

# Studies of Evolution Properties of Structure Functions in SIDIS



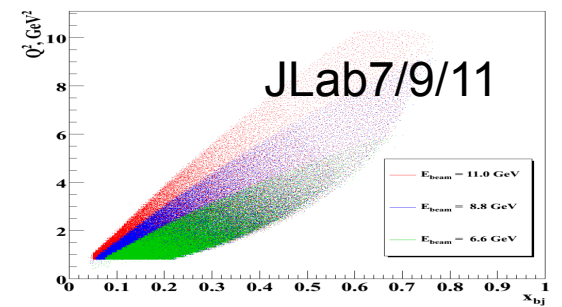
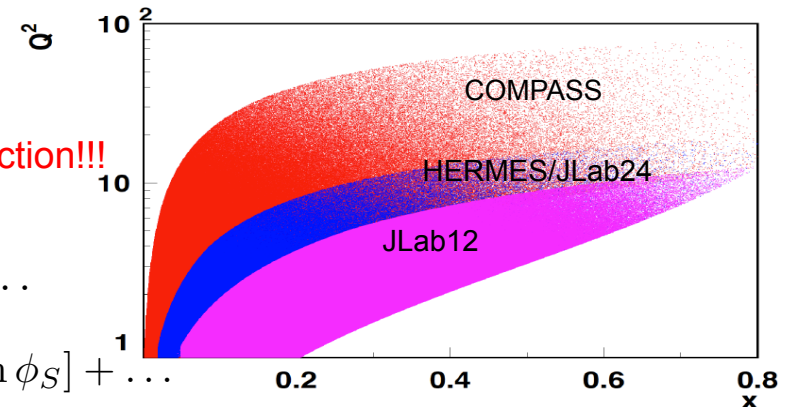
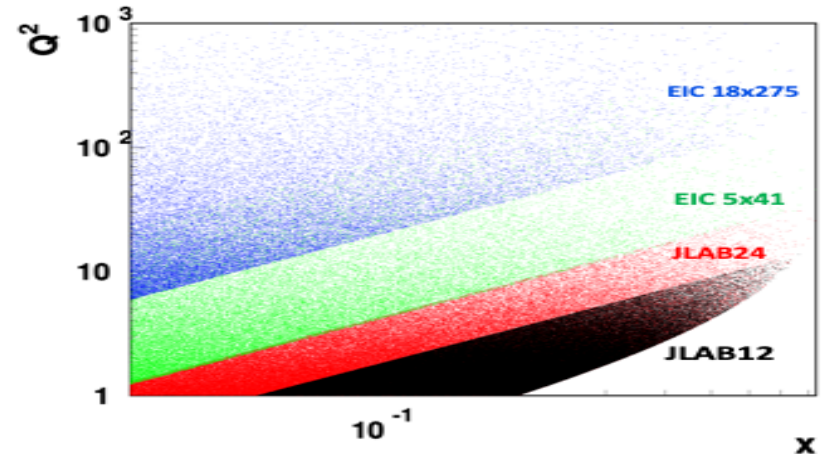
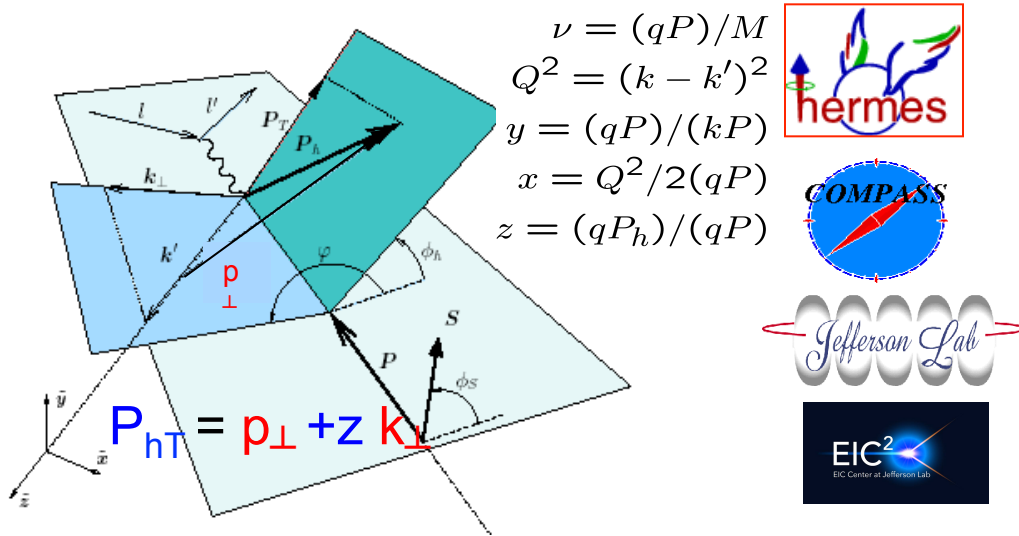
## Harut Avakian (JLab)

Understanding the QCD: from observables to QCD dynamics

- Testing the QCD based frameworks for finite energies in SIDIS with experiments with polarized beams and targets
- Studies of evolution properties of observables

Summary

# SIDIS kinematical coverage and observables



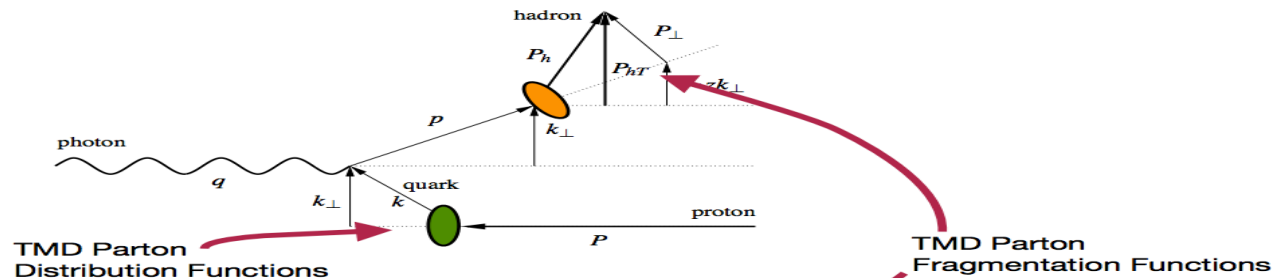
SIDIS experiments measure azimuthal dependence of the cross section!!!

$$\sigma \propto F_{UU} + P_b \sqrt{2\epsilon(1-\epsilon)} F_{LU}^{\sin \phi} \sin \phi + P_t \epsilon F_{UL}^{\sin 2\phi} \sin 2\phi + \dots$$

$$+ \epsilon F_{UU,L} + |S_{\perp}| [F_{UT}^{\sin \phi - \phi_S} \sin(\phi - \phi_S) + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin \phi_S} \sin \phi_S] + \dots$$

- Studies of azimuthal modulations give access to underlying 3D partonic distributions
- All azimuthal modulations increase with  $P_T$
- All assumptions involved in description of SFs, should be tested!
- QCD predicts only the  $Q^2$ -dependence of 3D PDFs

# TMDs IN SEMI-INCLUSIVE DIS



$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2) = \underbrace{x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2)}_{\text{hard factor}} \underbrace{\int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)}_{\text{W term}} + Y_{UU,T}(Q^2, \mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

The W term, dominates at low transverse momentum  $q_T = P_{hT}/z \ll Q$   
 So far, the Y term has been neglected in TMD extractions

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, k_\perp^2; \mu_f, \zeta_f)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = \underbrace{[C \otimes f_1]}_{\text{collinear PDF}}(x, \mu_{b_*}) \underbrace{e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})}}_{\text{perturbative Sudakov form factor}} \underbrace{\left( \frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K}}_{\text{Collins-Soper kernel (perturbative and nonperturbative)}} \underbrace{f_{1NP}(x, b_T^2; \zeta_f, Q_0)}_{\text{nonperturbative part of TMD}}$$

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

CS-kernel  $\rightarrow$  independent on any other variables

CS kernel describes the interaction of out-going parton with the confining potential  
 Provides nonperturbative part of evolution for TMDs

# Theory-Experiment: SIDIS Validation tests

$$\frac{d\sigma^{lN \rightarrow l' h X}}{dx_B dQ^2 dz dP_{h\perp}^2 d\phi_h} = \frac{\pi\alpha^2 y}{x_B y Q^2 (1-\epsilon)} [F_{UU,T} + \dots]$$

$$\left. \begin{array}{l} lN \rightarrow l' \pi^+ X \\ lN \rightarrow l' \rho^0 X \\ \dots \end{array} \right\} lN \rightarrow l' \pi^+ X$$

Fragmentation Function  
→ probability to produce  $\pi^+$

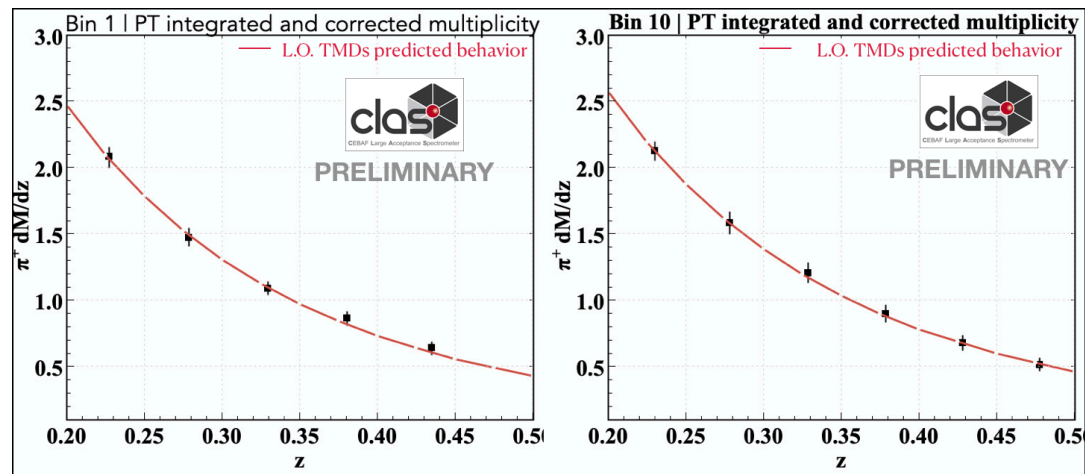
theory



$$F_{XY}^h(x, z, P_T, Q^2) \propto \sum H^q \times f^q(x, k_T, ..) \otimes D^{q \rightarrow h}(z, p_T, ..) + Y(Q^2, P_T) + \mathcal{O}(M/Q)$$

Integrated over transverse momentum are well described by the formalism

the CLAS data at 10.6 GeV follows global fits (also at 6 GeV).



# Azimuthal distributions in SIDIS (unpolarized)

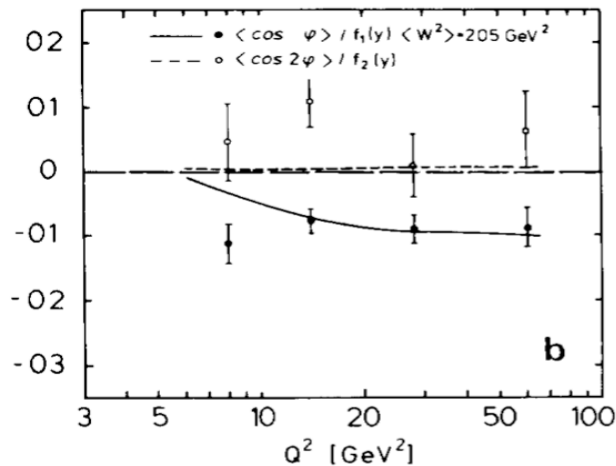
$$\frac{d\sigma}{dx_B dy d\psi dz d\phi_h dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right.$$

$$\left. + \epsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\epsilon(1-\epsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \right\},$$

H.T. ↓
H.T. ↓
H.T.

EMC-1983 (PL,v130,118)



**Observables: - Azimuthal Moments - Multiplicity**

$$\frac{d^4 M^{\pi^\pm}(x, Q^2, z, P_T^2)}{dx dQ^2 dz dP_T^2} = \left( \frac{d^4 \sigma^{\pi^\pm}}{dx dQ^2 dz dP_T^2} \right) / \left( \frac{d^2 \sigma^{DIS}}{dx dQ^2} \right)$$

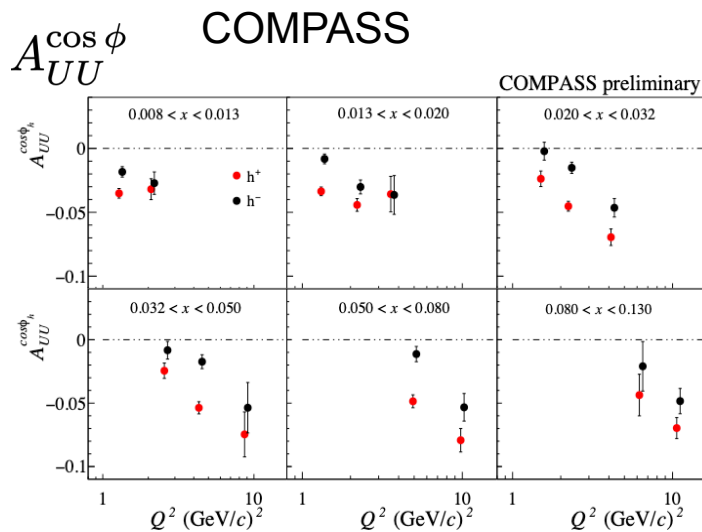
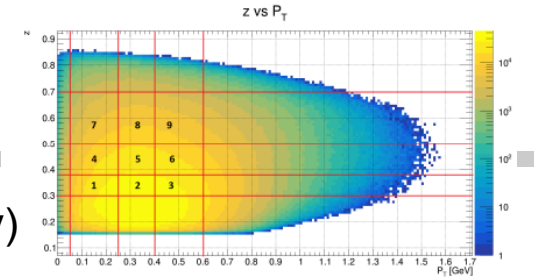
$$m^h(x, z, P_T^2, Q^2) = \frac{\pi F_{UU,T}(x, z, P_T^2, Q^2) + \pi \epsilon F_{UU,L}(x, z, P_T^2, Q^2)}{F_T(x, Q^2) + \epsilon F_L(x, Q^2)}$$

- Quark-gluon correlations are significant in electro production experiments (even at high energies).
- Large  $\cos\phi$  modulations observed in electroproduction (EMC, COMPASS, HERMES) may be a key in understanding of the QCD dynamics.
- What we know about the  $P_T$ -dependence of the  $F_{UU,L}$  (most likely increasing fast with  $P_T$ )?

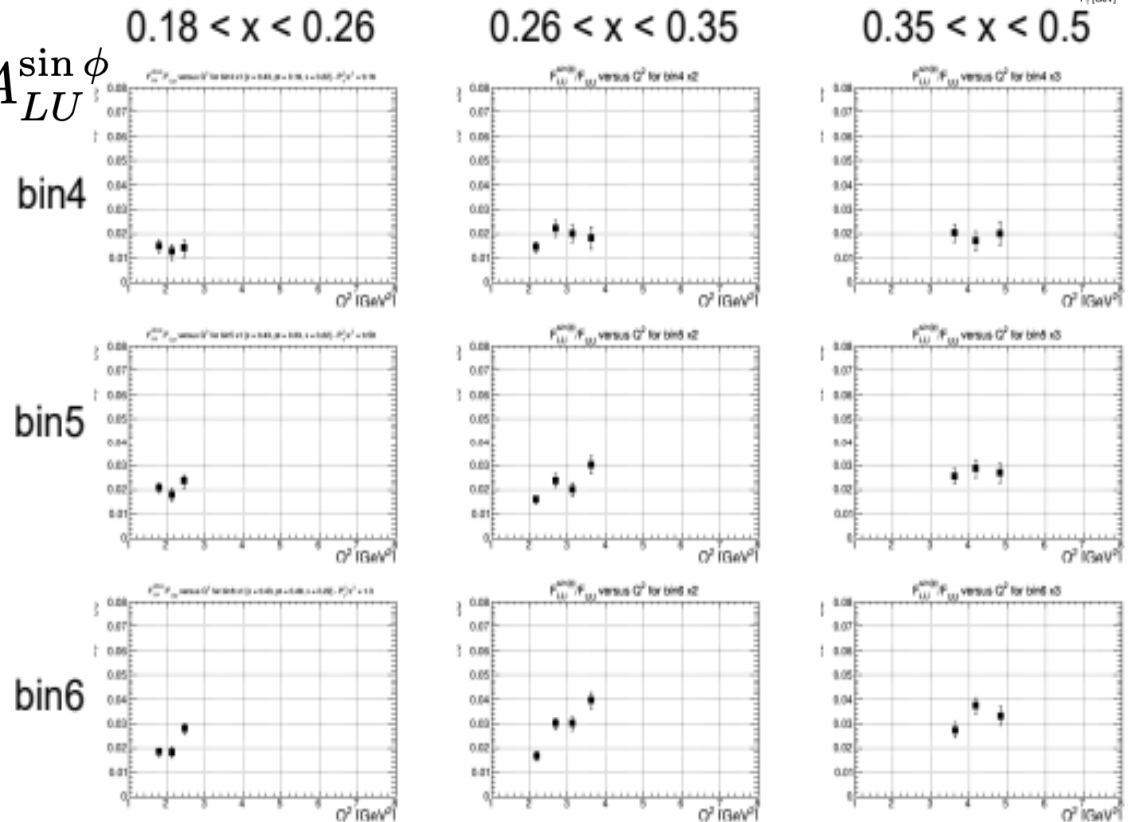


# Attempts to understand $Q^2$ -dependence of HT

CLAS12(preliminary)



$A_{LU}^{\sin \phi}$



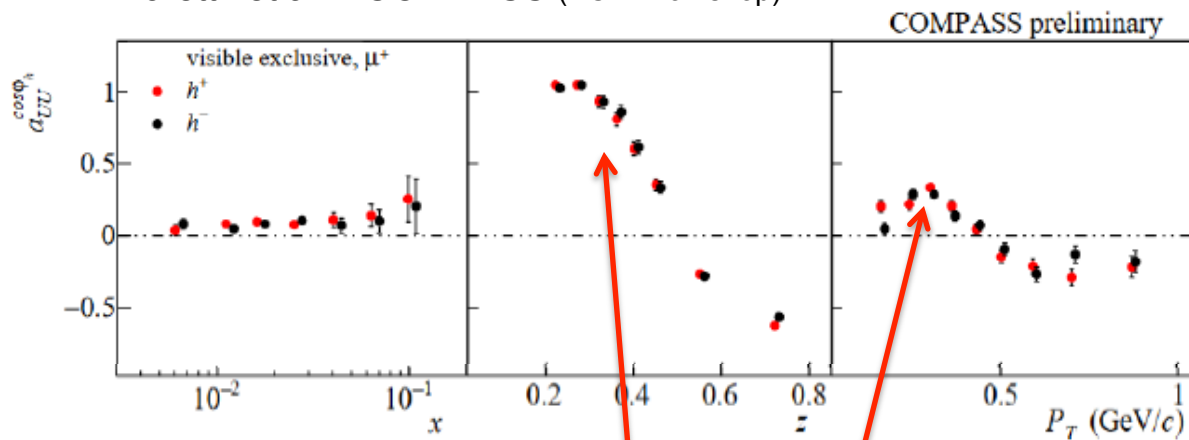
- We always measure ratio to

$$F_{UU,T} + \epsilon F_{UU,L}$$

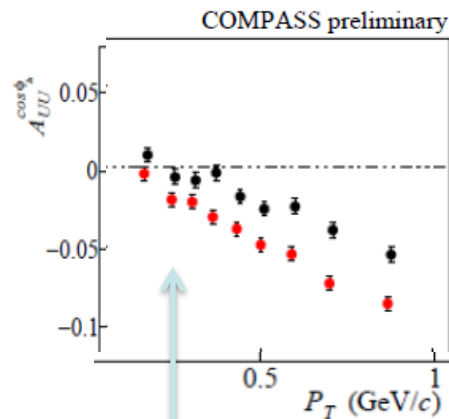
- The moments defined as a ratio to  $\phi$ -independent x-section(to  $F_{UU,T}$ ), are not decreasing with  $Q$ !!!
- The HT observables, don't look much like HT observables, something missing in understanding
- Understanding of these behavior can be a key to understanding of other inconsistencies**
- Checking the  $Q^2$  and  $P_T$ -dependences of the  $F_{UU,L}$  may provide crucial input for validation

# COMAPASS multiplicities and cosine modulations

Moretti et al → COMPASS (ECT\* workshop)

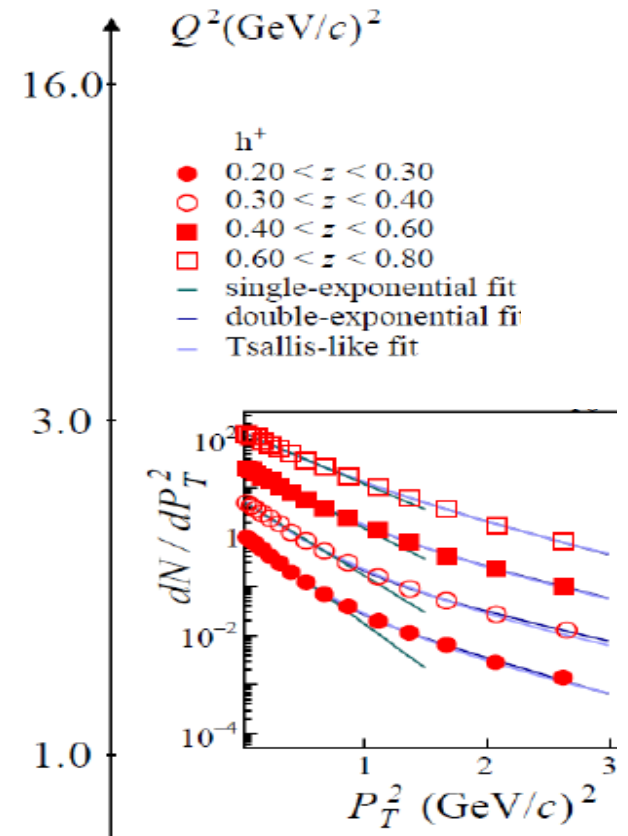


Negative cos of  $\rho^0$  converts to positive for low  $P_T$  pions (sign flip  $\sim z=0.5$ )



Indication of dominant VM contributions in the inclusive hadron samples, in particular at low  $P_T$ , critical for understanding of the QCD dynamics

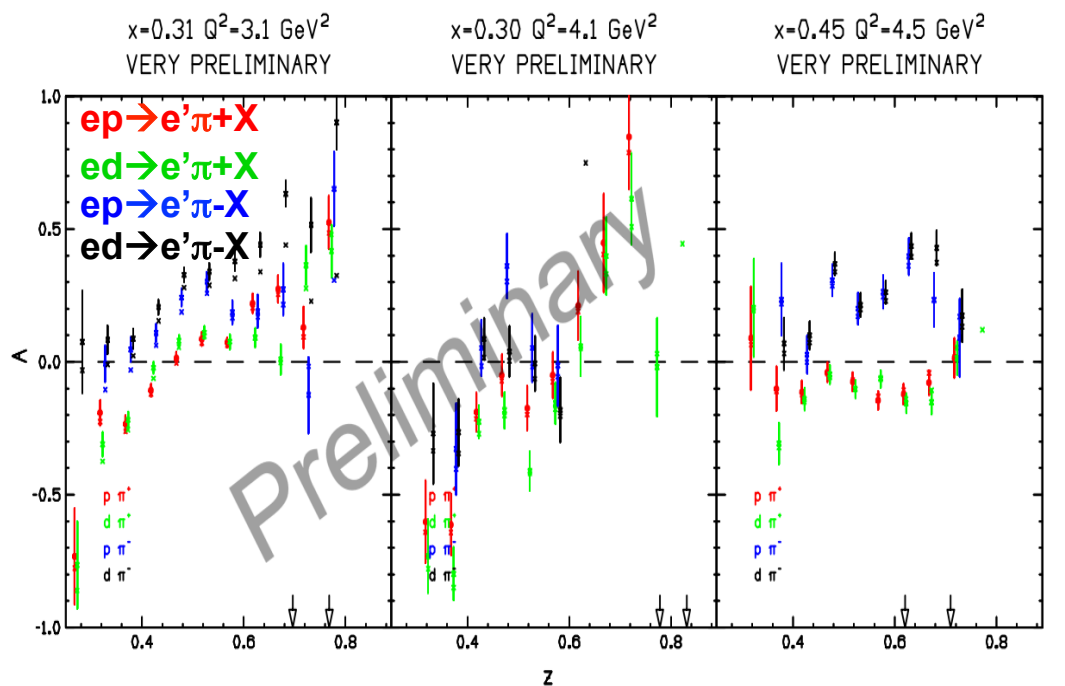
$\rho$ -decay pions mess up linear dependence at low  $P_T$



Theory is not able to explain the large  $P_T$  behavior of pion multiplicities !!!

# sin and cos azimuthal modulations from JLab

P. Bosted et al → Hall-C (ECT\*-2022)

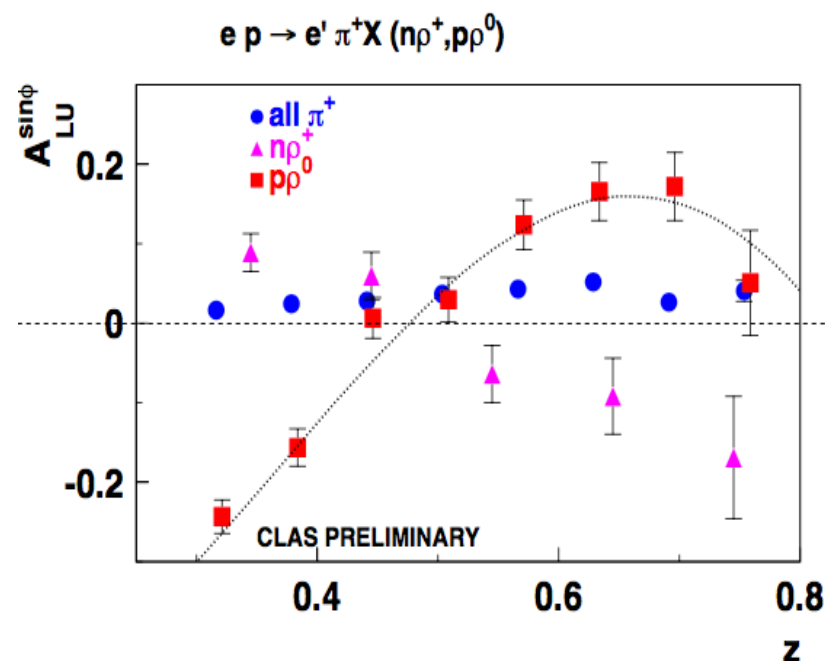


$$y = M_0 b e^{-b p_T^2} (1 + A p_T \cos \phi)$$

- "A" (cosine modulation) generally close to zero or positive.
- Cahn effect should give negative cosine

- Indication of dominant rho contributions, in particular for pions at low  $P_T$ , specially for  $\pi^-$ ,
- Understanding of the production mechanism is critical in understanding of QCD dynamics

H.A. → Hall-B (GDH-2004)

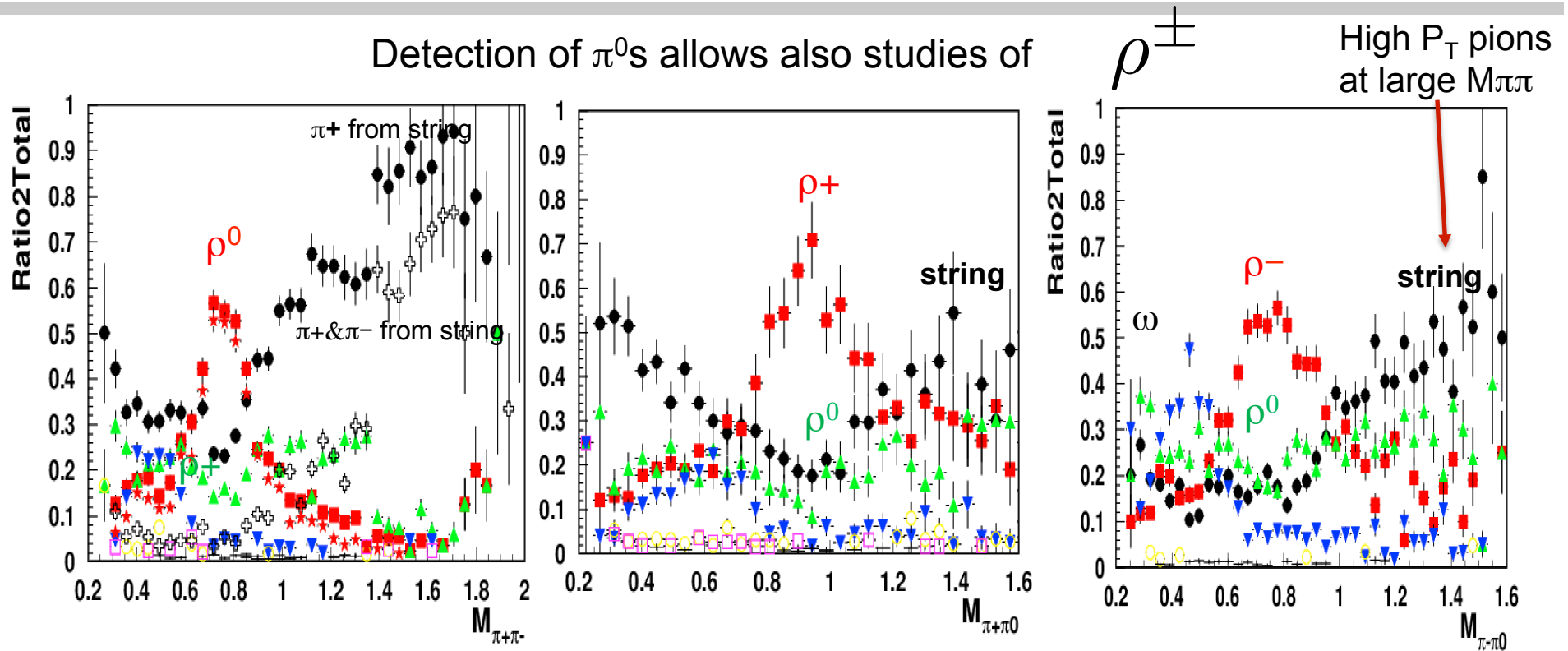


rho decay pions produce pions with SSA flipping the sign  $\sim z=0.5$ )



# Sources of inclusive pions: CLAS12 MC

Detection of  $\pi^0$ s allows also studies of



Dominant fraction of 2 pion combinations come from VM decays

- $\rho$
- string
- $\omega$

All measured 2 pion combinations are dominated by VM decays, indicate that all inclusive pions are dominated by VM decays at small  $P_T$ s, and in particular at lower  $z$ !!!

# SIDIS Validation tests: Collins-Soper kernel

$$\frac{d\sigma}{dQ^2 dx dz dk_{\perp}^2} = \frac{\pi\alpha_{\text{em}}^2(Q)}{Q^4} \frac{y^2}{1-\varepsilon} W(Q, x, z, k_{\perp})$$

$$\int_0^\infty \frac{bdb}{(2\pi)^2} J_0\left(\frac{k_{\perp} b}{z}\right) R[b, Q \rightarrow \mu] |C_V(Q)|^2 \sum_f e_f^2 f_1(x, b; \mu) d_1(z, b; \mu)$$

Evol.factor  
**our goal!**

**TMDs**

A. Vladimirov

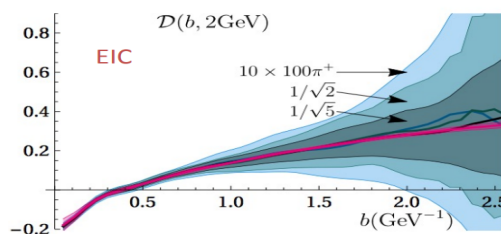
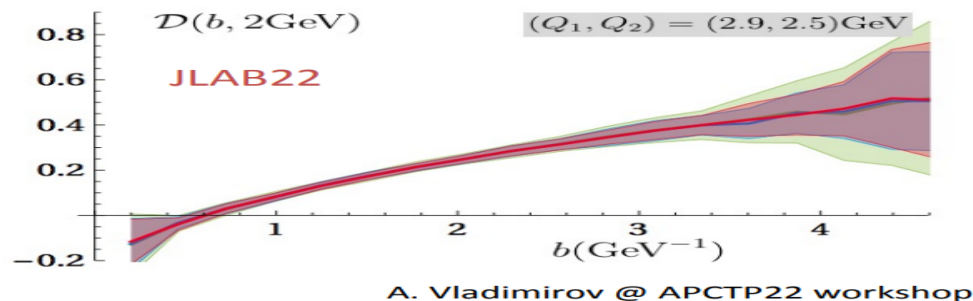
nonperturbative  $Q$  and  $x$  can be factorized

$$F(x, b; Q) = R[\mathcal{D}, Q] F(x, b)$$

- ▶  $R$  is known function
- ▶  $\mathcal{D}$  can be determined directly from data
  - ▶ requires dense coverage in  $p_T$
  - ▶ requires proper adjustments of  $(x, z, Q)$

## ▶ Ultimate test of factorization hypothesis

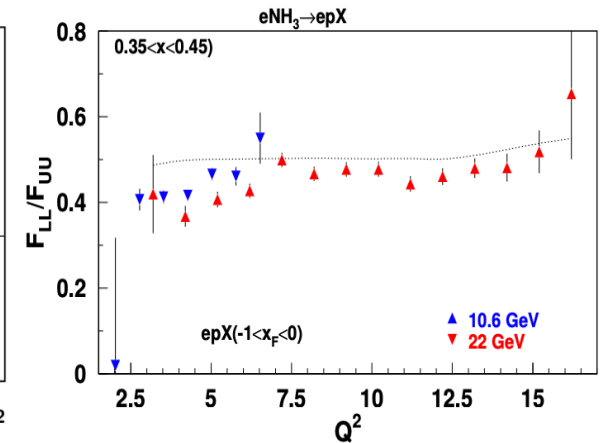
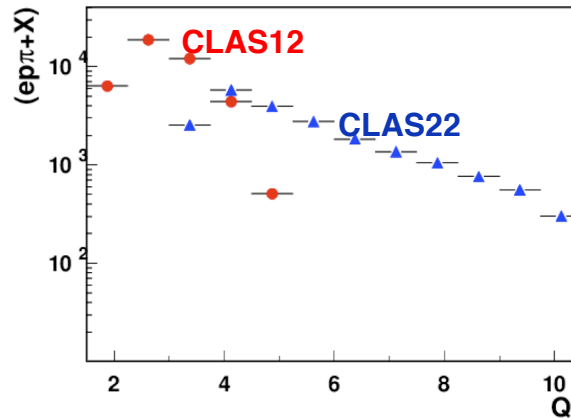
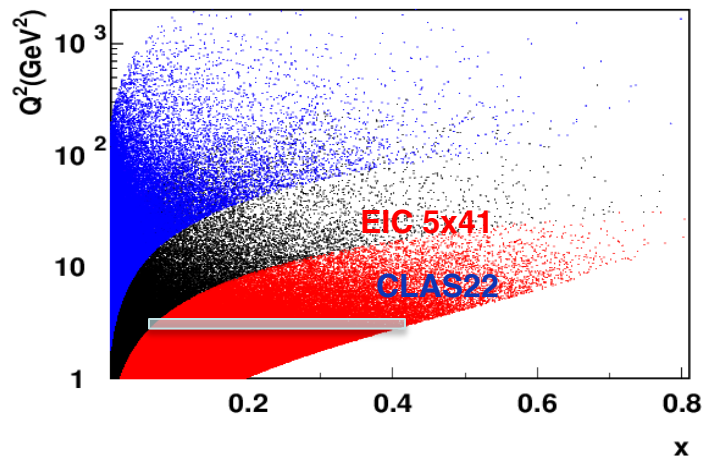
- ▶ Different  $(Q, x, z)$  MUST result into the same curve
  - ▶ Different final states  $(\pi^\pm, K^\pm)$  MUST result into the same curve
- ⇒ comparing Collins-Soper kernel obtained in different regimes we can scan the kinematic range and determine size of **TMD-factorization violation**



Different experiments most sensitive to different ranges in  $b$

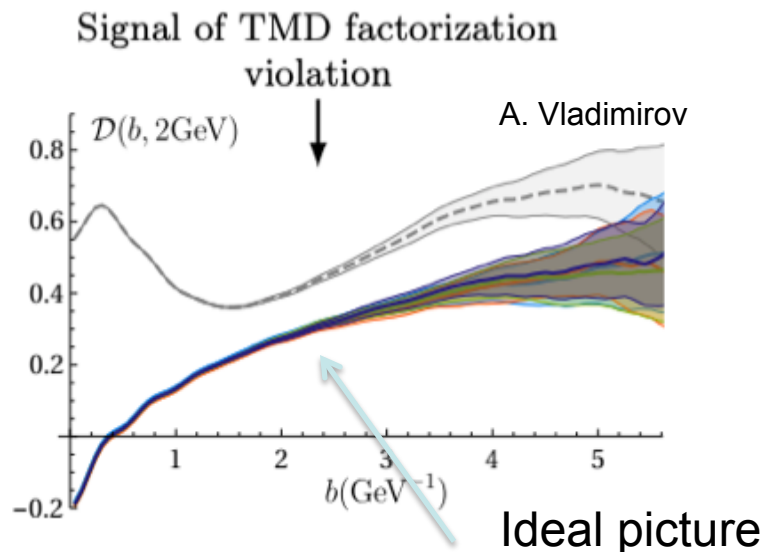
- JLab  $\sim 1 < b < 4$
- EIC  $\sim 0.5 < b < 1.2$
- LHC  $b < 0.5$
- COMPASS overlaps

# Accessing CS-kernel directly or through extraction of SFs



Use slices in  $Q^2$  (good resolution needed)

- Wide  $Q^2$  range and high luminosity is the key for a validating separation of twist-2 contributions



- $Q^2$  evolution studies possible, provide superior access to critical Collins-Soper (CS) kernel
- CLAS12 at JLab20+ can provide a wide range in  $Q^2$  combined with high lumi and superior resolution

- Test the CS-kernel from different experiments, and for different kinematics in a given experiment
- Evaluate the systematics due to factorization violation and define possible reasons (some can be easy to fix)

# Theory-Experiment: SIDIS Validation tests

$$\frac{d\sigma^{lN \rightarrow l' h X}}{dx_B dQ^2 dz dP_{h\perp}^2 d\phi_h} = \frac{\pi\alpha^2 y}{x_B y Q^2 (1-\epsilon)} [F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} F_{UU}^{\cos\phi_h} \cos\phi_h + \dots]$$

$$\left. \begin{array}{l} lN \rightarrow l' \pi^+ X \\ lN \rightarrow l' \rho^0 X \\ \dots \end{array} \right\} lN \rightarrow l' \pi^+ X$$

Fragmentation Function  $\rightarrow$  probability to produce  $\pi^+$

$$F_{XY}^h(x, z, P_T, Q^2) \propto \sum H^q \times f^q(x, k_T, ..) \otimes D^{q \rightarrow h}(z, p_T, ..) + Y(Q^2, P_T) + \mathcal{O}(M/Q)$$

- All azimuthal modulations, including SSA increase with  $P_T$ , and contributions with direct pions also increase with  $P_T$ , dominating from  $P_T > 0.7-0.8$  ( $F_{UU,L}/F_{UU,T}$  maybe  $> 1$  at large  $P_T$ )
- Fits to data were predominantly performed at low  $P_T$  ( $P_T < 0.8$ )
- Azimuthal modulations of cosine type most likely have the same sign for pions and rhos, while sine-type (ex. Collins) likely to have opposite signs
- Decay pions will have completely different azimuthal dependences, changing the sign from lower to higher  $z$  (energy fractions), may have different  $Q^2$ -dependences
- The effective fragmentation functions will have very complex dependence on kinematics, and will require precision multidimensional measurements to understand

What exactly we learn about the spin-dependent hadronization from extraction of these functions?

Suggestion  $\rightarrow$  study SIDIS at larger  $P_T$  ( $0.8 < P_T < 1.5$ ) with negligible rho contributions

# SUMMARY

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- Measurements of azimuthal modulations of inclusive pions, and multiplicities of pion pairs indicate very significant part of hadrons come from decays of VMs (even more in kaon case) and can provide important insight in understanding the “leading twist” observables
- Evolution studies and understanding the production mechanism will help in defining the validity region of the formalism, which may be much wider than currently anticipated
- To evaluate the systematics of extracted TMDs, it is critical to validate the formalism, and understand main contributions violating the factorized picture based on the dominance of the leading twist contributions

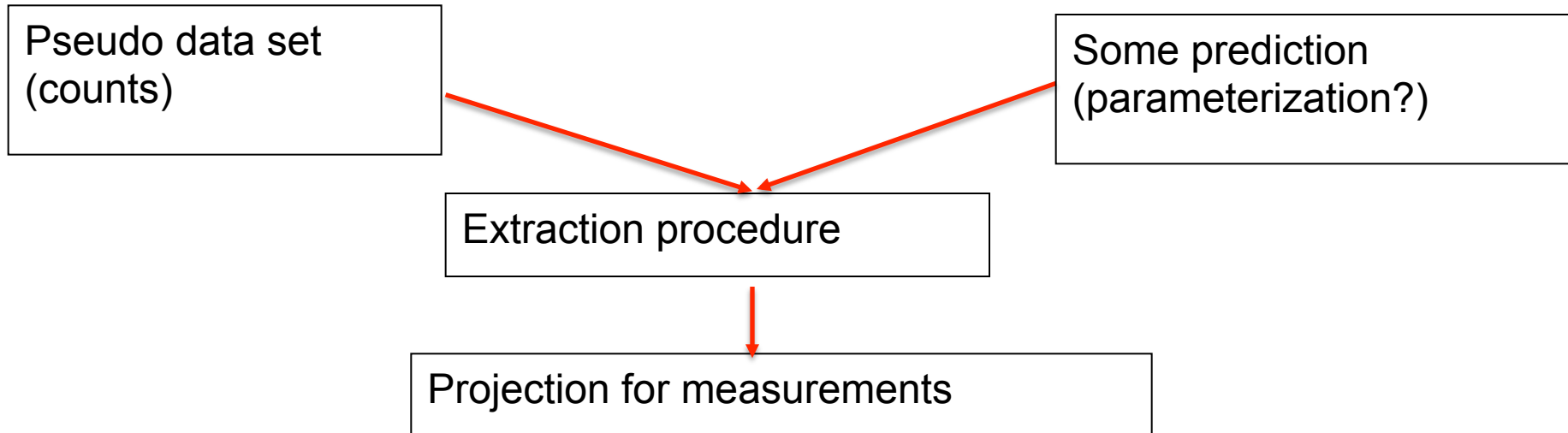
We need analysis frameworks with controlled systematics to make credible projections for future measurements!!!



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# Support slides

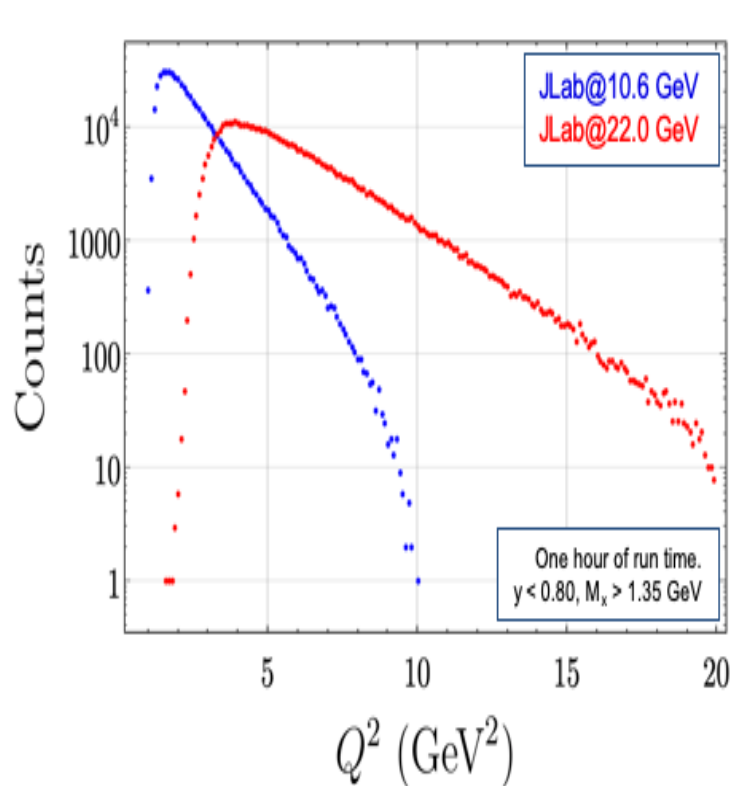
# Making projections: extraction procedure



Extraction procedure should have clear definition of systematics related to assumptions and approximations!!!!

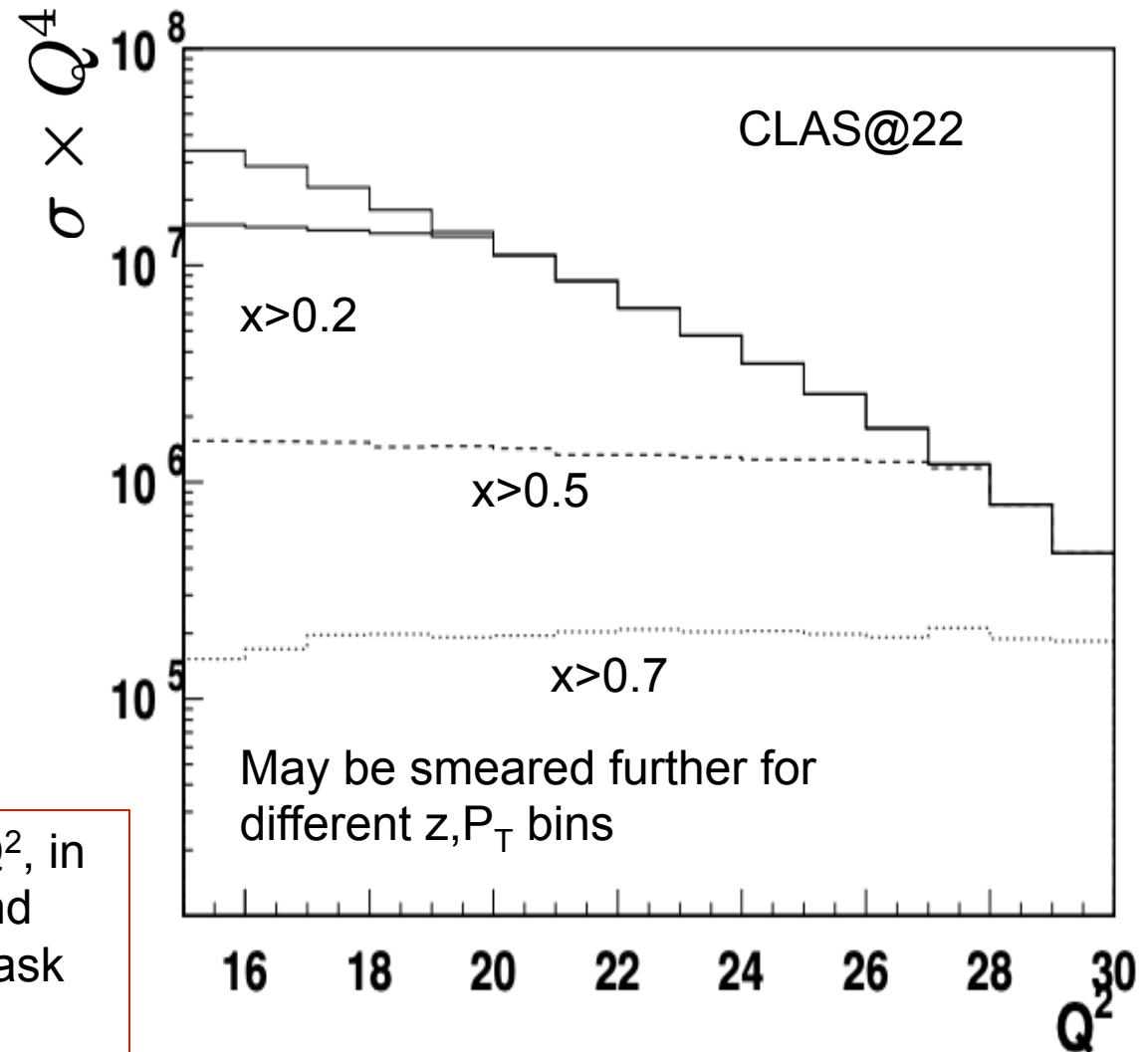
- The role of multidimensional measurements should be well defined, accounted in the extraction
  - The same parameterization used in production of data and extraction of TMDs will have practically unconstrained systematics
  - Using statistical errors from simulation to evaluate the errors on a given TMDs can produce absolutely unrealistic projections, in particular in boundaries.

# Finite energy: Kinematic limitations



Kinematic correlations, ( $P_T$  and  $Q^2$ , in particular) due to trivial energy and momentum conservation, may mask the real dependences

- Can be easily accounted

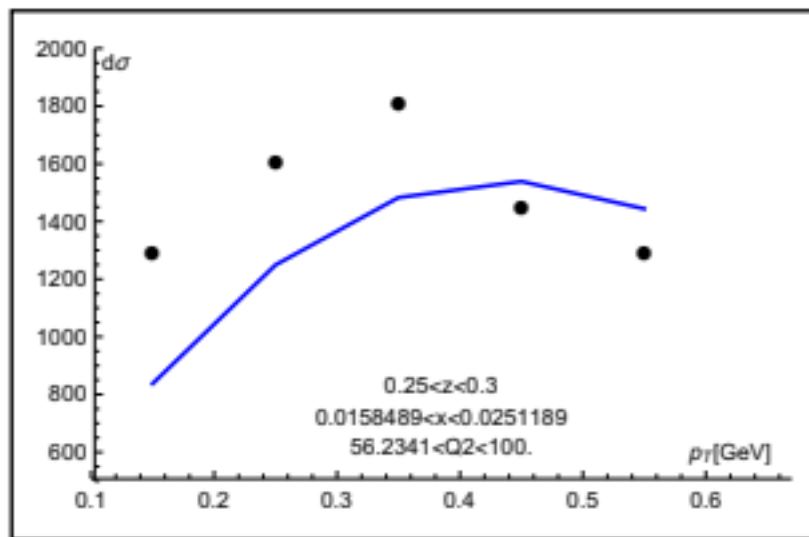


# What we learned: missing parts of the mosaic

- SIDIS, with hadrons detected in the final state, from experimental point of view, is a measurement of observables in 5D space  $(x, Q^2, z, P_T, \phi)$ , 6D for transverse target,  $+\phi_S$   
Collinear SIDIS, is just the proper integration, over  $P_T, \phi, \phi_S$
- SIDIS observations relevant for interpretations of experimental results:
  1. Understanding the kinematic domain where non-perturbative effects of interest are significant (ex.  $x, P_T$ -range)
  2. Understanding of  $P_T$ -dependences of observables in the full range of  $P_T$  dominated by non-perturbative physics is important
  3. Understanding of phase space effects is important (additional correlations)
  4. Understanding the role of vector mesons is important
  5. Understanding of evolution properties and longitudinal photon contributions
  6. Understanding of radiative effects may be important for interpretation
  7. Overlap of modulations (acceptance, RC,...) is important in separation of SFs
  8. Multidimensional measurements with high statistics, critical for separation of different ingredients
- QCD calculations may be more applicable at lower energies when 1)-7) clarified
- Need a realistic chain for MC simulations of SIDIS to produce realistic projections with controlled systematics

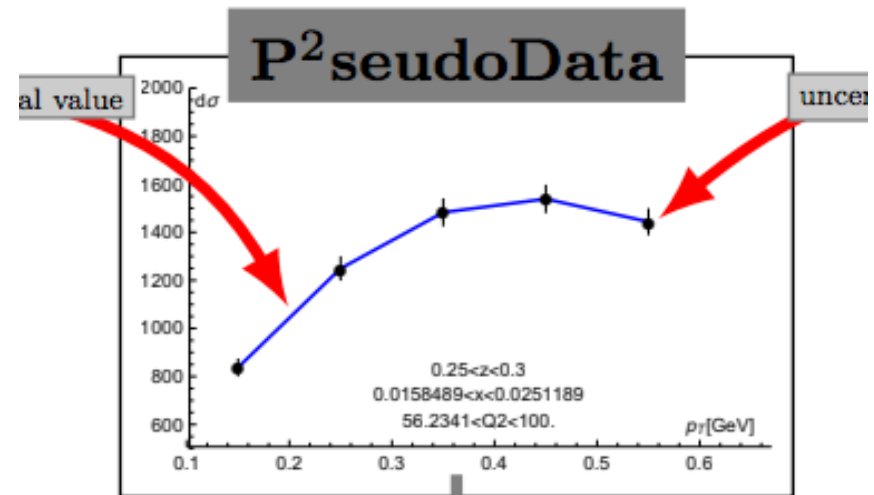
# Tests of formalism crucial

How it works in theory



**Does not work!**

$$\chi^2 / N_{pt} \gg 1$$



$$\chi^2 = 0$$