Applying different angular ordering constraints and kt-factorization approaches to the single inclusive hadron production in the e + e - annihilation processes

University of TEHRAN

The Physics department

Outline

- The kt- factorization approach and different AOC methods.
- TMD FFs (Transverse Momentum Dependent Fragmentation Functions)
- The cross section of the single inclusive $e^+ e^-$ annihilation processes
- Results and Discussion

❖ **The evolution of Non-singlet quark distribution:**

$$\frac{d}{dt} q^{NS}(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dz}{z}\, q^{NS}\left(\frac{x}{z},t\right) P_{qq}(z).$$

$$t \equiv \ln Q^2/Q_0^2$$

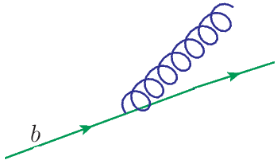
$$P_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z)\right)$$

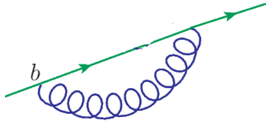
$$\int_0^1 \frac{f(z)}{(1-z)_+}\,dz \equiv \int_0^1 \frac{f(z)-f(1)}{1-z}\,dz$$

$$\frac{dq^{NS}}{dt}\Bigg/\frac{C_F\,\alpha_s(t)}{2\pi} = \frac{3}{2}\,q^{NS}(x,t) + \int_0^1 \frac{dz}{1-z}\left(\frac{1+z^2}{z}\,q^{NS}\left(\frac{x}{z},t\right) - 2q^{NS}(x,t)\right)$$

$$= \int_x^1 \frac{dz}{z}\,\frac{1+z^2}{1-z}\,q^{NS}\left(\frac{x}{z},t\right) - q^{NS}(x,t)\int_0^1 \frac{1+z^2}{1-z}\,dz,$$

$$\frac{dq^{NS}}{dt}(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^{1-\Delta}\frac{dz}{z}\,\hat{P}_{qq}(z)\,q^{NS}\left(\frac{x}{z},t\right) - \frac{\alpha_s(t)}{2\pi}\,q^{NS}(x,t)\int_0^{1-\Delta} dz\,\hat{P}_{qq}(z).$$





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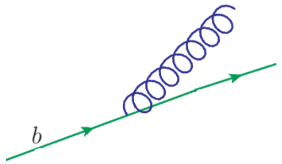
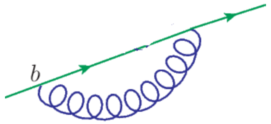
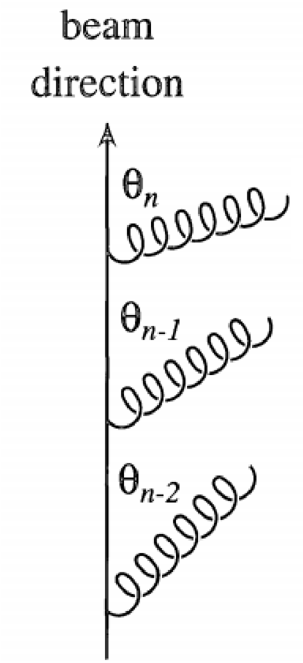






$$\cdots\ \theta_n > \theta_{n-1} > \theta_{n-2}\ \cdots$$

$$z < \frac{Q}{Q + k_t}$$

Angular ordering enforces increasing angles from the beam axis for the radiated gluons

- **DGAP evolution equation for PDFs:**

$$\frac{\partial a(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S}{2\pi}\left[\int_x^{1-\Delta} P_{aa'}(z)\, a'\left(\frac{x}{z},\mu^2\right) dz - a(x,\mu^2)\sum_{a'}\int_0^{1-\Delta} P_{a'a}(z')dz'\right]$$

$xg\left(x,\lambda^2\right)$ or $xq\left(x,\lambda^2\right)$

$$T_a(k_t^2,Q^2) = \exp\left(-\int_{k_t^2}^{Q^2}\frac{d\kappa_t^2}{\kappa_t^2}\frac{\alpha_s(\kappa_t^2)}{2\pi}\sum_b\int_0^{1-\Delta} dz P_{ba}(z)\right)$$

$$\frac{1}{T_a(k_t^2,Q^2)}\frac{\partial T_a(k_t^2,Q^2)}{\partial \ln k_t^2} = \frac{\alpha_s(k_t^2)}{2\pi}\sum_b\int_0^{1-\Delta} dz P_{ba}(z)$$

$$\begin{cases} T_a(k_t,\mu) = \exp\left(-\int_{k_t^2}^{\mu^2}\frac{\alpha_S(k_t'^2)}{2\pi}\frac{dk_t'^2}{k_t'^2}\sum_{a'}\int_0^{1-\Delta} P_{a'a}(z')\,dz'\right), \\ f_a(x,k_t^2,\mu^2) = T_a(k_t,\mu)\left[\frac{\alpha_S(k_t^2)}{2\pi}\int_x^{1-\Delta} P_{aa'}(z)\, a'\left(\frac{x}{z},k_t^2\right) dz\right]. \end{cases}$$

Analyzing Drell Yan process

PHYSICAL REVIEW D 104, 056005 (2021)

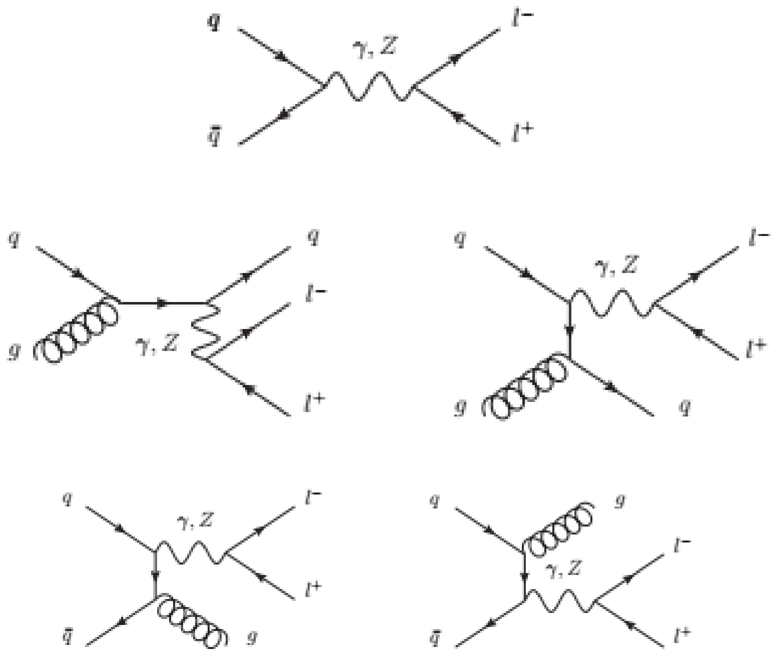
(1) $q^* + \bar{q}^* \to \gamma^*/Z \to l^+ + l^-$

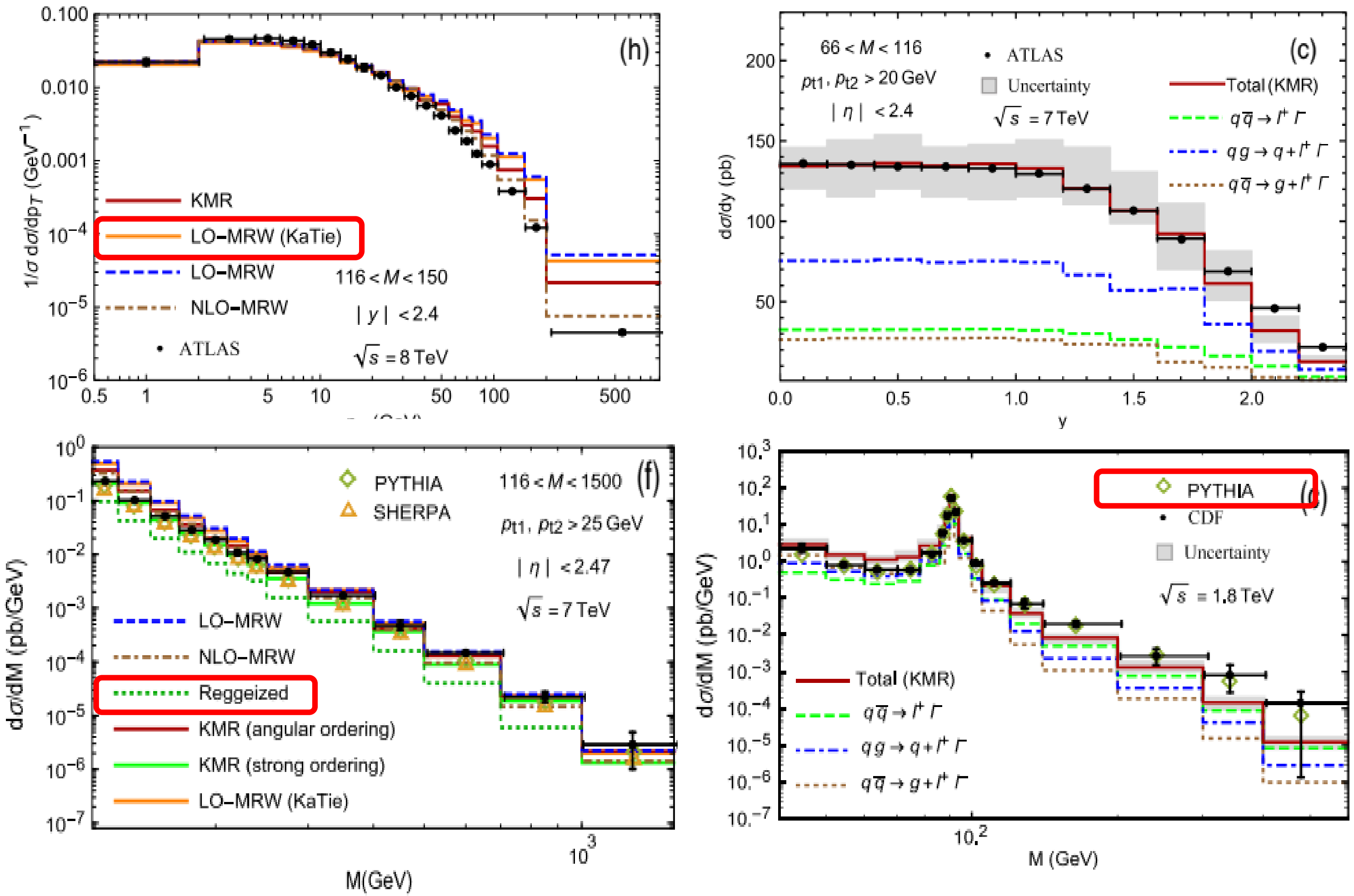
(2) $q^* + g^* \to \gamma^*/Z \to l^+ + l^- + q$

(3) $q^* + \bar{q}^* \to \gamma^*/Z + g \to l^+ + l^- + g$

$$\sigma_1 = \sum_q \int \frac{1}{16\pi(x_1x_2s)^2}|\mathcal{M}_1^{\gamma^*} + \mathcal{M}_1^Z|^2 \times f_q(x_1, k_{1t}^2, \mu^2) f_{\bar{q}}(x_2, k_{2t}^2, \mu^2) \frac{dk_{1t}^2}{k_{1t}^2} \times \frac{dk_{2t}^2}{k_{2t}^2} dp_{1t}^2 dp_{2t}^2 dy_1 dy_2 \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi},$$

$$\sigma_{2(3)} = \sum_q \int \frac{1}{256\pi^3(x_1x_2s)^2}|\mathcal{M}_{2(3)}^{\gamma^*} + \mathcal{M}_{2(3)}^Z|^2 \times f_q(x_1, k_{1t}^2, \mu^2) f_{g(\bar{q})}(x_2, k_{2t}^2, \mu^2) \frac{dk_{1t}^2}{k_{1t}^2} \times \frac{dk_{2t}^2}{k_{2t}^2} dp_{1t}^2 dp_{2t}^2 dy_1 dy_2 dy_3 \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi}\frac{d\psi_1}{2\pi}\frac{d\psi_2}{2\pi},$$




(h)
KMR
LO-MRW (KaTie)
LO-MRW
NLO-MRW
ATLAS
116 < M < 150
| y | < 2.4
√s = 8 TeV
1/σ dσ/dp_T (GeV⁻¹)
(c)
66 < M < 116
p_t1, p_t2 > 20 GeV
| η | < 2.4
ATLAS
Uncertainty
√s = 7 TeV
Total (KMR)
dσ/dy (pb)
y
(f)
PYTHIA
SHERPA
116 < M < 1500
p_t1, p_t2 > 25 GeV
| η | < 2.47
√s = 7 TeV
LO-MRW
NLO-MRW
Reggeized
KMR (angular ordering)
KMR (strong ordering)
LO-MRW (KaTie)
dσ/dM (pb/GeV)
M(GeV)
PYTHIA
CDF
Uncertainty
√s = 1.8 TeV
Total (KMR)
dσ/dM (pb/GeV)
M (GeV)

Initial question

Are the kt-factorization and different AOC methods applicable formalisms to reach the TMD Fragmentation Functions or NOT?

❖ **DGAP evolution equation for Fragmentation Function:**

$$\frac{\partial q\left(x, Q^2\right)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left[P_{qq}(z)\, q\left(\frac{x}{z}, Q^2\right) + P_{qg}(z)\, g\left(\frac{x}{z}, Q^2\right) \right],$$

$$\frac{\partial D_q^H(z, \mu^2)}{\partial ln\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{dx}{x} \left[P_{qq}(x) D_q^H\left(\frac{z}{x}, \mu^2\right) + P_{gq}(x) D_g^H\left(\frac{z}{x}, \mu^2\right) \right],$$

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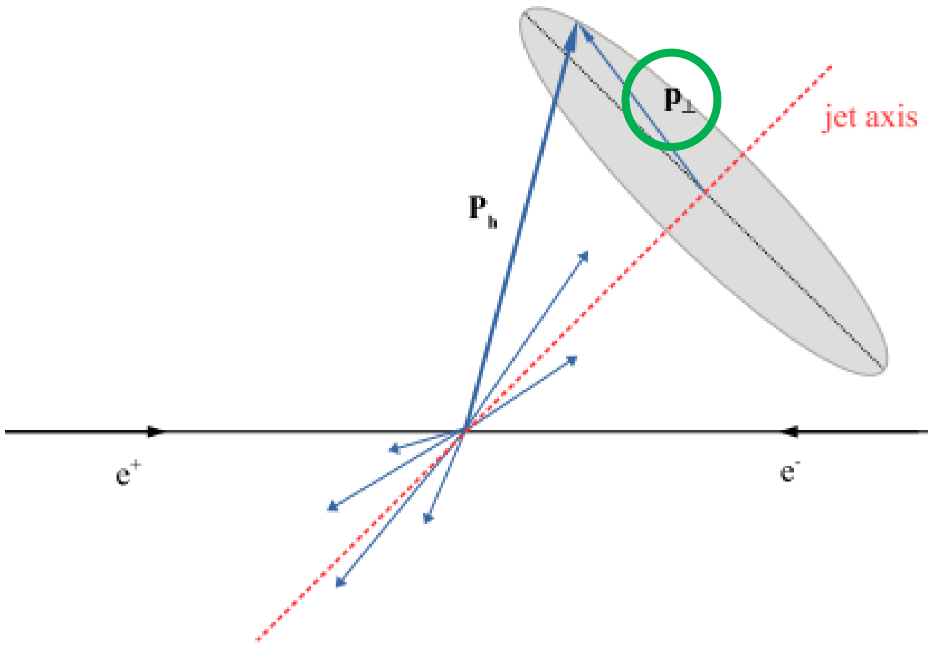
$$\frac{\partial D_q^H\left(z, \mu^2\right)}{\partial l n \mu^2}=\frac{\alpha_S\left(\mu^2\right)}{2 \pi} \int_x^1 \frac{d x}{x}\left[P_{q q}(x) D_q^H\left(\frac{z}{x}, \mu^2\right)+P_{g q}(x) D_g^H\left(\frac{z}{x}, \mu^2\right)\right],$$

TMD FFs

❖ **TMD FFs**, $D_i^H(z, Q, p_t)$ **, indicate the probability of transition of a parton 'i' at sacle Q into a hadron 'H' carrying away a fraction 'z' of the parton momentum.**



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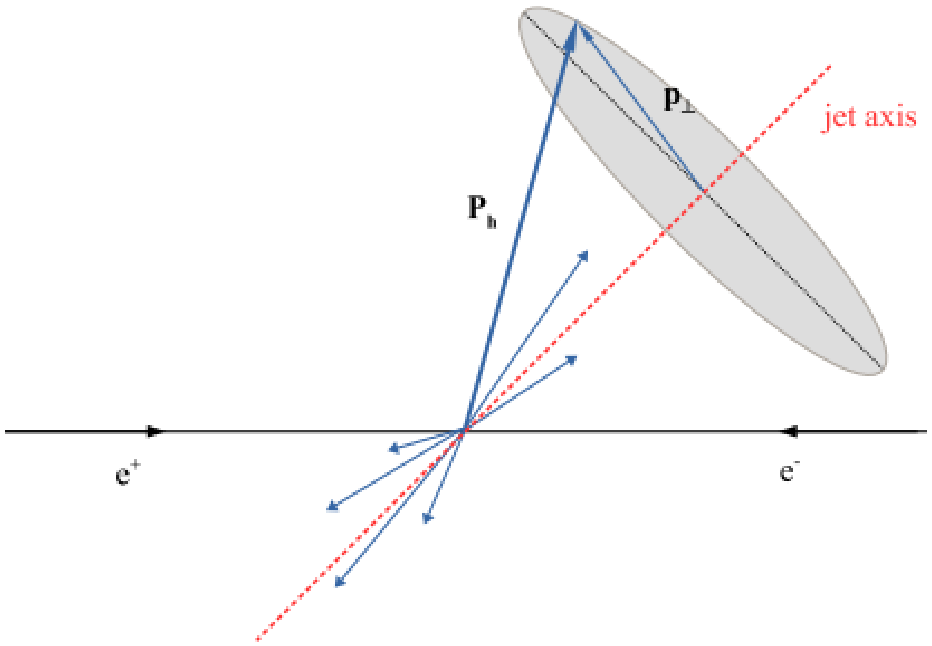

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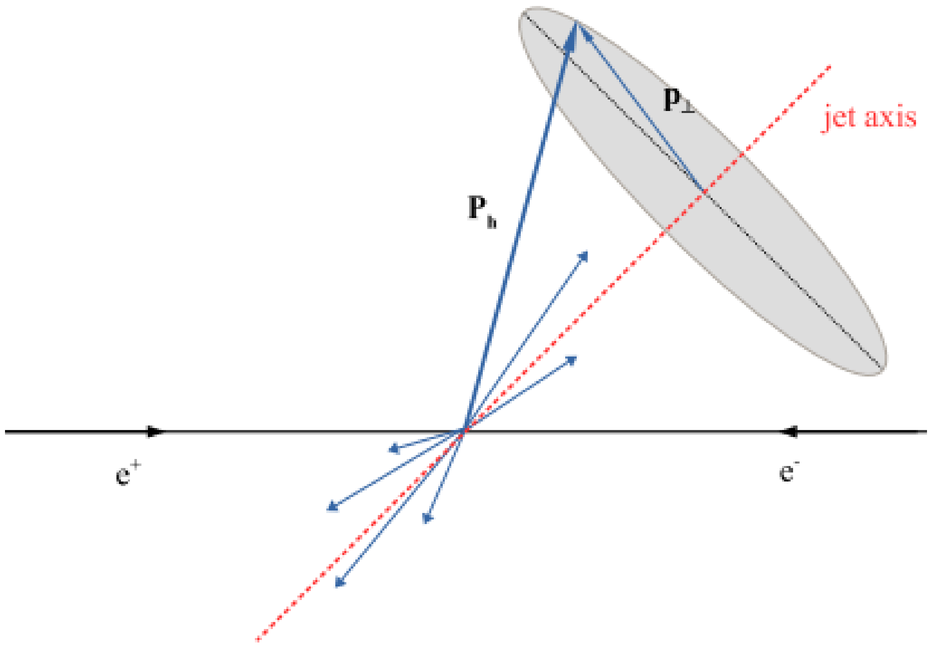

$$e^+e^- \to \gamma \to XH$$

- **The studies over TMD FFS**
  - **non-perturbative (valid in the $p_t \leq 1GeV$)**
  - **perturbative (valid in the $p_t > 1GeV$)**

- **DGAP evolution equation for Fragmentation Function:**

$$\frac{\partial D_q^H(z,\mu^2)}{\partial ln\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}\int_x^1 \frac{dx}{x}\left[P_{qq}(x)D_q^H\left(\frac{z}{x},\mu^2\right) + P_{gq}(x)D_g^H\left(\frac{z}{x},\mu^2\right)\right],$$

- **By inserting these splitting kernels and using the *plus* prescription in a straightforward way, one could have the LO DGLAP equation evaluated at a scale k_t:**

$$\frac{\partial \mathcal{D}_q^H(z,k_t^2)}{\partial ln k_t^2} = \frac{\alpha_S(k_t^2)}{2\pi}\{\sum_a \int_z^{1-\Delta} P_{aq}(x)\mathcal{D}_a^H\left(\frac{z}{x},k_t^2\right)dx - \mathcal{D}_q^H\left(z,k_t^2\right)\sum_a \int_x^{1-\Delta} P_{qa}(z')dz'\}.$$

$$\Rightarrow \quad D_q^H(z,k_t,\mu^2) = T_q(k_t,\mu^2)\sum_{b=q,g}\left[\frac{\alpha_S(k_t^2)}{2\pi k_t^2}\int_z^{1-\Delta} dz' P_{bq}^{(0)}(z')\mathcal{D}_b^H\left(\frac{z}{z'},k_t^2\right)\right],$$

$$T_q(k_t,\mu^2) = exp\left(-\int_{k_t^2}^{\mu^2}\frac{\alpha_S(\kappa_t^2)}{2\pi}\frac{d\kappa_t^2}{\kappa_t^2}\sum_b\int_0^{1-\Delta} dz' P_{qb}^{(0)}(z')\right). \qquad k_t = p_\perp/z$$

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$$D_q^H(z,k_t,\mu^2) = T_q(k_t,\mu^2)\sum_{b=q,g}\left[\frac{\alpha_S(k_t^2)}{2\pi k_t^2}\int_z^{1-\Delta} dz' P_{bq}^{(0)}(z')\mathcal{D}_b^H\left(\frac{z}{z'},k_t^2\right)\right],$$

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$$k_t = p_\perp / z$$

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DSS libray; D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D 76, 074033 (2007).

- **The LO-MRW method:**

$$D_q^{H,LO}(z,k_t,\mu^2) = T_q(k_t,\mu^2)\frac{\alpha_S(k_t^2)}{2\pi k_t^2}\int_z^1 dz'\left[P_{qq}^{(0)}(z')\frac{z}{z'}D_q^H\left(\frac{z}{z'},k_t^2\right)\Theta\left(\frac{\mu}{\mu+k_t}-z'\right)\right.$$

$$\left.+P_{gq}^{(0)}(z')\frac{z}{z'}D_g\left(\frac{z}{z'},k_t^2\right)\right],$$

with

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By expanding the MRW formalism to the NLO level, we have:

$$D_q^{H,NLO}(z,k_t,\mu^2) = \int_z^1 dz' T_q\left(k,\mu^2\right)\frac{\alpha_S(k^2)}{2\pi k_t^2}\sum_{b=q,\bar{q},g}\tilde{P}_{bq}^{(0+1)}(z')\ \times D_b^{H,NLO}\left(\frac{z}{z'},k^2\right)\Theta\left(1-z'-\frac{k_t^2}{\mu^2}\right),$$

Where $k^2 = \frac{k_t^2}{(1-z')}$. In the above formula, the Sudakov form factor is defined as:

$$T_q(k,\mu^2) = exp\left(-\int_{k^2}^{\mu^2}\frac{\alpha_S(\kappa_t^2)}{2\pi}\frac{d\kappa_t^2}{\kappa_t^2}\int_0^1 dz'z'\left[\tilde{P}_{qq}^{(0+1)}(z')+\tilde{P}_{qg}^{(0+1)}(z')\right]\right),$$

The cross section and experimental data

❖ **The cross section of the single inclusive hadron production in the *e+ e-* annihilation processes :**

$$\frac{1}{\sigma_{tot}}\frac{d\sigma^H}{dzd^2\vec{p}_\perp} = \frac{1}{\sum_q {e_q}^2}\sum_q {e_q}^2[D_q^H(z,p_\perp;\mu^2) + D_{\bar{q}}^H(z,p_\perp;\mu^2)].$$

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The experimental data

- $TASSO \rightarrow \sqrt{s}$=14, 22, 35, 44 GeV
- $MARKII \rightarrow \sqrt{s}$=29 GeV
- $CELLO \rightarrow \sqrt{s}$=34 GeV
- $AMY \rightarrow \sqrt{s}$=52-57 GeV
- $BELLE \rightarrow \sqrt{s}$=10.58 GeV (2019)
- $DELPHI, SLD\ and\ ALEPH \rightarrow \sqrt{s}$=91 GeV

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$$\frac{1}{\sigma_{tot}}\frac{d\sigma^H}{dz d^2\vec{p}_\perp} = \frac{1}{\sum_q e_q{}^2}\sum_q e_q{}^2[D_q^H(z,p_\perp;\mu^2) + D_{\bar{q}}^H(z,p_\perp;\mu^2)].$$

The experimental data

- $TASSO \rightarrow \sqrt{s}$=14, 22, 35, 44 GeV
- $MARKII \rightarrow \sqrt{s}$=29 GeV
- $CELLO \rightarrow \sqrt{s}$=34 GeV
- $AMY \rightarrow \sqrt{s}$=52-57 GeV
- $BELLE \rightarrow \sqrt{s}$=10.58 GeV (2019)
- $DELPHI, SLD\ and\ ALEPH \rightarrow \sqrt{s}$=91 GeV

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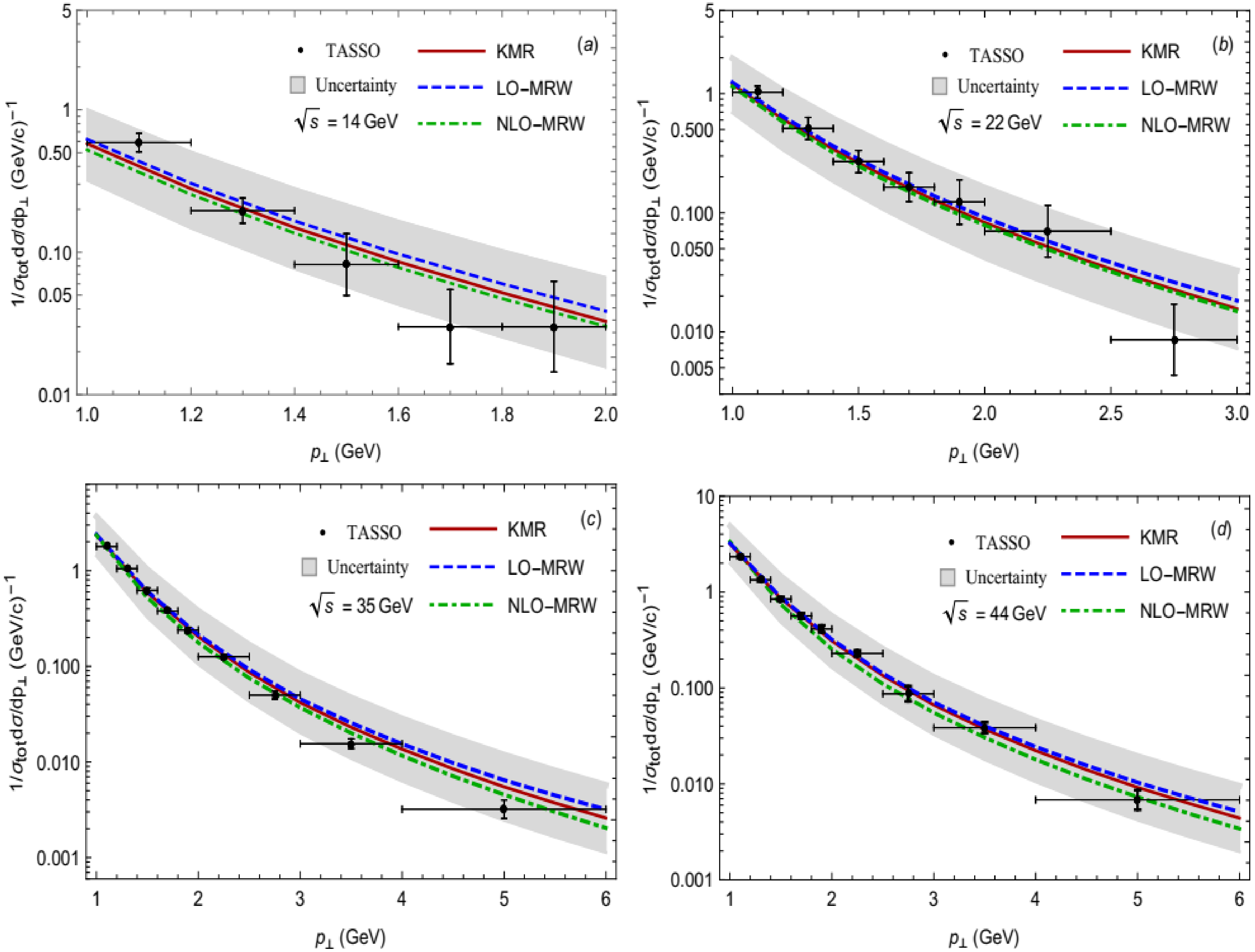

Fig2: The normalized differential cross sections $(1/\sigma_{tot})d\sigma/dp_\perp$ with respect to $p_\perp$ compared to the experimental data of TASSO at the different CM energies. The shaded uncertainty grey bands are belong to the KMR prescription.

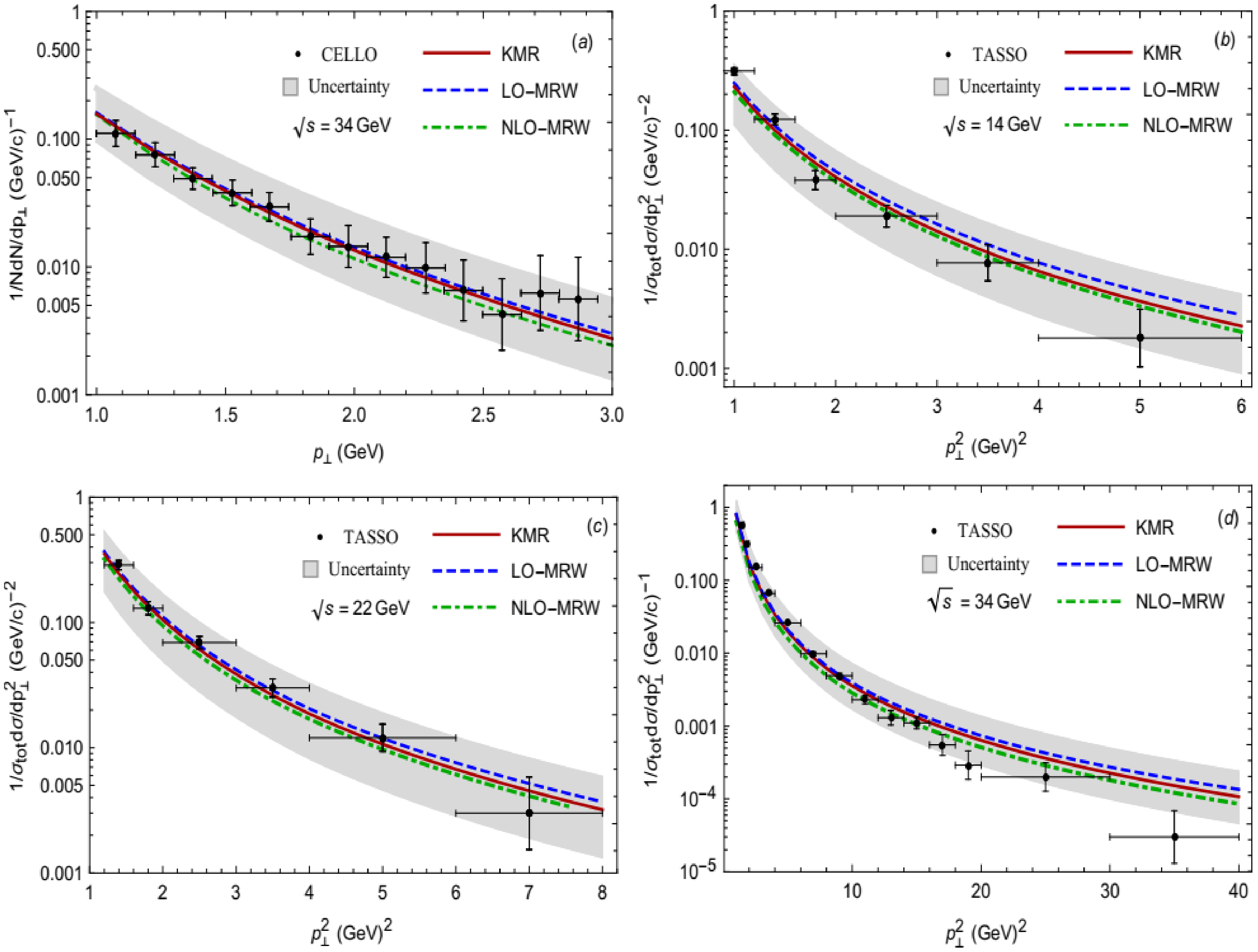

Fig3: The panel (a): the normalized distribution of the multiplicity with respect to $p_{\perp}$ for charged particles is compared to the experimental data of CELLO. The panels (b)-(d): the normalized differential cross sections with respect to $p_{\perp}$ squre compared to the experimental data of TASSO at the different CM energies. The shaded uncertainty grey bands are belong to the KMR prescription. The results are more or less the same as previous plots.

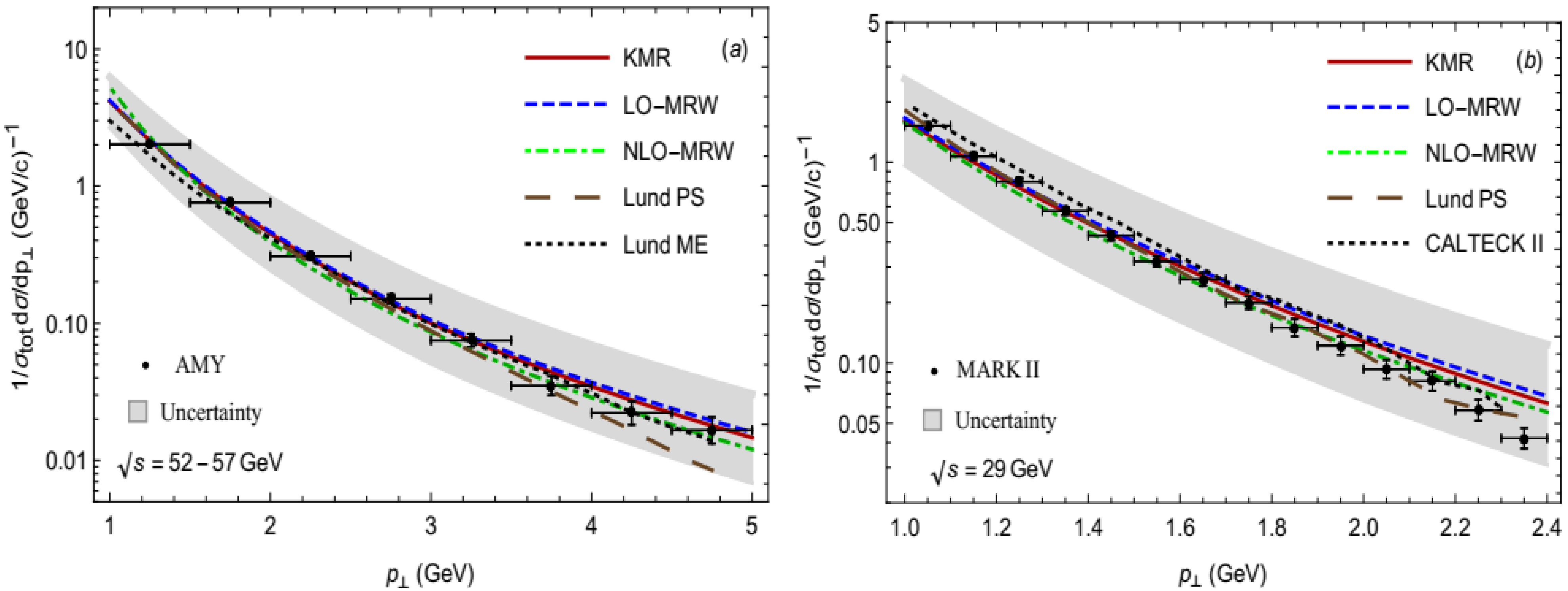

FIG. 4: The normalized differential cross sections with respect to $p_\perp$ for charged particles is compared to the experimental data of AMY [38] (the left panel), of MARK II [37] (the right panel), and some "QCD+fragmentation" models predictions. The shaded uncertainty grey bands are belong to the KMR prescription.

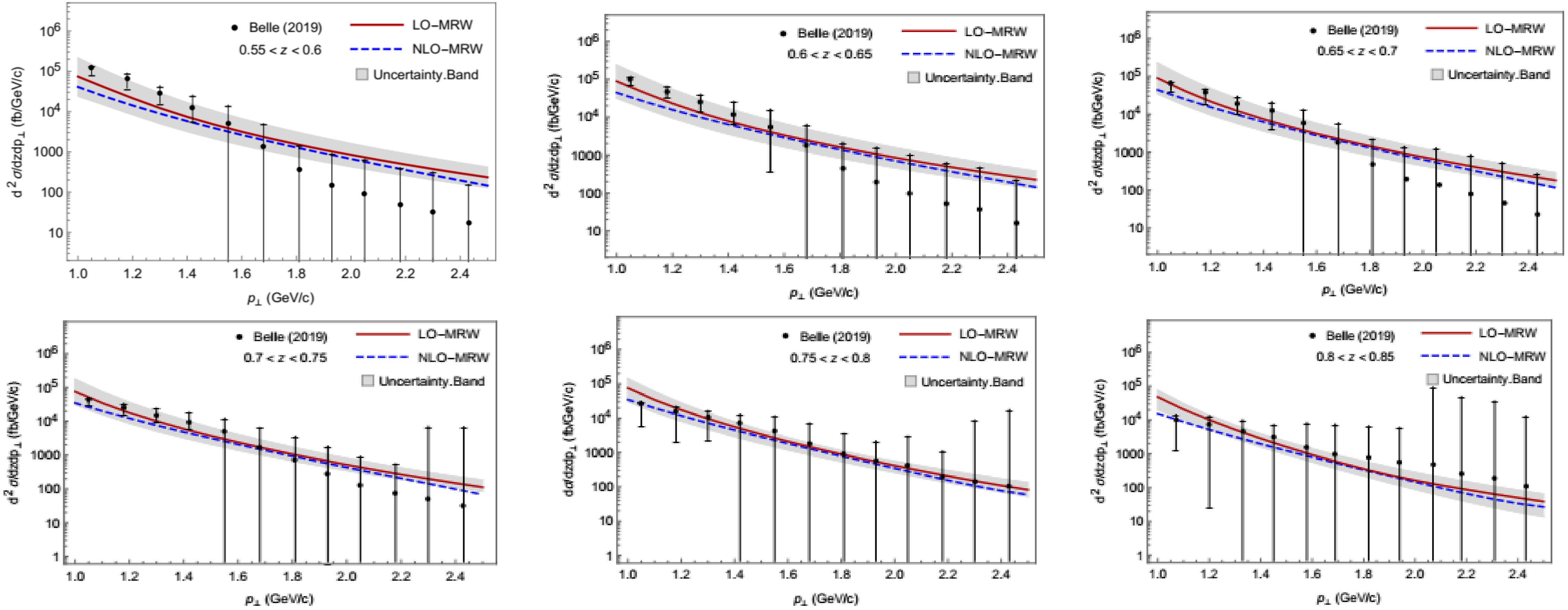

FIG. 5: The differential cross sections for pions as a function of $p_\perp$ for the indicated z bins and thrust $0.85 < T < 0.9$. The error grey bands represent the uncertainties for LO-MRW formalism. The results are compared to the experimental data of Belle collaboration [44] in the $\sqrt{s} = 10.58$ GeV center of mass energy.

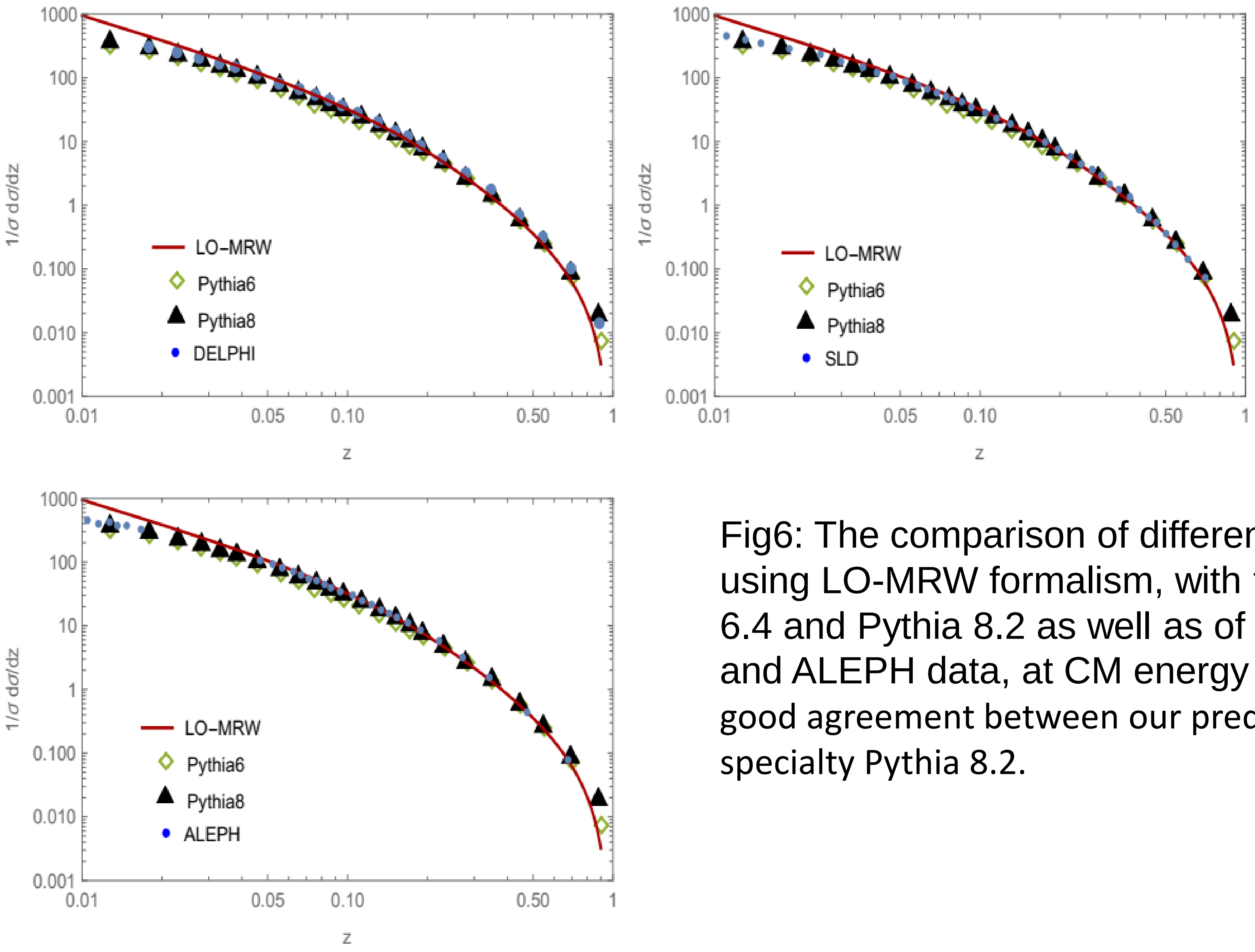

Fig6: The comparison of differential cross section using LO-MRW formalism, with those of Pythia 6.4 and Pythia 8.2 as well as of DELPHI, SLD and ALEPH data, at CM energy 91 GeV.. There is good agreement between our prediction and specialty Pythia 8.2.

FINAL REMARKS

- **Many studies on the kt-factorization and different AOC methods show that this approach could be used in calculating unintegrated parton distribution functions and in predicting the experimental data properly.**
- **We showed that with some considerations, it could be used in finding the transverse momentum dependent fragmentation functions.**
- **We hope to apply these formalisms to other hadrons and in higher accuracy levels in our future works.**

Thank you