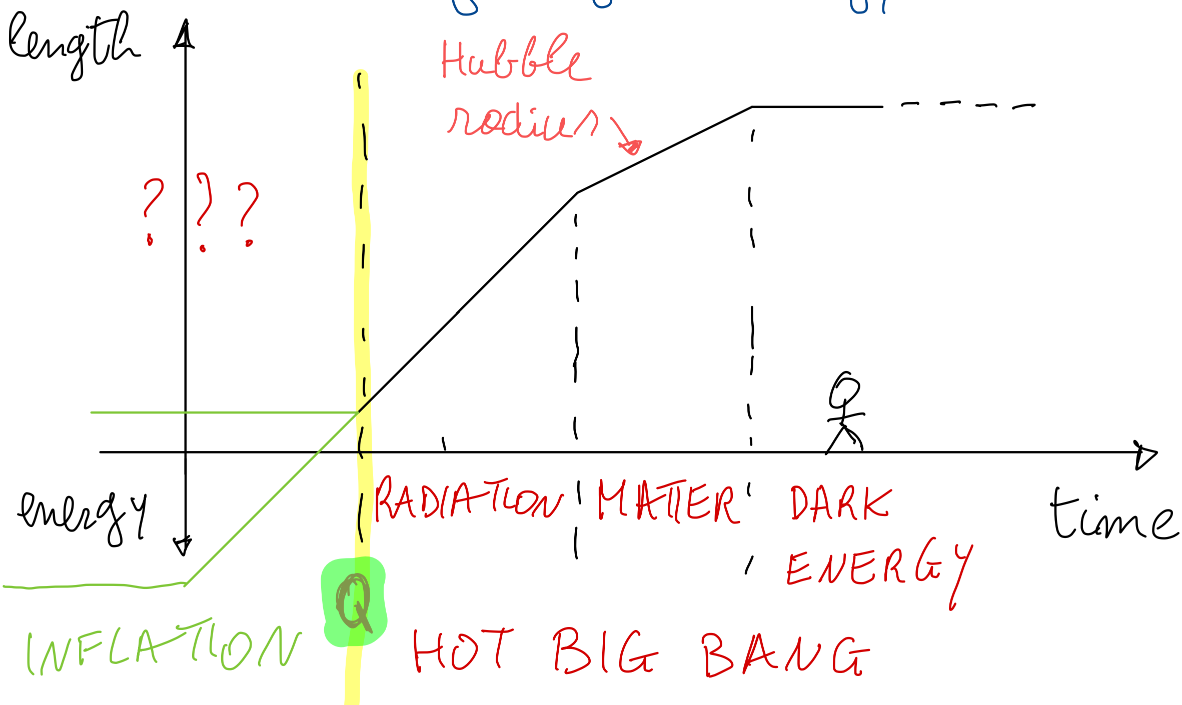
AN INTRODUCTION TO THE COSMOLOGICAL BOOTSTRAP

1 MOTIVATIONS & GENERALITIES

Cosmology is the ultimate observational mode of high energy physics



$$10^{13}\,\text{GeV} > \left(H = 1/L_H = \frac{\dot a}{a} \right) \gg \text{MeV}$$

relevant energy scale in cosmo

where $ds^2 = -dt^2 + a^2 d\bar{x}^2$ (FLRW metric)

- All perturbations in initial conditions are the same!

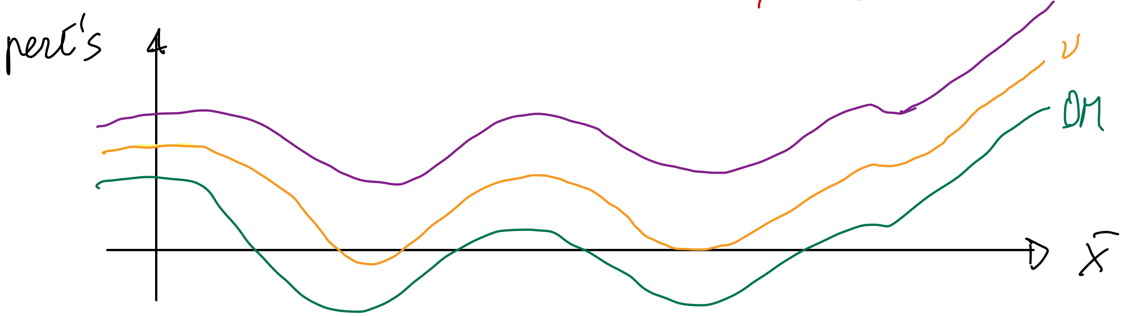
$$\left(\frac{\delta\rho(\bar{x})}{\bar{\rho}+\bar{p}}\right)_a = \left(\frac{\delta\rho(\bar{x})}{\bar{\rho}+\bar{p}}\right)_b$$

$a, b =$ baryons, neutrinos, Dark matter, "gravity"

average density & pressure

$$\simeq \zeta(\bar{x})$$

the adiabatic mode.



Prediction of single-clock inflation (a simple early universe vs flux compactifications)

In coords s.t. $\delta\rho = 0$

gravitons (not seen)

$$ds^2 = -dt^2 + a^2 e^{2\zeta(\bar{x})} \left(e^{\gamma}\right)_{ij} dx^i dx^j$$

- Cosmo observations on <u>large</u> scales measure correlators of $\zeta(\bar{x})$.

Say $Y = \{ T_{CMB}, E_{CMB}, \delta_{DM}, \delta_{\nu}, \delta_{gal}, \dots \}$

$$Y(\bar{x}) = \int_k e^{i\bar{k}\bar{x}} \cdot \Delta^Y(\underset{\uparrow}{k})\, \zeta(\bar{k}) + \mathcal{O}(\zeta^2)$$

transfer fct

Q $\Delta^T = ?$

Then

$$\left\langle \prod_a^n Y(\bar{x}_a) \right\rangle_{obs.} = \int \left[\prod_a^n d^3k_a \, \Delta^Y(\bar{k}_a) \right] \left\langle \prod_a^n \zeta(\bar{k}_a) \right\rangle_{Th.}$$

Cosmo observations	$\sim$	QFT correlators in curved spacetime

- Primordial spacetime is approx de Sitter

$$ds^2 = -dt^2 + e^{2Ht} d\bar{x}^2$$

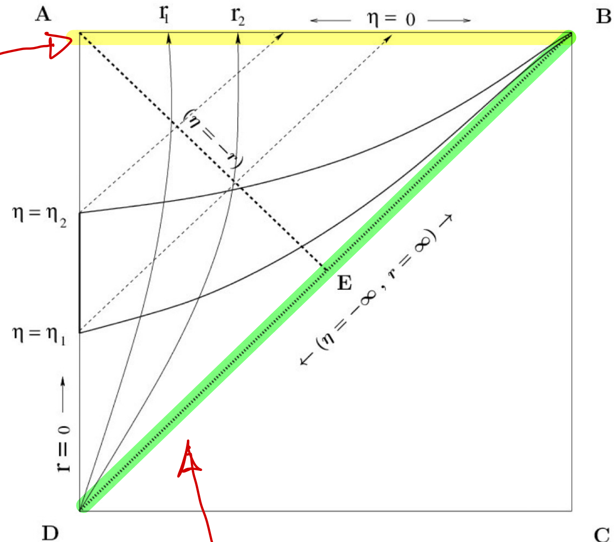
$$= \frac{-d\eta^2 + d\bar{x}^2}{\eta^2 H^2} = \frac{-d\eta^2 + dr^2 + r^2 d\Omega^2}{\eta^2 H^2}$$

(Evidence: coherent superHubble pert's $\langle TE \rangle$ + scale inv. $\langle TTT \rangle$ = inflation)

• dS conformal diagram

$-\infty < \eta < 0$
$0 < r < +\infty$

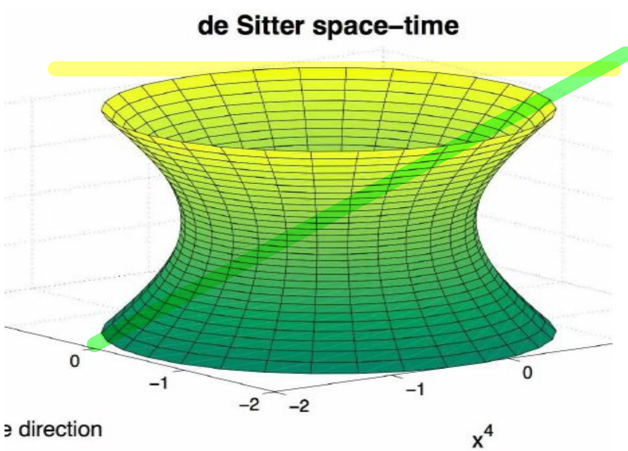
asymptotic future (conformal boundary) ~ reheating surface



asymptotics

asymptotic past

dS hyperboloid Mink(d+1, 1)



No spatial boundary, No t-like killing v.

Goal of cosmology: understand QFT & QG. in de Sitter (asymptotically, quasi)

Approaches:

1) Bottom-up: derive general properties from perturbation theory in dS & build a UV completion. (like Veneziano's amplitude for S.T.)

2) Top-down: find dS in S.T..

QFT IN DS

Perturbative quantization

$$\phi(\bar{k}, \eta) = f_k(\eta)\, a_{\bar{k}} + f_k^*(\eta)\, a^+_{\bar{k}}$$

mode functions

momentum cons

$$(\Box - m^2)\, f = 0$$ free e.o.m.

For example:

Mink: $f_k = e^{-i\omega t}$ $\qquad \omega = \sqrt{k^2+m^2}$

dS: $f_k = \frac{\sqrt{\pi}}{2} H (-\eta)^{3/2} H_\nu^{(1)}(-k\tau)$ ← Hankel fct.

$$\nu = \sqrt{\left(\frac{d}{2}\right)^2 - \frac{m^2}{H^2}}$$

Energy not conserved.

dS, $m=0$, $\nu = 3/2$ $\qquad f_k = \frac{H}{\sqrt{2k^3}}(1+ik\eta)\, e^{-ik\eta}$ massless

Reduces to Mink at early time = short scales

dS, $m=\sqrt{2}H$ $\qquad f_k = \frac{H\eta}{\sqrt{2k}}\, e^{-ik\eta}$ conformally coupled (toy model)

Infinite dimensional Fock space

$$\mathcal{H}_{\bar k} = \{ |0\rangle,\ a^{+}_{\bar k}|0\rangle,\ a^{+}_{\bar k} a^{+}_{\bar k}|0\rangle,\ \dots \}$$

$$\mathcal{H} = \prod_{\bar k} \mathcal{H}_{\bar k}$$

dim $\mathcal{H}_k = \infty$

dim $\mathcal{H} = (\infty)^{\infty}$

OBSERVABLES

expectation values

$$\langle \prod^{n} \mathcal{O} \rangle \equiv \langle \Omega | \mathcal{O}(x_1)\, \mathcal{O}(x_2) \dots \mathcal{O}(x_n) | \Omega \rangle$$

local operators

some state

We choose some initial state $|\Omega(\eta \to -\infty)\rangle$ and evolve with Hamiltonian. In the interaction picture

$$\langle \prod^{n} \mathcal{O} \rangle = \langle \Omega(-\infty) | U_I^{+}(\eta,-\infty) \prod \mathcal{O}_I\, U_I(\eta,-\infty) | \Omega(-\infty) \rangle$$

evolution operator

$$U_I(\eta,-\infty) = T e^{-i\int_{-\infty}^{\eta} d\eta'\, H_{int}(\eta')}$$

time ordering.

interaction Hamilt.

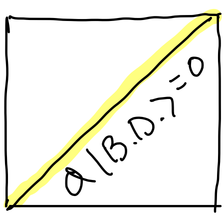
The same as for amplitudes:

$$A = \langle in | T e^{-i\int H dt} | out \rangle$$

Amplitudes: in-out, one-way trip U.

Correlators: in-in, return trip $U^{\dagger} \dots U$.

Initial state: $|\Omega\rangle$ = Bunch Davies vac.



$$\lim_{\eta \to -\infty} |\Omega(\eta)\rangle = |0\rangle$$

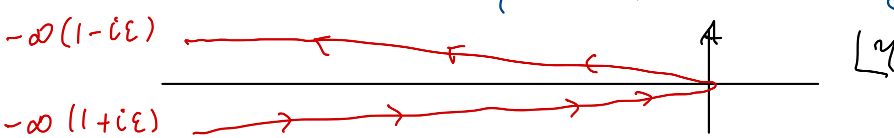
Fock vacuum as in Minkowski.

$$a_{\vec{k}} |0\rangle = 0 \quad \forall \vec{k}$$

To choose this vacuum we project

$$U_I(\eta, -\infty) \to T \exp\left[\int_{-\infty(1-i\varepsilon)}^{\eta} d\eta' \, H_{int}(\eta') \right]$$

[Can be computed by a path integral.



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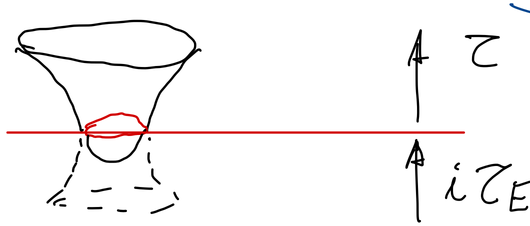
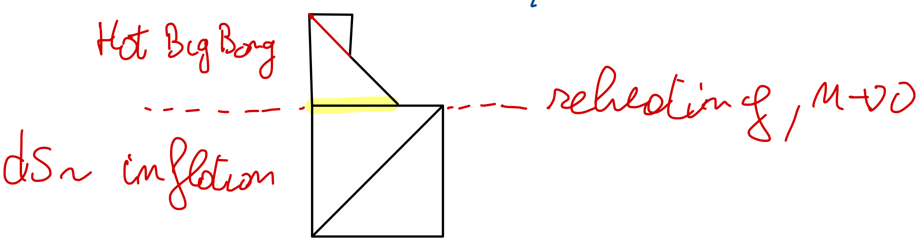
[Bunch-Davies on Poincaré patch extends to Hartle-Hawking on global dS.]



In practice:

- We only observe correlators at the future conformal boundary (meta-observables)



- massive fields decay at $\eta = 0$ as η^{α}

$$0 < \alpha = \frac{3}{2} - \nu < 3/2$$

- All derivatives of ϕ also decay.

- We only see $\lim_{\eta \to 0} \langle \Pi \phi^n \rangle$ $n=0$.

Alternative we use the Schroedinger picture for QFT. The wave fct is.

$$\Psi[\phi(\bar{x}); \eta] = \langle \phi(\bar{x}) | \Omega(\eta) \rangle$$

in-out only one U_I.

(bulk path integral)

$$= \int_{\text{initial state}}^{\phi(\bar{x},\eta) = \phi(\bar{x})} [d\phi] \, e^{iS}$$

Observables: boundary "path integral"

$$\langle \mathcal{O} \rangle = \int [d\phi(\bar{x})] \, \Psi^* \mathcal{O} \, \Psi$$

think $\sigma \simeq |A|^2$

Parameterize

$$\Psi[\phi, \eta] = \exp\left[- \sum_{n=2}^{\infty} \frac{1}{n!} \int \psi_n \, \phi(\bar{k}_1) \dots \phi(\bar{k}_n) \right]$$

wavefunction coefficients

In P.T. ψ_n are related to $\langle \phi^n \rangle$ as

$$\langle \phi^2 \rangle = \frac{1}{2 \, \mathrm{Re} \, \psi_2} ;$$

$$\langle \phi^3 \rangle = [\psi_3(\bar{k}) + \psi_3^*(-\bar{k})] / \Pi^3 \, \mathrm{Re} \, \psi_2(k_a)$$

$\psi : B = A : \sigma$