Gil Badel, EPFL

- 28 years old
- Swiss
- Studied at EPFL, Lausanne, Switzerland
- Now 4th (final) year PhD student in the Laboratory of Theoretical Particle Physics, EPFL
- Supervisor : Riccardo Rattazzi
- Speak French, English, some German
- Hobbies : reading, bouldering, volunteering in local associations

Large charge operators in CFTs [1909.01269, 1911.08505]

Consider the following theory at the Wilson-Fisher fixed point, in $4-\varepsilon$ euclidean dimensions.

$$\mathcal{L} = \partial\bar{\phi}\partial\phi + \frac{\lambda}{4}(\bar{\phi}\phi)^2 \tag{1}$$

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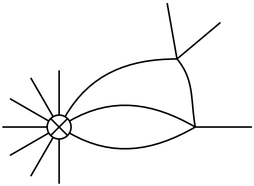

$$\sim \lambda^2 n(n-1)(n-2) \sim \lambda^2 n^3$$

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Consider the following theory at the Wilson-Fisher fixed point, in $4-\varepsilon$ euclidean dimensions.

$$\mathcal{L} = \partial\bar{\phi}\partial\phi + \frac{\lambda}{4}(\bar{\phi}\phi)^2 \tag{2}$$

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"Naive" Feynman diagrams computation :

$$\gamma_{\phi^n} = n\left[\frac{\lambda}{16\pi^2}\frac{(n-1)}{2} - \left(\frac{\lambda}{16\pi^2}\right)^2\frac{(2n^2-2n-1)}{4}\right] \tag{3}$$

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Perturbation theory breaks down at $\lambda n \gg 1$!

Semiclassical Expansion

$\lambda \to 0$, n finite, $\lambda n \ll 1$

- quantize fluctuations around the vacuum $\phi(x) = 0 + \delta\phi(x)$.
- $\gamma_{\phi^n} = n \left[\frac{\lambda}{16\pi^2} \frac{(n-1)}{2} - \left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2n^2-2n-1)}{4} \right]$

$\lambda \to 0$, $n \to \infty$, λn finite

- quantize fluctuations around a non-trivial saddle $\phi(x) = \boldsymbol{\phi}_{\text{cl.}}(\boldsymbol{x}) + \delta\phi(x)$.

$$\gamma_{\phi^n} = n \sum_{\ell=0}^{\infty} \lambda^{\ell} P_{\ell}(n) = n \sum_{\kappa=0} \lambda^{\kappa} F_{\kappa}(\lambda n) \tag{4}$$

[Backup] Semiclassical expansion

Compute dimension from 2-point function

$$\langle \bar{\phi}^n(x_f)\phi^n(x_i)\rangle = \frac{1}{|x_f - x_i|^{2\Delta_{\phi^n}}} \tag{5}$$

The Path Integral can be rewritten as

$$\begin{aligned}\langle \bar{\phi}^n(x_f)\phi^n(x_i)\rangle &\sim \int \mathcal{D}\bar{\phi}\mathcal{D}\phi\ \bar{\phi}^n(x_f)\phi^n(x_i) e^{-\int \mathrm{d}^d x \left(\partial\bar{\phi}\partial\phi + \frac{\lambda}{4}(\bar{\phi}\phi)^2\right)} \\ &\sim \int \mathcal{D}\bar{\phi}\mathcal{D}\phi\ e^{-\int \mathrm{d}^d x \left(\partial\bar{\phi}\partial\phi + \frac{\lambda}{4}(\bar{\phi}\phi)^2\right) + n(\ln\bar{\phi}(x_f) - \ln\phi(x_i))} \\ &\sim \int \mathcal{D}\bar{\phi}\mathcal{D}\phi\ e^{-\frac{1}{\lambda}\left[\int \mathrm{d}^d x \left(\partial\bar{\phi}\partial\phi + \frac{1}{4}(\bar{\phi}\phi)^2\right) + \lambda n(\ln\bar{\phi}(x_f) - \ln\phi(x_i))\right]}\end{aligned}$$

where in the last line we redefined the fields $\phi \to \frac{1}{\sqrt{\lambda}}\phi$.