

NLO corrections to single and double inclusive hadron production  
in DIS at small  $x$

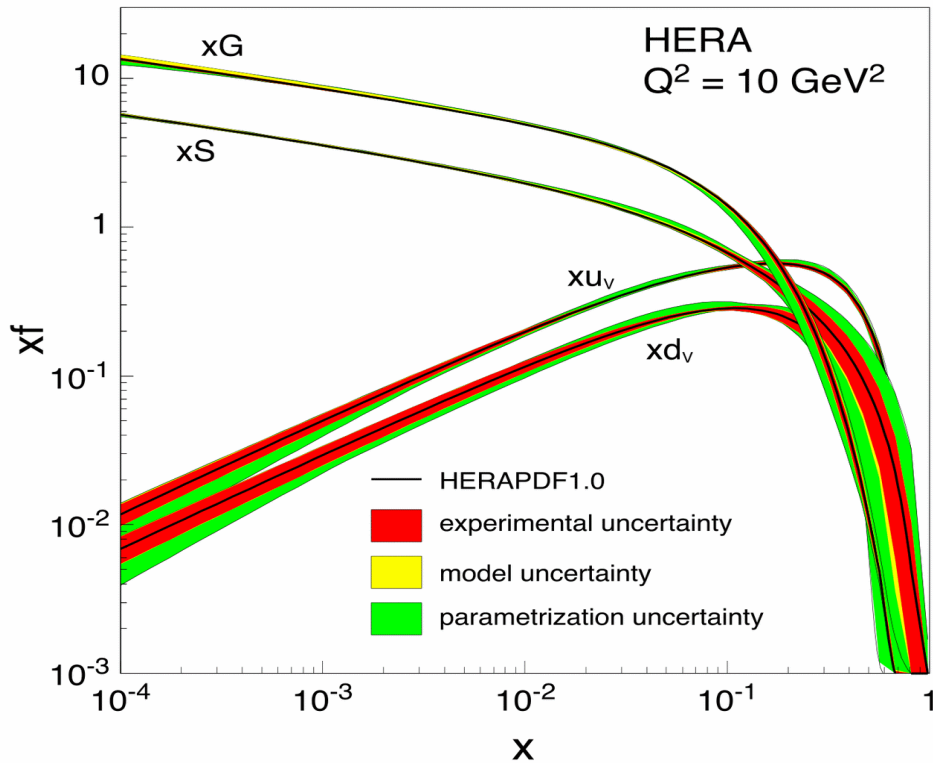
*Jamal Jalilian-Marian*

Baruch College and the City University of New York Graduate Center

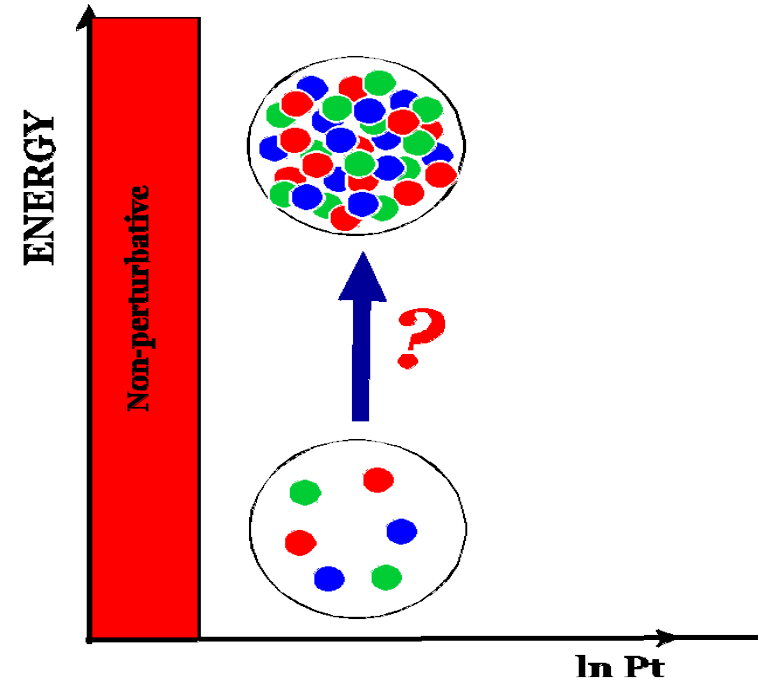
*Workshop on Resummation, Evolution, Factorization 2022*

*Online, October 31st – November 4th, 2022*

# dynamics of *universal gluonic matter*: *gluon saturation*



$$P_{gg} \sim P_{gq} \sim \frac{1}{x}$$



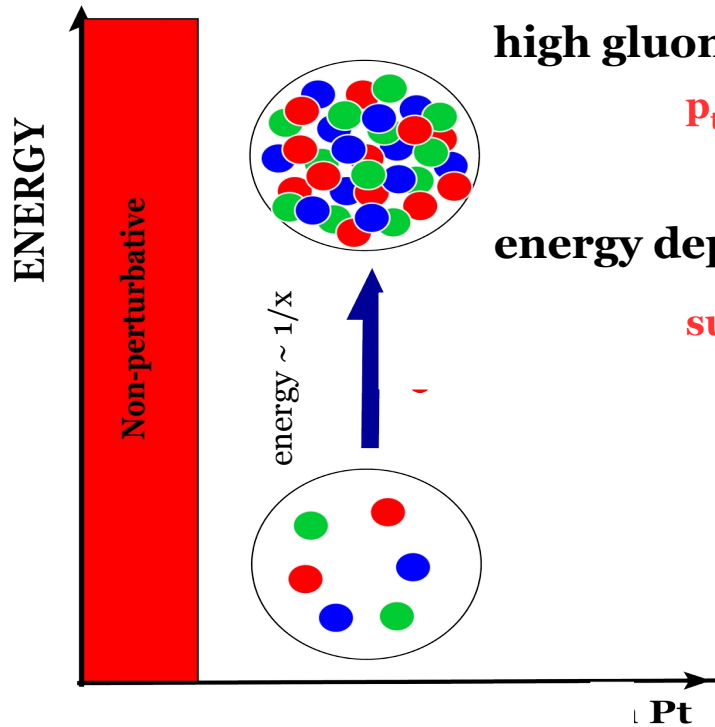
How does this happen ?

How do correlation functions evolve ?

Is there a universal fixed point for the evolution ?

Are there scaling laws ?

# QCD at high energy/small x: gluon saturation



high gluon density: Eikonal multiple scattering

$p_t$  broadening (generic to multiple scattering)

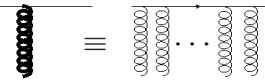
energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$Q_s^2(\mathbf{x}, \mathbf{b}_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1 \text{ GeV}^2$$

for a proton target (quarks)



a framework for multi-particle production in QCD at small x/low  $p_t$

*Shadowing/Nuclear modification factor*

*Azimuthal angular correlations*

*Long range rapidity correlations (ridge,...)*

*Initial conditions for hydro*

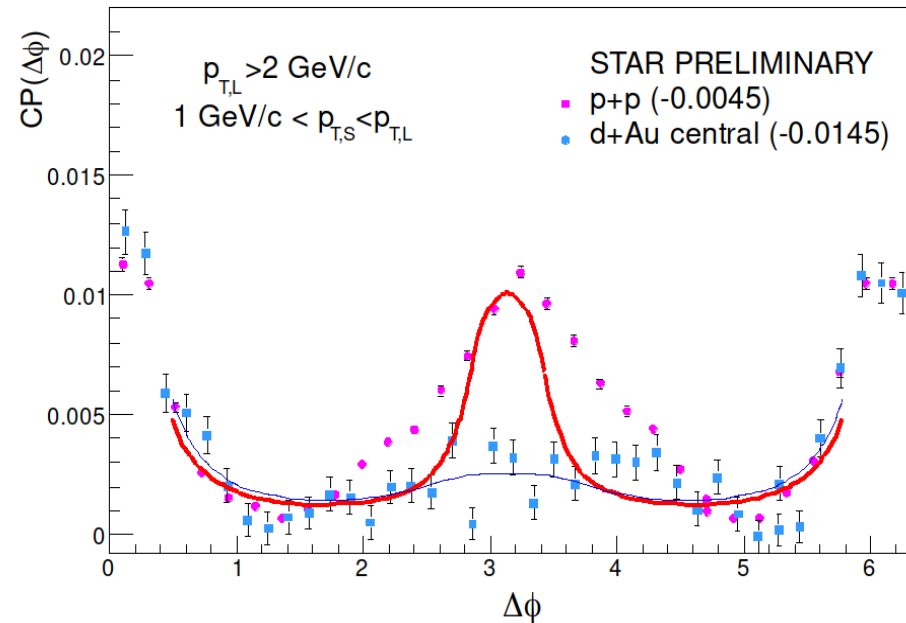
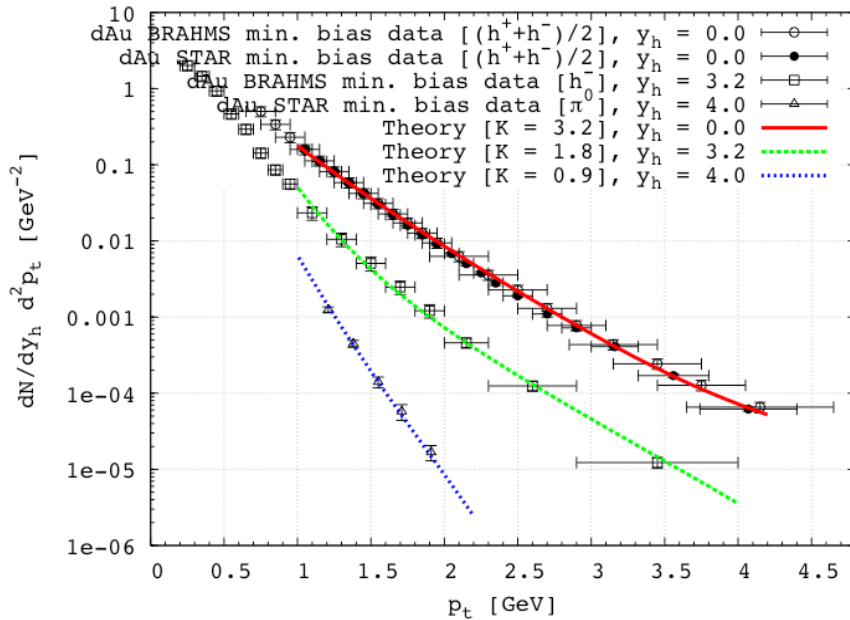
*Thermalization (?)*

$$x \leq 0.01$$

$$\alpha_s \ln(x_v/x) \sim 1$$

# CGC at RHIC

## Single and double inclusive hadron production in dA collisions



Dumitru, Hayashigaki, JJM, NPA770 (2006) 57

Albacete, Marquet, PRL105 (2010) 162301

For more recent work see talks by S. Wei on Wednesday and P. Kotko on Thursday

# NLO corrections to inclusive dihadron production in DIS at small $x$

Based on F. Bergabo and JJM:

NPA 1018 (2022) 122358

PRD 106 (2022) 5, 054035

Closely related talks in this conference:

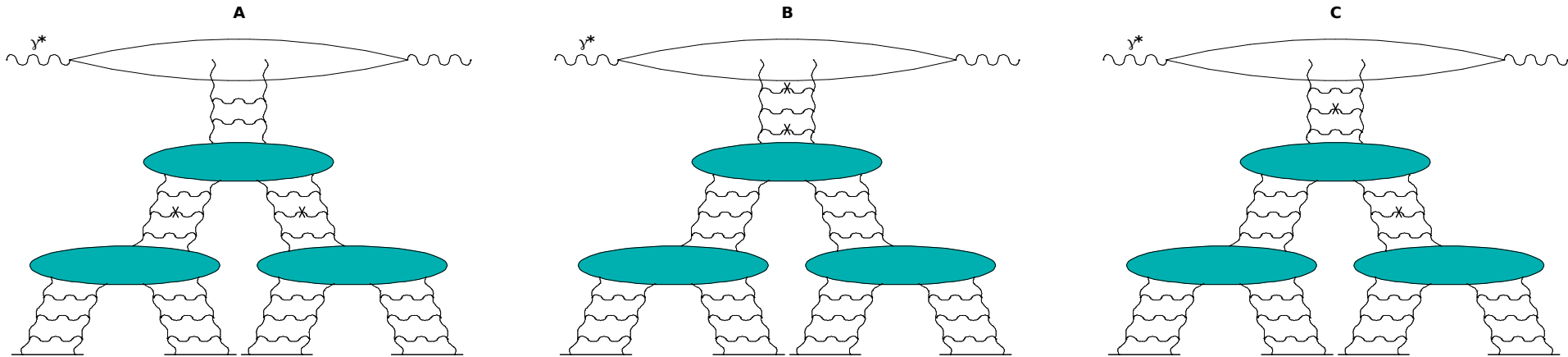
Sudakov effects in dijets: P. Taels, Wednesday

NLO dijets: F. Salazar, Thursday

NLO diffractive dihadrons: E. Li, Thursday

# LO: inclusive dihadron production in DIS at small $x$

JJM, Yu. Kovchegov  
PRD70 (2004) 114017



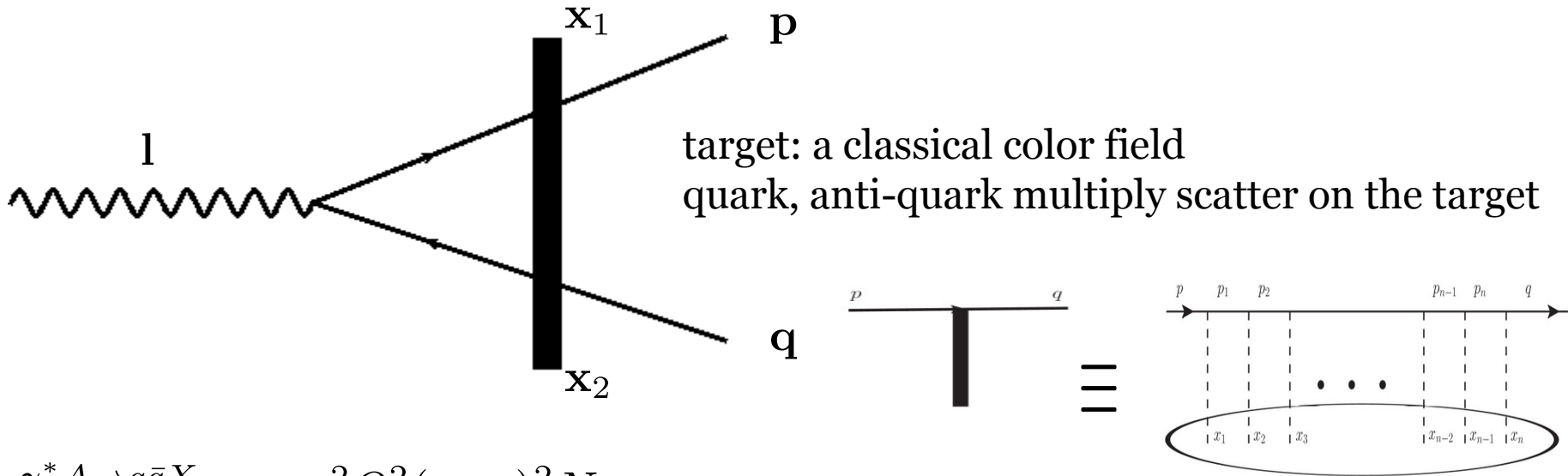
dominant diagrams in the **mid-rapidity** region

target is treated as a classical color field  $\mathbf{A}_a^\mu = \delta^{\mu-} n^\mu S_a(x^+, \mathbf{x})$

scatterings of gluons on the target encoded in Wilson lines  $U(\mathbf{x}_1), U^\dagger(\mathbf{x}_2)$

leading log evolution included

# LO: inclusive dihadron production in DIS at small x



$$\frac{d\sigma^{\gamma^* A \rightarrow q\bar{q}X}}{d^2p d^2q dy_1 dy_2} = \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7} \delta(1 - z_1 - z_2)$$

$$\int d^8 x_{\perp} e^{ip \cdot (x'_1 - x_1)} e^{iq \cdot (x'_2 - x_2)} [S_{122'1'} - S_{12} - S_{1'2'} + 1]$$

$$\left\{ 4z_1 z_2 K_0(|x_{12}|Q_1) K_0(|x_{1'2'}|Q_1) + (z_1^2 + z_2^2) \frac{x_{12} \cdot x_{1'2'}}{|x_{12}| |x_{1'2'}|} K_1(|x_{12}|Q_1) K_1(|x_{1'2'}|Q_1) \right\}$$

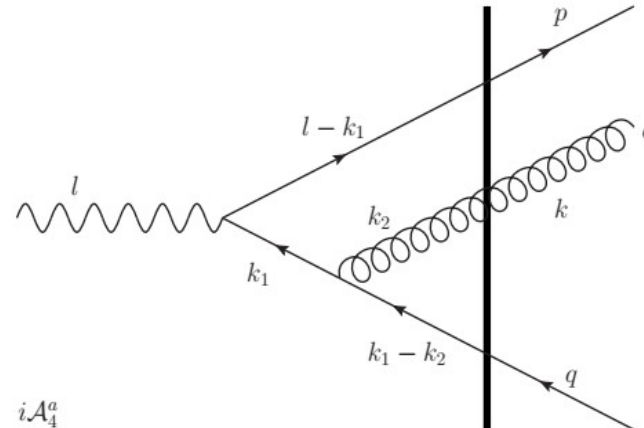
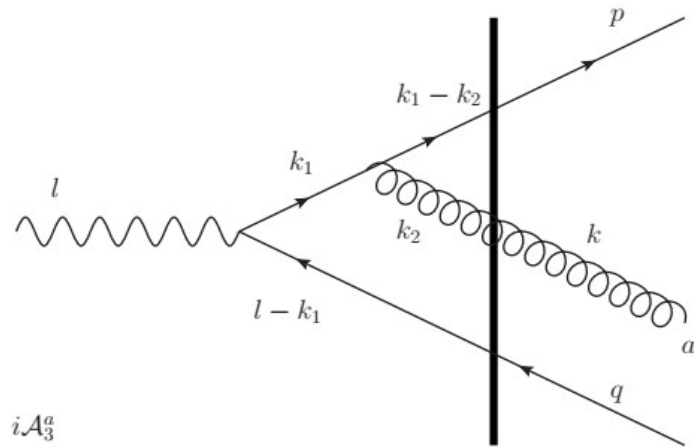
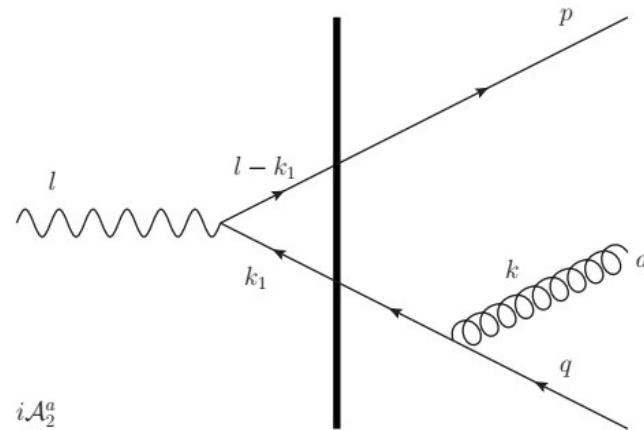
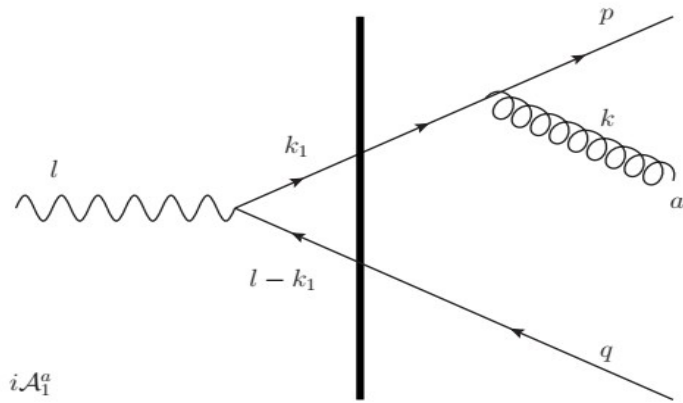
with

$$S_{12} \equiv \frac{1}{N_c} \text{Tr} V(x_1) V^\dagger(x_2)$$

$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

$$S_{122'1'} \equiv \frac{1}{N_c} \text{Tr} V(\mathbf{x}_1) V^\dagger(\mathbf{x}_2) V(\mathbf{x}_2') V^\dagger(\mathbf{x}_1')$$

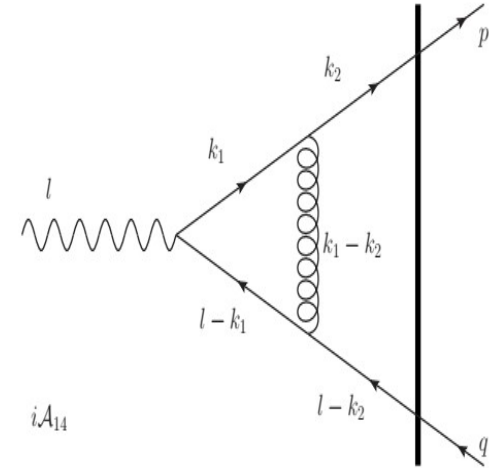
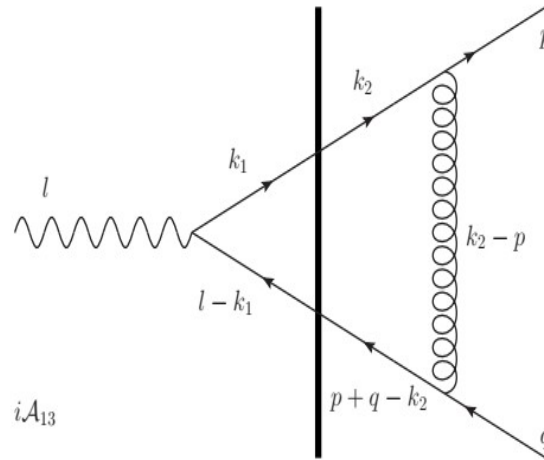
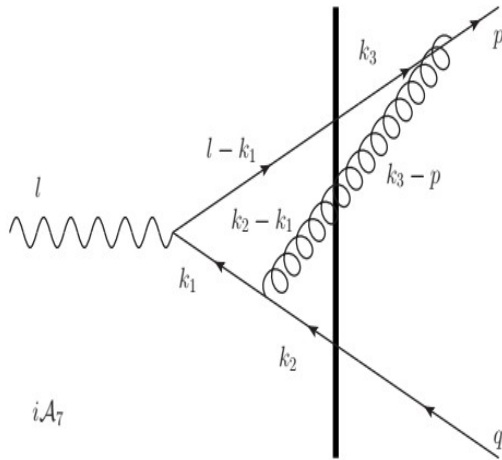
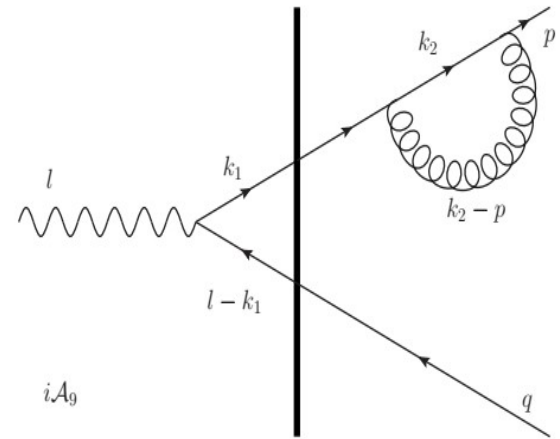
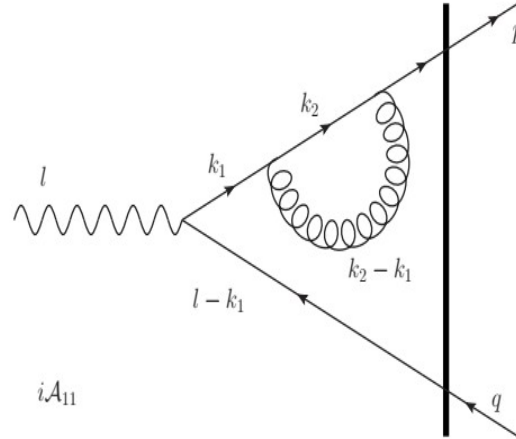
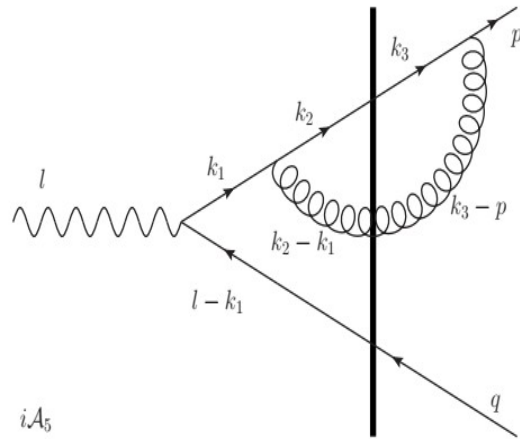
# one loop corrections - real diagrams



3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans  
PLB 761 (2016) 229 and NPB 920 (2017) 232



# One loop corrections – virtual diagrams



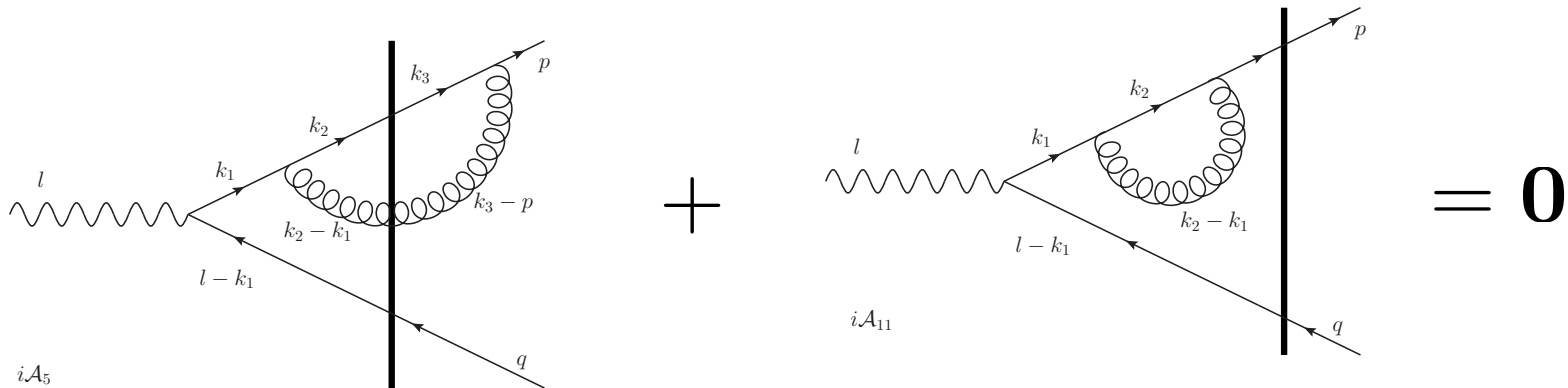
F. Bergabo and JJM, dihadrons, 2207.03606  
 P. Taelis et al., dijets, 2204.11650  
 P. Caucal et al., dijets, 2108.06347

# *divergences*

- *Ultraviolet:*

Real corrections are UV finite

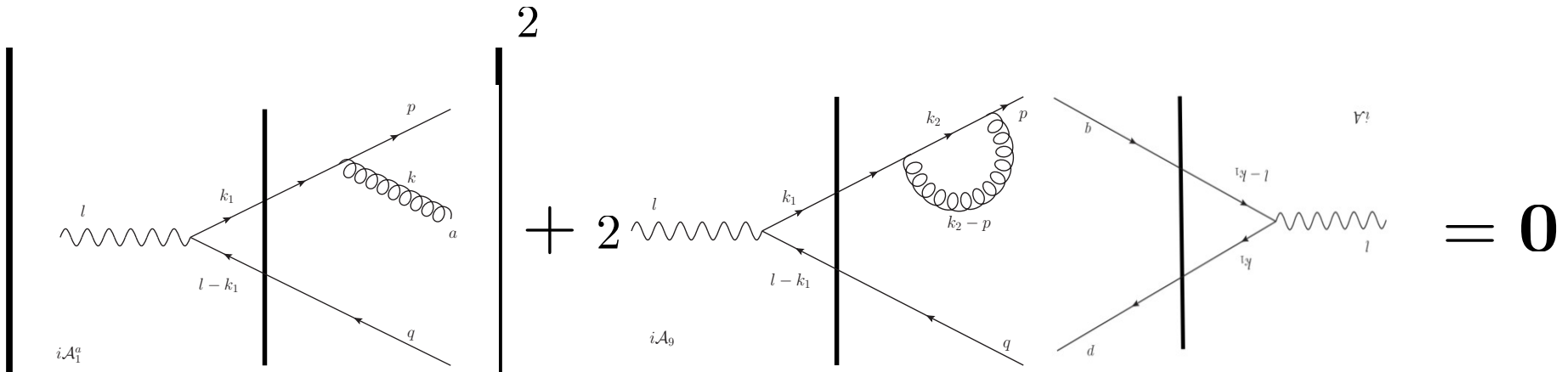
UV divergences cancel among virtual corrections



# *divergences*

- **Soft:**

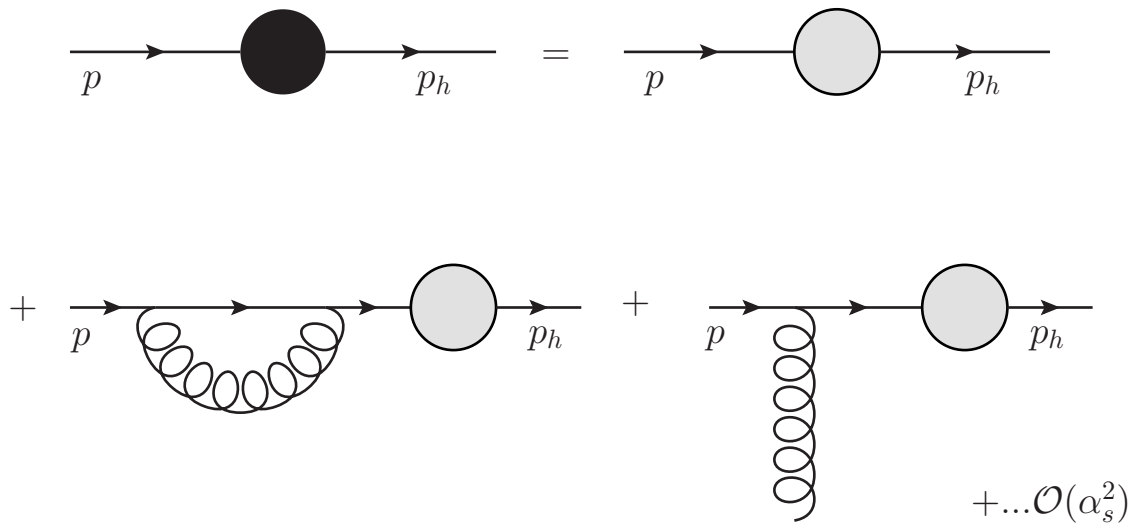
Soft divergences cancel between real and virtual corrections



# *divergences*

- Collinear:**

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



$$D_{h_1/q}(z_{h_1}, \mu^2) = \int_{z_{h_1}}^1 \frac{d\xi}{\xi} D_{h_1/q}^0\left(\frac{z_{h_1}}{\xi}\right) \left[ \delta(1 - \xi) + \frac{\alpha_s}{2\pi} P_{qq}(\xi) \log\left(\frac{\mu^2}{\Lambda^2}\right) \right]$$

# *divergences*

- **Rapidity:**

Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma_{\text{NLO}}^L}{d^2\mathbf{p} d^2\mathbf{q} dy_1 y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}} \delta(1 - z_1 - z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x} K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$

$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \left\{ \begin{aligned} & \left( \tilde{\Delta}_{12} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{12'} \right) S_{132'1'} S_{23} + \left( \tilde{\Delta}_{1'2'} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{21'} \right) S_{1'321} S_{2'3} \\ & + \left( \tilde{\Delta}_{12} + \tilde{\Delta}_{11'} - \tilde{\Delta}_{21'} \right) S_{322'1'} S_{13} + \left( \tilde{\Delta}_{1'2'} + \tilde{\Delta}_{11'} - \tilde{\Delta}_{12'} \right) S_{32'21} S_{1'3} \\ & - \left( \tilde{\Delta}_{11'} + \tilde{\Delta}_{22'} + \tilde{\Delta}_{12} + \tilde{\Delta}_{1'2'} \right) S_{122'1'} - \left( \tilde{\Delta}_{12} + \tilde{\Delta}_{1'2'} - \tilde{\Delta}_{12'} - \tilde{\Delta}_{21'} \right) S_{12} S_{1'2'} \\ & - \left( \tilde{\Delta}_{11'} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{12'} - \tilde{\Delta}_{21'} \right) S_{11'} S_{22'} - 2\tilde{\Delta}_{12} (S_{13} S_{23} - S_{12}) - 2\tilde{\Delta}_{1'2'} (S_{1'3} S_{2'3} - S_{1'2'}) \end{aligned} \right\}$$

JIMWLK evolution of quadrupoles

JIMWLK evolution of dipoles

# *Cancellation of divergences*

## • *Ultraviolet:*

Real corrections are UV finite

UV divergences cancel among virtual corrections

## • *Soft:*

Soft divergences cancel between real and virtual corrections

## • *Collinear*

Collinear divergences are absorbed into hadron fragmentation functions

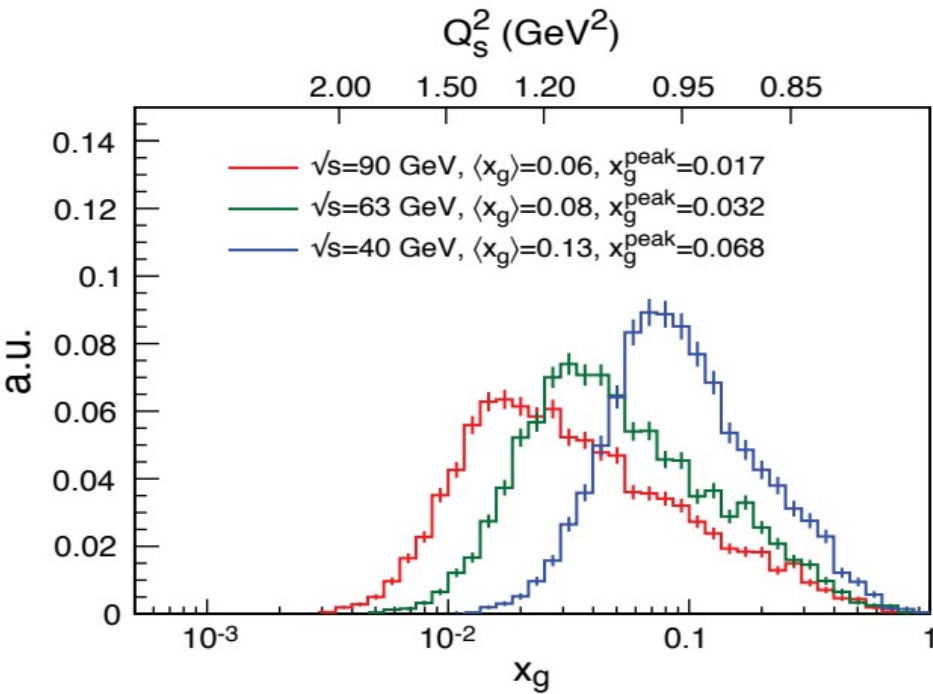
## • *Rapidity*

rapidity divergences are absorbed into JIMWLK evolution of dipoles, quadrupoles

$$\sigma^{\gamma^* A \rightarrow h_1 h_2 X} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h_1/q}(z_1, \mu^2) D_{h_2/\bar{q}}(z_2, \mu^2) + \sigma_{NLO}^{\text{finite}}$$

# EIC

## kinematics of double inclusive hadron production



Aschenauer et al. arXiv:1708.01527

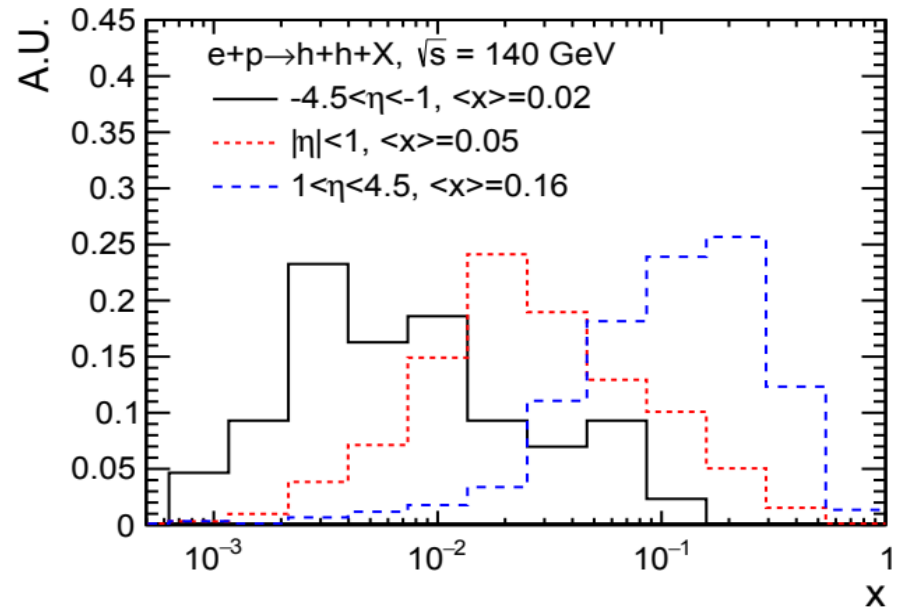


Fig. courtesy of Xiaoxuan Chu

Talk by E. Aschenauer today

# NLO corrections to single inclusive hadron production in DIS at small $x$

Larger kinematic phase space at EIC

No Sudakov

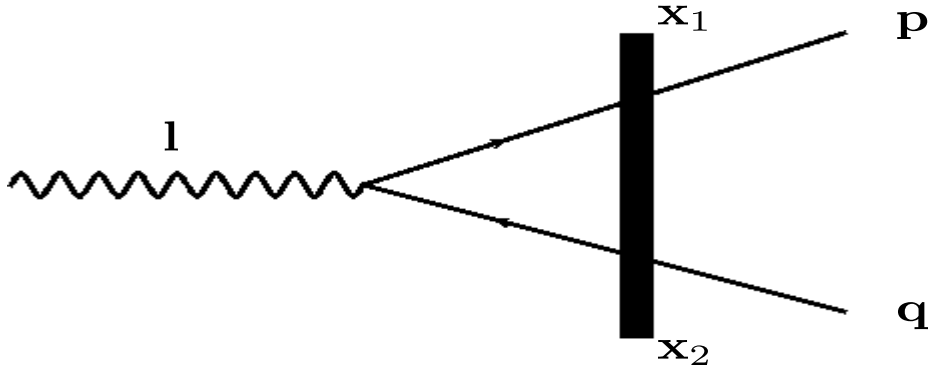
Dipoles only

Forward rapidity: quark or antiquark production

**F. Bergabo and JJM: arXiv:2210.03208**



# LO: antiquark production in DIS at small x



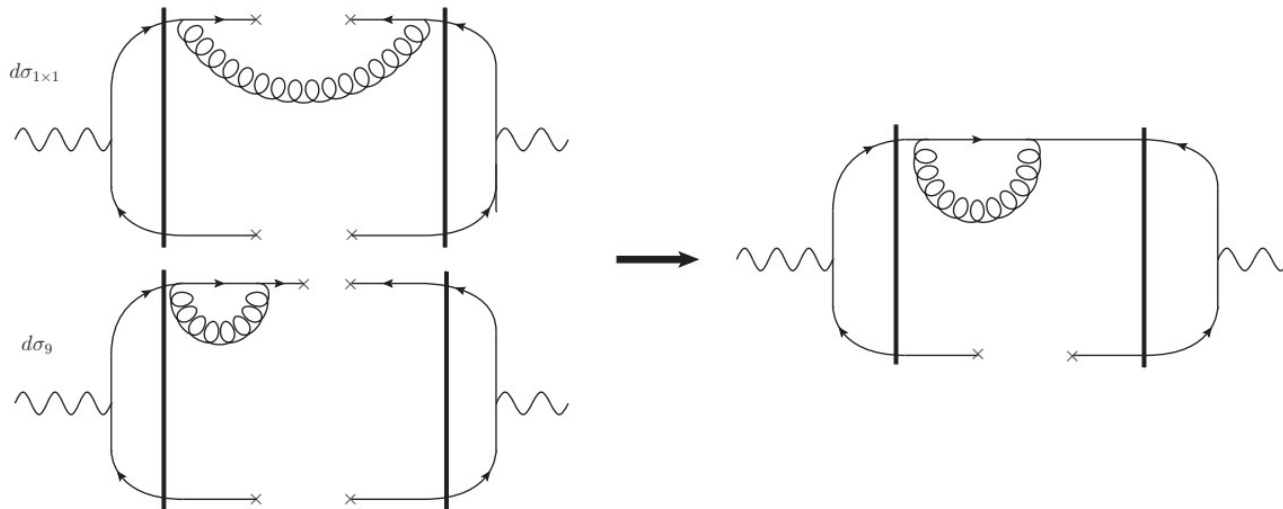
integrate out quark

$$\begin{aligned}
 \frac{d\sigma^{\gamma^* A \rightarrow \bar{q} X}}{d^2\mathbf{q} dy_2} &= \frac{e^2 Q^2 z_2^2 (1 - z_2) N_c}{(2\pi)^5} \int d^6\mathbf{x} [S_{22'} - S_{12} - S_{12'} + 1] \\
 &e^{i\mathbf{q} \cdot (\mathbf{x}'_2 - \mathbf{x}_2)} \left[ 4z_2(1 - z_2) K_0(|\mathbf{x}_{12}| Q_2) K_0(|\mathbf{x}_{12'}| Q_2) + \right. \\
 &\left. [z_2^2 + (1 - z_2)^2] \frac{\mathbf{x}_{12} \cdot \mathbf{x}_{12'}}{|\mathbf{x}_{12}| |\mathbf{x}_{12'}|} K_1(|\mathbf{x}_{12}| Q_2) K_1(|\mathbf{x}_{12'}| Q_2) \right]
 \end{aligned}$$

# NLO corrections to single inclusive hadron production in DIS at small $x$

start with NLO corrections to dihadron production and integrate out quark

cancellations among diagrams



# NLO corrections to single inclusive hadron production in DIS at small $x$

all terms with quadrupoles cancel; only dipoles contribute to the cross section

cancellations of divergences as before

$$\sigma^{\gamma^* A \rightarrow hX} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h/\bar{q}}(z_h, \mu^2) + \sigma_{NLO}^{\text{finite}}$$

phenomenology: need to consider hadronization of any of the 3 partons

relation to TMD,...

# ***SUMMARY***

***CGC is a systematic approach to high energy collisions***

***strong hints from RHIC, LHC,...***

***to be probed extensively at EIC***

***toward precision: NLO, sub-eikonal corrections, ...***

***structure functions***

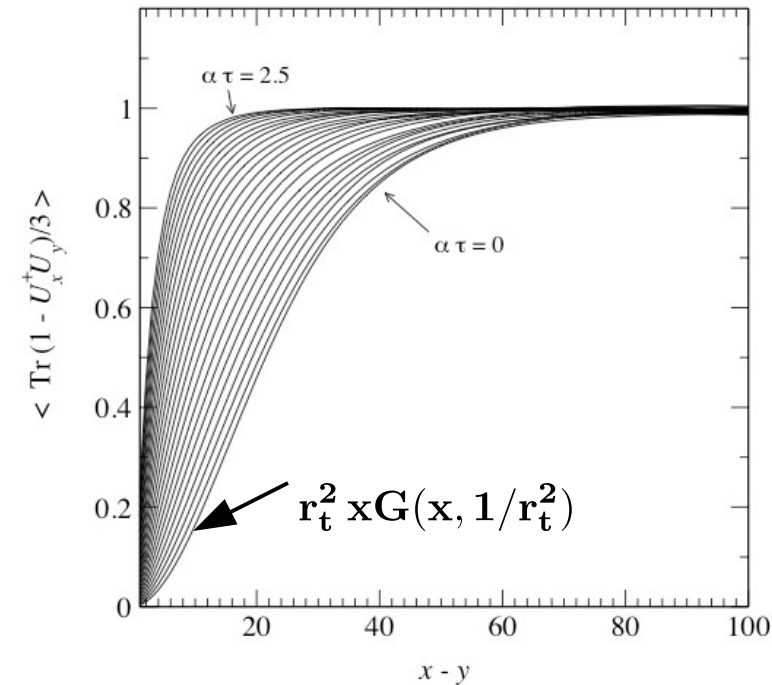
***SIDIS***

***dihadron/dijet production***

# Dipoles at large $N_c$ : BK equation

$$\frac{d}{dy} \mathbf{T}(\mathbf{x}_t - \mathbf{y}_t) = \frac{\bar{\alpha}_s}{2\pi} \int d^2 \mathbf{z}_t \frac{(\mathbf{x}_t - \mathbf{y}_t)^2}{(\mathbf{x}_t - \mathbf{z}_t)^2 (\mathbf{y}_t - \mathbf{z}_t)^2} [\mathbf{T}(\mathbf{x}_t - \mathbf{z}_t) + \mathbf{T}(\mathbf{z}_t - \mathbf{y}_t) - \mathbf{T}(\mathbf{x}_t - \mathbf{y}_t) - \mathbf{T}(\mathbf{x}_t - \mathbf{z}_t) \mathbf{T}(\mathbf{z}_t - \mathbf{y}_t)]$$

$$\mathbf{T}(\mathbf{x}_t, \mathbf{y}_t) \equiv \mathbf{1} - \mathbf{S}(\mathbf{x}_t, \mathbf{y}_t) = \frac{1}{N_c} \text{Tr} \langle \mathbf{1} - \mathbf{V}(\mathbf{x}_t) \mathbf{V}^\dagger(\mathbf{y}_t) \rangle$$



$$\tilde{\mathbf{T}}(\mathbf{p}_t) \rightarrow \log \left[ \frac{Q_s^2}{p_t^2} \right]$$

**saturation region**

$$\tilde{\mathbf{T}}(\mathbf{p}_t) \rightarrow \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right]^\gamma$$

**extended scaling region**

$$\tilde{\mathbf{T}}(\mathbf{p}_t) \rightarrow \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right]$$

**pQCD region**

*Rummukainen-Weigert, NPA739 (2004) 183*

*NLO: Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)*