Modified TMD Factorization and Sub-leading Power Corrections

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∫dkΠ Doktoratskolleg Particles and Interactions

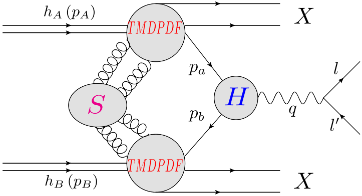
FWF Der Wissenschaftsfonds.

universität wien

Outline

TMD factorization in Drell-Yan

- The emerging partons are not parallel to the incoming hadron and are off-shell.
- The partons from the TMDPDFs have a non-negligible transverse momentum $\mathbf{p}_{Ta(b)}$.
- The transverse momentum has to be smaller than the collinear component of the emerging parton: $p^2_{a(b)T}/Q^2 \sim q_T^2/Q^2 \ll 1$ up to power corrections.
- In the Drell-Yan cross section the leptonic tensor $L^{\mu\nu}$ is projected onto the transverse plane by $-g_\perp^{\mu\nu}$, i.e. $\hat{L} = -g_\perp^{\mu\nu} L_{\mu\nu}$.

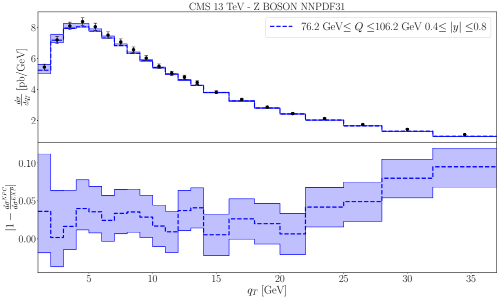


$$\frac{d\sigma^{\text{TMD}}_{h_A h_B \to l l' X}}{dQ^2\, dy\, dq_T^2} = \sum_c \hat{L}\sigma^{\text{Born}} H\left(\alpha_s, Q^2\right) \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T\cdot\mathbf{q}_T} F_{c\leftarrow h_A}\left(\alpha_s, x_A, b_T^2\right) F_{\bar{c}\leftarrow h_B}\left(\alpha_s, b_T^2, x_B\right) S^{-1} + Y$$

Y includes the $1/Q$ power corrections to the SCET factorization formula, dubbed by CSS
Collins et al. Nucl. Phys. B **250** (1985), 199-224; Collins et al. Phys. Rev. D **94** (2016) no.3, 034014

TMD factorization vs Drell-Yan data

DATA from CMS collaboration JHEP **12** (2019), 061



- At $|\mathbf{q}_T|/Q \ll 1$ the agreement is remarkable, nonetheless at intermediate/large $|\mathbf{q}_T|$, i.e. $|\mathbf{q}_T| \geq 0.25Q$, we start to appreciate a tension between theory and experiment.
- The non-pertubative model used in the uTMDPDF should not have a great impact on the points at intermediate/large $|\mathbf{q}_T|$.
- Might power corrections help to improve the predictions? Balitsky et al. JHEP **05** (2018), 150; Balitsky et al. JHEP **05** (2021), 046; Nefedov et al. Phys. Lett. B **790** (2019), 551-556; Ebert et al. 2112.07680 [hep-ph]; Luke et al. Phys. Rev. D **104** (2021) no.7, 076018, Beneke et al. JHEP **03** (2018), 001, Mulders et al. Nucl. Phys. B **667** (2003), 201-241...

The full hadronic tensor in Drell-Yan

We can write the hadronic tensor as Soper et al. Nucl. Phys. B **152** (1979), 109; Ellis et al. Nucl. Phys. B **207** (1982), 1-14, Nucl. Phys. B **212** (1983), 29; Sterman et al. Nucl. Phys. B **244** (1984), 221-246; Mulders et al. Phys. Rev. D **51** (1995), 3357-3372; Phys. Rev. D **57** (1998), 3057-3064

$$W^{\mu\nu} = \frac{1}{N_c} \sum_{a,b} \delta_{a\bar{b}} \int d^4 p_a d^4 p_b \delta^{(4)} \left(p_a + p_b - q\right) \mathrm{Tr} \left[\Phi_{a/A} \gamma^{\mu} \Phi_{b/B} \gamma^{\nu}\right]$$

where the quark-quark correlator is defined as

$$\left(\Phi_{a/A}\right)_{ij} \left(p_A, s_A; p_a\right) = \int \frac{d^d z}{(2\pi)^d} e^{ip_a \cdot z} \langle p_A, s_A | \bar{\psi}_j \left(0\right) \mathcal{W} \left(0, z | \bar{n}\right) \psi_i \left(z\right) | p_A, s_A \rangle$$

which has the general decomposition Goeke et al. Phys. Lett. B **618** (2005), 90-96

$$\Phi_{a/A} \left(p_A, s_A; p_a\right) = p_A^{+} \sum_{\Gamma^{\text{ctwist-2}}} \mathrm{Tr} \left[\Phi_{a/A} \Gamma^{ctwist-2}\right] \Gamma^{ctwist-2} + \sum_{\Gamma^{\text{ctwist-3}}} \mathrm{Tr} \left[\Phi_{a/A} \Gamma^{ctwist-3}\right] \Gamma^{ctwist-3}$$
$$+ \frac{1}{p_A^{+}} \sum_{\Gamma^{\text{ctwist-4}}} \mathrm{Tr} \left[\Phi_{a/A} \Gamma^{ctwist-4}\right] \Gamma^{ctwist-4}$$

where $\Gamma^{\text{ctwist-2}} = \{\gamma^{+}, i\gamma^{+}\gamma_5, i\sigma^{i+}\gamma_5\}$, $\Gamma^{\text{ctwist-3}} = \{1, i\gamma_5, \gamma^{i}, i\gamma^{i}\gamma_5, i\sigma^{ij}\gamma_5, i\sigma^{+-}\gamma_5\}$ and $\Gamma^{\text{ctwist-4}} = \{\gamma^{-}, i\gamma^{-}\gamma_5, i\sigma^{i-}\gamma_5\}$. For $\Phi_{b/B}$, $- \rightleftarrows +$

The full hadronic tensor in Drell-Yan

With the previous decomposition of the quark-quark correlator we have for unpolarised hadrons

$$W^{\mu\nu} = \frac{1}{N_c}\sum_{ab}\delta_{a\bar{b}}\int d^4p_a d^4p_b \delta^{(4)}(p_a+p_b-q)\cdot\Big[-g_T^{\mu\nu}\Big\{\Phi_{a/A}^{[\gamma^+]}\Phi_{b/B}^{[\gamma^-]}+\mathcal{O}\Big(\frac{1}{p_A^+p_B^-}\Big)\Big\}$$

$$+T^{\mu\nu}_{\text{Boer-Mulders}}\Big\{\Phi_{a/A}^{[i\sigma^{i+}\gamma_5]}\Phi_{b/B}^{[i\sigma^{i-}\gamma_5]}+\mathcal{O}\Big(\frac{1}{p_A^+p_B^-}\Big)\Big\}+\sum_i T_i^{\mu\nu}\underbrace{A^{[T_i]}}_{\sim\frac{1}{p_A^+p_B^-}}\Big]$$

- $g_T^{\mu\nu}$ and $T^{\mu\nu}_{Boer-Mulders}$ are orthogonal to each other and to $T_i^{\mu\nu}$.
- The blue term in the limit $p^{\pm}_{A(B)}\to\infty$ leads to the standard uTMDPDF.
- The purple term in the limit $p^{\pm}_{A(B)}\to\infty$ leads to the Boer-Mulders distribution.
- The orange terms may be understood as contributions from higher collinear twist structure functions.
- By contracting $W^{\mu\nu}$ with the appropriate tensor we can compute the contributions from different structure functions.

TMD limit and behaviour at $\mathbf{p}_{a(b)T} \to \mathbf{0}$

- In the limiting case $p^{\pm}_{A(B)} \to \infty$:
  $\delta^{(4)}(p_a + p_b - q) \simeq \delta\left(p_a^+ - q^+\right)\delta\left(p_b^- - q^-\right)\delta^{(2)}(\mathbf{p}_{aT} + \mathbf{p}_{bT} - \mathbf{q}_T)$;
  $x_A = p_a^+/p_A^+ \simeq q^+/p_A^+$ and $x_B = p_b^-/p_B^- \simeq q^-/p_B^-$
- The uTMDPDF and Boer-Mulders distributions are given by

$$F_{a(b)/A(B)}\left(x_{A(B)}, \mathbf{p}_{a(b)T}\right) = \int dp^{\mp}_{a(b)}\, \Phi^{[\gamma^{\pm}]}_{a(b)/A(B)}$$

$$h^{\perp}_{1,a(b)/A(B)}\left(x_{A(B)}, \mathbf{p}_{a(b)T}\right) = \int dp^{\mp}_{a(b)}\, \Phi^{\left[i\sigma^{i\pm}\gamma_5\right]}_{a(b)/A(B)}$$

- F goes as $1/\mathbf{p}_T^2$ and $h_1^{\perp}$ as $1/|\mathbf{p}_T|$ in the limit $\mathbf{p}_T \to 0$.
- Only the blue term in the hadronic tensor reproduces the $1/\mathbf{q}_T^2$ behaviour of the partonic cross section, everything else is less singular.
- By evaluating $g_T^{\mu\nu} W_{\mu\nu}$ at finite $\mathbf{q}_T/Q$ we are able to compute power corrections, in the collinear limit $p^{\pm}_{A(B)} \to \infty$, to uTMDPDF. We arrange all these power corrections into a matching coefficient.

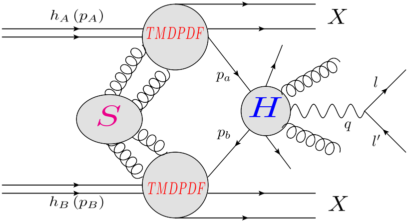
Modified Factorization Formula

$$\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2 dy d\mathbf{q}_T^2} = \sum_{a,b,c} \hat{L}\sigma_c^{\text{Born}} \int d^2\mathbf{p}_{Ta} d^2\mathbf{p}_{Tb} d^2\mathbf{q}'_T \delta^{(2)}\left(\mathbf{q}_T - \mathbf{p}_{Ta} - \mathbf{p}_{Tb} - \mathbf{q}'_T\right)$$

$$\int_{x_A}^{1} \frac{dz_a}{z_a} \int_{x_B}^{1} \frac{dz_b}{z_b} \theta\left(\frac{(z_a - x_A)(z_b - x_B)}{x_A x_B} - \frac{\mathbf{q}_T^{2\prime}}{Q^2 + \mathbf{q}_T^2}\right) \tilde{H}_{c\leftarrow a, \bar{c}\leftarrow b}\left(\alpha_s, Q^2, \frac{x_A}{z_a}, \frac{x_B}{z_b}, \mathbf{q}_T^{2\prime}, \mathbf{q}_T^2\right)$$

$$F_{a\leftarrow h_A}\left(\alpha_s, z_a, \mathbf{p}_{Ta}^2\right) F_{b\leftarrow h_B}\left(\alpha_s, z_b, \mathbf{p}_{Tb}^2\right) S^{-1}$$

- The origin of θ is purely kinematic.
- The coefficient $\tilde{H}$ is free of large logarithmic contributions. All of these are absorbed by the TMDPDF.
- We impose cuts on $\mathbf{p}_{a(b)T}$; $\mathbf{q}'_T$ is unrestricted.
- The contraction with the leptonic tensor $T^{\mu\nu}_{\text{Boer-Mulders}} L_{\mu\nu}$ is proportional to angular terms that give a negligible contributions with symmetric phase space cuts (and zero without cuts).
- A similar argument follows for the higher collinear twist contributions.



Our approach

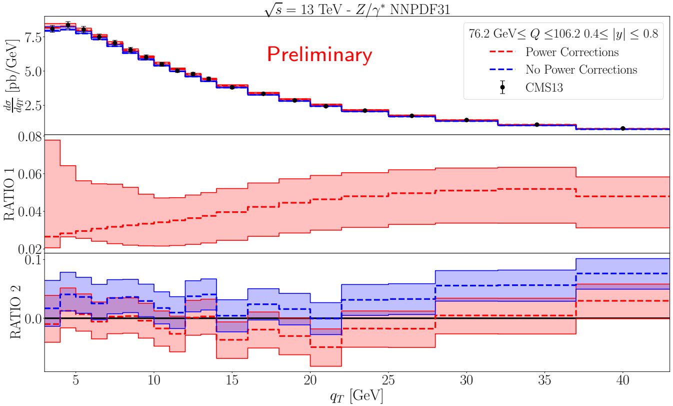
We use ideas from q_T-subtraction method: Catani, Grazzini et al. Nucl. Phys. B **596** (2001), 299-312; Catani, Grazzini et al. Phys. Lett. B **696** (2011), 207-213; Catani, Grazzini et. al. Phys. Rev. Lett. **98** (2007), 222002

$$d\sigma = \lim_{q_T \to 0} d\sigma + \left[d\sigma - \lim_{q_T \to 0} d\sigma \right]$$

- In our case the first term is well described by TMD factorization.
- It contains large logs (due to the expansion) that need to be resummed. TMD formalism is quite convenient for this task.
- The second term includes our power corrections as the difference between partonic level and fixed order cross section
- Typically the second term is computed using Monte-Carlo event generators. We provide an analytical computation at NLO+NLL.
- We modified the TMD factorization formula for DY to include this second term.

Power corrections vs Leading power

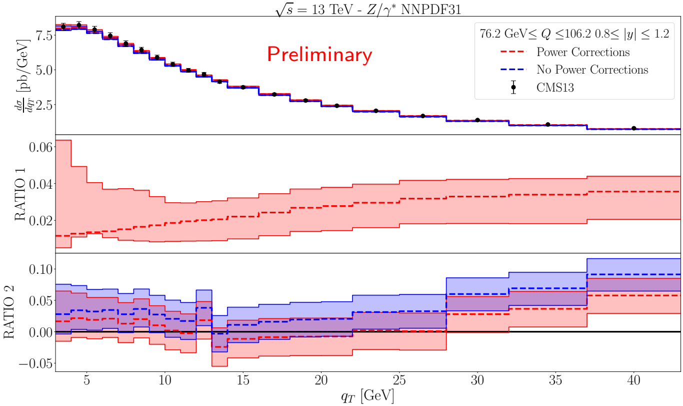
Data from CMS collaboration JHEP **12** (2019), 061



RATIO 1 $= 1 - d\sigma^{\text{NPC}}/d\sigma^{\text{PC}}$, RATIO 2 $= 1 - d\sigma^{PC(NPC)}/d\sigma^{DATA}$. Bigger than electroweak corrections Grazzini et al. Phys. Rev. Lett. **128** (2022) no.1, 012002; Sborlini et al. JHEP **08** (2018), 165

Power corrections vs Leading power

Data from CMS collaboration JHEP **12** (2019), 061



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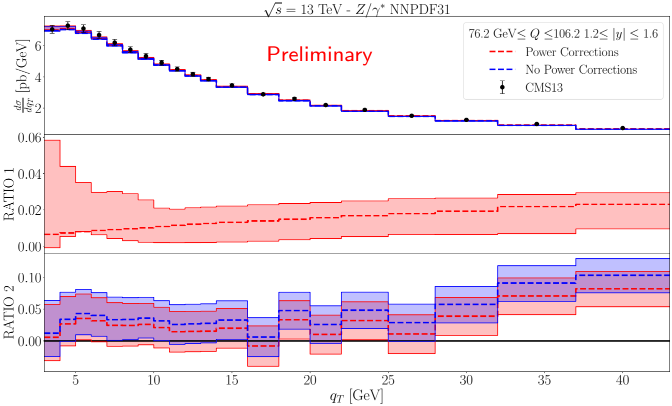
Power corrections vs Leading power

Data from CMS collaboration JHEP **12** (2019), 061



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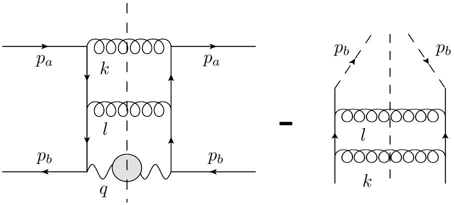
Computation at NNLO+NNLL

- We obtain a cancellation of the collinear and soft divergences at NNLO for the unpolarized case.
- At the level of $W^{\mu\nu}$ the tensor structures beyond $g_T^{\mu\nu}$ do not lead to any further IR divergences.
- We are able to reduce the calculation to a set of integrals in 1 and 2 dimensionless variables, which can be evaluated numerically.
- All the logarithms at leading power are resummed by the uTMDPDFs.



Summary & Outlook

SUMMARY

- At small $\mathbf{q}_T^2/Q^2$ our factorization formula reproduces TMD factorization.
- At $|\mathbf{q}_T| = Q \cdot 0.10$ we start to appreciate the effects of power corrections.
- By contracting $W^{\mu\nu}$ with the appropriate tensor we can compute the different power corrections.
- The power corrections increase the cross section at intermediate/large q_T, making it closer to the experimental data.
- Electroweak corrections are subleading compared to power corrections

OUTLOOK

- Improvement of the code for the numerics at NNLO.
- Inclusion of Boer-Mulders and higher collinear twist.
- Inclusion of evolution kernels at twist-3. Rodini & Vladimirov, JHEP **08** (2022), 031
- New extraction of TMDPDFs.
- Study of polarized processes.
- Extension to SIDIS.

THANK YOU FOR YOUR ATTENTION

Backup

Large logs

- Momentum Space $\mathbf{q}_T$

$$\frac{d\sigma}{dQ^2 dy d\mathbf{q}_T^2} \sim c_1^{[1]} \frac{\alpha_s}{\mathbf{q}_T^2} \log \frac{Q^2}{\mathbf{q}_T^2} + \frac{\alpha_s^2}{\mathbf{q}_T^2} \left(c_1^{[2]} \log \frac{Q^2}{\mathbf{q}_T^2} + c_2^{[2]} \log^2 \frac{Q^2}{\mathbf{q}_T^2} + c_3^{[2]} \log^3 \frac{Q^2}{\mathbf{q}_T^2} \right) + \cdots$$

- Impact parameter space $\mathbf{b}_T$

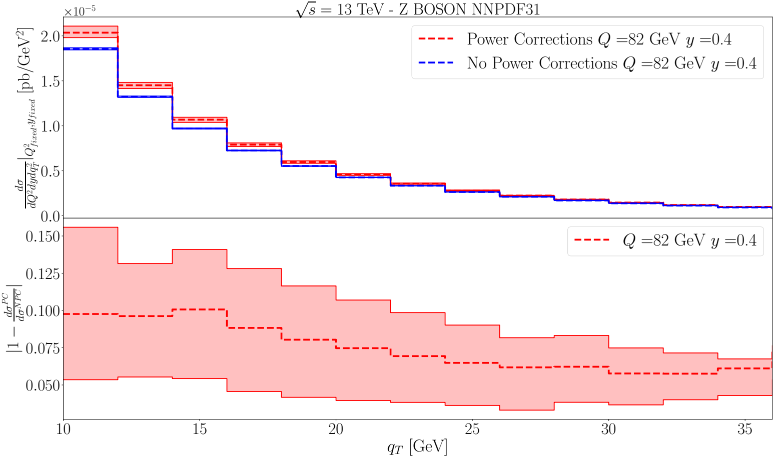
$$\frac{d\sigma}{dQ^2 dy d\mathbf{b}_T^2} \sim \alpha_s \left(c_0^{[1]} \log \frac{Q^2 \mathbf{b}_T^2}{4e^{-2\gamma_E}} + c_1^{[1]} \log^2 \frac{Q^2 \mathbf{b}_T^2}{4e^{-2\gamma_E}} \right) +$$
$$\alpha_s^2 \left(c_0^{[2]} \log \frac{Q^2 \mathbf{b}_T^2}{4e^{-2\gamma_E}} + c_1^{[2]} \log^2 \frac{Q^2 \mathbf{b}_T^2}{4e^{-2\gamma_E}} + c_2^{[2]} \log^3 \frac{Q^2 \mathbf{b}_T^2}{4e^{-2\gamma_E}} + c_3^{[2]} \log^4 \frac{Q^2 \mathbf{b}_T^2}{4e^{-2\gamma_E}} \right) + \cdots$$

Small $\mathbf{q}_T$ expansion at NLO.

Using the methods presented in Bacchetta et al. JHEP **08** (2008), 023; Soper et al. Phys. Rev. D **54** (1996), 1919-1935

$$\delta\left((p_a - p_b - q)^2\right) =$$

$$\frac{1}{Q^2+\mathbf{q}_T^2}\left[\frac{1}{(1-x_a)_+}\delta(1-x_b)+\frac{1}{(1-x_b)_+}\delta(1-x_a)-\delta(1-x_a)\,\delta(1-x_b)\ln\frac{\mathbf{q}_T^2}{Q^2+\mathbf{q}_T^2}\right]+\mathcal{O}\left(\frac{\mathbf{q}_T^2}{Q^2}\right)$$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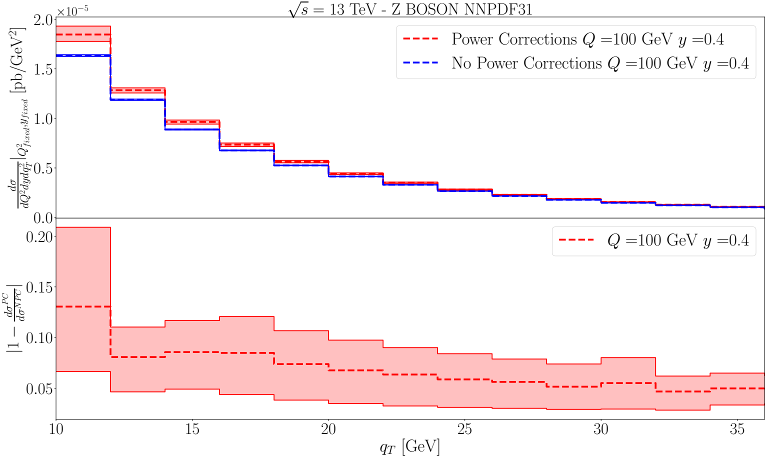
$\times 10^{-5}$
$\frac{d\sigma}{dQ^2 dy dq_T^2}|_{Q^2_{fixed}, y_{fixed}}$ [pb/GeV^2]
Power Corrections Q =82 GeV y =0.4
No Power Corrections Q =82 GeV y =0.4
2.0
1.5
1.0
0.5
0.0
$|1 - \frac{d\sigma^{PC}}{d\sigma^{NPC}}|$
Q =82 GeV y =0.4
0.150
0.125
0.100
0.075
0.050
10
15
20
25
30
35
q_T [GeV]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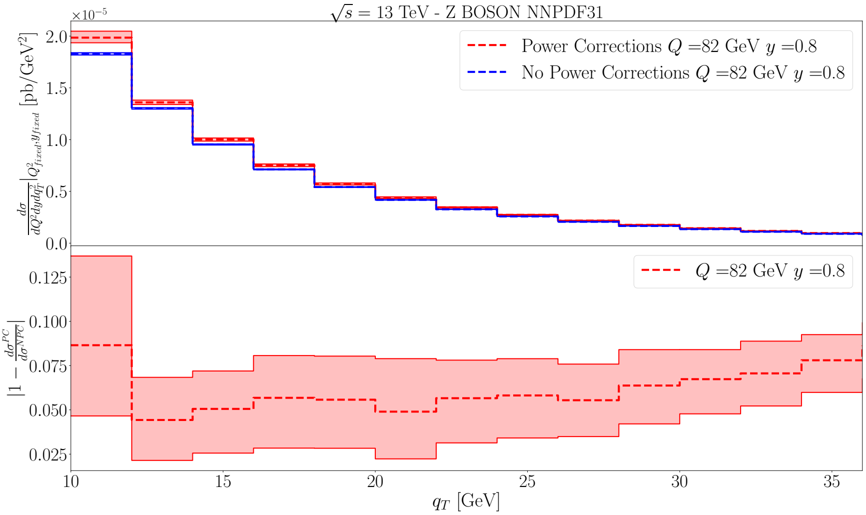
$\times 10^{-5}$
$\frac{d\sigma}{dQ^2 dy dq_T^2}\Big|_{Q^2_{fixed}, y_{fixed}}$ [pb/GeV^2]
Power Corrections Q =100 GeV y =0.4
No Power Corrections Q =100 GeV y =0.4
$|1 - \frac{d\sigma^{PC}}{d\sigma^{NPC}}|$
Q =100 GeV y =0.4
q_T [GeV]
2.0
1.5
1.0
0.5
0.0
0.20
0.15
0.10
0.05
10
15
20
25
30
35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$\times 10^{-5}$
Power Corrections Q =82 GeV y =0.8
No Power Corrections Q =82 GeV y =0.8
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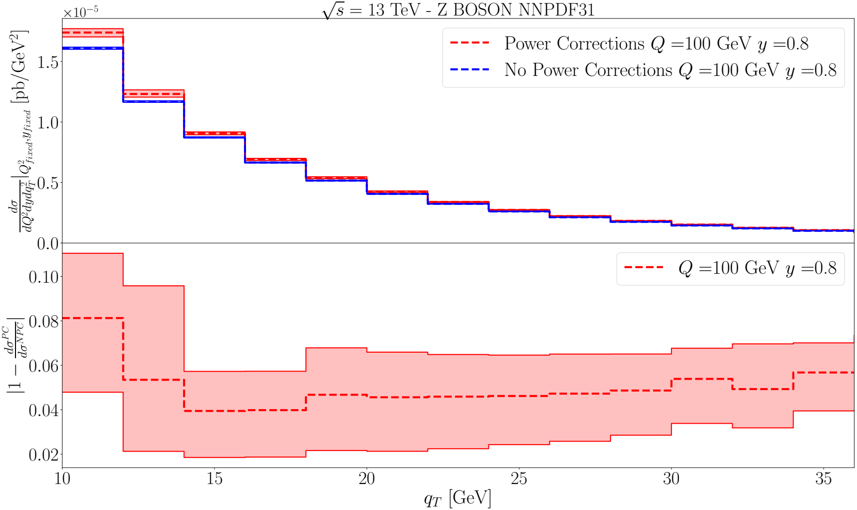
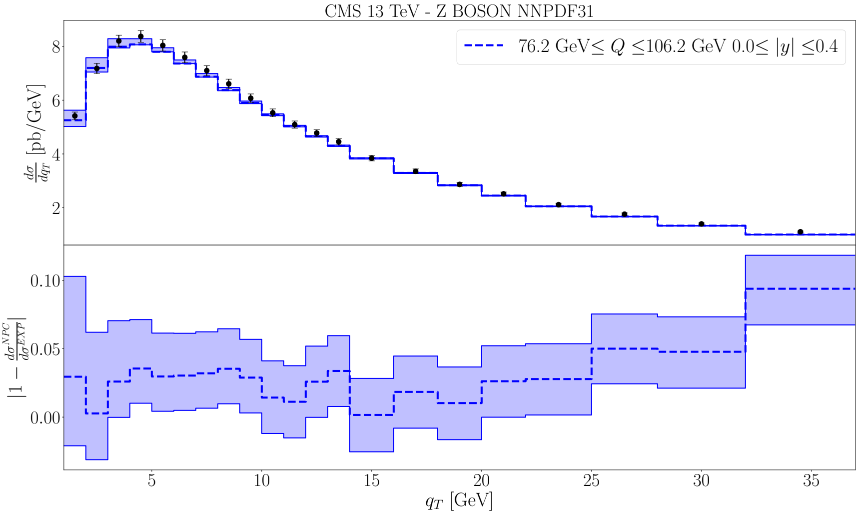
0.0
0.5
1.0
1.5
2.0
Q =82 GeV y =0.8
$|1 - \frac{d\sigma^{PC}}{d\sigma^{NPC}}|$
0.025
0.050
0.075
0.100
0.125
10
15
20
25
30
35
q_T [GeV]


$\sqrt{s}$ = 13 TeV - Z BOSON NNPDF31
$\times 10^{-5}$
$\frac{d\sigma}{dQ^2 dy dq_T^2}|_{Q^2_{fixed}, y_{fixed}}$ [pb/GeV^2]
Power Corrections Q =100 GeV y =0.8
No Power Corrections Q =100 GeV y =0.8
$|1 - \frac{d\sigma^{PC}}{d\sigma^{NPC}}|$
Q =100 GeV y =0.8
q_T [GeV]


CMS 13 TeV - Z BOSON NNPDF31
76.2 GeV≤ Q ≤106.2 GeV 0.0≤ |y| ≤0.4
$\frac{d\sigma}{dq_T}$ [pb/GeV]
$|1 - \frac{d\sigma^{NPC}}{d\sigma^{EXP}}|$
q_T [GeV]
2
4
6
8
0.00
0.05
0.10
5
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15
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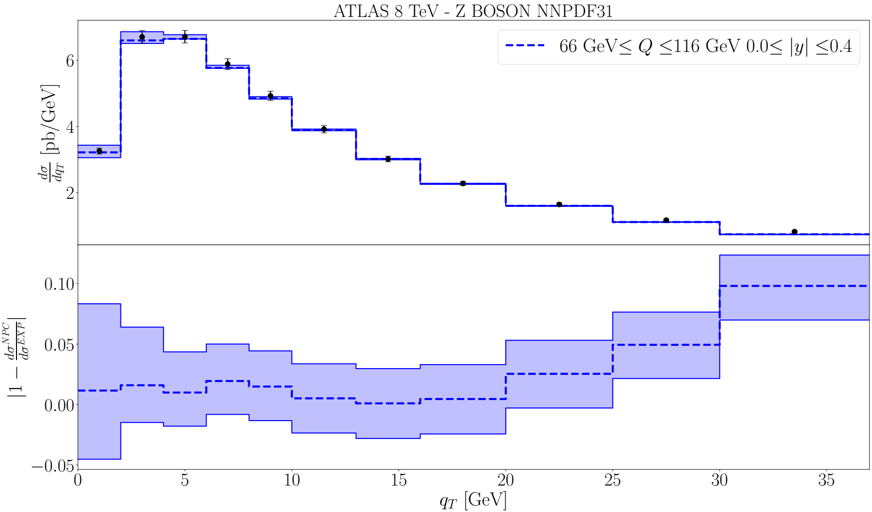
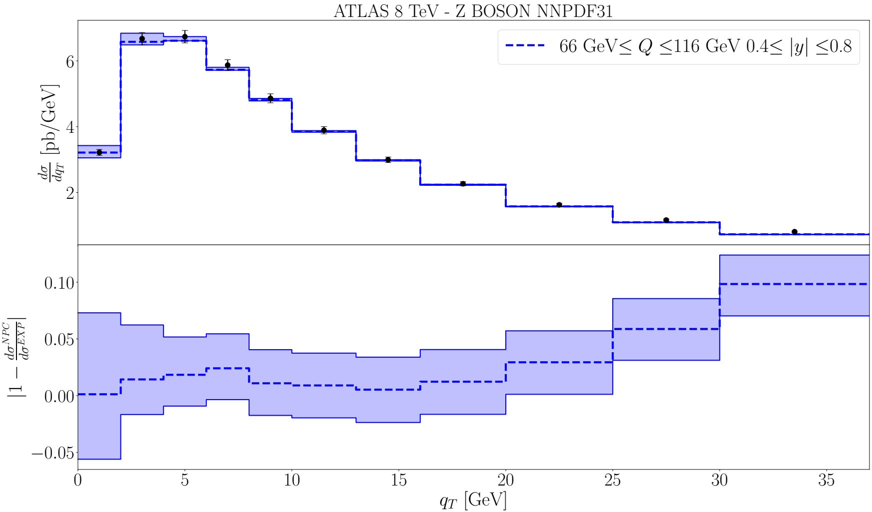
25
30
35


ATLAS 8 TeV - Z BOSON NNPDF31
66 GeV≤ Q ≤116 GeV 0.0≤ |y| ≤0.4
dσ/dq_T [pb/GeV]
|1 − dσ^NPC/dσ^EXP|
q_T [GeV]
0
5
10
15
20
25
30
35
2
4
6
−0.05
0.00
0.05
0.10


ATLAS 8 TeV - Z BOSON NNPDF31
66 GeV≤ Q ≤116 GeV 0.4≤ $|y|$ ≤0.8
$\frac{d\sigma}{dq_T}$ [pb/GeV]
$|1-\frac{d\sigma^{NPC}}{d\sigma^{EXP}}|$
q_T [GeV]
2
4
6
0.10
0.05
0.00
−0.05
0
5
10
15
20
25
30
35