New approach to color-coherent parton evolution

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Resummation, Evolution, Factorization 2022

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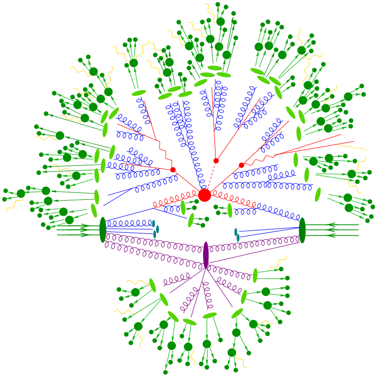
LHC event generators

[Buckley et al.] arXiv:1101.2599
[Campbell et al.] arXiv:2203.11110



- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1p_2\to X} = \sum_{i,j\in\{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1,\mu_F^2) f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij\to X}(x_1 x_2,\mu_F^2)}_{\text{short distance}}$$

Connection to QCD theory

- $\hat{\sigma}_{ij\to n}(\mu_F^2) \to$ Collinearly factorized fixed-order result at N^{x}LO
  Implemented in fully differential form to be maximally useful
- $f_i(x,\mu_F^2) \to$ Collinearly factorized PDF at N^{y}LO
  Evaluated at $O(1\mathrm{GeV}^2)$ and expanded into a series above 1GeV^2
  DGLAP: $\frac{\mathrm{d}x\,xf_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\,\frac{\alpha_s}{2\pi}\big[zP_{ab}(z)\big]_+\,\tau f_b(\tau,t)\,\delta(x-\tau z)$
  Implemented by parton showers, dipole showers, antenna showers, ...
  - In the spotlight recently due to missing systematic error budget
    - Logarithmic precision [PanScales,Deductor,Herwig,Sherpa,...]
    - Higher-order corrections [Vincia,Sherpa,Herwig,...]
  - Uncontrollable effects from on-shell kinematics mapping
  - Various possibilities of matching to collinear limit

$$P_{aa}(z) = C_a\,\frac{2z}{1-z} + \ldots \qquad \leftrightarrow \qquad J^\mu = \sum_i \mathbf{T}_i\,\frac{p_i^\mu}{p_ip_j}$$

This talk: Novel soft-collinear matching & kinematics mapping

The semi-classical matrix element – Angular ordering

[Marchesini,Webber] NPB310(1988)461

- Soft gluon radiator can be written in terms of energies and angles

$$J_\mu J^\mu \to \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{W_{ik,j}}{E_j^2}$$

Angular "radiator" function

$$W_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})}$$

- Divergent as $\theta_{ij} \to 0$ and as $\theta_{jk} \to 0$

$\to$ Expose individual collinear singularities using $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

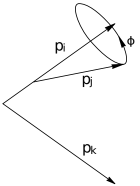
$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left[\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{kj})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{kj}}\right]$$

  - Divergent as $\theta_{ij} \to 0$, but regular as $\theta_{kj} \to 0$
  - Convenient properties upon integration over azimuthal angle

The semi-classical matrix element – Angular ordering

- Work in a frame where direction of $\vec{p}_i$ aligned with z-axis

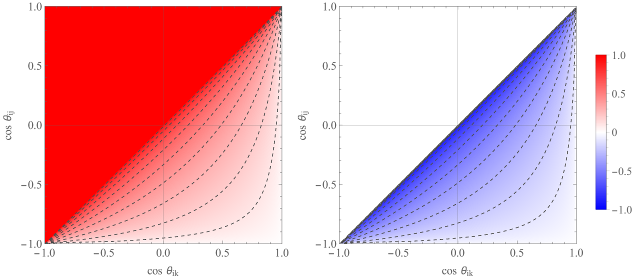
$$\cos\theta_{kj} = \cos\theta_k^i \cos\theta_j^i + \sin\theta_k^i \sin\theta_j^i \cos\phi_{kj}^i$$



- Integration over ϕ_j yields

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^i \tilde{W}_{ik,j}^i = \frac{1}{1-\cos\theta_j^i} \times \begin{cases} 1 & \text{if} \quad \theta_j^i < \theta_k^i \\ 0 & \text{else} \end{cases}$$

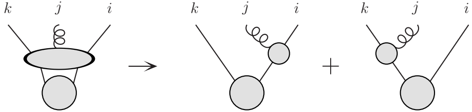
  - On average, no radiation outside cone defined by parent dipole
  - Differential radiation pattern more intricate:
    Positive & negative contributions outside cone sum to zero



The semi-classical matrix element – Alaric approach

- Alternative approach: partial fraction matrix element & match to collinear sectors [Ellis,Ross,Terrano] NPB178(1981)421, [Catani,Seymour] hep-ph/9605323

$$\frac{W_{ik,j}}{E_j^2} \rightarrow \frac{1}{p_ip_j}\frac{p_ip_k}{(p_i+p_k)p_j} + \frac{1}{p_kp_j}\frac{p_ip_k}{(p_i+p_k)p_j}$$



  - Captures matrix element both in angular ordered and unordered region
  - Caveat: Oversampling difficult for certain kinematics maps

- Separate into energy & angle first [Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057 Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

$$\bar{W}^i_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{kj})}$$

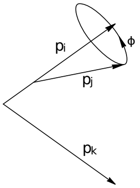
  - Bounded by $(1-\cos\theta_{ij})\bar{W}^i_{ik,j} < 2$
  - Strictly positive

The semi-classical matrix element – Alaric approach

- Integration over ϕ_j yields

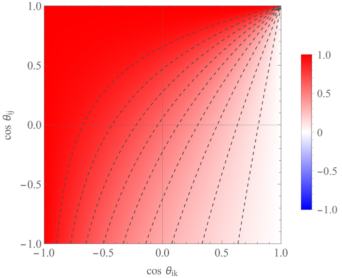
$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^i \bar{W}_{ik,j}^i = \frac{1}{1-\cos\theta_j^i}\frac{1}{\sqrt{(\bar{A}_{ij,k}^i)^2 - (\bar{B}_{ij,k}^i)^2}}$$

  - Radiation across all of phase space
  - Probabilistic radiation pattern

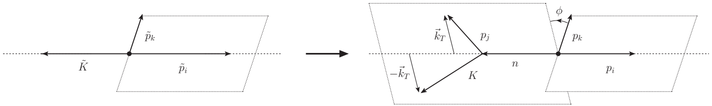


$$\bar{A}_{ij,k}^i = \frac{2-\cos\theta_j^i(1+\cos\theta_k^i)}{1-\cos\theta_k^i}$$

$$\bar{B}_{ij,k}^i = \frac{\sqrt{(1-\cos^2\theta_j^i)(1-\cos^2\theta_k^i)}}{1-\cos\theta_k^i}$$



Kinematics mapping – Alaric approach



- In collinear limit, splitting kinematics defined by ($n \to$ auxiliary vector)

$$p_i \xrightarrow{i||j} z\,\tilde{p}_i\;, \qquad p_j \xrightarrow{i||j} (1-z)\,\tilde{p}_i \qquad \text{where} \qquad z = \frac{p_i n}{(p_i+p_j)n}$$

- Parametrization, using hard momentum $\tilde{K}$

$$p_i = z\,\tilde{p}_i\;, \qquad n = \tilde{K} + (1-z)\,\tilde{p}_i$$

- Using on-shell conditions & momentum conservation ($\kappa = \tilde{K}^2/(2\tilde{p}_i\tilde{K})$)

$$\begin{aligned} p_j &= (1-z)\,\tilde{p}_i + v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) + k_\perp \\ K &= \qquad\qquad \tilde{K} - v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) - k_\perp \end{aligned}$$

- Momenta in $\tilde{K}$ Lorentz-boosted to new frame K [Catani,Seymour] hep-ph/9605323

$$p_l^\mu \to \Lambda^\mu{}_\nu(K,\tilde{K})\,p_l^\nu\;, \qquad \Lambda^\mu{}_\nu(K,\tilde{K}) = g^\mu{}_\nu - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})_\nu}{(K+\tilde{K})^2} + \frac{2\tilde{K}^\mu K_\nu}{K^2}\;.$$

Logarithmic accuracy – The $\alpha_s \to 0$ limit

- Framework to quantify log accuracy of parton showers established in [Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] arXiv:1805.09327, arXiv:2002.11114
- Example: Thrust or $FC_{1-\beta}$ in $e^+e^- \to$hadrons

- Define a shower evolution variable $\xi = k_T^2/(1-z)$
- Parton-shower one-emission probability for $\xi > Q^2\tau$

$$R_{\rm PS}(\tau) = 2\int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \int_{z_{\rm min}}^{z_{\rm max}} dz\, \frac{\alpha_s\big(k_T^2\big)}{2\pi}\, C_F \left[\frac{2}{1-z} - (1+z)\right] \Theta(\eta)$$

- Approximate to NLL accuracy

$$R_{\rm NLL}(\tau) = 2\int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \left[\int_0^1 dz\, \frac{\alpha_s\big(k_T^2\big)}{2\pi} \frac{2\,C_F}{1-z}\Theta(\eta) - \frac{\alpha_s(\xi)}{\pi}\, C_F B_q\right]$$

Logarithmic accuracy – The $\alpha_s \to 0$ limit

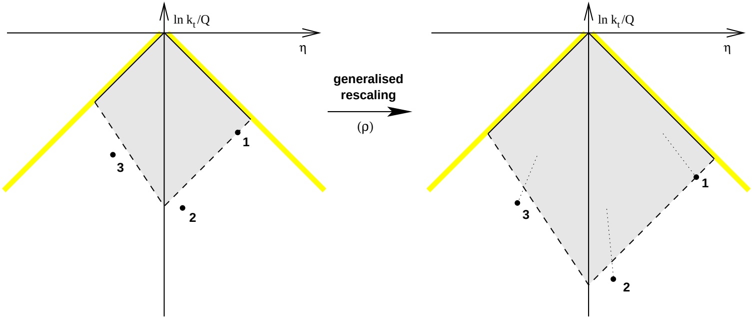
- Cumulative cross section $\Sigma(\tau) = e^{-R(\tau)}\mathcal{F}(\tau)$ obtained from all-orders resummed result by Taylor expansion of virtual corrections in cutoff ε

$$\mathcal{F}(\tau) = \int \mathrm{d}^3k_1 |M(k_1)|^2 \, e^{-R' \ln \frac{\tau}{\varepsilon v_1}} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=2}^{m+1} \int_{\varepsilon v_1}^{v_1} \mathrm{d}^3k_i |M(k_i)|^2 \right) \times \Theta\big(\tau - V(\{p\}, k_1, \ldots, k_n)\big)$$

  - $\mathcal{F}(\tau)$ is pure NLL & accounts for (correlated) multiple-emission effects
- In order to make $\mathcal{F}(\tau)$ calculable, make the following assumptions
  - Observable is recursively infrared and collinear safe
  - Hold $\alpha_s(Q^2)\ln\tau$ fixed, while taking limit $\tau \to 0$
    $\to$ Can factorize integrals and neglect kinematic edge effects
- Breaks momentum conservation and unitarity for finite τ
  $\to$ Clean NLL result, but unknown kinematic corrections
  [Reichelt,Siegert,SH] arXiv:1711.03497

Logarithmic accuracy – Analytic proof

[Banfi,Salam,Zanderighi] hep-ph/0407286



- $\alpha_s \to 0$ limit corresponds to similarity transformation of Lund plane
- Can be parametrized in terms of scaling parameter ρ
  a, b – observable-dependent resummation parameters

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln\rho}{a+b}\,, \qquad \text{where} \qquad \xi = \frac{\eta}{\eta_{\max}}$$

- NLL precision requires this to be maintained after full evolution

Logarithmic accuracy – Analytic proof

- Lorentz transformation defined by shift $\tilde{K} \to K$

$$K^\mu = \tilde{K}^\mu - X^\mu \;, \qquad \text{where} \qquad X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$

- X is small, but is it small enough? Rewrite

$$\Lambda^\mu_{\ \nu}(K,\tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

- In NLL limit, coefficients scale as

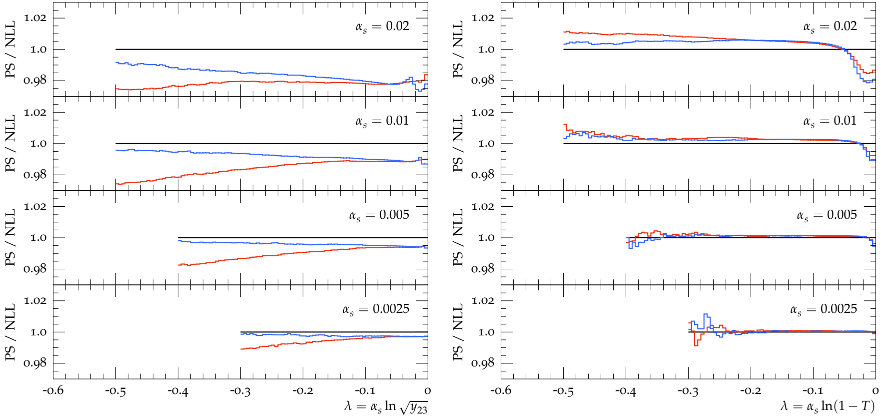
$$A^\nu \overset{\rho\to 0}{\longrightarrow} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2}\;, \qquad \text{and} \qquad B^\nu \overset{\rho\to 0}{\longrightarrow} \frac{\tilde{K}^\nu}{\tilde{K}^2}\;.$$

- Relative momentum shift of soft emission particle l becomes

$$\begin{aligned}
\Delta p_l^0/\tilde{p}_l^0 &\sim 2X^0 + \rho^{1-\max(\xi_i,\xi_j)}\,\tilde{K}^0 &&\sim \rho^{1-\max(\xi_i,\xi_j)}\\
\Delta p_l^3/\tilde{p}_l^3 &\sim X^3 &&\sim \rho^{1-\max(\xi_i,\xi_j)}\\
\Delta p_l^{1,2}/\tilde{p}_l^{1,2} &\sim \rho^{-\xi_l}X^{1,2} &&\sim \rho^{1-\xi_l}
\end{aligned}$$

- For hard momenta, leading terms in X^μ cancel exactly
  Remaining components scale as ρ or stronger

Logarithmic accuracy – Numerical checks

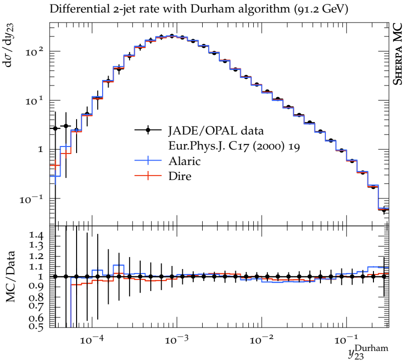


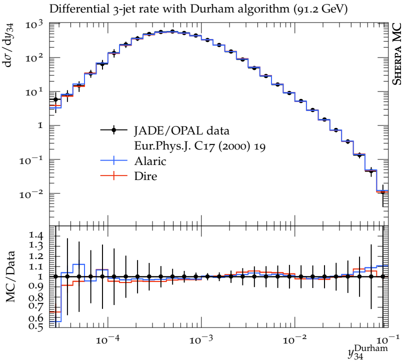
- Durham $2 \to 3$ rate in Cambridge jets and thrust in $e^+e^- \to$ hadrons
- At fixed $\lambda = \alpha_s \log v$, deviation from NLL should scale as α_s
- Dire algorithm (red) fails, Alaric (blue) passes

Comparison to experimental data: LEP I

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

- Comparison to experimental data from LEP

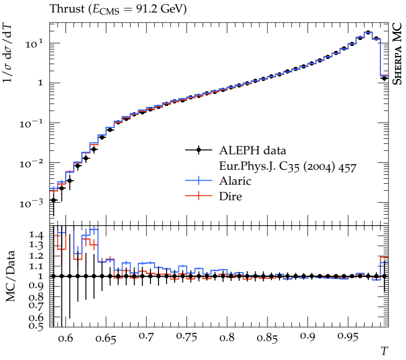


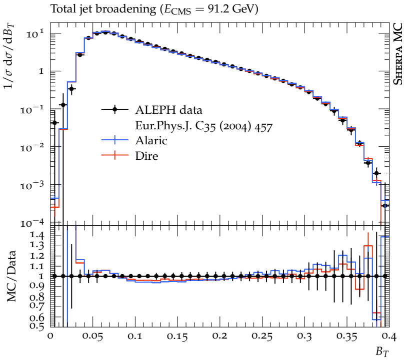


Comparison to experimental data: LEP I

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

- Comparison to experimental data from LEP

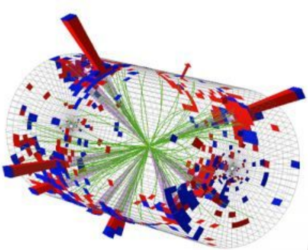


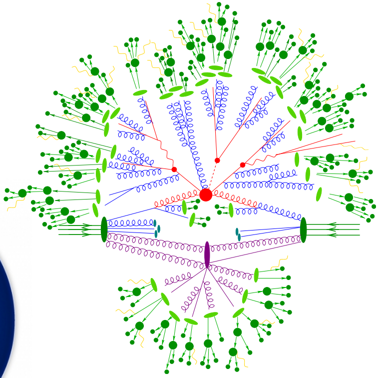


Summary and Outlook

- Lots of activity in parton shower development right now
  - Logarithmic precision [PanScales,Deductor,Herwig,Sherpa,...]
  - Higher-order kernels [Vincia,Sherpa,Herwig,...]
  - Interplay w/ NNLL [PanScales,...]
- New Alaric scheme contributes
  - Intuitive understanding & connection to angular ordering
  - Simple, analytic proof of NLL precision
  - Unified treatment of FSR & ISR
  - Easy matching to NLO
- Next steps
  - Spin correlations & $1/N_c$ terms
  - Higher-order splitting functions
  - Heavy flavor evolution

Exciting times ahead!

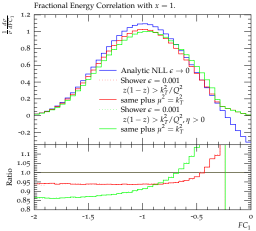




$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + h.c.$$

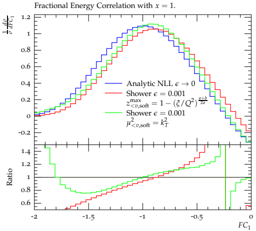
Numerical effects away from the $\alpha_s \to 0$ limit

[Reichelt,Siegert,SH] arXiv:1711.03497



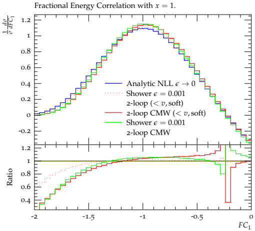
Single emission effects

- 4-mom conservation
- PS sectorization
- k_T scale in coll. terms



Multiple emission effects

- z bounds by unitarity
- k_T scale by unitarity



Effects of scale choice

- 2-loop CMW in all soft terms
- 2-loop CMW overall

- Simplest process and simplest type of observable, still sizable differences away from $\tau \to 0$ limit
- How do we proceed to quantify precision in the intermediate region (“between” NLL and NLO) ?