# Monopoles in Cosmology
# DESY Theory Workshop Seminar

Jorinde van de Vis

February 22, 2022

## 1 Introduction

The gauge group of the Standard Model of particle physics is $SU(3)_c \times SU(2)_L \times U(1)_Y$. At energies below $\sim$ 100GeV, the electroweak gauge group $SU(2)_L \times U(1)_Y$ gets broken to $U(1)_{\rm em}$. It is conceivable that the Standard Model gauge group is also the product of the symmetry breaking of some theory with higher symmetry. A theory in which the gauge groups of the Standard Model get unified at some high energy scale, is called a Grand Unified Theory (GUT). Many different kinds of GUTs have been proposed, based on different gauge groups (e.g. $SU(5), SO(10), E(6), ...$ [1, 2, 3] respectively). The energy scale at which grand unification occurs depends on the model, but is usually around $10^{15}$GeV

Since the temperature of the early universe was very high (we know that the temperature was 10MeV during Big Bang Nucleosynthesis, but it could have been many orders of magnitude higher before), it is possible that the GUT symmetry was restored. As we will see in this seminar, magnetic monopoles might have been formed in the GUT-breaking phase transition. Unfortunately, the density of monopoles is predicted to be much too large to be consistent with observations. This is the so-called monopole problem, for which inflation forms the most well-known solution.

The outline of these lecture notes is as follows:

- Some basic relations of cosmology are summarized in Section 2.
- Section 3 discusses the formation of magnetic monopoles in a cosmological phase transition, and their subsequent evolution.
- In Section 4 two solutions to the monopole problem are discussed.

Some reviews on monopoles in cosmology can be found in Chapter 8 of [4], Chapter 7 of [5], and [6] and [7].

## 2 FRW cosmology

In this section, we will summarize some cosmological relations that are important for the rest of the discussion. The treatment is based on [8]. We will describe the spacetime of the expanding universe by the famous Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \,, \tag{1}$$

where $a(t)$ denotes the scale-factor and we have assumed that the universe is spatially flat, isotropic and homogeneous. The scale factor relates so-called comoving coordinates $x^i = \{x^1, x^2, x^3\}$ to physical coordinates $x^i_{\rm phys} = a(t)x^i$. The physical velocity of an object thus gets two contributions:

$$v^i_{\rm phys} = \frac{dx^i_{\rm phys}}{dt} = a(t)\frac{dx^i}{dt} + \frac{da}{dt}x^i \equiv v^i_{\rm pec} + Hx^i_{\rm phys} \,, \tag{2}$$

where the first contribution is the peculiar velocity, which is measured by a comoving observer, and the second is called the Hubble flow, with the Hubble parameter defined as

$$H \equiv \frac{\dot{a}}{a} \,. \tag{3}$$

We will now assume that the constituents of the universe can be described by the stress-energy tensor of a perfect fluid

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu - Pg_{\mu\nu} \,, \tag{4}$$

where $\rho$ is the energy density of the fluid, $P$ the pressure and $u^\mu$ the four-velocity. The 0-th component of energy-momentum conservation $\partial_\mu T^{\mu\nu}$ in a FRW-background yields the following continuity equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0 \,. \tag{5}$$

The relation between the pressure and energy density defines the equation of state $w$

$$w \equiv \frac{P}{\rho} \,. \tag{6}$$

The most commonly encountered equations of state are cold (dark) matter ($w = 0$), radiation ($w = 1/3$) and vacuum energy ($w = -1$). Plugging these relations into the continuity equations, we find

$$\rho \propto a^{-3} \,, \qquad \text{matter} \,, \tag{7}$$

$$\rho \propto a^{-4} \,, \qquad \text{radiation} \,, \tag{8}$$

$$\rho \propto a^{0} \,, \qquad \text{vacuum energy} \,. \tag{9}$$

By plugging the stress-energy tensor of the perfect fluid into the Einstein equations in the FRW background, we find the Friedmann equations

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{1}{3M_P^2}\rho, \tag{10}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = -\frac{1}{6M_P^2}(\rho + 3P) \,, \tag{11}$$

with $G$ Newton's constant and $M_P = (8\pi G)^{-1/2} = 2.435 \times 10^{18}\text{GeV}$ the reduced Planck mass. From the Friedmann equations, we find for a single component universe (with $w \neq -1$)

$$a(t) \propto t^{\frac{2}{3(1+w)}} . \tag{12}$$

**Radiation** The energy density of a relativistic gas of particles depends on the temperature via

$$\rho = \frac{\pi^2}{30} g_*(T) T^4 , \tag{13}$$

where $g_*(T)$ denotes the number of relativistic degrees of freedom at temperature $T$. Combining eqs. (8), (12) and (13) for a radiation-dominated universe, we find that

$$\rho \propto t^{-2} , \qquad T \propto t^{-1/2} , \tag{14}$$

implying that the energy density and temperature of the early universe increase as we go back in time.

# 3 Formation of monopoles: Kibble-Zurek mechanism

The Kibble-Zurek mechanism was proposed by Kibble [9] in 1976 (see also [10]) and refined by Zurek in 1985 [11]. Their works describe how magnetic monopoles (and other cosmological defects) can get formed in spontaneous symmetry breaking of GUTs, and estimate the value of the correlation length around the critical temperature, which sets the density of the monopoles.

## 3.1 Symmetry-breaking phase transition

As we have seen above, for a radiation-dominated universe, the temperature could be very high in the early universe. At large temperature, the effective potential gets loop corrections from interactions with the thermal bath. Following [9] we consider a gauge theory described by

$$\mathcal{L} = \frac{1}{8} \mathrm{Tr} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} D_\mu \phi \cdot D^\mu \phi - V(\phi) , \tag{15}$$

with $\phi$ belonging to the $N$-dimensional vector representation of $O(N)$[1], and

$$D_\mu \phi = \partial_\mu \phi - e B_\mu \phi , \tag{16}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\mu B_\nu + e[B_\mu, B_\nu] , \tag{17}$$

$$V(\phi) = \frac{\lambda}{8} (\phi^2 - \eta^2)^2 , \tag{18}$$

[1]This is a simpler set-up than the Georgi-Galshow $SU(5)$ GUT [1], in which the heavy symmetry-breaking scalar is taken to be a scalar in the adjoint representation.

at zero temperature the $O(N)$ symmetry is spontaneously broken to $O(N-1)$ by the vacuum expectation value of $\phi$, which satisfies (at tree level)

$$\langle\phi\rangle^2 = \eta^2 \,. \tag{19}$$

At finite temperature the potential obtains one-loop corrections from interactions with the thermal plasma. We neglect the fermions. At leading order in the temperature the potential becomes

$$V_T(\phi) = \frac{\lambda}{8}(\phi^2 - \eta^2)^2 + \frac{1}{48}[\lambda(N+2) + 6(N-1)e^2]T^2\phi^2 \,. \tag{20}$$

A potential of this shape gives rise to a second-order phase transition. For $T > T_c$, with

$$T_c = \eta\left(\frac{N+2}{12} + \frac{N-1}{2}\frac{e^2}{\lambda}\right)^{-1/2} , \tag{21}$$

the minimum of the potential occurs at $\phi = 0$. Below that temperature, the field acquires a vacuum expectation value

$$\langle\phi\rangle^2 = \eta^2\left(1 - \frac{T^2}{T_c^2}\right) . \tag{22}$$

The vev thus spontaneously breaks the $O(N)$-symmetry to $O(N-1)$ symmetry.

### 3.2 When do monopoles get formed?

Suppose that some gauge theory with symmetry group $G$ gets spontaneously broken down to a subgroup $H$. 't Hooft-Polyakov monopoles arise when the second homotopy group $\pi_2(G/H)$ is nontrivial. This occurs when a Grand Unified Theory $G = SU(5), SO(10), ...$ gets broken to the Standard Model gauge group $H = SU(3)_C \times SU(2)_L \times U(1)_Y$, for which $\pi_2(G/H) = \mathbb{Z}$ . Such a nontrivial second homotopy group does *not* occur when the original group contains a $U(1)$, implying that no 't Hooft-Polyakov monopoles are formed during the electroweak phase transition. See Jeremy's notes for a more extensive discussion.

**Quick reminder of 't Hooft-Polyakov monopoles** Let us consider a Higgs-field $\varphi$ in the vector representation of $SO(3)$. The monopole corresponds to the 'Hedgehog' field configuration.

$$\varphi^a(\vec{r}) = v f(r)\frac{r_a}{r} , \qquad \text{with} \qquad r = |\vec{r}| , \tag{23}$$

and $r_a$ denotes the $a$th component of the position vector $\vec{r}$. The function $f$ describes the 'length' of the Higgs field vector, and equals $f = 0$ at the origin and $f = 1$ at $r \to \infty$. In order to satisfy the condition $D_\mu\varphi^a = 0$, the gauge field should satisfy

$$A_\mu^a \to \epsilon_{\mu ab}\frac{r_b}{er^2} , \tag{24}$$

which corresponds to a magnetic monopole.

The mass of the monopole is approximately given by (this is the Prasad-Sommerfield solution [12], which provides a lower bound)

$$m_{\rm mon} = \frac{M_V}{\alpha}\,, \tag{25}$$

where $M_V$ is the mass of the vector boson of the broken symmetry group and $\alpha = e^2/(4\pi)$ is the fine structure constant.

### 3.3 Estimate of the correlation length and initial density

During the phase transition, we expect that the field $\phi$ will find a different orientation in different regions in space (if the regions are sufficiently separated from each other). These regions with expectation values of $\phi$, are called 'protodomains', and 't Hooft-Polyakov monopoles can form at the points where different domains meet. We will now estimate the sizes of the protodomains, following Kibble [9].

**Second-order phase transition** We first study the second-order phase transition caused by the potential of eq.(20). At $T \lesssim T_c$, a fluctuation back from the minimum of the potential to $\phi = 0$ corresponds to a difference in the potential energy

$$\Delta f = \frac{\lambda}{8}(\langle\phi\rangle^2)^2\,. \tag{26}$$

The scale of these fluctuations is set by $\xi$, the correlation length, which is given by

$$\xi^{-1} = m_\phi = \sqrt{\lambda}|\langle\phi\rangle|\,, \tag{27}$$

where $m_\phi$ is the mass of the field undergoing the phase transition. Fluctuations with a scale $\xi$, can frequently occur as long as

$$\xi^3 \Delta f \lesssim T\,. \tag{28}$$

At the temperature $T$ for which (neglecting an $\mathcal{O}(1)$ factor)

$$\frac{1}{T^2} \sim \frac{1}{T_c^2} + \frac{\lambda}{\eta^2}\,, \tag{29}$$

fluctuations become energetically unfavorable. At that moment the correlation length is given by

$$\xi^{-1} = \sqrt{\lambda}\langle\phi\rangle = \sqrt{\lambda}\eta\sqrt{\frac{\lambda T_c^2}{\eta^2 + \lambda T_c^2}}, \tag{30}$$

which is roughly equal to $\lambda T_c$ or $\lambda\eta$ (depending on the value of $\lambda$ and $N$). This value of the correlation length sets the initial scale of the protodomains. Monopoles form where different domains meet. The initial density of monopoles is thus given by [2]

$$n_{\rm mon} = \frac{1}{\xi^3} \sim \lambda^3\eta^3\,. \tag{31}$$

[2]Strictly speaking, only one monopole can form at the point where four domains meet, and the probability that the field configurations are such that a monopole forms is somewhat smaller than one. The real initial density will thus be somewhat smaller.

**First-order phase transition** A first order phase transition occurs when a potential barrier exists between the symmetric phase minimum and the broken phase minimum (such a barrier can e.g. be generated by radiative corrections). In this case, the phase transition proceeds by the nucleation of bubbles of broken symmetry. These bubbles expand and collide, until the entire universe is in the broken vacuum. Monopoles can be produced when bubbles with different field orientations collide. The number density of monopoles is set by the typical bubble size at the time of collision. This quantity is not so easy to obtain (and model-dependent). An upper bound on the bubble size is given by the typical horizon size during the phase transition. We thus obtain a lower bound on the monopole density of

$$n_{\text{mon}} > p d_H^{-3} \,, \tag{32}$$

where $p$ is the probability factor for monopole formation and $d_H$ is the horizon size, given by[3]

$$d_H \sim \frac{1}{H} = \frac{M_P}{T_c^2}\sqrt{\frac{90}{g_* \pi^2}}\,. \tag{33}$$

Note that $d_H$ is larger than the value of $\xi$ that we obtained for a second-order phase transition.

## 3.4 Zurek mechanism

Let's consider the case of a second-order phase transition again. In the Kibble mechanism, the time that the system needs to adjust to a fluctuation was not taken into account, *i.e.* it was assumed that the system can react instantaneously. A more careful treatment of the finite response time was done by Zurek [11] (see also [13, 14]).

Close to the critical temperatue, the correlation length and the relaxation time $\tau$ diverge

$$\xi = \xi_0 |\epsilon|^{-\nu} \,, \qquad \tau = \tau_0 |\epsilon|^{-\mu} \,, \tag{34}$$

where $\mu, \nu$ are critical exponents that are determined by the universality class. $\epsilon$ parameterizes the distance from the critical temperature

$$\epsilon \equiv \frac{T_c - T}{T_c} \,. \tag{35}$$

Let us assume a linear relation between $\epsilon$ and $t$ when the system passes through the critical temperature:

$$\epsilon = \frac{t - t_c}{\tau_Q} \,, \tag{36}$$

where $1/\tau_Q$ is called the quenching rate. There is a particular time $t_*$, for which the distance to the critical time $t_c$ equals $\tau(t_*)$. For $|t - t_c| < |t_* - t_c|$, the system can not keep up with

[3] It should be noted here that we assumed no strong supercooling. If the temperature at bubble collision is much smaller than the critical temperature, the monopole density can be suppressed.

the temperature change, and fluctuations are frozen. We find

$$|t_* - t_c| = \tau_0 |\epsilon(t_*)|^{-\mu} = |\epsilon(t_*)|\tau_Q\,, \quad \to \quad |\epsilon(t_*)| = \left(\frac{\tau_Q}{\tau_0}\right)^{-\frac{1}{\mu+1}} . \tag{37}$$

We arrive at an updated prediction for the correlation length:

$$\xi(t_*) \sim \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^{\frac{\nu}{1+\mu}} . \tag{38}$$

We should notice that plugging in $\mu, \nu = 1/2$ and $\xi_0 \sim \tau_0 \sim 1/(\sqrt{\lambda} T_c)$, and $\tau_Q = 2t_c$ the correlation length is

$$\xi \sim \left(\frac{T_c}{H}\right)^{1/3} \frac{1}{\lambda^{2/3} T_c}\,, \tag{39}$$

which is comparable to the value obtained in eq.(30). An interesting aspect of [11] is that Zurek draws an analogue between cosmological defect formation and formation of vortex lines in superfluid helium. The mechanism has now been demonstrated experimentally in different systems (see e.g. [15]).

### 3.5 Monopole problem

We have now made several estimates for the sizes of the protodomains. We will now estimate the corresponding monopole density. The most conservative estimate comes from the case of a first-order phase transition $\xi = d_H$. At the time of formation, we find

$$\left.\frac{n}{T^3}\right|_{\mathrm{in}} \gtrsim p \left(\frac{T_c}{M_P}\sqrt{\frac{90}{g_* \pi^2}}\right)^3 . \tag{40}$$

Taking $T_c \sim 10^{16}$GeV, $g_* \sim 100$ and $p \sim 1/10$ we obtain $n/T^3 \gtrsim 10^{-10}$ (and $n/s \gtrsim 10^{-12}$). Since monopoles are stable (up to annihilations, which we will discuss in Subsection 3.6), their number density is only affected by the expansion of the universe. The ratio $n/s$ thus remains constant. The ratio of the monopole energy density to the critical energy density is given by [5]

$$\Omega_{\mathrm{mon}} h^2 \simeq 10^{24} \left(\frac{n_{\mathrm{mon}}}{s}\right) \frac{m_{\mathrm{mon}}}{10^{16}\mathrm{GeV}} . \tag{41}$$

Remembering that the mass of the monopole is $\sim 10^{16}$GeV, and that $\Omega_m = 0.32$ and $h = 0.67$, we conclude that the energy density in monopoles is unacceptably large. For the case of a second-order phase transition, $\xi < d_H$, so the problem is even more stringent.

### 3.6 Monopole annihilation

We have just seen that, if the monopoles get produced during a symmetry breaking phase transition around the GUT scale, and their energy just redshifts thereafter, the density of monopoles is much too large. But maybe monopole-antimonopole annihilation can solve the problem? The effect of annihilation on the monopole density was studied by Preskill in 1979 [16]. The evolution equation for the monopole density (assuming that the density of monopoles and antimonopoles is equal) is given by

$$\frac{dn_{\rm mon}}{dt} = -An_{\rm mon}^2 - 3Hn_{\rm mon}\,, \tag{42}$$

where $A$ characterizes the annihilation process. It can be estimated as

$$A = \frac{1}{e^2 b T^2}\,, \tag{43}$$

where $b \sim 100$. Eq.(42) can be integrated, and we find

$$\frac{n_{\rm mon}(T)}{T^3} = \left\{ \left( \frac{n_{\rm mon}}{T^3}\Big|_{\rm in} \right)^{-1} + \frac{M_P}{e^2 b}\sqrt{\frac{g_* \pi^2}{90}} \left[ \frac{1}{T} - \frac{1}{T_{\rm in}} \right] \right\}^{-1}\,. \tag{44}$$

The annihilation rate cuts off when the mean free path of the monopoles becomes larger than the capture rate, which occurs at a temperature

$$T_f \sim \frac{e^4 m_{\rm mon}}{b}\,, \tag{45}$$

which determines the final density

$$\frac{n_{\rm mon}(T_f)}{T^3} = \sqrt{\frac{90}{g_* \pi^2}} \frac{e^6 m_{\rm mon}}{M_P}\,. \tag{46}$$

This density is still orders of magnitude too large, so the monopole problem is not solved by annihilations. Note that annihilation only occurs if the initial density is larger than eq.(46), otherwise the initial density simply remains constant.

# 4 Solutions to the monopole problem

We have seen that the amount of monopoles from a GUT-breaking phase transition is unacceptably high, even when annihilations are taken into account. We will now consider several solutions to the monopole problem.

## 4.1 No GUT

The most obvious solution is to assume that monopoles just do not get formed, either because the universe does not reach a temperature high enough for restoration of GUT, or because the forces simply do not get unified in a simple gauge group.

## 4.2 Inflation

A possibly more attractive solution is cosmological inflation [17, 18], which actually solves three cosmological problems at once: the flatness, horizon and monopole problem.

Inflation is a period of shrinking comoving Hubble radius,

$$\frac{d}{dt}(aH)^{-1} < 0\,, \tag{47}$$

or accelerated expansion

$$\ddot{a} > 0\,. \tag{48}$$

The result of inflation is that initial number densities get strongly diluted, and that the entire observable universe (today) must have been in causal contact. The conditions for inflation can only be fulfilled when

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} < 1\,, \tag{49}$$

where we defined $dN = d\ln a$, measuring the numbers of e-folds of expansion. In order to solve the horizon problem (why is the temperature of the CMB so uniform), inflation has to persist for at least $N \sim 50-60$ e-folds. Such a long period of inflation also sufficiently dilutes the monopole density.

A straightforward way to satisfy the conditions for inflation is a domination of the energy density by the potential energy of a scalar field, see e.g. [8] for further details. When inflation ends, the energy density of the inflaton field gets transferred to radiation during the 'reheating' era. If this process is very efficient, the temperature of the universe becomes equal to

$$T \sim \left(\frac{30}{\pi^2 g_*} V_{\rm inf}\right)^{1/4}\,, \tag{50}$$

where $V_{\rm inf}$ is the inflationary energy scale. In order to prevent formation of monopoles *after* inflation, this energy scale can not be too high (or reheating should be slow).

## 4.3 Black hole solution

A solution that could reduce the number of monopoles even *after* inflation, was presented by Stojkovic and Freese [19]. The mechanism relies on capture of the monopoles by black holes. The evolution equation for the monopole density now obtains an additional contribution

$$\frac{dn_{\rm mon}}{dt} = -An_{\rm mon}^2 - n_{\rm bh}\sigma g v_{\rm m} n_{\rm mon} - 3Hn_{\rm mon}\,, \tag{51}$$

where $n_{\rm bh}$ is the number density of black holes, $\sigma_g$ the capture cross section and $v_{\rm m}$ the monopole velocity. The quantity $n_{\rm bh}\sigma_g v_{\rm m}$ is estimated as

$$n_{\rm bh}\sigma_g v_{\rm m} = f\beta^2 b m_{\rm mon}\,, \tag{52}$$

where $f < 1$ is the fraction of the energy density in black holes and $\beta < 1$ parameterizes the mass of the black hole (larger $\beta$ corresponds to larger mass). Under the assumption that the black hole capture rate persists until low temperatures, the solution for the monopole density becomes

$$\frac{n_{\rm mon}(t)}{s} = \left.\frac{n_{\rm mon}(t)}{s}\right|_{\rm in} e^{-0.1 f\beta^2 (t-t_{\rm in})/t_P} \,. \tag{53}$$

After estimating $f$ and $\beta$, the authors conclude that black holes with a mass of $M_{\rm bh} < 10^9$gm can solve the monopole problem, without violating any observational constraints.

## References


[1] H. Georgi and S. L. Glashow, *Unity of All Elementary Particle Forces*, *Phys. Rev. Lett.* **32** (1974) 438.

[2] H. Fritzsch and P. Minkowski, *Unified Interactions of Leptons and Hadrons*, *Annals Phys.* **93** (1975) 193.

[3] F. Gursey, P. Ramond and P. Sikivie, *A Universal Gauge Theory Model Based on E6*, *Phys. Lett. B* **60** (1976) 177.

[4] J. Preskill, *MAGNETIC MONOPOLES*, *Ann. Rev. Nucl. Part. Sci.* **34** (1984) 461.

[5] E. W. Kolb and M. S. Turner, *The Early Universe*.

[6] Alan Guth, *The magnetic monopole problem*, *MIT Lecture notes* (2018) .

[7] A. Rajantie, *Introduction to Magnetic Monopoles*, *Contemp. Phys.* **53** (2012) 195 [1204.3077].

[8] Daniel Baumann, *Cosmology*, *Part III Mathematical Tripos* (2015) .

[9] T. W. B. Kibble, *Topology of Cosmic Domains and Strings*, *J. Phys. A* **9** (1976) 1387.

[10] T. W. B. Kibble, *Some Implications of a Cosmological Phase Transition*, *Phys. Rept.* **67** (1980) 183.

[11] W. H. Zurek, *Cosmological Experiments in Superfluid Helium?*, *Nature* **317** (1985) 505.

[12] M. K. Prasad and C. M. Sommerfield, *Exact classical solution for the 't hooft monopole and the julia-zee dyon*, *Phys. Rev. Lett.* **35** (1975) 760.

[13] H. Murayama and J. Shu, *Topological Dark Matter*, *Phys. Lett. B* **686** (2010) 162 [0905.1720].

[14] A. del Campo and W. H. Zurek, *Universality of phase transition dynamics: Topological Defects from Symmetry Breaking*, *Int. J. Mod. Phys. A* **29** (2014) 1430018 [1310.1600].

[15] P. C. Hendry, N. S. Lawson, R. A. M. Lee, P. V. E. McClintock and C. D. H. Williams, *Generation of defects in superfluid $^4$He as an analogue of the formation of cosmic strings*, *Nature* **368** (1994) 315.

[16] J. Preskill, *Cosmological Production of Superheavy Magnetic Monopoles*, *Phys. Rev. Lett.* **43** (1979) 1365.

[17] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, *Phys. Rev. D* **23** (1981) 347.

[18] A. A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett. B* **91** (1980) 99.

[19] D. Stojkovic and K. Freese, *A Black hole solution to the cosmological monopole problem*, *Phys. Lett. B* **606** (2005) 251 [hep-ph/0403248].