

Transverse momentum broadening from NLL BFKL to all orders in pQCD

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Resummation Evolution Factorization 2022

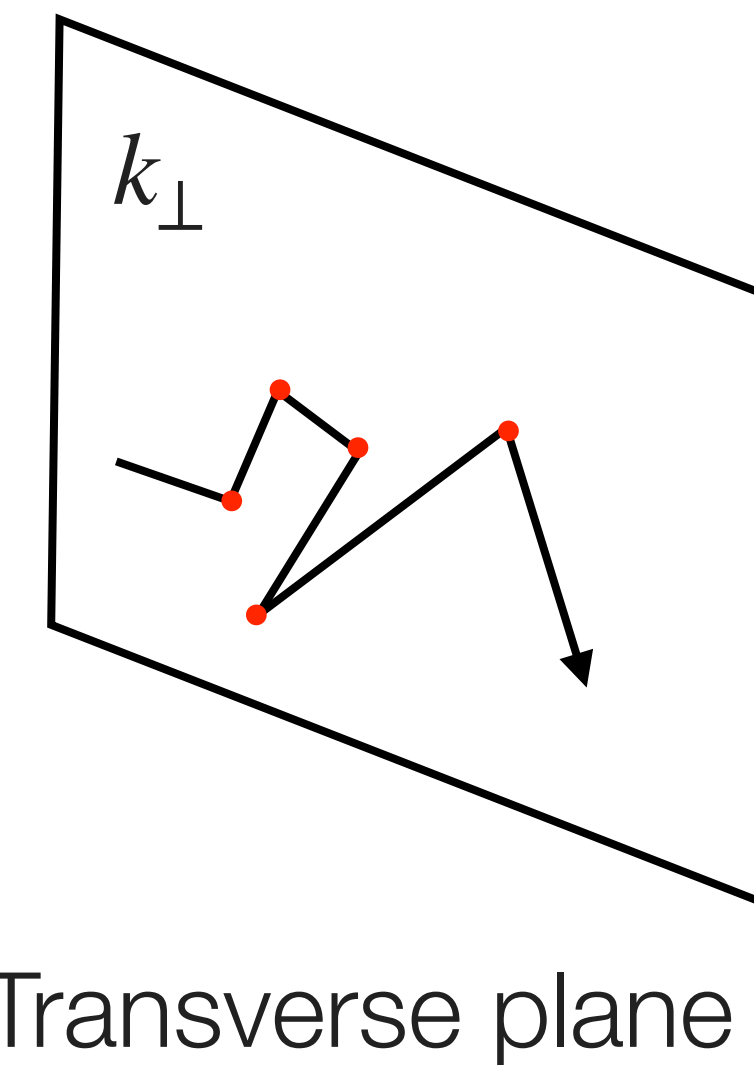
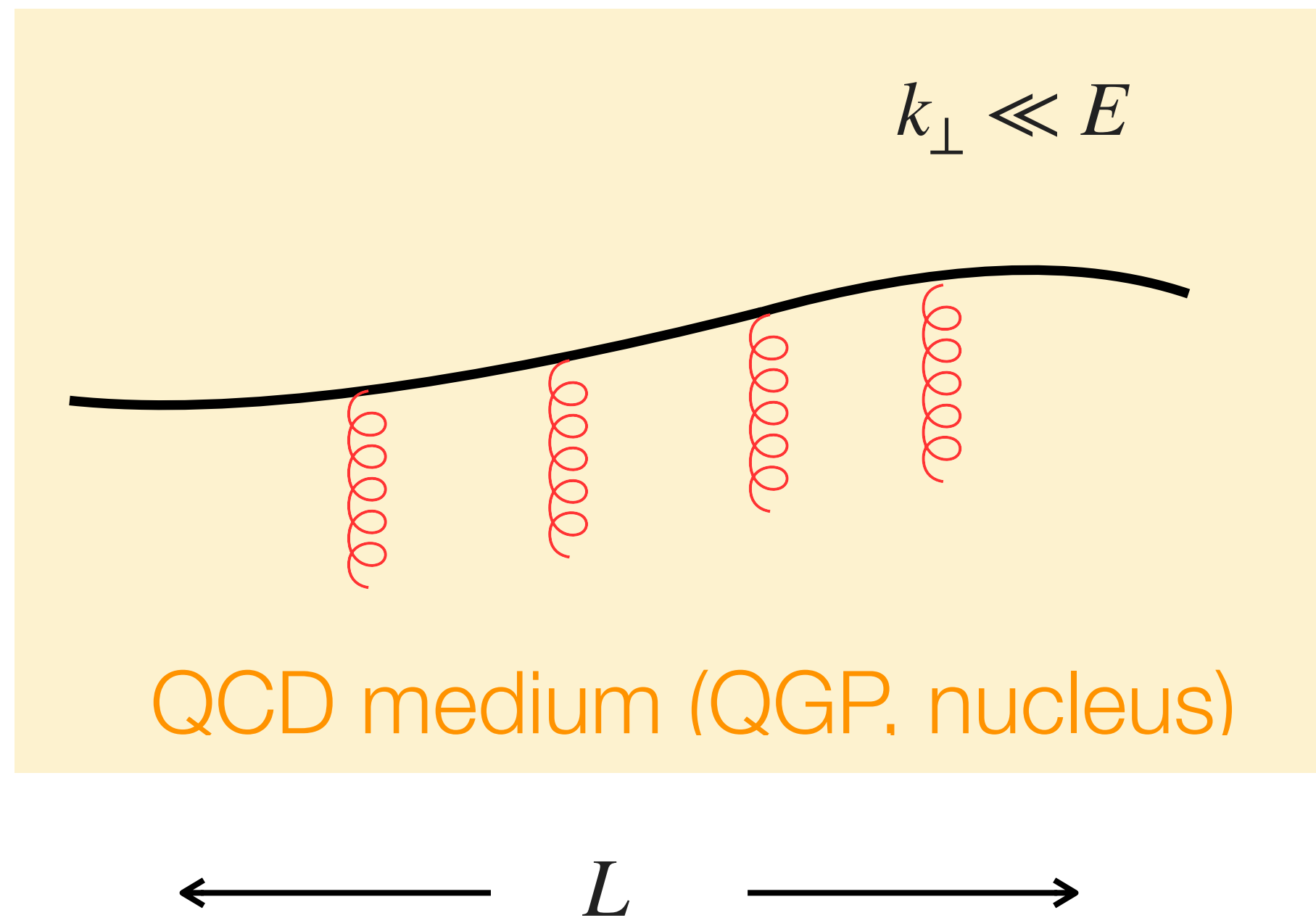
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In collaboration with Paul Caucal

2109.12041 [hep-ph] 2203.09407 [hep-ph] 2209.08900 [hep-ph]

Transverse momentum broadening (TMB)

- High energy partons experience random kicks in hot or cold nuclear matter that cause their transverse momentum to increase over time



- Normal diffusion scaling at LO (from 2 to 2 matrix element)

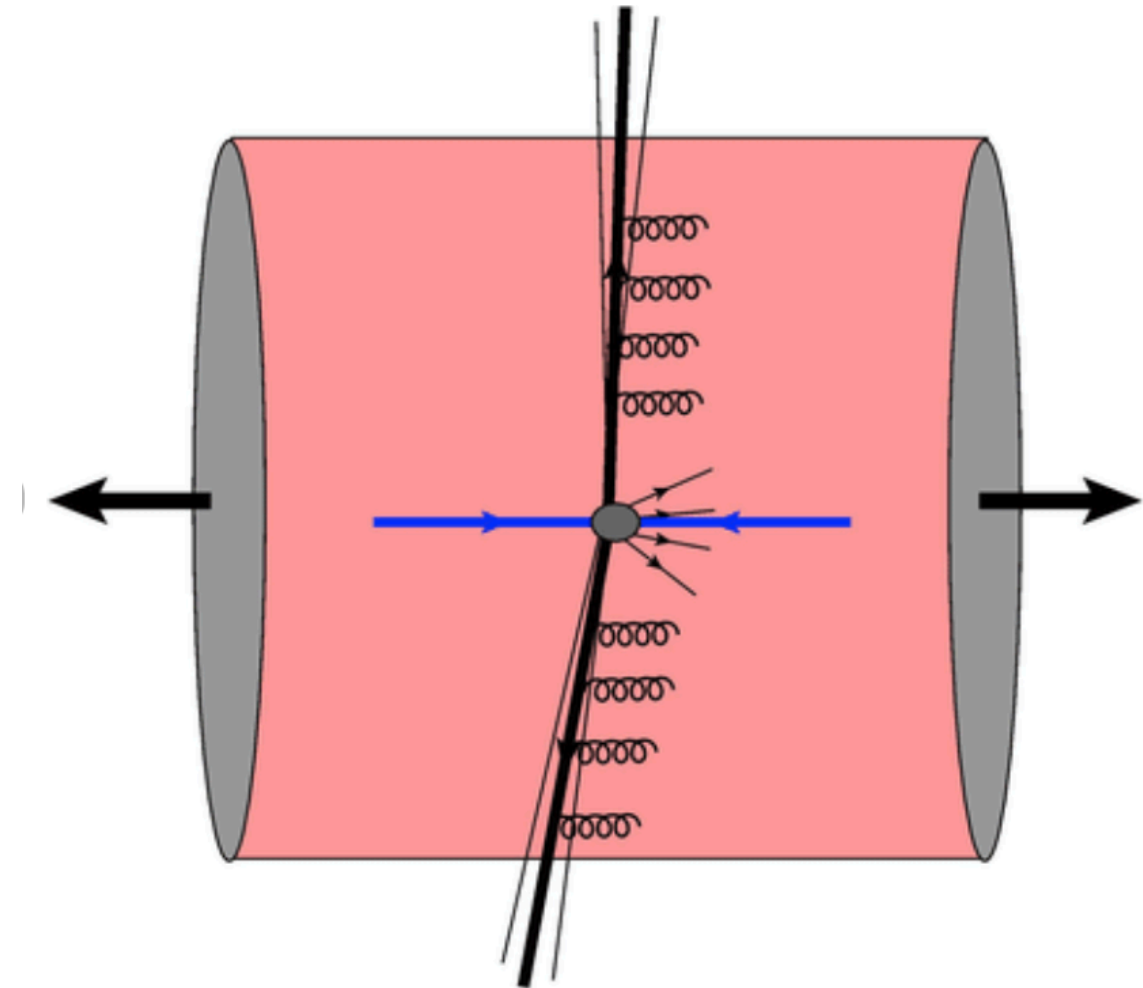
$$\langle k_{\perp}^2 \rangle_{\text{typ}} \propto \hat{q} t$$

Q: What are the effects of quantum corrections on transverse momentum broadening?

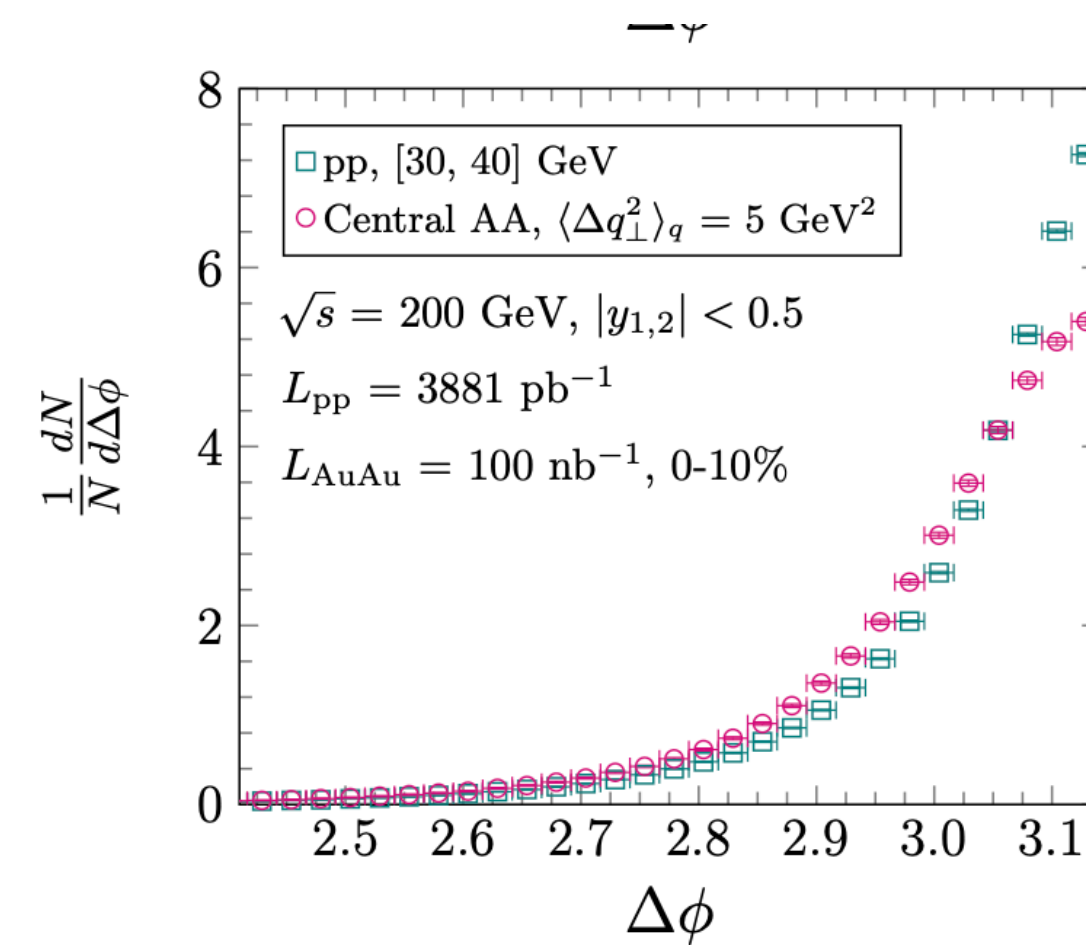
Transverse momentum broadening (TMB)

- Probe the QGP in Heavy Ion Collisions: [dijet azimuthal de-correlation](#), [jet quenching](#)

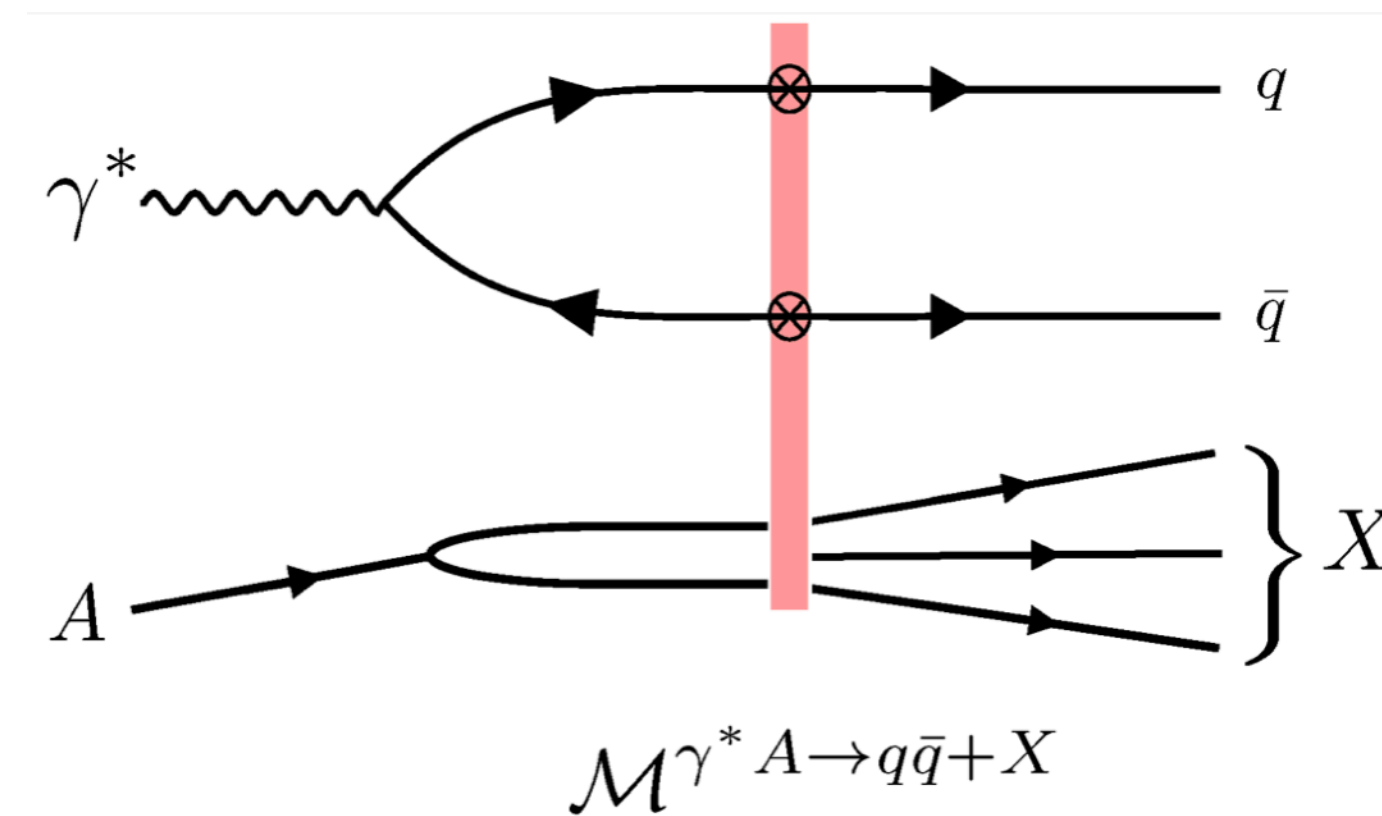
Mueller, Wu, Xiao, Yuan (2016)



Jia, Xiao, Yuan (2019)



- Probe cold nuclear matter: SIDIS and forward dijet production in eA



TMB and Wilson lines

- TMB is related to the scattering of color dipole off a **strong background field** A^μ

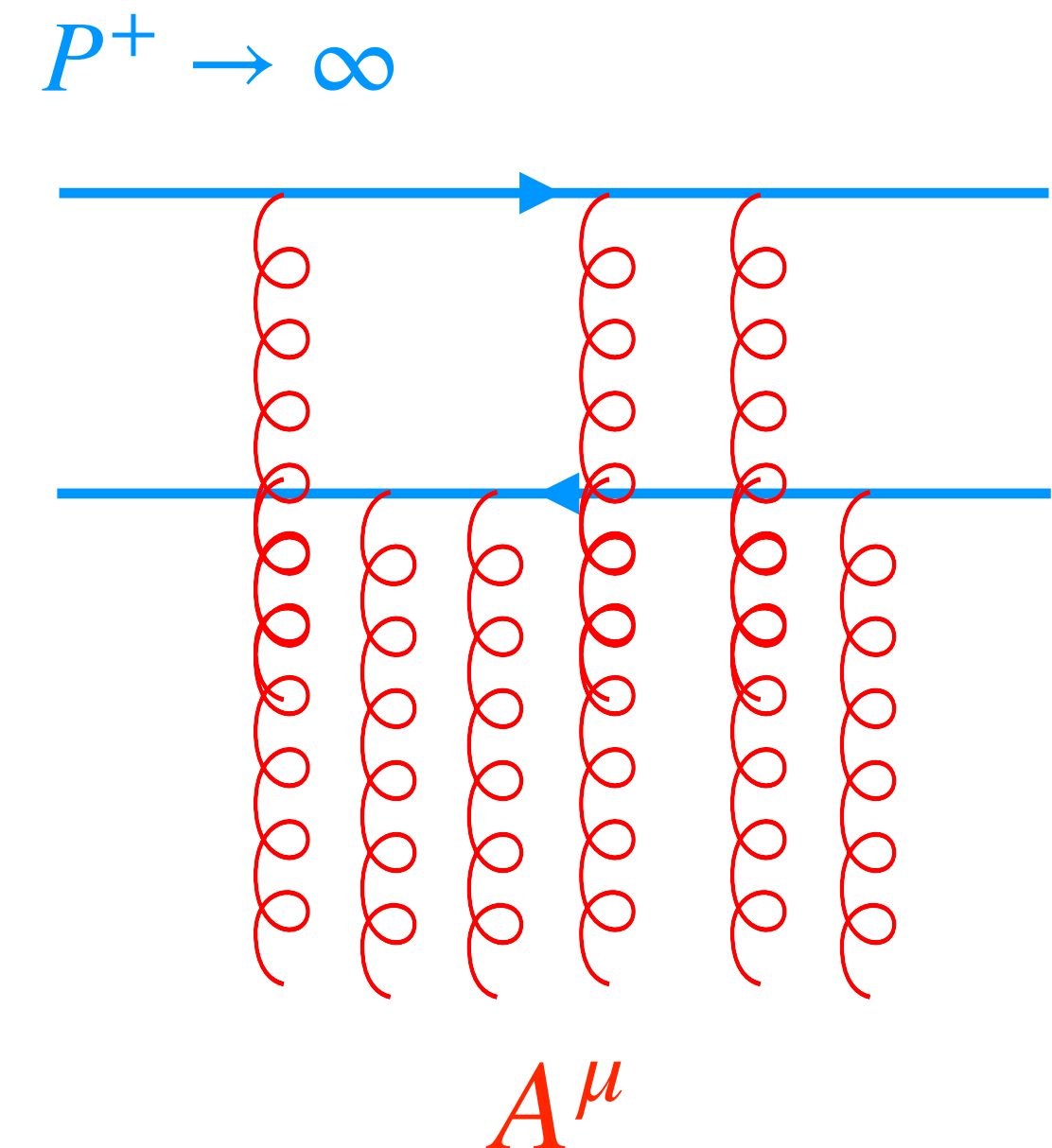
$$\mathcal{P}(\mathbf{k}_\perp) \equiv \frac{dN}{d^2\mathbf{k}_\perp} = \int d^2\mathbf{x}_\perp e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} S(\mathbf{x}_\perp).$$

- Dipole S-matrix

$$S(\mathbf{x}_\perp) \equiv \frac{1}{N_c} \text{Tr} \langle U(\mathbf{x}_\perp) U^\dagger(\mathbf{0}) \rangle,$$

Path ordered exponential:

$$U(\mathbf{x}_\perp) \equiv \mathcal{P} \exp \left[ig \int_{-\infty}^{+\infty} dx^+ t^a A_a^-(x^+, \mathbf{x}_\perp) \right]$$



Relation to \hat{q}

$$S(x_\perp) = e^{-\frac{1}{4}x_\perp^2 L \hat{q}(1/x_\perp, L)}$$

Quantum corrections to \hat{q}

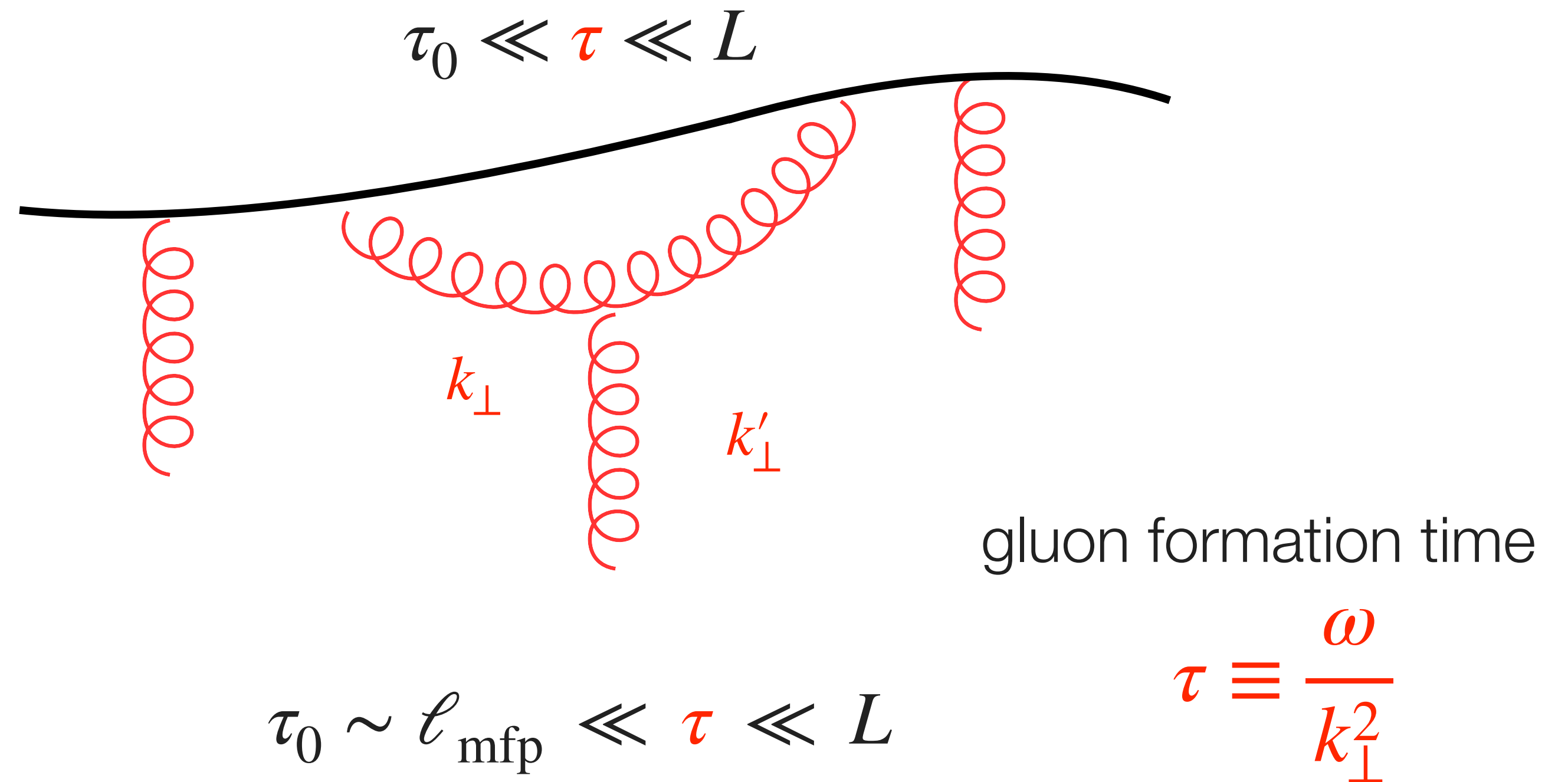
- Potentially large double logs (DL) in transverse momentum broadening at NLO

$$\text{NLO} \sim \bar{\alpha} \int \frac{dk_{\perp}^2}{k_{\perp}^2} \int \frac{d\tau}{\tau}$$

[Liou, Mueller, Wu (2013)]

[Blaizot, Dominguez, Iancu, MT (2014)]

$$\langle k_{\perp}^2 \rangle = \hat{q}_0 L \left(1 + \frac{\bar{\alpha}}{2} \log^2 \frac{L}{\tau_0} \right)$$



- Not the standard DGLAP double log: the factor $1/2$ reflects the presence of multiple scattering constraint $k_{\perp} > \hat{q} \tau \equiv Q_s^2$ (saturation boundary)

Geometric scaling and superdiffusion

- **Traveling waves solutions:** Derive sub-asymptotic behavior. We follow Brunet and Derrida's (1988) and U. Ebert and W. van Saarloos (2000) approaches to FKPP equation (Fisher-Kolmogorov-Petrovsky, Piskunov) - (population growth, wave propagation, etc)
- First application to QCD in small x (Balitsky-Kovchebov equation) by Munier and Peschanski (2003)
- The all order results was first derived in the DL approximation

Asymptotically: Lévy distribution

$$S(r_{\perp}, L) \rightarrow e^{-(r_{\perp}^2 Q_s^2(L))^{1-2\sqrt{\alpha}}}$$

↪ non-Gaussian

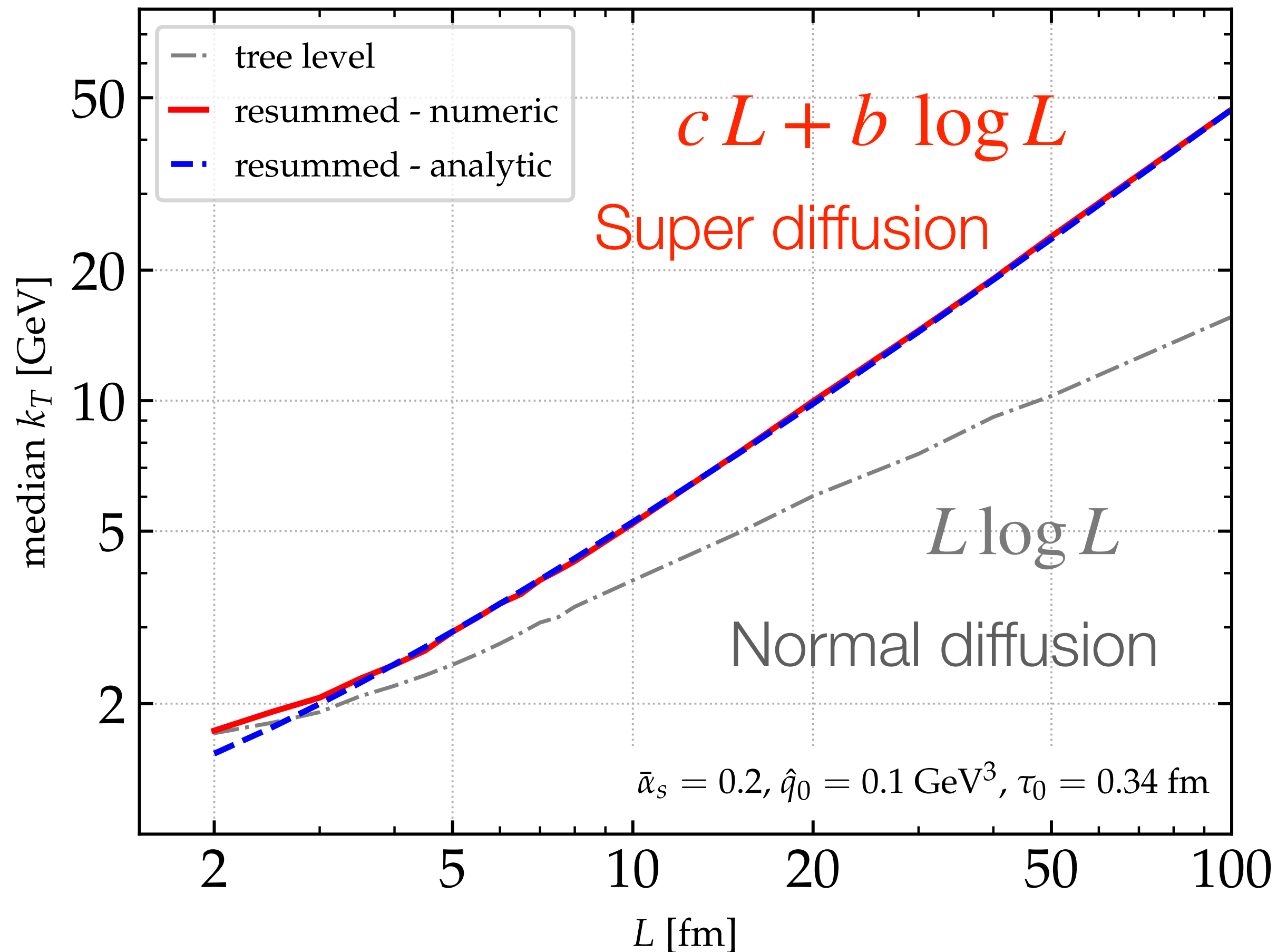
Anomalous scaling: super diffusive process

$$Q_s^2(L) \propto L^{1+2\sqrt{\alpha}}$$

Time dependence of the typical transverse momentum

$$\rho_s(Y) = \log Q_s^2(Y) = cY + b \log Y + \text{const.}$$

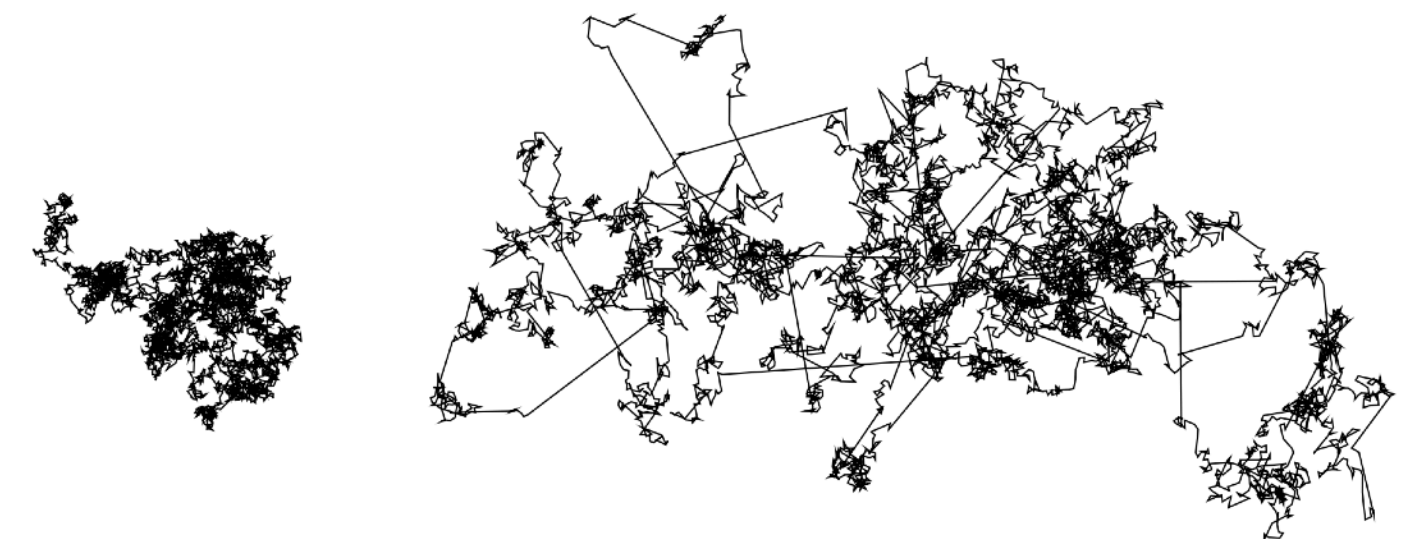
$$Y \equiv \log \frac{L}{\tau_0} \quad c \simeq 1 + 2\sqrt{\bar{\alpha}}$$



Nonlocal quantum corrections: **anomalous system size dependence (super diffusion)**

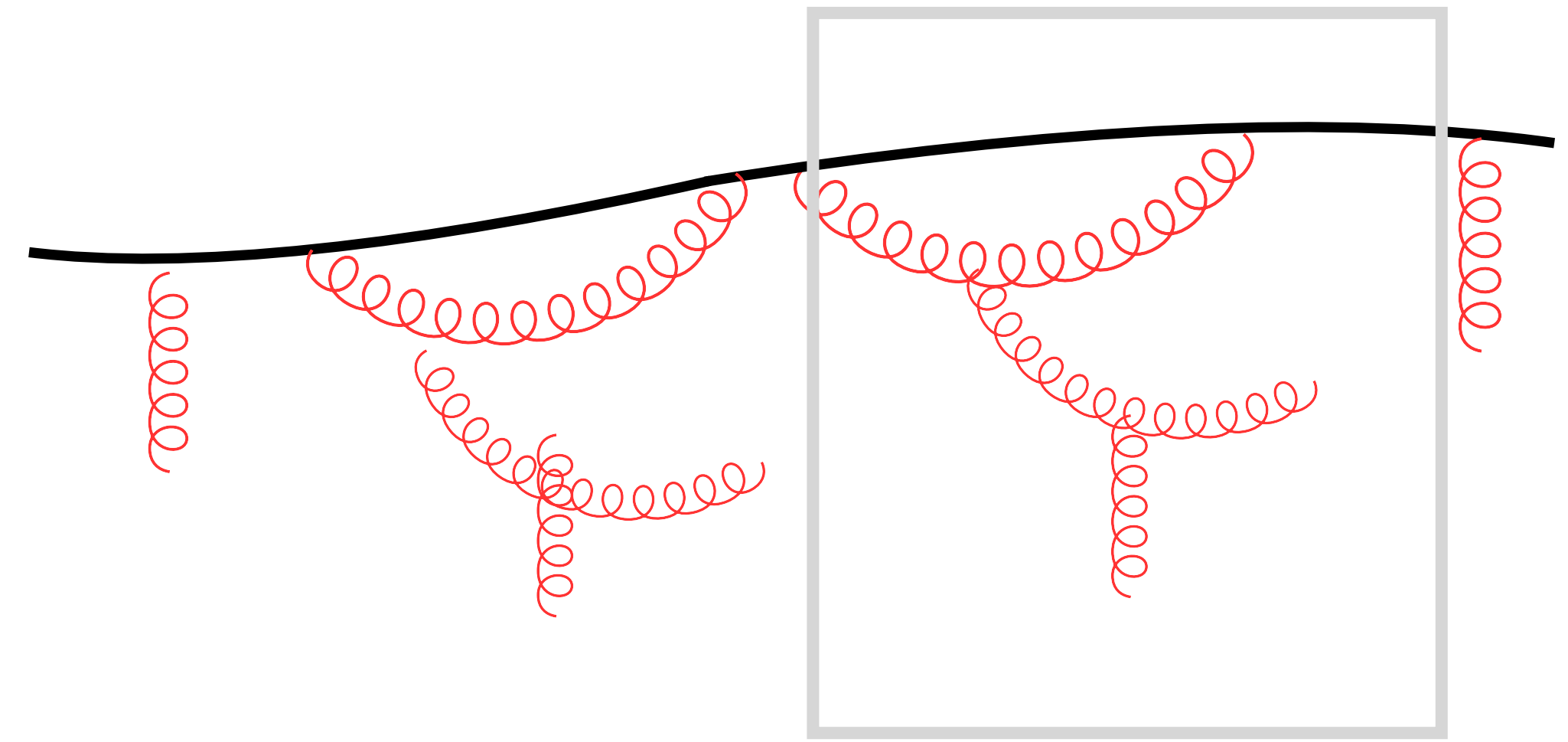
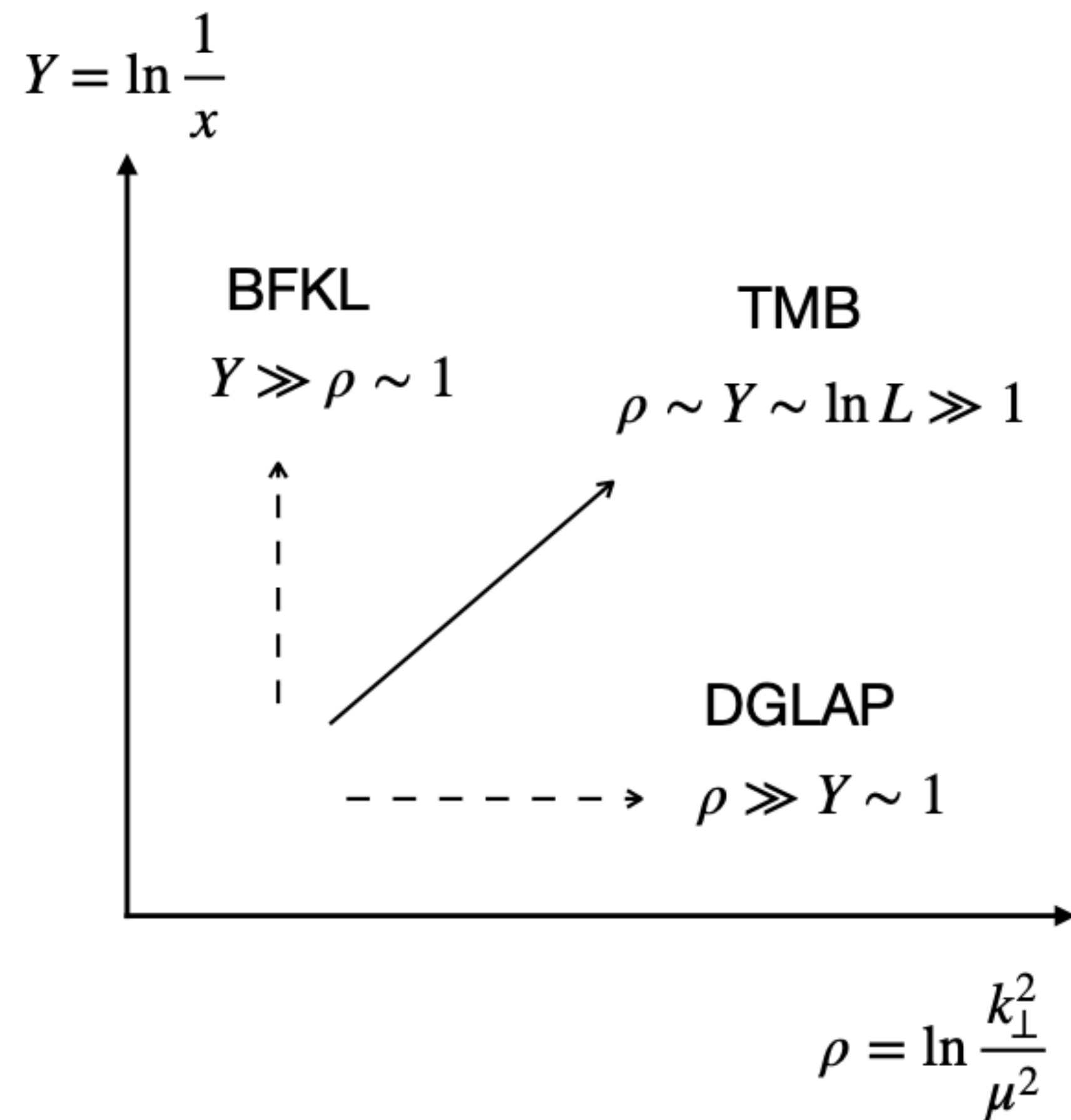
$$Q_s^2 \simeq \langle k_{\perp}^2 \rangle_{\text{median}} \propto L^{1+2\sqrt{\bar{\alpha}}}$$

Universal behavior observed down to $L \simeq 3 \text{ fm}$



Next to double logarithmic evolution

- Resummation of NDLA from BFKL or DGLAP + saturation boundary



BFKL or DGLAP evolution

$$\frac{d}{dY} \rho_s(Y) \simeq 1 + 2\sqrt{\bar{\alpha}} + \mathcal{O}(\bar{\alpha})$$

DLA NDLA

Next to double logarithmic evolution

- Solve linear NLL-BFKL with saturation boundary $\rho > \rho_s(Y)$ (non-linear effects)

$$\frac{\partial \hat{q}}{\partial Y} = \chi_{\text{LL}}(\partial_\rho) [\bar{\alpha}_s(\rho) \hat{q}(\rho, Y)] + \bar{\alpha}_s^2(\rho) \tilde{\chi}_{\text{NLL}}(\partial_\rho) \hat{q}(\rho, Y) \quad \rho \equiv \ln \frac{k_\perp^2}{\mu^2} \quad Y \equiv \ln \frac{L}{\tau_0}$$

Iancu (2014) MT, Blaizot (2014) Vaidya (2021)

- Saturation condition $\rho > \rho_s(Y) \equiv \ln \frac{Q_s^2(L)}{\mu^2}$
 - N.B. $Q_s^2(L) \equiv \hat{q}(L) L$
- The details of non-linear terms are not relevant to derive the asymptotics of the saturation scale and the behavior of the TMB distribution at $k_\perp > Q_s(L)$ [Mueller, Triantafyllopoulos (2002) (Iancu, Itakura, McLerran (2002))]
- Expansion of BFKL kernel around DLA: $\gamma = 0$

$$\chi_{\text{LL}}(\gamma) = \frac{1}{\gamma} + 2\zeta(3)\gamma^2 + \mathcal{O}(\gamma^4),$$

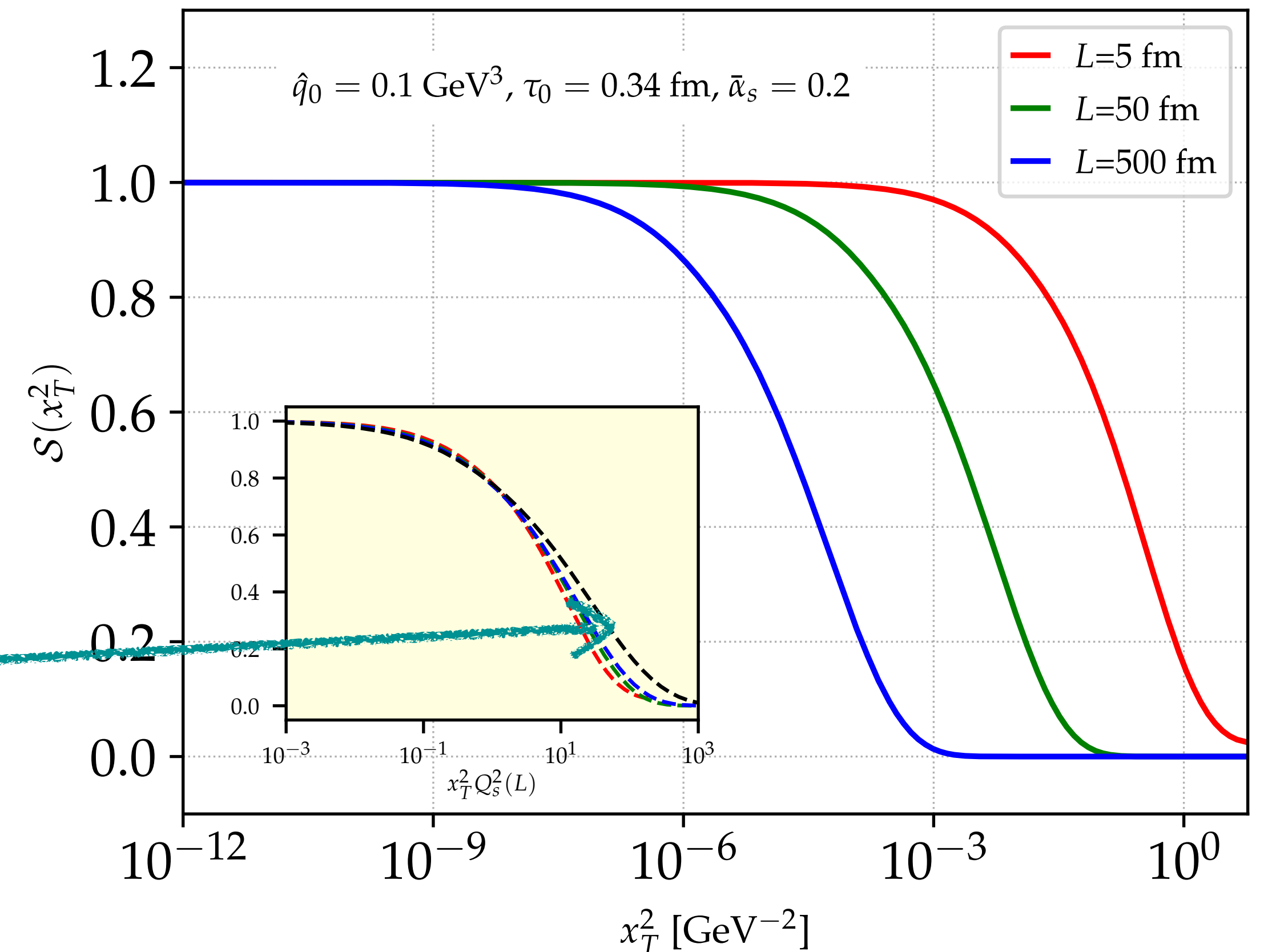
All orders - Geometric scaling

- We look for a solution of the form (solve around the scaling solution for running coupling)

$$\hat{q}(\rho, Y) = e^{\rho_s(Y) - Y} e^{\beta x} f(x, Y), \quad x = \frac{\rho - \rho_s(Y)}{\sqrt{Y}}$$

- To obtain the sub-asymptotic corrections we use the so-called leading-edge expansion

$$f(x, Y) = \sum_{n=-1}^{\infty} Y^{-n/6} G_n \left(\frac{x}{Y^{1/6}} \right)$$



U. Ebert and W. van Saarloos (2000)

Universal terms up to $Y^{-3/2}$ for

$$\begin{aligned} \dot{\rho}_s = & 1 + \frac{4b_0}{\bar{Y}^{1/2}} + \frac{2\xi_1 b_0}{\bar{Y}^{5/6}} + b_0 (1 - 8b_0 + 4b_0 B_g) \frac{1}{\bar{Y}} \\ & - \frac{7\xi_1^2 b_0}{270} \frac{1}{\bar{Y}^{7/6}} - (5 + 1944b_0) \frac{\xi_1 b_0}{81} \frac{1}{\bar{Y}^{4/3}} \\ & - 2b_0^2 (1 - 8b_0 + 4b_0 B_g) \frac{\ln(\bar{Y})}{\bar{Y}^{3/2}} + \mathcal{O}(\bar{Y}^{-3/2}), \end{aligned}$$

$$\bar{Y} = 4b_0 Y$$

$$b_0 = \frac{1}{\beta_0} = -\frac{1}{B_g}$$

$$B_g = -\frac{11}{12} - \frac{N_f}{6N_c^3} \approx -\frac{11}{12}$$

NLL BFKL

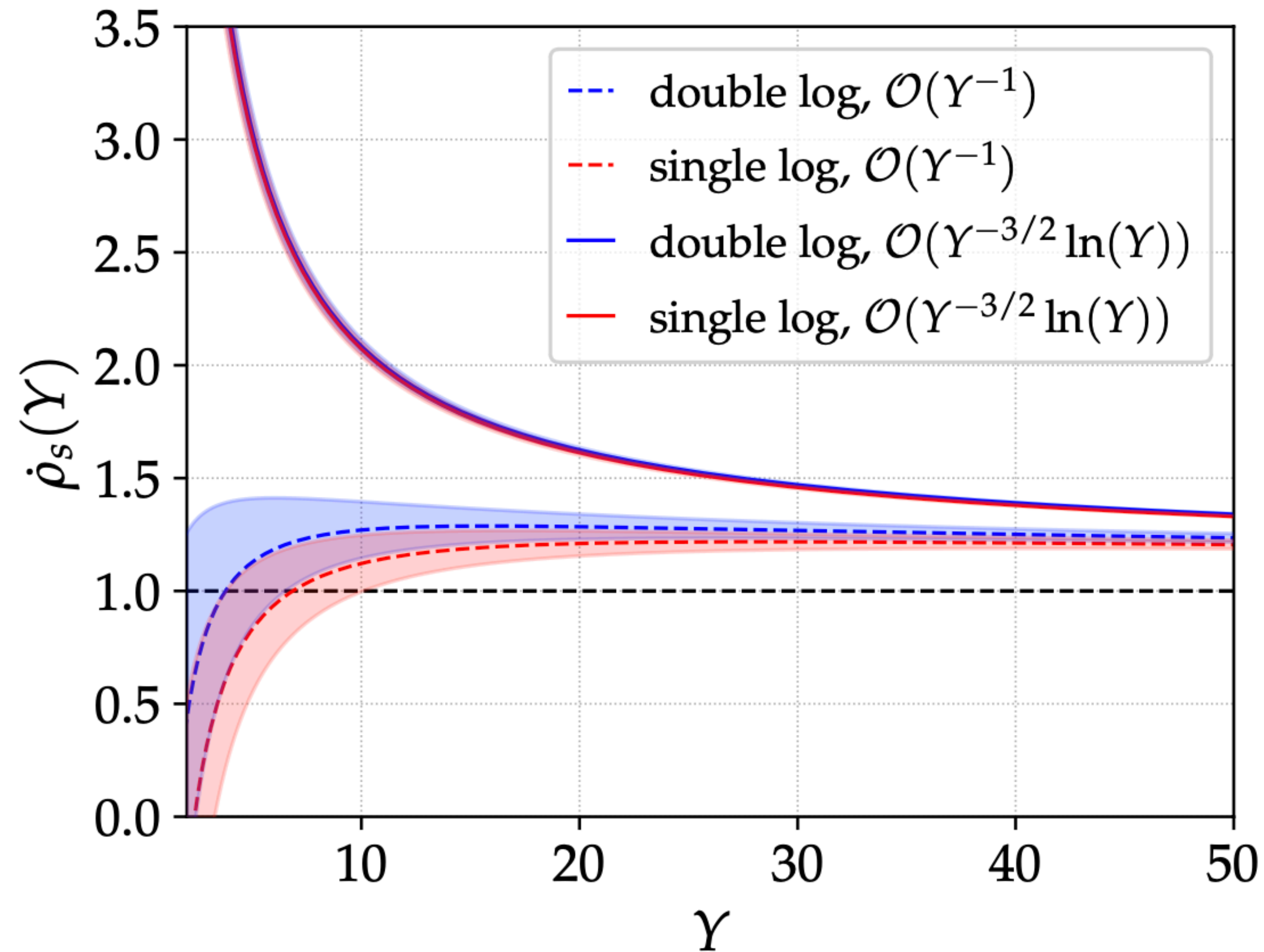
$$\rho_s(Y) \equiv \ln \frac{Q_s^2(L)}{\mu^2} \equiv \ln \frac{\hat{q}(L)L}{\mu^2}$$

MT, Caucal 2209.08900 [hep-ph]

- Non universal terms start at order $Y^{-3/2}$ as can be seen by the substitution $Y \rightarrow Y + Y_0$
- NNLO BFKL and beyond do not contribute to the universal terms

Universal terms up to $Y^{-3/2}$

MT, Caucal 2209.08900 [hep-ph]



Summary

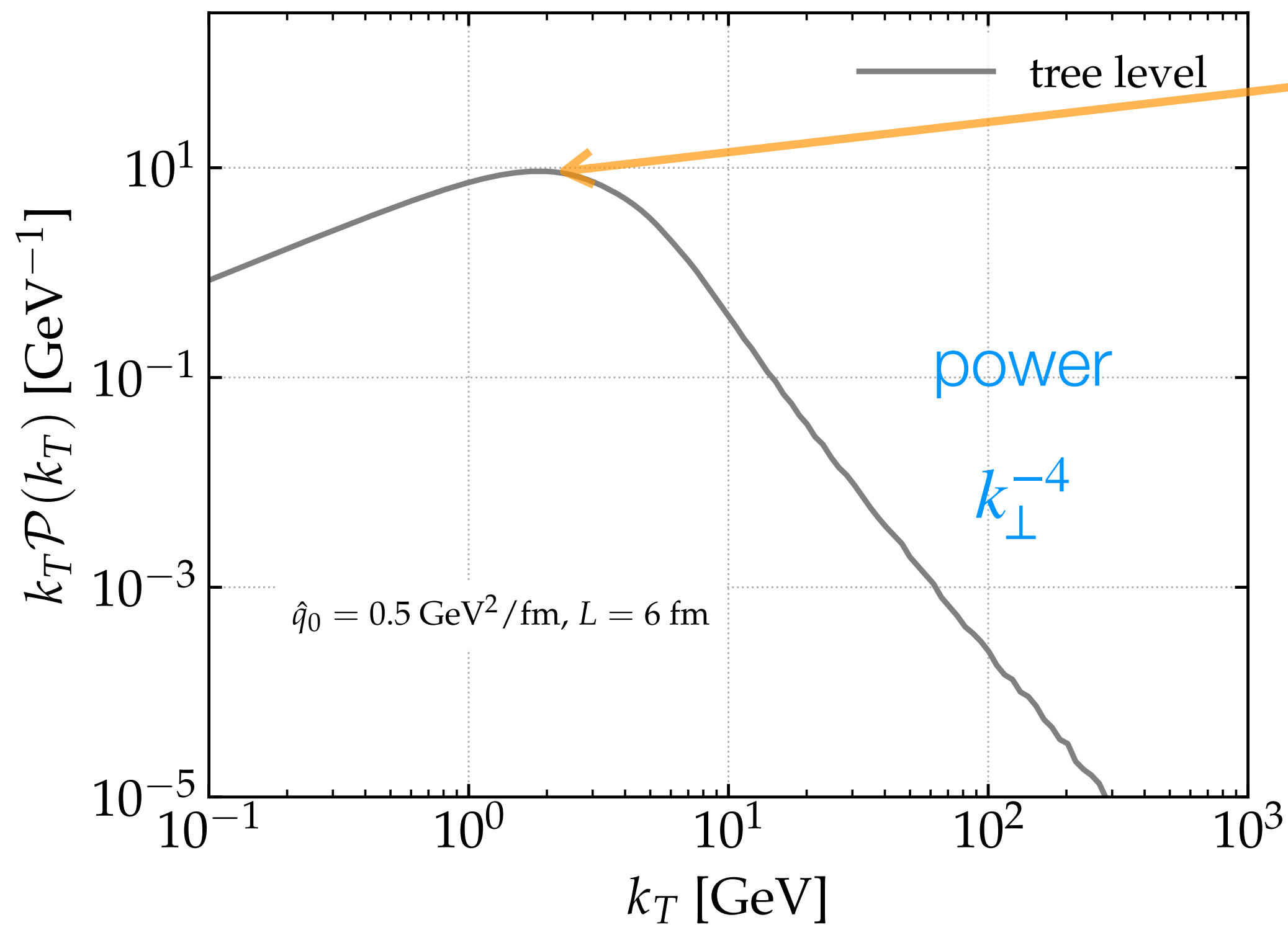
- TMB in QCD is a **super-diffusive process** (non-gaussian) due to logarithmically enhanced **quantum corrections** \longrightarrow **anomalous system size dependence**
- Exhibits **geometric scaling** and heavy tails akin to **Lévy random walks** \longrightarrow
Substantial departure from the LO Molière scattering
- Systematic approach for computing **universal asymptotic** and pre-asymptotic solutions for transverse momentum broadening
- Using DGLAP or **NLL BFKL** (due to the DL nature of the problem) we have **computed all of the universal terms** in the asymptotic expansion of the **saturation scale**

Backup

Tree Level and large media

TMB distribution at leading order - resummation of multiple scattering

Gaussian for $k_{\perp} < Q_s \sim \hat{q}L$ and exhibits the power law tail k_{\perp}^{-4} for $k_{\perp} > Q_s$



multiple scattering

$$P(k_{\perp}) = \frac{4\pi}{\hat{q}L} e^{-\frac{k_{\perp}^2}{\hat{q}L}}$$

Normal diffusion

$$\langle k_{\perp}^2 \rangle_{\text{typ}} \propto L$$

L : Medium size

Diffusion coefficient at tree level

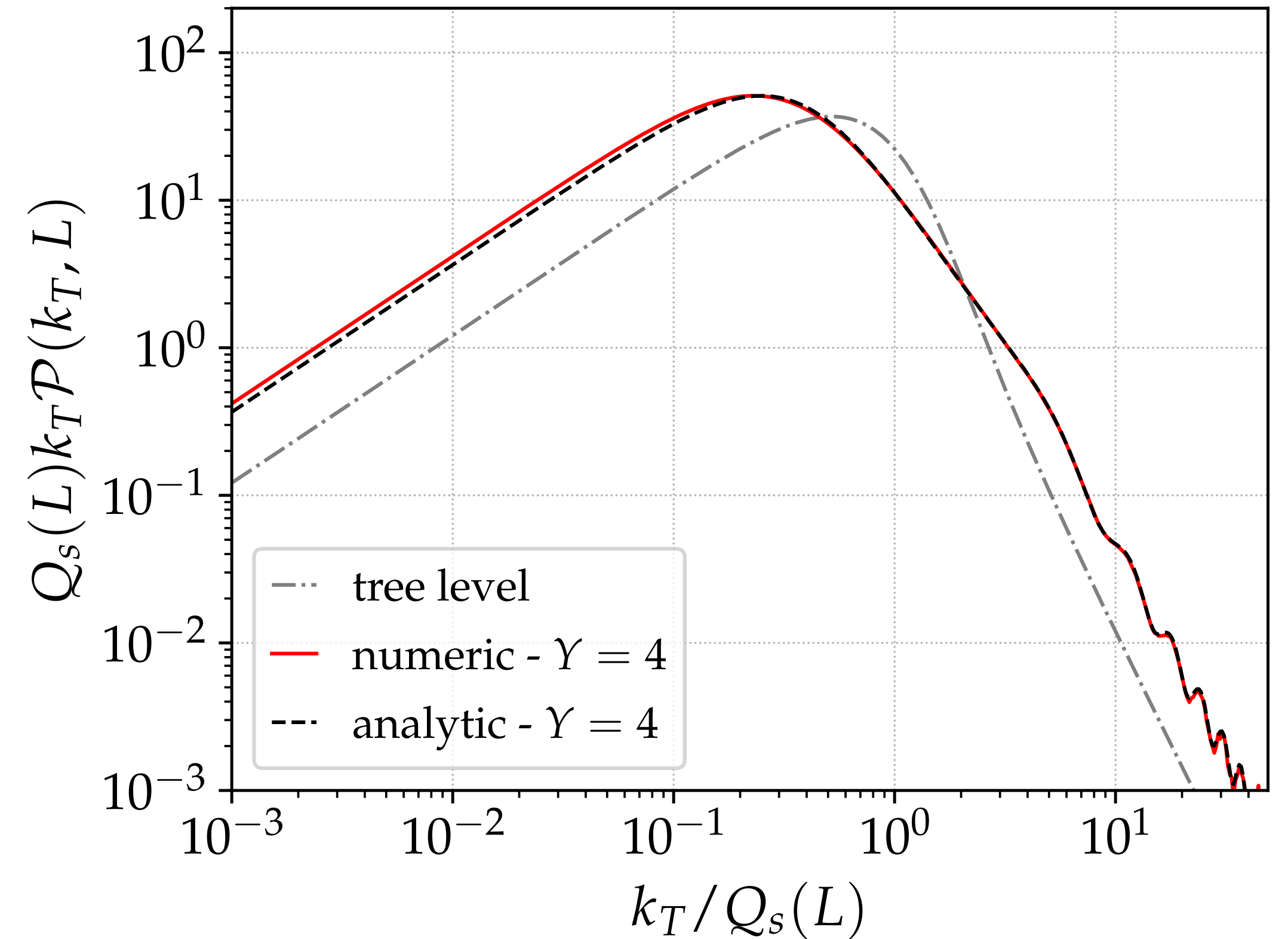
$$\hat{q} = C_R \int_{q_{\perp}} q_{\perp}^2 \frac{d^2\sigma}{d^2q_{\perp}} \simeq 4\pi\alpha_s^2 n \log \frac{q_{\text{max}}}{\mu^2}$$

Running coupling case (vs numerics)

- At large Y (leading contribution):

$$G(\zeta) = \frac{2^{1/3} b_0^{1/6}}{\text{Ai}'(\xi_1)} \text{Ai} \left[\xi_1 + 2^{-1/3} b_0^{1/3} \zeta \right]$$

- Small Y (leading edge expansion diverges). Instead expand around the saturation line



$$\hat{q}_{<}(Y, x) = \hat{q}_0 e^{\rho_s(Y)-Y} \exp \left(\frac{\dot{\rho}_s - 1}{\dot{\rho}_s} x + \frac{1}{2} \frac{\ddot{\rho}_s}{\dot{\rho}_s^3} x^2 + \mathcal{O}(x^3) \right)$$

Systematic approach for universal sub-leading terms

- Fixed coupling

Caucal, MT, 2109.12041 [hep-ph]

$$\rho_s(Y) = cY - \frac{3c}{1+c} \ln(Y) + \kappa - \frac{6c\sqrt{2\pi(c-1)}}{(1+c)^2} \frac{1}{\sqrt{Y}} + \mathcal{O}\left(\frac{1}{Y}\right)$$

- Running coupling

Caucal, MT, 2203.09407 [hep-ph]

$$\begin{aligned} \rho_s(Y) = & Y + 2\sqrt{4b_0Y} + 3\xi_1(4b_0Y)^{1/6} + \left(\frac{1}{4} - 2b_0\right) \ln(Y) + \kappa \\ & + \frac{7\xi_1^2}{180} \frac{1}{(4b_0Y)^{1/6}} + \xi_1 \left(\frac{5}{108} + 18b_0\right) \frac{1}{(4b_0Y)^{1/3}} + b_0(1 - 8b_0) \frac{\ln(Y)}{\sqrt{4b_0Y}} + \mathcal{O}(Y^{-1/2}) \end{aligned}$$

First four terms conjectured by Iancu and Triantafyllopoulos (2015)

Running coupling case

- One-loop running coupling:

$$\bar{\alpha}(k_{\perp}) \simeq \frac{b}{\ln k_{\perp}^2 / \Lambda_{QCD}^2} \quad \text{with} \quad b = \frac{12N_c}{11N_c - 2N_f}$$

- Slower evolution w.r.t. $Y \equiv \log \frac{L}{\tau_0}$

- Modified scaling variable $x \equiv \log \frac{k_{\perp}^2}{Q_s^2} \rightarrow x \equiv \frac{\log \frac{k_{\perp}^2}{Q_s^2}}{\sqrt{Y}} \sim \sqrt{\bar{\alpha}(Y)} \log \frac{k_{\perp}^2}{Q_s^2}$