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Parton showers from old to new paradigms

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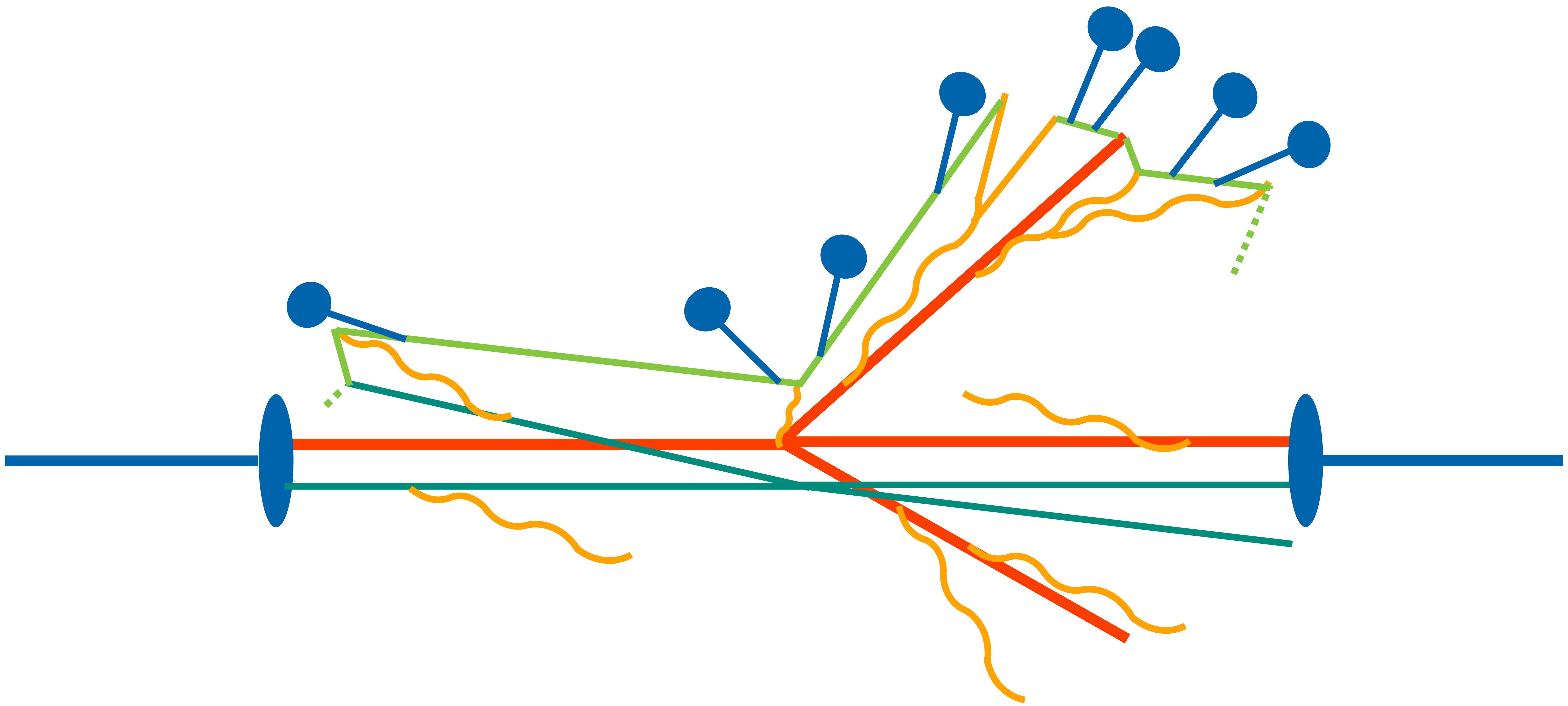
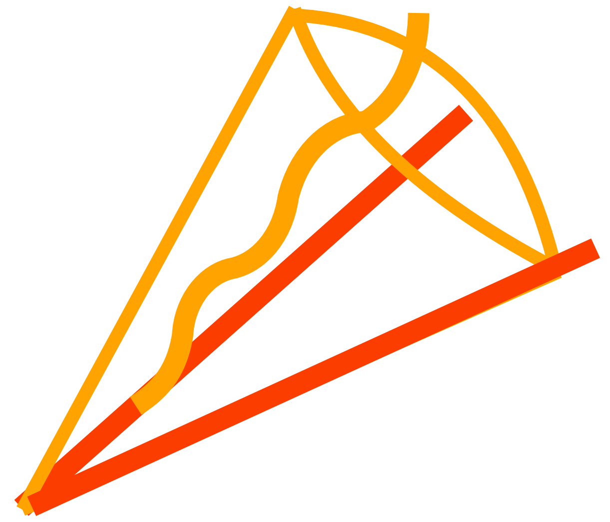
At the
ECFA workshop on e+e- colliders
Hamburg | 6 October 2022

QCD event generators



$$d\sigma \sim L \times d\sigma_H(Q) \times PS(Q \to \mu) \times MPI \times Had(\mu \to \Lambda) \times \ldots$$

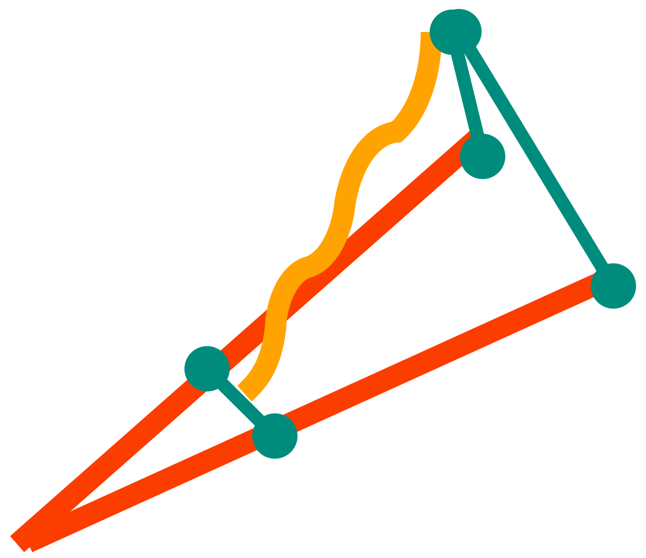
Shower & Parton Branching Paradigms



Parton branchings order in angle.

- Driven by QCD coherence
- Recoil global
- Links to analytic use of coherent branching

Herwig 7

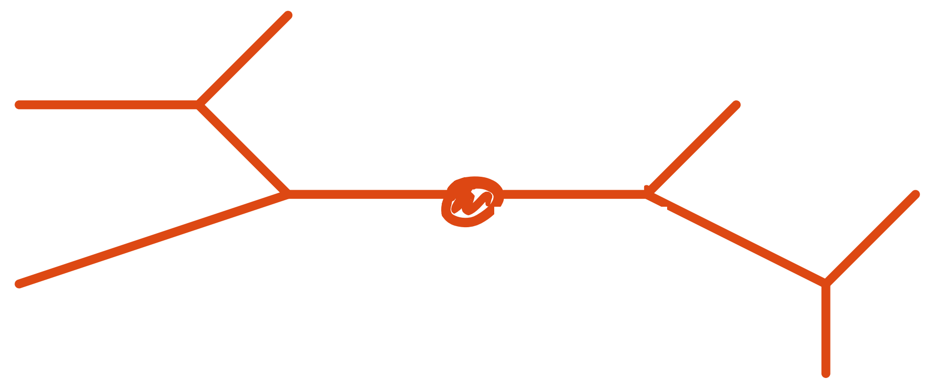


Dipole branchings order in transverse momentum.

- Driven by large-N dipole pattern and colour flows
- Momentum conservation for each emission
- Advantageous for matching & merging

Herwig 7, Pythia 8, Sherpa, PanScales, Deductor

Sequences of emission scales and momentum fractions as Markov process.
Restore momentum conservation per emissions or at end of evolution.
Deemed to **perform resummation** of logarithmic enhancements.



$$\mathrm{d}S = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\tilde{q}_i^2}{\tilde{q}_i^2}\mathrm{d}z P(z_i) \exp\left(-\int_{\tilde{q}_i^2}^{Q^2}\frac{\mathrm{d}q^2}{q^2}\int_{z_-(k^2)}^{z_+(k^2)}\mathrm{d}\xi \frac{\alpha_s}{2\pi}P(z)\right)$$

emission rate — no emission probability

$$\sigma(n\ \text{jets}, \tau) \sim \sum_k \sum_{l\leq 2k} c_{nkl}\ \alpha_s^k(Q)\ \ln^l \frac{1}{\tau}$$

LHC-age working horses

Current release series	Hard matrix elements	Shower algorithms	NLO Matching	Multijet merging	MPI	Hadronization	Shower variations
Herwig 7	Internal, libraries, event files	QTilde, Dipoles	Internally automated	Internally automated	Eikonal	Clusters, (Strings)	Yes
Pythia 8	Internal, event files	Pt ordered, DIRE, VINCIA	External	Internal, ME via event files	Interleaved	Strings	Yes
Sherpa 2	Internal, libraries	CSShower, DIRE	Internally automated	Internally automated	Eikonal	Clusters, Strings	Yes

LHC-age working horses

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	Current release series	Hard matrix elements	Shower algorithms	NLO Matching	Multijet merging	MPI	Hadronization	Shower variations
H7	Herwig 7	Internal, libraries, event files	QTilde, Dipoles	Internally automated	Internally automated	Eikonal		
	Pythia 8	Internal, event files	Pt ordered, DIRE, VINCIA					
	Sherpa 2	Internal, libraries	CSShower, DIRE				gs	Yes

General purpose — NLO matching & merging is settled.

The bottleneck are showers, also towards fully flexible matching with NNLO, as well as the uncertainties introduced by hadronization.

Accuracy of Parton Showers

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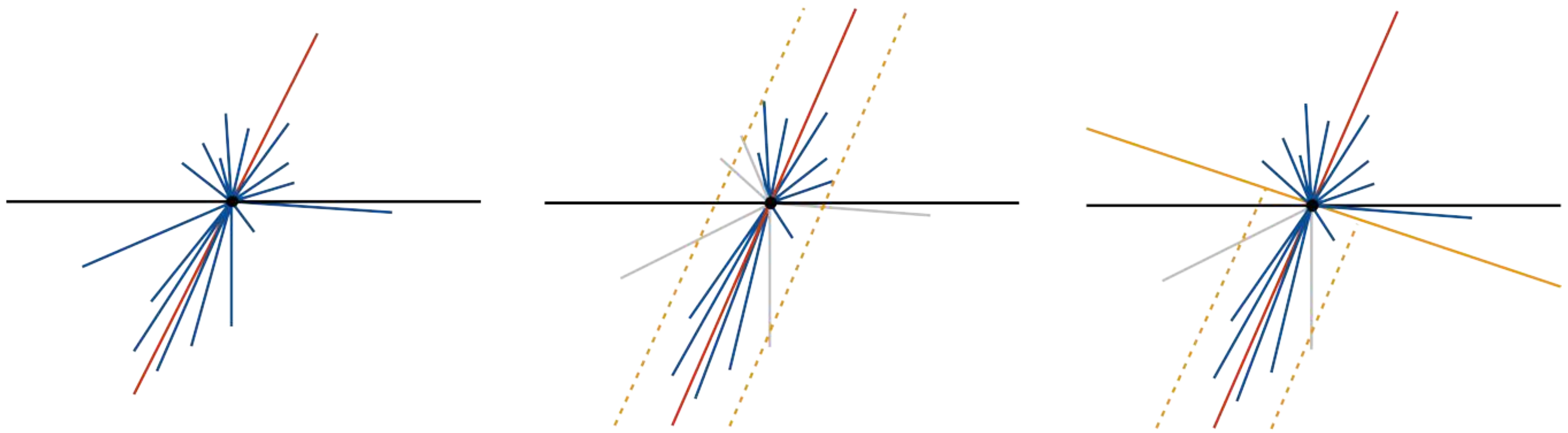

E.g. for exponentiating observables:

$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

LL — qualitative   NLL — quantitative   NNLL — precision

$\alpha_s L \sim 1$

Accuracy for massive event shapes

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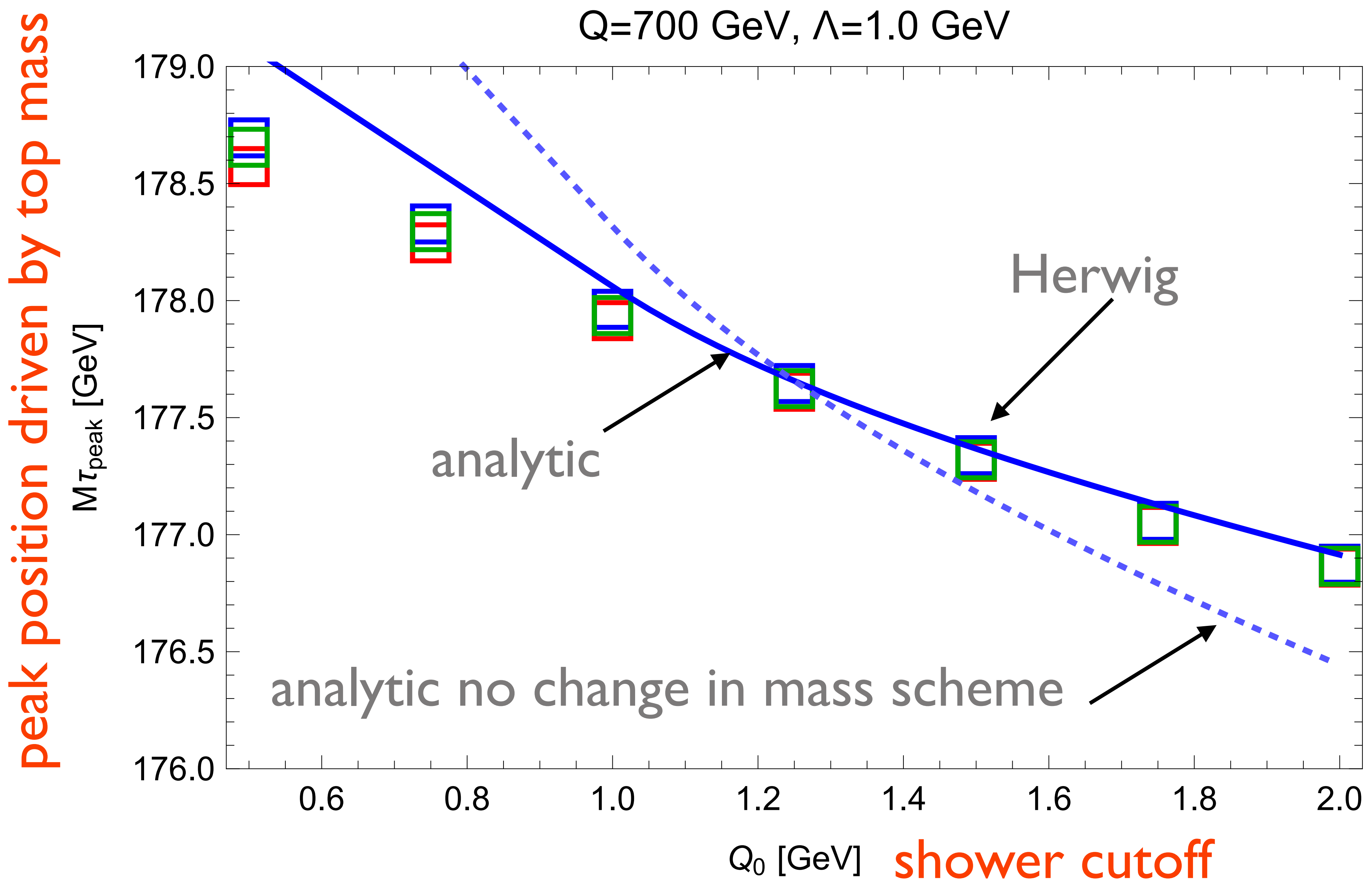
Coherent branching jet mass distribution including mass effects

$$z\,(1-z)\,\tilde{q}^2 = -m_{\widetilde{ij}}^2 + \frac{m_i^2}{z} + \frac{m_j^2}{1-z} - \frac{p_\perp^2}{z\,(1-z)}$$

$$P_{q\to qg} = \frac{C_F}{1-z}\left[1 + z^2 - \frac{2m_q^2}{z\tilde{q}^2}\right]$$

[Gieseke, Stephens, Webber – JHEP 0312 (2003) 045]

NLL accurate for global observables with massive quarks.



Analytically calculate perturbative correction to the top mass as predicted by parton branching algorithms

$$m_t^{\rm MC} = m_t^{\rm pole} + \Delta_m^{\rm pert} + \Delta_m^{\rm non\text{-}pert} + \Delta_m^{\rm MC}$$

$$m_t^{\rm CB}(Q_0) = m_t^{\rm pole} - \frac{2}{3} Q_0\, \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

[Hoang, Plätzer, Samitz — JHEP 1810 (2018) 200]

Accuracy of Parton Showers

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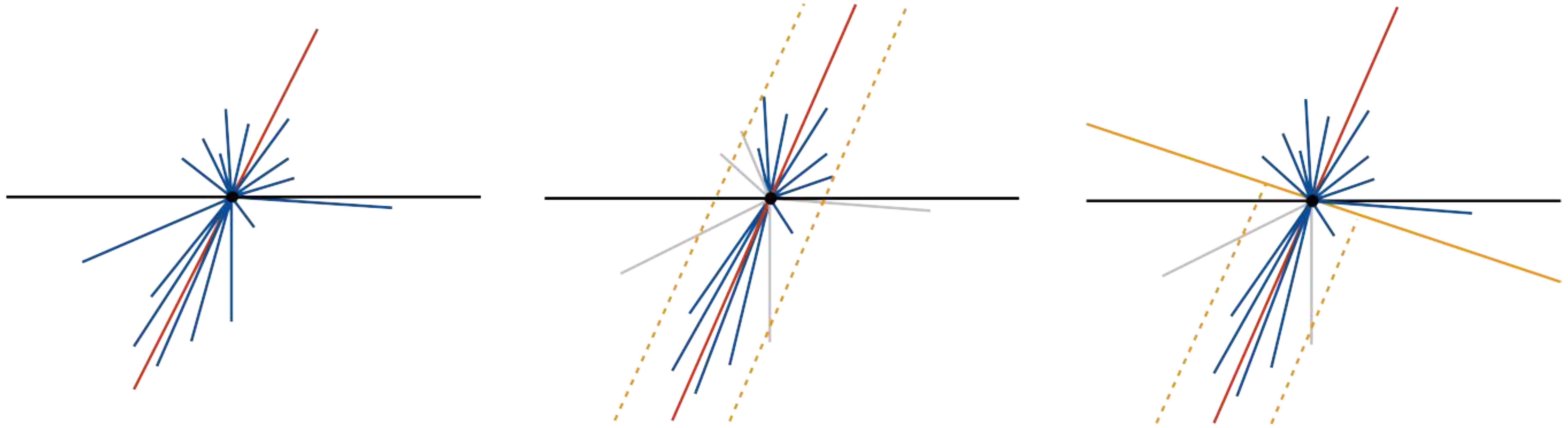
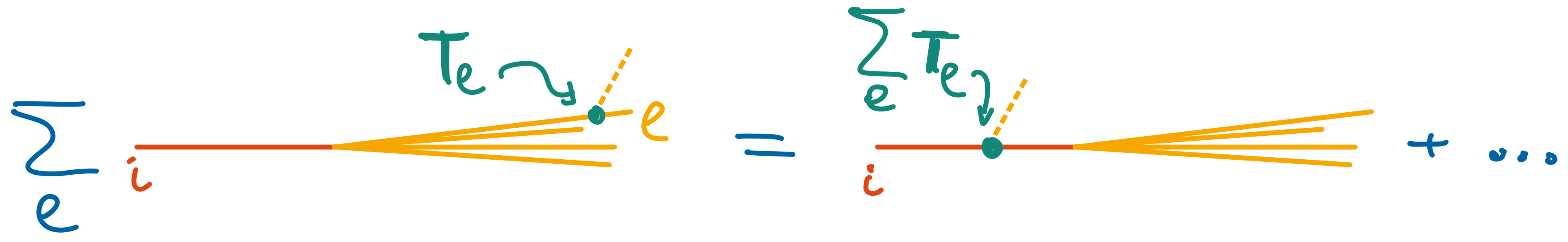


Global event shapes from coherent branching



[Catani, Trentadue, Webber, Marchesini ….]
basis of Herwig angular ordered shower

Non-globals for large N from dipole branching

$$\frac{\partial G_{ab}(t)}{\partial t} = -\int_{\text{in}} \frac{\mathrm{d}\Omega_k}{4\pi}\omega_{ab}(k)G_{ab}(t) + \int_{\text{out}} \frac{\mathrm{d}\Omega_k}{4\pi}\omega_{ab}(k)\Big[G_{ak}(t)G_{kb}(t) - G_{ab}(t)\Big]$$

[Banfi, Marchesini, Smye — JHEP 08 (2002) 006]
even at NLL: [Banfi, Dreyer, Monti — JHEP 10 (2021) 006]

Accuracy of Parton Showers

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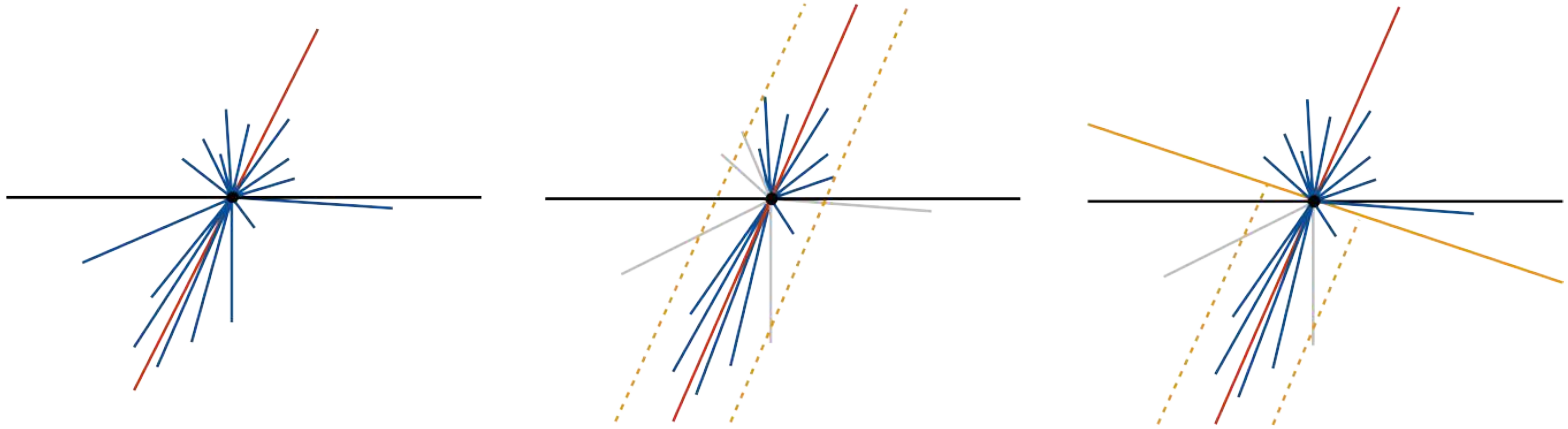
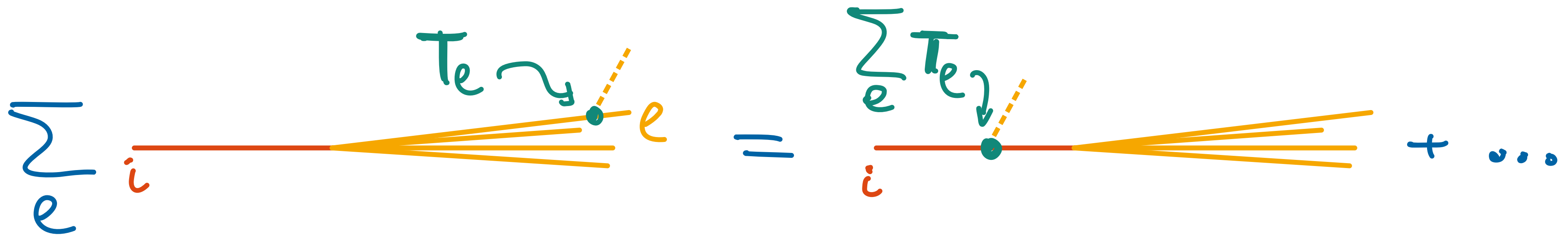


NLO with matching

NLL with coherent branching
Issues in dipole showers

Issues in coherent branching
LL with dipole showers

Global eve… …le branching



$$\frac{\partial G_{ab}(t)}{\partial t} = -\int_{\text{in}} \frac{d\Omega_k}{4\pi}\omega_{ab}(k) G_{ab}(t) + \int_{\text{out}} \frac{d\Omega_k}{4\pi}\omega_{ab}(k)\Big[G_{ak}(t)G_{kb}(t) - G_{ab}(t)\Big]$$

[Catani, Trentadue, Webber, Marchesini ….]
basis of Herwig angular ordered shower

[Banfi, Marchesini, Smye — JHEP 08 (2002) 006]
even at NLL: [Banfi, Dreyer, Monti — JHEP 10 (2021) 006]

Recoil

Various choices of recoil possible:

	Minimal number of recoilers.	Across the entire event.
After each emission.	"local"	
After all emissions.		"global"

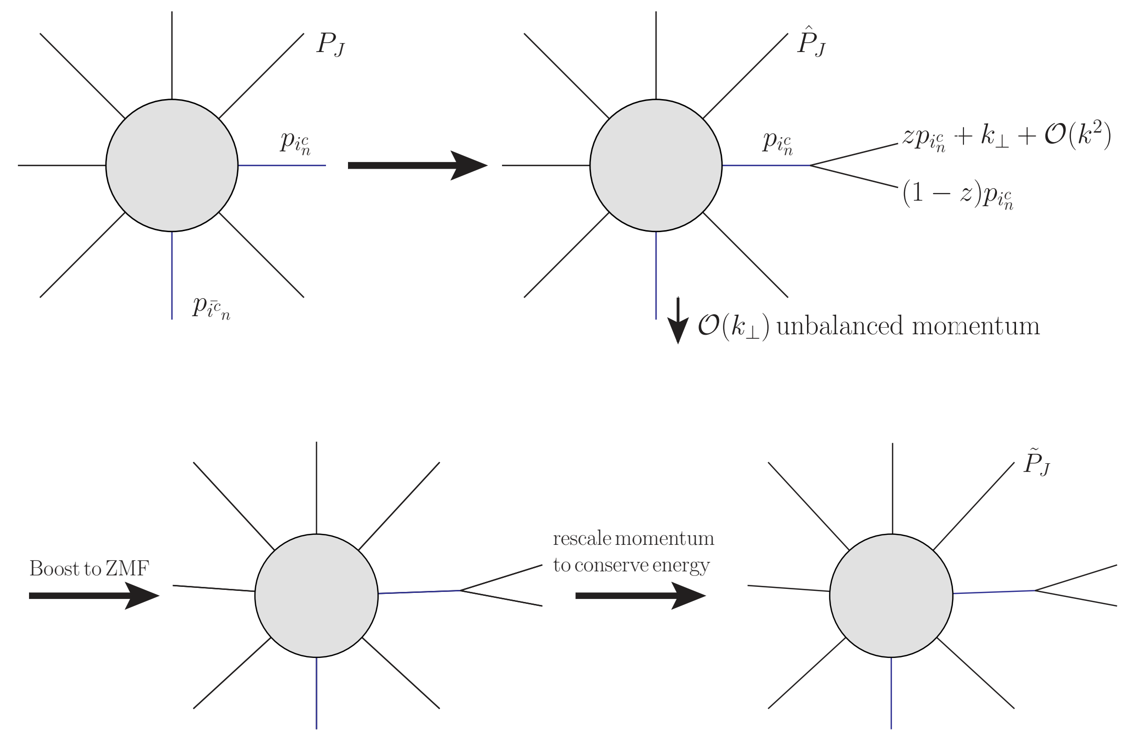
Recoil is crucial to an algorithm's accuracy.

Closely linked to how we implement the **soft radiation pattern and the initial conditions.** Some choices invert initially assumed strong ordering.

$$\frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\, p_{j_n}\cdot q_n} \longrightarrow \frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\, p_{j_n}\cdot q_n} - \frac{T\cdot p_{j_n}}{T\cdot q_n}\frac{1}{p_{j_n}\cdot q_n} + \frac{T\cdot p_{i_n}}{T\cdot q_n}\frac{1}{p_{i_n}\cdot q_n}$$

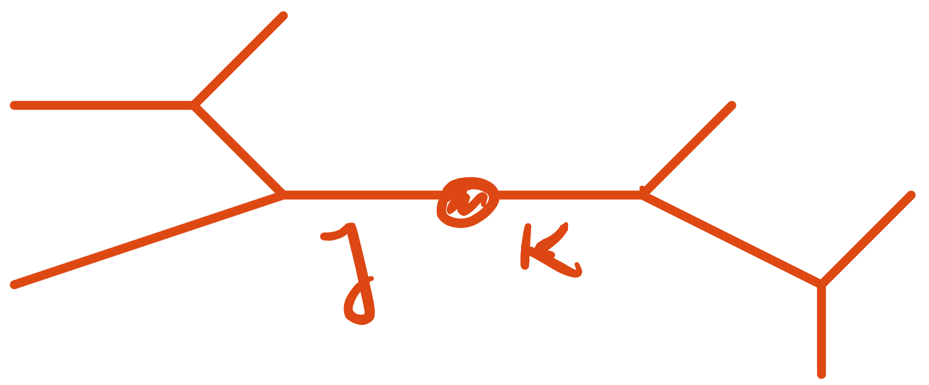
local

$$q_i = zp_i + \frac{p_\perp^2}{zs_{ij}}p_j + k_\perp$$
$$q = (1-z)p_i + \frac{p_\perp^2}{(1-z)s_{ij}}p_j - k_\perp$$
$$q_j = \left(1 - \frac{p_\perp^2}{z(1-z)s_{ij}}\right)p_j\ ,$$



local/global

global



$$q_{ji} = \alpha_i p_j + \beta_i \bar{n}_j + k_{\perp i}$$
$$q_{ji} \to \frac{1}{\alpha_i}\Lambda\, q_{ji}$$

[Dasgupta, Dreyer, Hamilton, Monni, Salam — PRL 125 (2020) 5]
[Forshaw, Holguin, Plätzer – JHEP 09 (2020) 014 & EPC C81 (2021) 4]

Improving Shower Accuracy

Demonstrate NLL accurate evolution:

- PanScales — numerical

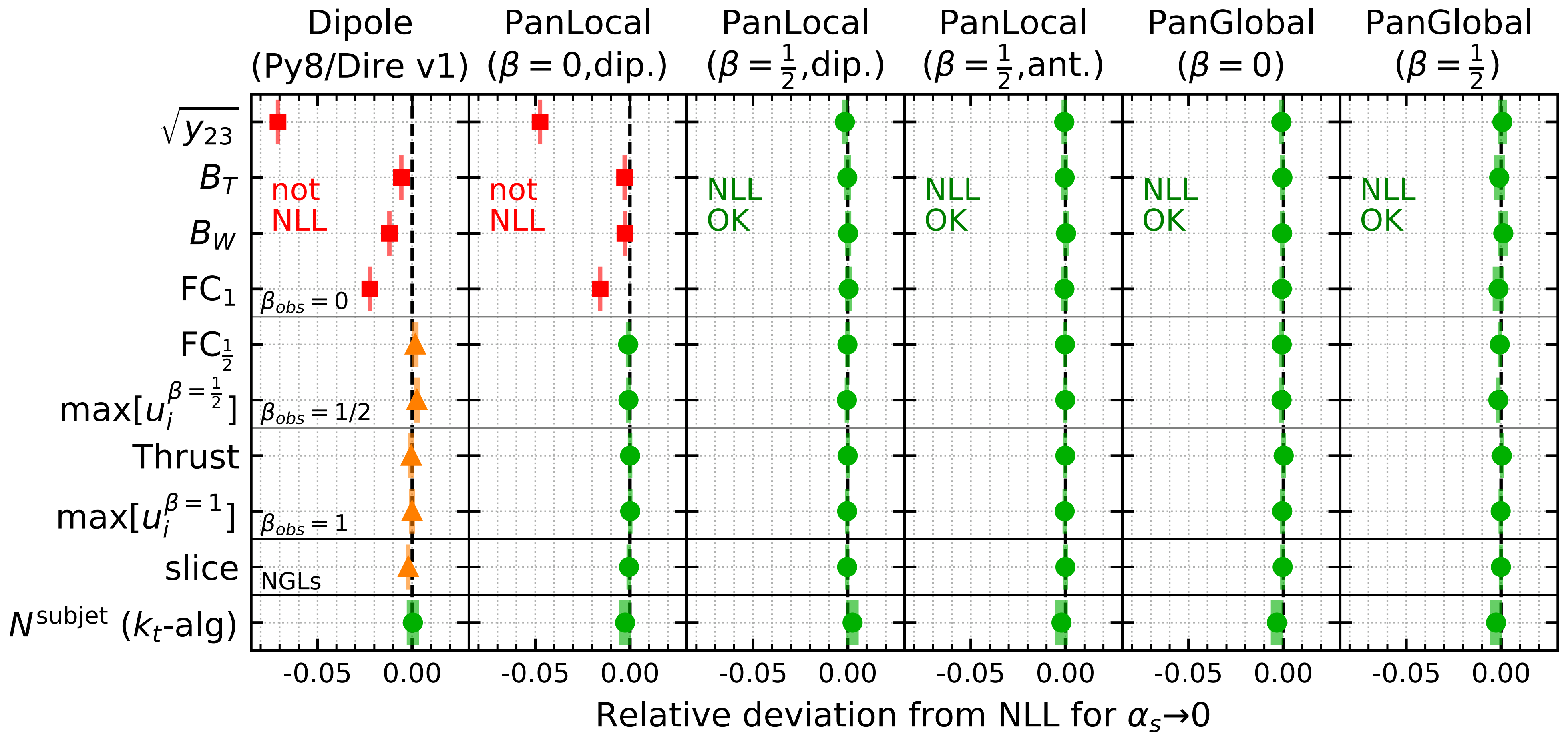
  [Dasgupta, Monni, Salam, Soyez + ….]
- Deductor — numerical/analytical

  [Nagy, Soper]
- Forshaw/Holguin/Plätzer — analytical

  [aim at improving Herwig 7 dipole shower]
- Sherpa — numerical/analytical

  [Herren, Höche, Krauss, Reichelt, Schönherr]

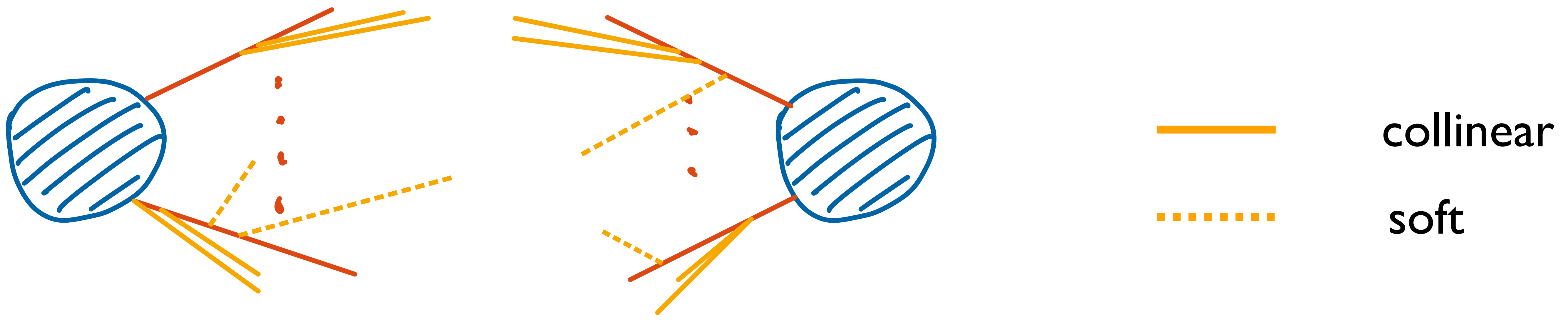
} Based on amplitude evolution.



[PanScales]

Analyses need to build on squaring amplitudes with many additional legs, then reduce by using coherence and/or large-N limit.

At least analyse two emissions.



Amplitude Conjugate Amplitude

Improving Shower Accuracy

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- PanScales — numerical
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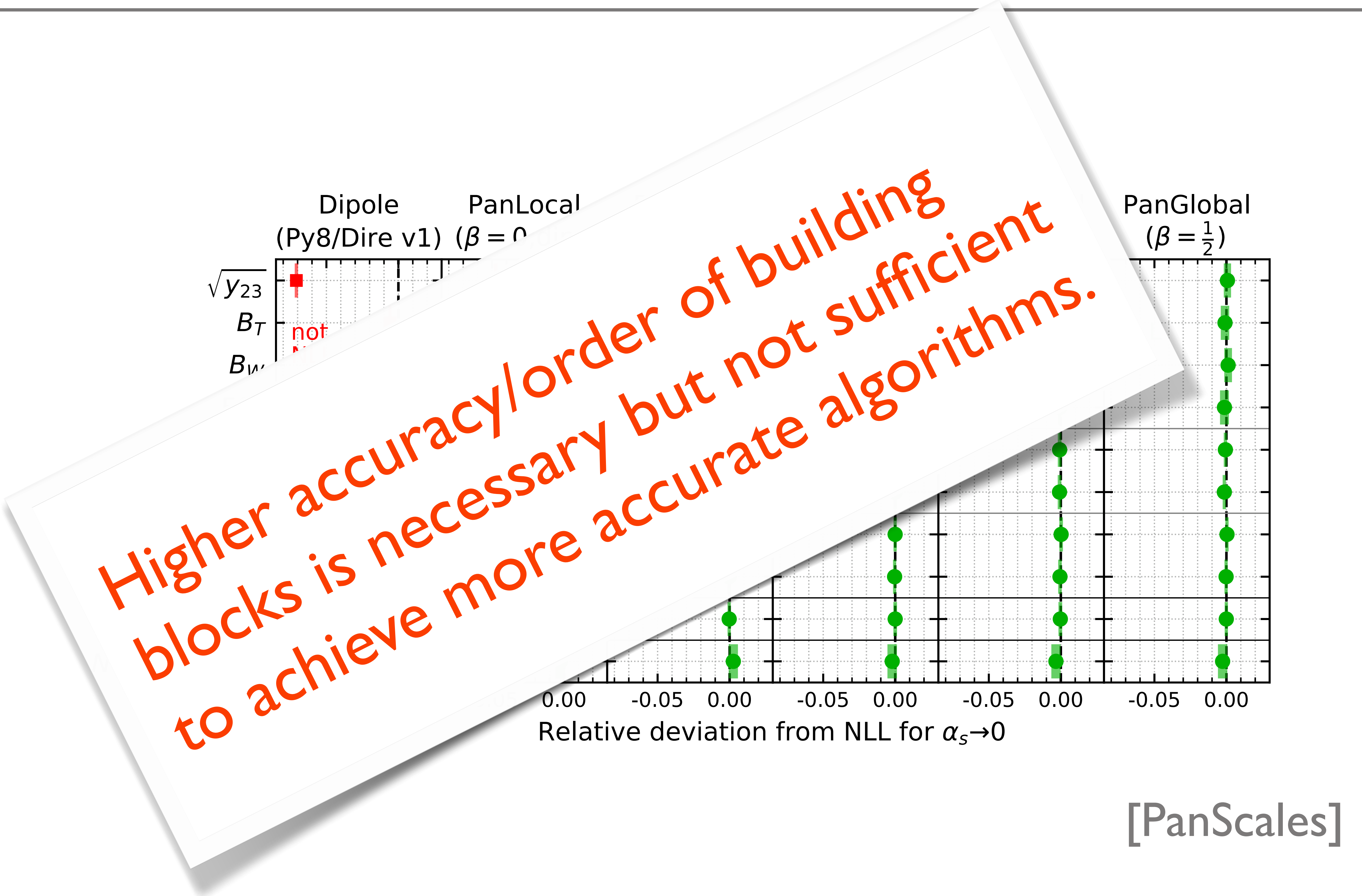
Based on amplitude evolution.

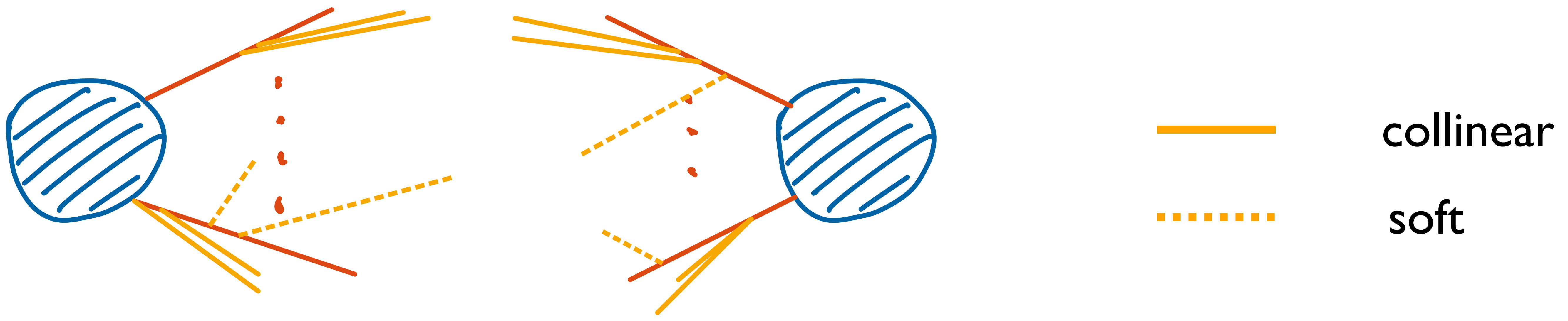


Higher accuracy/order of building blocks is necessary but not sufficient to achieve more accurate algorithms.

[PanScales]

Analyses need to build on squaring amplitudes with many additional legs, then reduce by using coherence and/or large-N limit.

At least analyse two emissions.



Amplitude

Conjugate Amplitude

Further lessons from amplitude evolution

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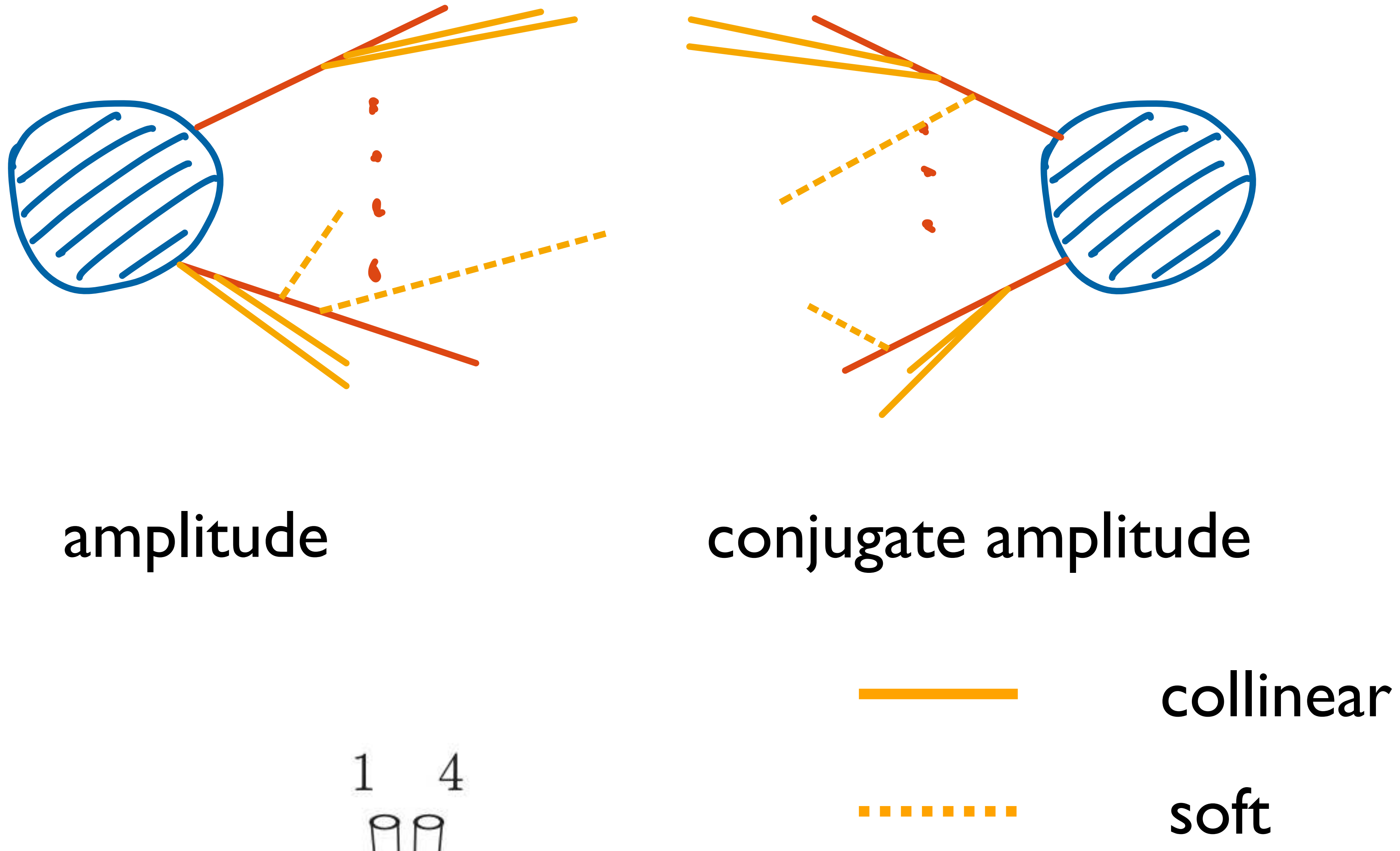
[Holguin, Forshaw, Plätzer — JHEP 05 (2022) 190]

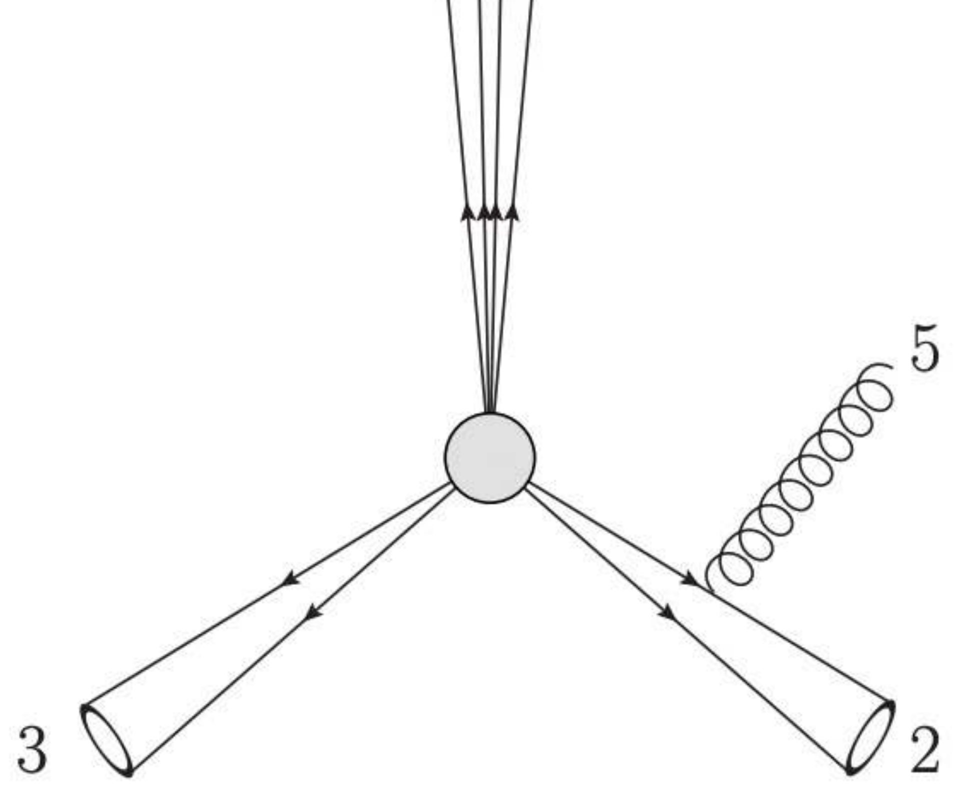
Coherent branching full-colour accurate for global observables:
Colour correlations for two (and three) partons are trivial.

Basis functions for soft radiation patterns then allow us to express three-jet global cross sections to the same level of accuracy:



$$\frac{\mathrm{d}(\cos\theta_{in})\mathrm{d}\phi_n}{1-\cos\theta_{in}}\Theta(\theta_{in}<\theta) \mapsto {}^{(n-1)}\mathrm{S}_i^{j,k}\, E_n^2\,\Theta(\theta_{in}<\theta)\,\mathrm{d}(\cos\theta_{in})\mathrm{d}\phi_n$$

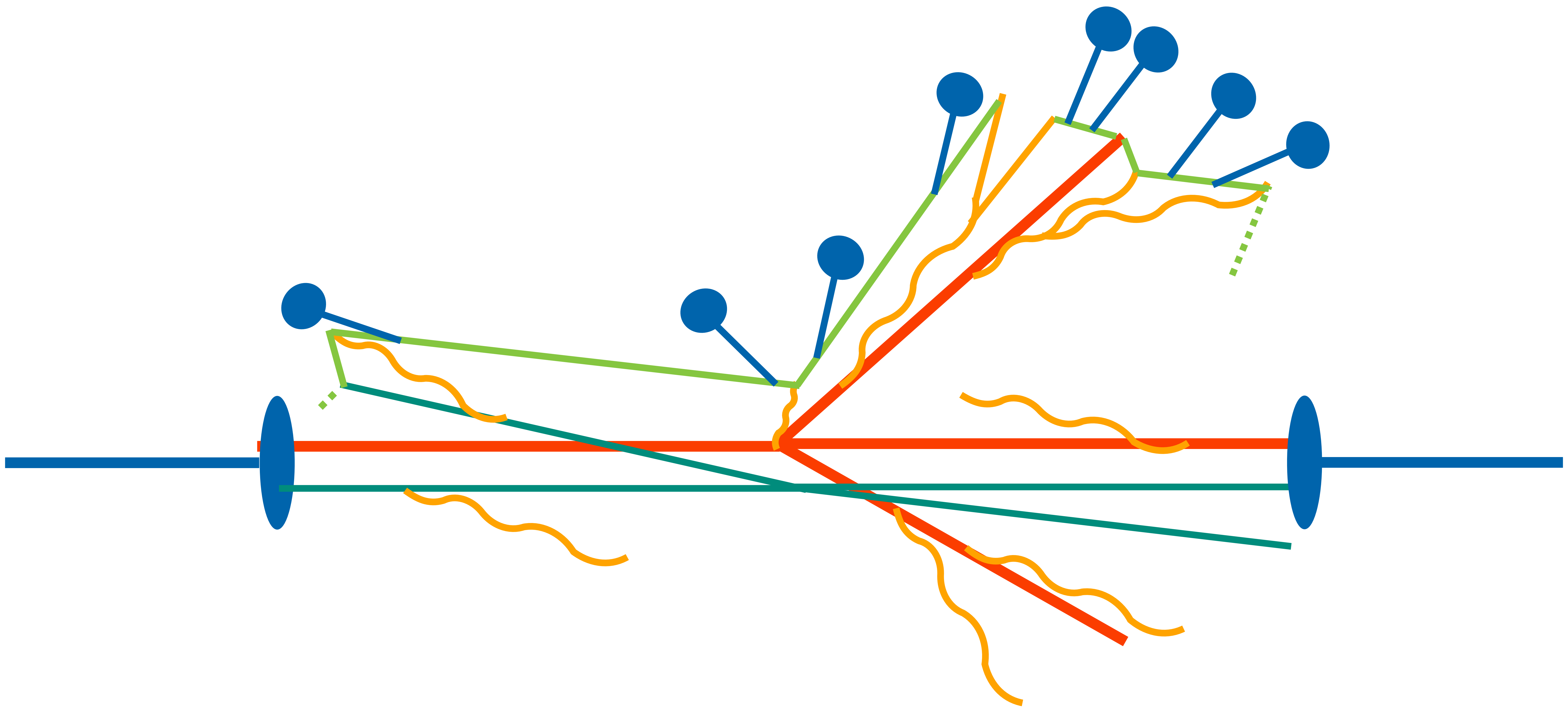
$${}^{(n-1)}\mathrm{S}_i^{j,k} = \frac{s_{ij}s_{nk}+s_{ik}s_{nj}-s_{jk}s_{ni}}{2s_{ni}s_{nj}s_{nk}}$$



Large-angle soft gluon only resolves three-jet system.

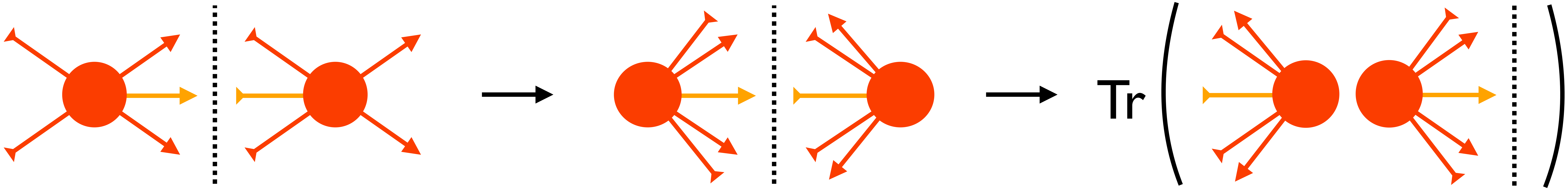
Simple generalisation of angular ordered showers.
Not a recipe for showers which are generally colour exact.

Complexity factorized?



$$d\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q \to \mu)\,d\mathbf{H}(Q)\,\mathbf{PS}^\dagger(Q \to \mu)\,\mathbf{Had}(\mu \to \Lambda)\right]$$

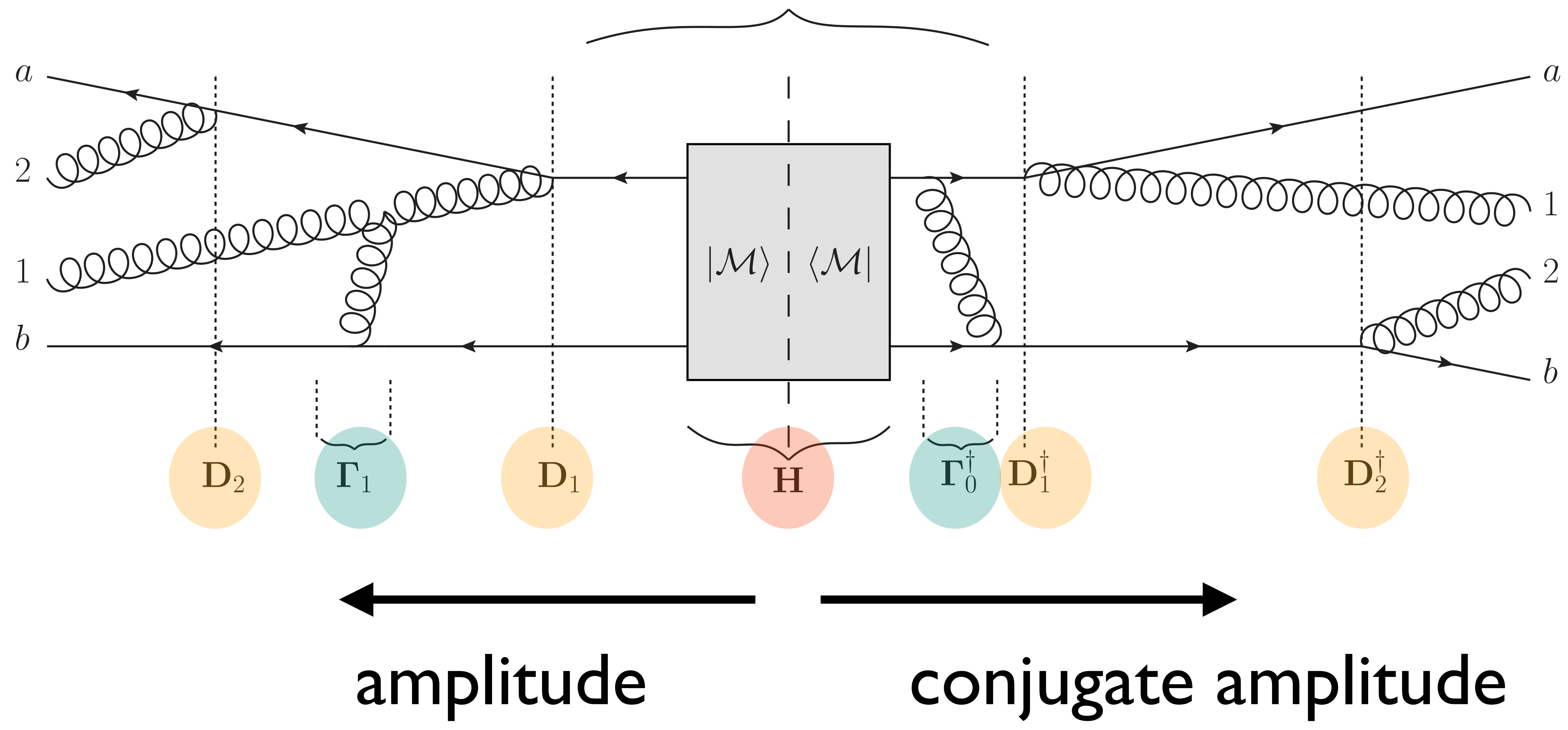
Amplitude evolution



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\ \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}(k')}\ \mathbf{D}_n(k)\ \mathbf{A}_{n-1}(k)\ \mathbf{D}_n^\dagger(k)\ \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

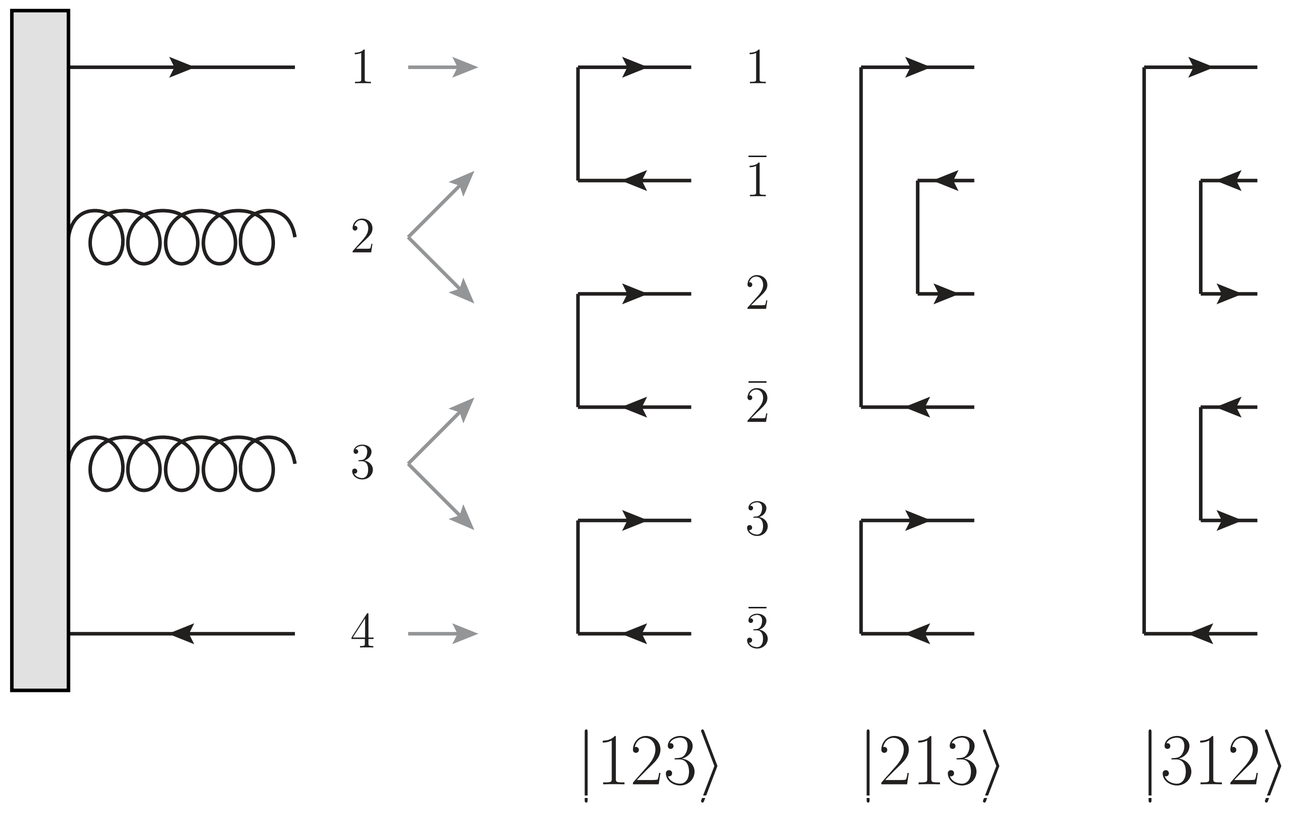
Different histories in amplitude and conjugate amplitude needed to include interference.



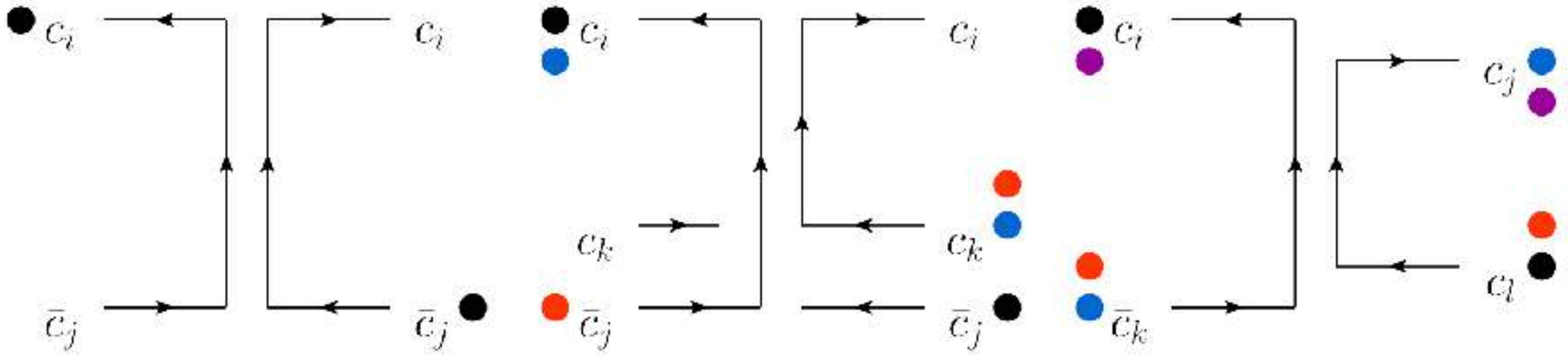
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

Colour flow evolution

Resummation of general, in particular, non-global observables requires colour evolution



Dipole, "string" and "ring" radiation patterns at subleading colour [Holguin, Forshaw, Plätzer — JHEP 05 (2022) 190]

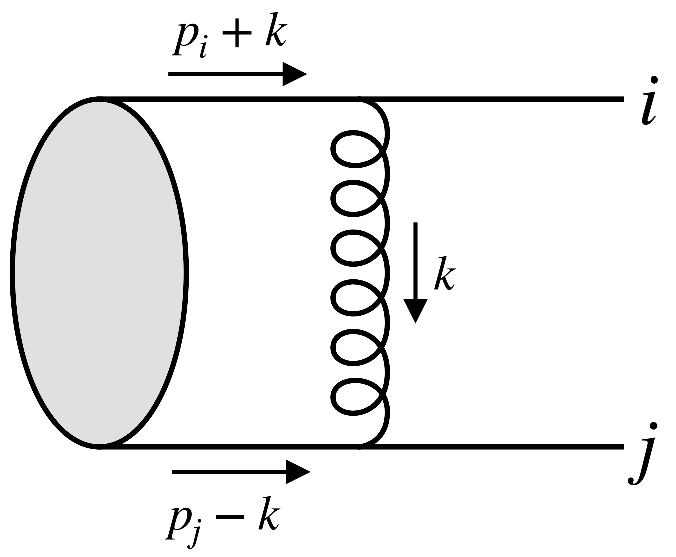


ω_{ij} $\quad\quad$ $\omega_{ij} + \omega_{ik} - \omega_{jk}$ $\quad\quad$ $\omega_{il} + \omega_{kj} - \omega_{kl} - \omega_{ij}$

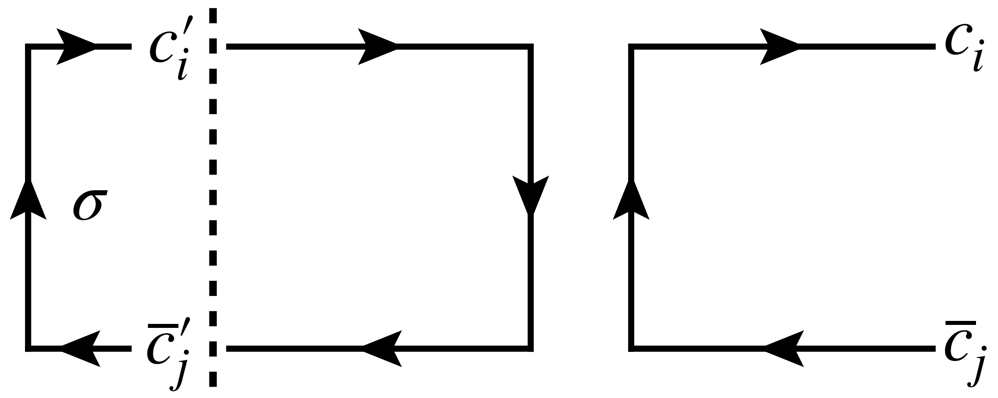
rings free of collinear singularities — related to conformal cross ratios

Dipole flips from virtual corrections, known up to two loop order.

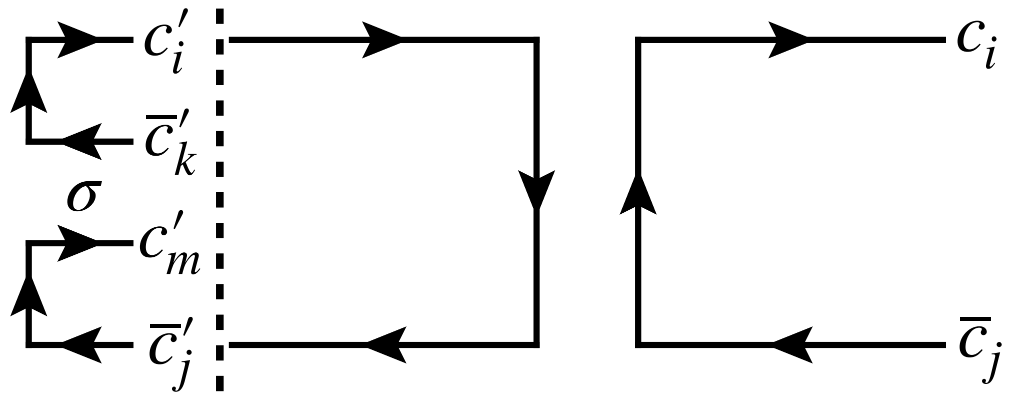
[Plätzer, Ruffa — JHEP 06 (2021) 007]
[Plätzer – EPJ C 74 (2014) 2907]



Leading N diagonal



Subleading N flips



Hard process evolution beyond large N

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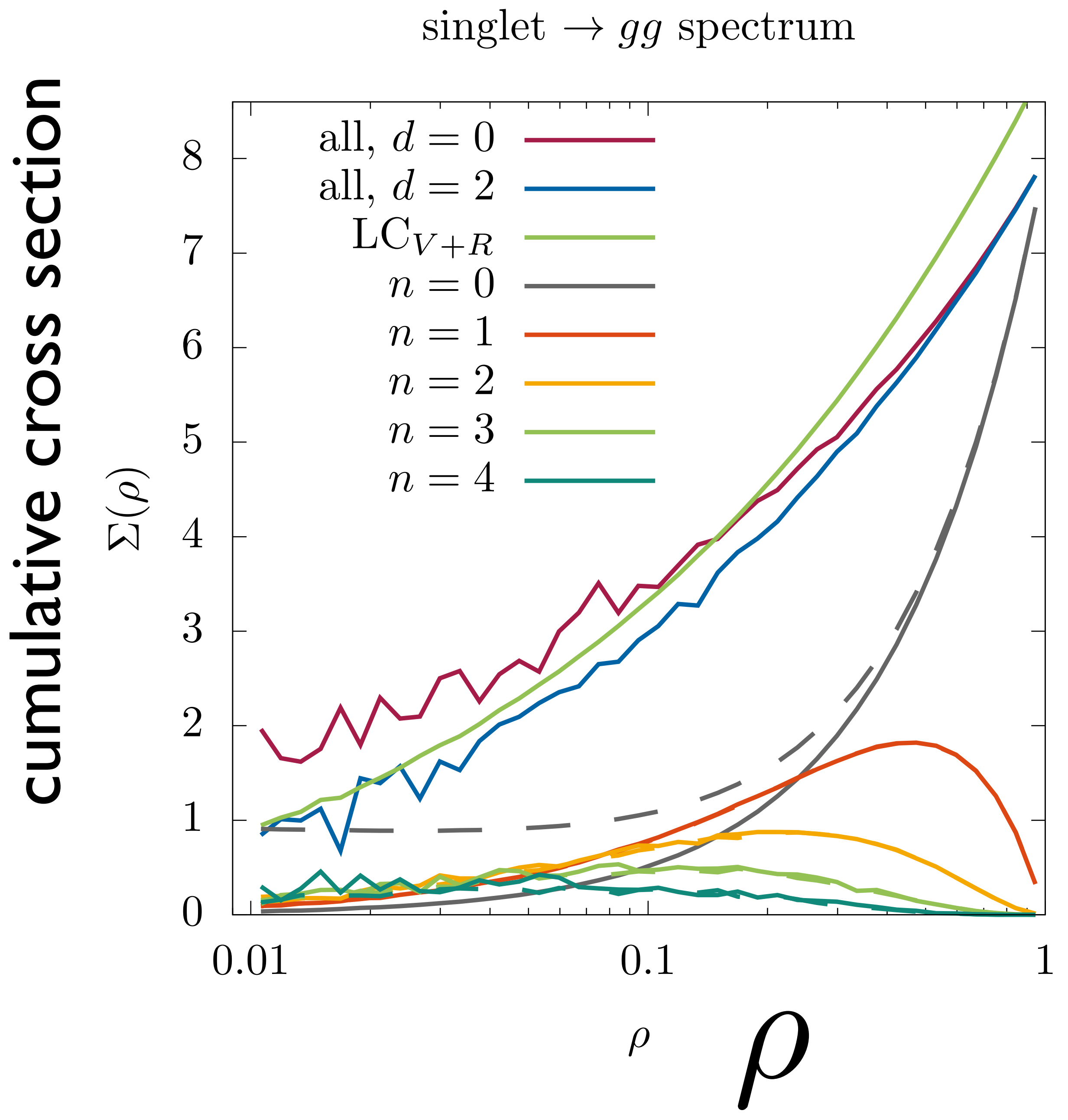
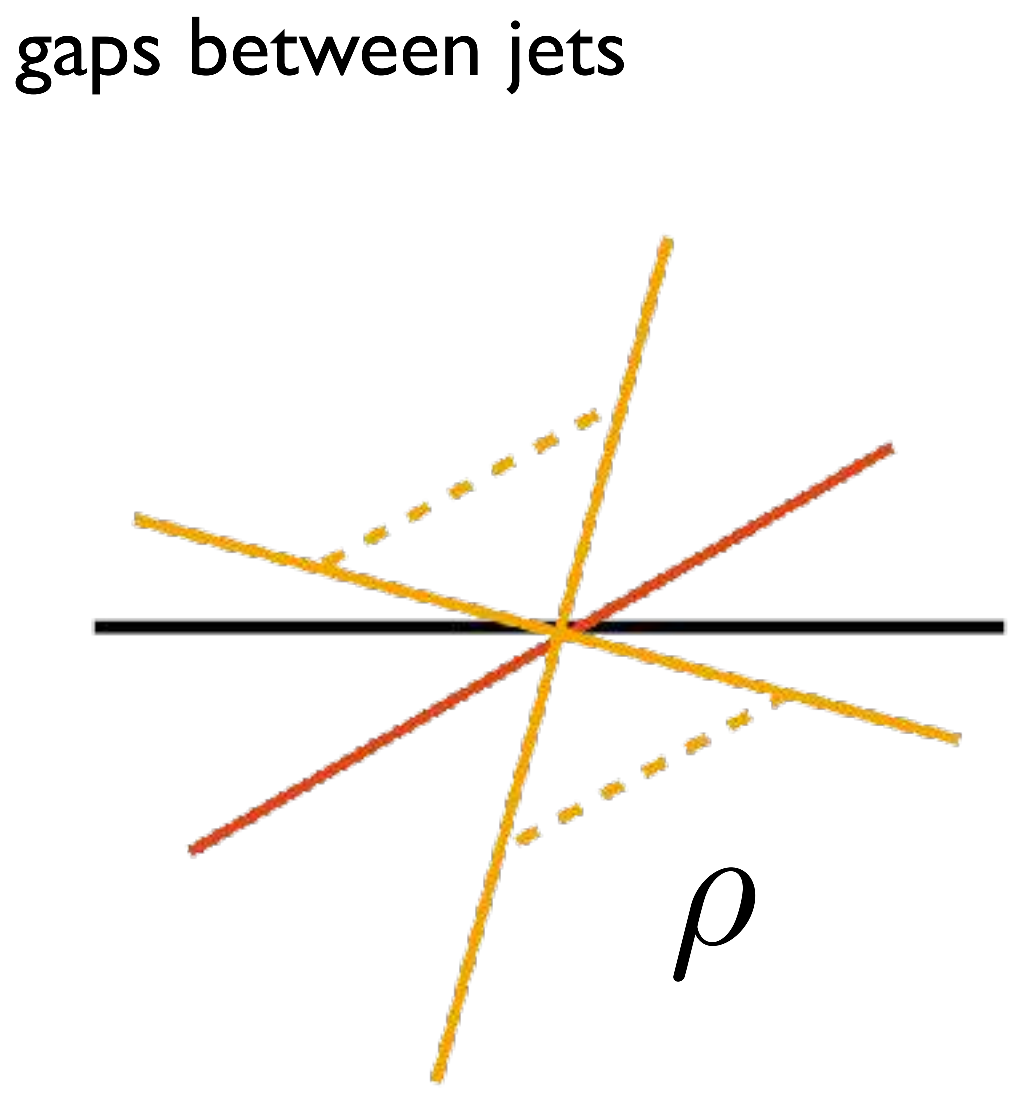
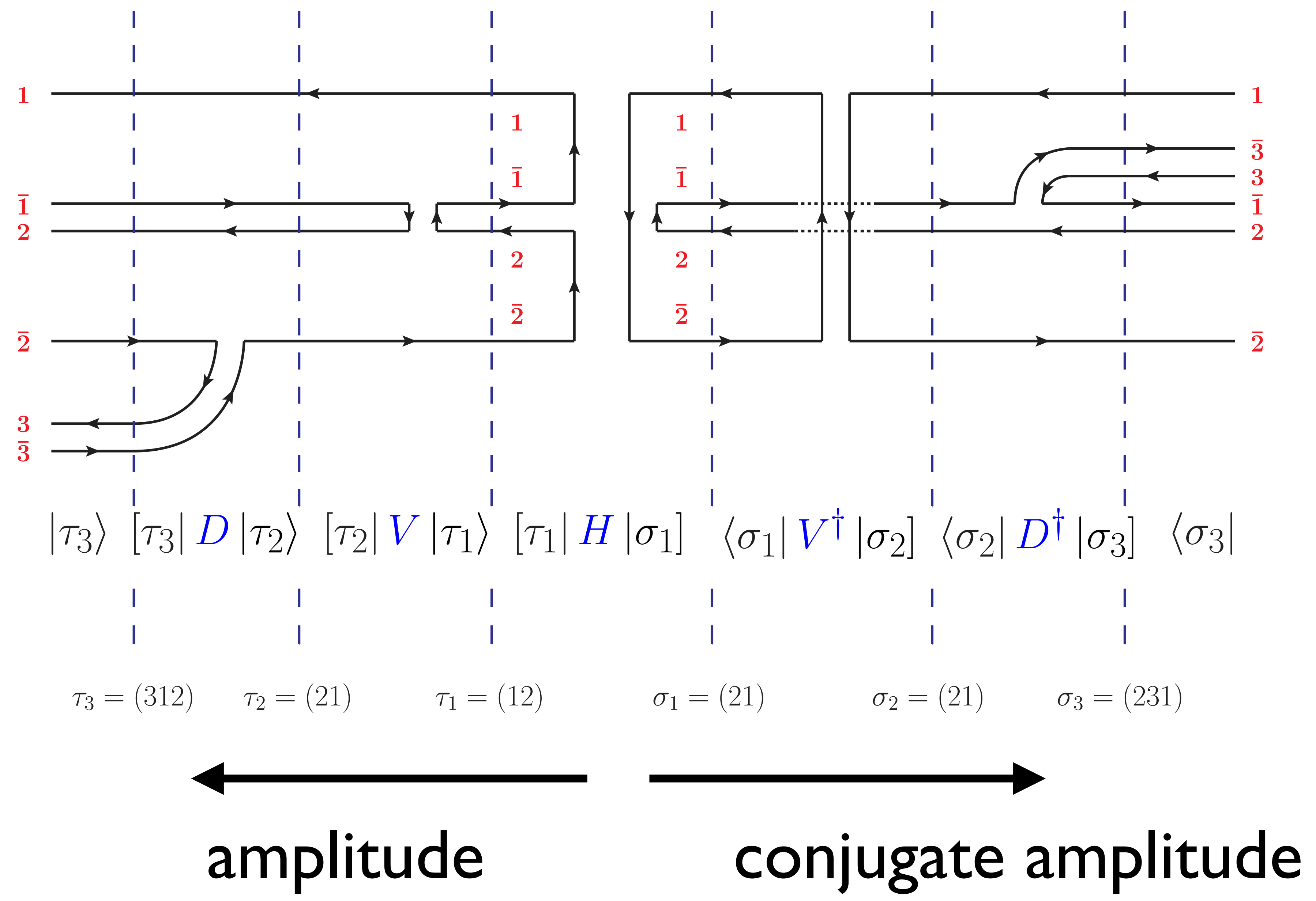
CVolver solves evolution equations in colour flow space

[De Angelis, Forshaw, Plätzer — PRL 126 (2021) 11]
based on [Plätzer – EPJ C 74 (2014) 2907]

$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'} \mathbf{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'} \mathbf{\Gamma}^\dagger(k')}$$







Agrees with Hatta & Ueda using equivalent Langevin formulation.
[Hatta et al. — Nucl.Phys.B 962 (2021) 115273]

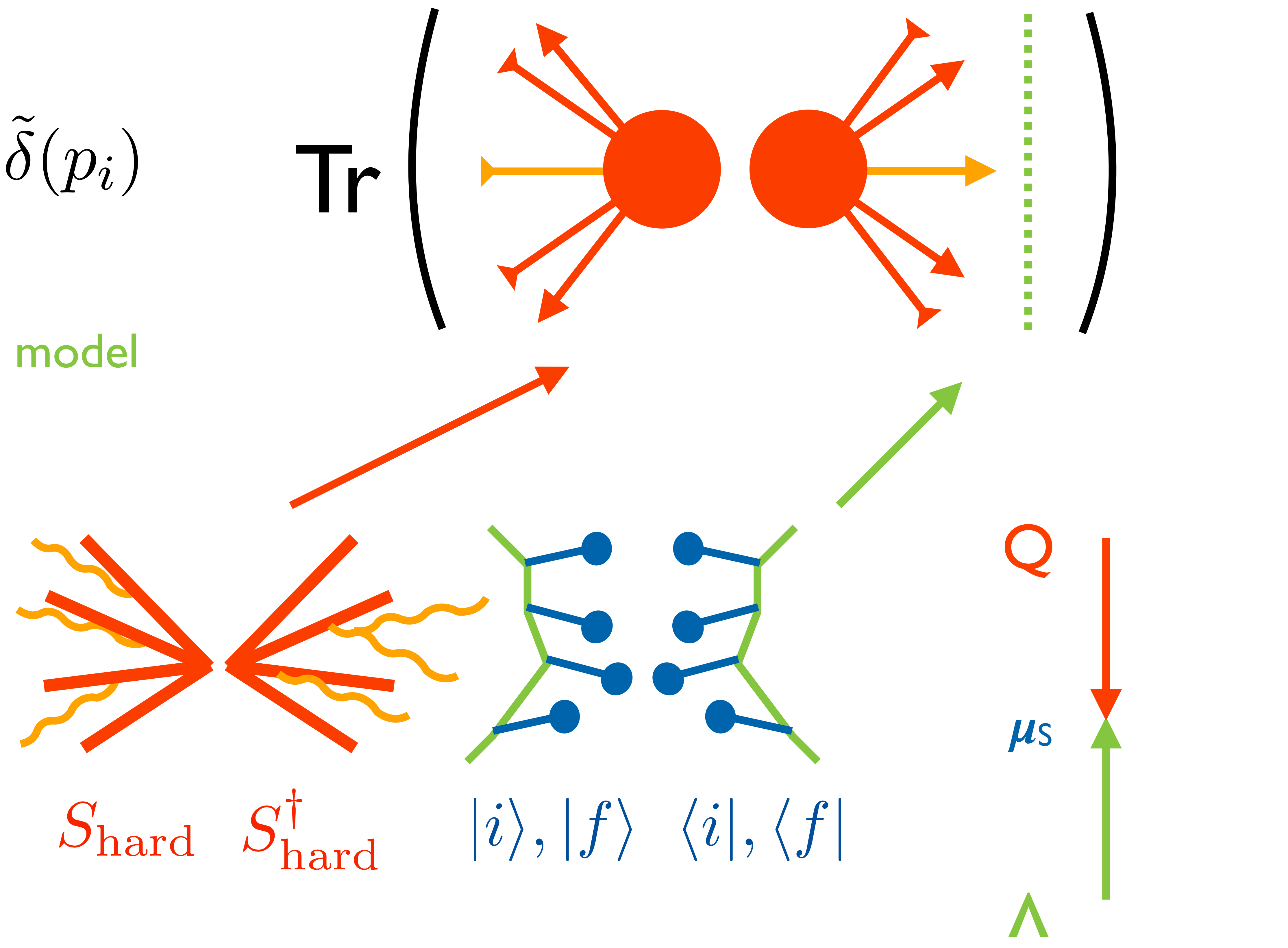
Amplitude evolution & hadronization

$$\sigma = \sum_n \int \alpha_S^n \, \mathrm{Tr}\left[(\mathbf{A}_n + \mathbf{\Delta}_n)\mathbf{S}_n\right] \mathrm{d}\phi(Q) \prod_{i=1}^{n} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

dressing of hard process ~ parton shower

soft evolution ~ hadronization model

α_s corrections to tower of logarithms present in A



Redefinitions of "bare" measurements provides infrared subtractions, renormalised hard operator can then be redefined to sum up logarithmic enhancements.

$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S} \mathbf{A}_n + \mathbf{A}_n \mathbf{\Gamma}^\dagger_{n,S} - \sum_{s\geq 1} \alpha_S^s \mathbf{R}^{(s)}_{S,n} \mathbf{A}_{n-s} \mathbf{R}^{(s)\dagger}_{S,n}$$

Framework dedicated to higher orders and accuracy investigation, currently focus on soft evolution and colour correlations. Second order under control.

[Plätzer – arXiv:2204.06956]

Evolution equations

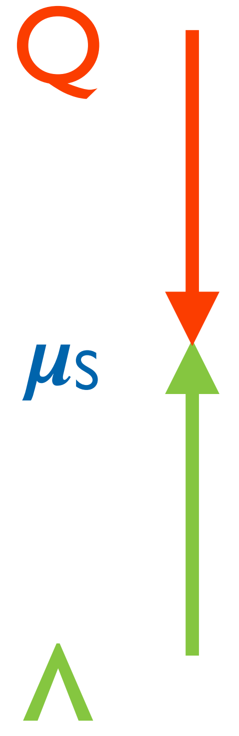
[Plätzer – arXiv:2204.06956]

We can now obtain the evolution equations we asked for:

$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S}\mathbf{A}_n + \mathbf{A}_n\mathbf{\Gamma}^\dagger_{n,S} - \sum_{s\geq 1}\alpha_S^s \mathbf{R}^{(s)}_{S,n}\mathbf{A}_{n-s}\mathbf{R}^{(s)\dagger}_{S,n}$$

$$\partial_S \equiv \partial/\partial \log \mu_S$$

$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

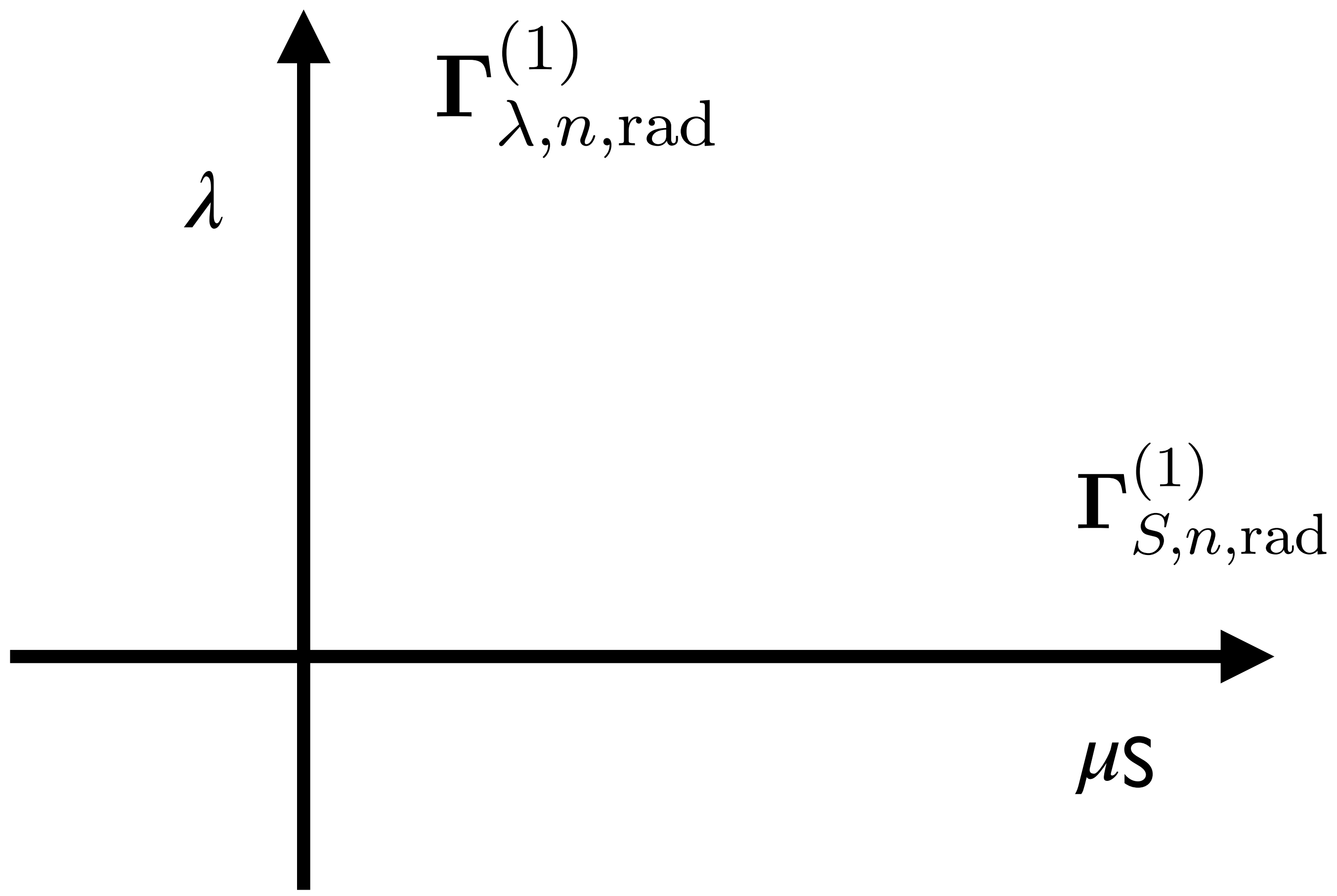


Coupled system of evolution equations: For each resolution we have chosen, we get one. Directions of evolution are different in scale and multiplicity.

Evolution variables are related to resolution criterion, defining anomalous dimensions at the level of the integrand:

$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1}\theta(E_n - \mu_S)$$

"soft or collinear"



Match these resolutions to observable to ensure accurate predictions.

Hadronization models

[Plätzer – arXiv:2204.06956]

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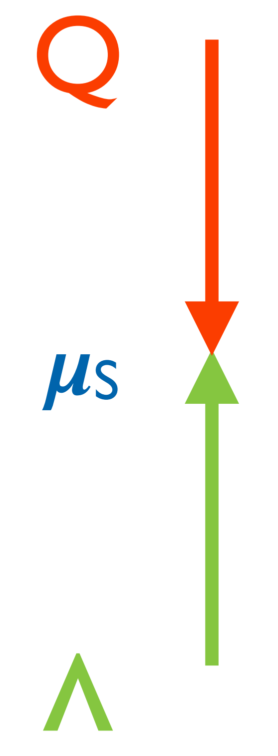
$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S}\mathbf{A}_n + \mathbf{A}_n\mathbf{\Gamma}^\dagger_{n,S} - \sum_{s\geq 1}\alpha_S^s \mathbf{R}^{(s)}_{S,n}\mathbf{A}_{n-s}\mathbf{R}^{(s)\dagger}_{S,n}$$

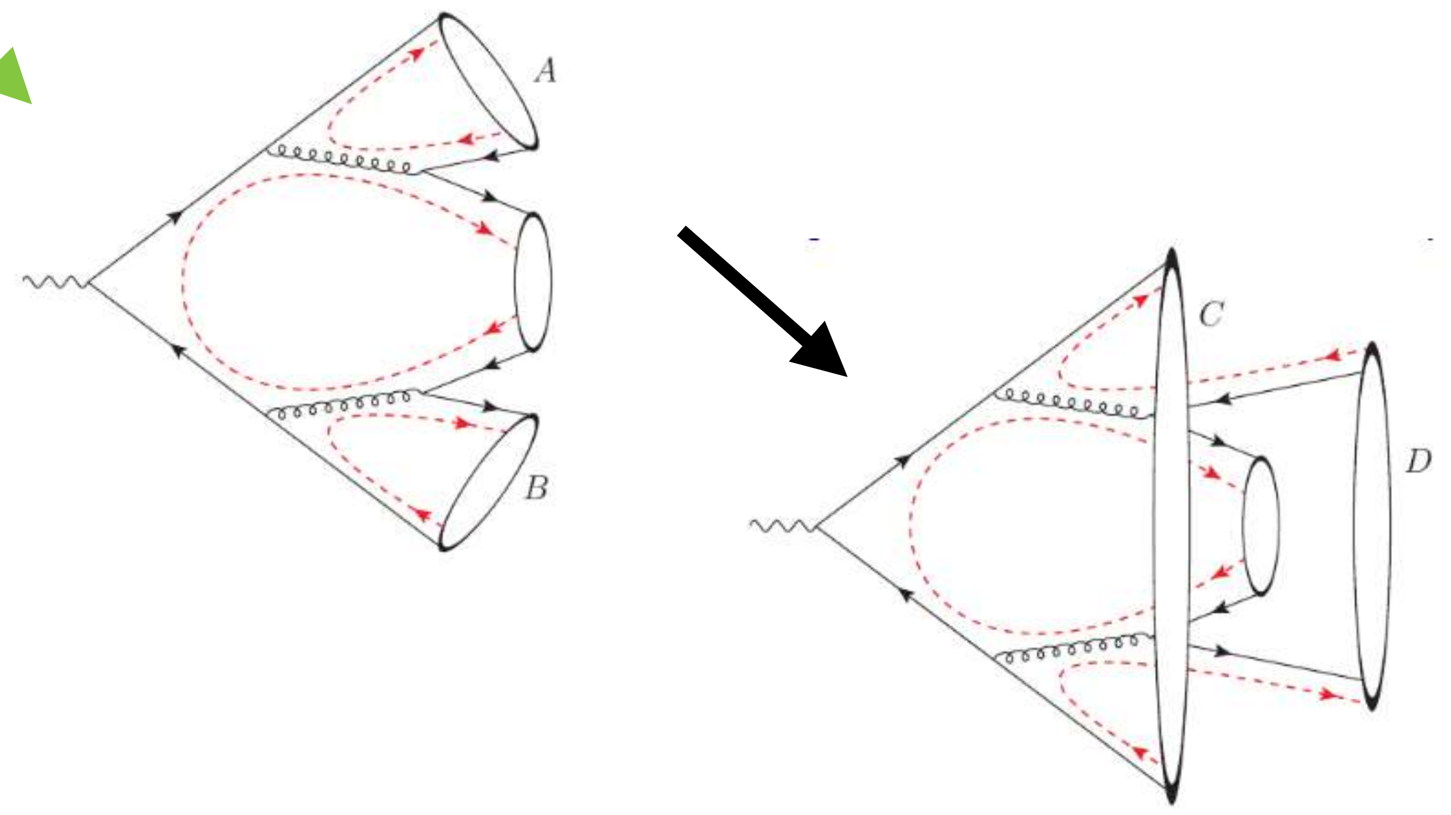
$$\partial_S \equiv \partial/\partial \log \mu_S$$

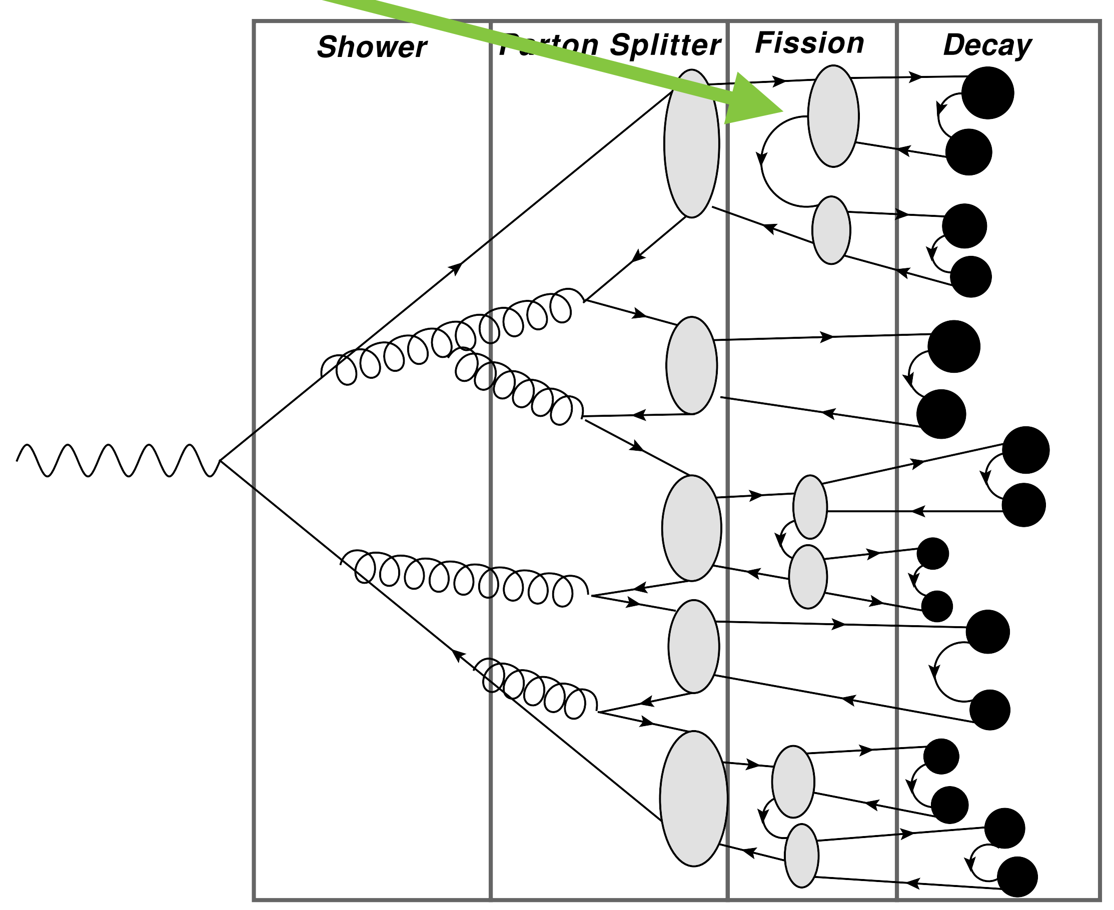


$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

Evolution equation for a hadronization model!

Features which relate to the high-energy dynamics of the Herwig cluster model.





[Gieseke, Kirchgaesser, Plätzer – EPJ C 78 (2018) 99]
[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]

Amplitude evolution & electroweak physics

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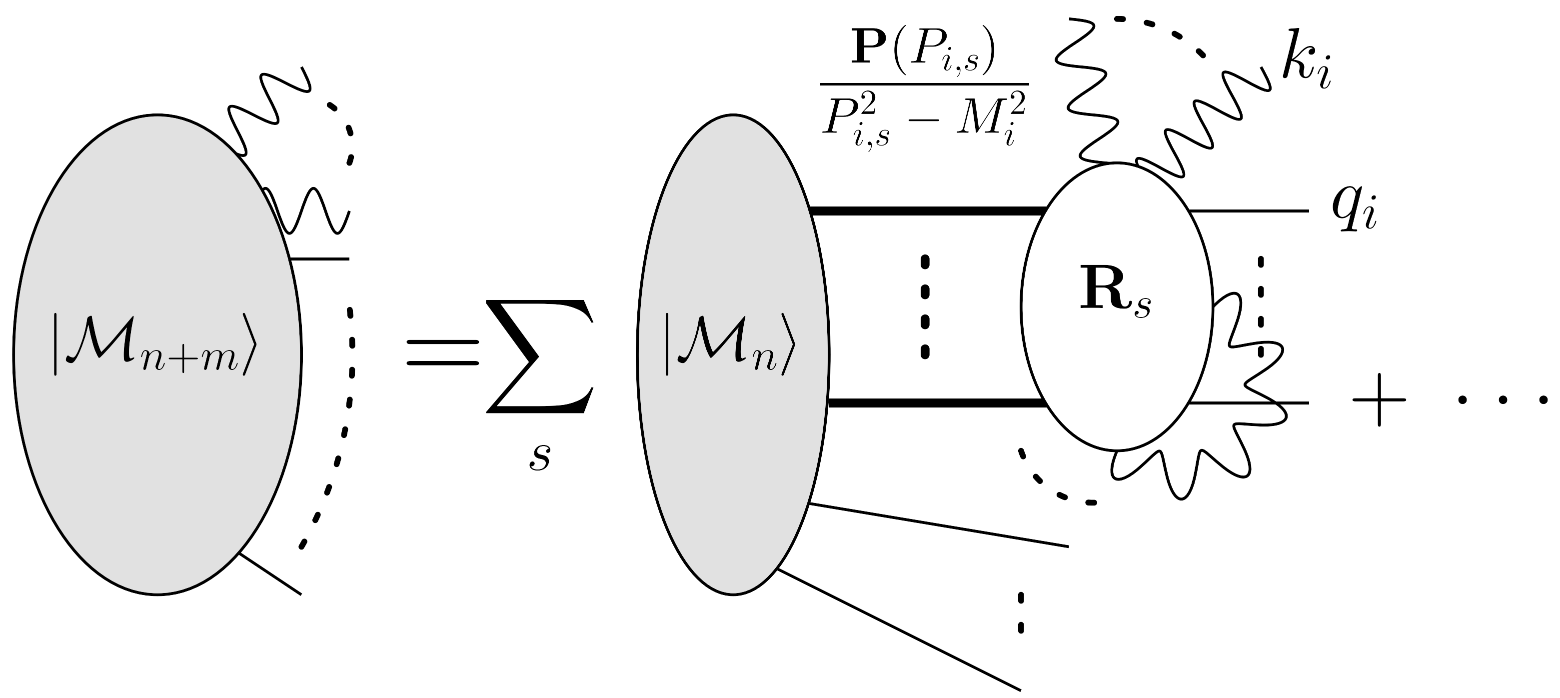
[Plätzer, Sjödahl — arXiv:2204.03258]

Reconsider factorisation properties of amplitudes in presence of electroweak physics.
Recoil and mass shell conditions are crucial in "quasi-soft" limit — also to obtain proper wave-function renormalisation for factorized building blocks.



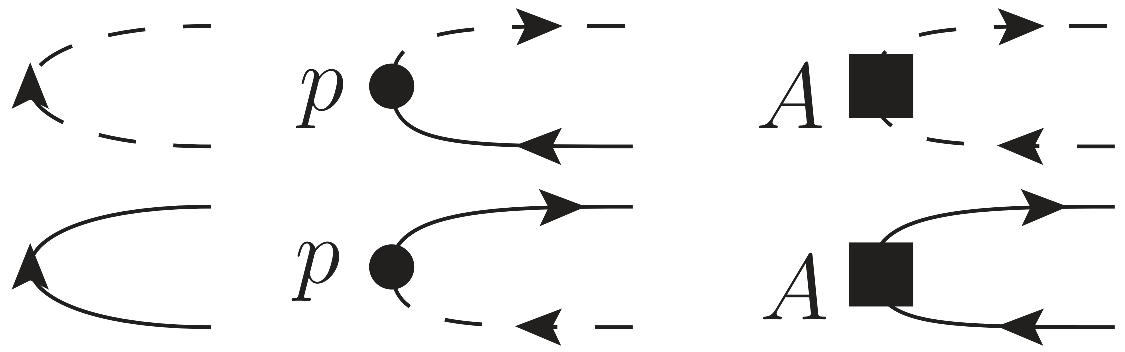
Systematic factorisation parametrised around Eikonal propagators:

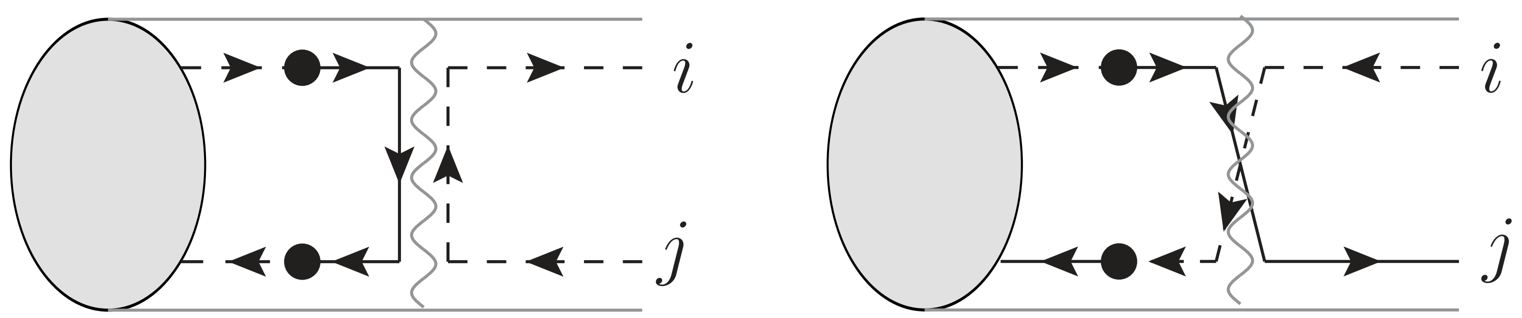
$$|\tilde{\mathcal{M}}(\{q_i\}_n;\{k_i\}_m)\rangle \simeq \sum_s \mathbf{S}_s(\{q\}_{i\in h_s},\{k_i\}_m)|\tilde{\mathcal{M}}(\{p_i\}_n)\rangle$$

$$(q_i + K_{i,s})^2 - M_i^2 = 2p_i\cdot Q_{i,s}$$

$$p_i\cdot Q_{i,s} \ll p_i\cdot n_{i,s} \equiv S_{i,s}$$

Amplitudes now become vectors in the space of colour & isospin flows, as well as chirality flow:





Framework to build and analyse electroweak showers at same level of understanding as QCD showers.

Summary & Outlook

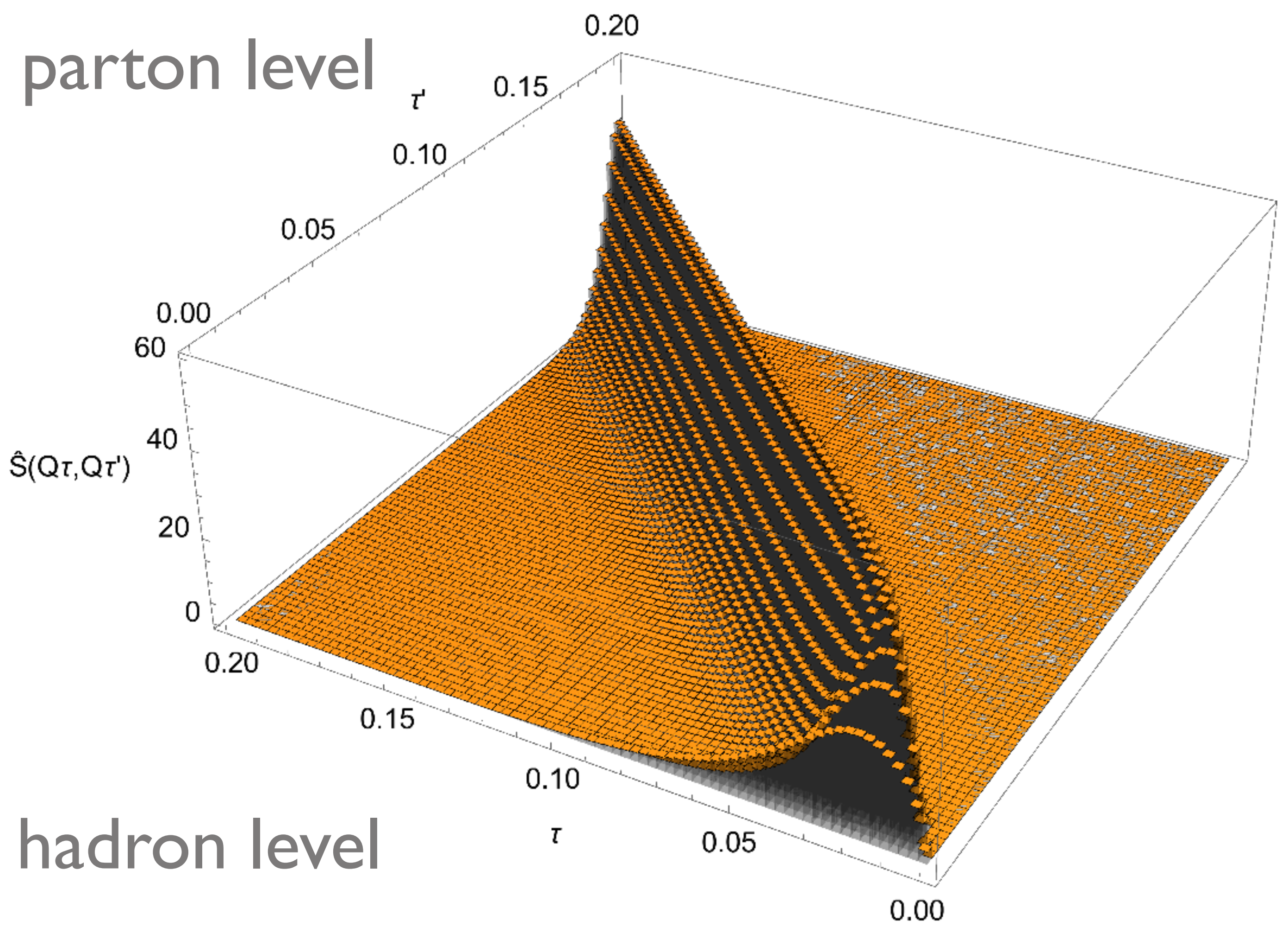
Multi-purpose event generators are working horses of collider phenomenology. Matching & merging has been focus of last decade.

As we aim to use more and more of the complex structures, shower accuracy becomes the bottleneck.

Resummation and design of parton showers needs to go **hand in hand**: amplitude evolution can serve as a theoretical tool and an algorithm in its own right.

The understanding of hadronization effects and models, and their interplay with parton showers will be one of the main topics in the future, not only in light of measuring fundamental parameters.

Herwig's cluster model versus factorisation



[Hoang, Plätzer, Samitz — in progress]

Thank you!