Benchmarking Variational Quantum Algorithms for the LUXE experiment

Arianna Crippa[1], Lena Funcke[2], Tobias Hartung[3,6], Beate Heinemann[1,4], Karl Jansen[1,5], **Annabel Kropf [1,4]**, Stefan Kühn[6], Federico Meloni[1], David Spataro[1,4], Cenk Tüysüz[1,5], Yee Chinn Yap[1]

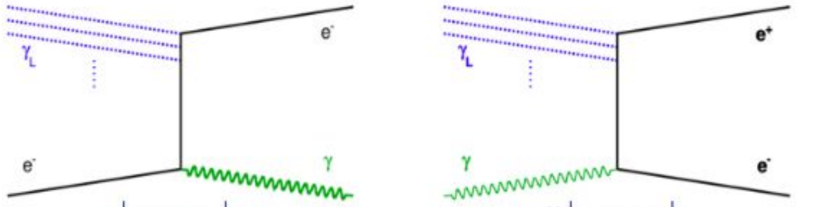
[1]Deutsches Elektron-Synchrotron DESY
[2]Massachusetts Institute of Technology, MIT
[3]University of Bath
[4]Albert-Ludwigs-Universität Freiburg
[5]Humboldt-Universität zu Berlin
[6]CaSToRC, The Cyprus Institute

DPG 2022 Heidelberg, 23.03.2022

HELMHOLTZ LUXE UNI FREIBURG FTX DESY.

LUXE

Laser und XFEL Experiment



Goal: Investigate the transition into the non-perturbative (strong-field) regime of QED

→ Precision measurement of photon-photon, photon-electron interactions

How: Use high intensity laser (40-350 TW) and the european XFEL's electron beam (16.5 GeV)

Crucial: Measure number of positrons as a function of the laser intensity parameter $\xi = \sqrt{4\pi\alpha}\left(\frac{\varepsilon_L}{\omega_L m_e}\right) = \frac{m_e \varepsilon_L}{\omega_L \varepsilon_{cr}}$

HELMHOLTZ

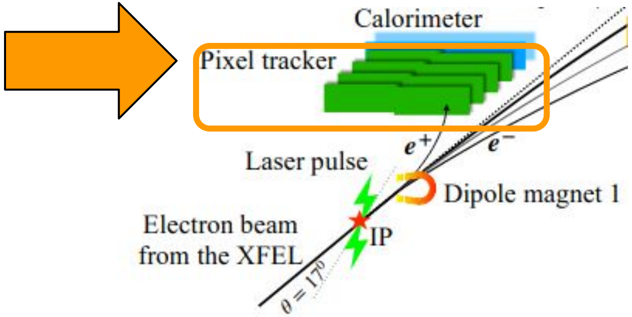
DESY.

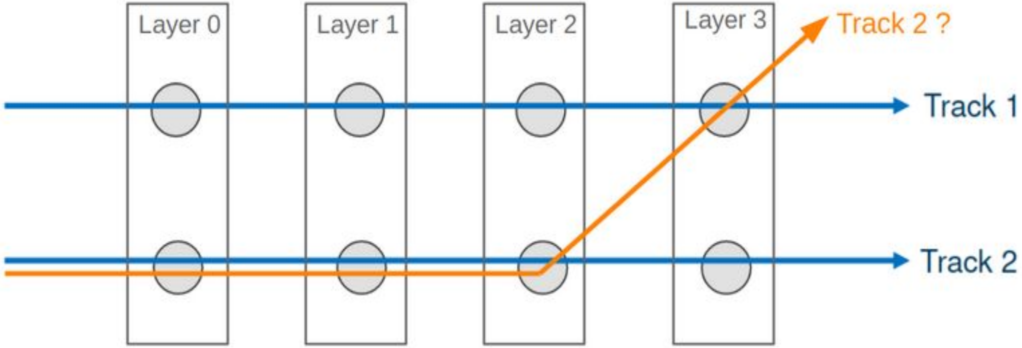
Track Reconstruction

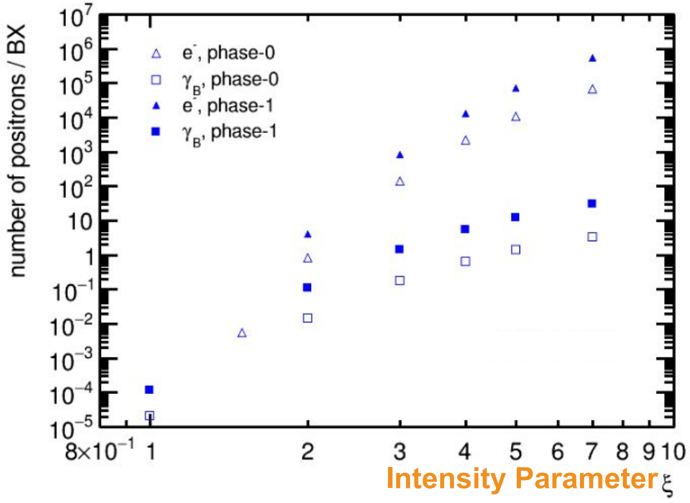
Positron Pixel Tracker

Positrons impinge on 4-layered binary hit/no-hit silicon detector (ALPIDE chip, 5 x 10^5 pixel of size 27 x 29 μm^2)

Challenge: Find tracks from a set of hits is computationally demanding







HELMHOLTZ

Track Reconstruction

Novel approach: Particle track reconstruction using quantum computers

A Pattern Recognition Algorithm for Quantum Annealers

Frédéric Bapst, Wahid Bhimji, Paolo Calafiura, Heather Gray, Wim Lavrijsen, Lucy Linder & Alex Smith

Computing and Software for Big Science **4**, Article number: 1 (2020) | Cite this article

1466 Accesses | **7** Citations | **14** Altmetric | Metrics

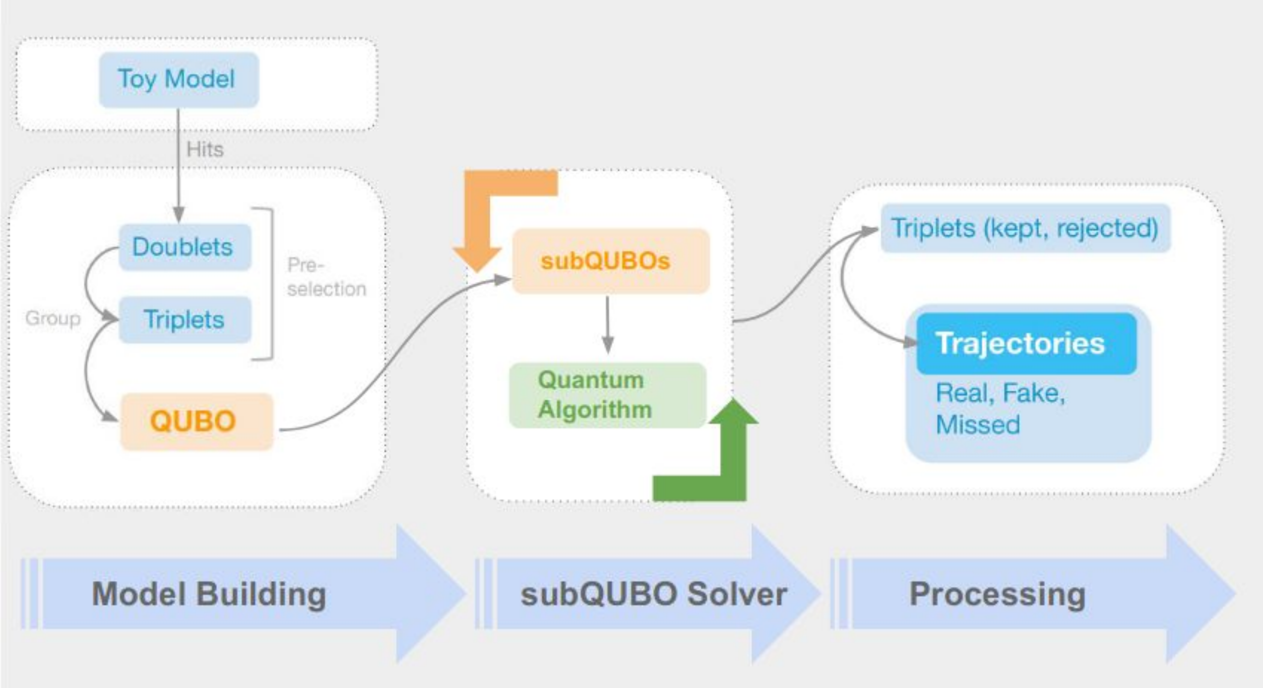
+ Classical benchmark!

QUBO (Quadratic Unconstrained Binary Optimization)

The goal is to benchmark both on basis of:

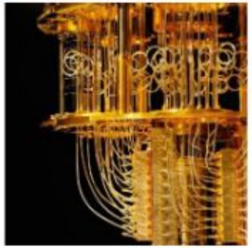
1. Generated signal interactions at the IP (T. G. Blackburn, A. J. MacLeod, B. King, arXiv:2103.06673v2),
2. Custom detector model where complexity can be controlled.

Quantum Algorithm Overview

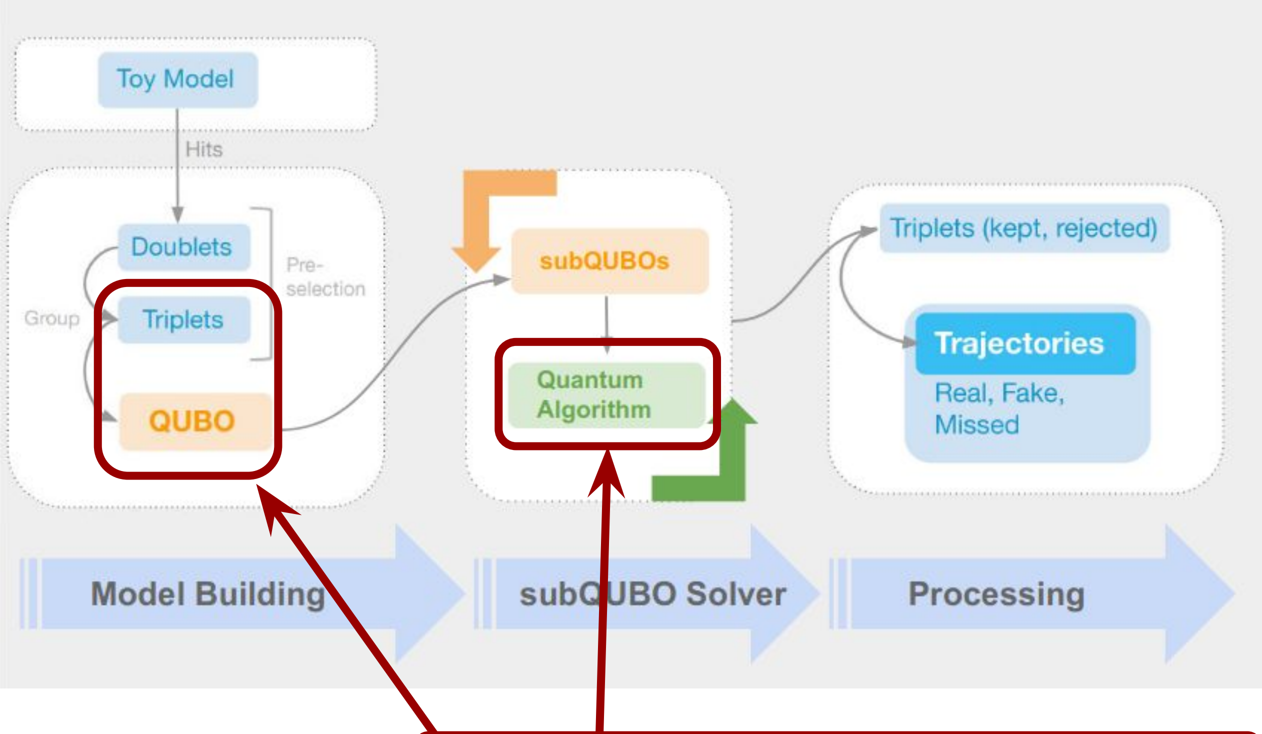


Devices used:
IBM Q
1-7 real qubit systems available + qc simulators

IBM Quantum Experience:
(https://quantum-computing.ibm.com/)

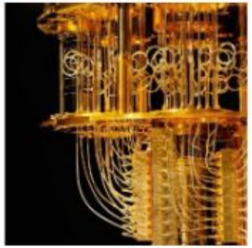


Quantum Algorithm Overview



Devices used:
IBM Q
1-7 real qubit systems available + qc simulators

IBM Quantum Experience:
(https://quantum-computing.ibm.com/)



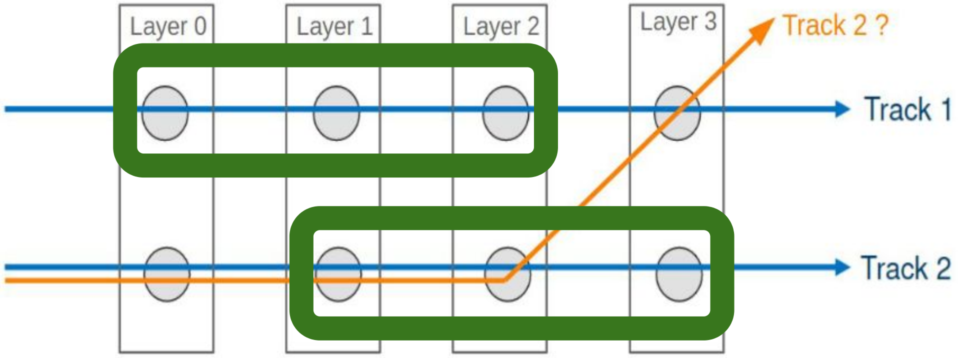
Closer look in this talk: Triplets, QUBO, QA

Tracking Formulation for a Quantum Computer

Grouping Hits

Task: Find formulation of the track reconstruction so that if simulated on a quantum computer, the ground state relates to correct tracks

Step 1:
Form sets of triplets
(three consecutive hits)

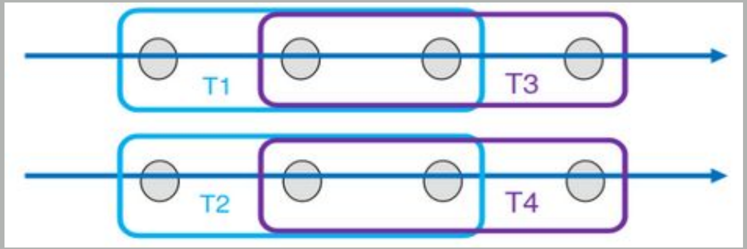


HELMHOLTZ

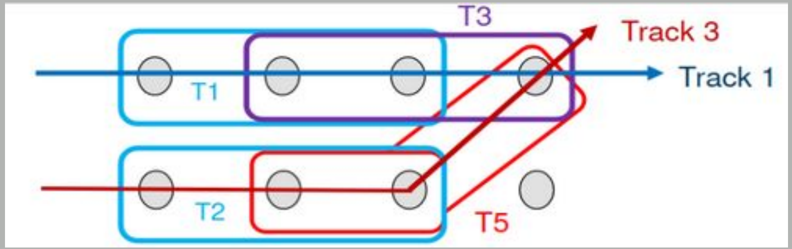
DESY.

QUBO

Quadratic Unconstrained Binary Optimization



Well-aligned triplets



Conflict between triplets T3 and T5

Step 2: Maximize number of triplets that form well-aligned tracks and minimize triplets that contribute a conflict

- Express problem as a QUBO
- Minimizing the QUBO returns the best set of track candidates

HELMHOLTZ

DESY.

QUBO

Quadratic Unconstrained Binary Optimization

In the QUBO formulation, triplets T_i are assigned a binary value, one if chosen and zero if discarded.
Closely related to Ising Model: Binary (0,1) instead of spins (-1,1).

$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \qquad T_i, T_j \in \{0,1\}$$

quality of triplet with quality parameter a_i

Compatibility between two triplets

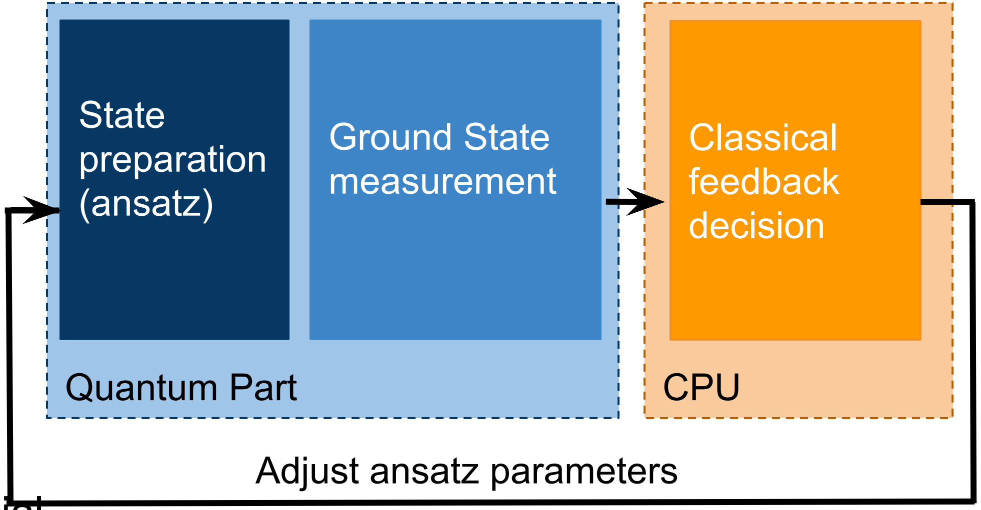
$$b_{ij} = \begin{cases} -S(Ti,Tj), & \text{if } (T_i,T_j) \text{ form a quadruplet,} \\ \zeta & \text{if } (T_i,T_j) \text{ are in conflict,} \\ 0 & \text{otherwise.} \end{cases}$$

Hybrid Algorithms

VQE/QAOA

Step 3: Minimize the QUBO

Hybrid: Quantum subroutine run inside of a classical optimization loop
Variational-Quantum-Eigensolver (VQE): hybrid algorithm used to find eigenvalues of a matrix
The **Quantum Approximate Optimization Algorithm (QAOA)** can be viewed as a special case of VQE

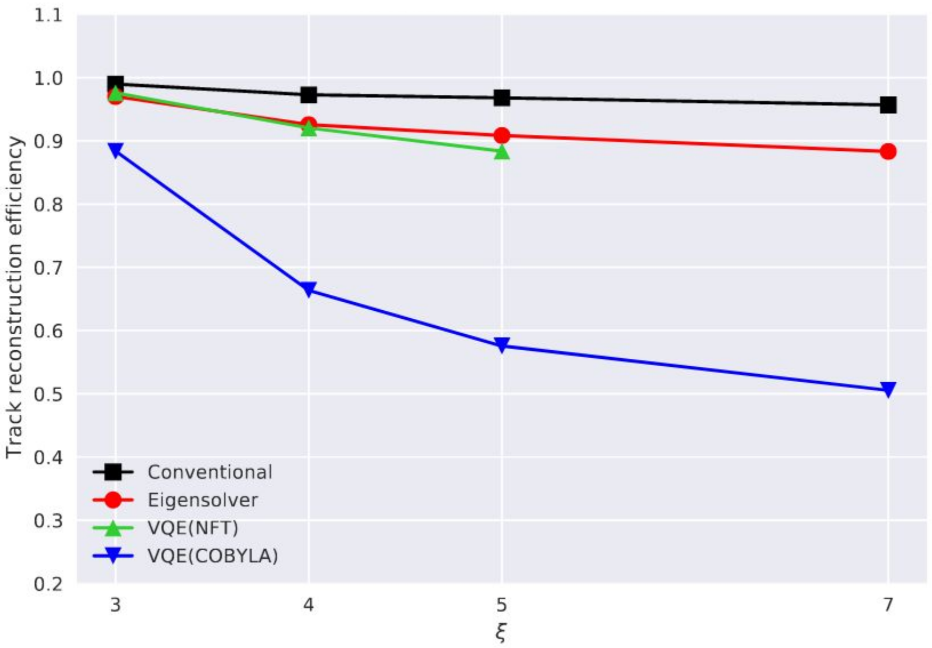


Track Reconstruction Efficiency

Benchmarking VQE(NFT) and VQE(COBYLA) on Simulator

- Two optimizers considered: NFT and COBYLA
- Eigensolver refers to solving subQUBO through diagonalizing of the Hamiltonian

VQE's performance depends on optimizer!



HELMHOLTZ

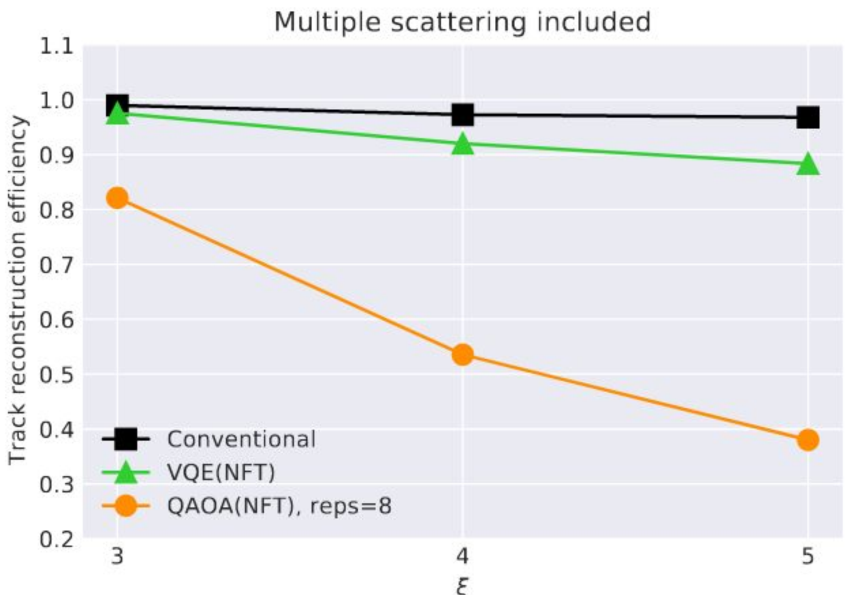
DESY.

Track Reconstruction Efficiency

Benchmarking QAOA(NFT) and VQE(NFT) on Simulator

- QAOA's efficiency depends on the number of repeats (reps)
- Both are benchmarked using NFT optimizer

VQE outperforms QAOA using the NFT optimizer



HELMHOLTZ

DESY.

Summary and Outlook

Goal: Study the limits for a quantum algorithm on **noisy device** to tackle particle track reconstruction for the LUXE experiment (positron tracker)

Here, VQE and QAOA were compared:

- VQE(NFT) depends on optimizer. NFT outperforms COBYLA
- VQE outperforms QAOA (8 reps)

Future outlook:

- Make use of real devices and develop error mitigation techniques
- Future study of solver's dependencies (reps, optimizer)
- Study impact of QUBO division method

Thank you

QAOA

Solving the subQubo

-QAOA can be viewed as a special case of VQE.

- Hamiltonian contains only Z terms, we do not need to change the basis for measurements.

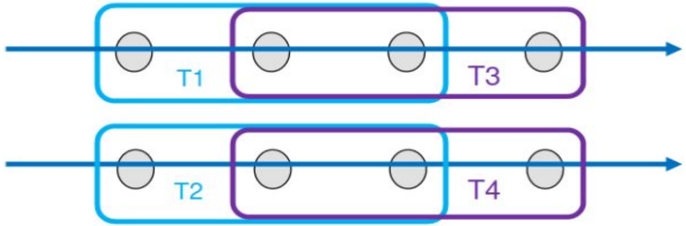
Differences to VQE:

- The form of the ansatz is limited
- Restricted to Ising Hamiltonians
- In QAOA our goal is to find the solution to the problem. To do that we don’t need to find the ground state.

QUBOs

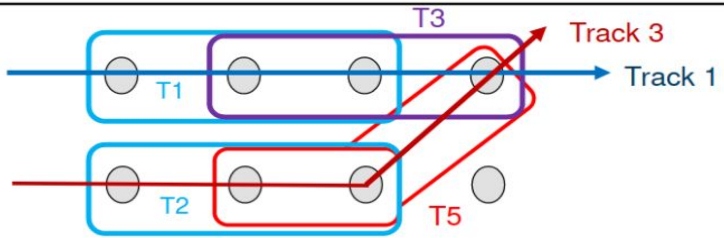
$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \quad T \in \{0,1\}$$

$$b_{ij} = \begin{cases} -S(Ti, Tj), & \text{if } (T_i, T_j) \text{ form a quadruplet,} \\ \zeta & \text{if } (T_i, T_j) \text{ are in conflict,} \\ 0 & \text{otherwise.} \end{cases}$$



[T1, T2, T3,T4]→combinations:

T1T2	T1T3	T1T4	T2T3	T2T4	T3T4
↓	↓	↓	↓	↓	↓
+0	-S	+0	+0	-S	+0



[T1, T2, T3,**T5**] → combinations:

T1T2	T1T3	T1**T5**	T2T3	T2**T5**	T3**T5**
↓	↓	↓	↓	↓	↓
+0	-S	+0	+0	-S	conflict

7

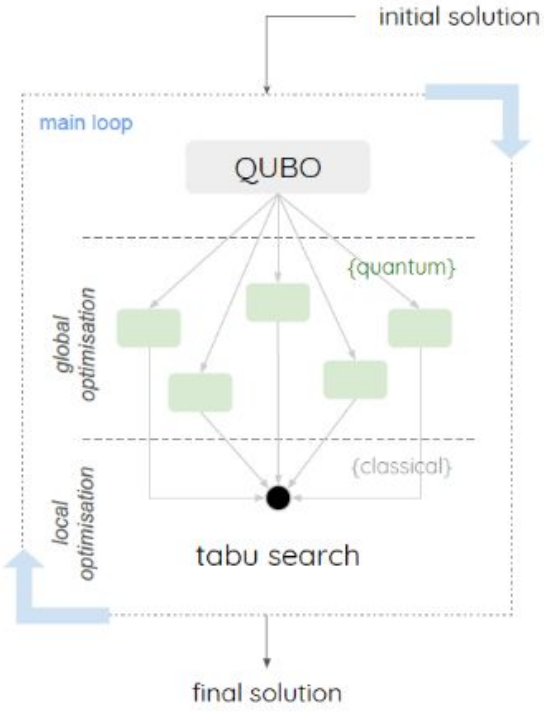
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

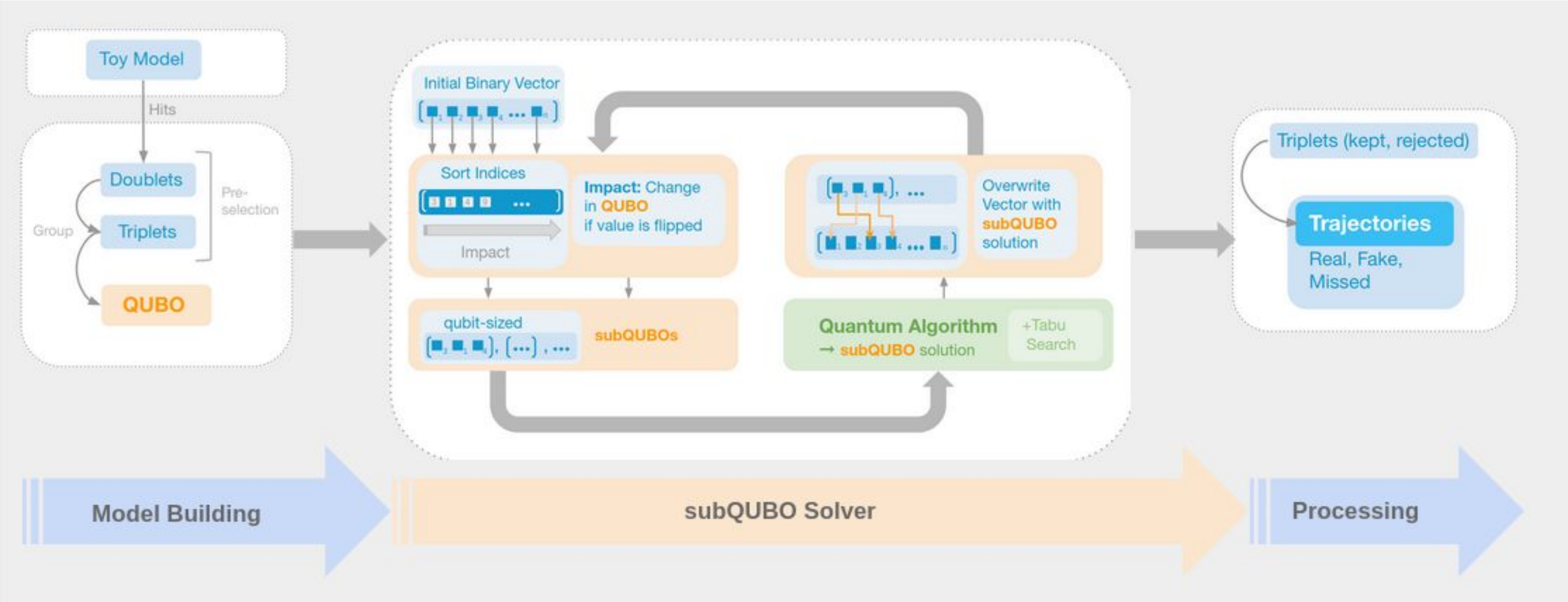
DESY.

SubQubos

Problem: Devices restricted to small number of qubits

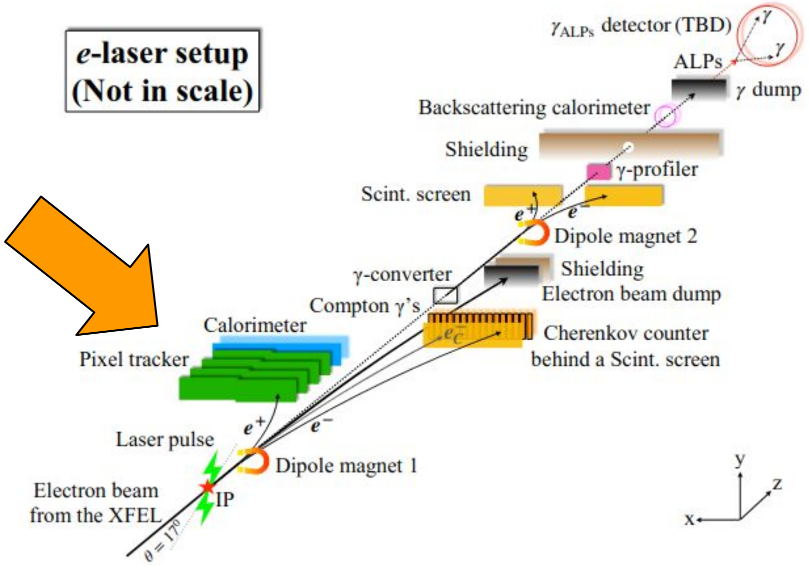
- Big QUBOS cannot be simulates
- Break QUBO into subsets → subQUBOS!
- Iterated vector converges to solution vector

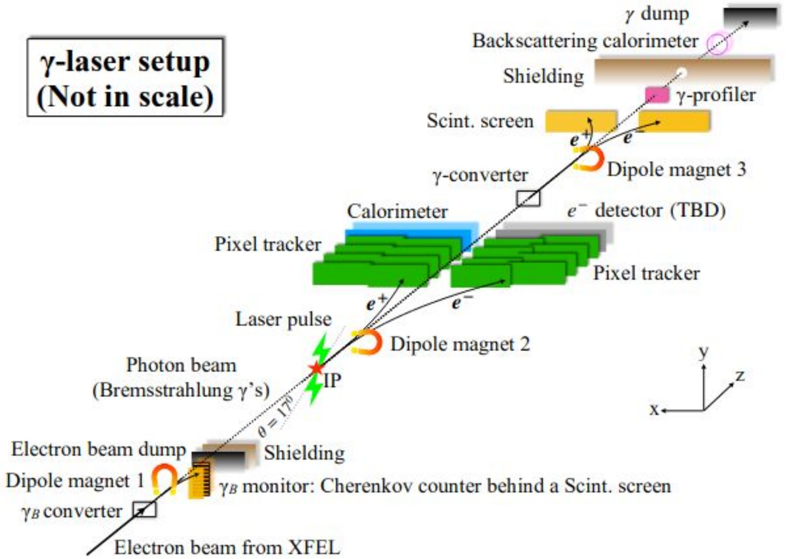




Toy Model
Hits
Doublets
Triplets
QUBO
Group
Pre-selection
Initial Binary Vector
Sort Indices
Impact
Impact: Change in QUBO if value is flipped
qubit-sized
subQUBOs
Overwrite Vector with subQUBO solution
Quantum Algorithm
→ subQUBO solution
+Tabu Search
Triplets (kept, rejected)
Trajectories
Real, Fake, Missed
Model Building
subQUBO Solver
Processing

LUXE setup





LUXE CDR: arXiv:2102.02032