: Gluon jets:

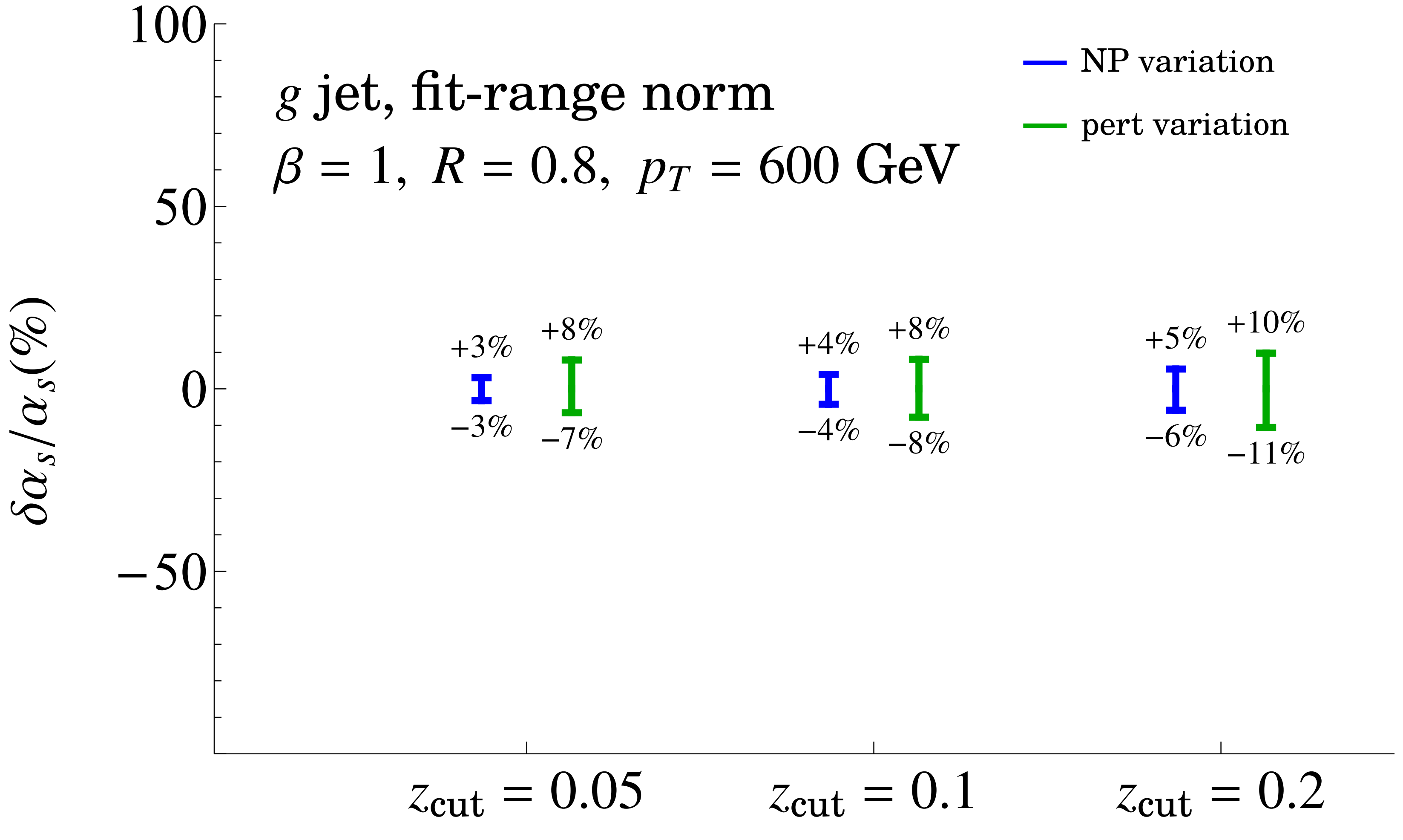
Vary β:





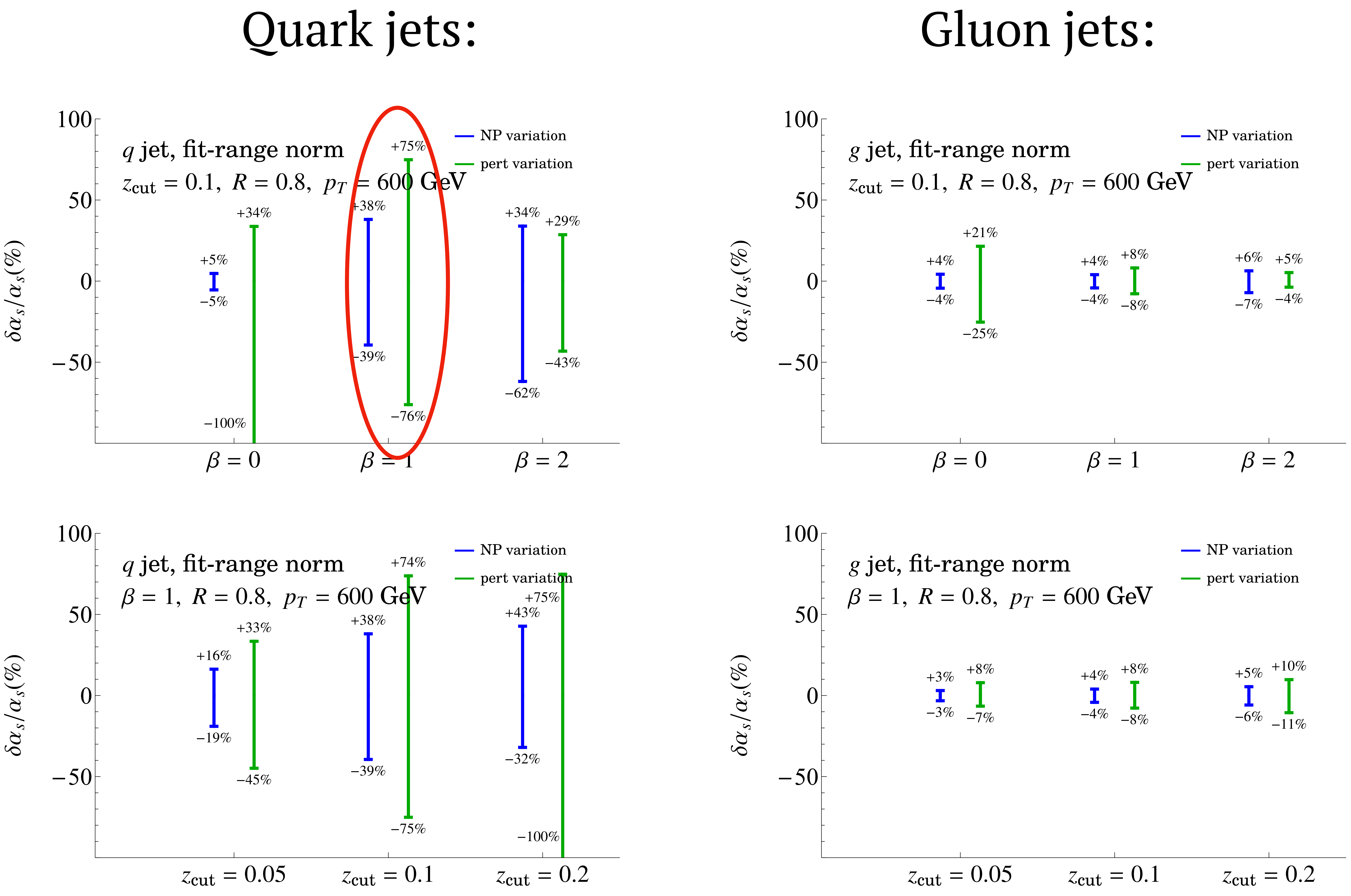
Vary z_{cut}:





Results for fixed-range normalization

Fixed-range normalization eliminates almost all α_s sensitivity of quark jets



Results for inclusive normalization

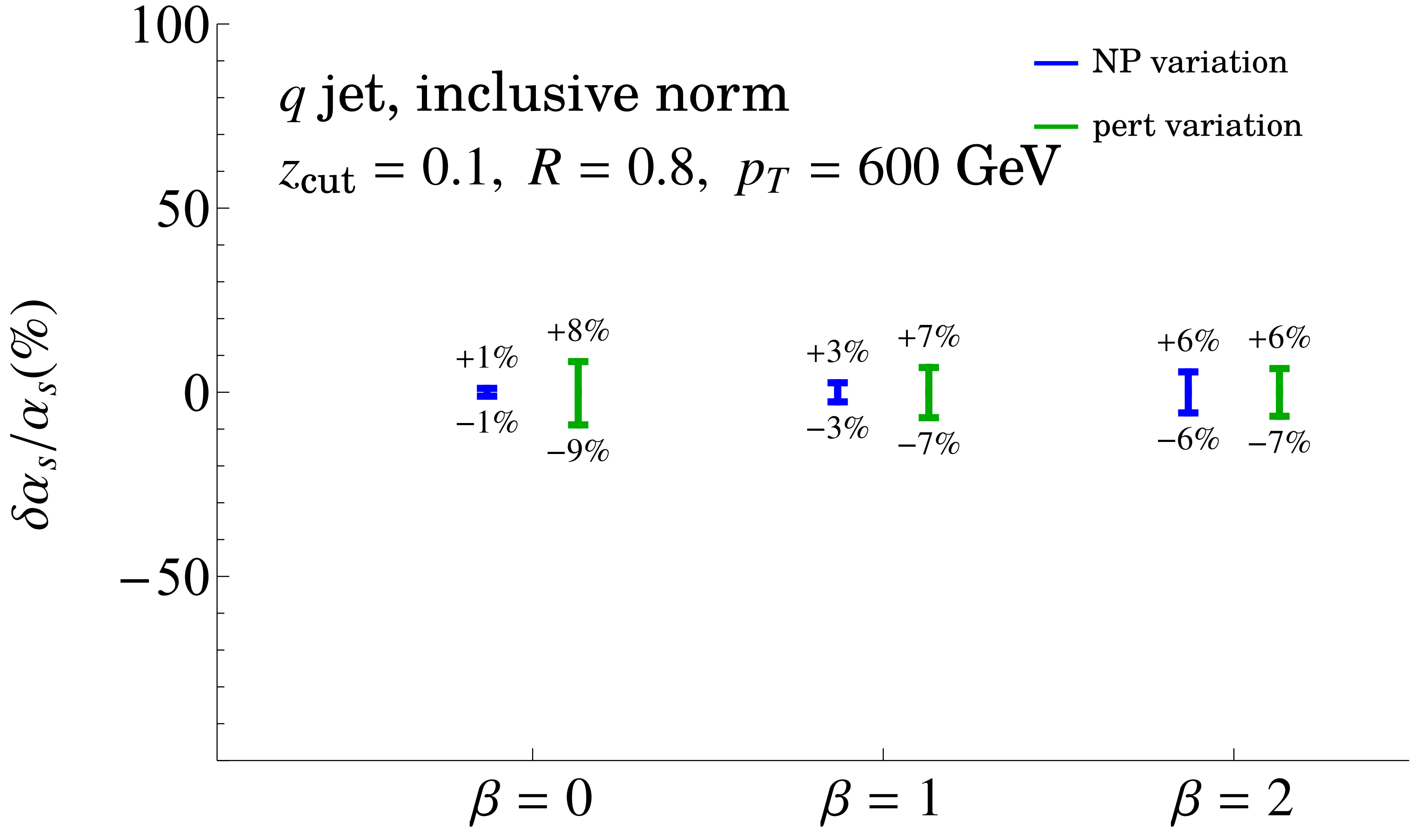
Normalize to inclusive cross section in the p_T-η bin: $\frac{1}{\sigma_{\text{incl}}} \frac{\mathrm{d}^3\sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}\xi}$

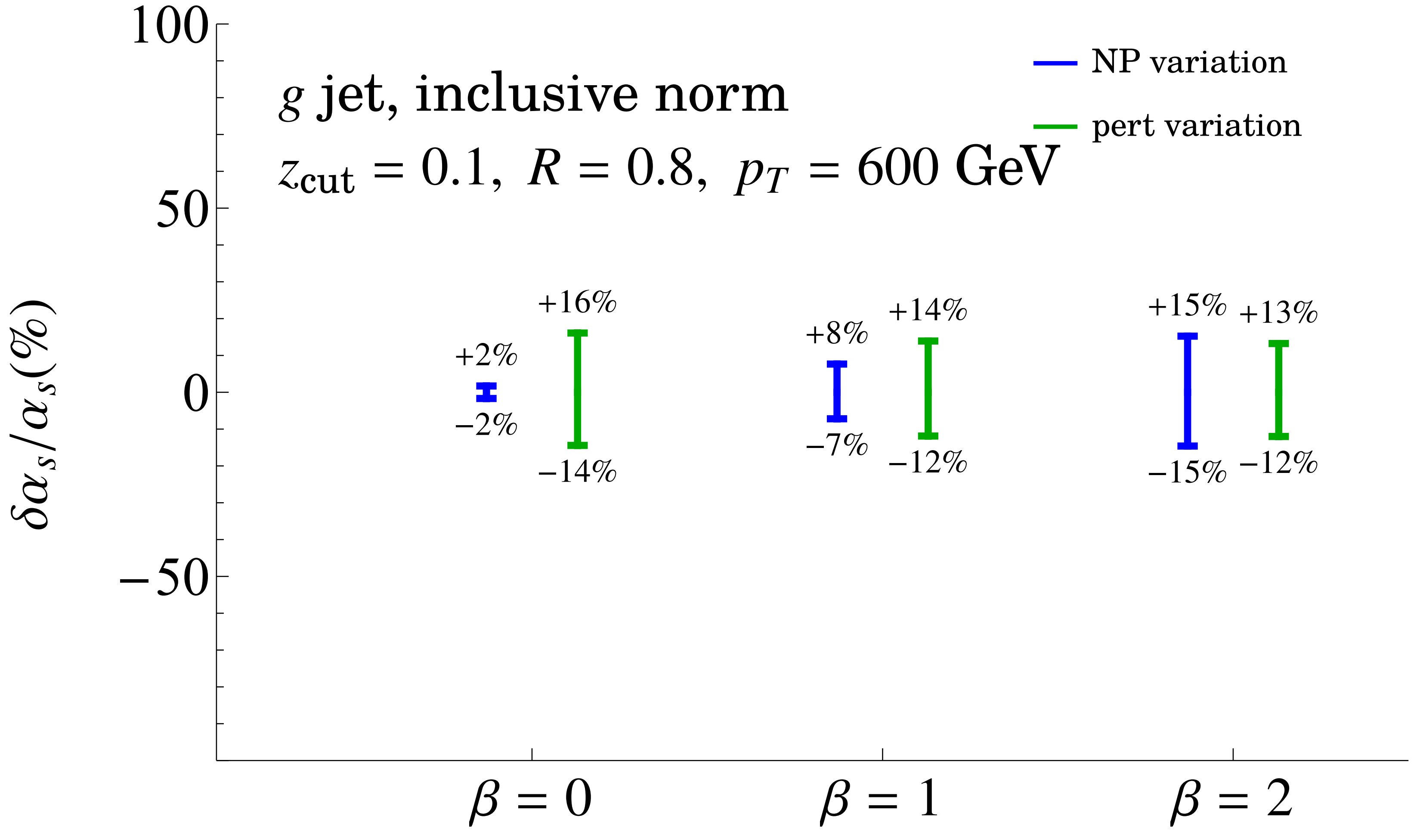
$$\sigma_{\text{incl}} = \frac{\mathrm{d}^2\sigma}{\mathrm{d}p_T \mathrm{d}\eta}$$

Results for inclusive normalization

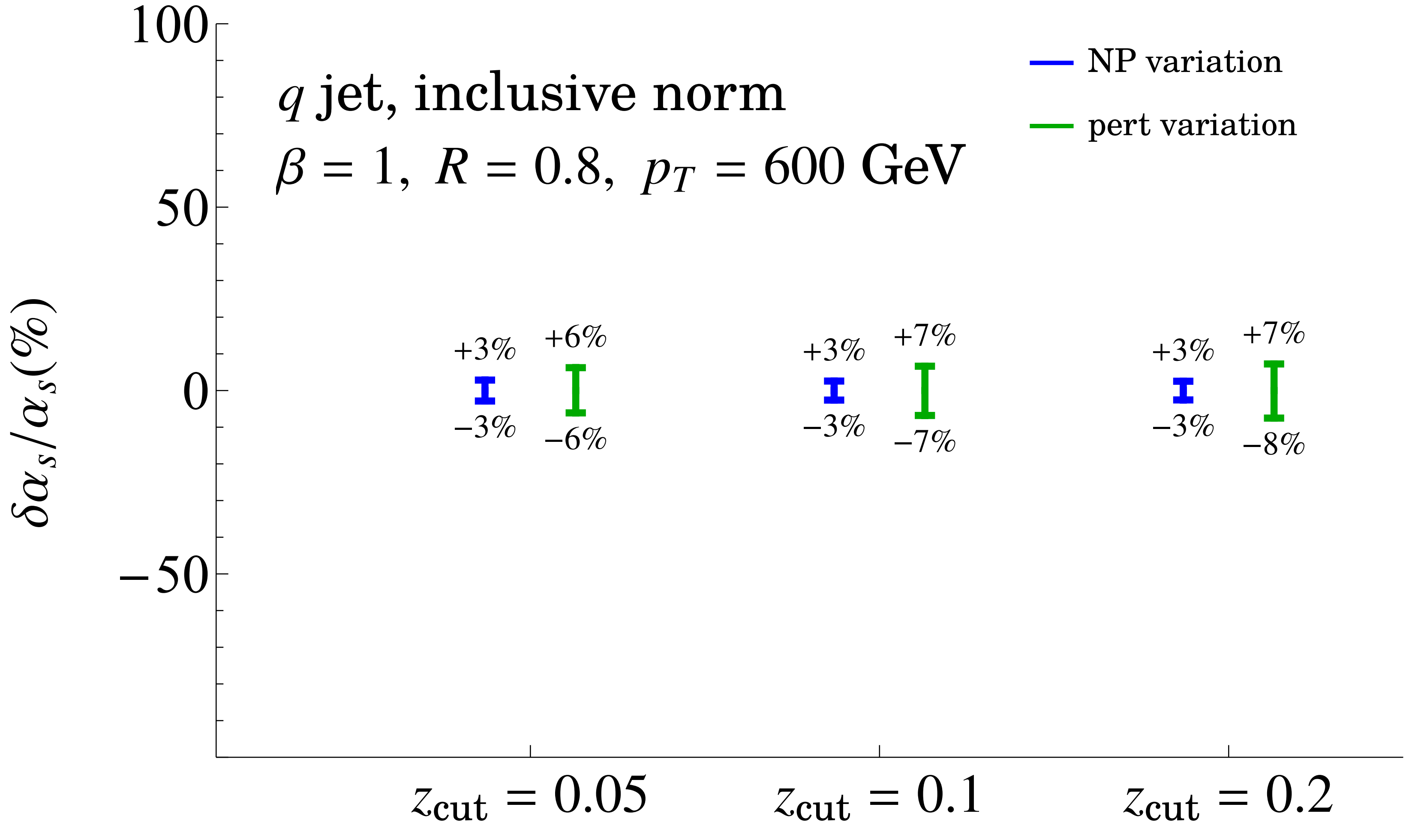
Quark jets: Gluon jets:

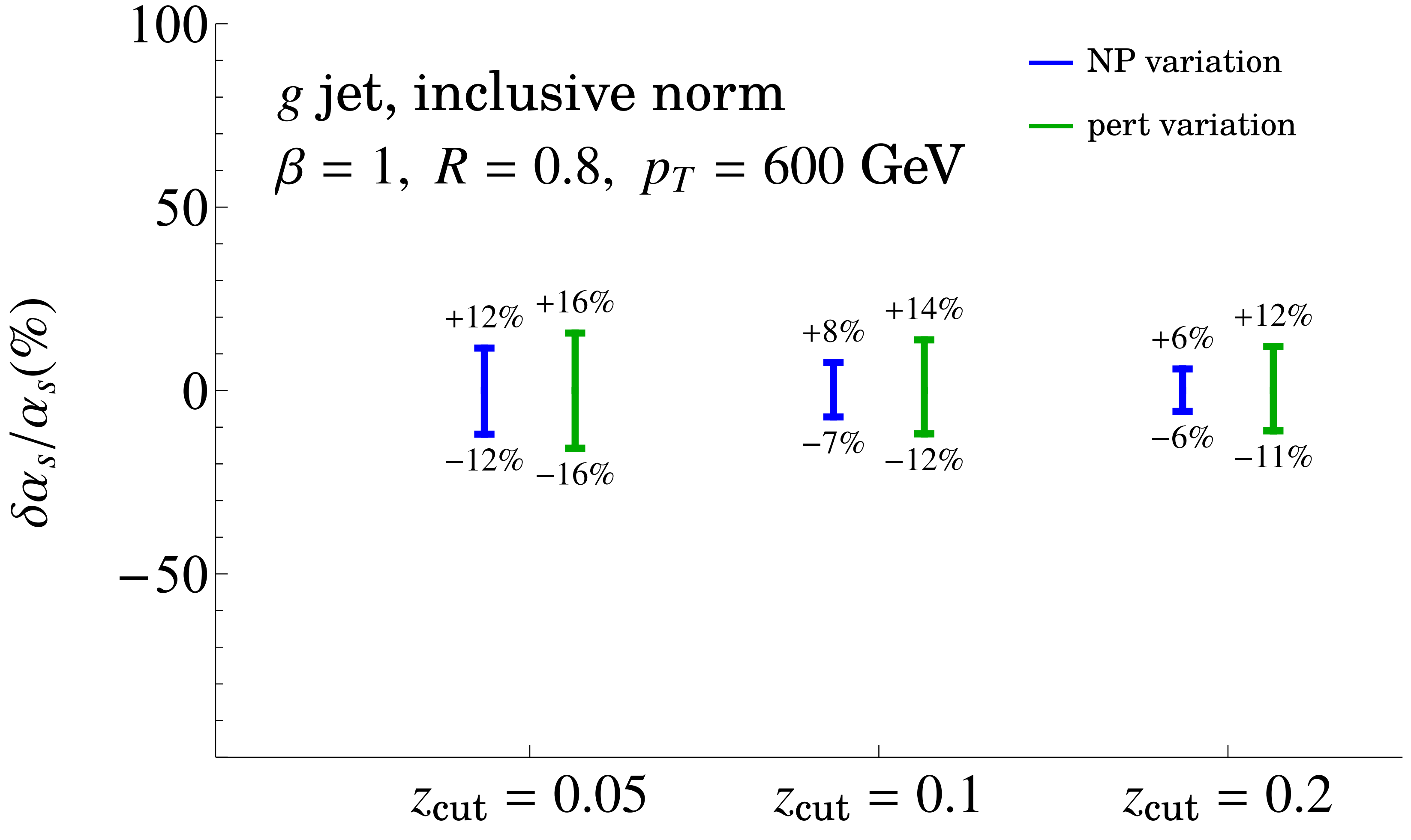
Vary β:





Vary z_{cut}:

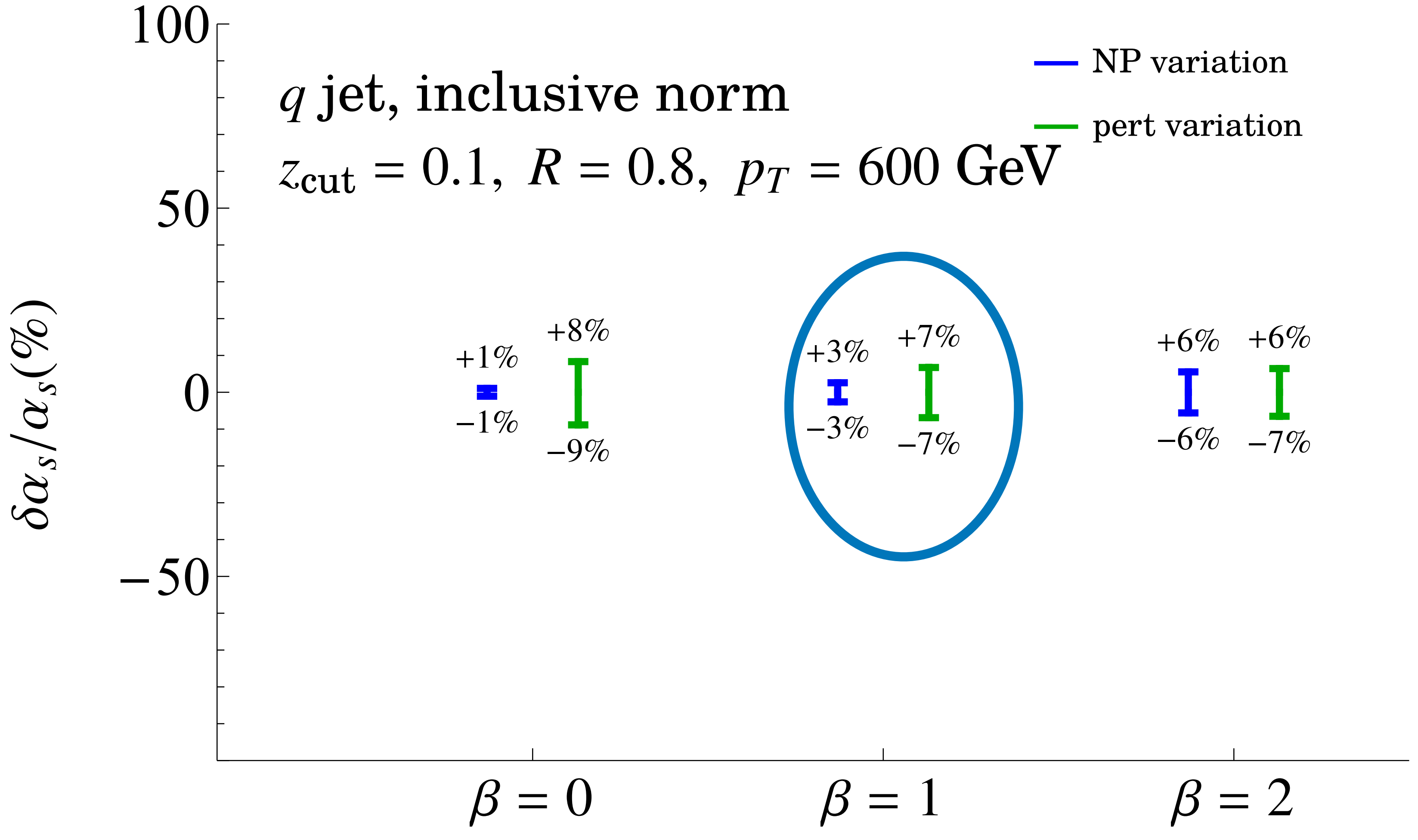


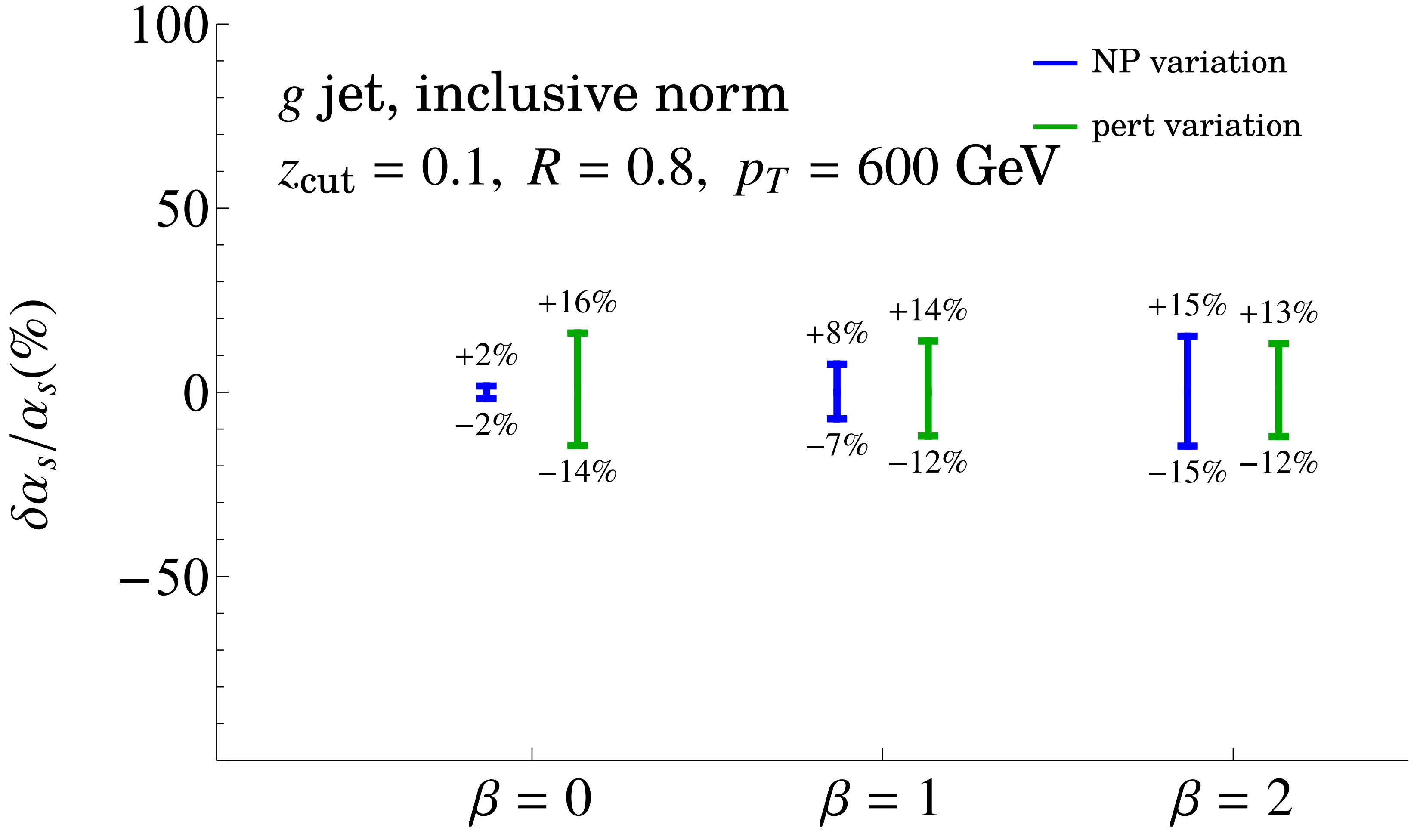


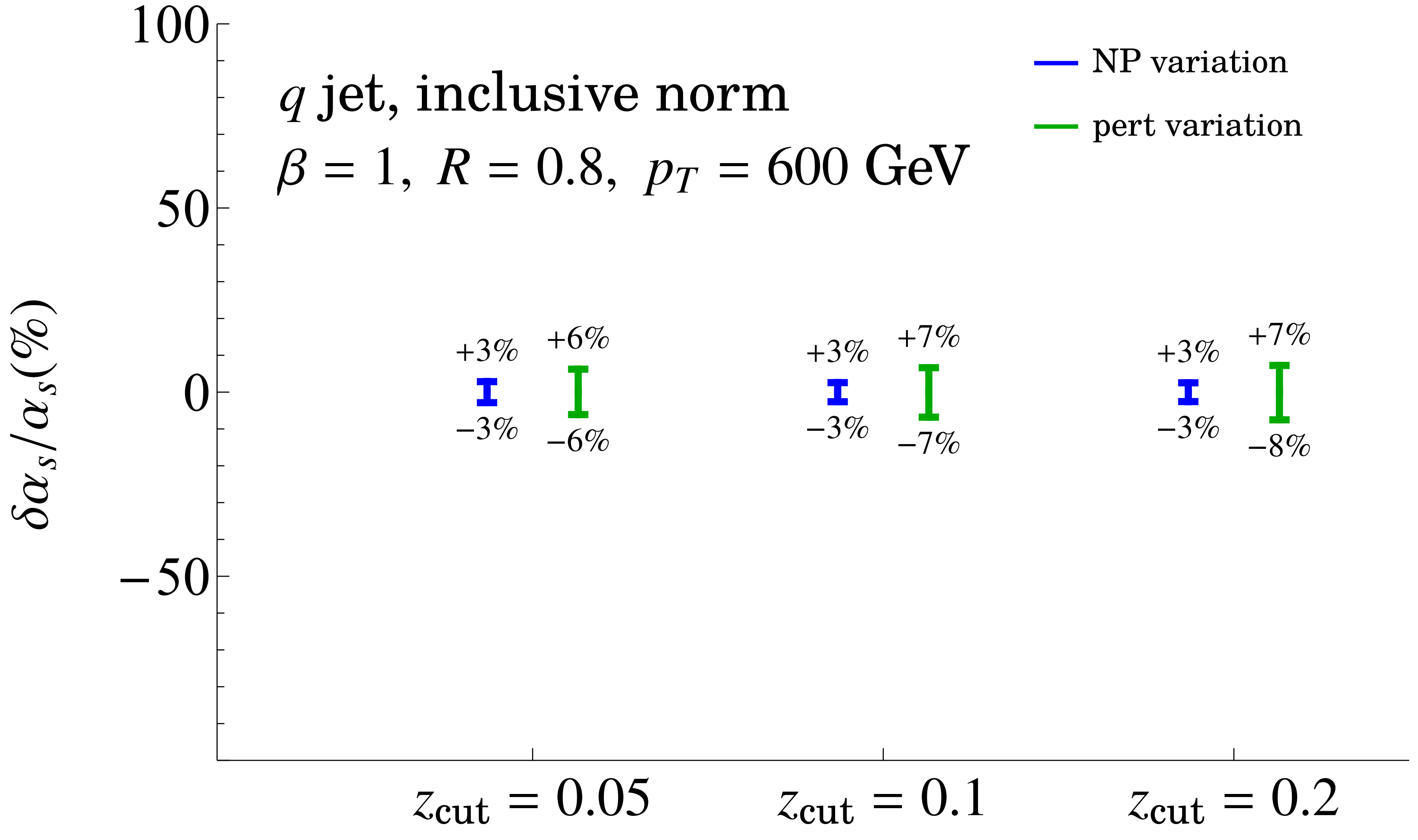
Results for inclusive normalization

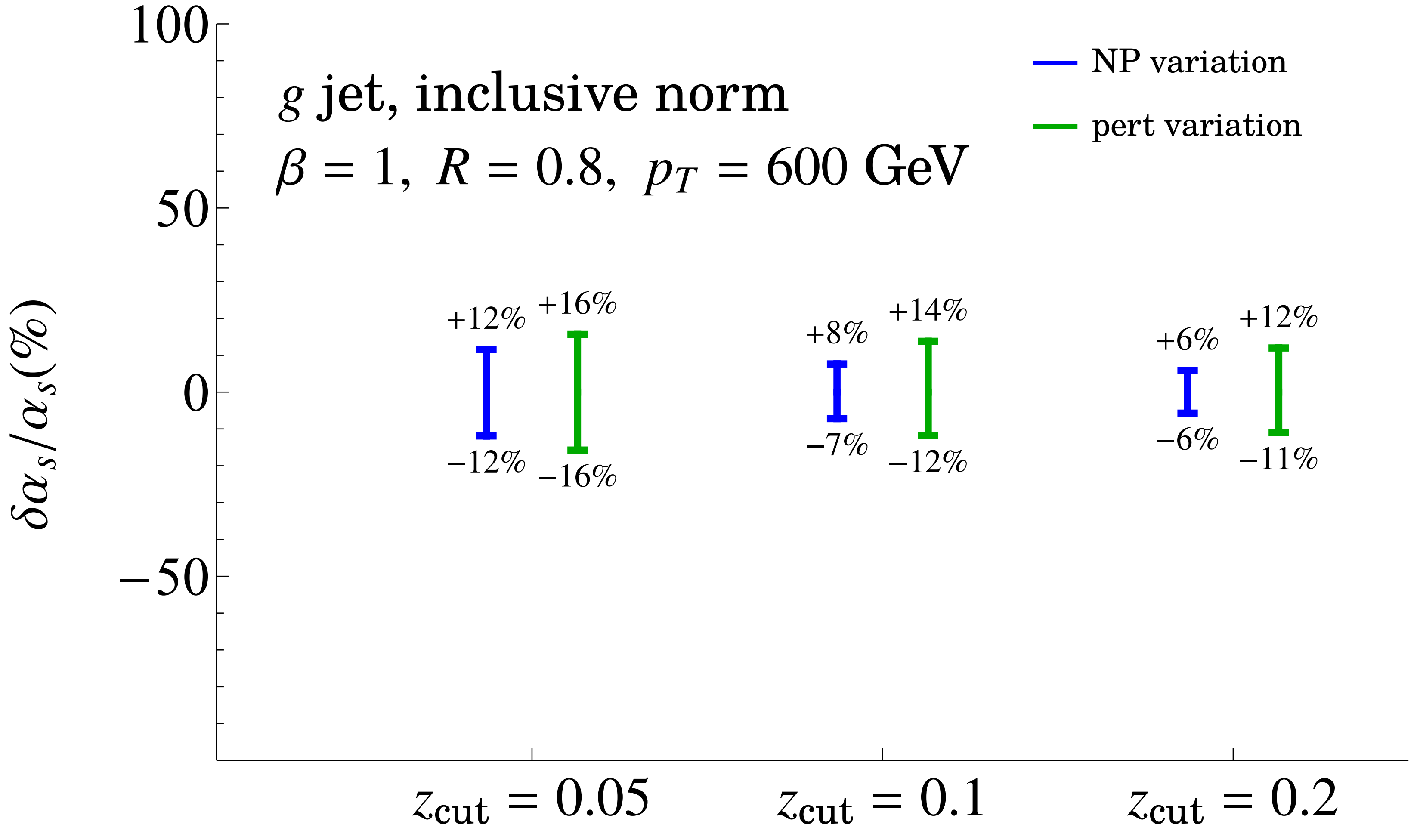
Quark jets: Gluon jets:

Normalizing to the full spectrum is essential









Prospects for improving precision

- Uncertainties can be reduced by going to **higher logarithmic orders**
- All **NNLL** data known

  [Bell, Rahn, Talbert 2018-2020] [Frye, Larkoski, Schwartz, Yan 2016]

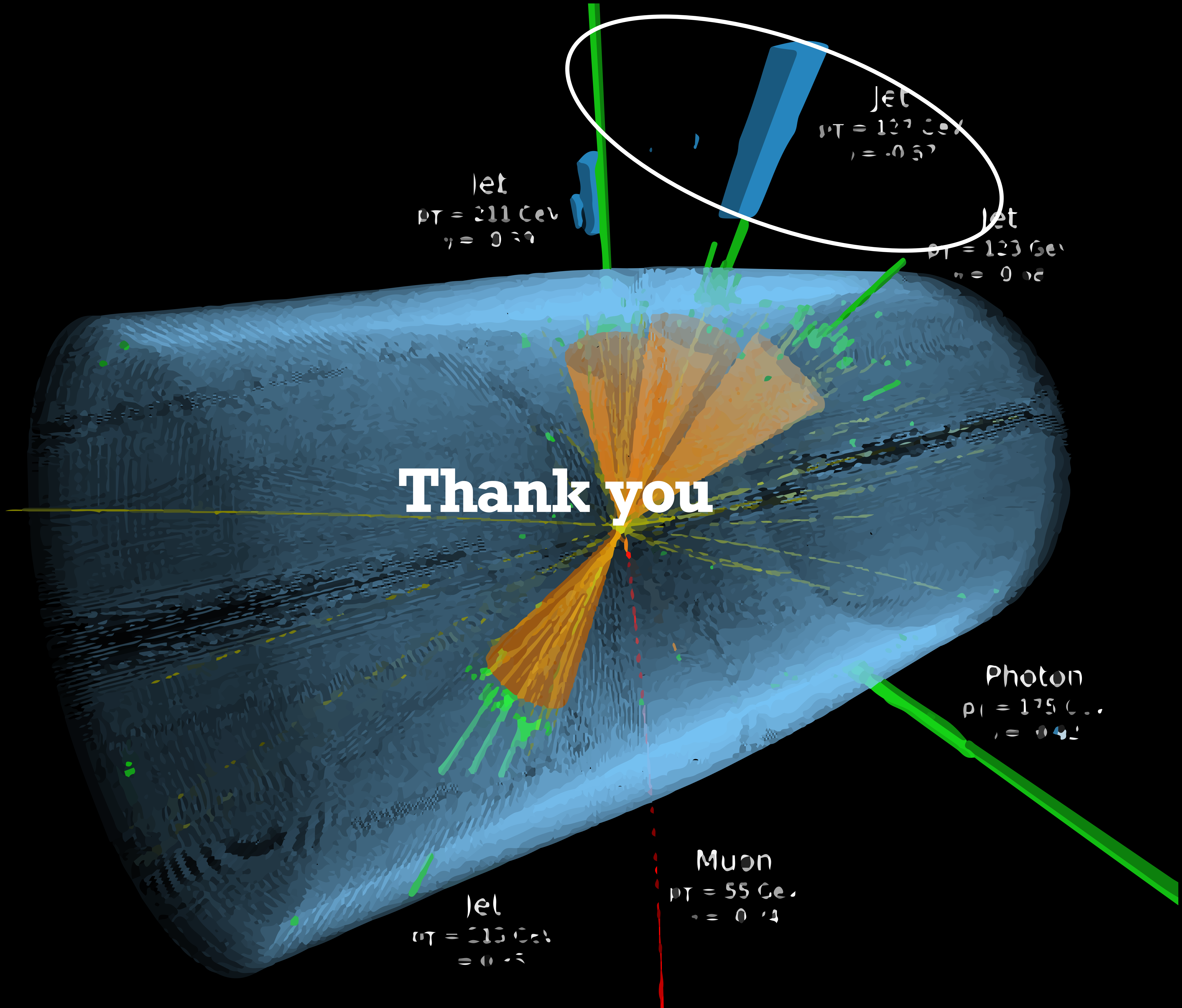
- Recent results at **N^{3}LL** for hemisphere jets in e^+e^- collisions

  [Kardos, Larkoski, Trócsányi]

- **Need 2-loop constant pieces** for extending to N^{3}LL
  - collinear-soft (gluon jets) and
  - global-soft (quark and gluon jets, $R < \pi/2$)

Conclusions

- Combined **perturbative** uncertainty of around **9% for quark jets and 16% for gluon jets.**
- For $\beta = 1$ we find **nonperturbative** uncertainty of **3% for quark jets and 8% for gluon jets**.
- q/g fraction **well defined** in theoretical calculations. PDF dependence **subdominant for normalized** cross sections.
- **Model-independent** estimate of nonperturbative power corrections
- Normalizing to **inclusive cross section** in p_T-η bin essential to retain α_s-sensitivity
- **N^{3}LL calculations** would reduce perturbative uncertainty
- **Constrain nonperturbative parameters** using multiple $z_{\rm cut}$, β values.



Thank you

Backup slides

Soft drop jet mass in inclusive jets

$$\xi = \frac{m_J^2}{p_T^2 R^2}$$

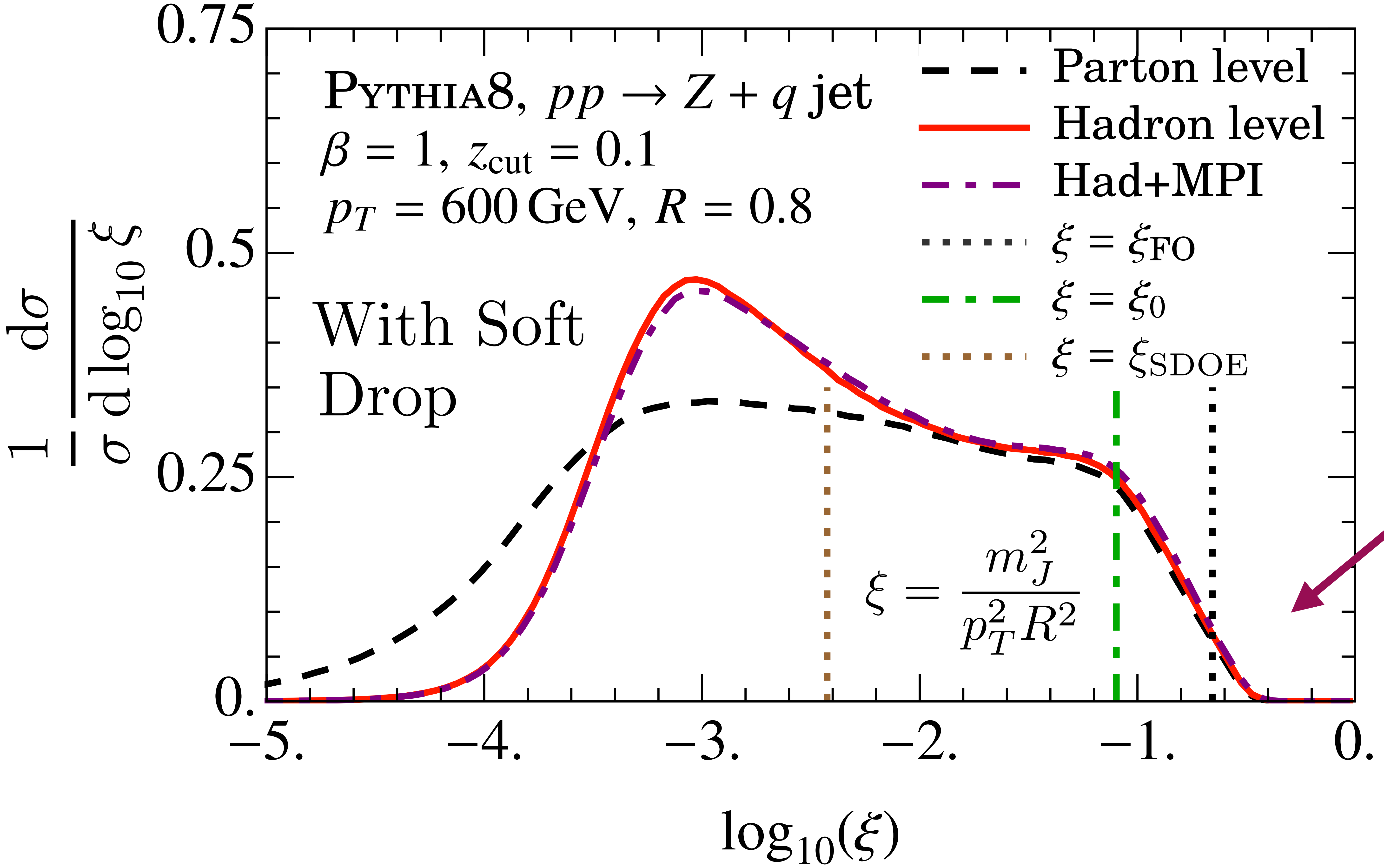
$$\tilde{\mathcal{G}}_\kappa(\xi, p_T R, \mu) \equiv \frac{1}{\sigma_\kappa^{\rm incl}} \frac{\mathrm{d}\sigma_\kappa}{\mathrm{d}\xi}(p_T, \eta) = N_{\rm incl}^\kappa(p_T R, \mu) \mathcal{J}_\kappa(\xi, p_T, \eta, R, \mu)$$

Energy scales:

- Hard-collinear scale: $Q = p_T R$
- Soft drop scale: $Q_{\rm cut} = z_{\rm cut} Q \left(\frac{R}{R_0}\right)^\beta$
- Nonperturbative scale: $\Lambda_{\rm QCD}$

Region for fitting to α_s

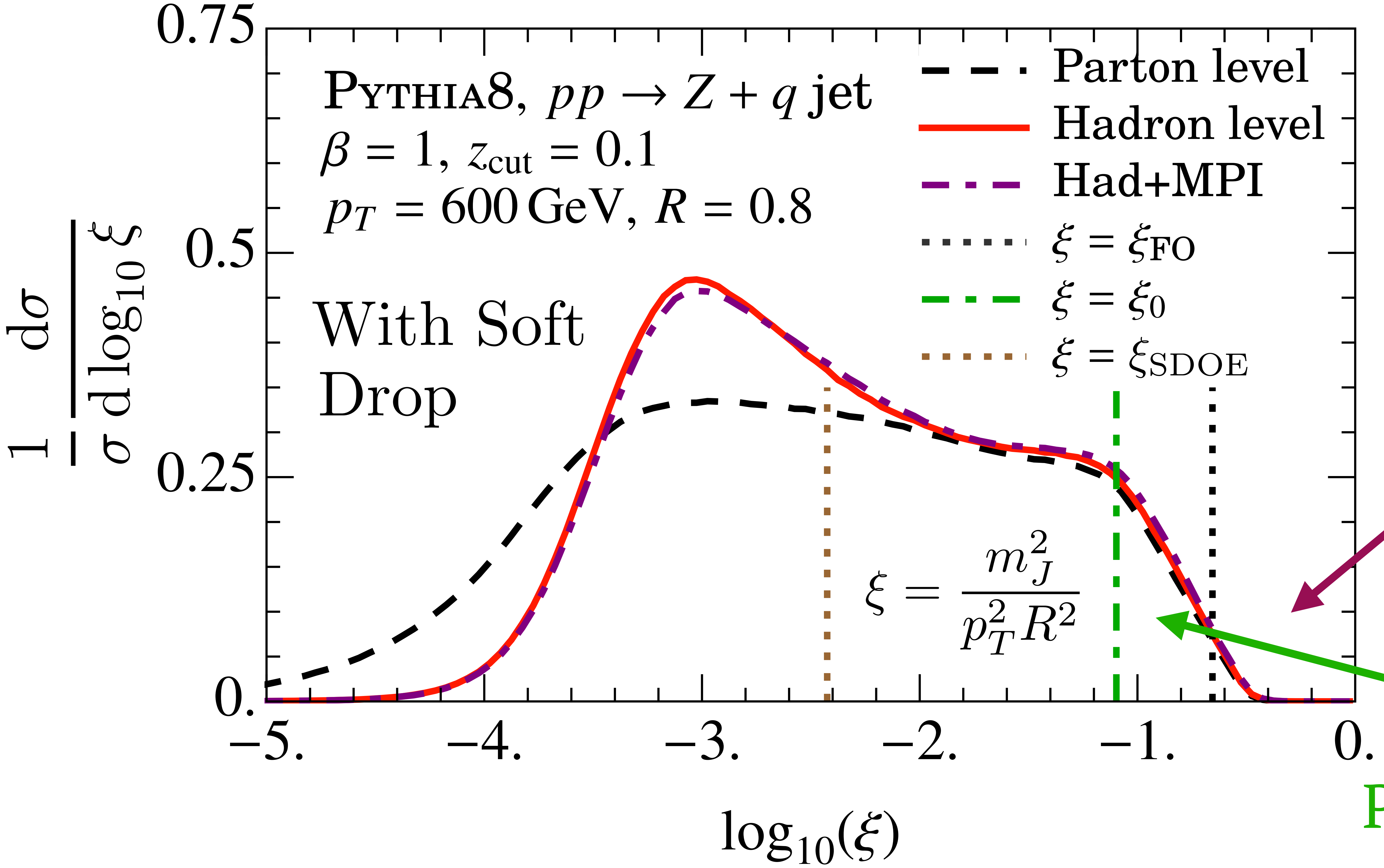
$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



Fixed order region: $\xi \lesssim 1$

Region for fitting to α_s

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



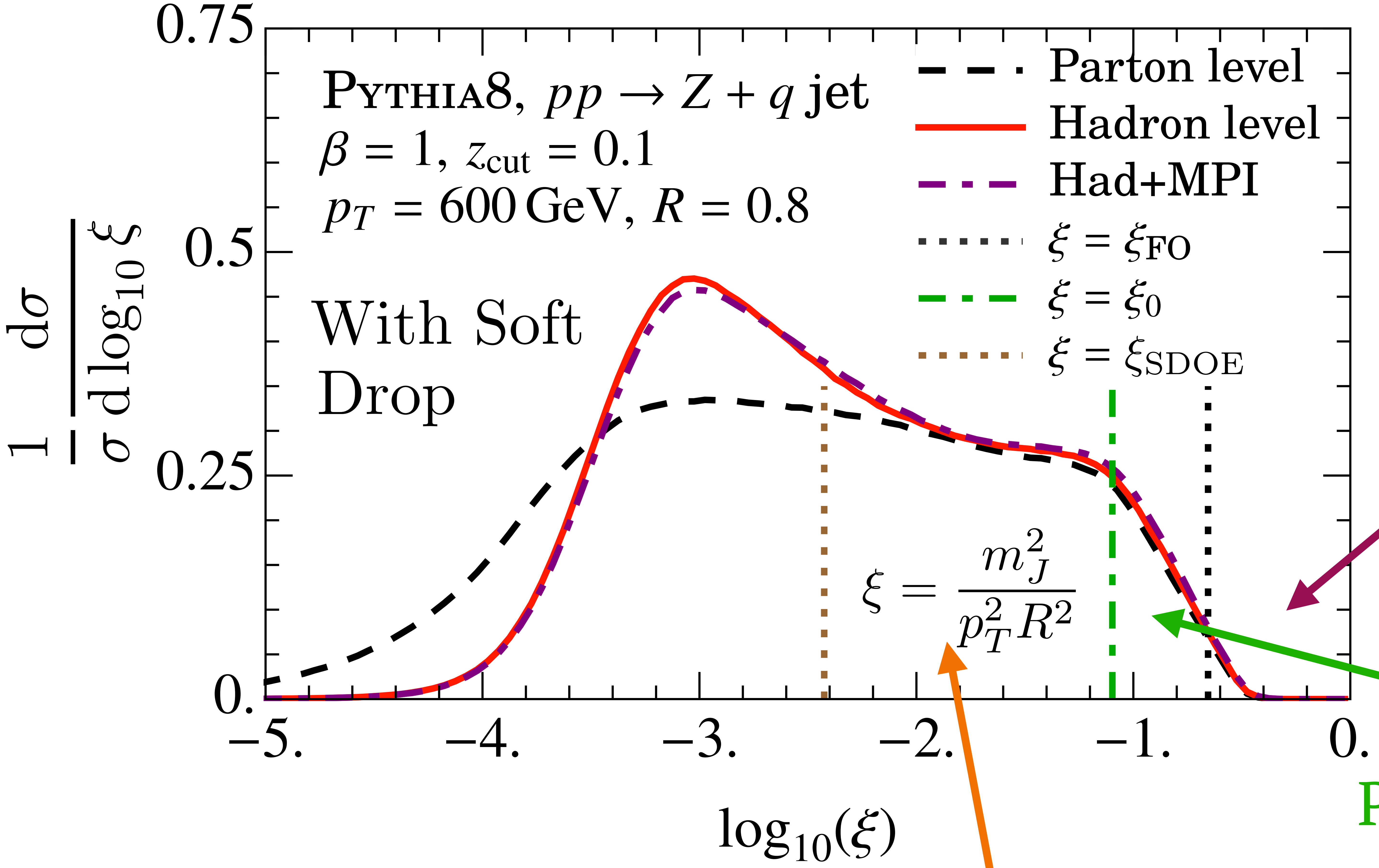
Fixed order region: $\xi \lesssim 1$

$$\xi_0 = \frac{Q_{\rm cut}}{Q}$$

Plain jet mass resummation region: $\xi_0 \lesssim \xi \ll 1$

Region for fitting to α_s

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



Fixed order region: $\xi \lesssim 1$

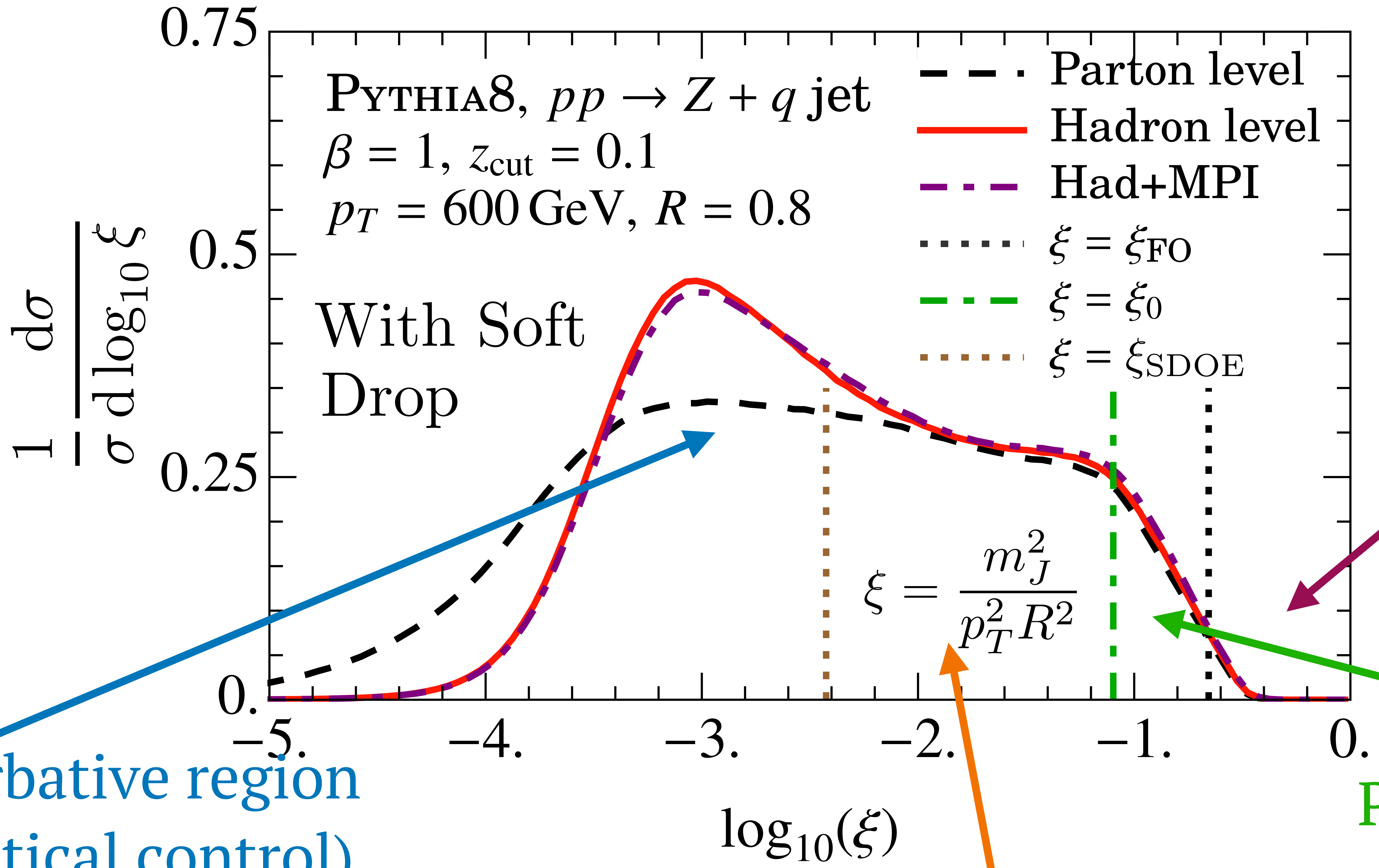
$$\xi_0 = \frac{Q_{\rm cut}}{Q}$$

Plain jet mass resummation region: $\xi_0 \lesssim \xi \ll 1$

Soft drop operator expansion region: $\xi_0 \left(\frac{\Lambda_{\rm QCD}}{Q_{\rm cut}}\right)^{\frac{2+\beta}{1+\beta}} \ll \xi \ll \xi_0$

Region for fitting to α_s

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



Fixed order region: $\xi \lesssim 1$

$$\xi_0 = \frac{Q_{\mathrm{cut}}}{Q}$$

Nonperturbative region (no theoretical control)

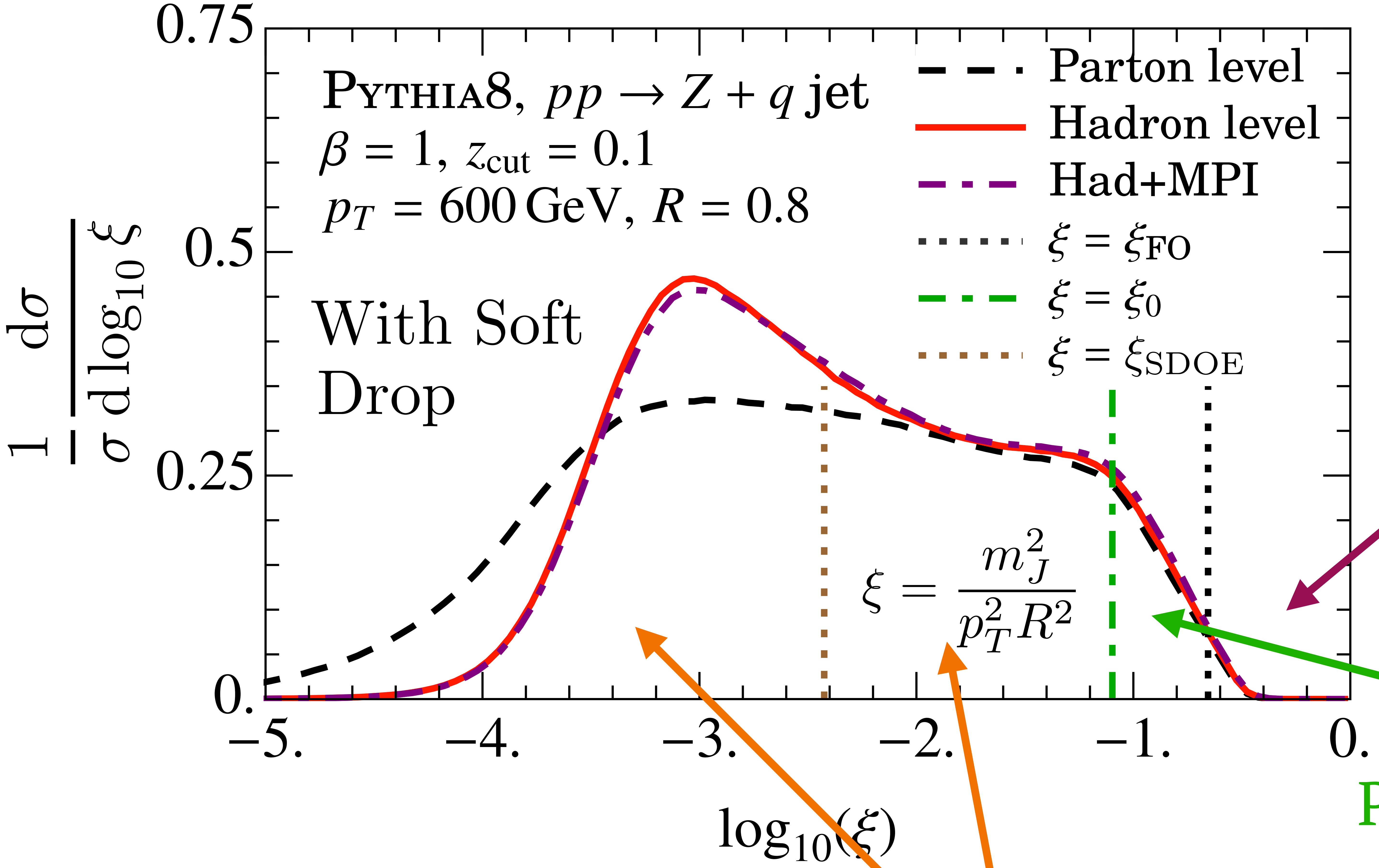
$$\xi \gtrsim \xi_0 \left(\frac{\Lambda_{\mathrm{QCD}}}{Q_{\mathrm{cut}}}\right)^{\frac{2+\beta}{1+\beta}}$$

Plain jet mass resummation region: $\xi_0 \lesssim \xi \ll 1$

Soft drop operator expansion region: $\xi_0 \left(\frac{\Lambda_{\mathrm{QCD}}}{Q_{\mathrm{cut}}}\right)^{\frac{2+\beta}{1+\beta}} \ll \xi \ll \xi_0$

Large logarithms

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



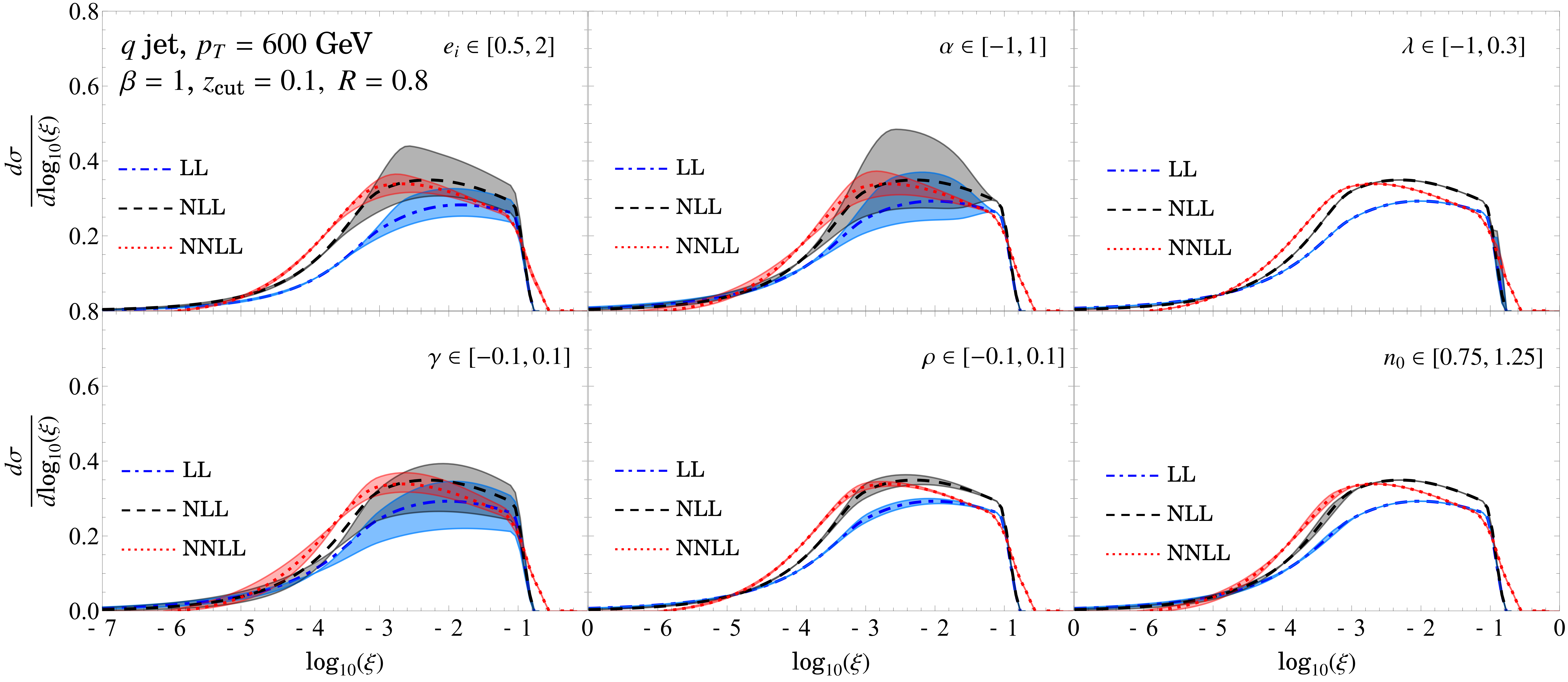
Fixed order region

$$\xi_0 = \frac{Q_{cut}}{Q}$$

Plain jet mass resummation region: $\alpha_s^n \ln^m(\xi)$

Soft drop resummation region: $\alpha_s^n \ln^m(\xi/\xi_0)$

Scale variations



Impact of scale variations: dependence on β