Gluon uPDF and K_T factorization: From the Higgs boson production at LHC to J/ψ production at EIC

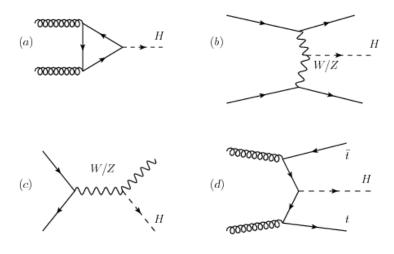
Vaibhav S. Rawoot, Amity University Mumbai, India

Resummation, Evolution and factorization 2022 (REF 2022)

R. Islam, M. Kumar and **VR**, Eur. Phys. J. C **79**, no.3, 181 (2019)

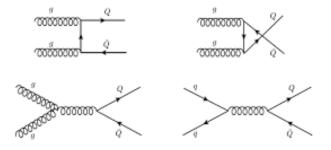


Gluon distribution and the Higgs boson production



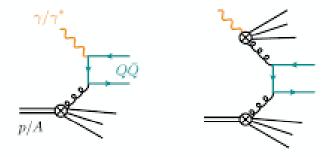
Gluon distributions and the J/ψ production

Hadron-hardon collision (Ex. LHC).



Gluon distributions and the J/ψ production

Electron-proton collision (Ex. EIC).



QCD factorization theorem

► Collinear factorization

$$\sigma(h_1h_2 \to F) = f_{a/h_1}(x_1, Q^2) \otimes f_{b/h_2}(x_2, Q^2) \otimes \hat{\sigma}_{(ab \to F)}(Q^2) + \mathcal{O}(\Lambda/Q)$$

Process dependent partonic cross section

$$\hat{\sigma}(Q^2) = \hat{\sigma}^{(0)} + \alpha_s(Q^2)\hat{\sigma}^{(1)} + \alpha_s^2(Q^2)\hat{\sigma}^{(2)} + \dots LO NLO NNLO$$

- Collinear approximation in parton model and evolution of parton densities discribed by Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation.
- $ightharpoonup k_T$ factorization

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k_T or TMD factorization

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 - S. Catani, F. Fiorani and G. Marchesini, Phys. Lett. B **234**, 339 (1990).
 - S. Catani, F. Fiorani and G. Marchesini, Nucl. Phys. B **336**, 18 (1990). G. Marchesini, Nucl. Phys. B **445**, 49 (1995)
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Heavy quark production

[hep-ph/0007238].

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 - D. de Florian, G. Ferrera, M. Grazzini and D. Tommasini, JHEP **1206**, 132 (2012) [arXiv:1203.6321 [hep-ph]].
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- ► HRes: Fixed order cross sections for SM Higgs boson production up to NNLO by consistently including all-order resummation of soft-gluon effects at small transverse momenta up to NNLL ⇒ NNLO + NNLL
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CASCADE

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· CASCADE1: Manual

CASCADE3

Hadron level Monte Carlo generator for ep and pp scattering applying Transverse Momentum Dependent (TMD) parton densities and parton shower.

The Monte Carlo program CASCADE generates a full hadron event record according to the HEP common standards.

CASCADE was originally intended for small x processes and used only gluon chains in the initial state cascade.

With the development of the Parton Branching TMDs, which are valid over a large range in x and Q2, new developments were possible in CASCADE3:

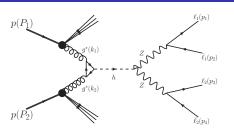
CASCADE3 (apart from the older features) makes use of LHE files (either from collinear NLO calculations like MC@NLO or off-shell calculations from KaTie/Pegasus) and has a full flavor initial state parton shower, which follows directly the TIMD from the Parton Branching method.

Different sets of TMDs are now accessed via TMDlib.

Ref: S. Baranov, A. Bermudez Martinez, L. I. Estevez Banos, F. Guzman, F. Hautmann, H. Jung, A. Lelek, J. Lidrych, A. Lipatov and M. Malyshev, *et al.* "CASCADE3 A Monte Carlo event generator based on TMDs," Eur. Phys. J. C **81**, no.5, 425 (2021)



$pp \to H \to ZZ \to l_1 l_1 l_2 l_2$



with

$$x_1 = \frac{|\boldsymbol{p}_{1T}|}{\sqrt{s}}e^{y_1} + \frac{|\boldsymbol{p}_{2T}|}{\sqrt{s}}e^{y_2} + \frac{|\boldsymbol{p}_{3T}|}{\sqrt{s}}e^{y_3} + \frac{|\boldsymbol{p}_{4T}|}{\sqrt{s}}e^{y_4}$$

and

and
$$x_2=rac{|m{p}_{1T}|}{\sqrt{s}}e^{-y_1}+rac{|m{p}_{2T}|}{\sqrt{s}}e^{-y_2}+rac{|m{p}_{3T}|}{\sqrt{s}}e^{-y_3}+rac{|m{p}_{4T}|}{\sqrt{s}}e^{-y_4}$$

$pp \to H \to ZZ \to l_1 l_2 l_2 l_2$

$$\begin{split} \frac{d\sigma}{dy_1 dy_2 dy_3 dy_4 d\boldsymbol{p}_{1T}^2 d\boldsymbol{p}_{2T}^2 d\boldsymbol{p}_{3T}^2} &= \int d\boldsymbol{k}_{1T}^2 d\boldsymbol{k}_{2T}^2 \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{1}{(2^{12})\pi^5 (x_1 x_2 s)^2} \\ &|\bar{\mathcal{M}}|^2 f_g(x_1, \boldsymbol{k}_{1T}^2) f_g(x_2, \boldsymbol{k}_{2T}^2) \end{split}$$

$$k_{1T} + k_{2T} = p_{1T} + p_{2T} + p_{3T} + p_{4T}$$



$\mathcal{M}(g * g * \to H \to ZZ \to 4l)$

$$\mathcal{M}(g*g*\to H\to ZZ\to 4l) = \mathcal{M}(g*g*\to H)\frac{1}{\hat{s}-m_H^2+i\Gamma_H m_H}\mathcal{M}(H\to ZZ\to l_1\,\bar{l}_1)$$

$$|\mathcal{M}|^{2} = \frac{2}{9} \frac{\alpha_{s}^{2}}{\pi^{2}} \frac{m_{Z}^{4}}{v^{4}} \frac{\left[(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2})^{2} + \hat{s} \right]^{2} \cos^{2} \phi}{(\hat{s} - m_{H}^{2})^{2} + \Gamma_{H}^{2} m_{H}^{2}}$$
$$\frac{\left[(p_{1} \cdot p_{4})(p_{2} \cdot p_{3}) \{ 2g_{L}^{2}g_{R}^{2} \} + (p_{1} \cdot p_{3})(p_{2} \cdot p_{4}) \{ g_{L}^{4} + g_{R}^{4} \} \right]}{\left[(2p_{1} \cdot p_{2} - m_{Z}^{2})^{2} + \Gamma_{Z}^{2} m_{Z}^{2} \right] \left[(2p_{3} \cdot p_{4} - m_{Z}^{2})^{2} + \Gamma_{Z}^{2} m_{Z}^{2} \right]}$$

$$g_L = \frac{g_W}{\cos\theta_W} \left(-\frac{1}{2} + \sin^2\theta_W \right), \quad g_R = \frac{g_W}{\cos\theta_W} \sin^2\theta_W, \quad \text{and} \quad v = (\sqrt{2}G_F)^{-1/2}$$



ATLAS data for $pp \to H \to 4\, leptons$ and k_T factorization approach

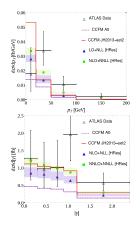


Figure: ATLAS, $\sqrt{s}=8~{\rm TeV}_{\rm color}$

ATLAS data for $pp \to H \to 4\, leptons$ and k_T factorization approach

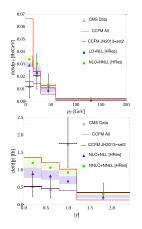


Figure: CMS, $\sqrt{s}=8~{\rm TeV}_{\rm color}$

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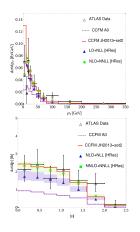


Figure: CMS, $\sqrt{s}=13~{\rm TeV}_{\rm color}$

Models for J/ψ Production

- ► Color Singlet Model (CSM).
- ► Color Evaporation Model (CEM).
- ► NRQCD factorization approach.
- ► Improved Color Evaporation Model (ICEM).



Color Singlet Model (CSM)

- Proposed shortly after the discovery of the J/ψ . [Einhorn and Ellis (1975)]
- ightharpoonup Qar Q pair is formed in the short-distance process in color-singlet state and has the same spin and angular momentum quantum numbers as the quarkonium.
- ightharpoonup Color singlet $Q\bar{Q}$ wave function at origin.

$$A(P) = \frac{1}{\sqrt{4\pi}} R_0 Tr[O(P, 0) P_{ss_z}(P, 0)]$$

 R_0 is the radial function at the origin.

▶ Recent studies shows the NLO and NNLO corrections to CSM improves the fits at TEVATRON and RHIC.
J.P. Lansberg, Eur. Phys. J. C 61, 693 (2009), Phys. Lett. B 695, 149 (2010).



Color Evaporation Model (CEM) H. Fritzsch (1977)

- A theory which is known to satisfy all-order factorization is the color evaporation model (CEM)
- Initially introduced in 1977 and was revived in 1996 by Halzen.
- ▶ The cross-section for a quarkonium state H is some fraction F_H of the cross-section for producing $Q\bar{Q}$ pair with invariant mass below the $M\bar{M}$ threshold

$$\begin{split} \sigma_{CEM}[h_A h_B \to H + X] &= F_H \sum_{i,j} \int_{4m^2}^{4m_M^2} d\hat{s} \int dx_1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \\ &\quad \times \hat{\sigma}_{ij}(\hat{s}) \delta(\hat{s} - x_1 x_2 s) \end{split}$$

- M is the lowest mass meson containing the heavy quark Q.
- $lackbox{ }Qar{Q}$ pair is assumed to neutralize its color by interaction.
- Good description of photoproduction data after inclusion of higher order QCD corrections.
 Eboli etal, arXiv: hep-ph/0211161 (2002).

NRQCD Factorization approach.

Bodwin, Braaten and Lepage (1995)

It is the effective theory based on a systematic expansion in both α_s and v, which is heavy quark velocity within the bound state.

$$\sigma[H] = \sum_{n} \sigma_n(\Lambda) \langle \mathcal{O}_n^H(\Lambda) \rangle$$

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- ► The NRQCD factorization approach to heavy-quarkonium production is by far the most sound theoretically and most successful phenomenologically.

Butenschon and Kniehl, Phys. Rev. Lett. 106, 022003 (2011)



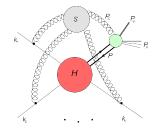
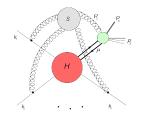


Figure: Charmonium production in High Energy production

Ref': Y. Q. Ma and R. Vogt, Phys. Rev. D 94, no.11, 114029 (2016) > 9



$$P = P_{\psi} + P_S + P_X$$
 $M_{\psi} < M - M_X < M$
 $\langle P_{\psi} \rangle = \frac{M_{\psi}}{M} P + O(\lambda^2/m_c)$

$$\begin{split} \frac{d\sigma_{\psi}(P)}{d^3P} &= F_{\psi} \int_{M_{\psi}}^{2M_D} d^3P' dM \frac{d\sigma_{c\bar{c}}(M,P')}{dMd^3P'} \delta^3(P - \frac{M_{\psi}}{M}P') \\ &= F_{\psi} \int_{M_{\psi}}^{2M_D} dM \frac{d\sigma_{c\bar{c}}(M,P' = (M/M_{\psi})P)}{dMd^3P}, \end{split}$$

How to take in to account the momentum conservation in kinematics?



$$\frac{d\sigma_{\psi}}{dyd^2p_{T\psi}} = \frac{F_{\psi}}{s} \int_{M_{\psi}^2}^{4M_D^2} dM^2 \frac{M_{\psi}}{M} \int d^2\mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{\gamma/p}(x_{\gamma}, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{p}_T - \mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{\gamma/p}(x_{\gamma}, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \frac{M}{M_{\psi}} [\mathbf{k}_{\perp g})] dM^2 \mathbf{k}_{\perp g} f_{\gamma/p}(x_{\gamma}, \mathbf{k}_{\perp g}) f_{\gamma/p}(x_{\gamma}, \mathbf{k}$$

- Mass dependence appearing in the PDFs.
- Including off-shell matrix element along with the CCFM evolved PDFs we will have a differential cross-section
- ▶ J/ψ production using ICEM will be studied in the context of Electron Ion collider
- ► The result will have potential to understand unintegrated PDFs using kT factorization approach.
- Comparison of the differential cross section with the results obtained from CASCADE will be cruicial.
- ► Implementing heavy quarkonium production model in CASCADE is important.



Conclusion

- The Higgs boson production and J/ψ production provide an excellent opportunity to test the k_T factorization together with uPDFs based on CCFM evolution.
- Out results are compared with experimental results from from ATLAS and CMS.
- ▶ EIC will provide an opportunity to test it at small x for J/ψ production.



Thank You for your attention.