Threshold Improved Transverse momentum dependent distribution functions

Resummation, Evolution, Factorization 2022

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Outline of the talk:

- Introduction
- Theoretical Formalism
  - For DY process
  - For DIA Process
  - For SIDIS Process
- Numerical results
- Conclusion

Introduction

- The parton distribution functions (PDFs) and fragmentation functions (FFs) provide the dynamics of the partons inside hadrons.

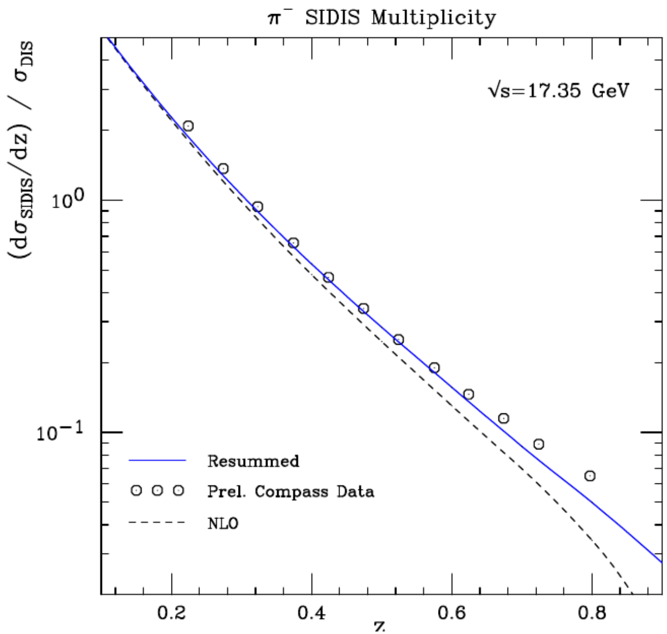
- In the last decade, hadron physics community extended the the investigation of parton dynamics by proposing new kind of parton distribution function, called transverse momentum dependent parton distribution function (TMDPDF).

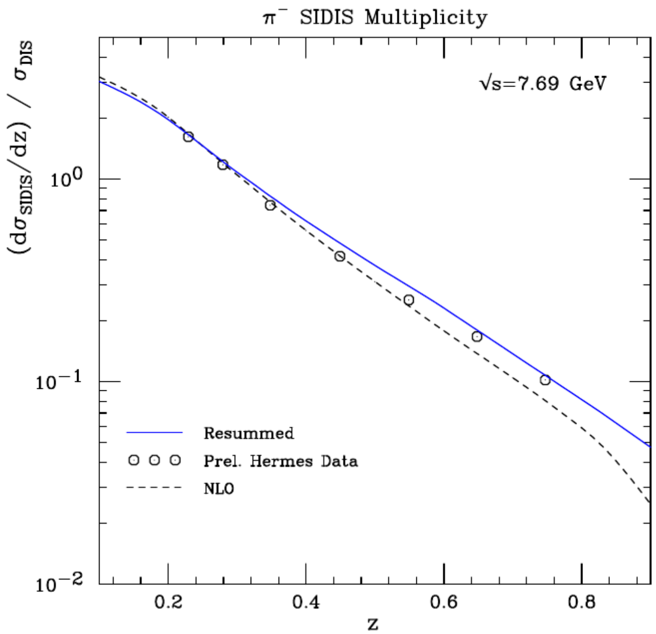
- The TMDPDFs can provide both the momentum and space information of a single parton inside a hadron. Therefore, it can provide 3D structure of a hadron.

- Apart from 3D structure of hadrons, it can also provide information about orbital motion of parton, spin-orbit correlations in QCD etc.

- In this work, we include the threshold effect in TMDPDFs and TMDFFs. The threshold effect is important for reliable theoretical prediction near the right edge of the phase space.

Introduction





D. Anderle, F. Ringer, W. Vogelsang
Phys.Rev.D 87 (2013) 3

The TMD factorization was first derived by Collin, Soper and Sterman (*hep-ph/0409313*) and later it has been also derived in Soft Collinear Effective Theory (SCET).

Theoretical Formalism for TMDPDF

$$h_1(P_1) + h_2(P_2) \to \gamma^*(q) \to l^+ + l^- + X$$

The Hadronic Threshold variable : $\tau^{\mathrm{DY}} \equiv Q^2/S \quad S = (P_1 + P_2)^2 \quad Q^2 = q^2$

In the small transverse momentum limit

$$\frac{\mathrm{d}^3\sigma^{\mathrm{DY}}}{\mathrm{d}^2\boldsymbol{q}_T\,\mathrm{d}\tau^{\mathrm{DY}}} = \sigma_0^{\mathrm{DY}} \int_{C_N} \frac{\mathrm{d}N}{2\pi i} \left(\tau^{\mathrm{DY}}\right)^{-N} \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{4\pi^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T} H^{\mathrm{DY}}(Q,\mu) \times \sum_q e_q^2 \tilde{f}_{q/h_1}^{\mathrm{TMD}}(N, b_T, \mu, \zeta) \tilde{f}_{\bar{q}/h_2}^{\mathrm{TMD}}(N, b_T, \mu, \zeta)$$

Born cross section

$$\sigma_0^{\mathrm{DY}} \equiv \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c Q^2}$$

The rapidity divegences are regularized and the Collin-Soper scale is determined

$$\zeta_f^{\mathrm{TMD}} = Q^2$$

The partonic threshold variable

$$\hat{\tau}^{\mathrm{DY}} \equiv \frac{\tau^{\mathrm{DY}}}{x_1 x_2}$$

Theoretical Formalism for TMDPDF

- When the partonic threshold variable is close to 1; the previous factorization is not complete because it does not include the threshold effect.

$$Q \gg Q(1-\hat{\tau}) \gg q_T$$

- In this kinematic regions, the new scale hyrarchy introduces an additinal degrees of freedom known as collinear-soft (csoft) degrees of freedom.

- The momentum scaling of the csoft mode:

$$k^{\mu}_{cs} \equiv (\bar{n}\cdot k_{cs},\, n\cdot k_{cs},\, k_{cs,\perp}) \sim \left(Q(1-\hat{\tau}), \frac{q_T^2}{Q(1-\hat{\tau})}, q_T\right)$$

Bauer, Tackmann, Walsh, Zuberi 2012; Procura, Waalewijn, Zeune 2015; Larkoski, Moult, Neill 2015; Becher, Neubert, Rothen, Shao 2015; Chien, Hornig, Lee 2016; Pietrulewicz, Tackmann, Waalewijn 2016

- One can combine csoft function with soft function to define rapidity regularized 'modified soft' function as;

$$\tilde{S}_c(N, b_T, \mu, \zeta) = \tilde{S}_c^{\mathrm{unsub}}(N, b_T, \mu, \zeta/\nu^2)\sqrt{S(b_T, \mu, \nu)}$$

Theoretical Formalism for TMDPDF

Refactorization of the TMDPDF takes the form:

$$\tilde{f}^{\mathrm{TMD}}_{i/h}(N, b_T, \mu) \xrightarrow{N\to\infty} \underbrace{\tilde{S}^{\mathrm{unsub}}_c(N, b_T, \mu, \zeta/\nu^2)\sqrt{S(b_T, \mu, \nu)}}_{\equiv\ \tilde{S}_c(N, b_T, \mu, \zeta)} \tilde{f}_{i/h}(N, \mu)$$

$$\tilde{f}_{i/h}(N, \mu) = \int_0^1 \mathrm{d}x\, x^{N-1} f_{i/h}(x, \mu)$$

The definition of the soft function is same as the usual TMD case.

Evolution of Collin-Soper scale is also same as usual TMD case because the threshold resummation will not effect the rapidity divergence.

$$\sqrt{\zeta}\frac{\mathrm{d}}{\mathrm{d}\sqrt{\zeta}}\, \tilde{S}_c(N, b_T, \mu, \zeta) = \kappa(b_T, \mu)\, \tilde{S}_c(N, b_T, \mu, \zeta)$$

Theoretical Formalism for TMDPDF

$$\kappa(b_T,\mu) \equiv -\Gamma_{\mathrm{cusp}}(\alpha_s)L_b + \mathcal{O}\left(\alpha_s^2\right) \quad L_b = \ln\left(\mu^2 b_T^2/b_0^2\right) \text{ with } b_0 = 2e^{-\gamma_E}$$

$$\tilde{S}_c\left(N,b_T,\mu_b,\zeta_f\right) = \tilde{S}_c\left(N,b_T,\mu_b,\zeta_i\right)\left(\sqrt{\frac{\zeta_f}{\zeta_i}}\right)^{\kappa(b_T,\mu_b)}$$

$\zeta_i = \mu_b^2 = b_0^2/b_T^2$ and ζ_f will be determined from RG consistency

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}H\left(Q,\mu\right) = \Gamma^h(\alpha_s)H\left(Q,\mu\right),$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\tilde{S}_c\left(N,b_T,\mu,\zeta_f\right) = \Gamma^{s_c}(\alpha_s)\tilde{S}_c\left(N,b_T,\mu,\zeta_f\right),$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\tilde{f}_q\left(N,\mu\right) = \Gamma^{f_q}(\alpha_s)\tilde{f}_q\left(N,\mu\right),$$

Theoretical Formalism for TMDPDF

$$\Gamma^h(\alpha_s) = 2\,\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{Q^2}{\mu^2} + 2\gamma_V(\alpha_s)$$

$$\Gamma^{s_c}(\alpha_s) = -\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{\zeta_f}{\mu^2} + \gamma_{S_c}(\alpha_s)$$

$$\Gamma^{f_q}(\alpha_s) = -2\,\Gamma_{\text{cusp}}(\alpha_s)\ln\bar{N} + 2\gamma_{f_q}(\alpha_s)$$

From RG consistency

$$\Gamma^h + 2\,\Gamma^{s_c} + 2\,\Gamma^{f_q} = 0$$

We obtain

$$\zeta_f^{\text{TTMD}} = \frac{Q^2}{\bar{N}^2} \qquad \bar{N} = N e^{\gamma_E}$$

The all order resummation formula can be obtained by solving RG equation form their intrinsic scale to a common scale. The NLL resum formula is given by

$$\frac{\mathrm{d}^3\sigma^{\text{DY}}}{\mathrm{d}^2\boldsymbol{q}_T\,\mathrm{d}\tau^{\text{DY}}} = \sigma_0^{\text{DY}} \int_{C_N} \frac{\mathrm{d}N}{2\pi i}\left(\tau^{\text{DY}}\right)^{-N} \int_0^\infty \frac{\mathrm{d}b_T\, b_T}{2\pi} J_0(q_T\, b_T)$$
$$\times \sum_q e_q^2 \tilde{f}_{q/h_1}^{\text{TTMD}}(N, b_T, Q)\, \tilde{f}_{\bar{q}/h_2}^{\text{TTMD}}(N, b_T, Q)$$

Theoretical Formalism for TMDPDF

$$\tilde{f}_{i/h}^{\mathrm{TTMD}}(N, b_T, Q) = \exp\left[-S_{\mathrm{pert}}(Q, \mu_{b_*}, \mu_F) - S_{\mathrm{NP}}\left(b_T, Q_0, \zeta_f^{\mathrm{TTMD}}\right)\right] \tilde{f}_{i/h}(N, \mu_F)$$

$$S_{\mathrm{pert}}(Q, \mu_{b_*}, \mu_F) = \int_{\mu_{b_*}}^{Q} \frac{d\mu}{\mu} \frac{\Gamma^h(\alpha_s)}{2} - \int_{\mu_F}^{\mu_{b_*}} \frac{d\mu}{\mu} \Gamma^{f_i}(\alpha_s)$$

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\mathrm{max}}^2}}, \qquad \text{with} \quad b_{\mathrm{max}} = 1.5\,\mathrm{GeV}^{-1}$$

Collins, Soper, Sterman
Nucl. Phys. B250 (1985) 199–224.

$$S_{\mathrm{NP}}\left(b_T, Q_0, \zeta_f\right) = g_1^f b_T^2 + \frac{g_2}{2} \ln \frac{\sqrt{\zeta_f}}{Q_0} \ln \frac{b_T}{b_*}$$

Sun, Isaacson, Yuan, Yuan
[1406.3073]

$$g_1^f = 0.106\,\mathrm{GeV}^2, \; g_2 = 0.8 \text{ and } Q_0^2 = 2.4\,\mathrm{GeV}^2$$

Theoretical Formalism for TMDFF

$$e^+ + e^- \to \gamma^*(q) \to h_1(P_1) + h_2(P_2) + X \qquad \tau^{\text{DIA}} \equiv \frac{(P_1+P_2)^2}{Q^2}$$

$$\frac{\mathrm{d}^3\sigma^{\text{DIA}}}{\mathrm{d}^2\boldsymbol{q}_T \mathrm{d}\tau^{\text{DIA}}} = \sigma_0^{\text{DIA}} \int_{C_N} \frac{\mathrm{d}N}{2\pi i} \left(\tau^{\text{DIA}}\right)^{-N} \int \frac{\mathrm{d}b_T\, b_T}{2\pi} J_0(q_T b_T) H^{\text{DIA}}(Q,Q) \times \sum_q e_q^2 \tilde{D}_{h_1/q}^{\text{TTMD}}(N, b_T, Q) \tilde{D}_{h_2/\bar{q}}^{\text{TTMD}}(N, b_T, Q)$$

$$\tilde{D}_{h/q}^{\text{TTMD}}(N,b,Q) = \exp\left[-S_{\text{pert}}\left(Q,\mu_{b_*},\mu_F\right) - S_{\text{NP}}\left(b, Q_0, \zeta_f^{\text{TTMD}}\right)\right] \tilde{D}_{h/q}(N,\mu_F)$$

$$S_{\text{pert}}\left(Q,\mu_{b_*},\mu_F\right) = \int_{\mu_{b_*}}^{Q} \frac{d\mu}{\mu} \frac{\Gamma^h(\alpha_s)}{2} - \int_{\mu_F}^{\mu_{b_*}} \frac{d\mu}{\mu} \Gamma^{d_q}(\alpha_s)$$

$$S_{\text{NP}}\left(b, Q_0, \zeta_f\right) = g_1^D b^2 + \frac{g_2}{2} \ln\frac{\sqrt{\zeta_f}}{Q_0} \ln\frac{b}{b_*} \qquad g_1^D = 0.042\,\text{GeV}^2$$

Sun, Isaacson, Yuan,Yuan
[1406.3073]

Numerical Result

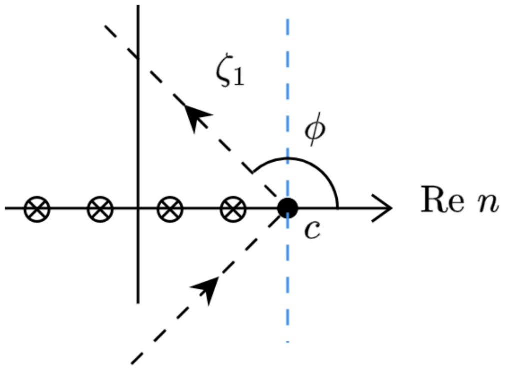
Modified Collin-Soper scale and Mellin inversion

For large valus of N, the collin-Soper scale may go down the non-perturbative scale which violates our factorization condition; $Q \gg Q(1-\hat{\tau}) \gg q_T$

$$\zeta_* \equiv \zeta_*(\zeta_f^{\rm TTMD}, Q_0) = \left(\frac{Q}{\bar{N}}\right)^2 \left(1 + \frac{Q_0^2 \bar{N}^2}{Q^2}\right)$$

The general Mellin invarsion formula reads

$$\varphi(x) = \frac{1}{\pi}\int_0^\infty \mathrm{d}z\ \mathrm{Im}\left[\exp(i\phi)x^{-c-z\exp(i\phi)}\varphi_{n=c+z\exp(i\phi)}\right]$$

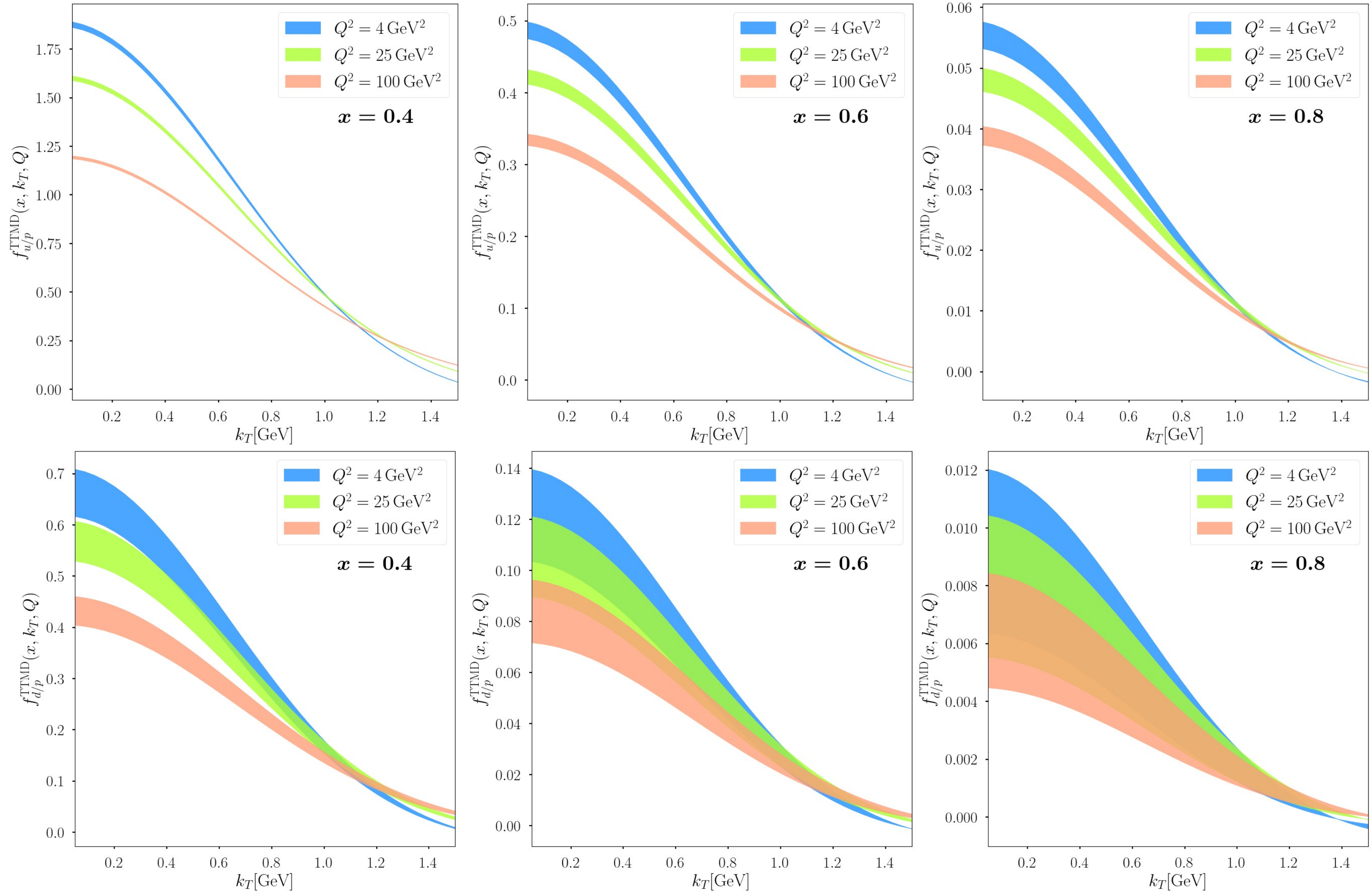


Modified Collin-Soper scale introduces new poles:

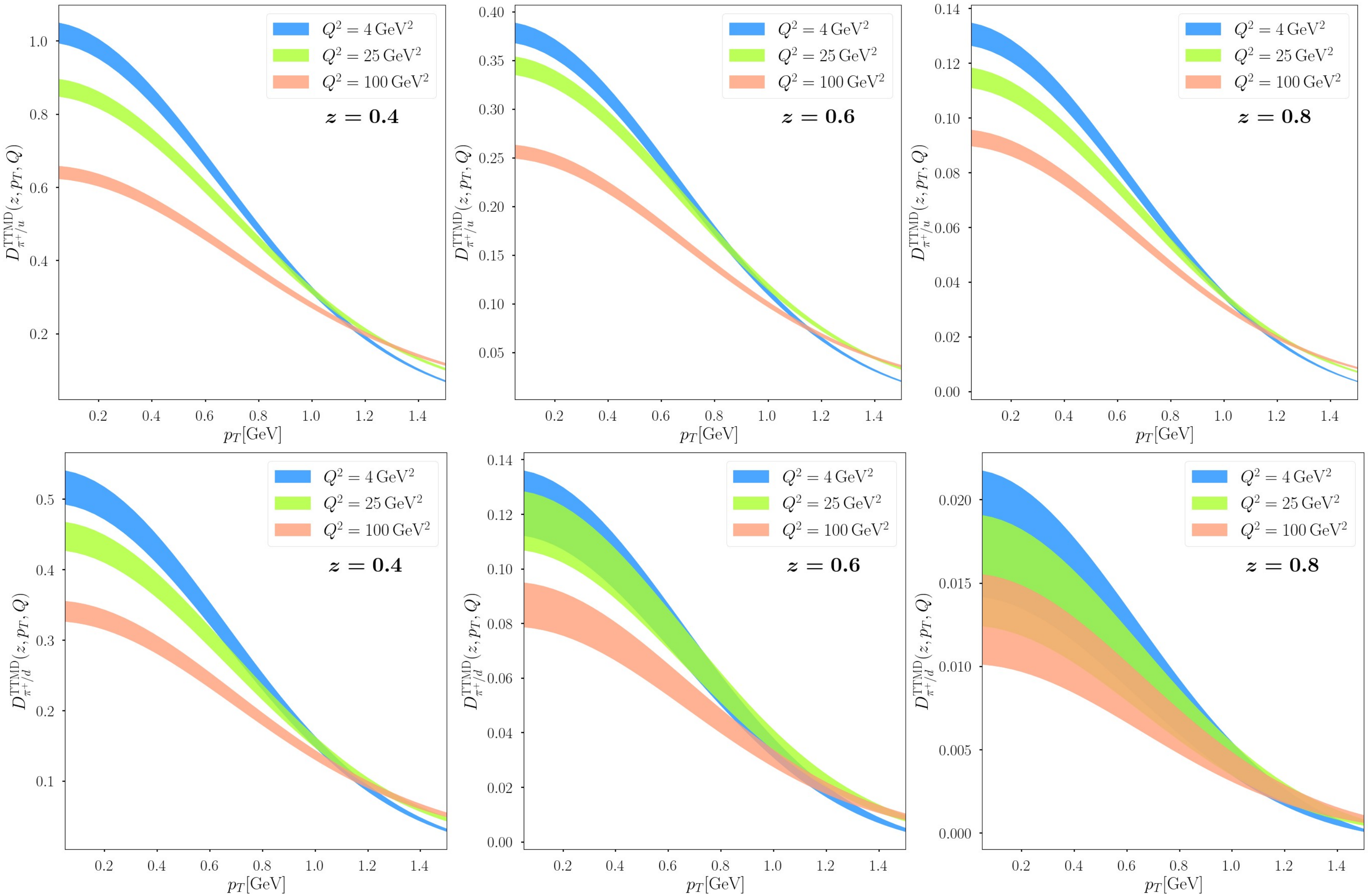
$$n = -c \pm i\frac{\bar{Q}}{Q_0}$$

We chose $\phi = \pi/2$ and $c = 1.6$

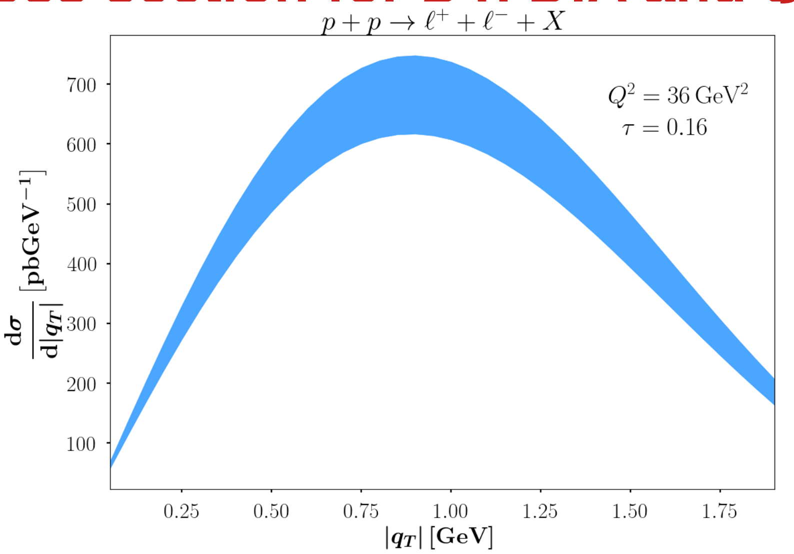
Numerical Result for TMDPDF

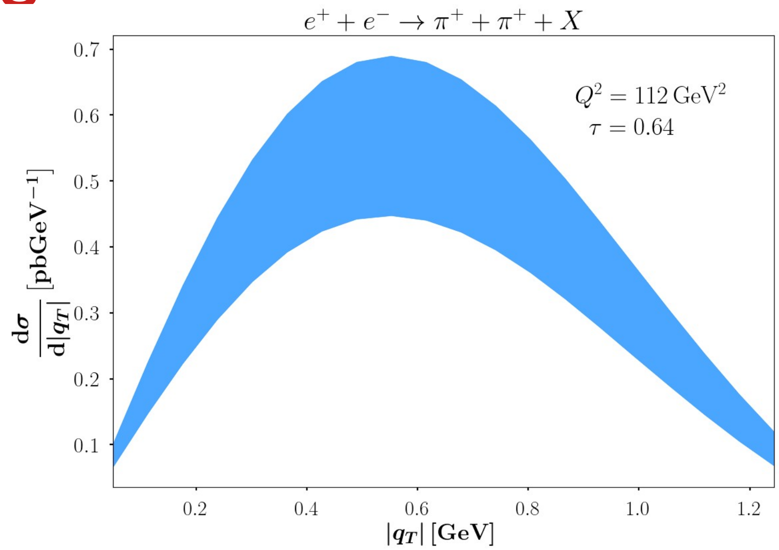


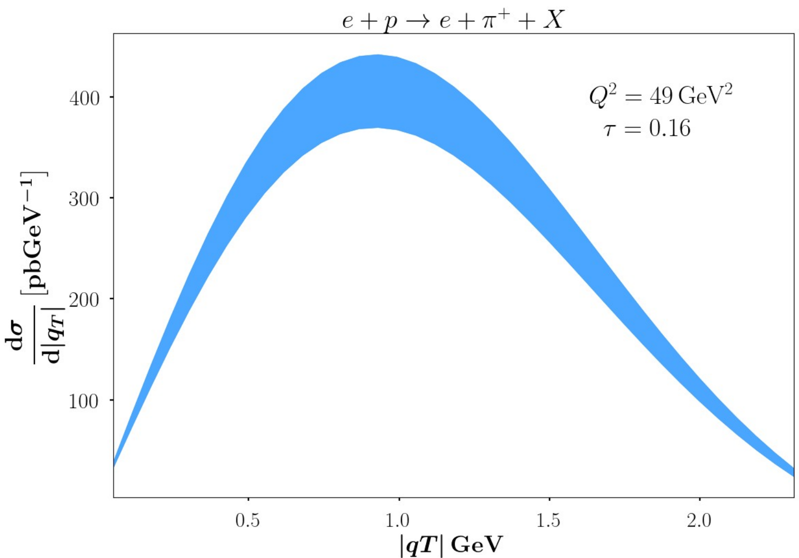
Numerical Result for TMDFF



Cross section for DY. DIA and SIDIS







Conclusion

- We provide theoretical formalism for for threshold improved TMDPDFs and TMDFFs.
- In our numerical analysis, we observe that to have a kinematic consistence result, one needs to modify Collins-Soper scale.
- The modified Collin-Soper scale will introduce two new poles. We provide Mellin inversion prescription to avoid all kind of ploes.
- Our formalism will serve as a reliable theoretical input for extracting the TMD functions at large x value.