Extraction of α_s using data from H1

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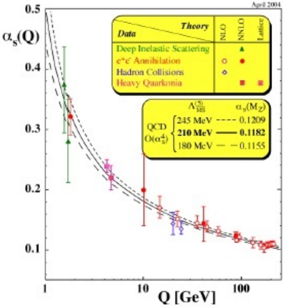
Introduction

- QCD is the theory of quarks, gluon and their interaction in the hadrons
- QCD has an important property which is the asymptotic freedom :
  "At high energy, quarks/gluons behave as quasi-free particles"
- The renormalization group equation predict that :

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1+\alpha(\mu^2)\underbrace{b}_{\frac{33-2n_f}{12\pi}}\ln(Q^2/\mu^2)}$$



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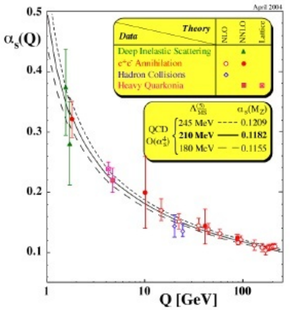
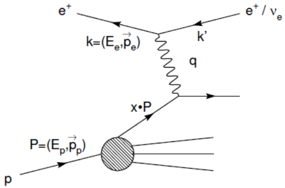


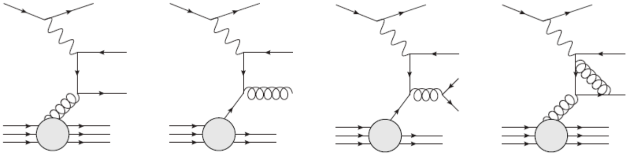
- So, the perturbative calculation at high energy scale is allowed
- Goal : How to measure α_s from the H1 data

Extraction of α_s using jet production

- Deep inelastic process DIS :



- To measure α_s, we have to use a measurement with more than one jet



- The next-to-leading (NLO) contribution is given by these diagrams

Theoretical overview of jet production at NLO

- Schematically, the NLO cross section is given by :

$$\sigma_{n-jet} = \sigma^{LO}_{n-jet} + \sigma^{HO}_{n-jet} = \sigma^{LO}_{n-jet} + \left[\int_{n+1} d\sigma^R + \int_n d\sigma^V \right] \quad (1)$$

- To extract the IR divergences we use the subtraction method
- The inclusive jet cross section in DIS is given :

$$\sigma = \sum_{a,n} \int_0^1 dx c_{a,n}\left(\frac{x_{B_j}}{x}, \mu_r, \mu_f\right). \otimes [\alpha_s^n(\mu_r).f_{a/h}(x,\mu_f)] \quad (2)$$

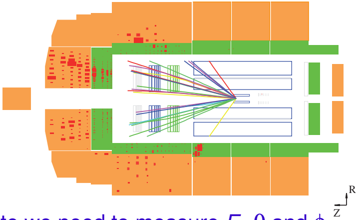
- Use fasNLO program to calculate σ
  - Program calculates the jet cross section in the next-to-leading order
  - Based on Fortran language (or C++), require NLOjet++ code
  - The basic idea is to write the cross section σ

$$\sigma = \sum_{a,n,l,k} \overbrace{\tilde{\sigma}_{n,a,k,l}}^{c_{n,a}(x,\mu).[e^{(k)}(x).b^{(l)}(\mu)]} .\alpha_s^n(\mu^{(l)}) f_{a/h}(x^{(k)}, \mu^{(l)}) \quad (3)$$

  - Where we separate the PDFs from the integration $\Rightarrow$ The computation is done very fast

Jet data

- A DIS NC event at high Q^2



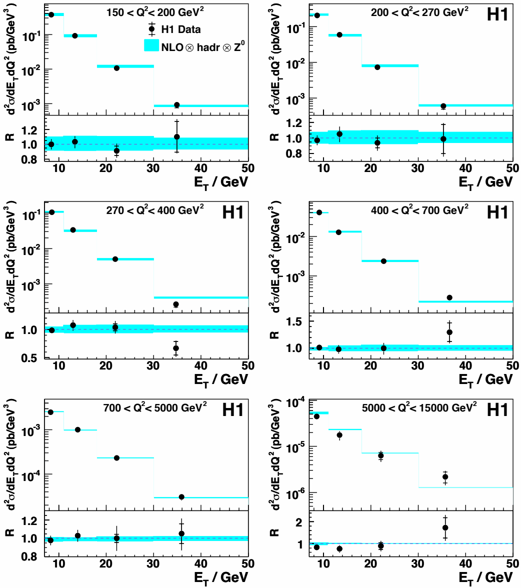
- To reconstruct a 4-vector in the final state we need to measure E, θ and ϕ
- The DIS kinematical variables :

$$150 < Q^2 < 15000 GeV^2 \qquad 0.2 < y < 0.7 \tag{4}$$

- The jet analysis is performed in Breit frame : $2x_{B_j}\vec{p} + \vec{q} = 0$
- Particles of the hadronic final state are clustered using the k_T algorithm
- The experimental cross section :

$$\sigma^i = \frac{Nev^i}{\mathcal{L}_{data}} \cdot \frac{f^i_{QED}}{A^i} \tag{5}$$

Inclusive Jet Cross Section



Fitting technique

The fit of α_s is performed by the minimization of the χ^2 using Minuit

- Source of correlated errors :
  - Positron energy scale
  - Positron polar angle
  - Model dependance of the data correction
  - HFS scale uncertainty
  - Luminosity measurement
  - QED radiative correction
- Source of uncorrelated errors :
  - Statistical uncertainty
  - model dependance
  - positron energy
  - positron polar angle
  - HFS energy scale uncertainty
- The model dependance and the HFS scale uncertainty are the dominated errors

Minimization of χ^2 using Minuit

- χ^2 definition

$$\chi^2(\alpha_s,\vec{\varepsilon}) = \sum_i \frac{[\sigma_i^{exp} - \sigma_i^{the}(\alpha_s)\{1-\sum_k \delta_{i,k}(\varepsilon_k)\}]^2}{\delta^2_{i,uncorr}} + \sum_k \varepsilon_k^2 \quad (6)$$

- i runs over measured cross section
- $\sigma_i^{the}(\alpha_s) = \sigma_i^{fastNLO}(\alpha_s) \otimes [z^0/hcor]$
- k runs over all sources of correlated errors
- $\delta_{i,k}$: contribution from kth correlated source to ith measurements
- α_s and ε_k are the fitted parameters, where :
  - $\varepsilon_1 =$ Positron energy scale
  - $\varepsilon_2 =$ Positron energy angle
  - $\varepsilon_3 =$ Model dependance of the data correction
  - $\varepsilon_4 =$ HFS uncertainty
  - $\varepsilon_5 =$ Luminosity measurement

- Minuit : is a collection of minimization libraries developed at CERN used to find the minimum value of a multi-parameter function.

Extraction of α_s

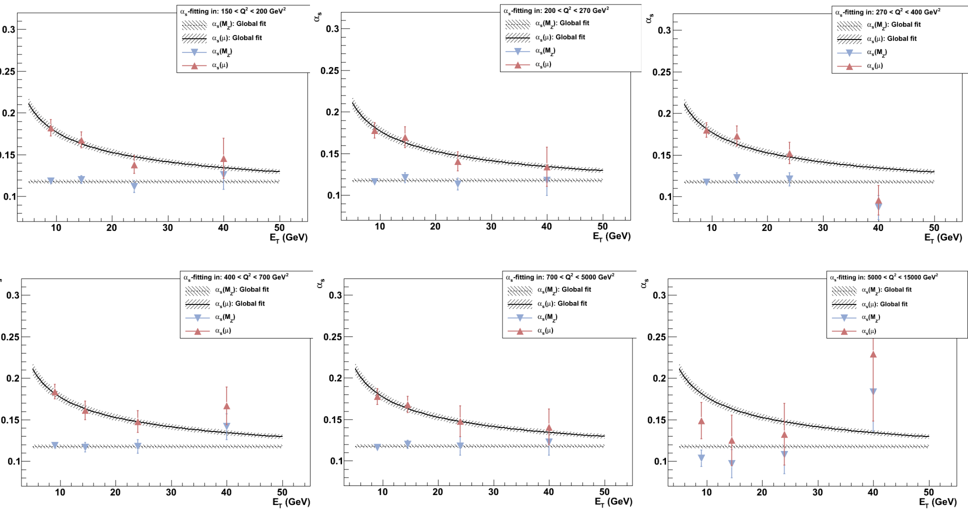
In this work, the central value of $\alpha_s(M_z)$ is obtained from :

- The fit of the double differential cross section $d^2\sigma_{jet}/dE_T dQ^2$
- Using the k_T algorithm
- Using CEQ65 parameterization for the PDFs
- The renormalization scale chosen to be E_T
- The DIS phase space covered by this analysis is defined by :

$$150 < Q^2 < 15000 GeV \qquad 0.2 < y < 0.7 \qquad (7)$$

- The data were collected with H1 detector in the years 1999 and 2000

Fitted value of $\alpha_s(E_T)$ in six regions of Q^2



Results

All the 24 measurements used in a common fit of the strong coupling constant yields :

$$\chi^2 = 20.275$$

$$\alpha_s(M_z) = 0.11781 \pm 0.00232$$

$$\varepsilon_1 = -0.33 \pm 0.67$$

$$\varepsilon_2 = 1.02 \pm 0.824$$

$$\varepsilon_3 = 0.31 \pm 0.97$$

$$\varepsilon_4 = 0.23 \pm 0.73$$

$$\varepsilon_5 = 0.07 \pm 0.99$$

The value α_s from the publication : $\alpha_s(M_z) = 0.1179 \pm 0.0024$


Thank you

Minimaization of χ^2 using Minuit

Minuit is a collection of minimization libraries developed at CERN by Fred James (1975)

- Fnd the minimum value of a multi-parameter function
- How to use minuit ?
  - Write the FCN subroutine : it is a multi-parameter Fortran function, calculates the χ^2 between the prediction and the data.
  - The parameters of FCN are the α_s and the ε_k
  - Give to the parameters a starting values ($\alpha_s = 0.118$ and $\varepsilon_k = 0$) and starting errors
  - Let all the parameters free
  - Call Minuit : Minuit commands (Migrad, Minos and hesse) request minuit to minimize FCN with respect to the parameters which corresponding to the lowest value of the χ^2
  - Get a value of α_s and ε_k