Infrared renormalon effects in color dipole distribution in small-x.

Ms. Nahid Vasim

Aligarh Muslim University, Aligarh
India

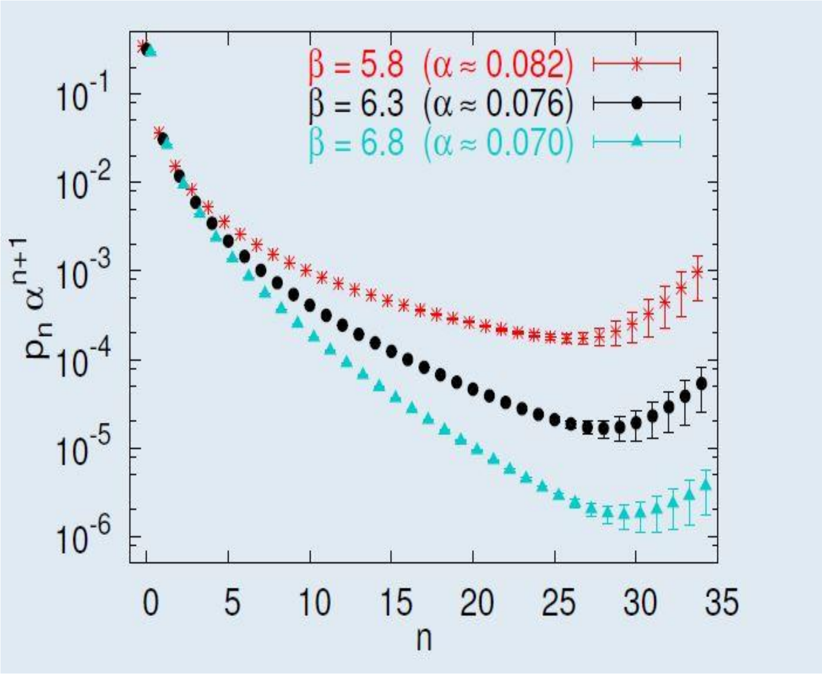
Diverging perturbation series

In Quantum Field Theory the perturbation series is divergent.

$$\sum_n C_n {\alpha_s}^n$$

Dyson's argument [Phys. Rev. 85, 631 (1952)]
Perturbation series are typically divergent with zero radius of convergence.

Each term in the series first decrease and gradually approaches to a minimum and then start to increase without any limit.



Diverging perturbation series

In Quantum Field Theory the perturbation series is divergent.

$$\sum_n c_n {\alpha_s}^n$$

Dyson's argument [Phys. Rev. 85, 631 (1952)]

Perturbation series are typically divergent with zero radius of convergence.

What does $\sum_n \mathbf{C_n {\alpha_s}^n} = \infty$ do ?

↓

- ✓ **No way restricts predictions from the perturbation series for practical applications.**

- ✓ **We want a good approximation by calculating just a few terms of the perturbation series, not all of them!**

Effect of running coupling of QCD

The strong coupling acts as the expansion parameter.

$$\sum_n C_n {\alpha_s}^n$$

The coefficient C_n have factorial-like growth; n!

There are 3 known sources of n! behavior:

- infrared (IR) renormalons
- ultraviolet (UV) renormalons
- instantons.

Best guess of perturbation theory

There are ways where a value can be assign to sum of divergent series

For a factorially divergent series **Borel summation** is mostly used.

Step 1 : Borel transform

$$\sum_{n=0}^{\infty} r_n \alpha^{n+1} \rightarrow B(t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}$$

Step 2: Corresponding Borel integral

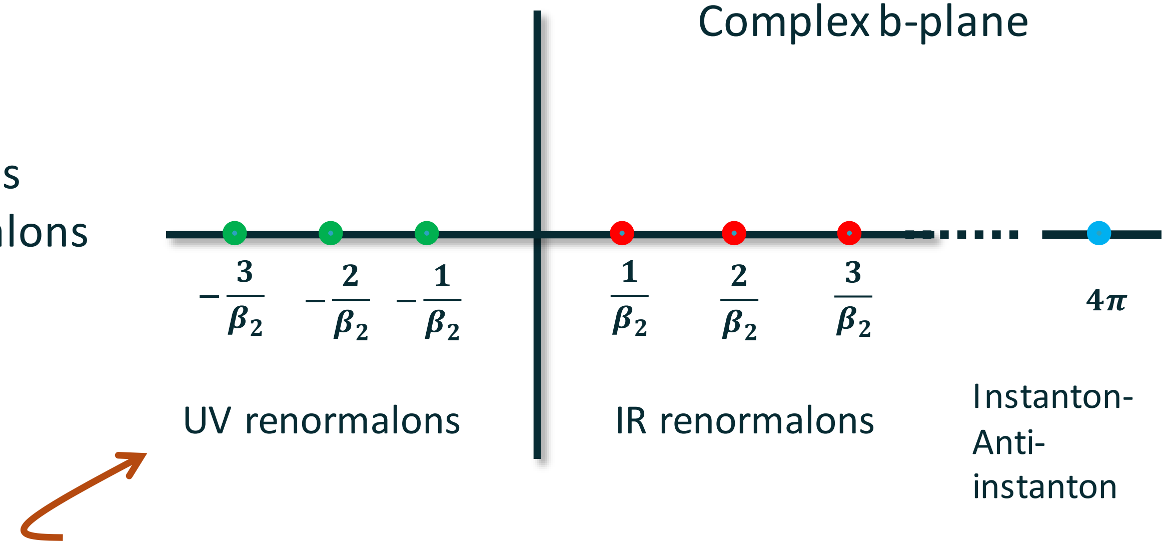
$$\int_{n=0}^{\infty} dt\, e^{-t/\alpha} B(t)$$

Divergent behavior is encoded in the singularities of Borel transform.

Divergence of perturbative series in QCD

The divergence of perturbative series in QCD is reflected through the pole singularities in Borel plane.

- Infrared (IR) renormalons
- Ultraviolet (UV) renormalons
- Instantons.



If the Borel integral has no singularity in the positive real axis and the terms in the series do not increase faster than the factorial growth, the divergent series is Borel summable.

Infrared renormalon

- IR renormalon are the first Borel non-summable singularity of the QCD expansions.

- The non perturbative contribution stems from the IR renormalon.

Low momenta $k_\perp$ $\longrightarrow$ Nonperturbative corrections

- It lie on the positive real axis of the Borel plane and the integral is not defined.

7

Ambiguity of the Borel integral!

Uncertainty due to renormalon

The analytical structure of the Borel transform is connected to the large order behavior of series

$$\sum_{n=0}^{\infty} r_n \alpha^{n+1} \rightarrow B(t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}$$

$$\downarrow$$

$r_n = n!\ a^n \longrightarrow$ For $a > 0$, fixed sign series.

- **The Borel integral still be defined by moving the contour above or below the singularities.**
- **Difference between various regularization prescriptions gives an estimate of the uncertainty due to the renormalon singularity.**

IR Renormalons in (color dipole) gluon distribution

Relation between dipole amplitude $\mathcal{N}$ and the unintegrated dipole gluon distribution $\mathbb{F}\,(x, k_\perp)$ **[**Levin and Ryskin;1987**]**

$$\int d^2b\, \mathcal{N}(r_\perp, b_\perp, x) = \frac{2\pi}{N_c} \int d^2k_\perp \left(1 - e^{ik_\perp . r_\perp}\right) \alpha_s\left({k_\perp}^2\right) \frac{1}{k_\perp} \mathbb{F}(x, k_\perp)$$

Running coupling.

$$\sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu^2)\beta_2}{2}\right)^n n!\, C \quad \xrightarrow{\text{Resumming}} \quad \int_0^\infty db\; e^{-b/\alpha_s(\mu^2)} \frac{1}{b - 2/\beta_2}$$

IR Renormalons in (color dipole) gluon distribution

Relation between dipole amplitude $\mathcal{N}$ and the unintegrated dipole gluon distribution $\mathbb{F}\,(x, k_\perp)$ **[**Levin and Ryskin;1987**]**

$$\int d^2b\, \mathcal{N}(r_\perp, b_\perp, x) = \frac{2\pi}{N_c} \int d^2k_\perp \left(1 - e^{ik_\perp . r_\perp}\right) \alpha_s\left({k_\perp}^2\right) \frac{1}{k_\perp} \mathbb{F}(x, k_\perp)$$

Running coupling.

$$\sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu^2)\beta_2}{2}\right)^n n!\, C$$

Resumming

$$\int_0^\infty db\; e^{-b/\alpha_s(\mu^2)} \frac{1}{b - 2/\beta_2}$$

Outside the saturation region.

$k^2{}_\perp < Q_s$

IR Renormalons in (color dipole) gluon distribution

Relation between dipole amplitude $\mathcal{N}$ and the unintegrated dipole gluon distribution $\digamma(x, k_\perp)$ [Levin and Ryskin;1987]

$$\int d^2b\, \mathcal{N}(r_\perp, b_\perp, x) = \frac{2\pi}{N_c} \int d^2k_\perp \left(1 - e^{ik_\perp . r_\perp}\right) \alpha_s({k_\perp}^2) \frac{1}{k_\perp} \digamma(x, k_\perp)$$

$$\mathcal{F}(x, k_\perp) \propto \frac{k_\perp^2}{Q_s^2(x)}$$

Outside the saturation region.

$${k^2}_\perp < Q_s$$

Running coupling.

$$\sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu^2)\beta_2}{2}\right)^n n!\, C$$

Resumming

$$\int_0^\infty db\, e^{-b/\alpha_s(\mu^2)} \frac{1}{b - 2/\beta_2}$$

IR Renormalons in (color dipole) gluon distribution at small-x

Relation between dipole amplitude $\mathcal{N}$ and the unintegrated dipole gluon distribution $\mathbb{F}(x, k_\perp)$ **[**Levin and Ryskin;1987**]**

$$\int d^2b\, \mathcal{N}(r_\perp, b_\perp, x) = \frac{2\pi}{N_c} \int d^2k_\perp \left(1 - e^{ik_\perp . r_\perp}\right) \alpha_s\left({k_\perp}^2\right) \frac{1}{k_\perp} \mathbb{F}(x, k_\perp)$$

$$\mathbf{x\mathbb{F}^{DP}(x, k_\perp)}|_{\mathbf{Q_s \gtrsim k_\perp \gg \Lambda_{QCD}}}$$

In the small momenta($\mathbf{k_\perp}$) regions, within the window of momentum scale $\Lambda_{\mathbf{QCD}}$ and saturation scale $\mathbf{Q_s}$ where the running coupling becomes large, infrared renormalons is believed to be the source of the divergence in the perturbation series.

Color dipole TMD at small-x and small transverse momentum

M. Siddiqah, N. Vasim, K. Banu, T. Bhattacharyya and R. Abir,
Phys.R D 97(2018),054009.

We use a special solution of BK equation or S-matrix (Levin-Tuchin solution) that is valid for small-x and large transverse separation (large $\mathbf{r}_\perp$) to derive Color dipole distribution that would be valid for small-x and small transverse momentum (small $\mathbf{k}_\perp$).

$$xG^{DP}(x, k_\perp) = \frac{S_\perp N_c}{2\pi^2 \alpha_s} k_\perp \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp . r_\perp} S(x, r_\perp)$$

Levin-Tuchin Solution

Gaussian in $\ln^2\left[r_\perp{}^2 Q_s{}^2(Y) \right]$ ← $\mathbf{S(r_\perp, Y) = \exp\left(-\tau \ln^2\left[r_\perp{}^2 Q_s{}^2(Y) \right] \right)}$

This is valid when $r_\perp{}^2 Q_s{}^2(Y) \gtrsim 1$, leading to large logarithms in the exponent.

Color dipole TMD at small-x and small transverse momentum

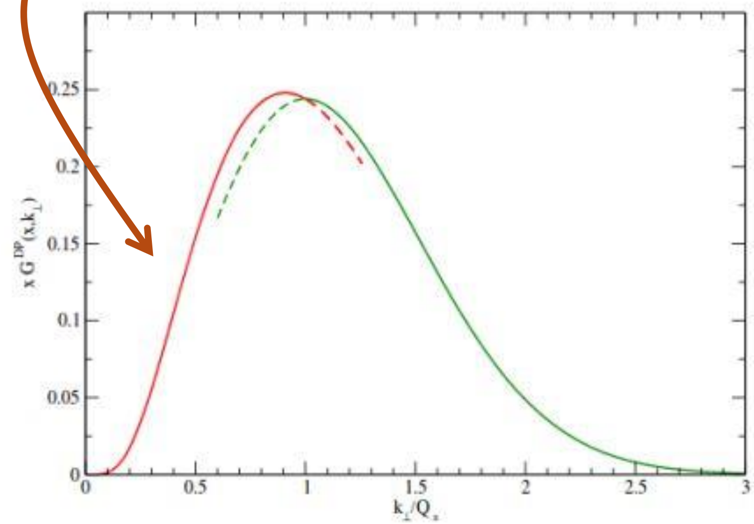
M. Siddiqah, N. Vasim, K. Banu, T. Bhattacharyya and R. Abir,
Phys.R D 97(2018),054009.

We developed new mathematical techniques to Fourier transform lognormal distributions in two dimension.

When resumming the series in leading log accuracy, the results showing up striking similarity with the Sudakov form factor.

$$xG^{DP}(x,k_\perp)|_{Q_s \gtrsim k_\perp \gg \Lambda_{QCD}} \approx -\frac{S_\perp N_c \tau}{\pi^3 \alpha_s} \ln\left(\frac{k_\perp^2}{4Q_s^2(Y)}\right) \exp\left[-\tau \ln^2\left(\frac{k_\perp^2}{4Q_s^2(Y)}\right)\right]$$

Inside the saturation region, the dipole gluon distribution is expected to go to zero in zero momentum.

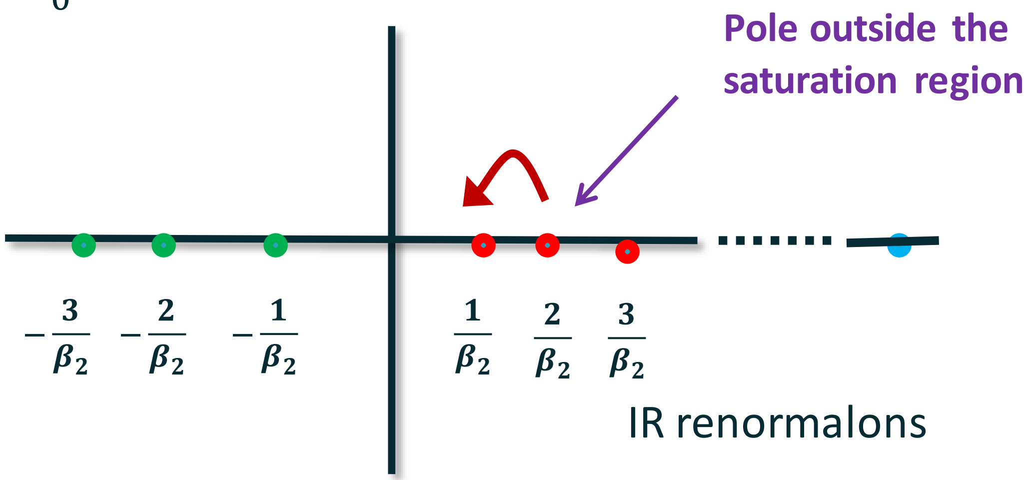


IR Renormalons in (color dipole) gluon distribution

N.Vasim, R.Abir, Nuclear Physics B 953 (2020), 114961.

On resumming the contribution inside the saturation region , the effect of renormalon in the Borel integral as

$$\int_0^\infty db \exp\left(-\frac{(1+\epsilon)b}{\alpha(\mu^2)}\right)\frac{1}{b-\mathbf{1/\beta_2}}$$



The non-linear saturation effects at small-x shift the first IR pole at the Borel plane towards zero from 2/β2 to 1/β2

IR Renormalons in (color dipole) gluon distribution

N.Vasim, R.Abir, Nuclear Physics B 953 (2020), 114961.

Associated uncertainty:

- An enhanced non-perturbative uncertainty ⟶ $\mathcal{O}\left(\Lambda_{QCD}^2\right)$

$$\sim \frac{r_\perp{}^2}{\beta_2}\,\Lambda_{QCD}{}^2\;\frac{\ln \Lambda_{QCD}{}^2}{4Q_s{}^2}\;\exp\left(-\tau\,\ln^2\frac{\Lambda_{QCD}{}^2}{4Q_s{}^2}\right)$$

- Presence of the Sudakov factor indicates that the saturation effect tend to suppress the IR renormalon effects at small-x.

Summary

- Non-perturbative effects in QCD actually stems from the diverging nature of the perturbative series.
- The non-linear saturation effects at small-x shift the first IR pole at the Borel plane towards zero.
- The saturation effects suppress the renormalon effect through a Sudakov type of soft factor.

Summary

- Non-perturbative effects in QCD actually stems from the diverging nature of the perturbative series.
- The non-linear saturation effects at small-x shift the first IR pole at the Borel plane towards zero.
- The saturation effects suppress the renormalon effect through a Sudakov type of soft factor.

Thank you.