

# The Top Threshold & QCD Precision at $e^+e^-$ Higgs-Top-Electroweak Factories

Frank Simon  
KIT & MPI for Physics

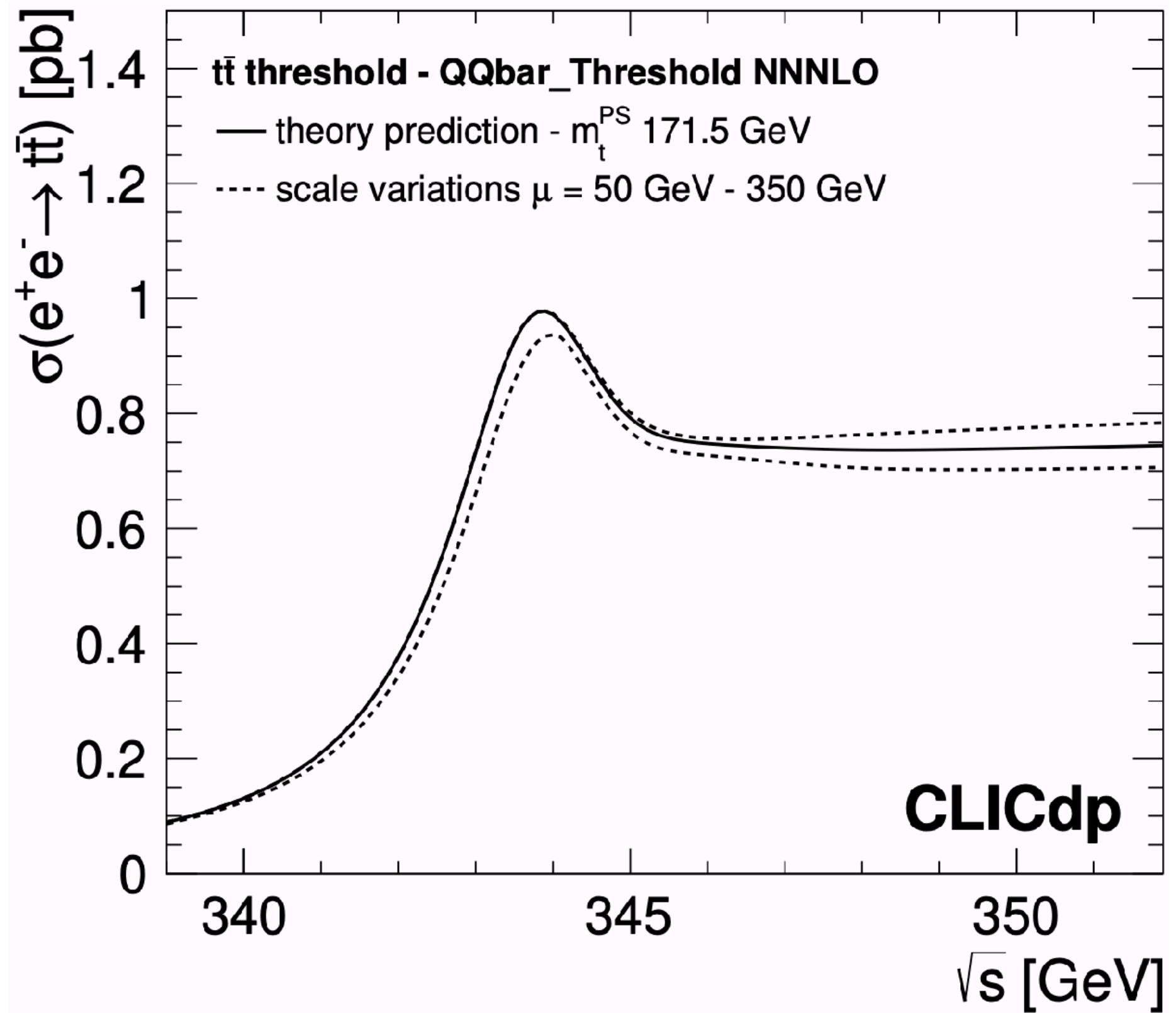
# Outline

- The Top quark pair production threshold [*the emphasis of this talk - sorry, personal bias!*]
  - Threshold overview
  - Measuring the mass
  - Threshold beyond mass
- QCD precision measurements
  - The strong coupling constant
  - Gluon jets, colour reconnection

# **Part I: The Top Threshold**

# The Top Quark Pair Production Threshold

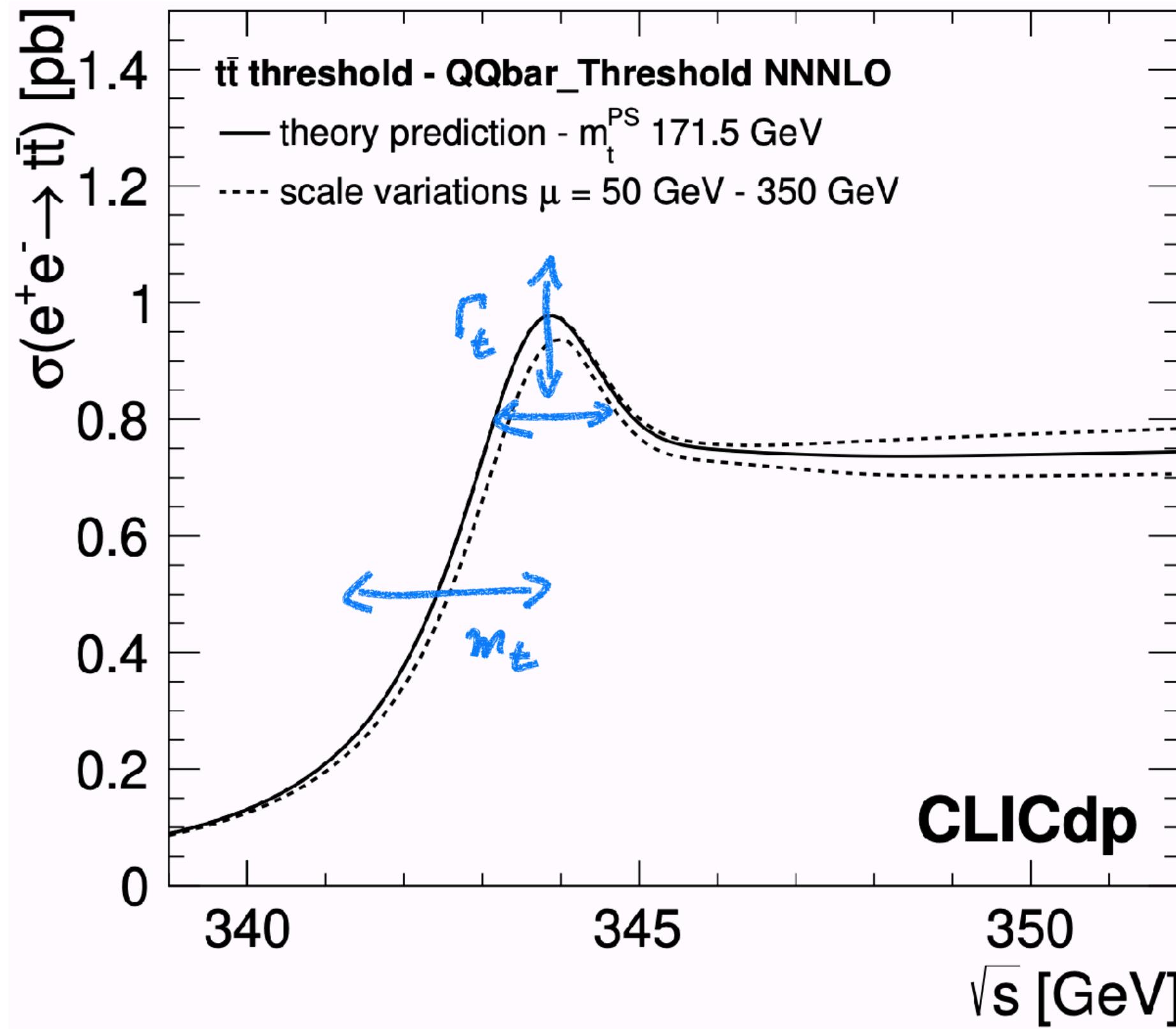
Sensitivity to Top Quark Properties and Beyond



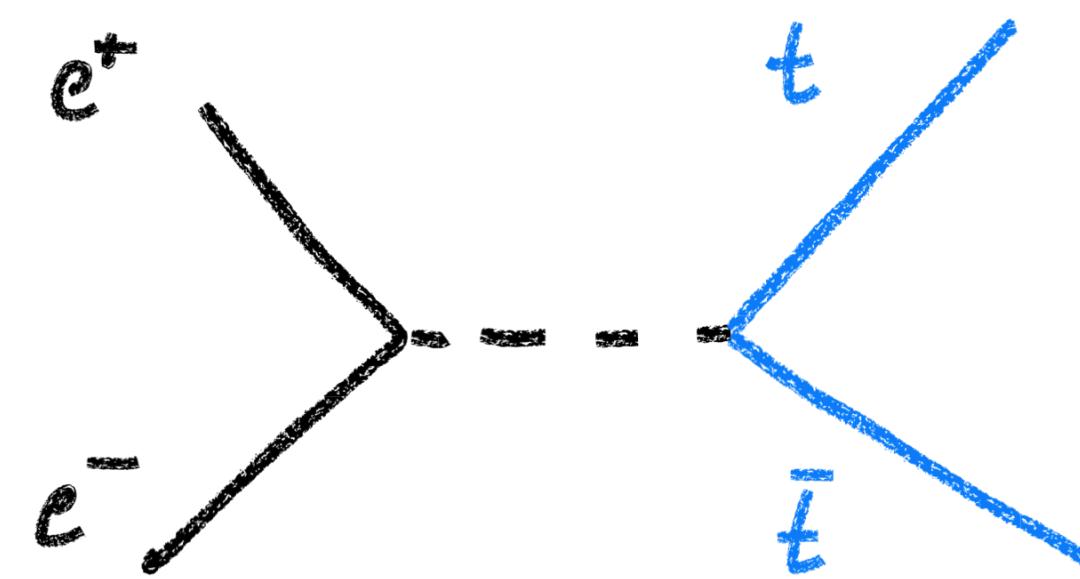
- The cross section of top quark pair production in the threshold region depends on top quark properties, and on QCD
- Precise theoretical calculations of cross section in the threshold region, in well-defined mass schemes ( $m_t^{\text{PS}}$ ,  $m_t^{1S\dots}$ ) -> Can be converted directly into MSbar mass.

# The Top Quark Pair Production Threshold

Sensitivity to Top Quark Properties and Beyond

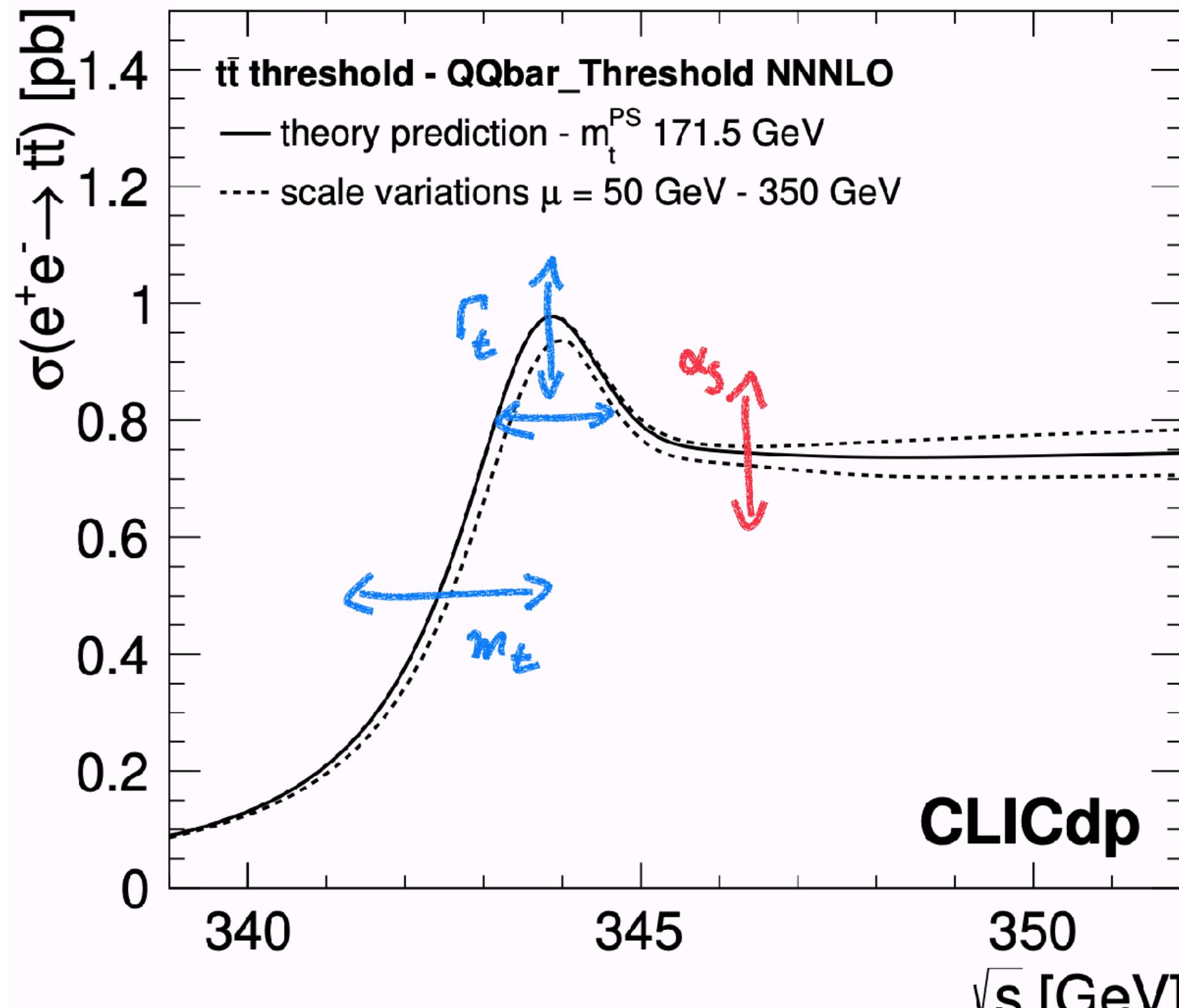


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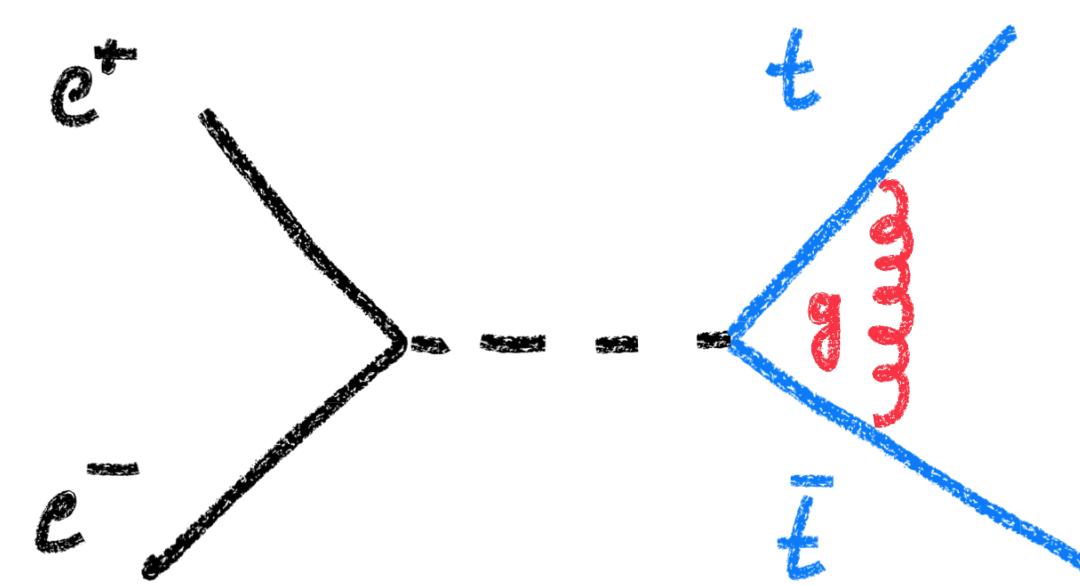


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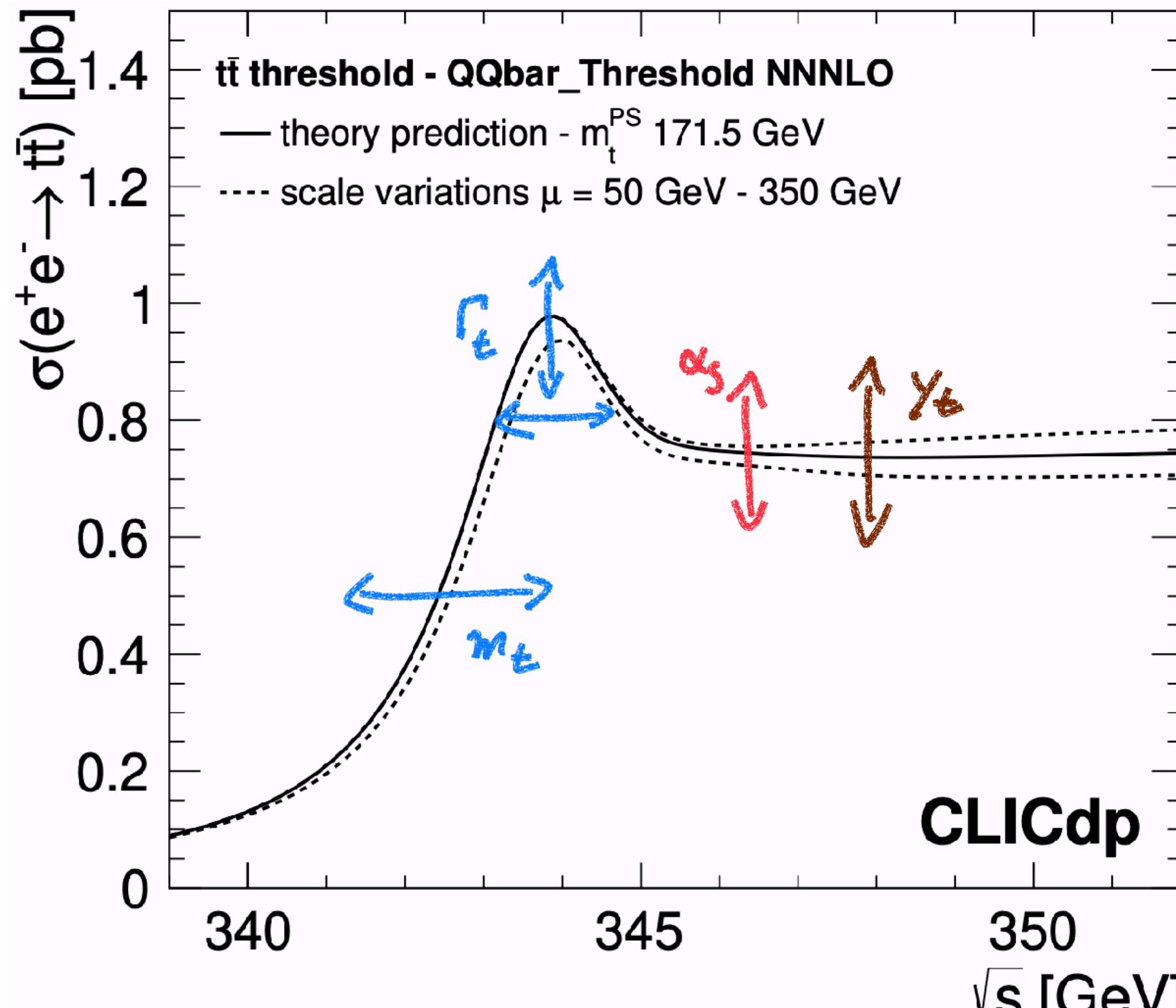


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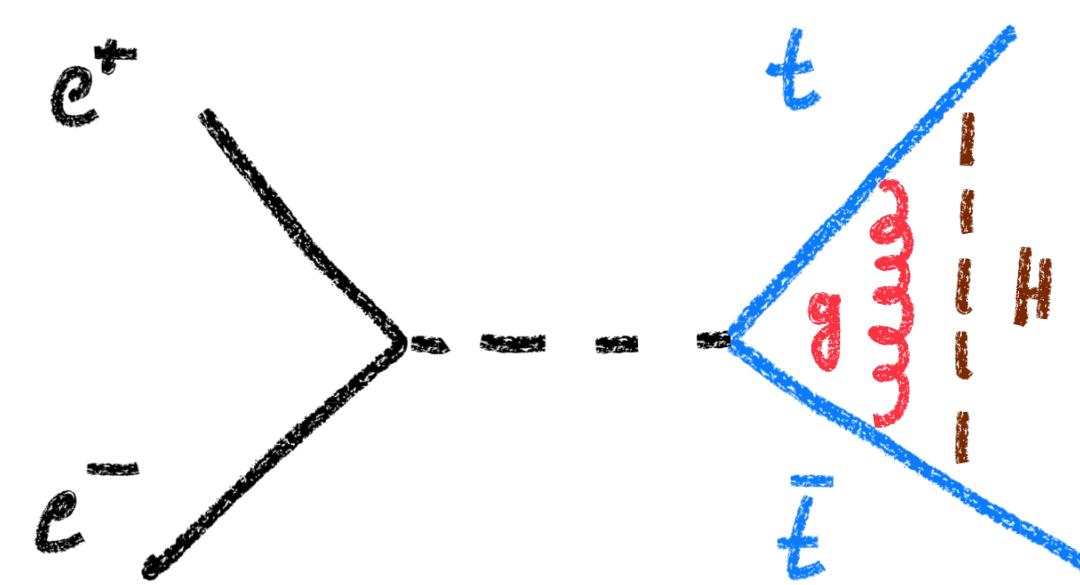


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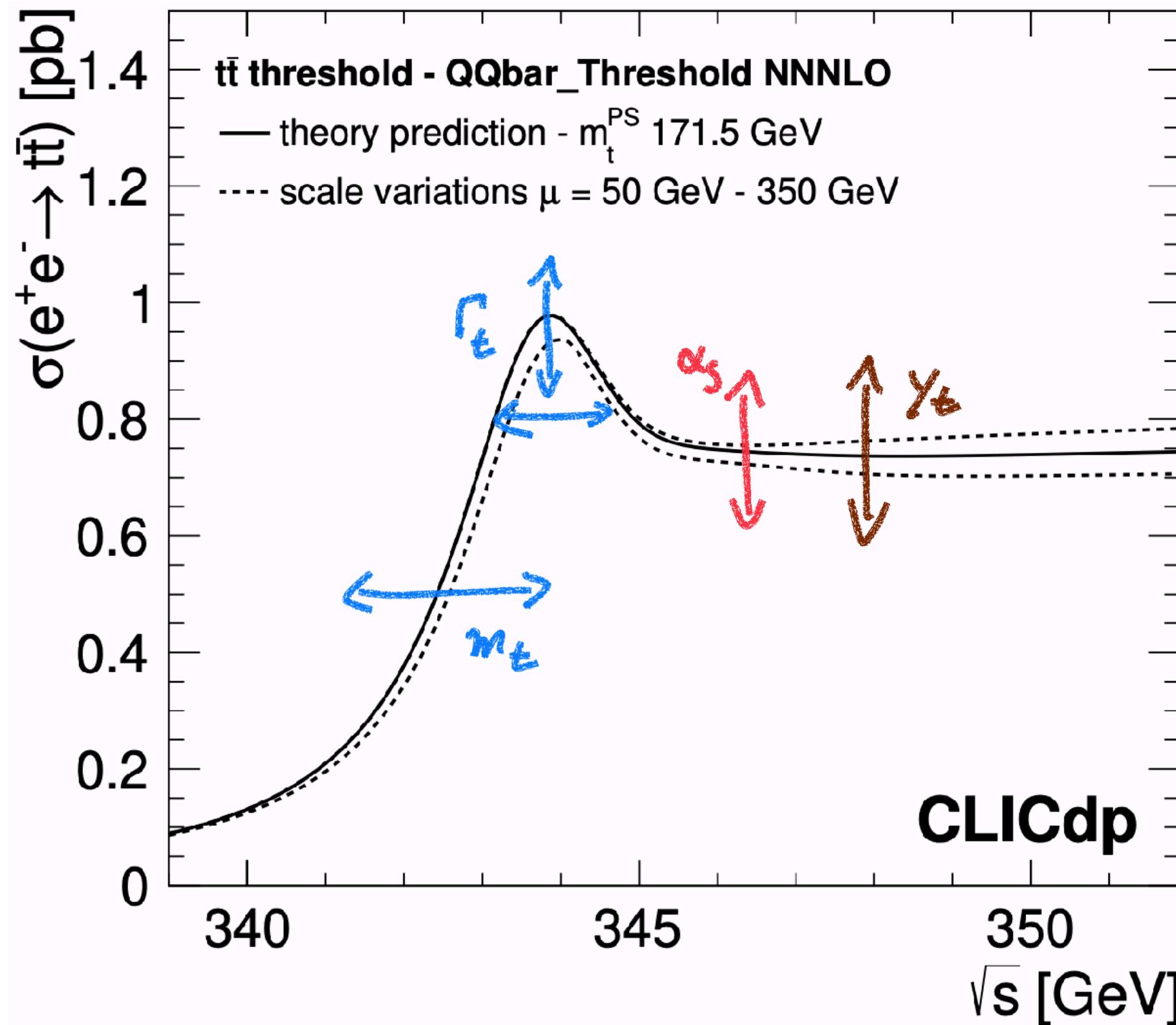


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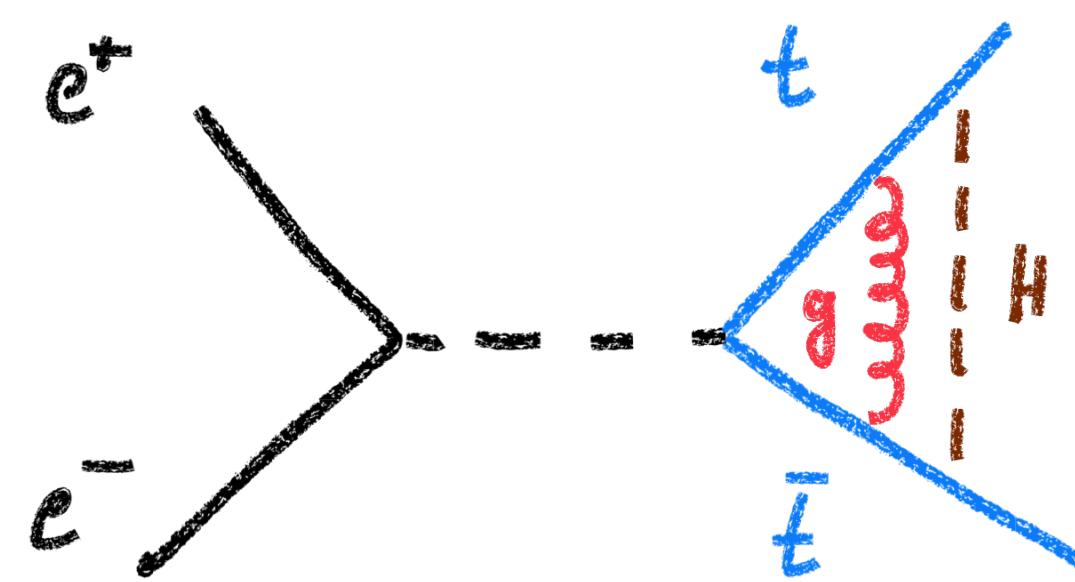


# The Top Quark Pair Production Threshold

## Sensitivity to Top Quark Properties and Beyond

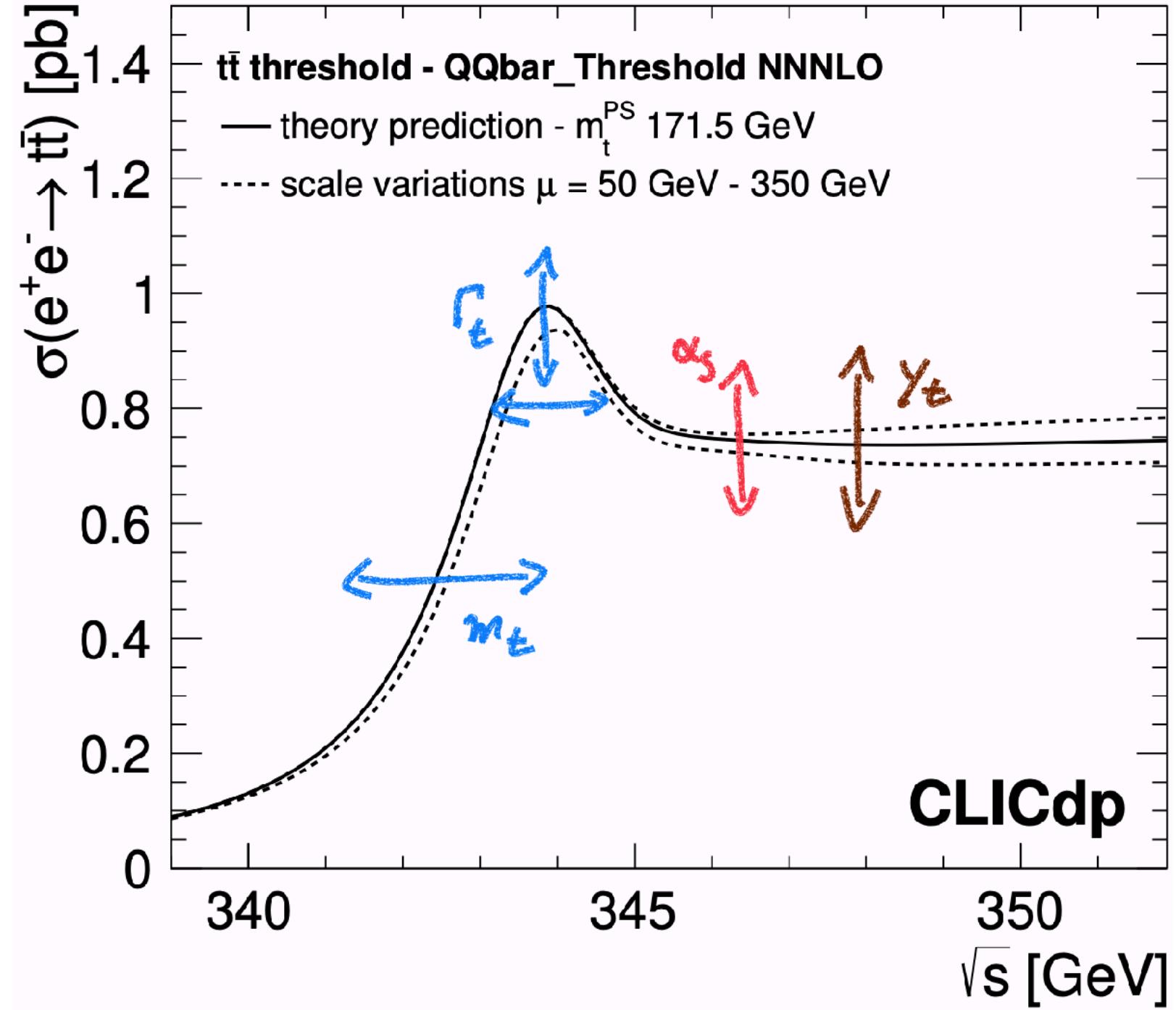


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- Precise theoretical calculations of cross section in the threshold region, in well-defined mass schemes ( $m_t^{\text{PS}}$ ,  $m_t^{1S\dots}$ ) -> Can be converted directly into MSbar mass.
- In principle,  $\alpha_s$  can be extracted from the threshold, but the precision is typically less than the current world average - using external input more efficient.

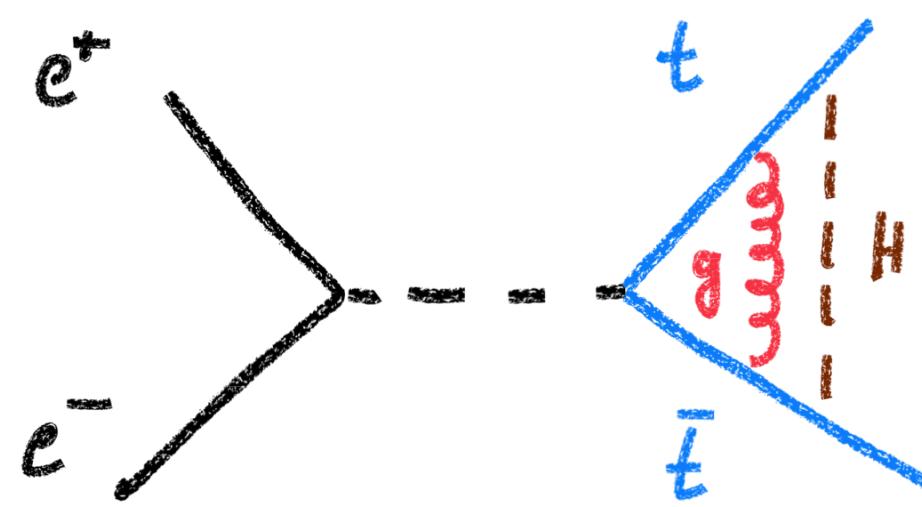


# The Top Quark Pair Production Threshold

From Theory to Experiment

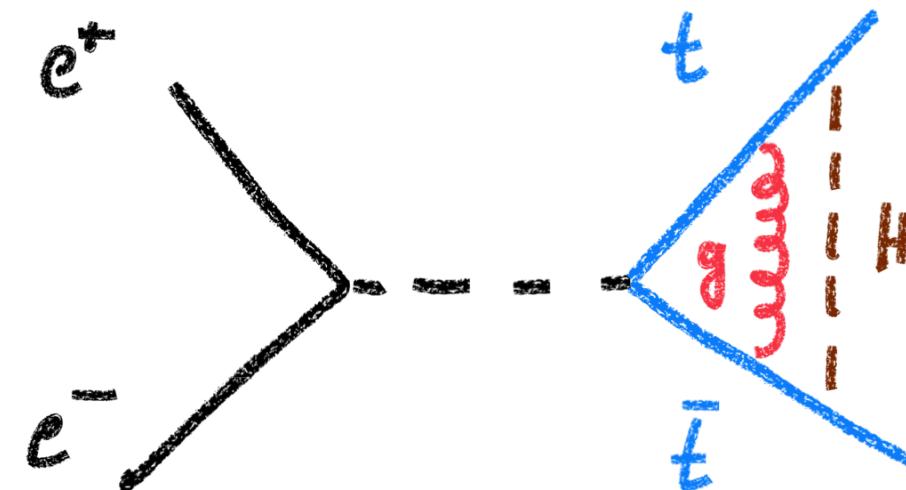
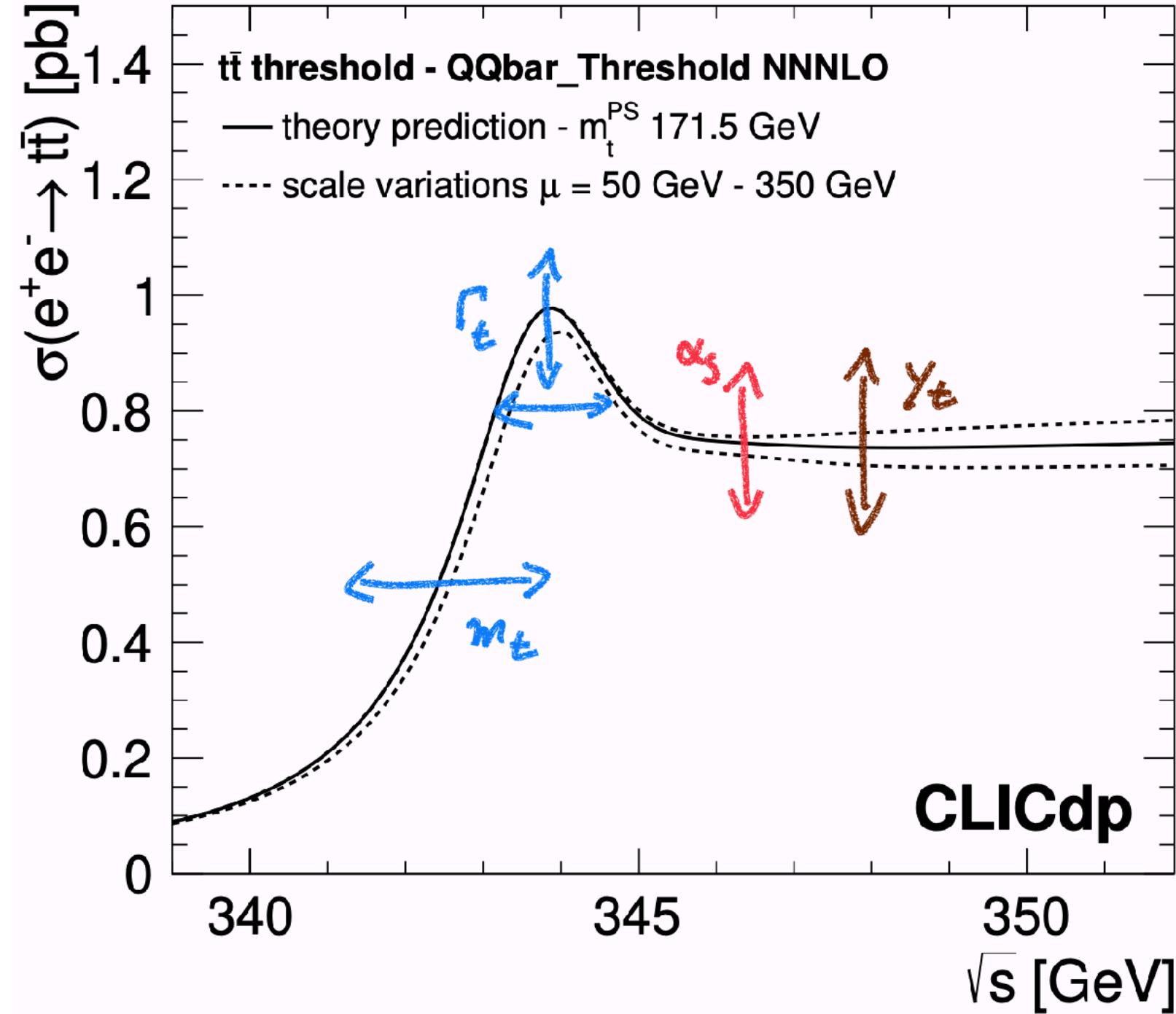


- The threshold shape is modified by
  - initial state radiation
  - collider luminosity spectrum



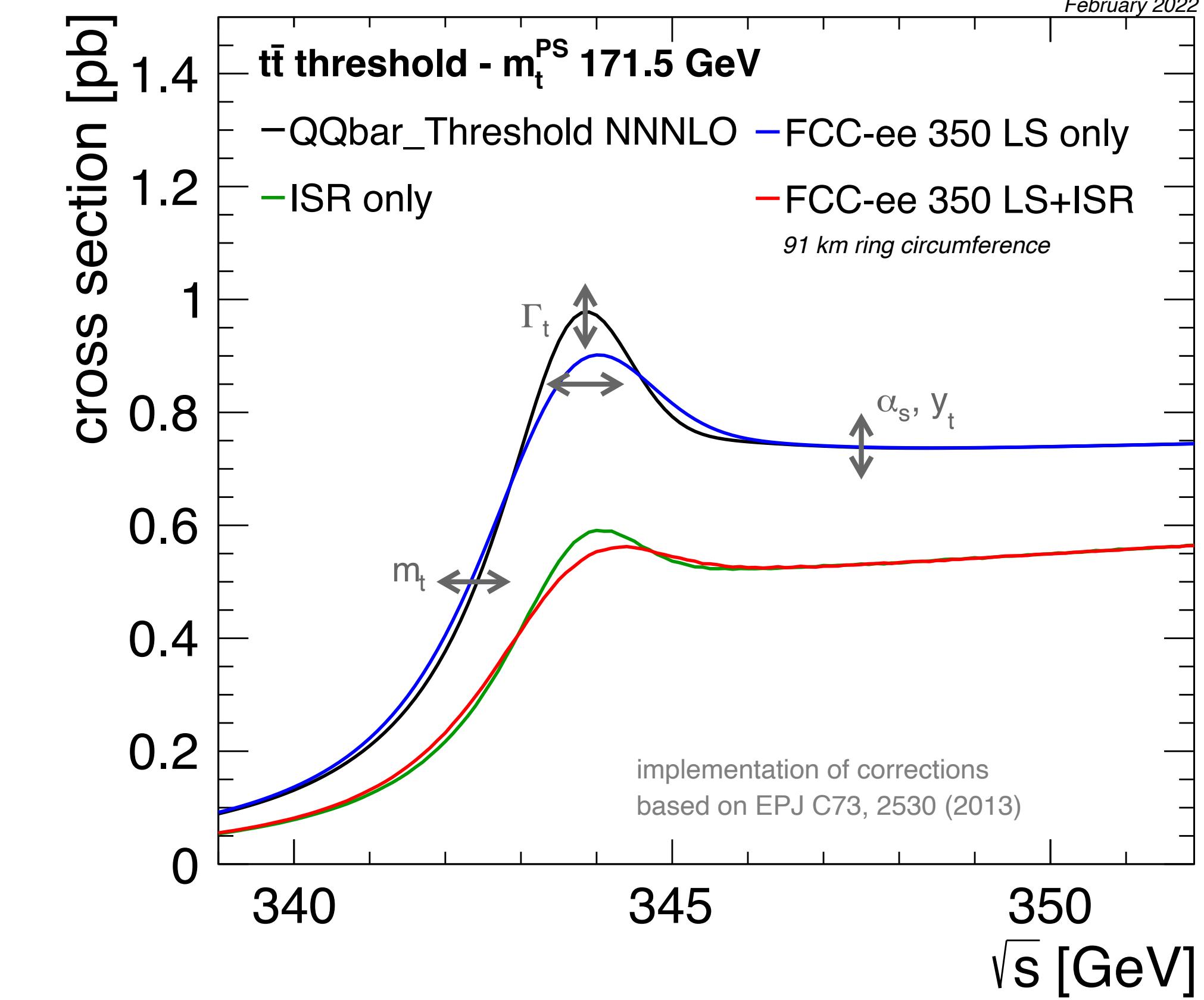
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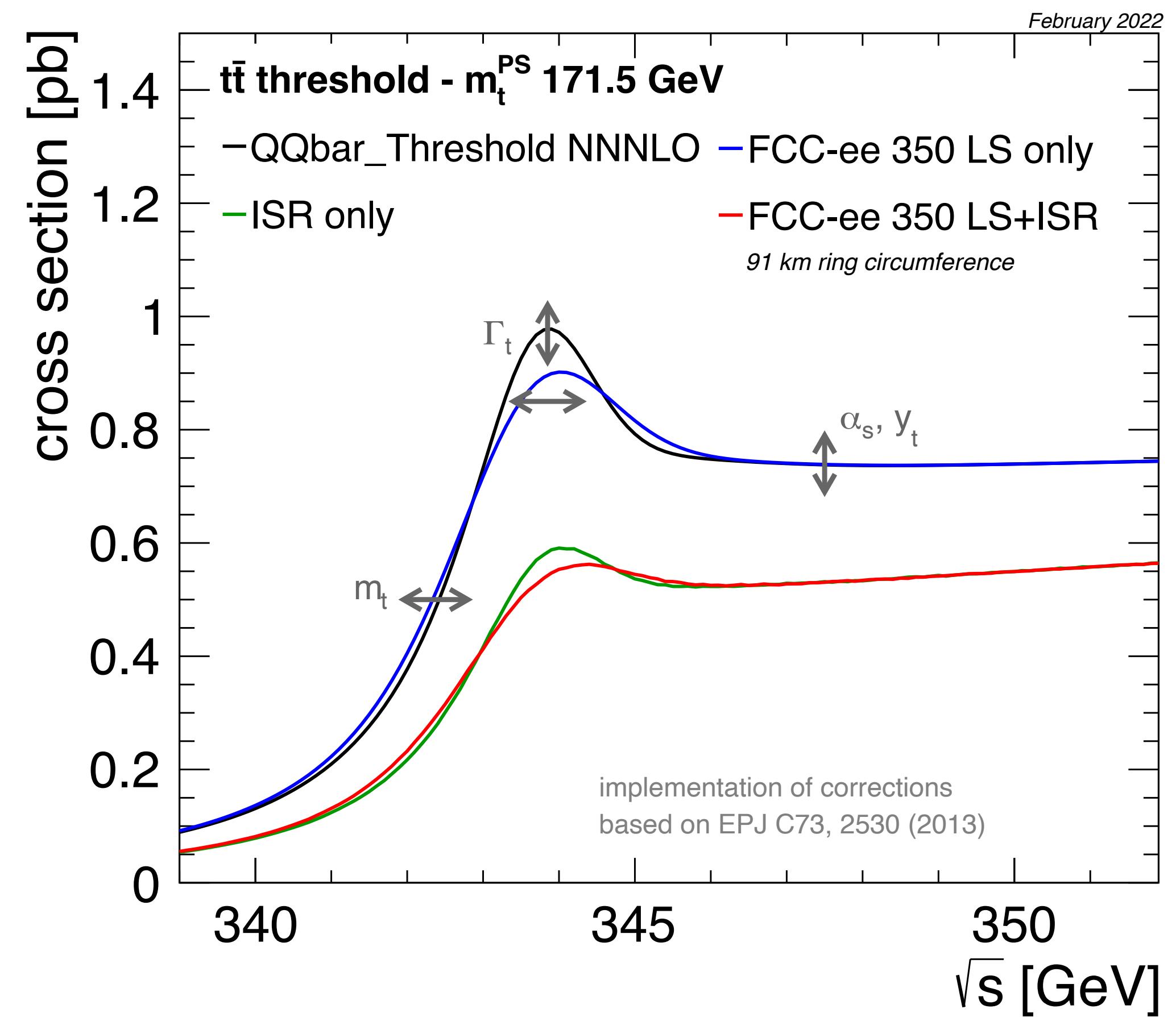
ISR, luminosity spectrum



# Differences between Colliders

## The Luminosity Spectrum

- Linear collider luminosity spectra are characterized by a beamstrahlung tail, FCC-ee is close to Gaussian

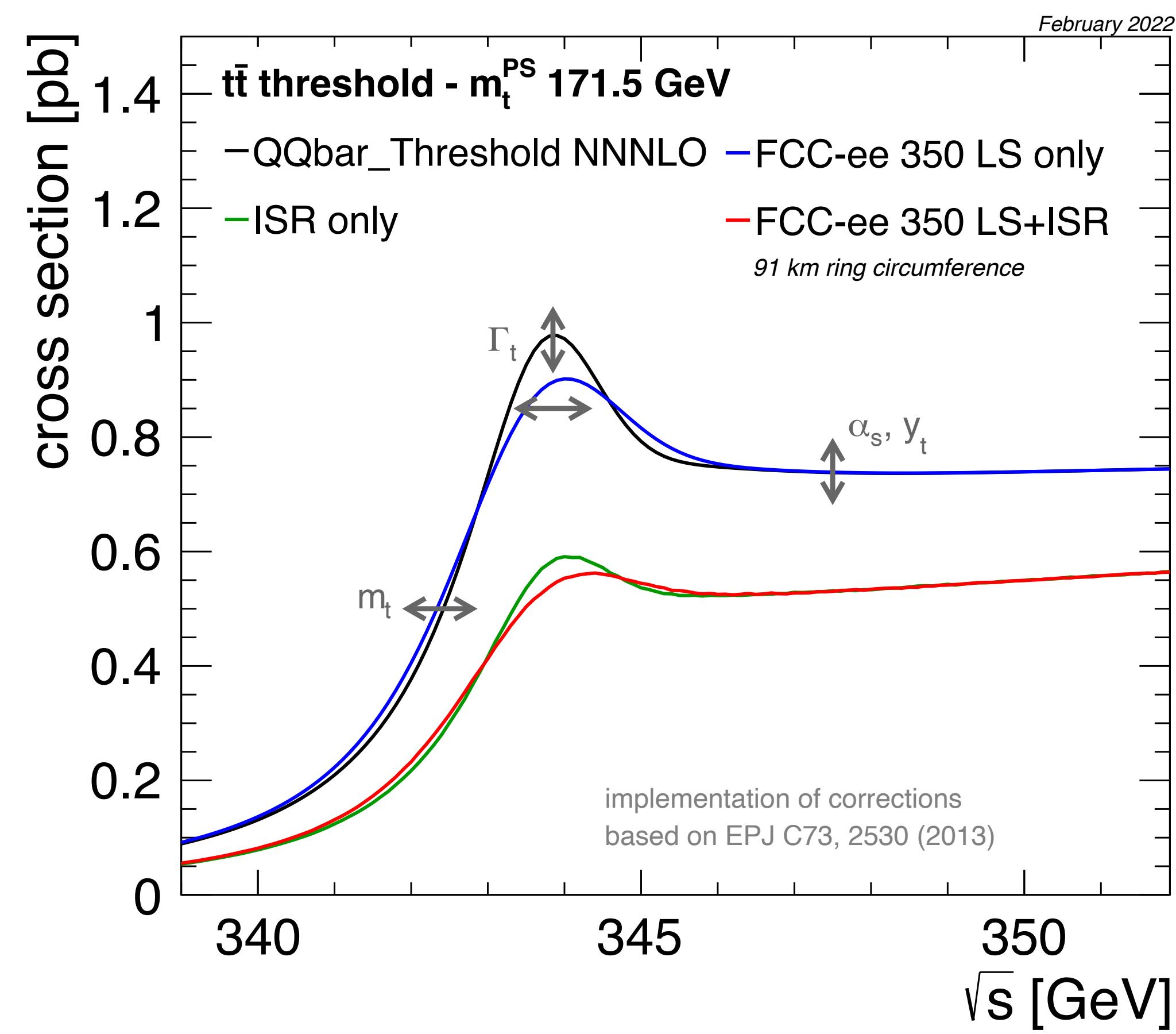


FCC vs

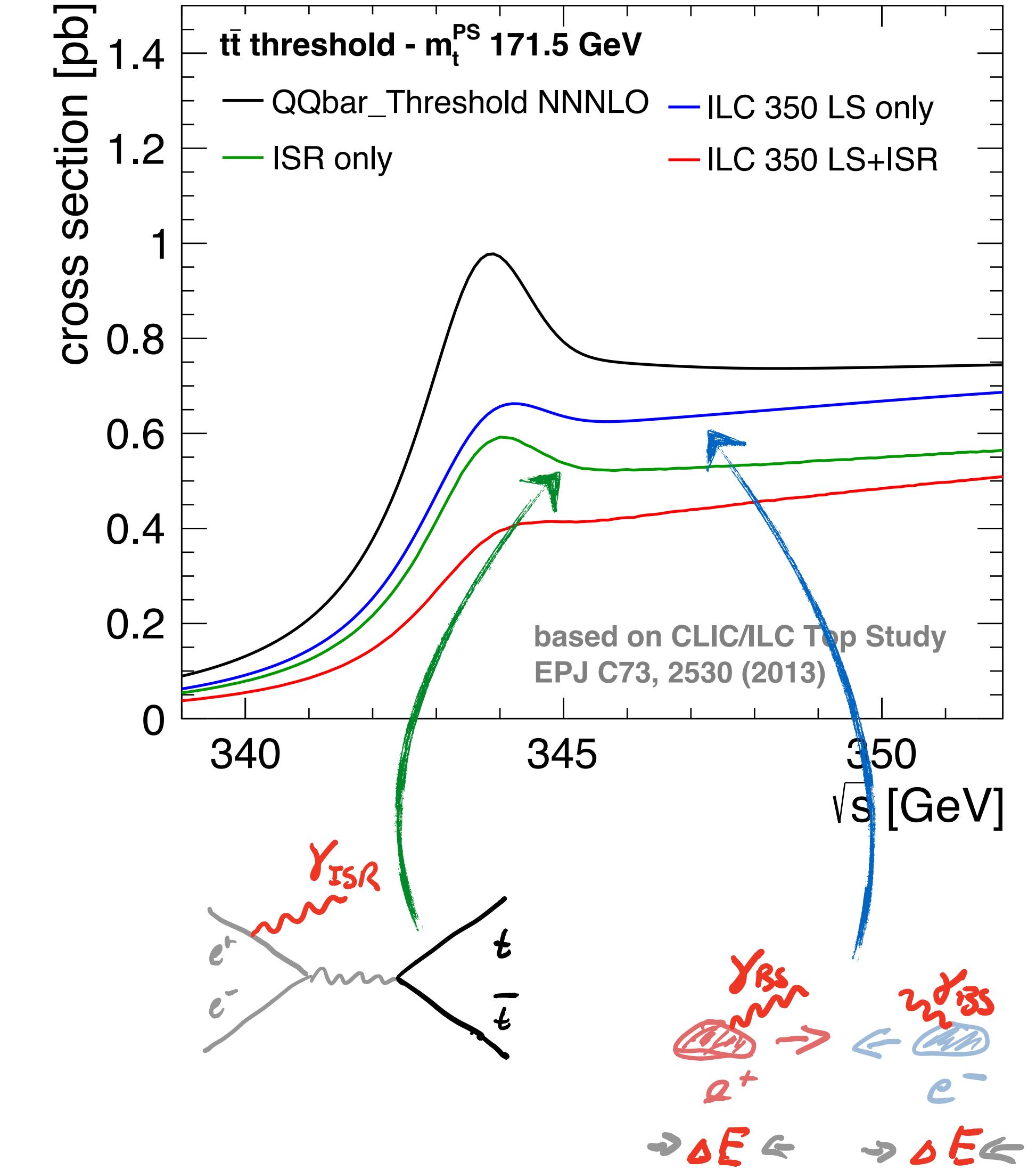
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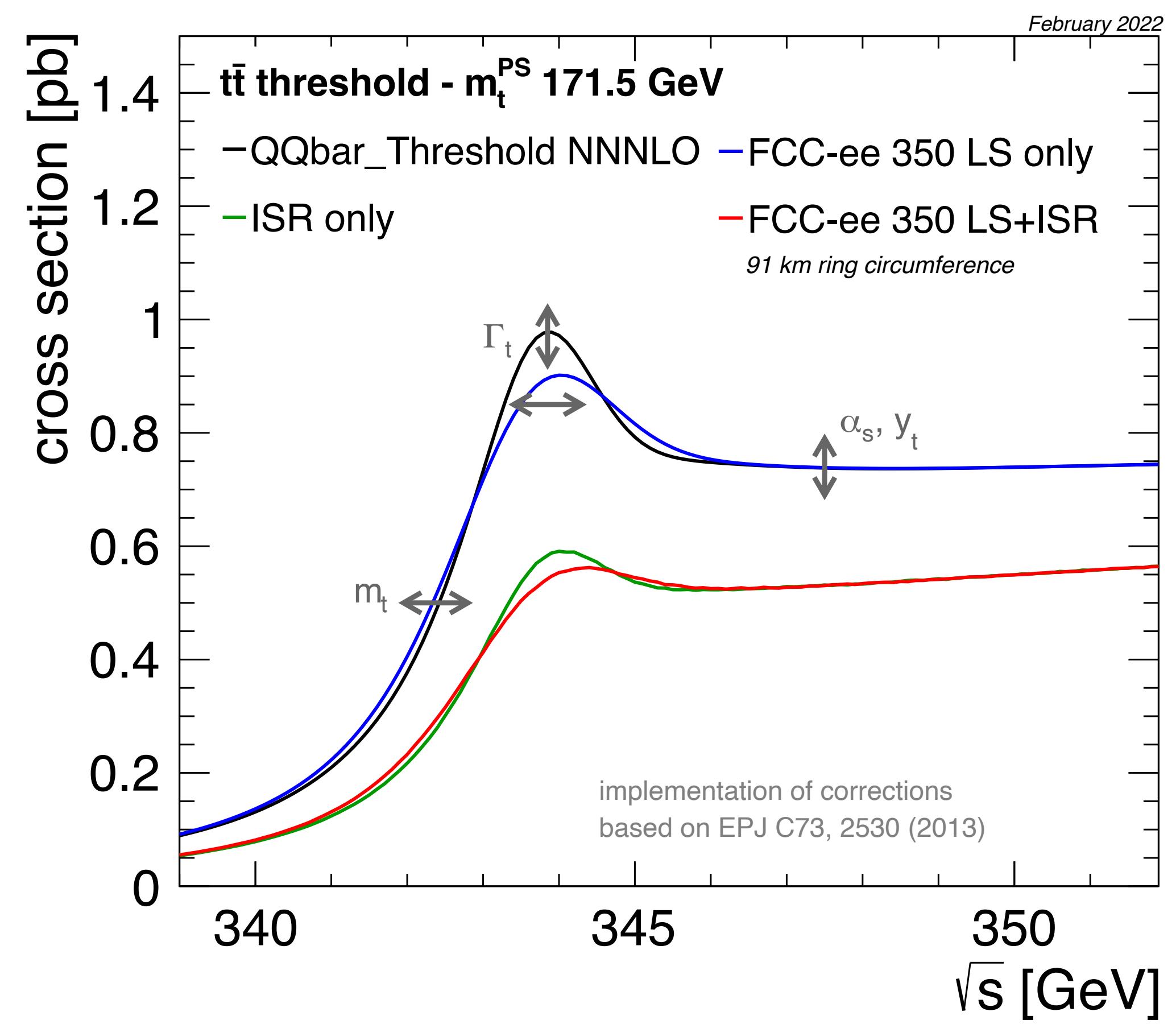
FCC vs  
ILC



# Differences between Colliders

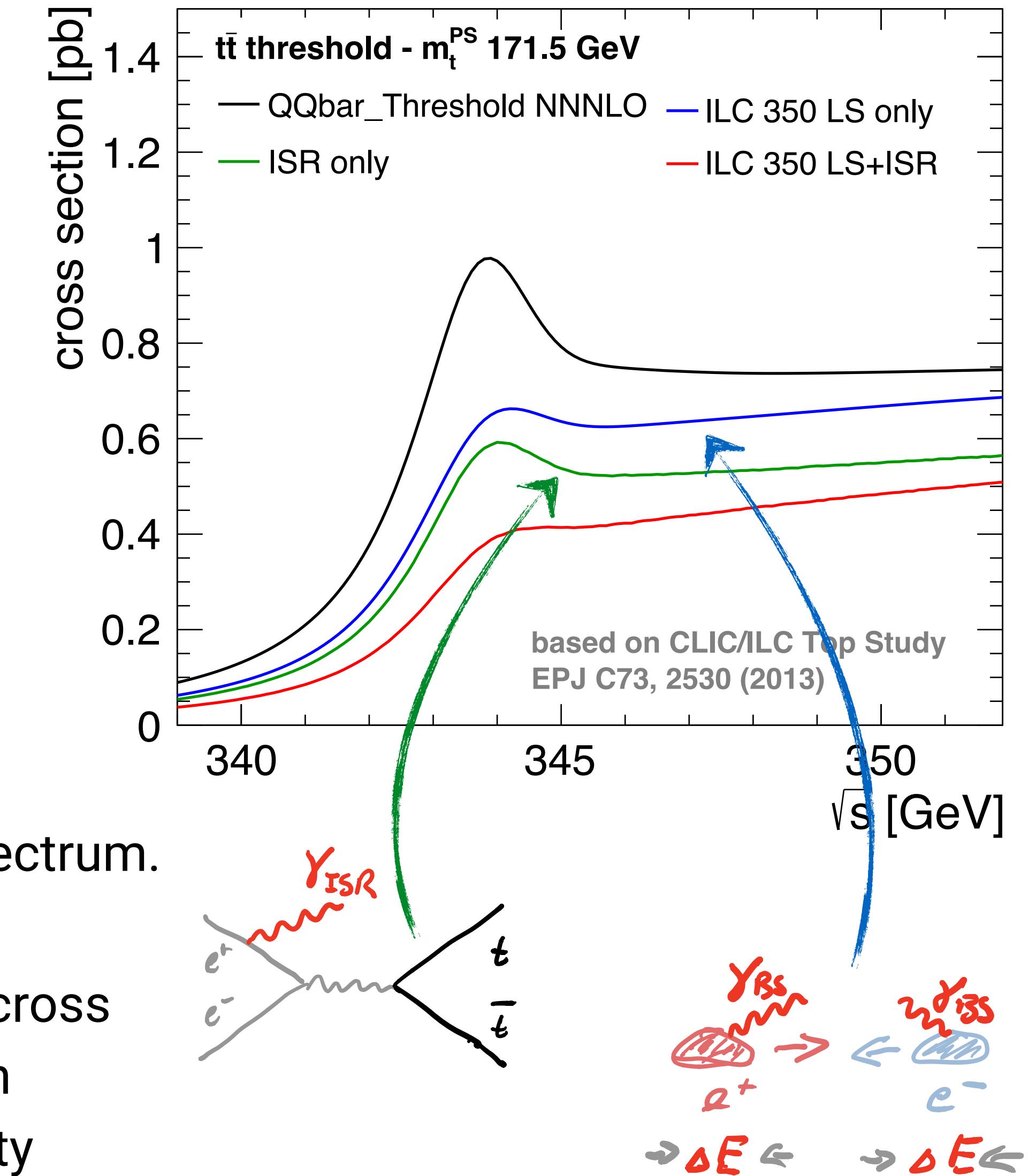
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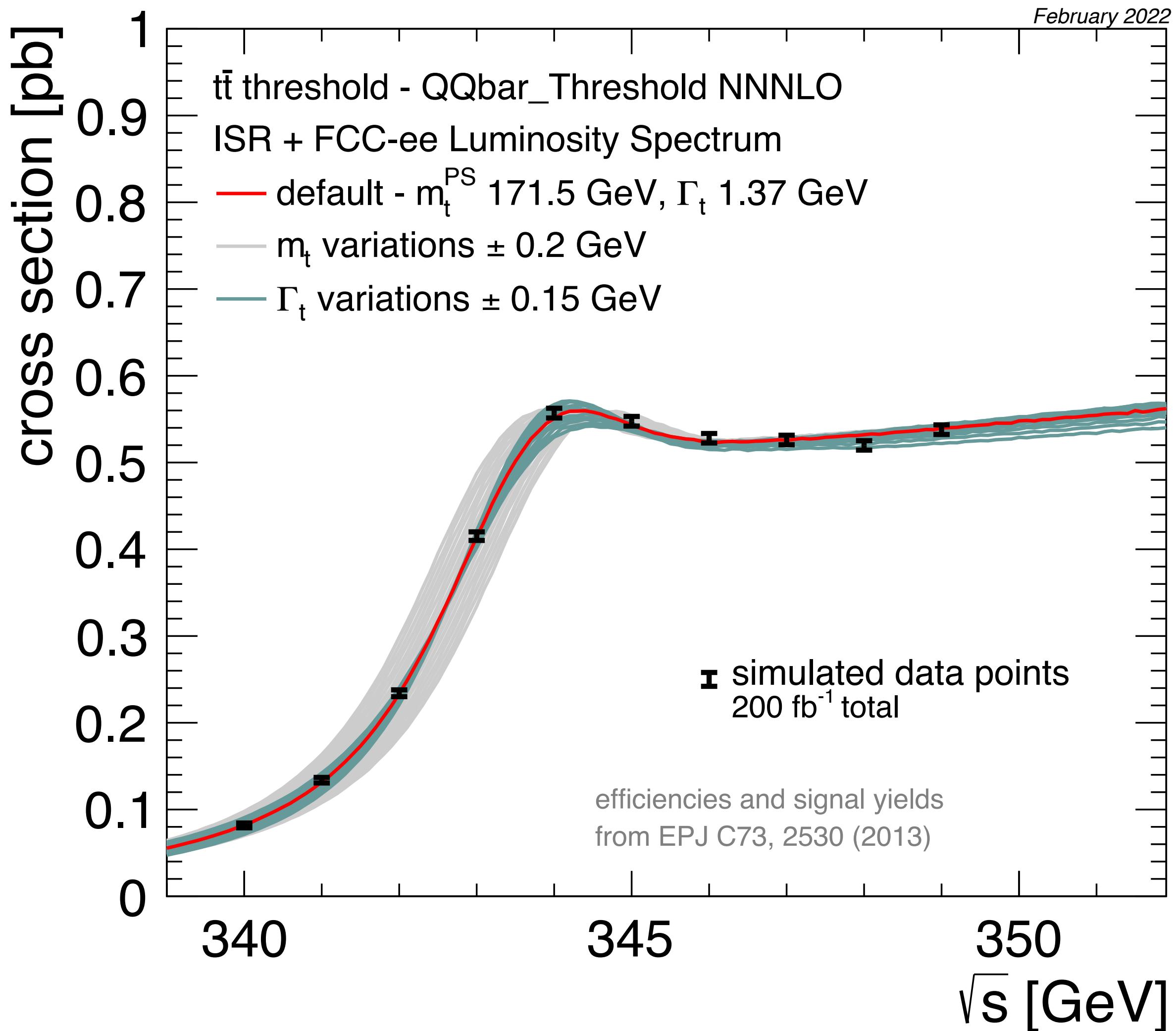
FCC vs  
ILC

Requires: Precise understanding and measurement of spectrum.  
In this case:  
~ 30% reduction of cross section -> 15% hit on statistical uncertainty



# The Standard Threshold Scan

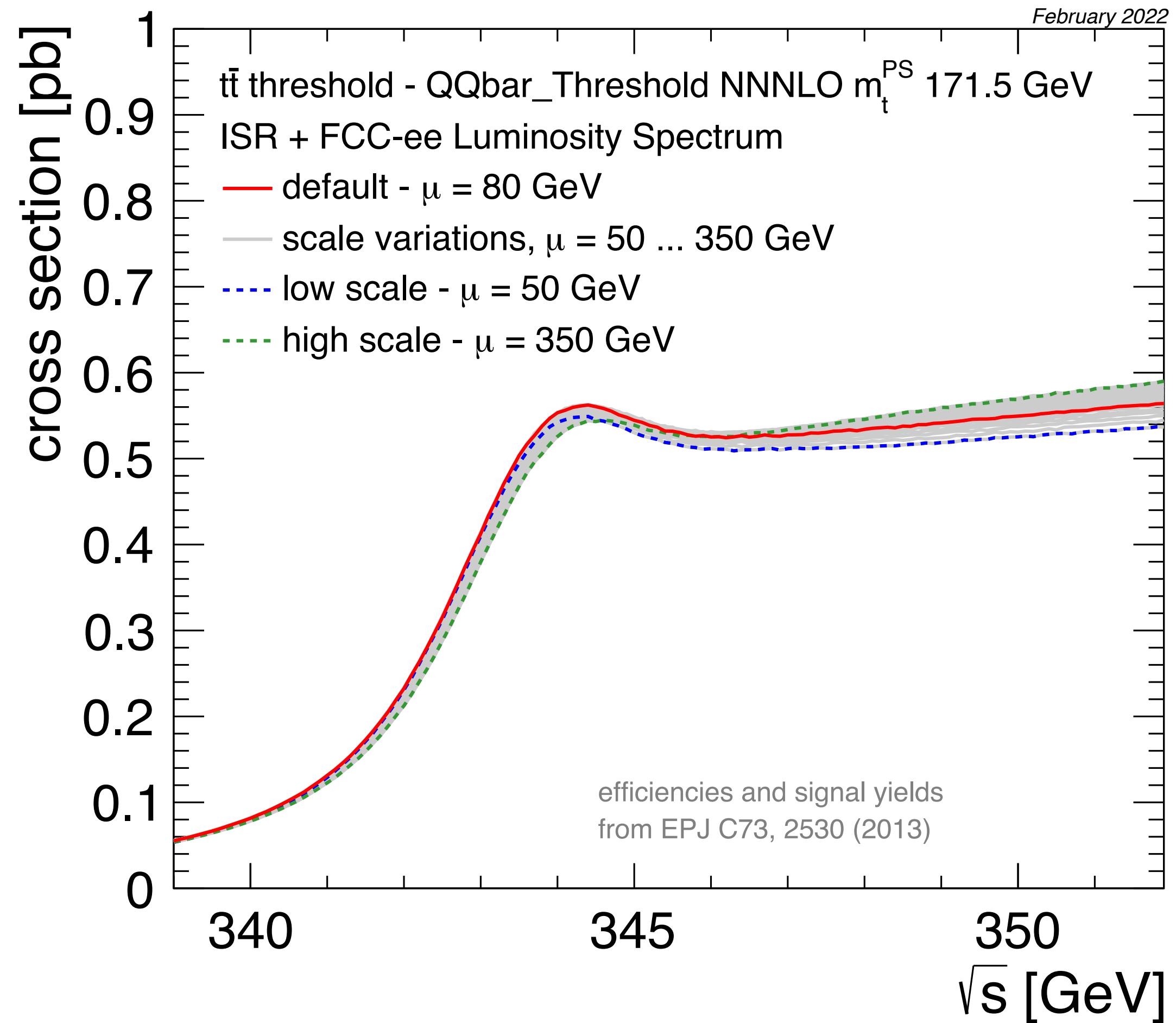
## Experimental Assumptions



- The standard assumptions:  
Efficiency, signal and background yields taken from EPJ C73, 2530 (2013):  
70.2% signal efficiency, 73 fb effective background cross section after selection
  - A 10-point threshold scan, with equal luminosity sharing, spacing by 1 GeV, from 340 - 349 GeV
  - ILC, FCC-ee assume 200  $\text{fb}^{-1}$  total, CLIC 100  $\text{fb}^{-1}$   
(for easier comparisons, 200  $\text{fb}^{-1}$  numbers are often also quoted for CLIC)
  - Top mass (and other parameters, such as  $\Gamma_t$ ,  $y_t$ ,  $a_s$ ) extracted via template fits of predicted cross sections with different input parameters.
- Theory essential** - here NNNLO QCD [Beneke et.al.]

# Theory Uncertainties

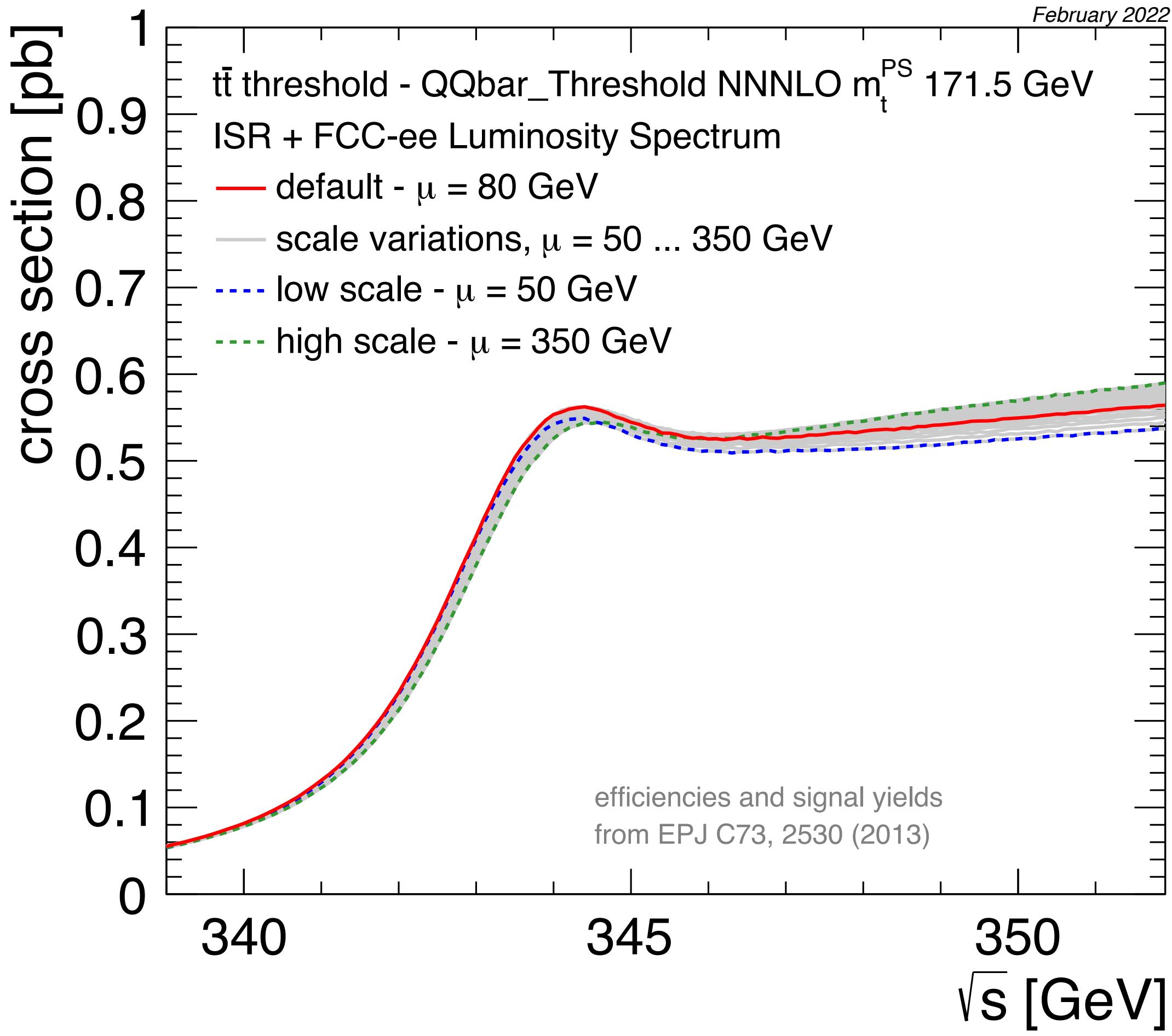
A key factor



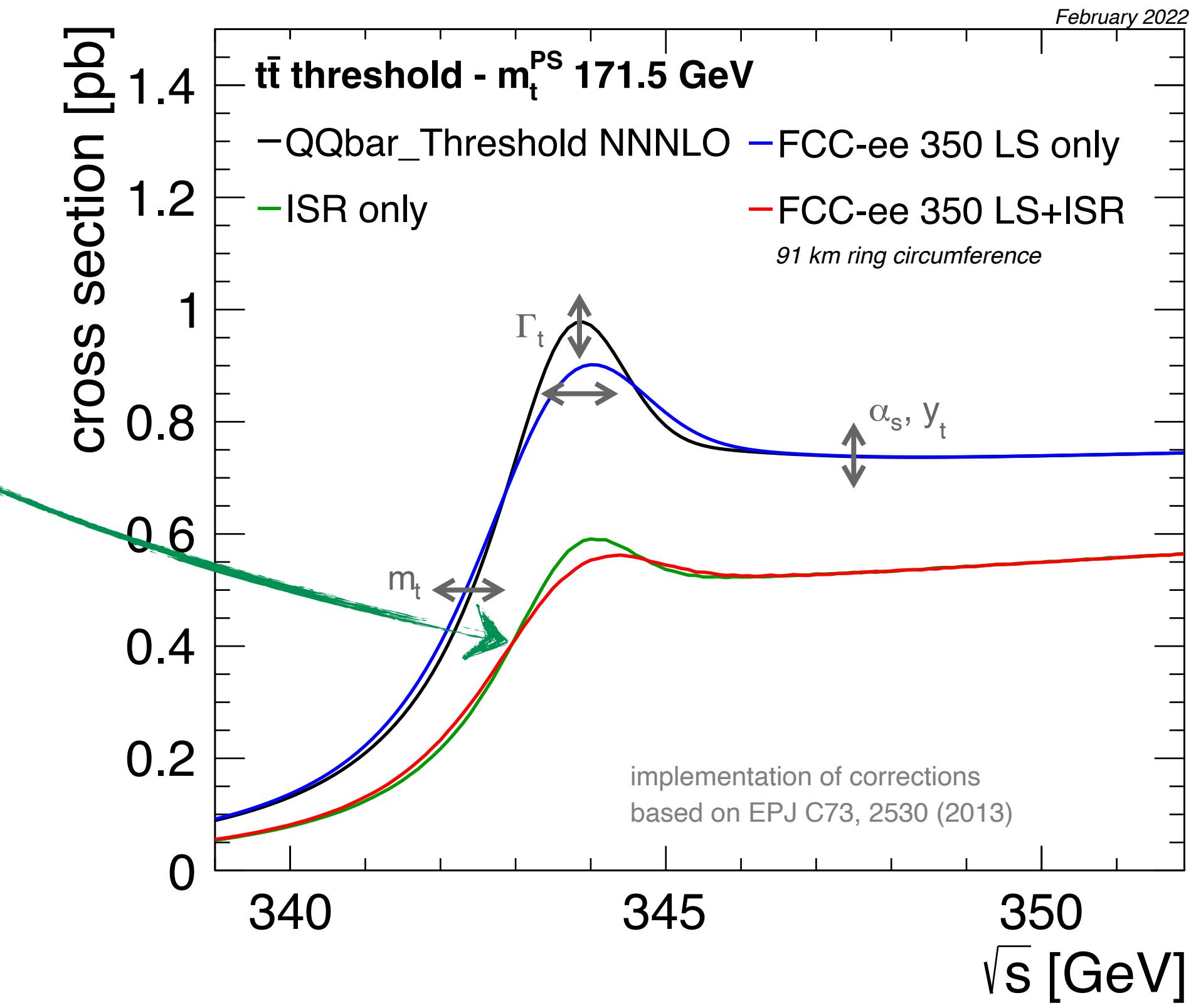
- QCD scale uncertainties highly relevant.

# Theory Uncertainties

A key factor

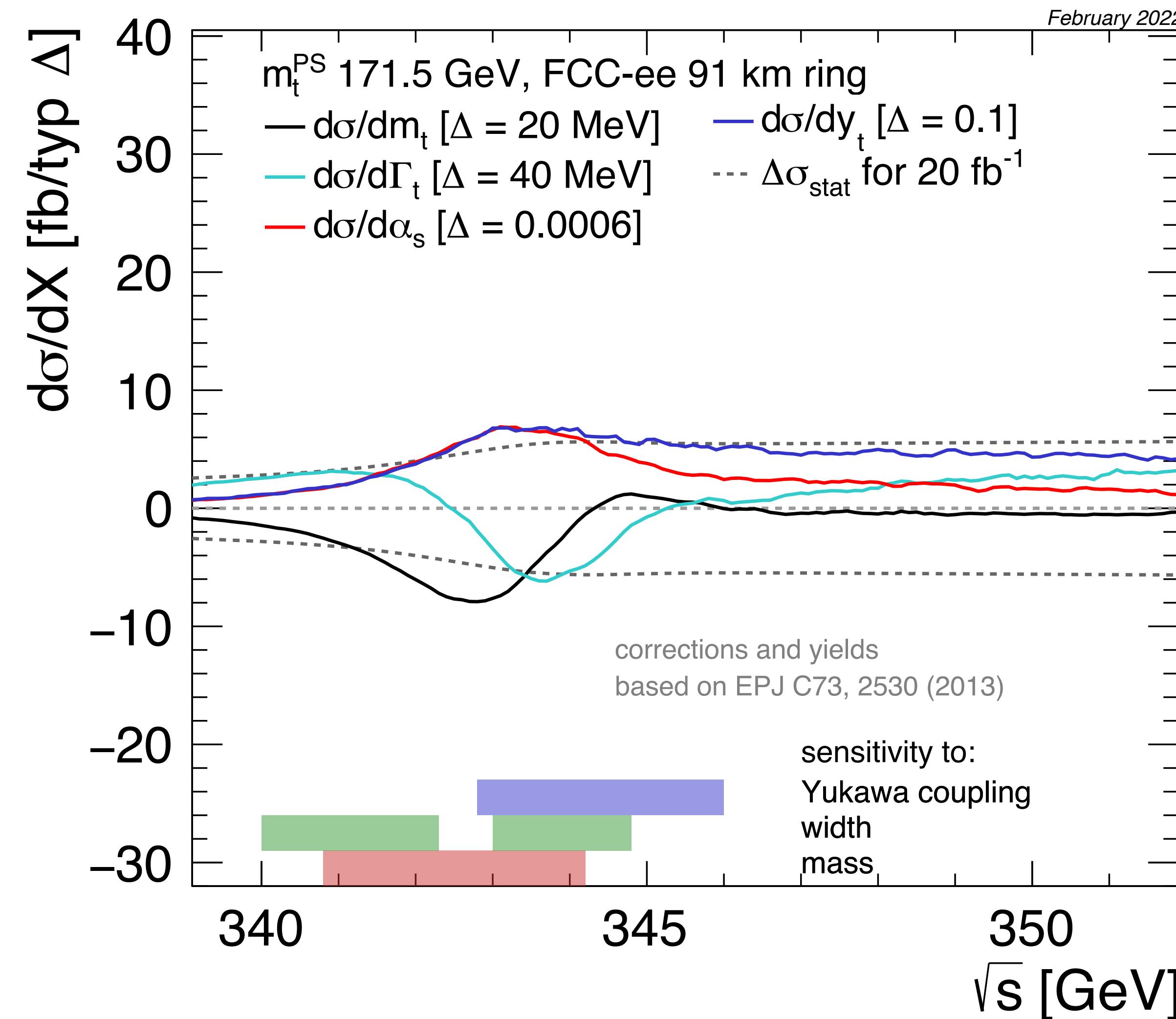


- QCD scale uncertainties highly relevant.
  - Also need to calculate other effects, such as ISR, to the required precision!
- the step from black to green  
[only approximate in current experimental studies]



# Choosing the Scan Range

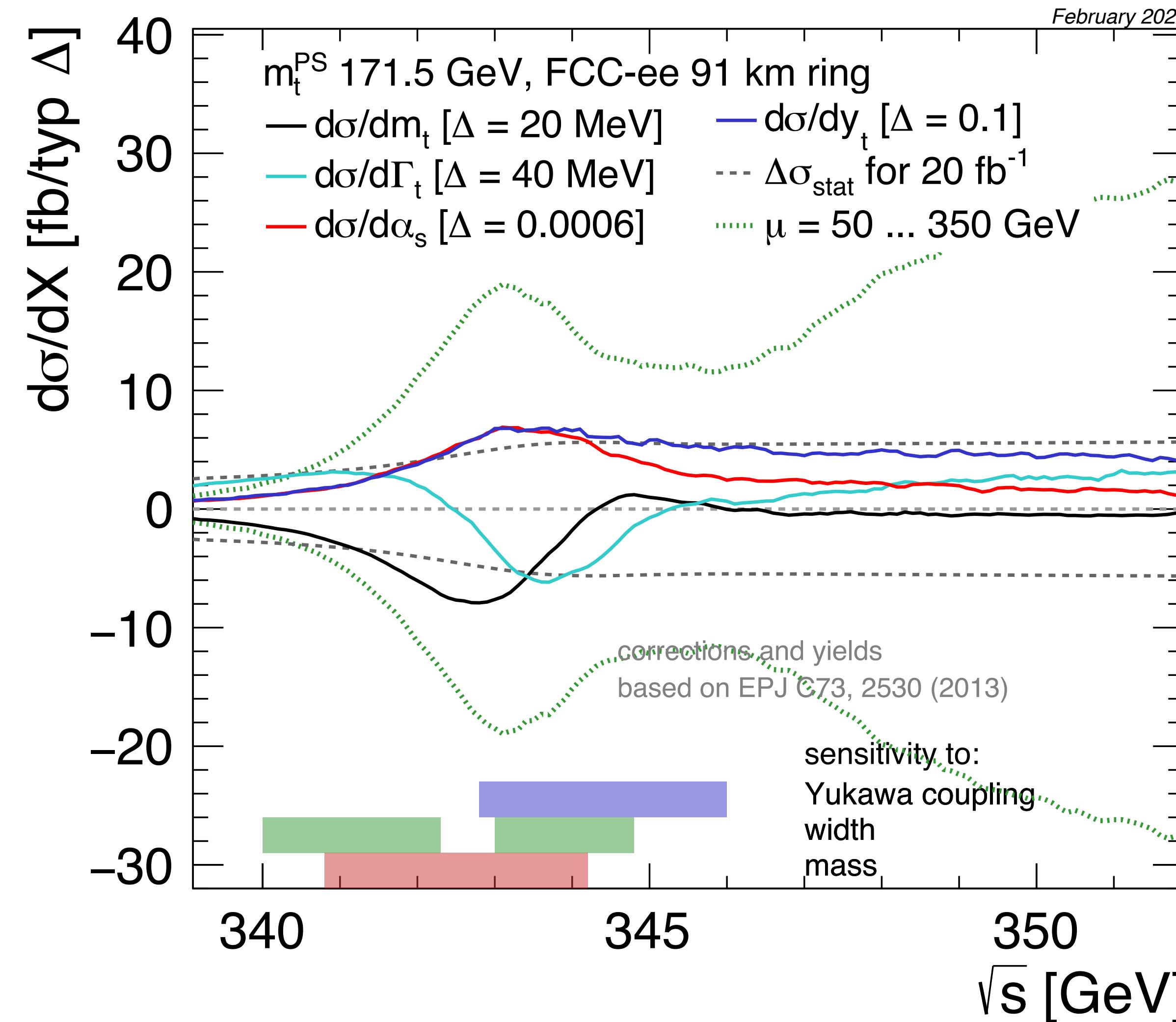
## Parameter Sensitivity



- Plot shows the derivative of the cross section for various parameters - to make this understandable this is normalised to typical changes of these parameters
- Full use to optimize scan range requires knowledge of mass to  $\sim 200$  MeV in PS scheme. Can be achieved with  $2 \times 5 \text{ fb}^{-1}$ :
  - point 1:  $\sqrt{s} = 2 \times m_t^{\text{PS,LHC}} - 1.5 \text{ GeV}$
  - point 2:  $\sqrt{s} = 2 \times m_t^{\text{PS,LHC}} + 0.5 \text{ GeV}$  [[arXiv:1902.07246](https://arxiv.org/abs/1902.07246)]
 (N.B.: This is safe also when taking theory uncertainties into account)
- Optimizing for particular parameters can reduce the statistical uncertainty by  $\sim 25\%$  [JHEP 7, 70 (2021)]

# Choosing the Scan Range

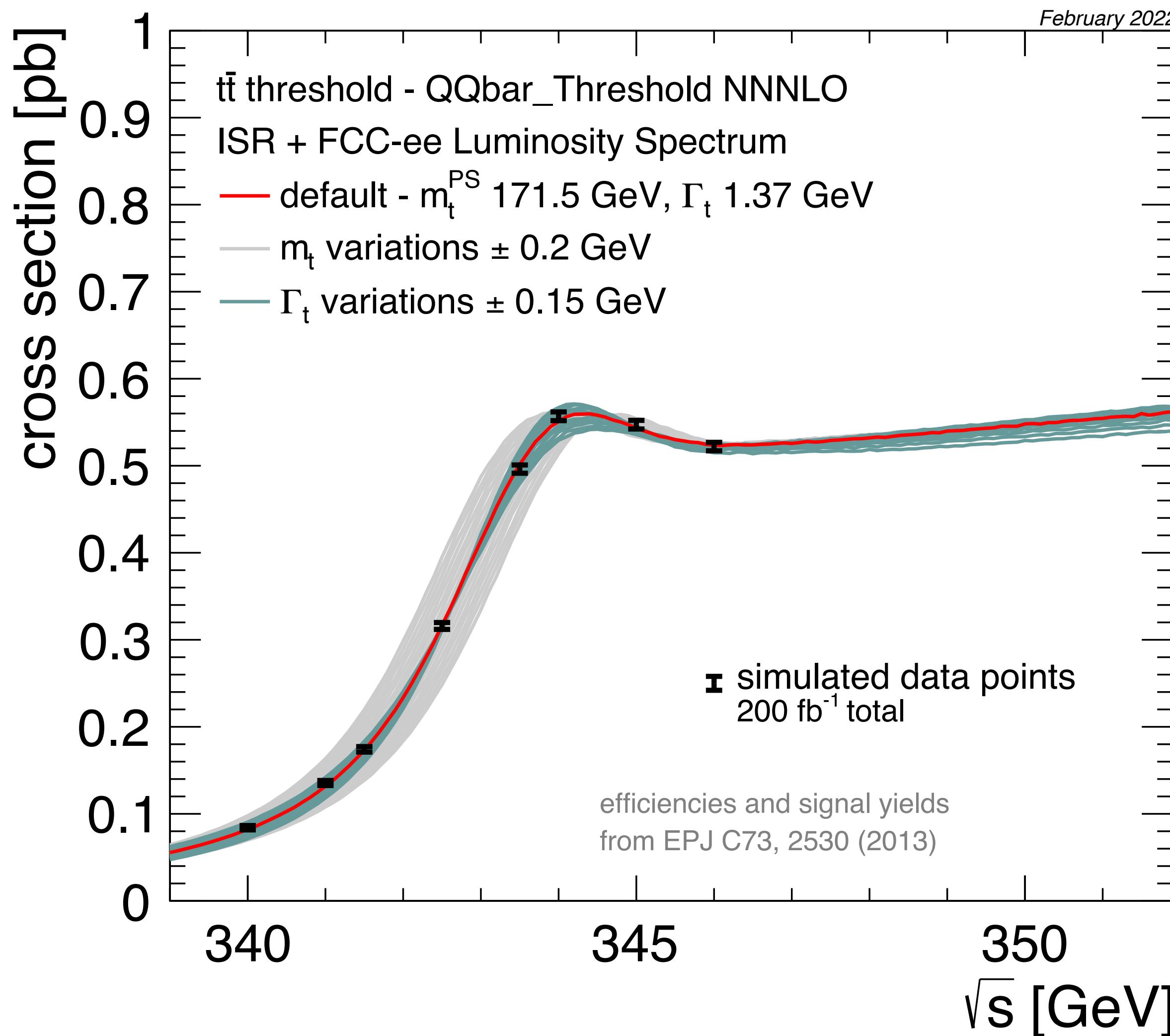
*Enter theory uncertainty*



- QCD scale uncertainties dominate over point-by-point statistical uncertainties for typical threshold scans: At this point optimising scan strategies to reduce statistical uncertainties does not improve the total uncertainty - in fact concentrating on a very small range may make systematic control more difficult.
- In general: Also to separate contributions from different parameters, the most relevant range is 340 - 346 GeV.  
Higher energy points would primarily benefit a  $y_t$  measurement.

# Choosing the Scan Range

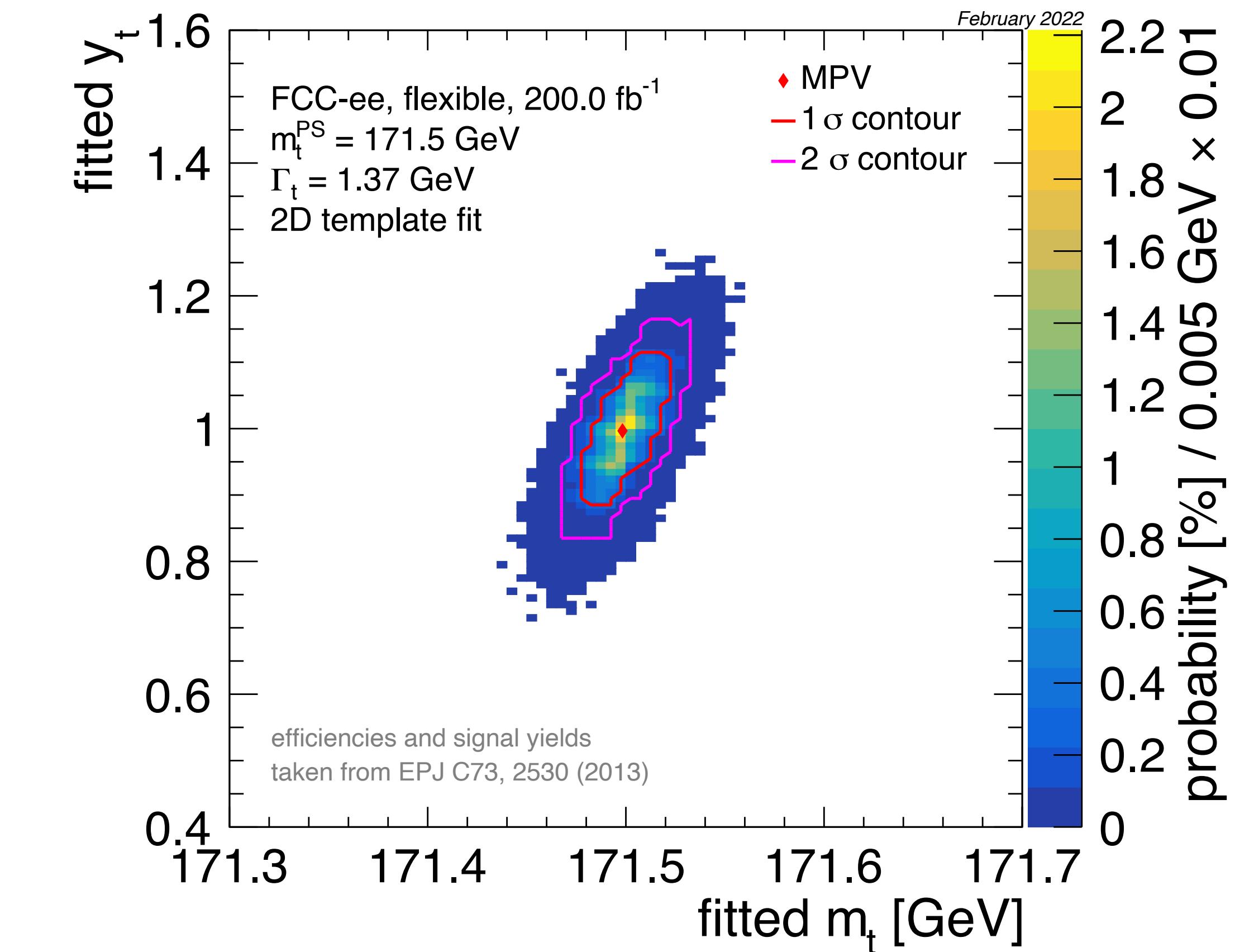
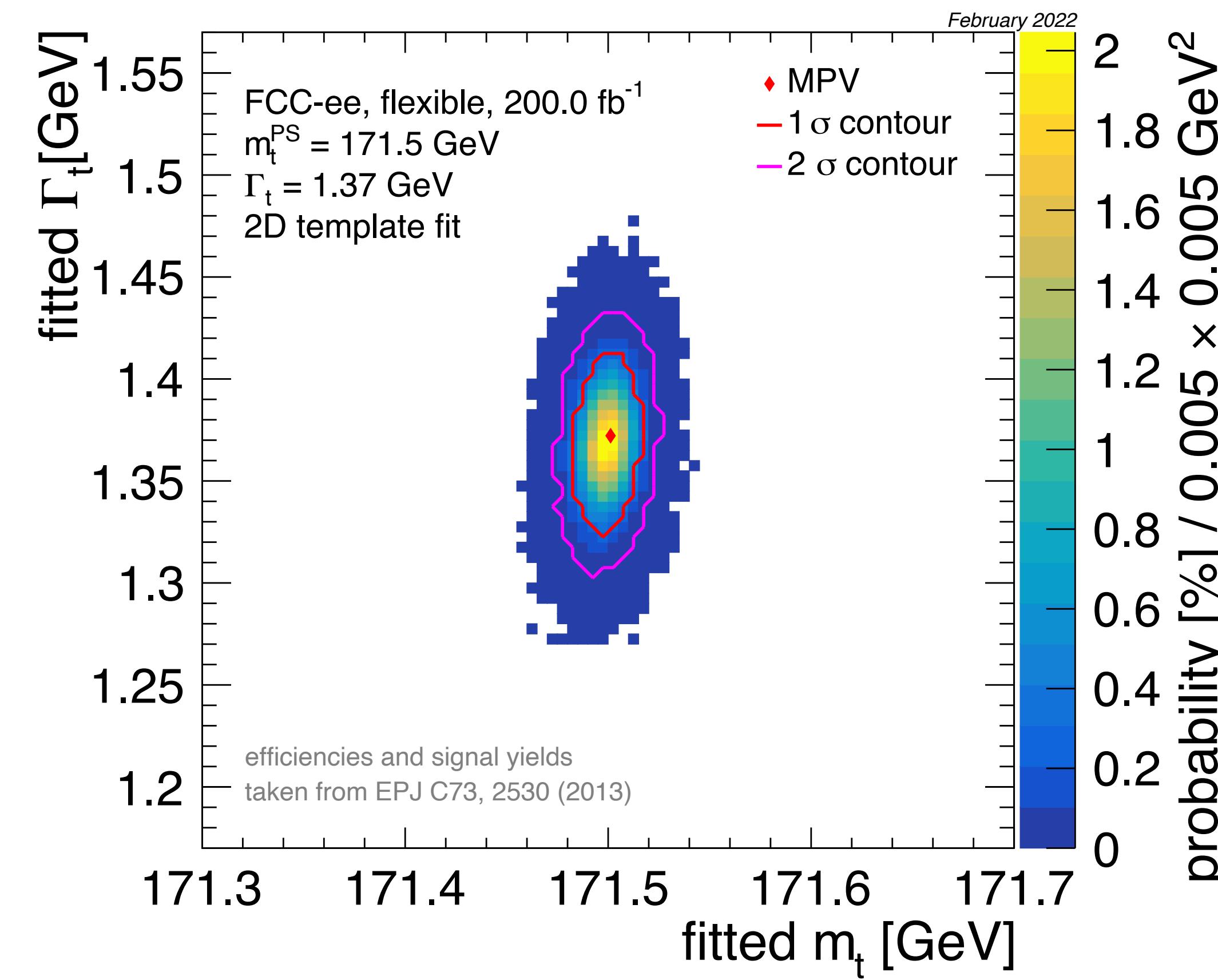
*Bottom line for FCC-ee studies (very similar for ILC)*



- Mildly optimized scan (mass & width) for FCC-ee as a balance between different sensitivities: 8 points in the range of 340 - 346 GeV assumed for most results in the following

# Fitting Multiple Parameters

Mass, Width, Yukawa Coupling



- $\sim 45 \text{ MeV}$  on width

- $\sim 11.5\%$  on Yukawa coupling

# Uncertainties Overview

ILC & FCC-ee

- Relatively thorough evaluation for ILC:

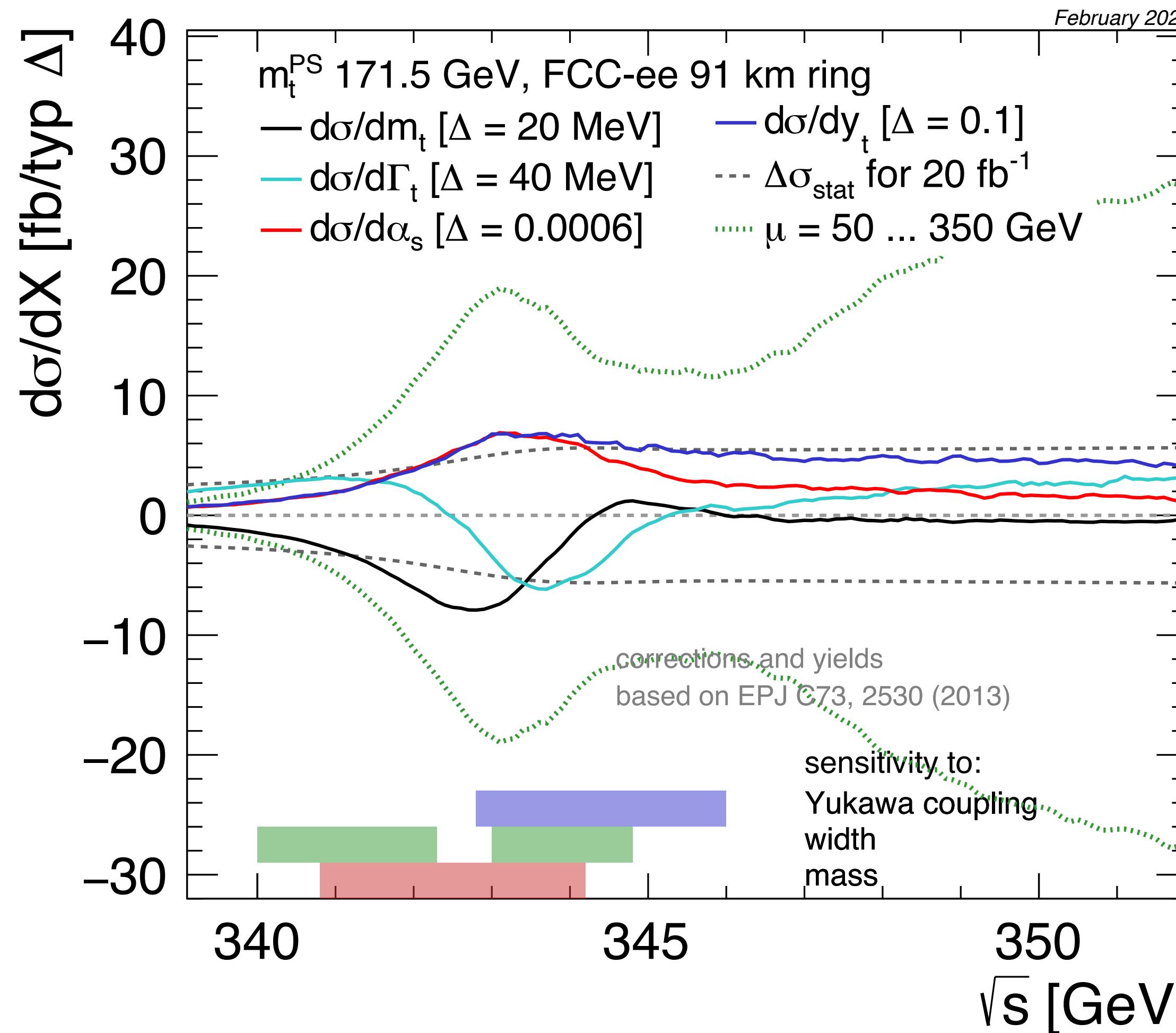
For FCC-ee

error source	$\Delta m_t^{\text{PS}} [\text{MeV}]$
stat. error (200 fb <sup>-1</sup> )	13
theory (NNNLO scale variations, PS scheme)	40
parametric ( $\alpha_s$ , current WA: $9 \times 10^{-4}$ )	26
non-resonant contributions (such as single top)	< 40
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	< 10
beam energy uncertainty	< 17
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 - 50
total (stat. + syst.)	40 – 75

9 (compressed scan)  
40 - 45, depending on scan range  
3.2 with ultimate  $\alpha_s$  ( $1.2 \times 10^{-4}$ )  
< 40 (no new evaluation)  
10 - 20 (no new evaluation, ~ % level on selection)  
negligible  
3 (for 5 MeV energy uncertainty)

# Uncertainties - Parametric

A few more details



Correlation of mass with  $\alpha_s, y_t$

Uncertainty scales with input precision:

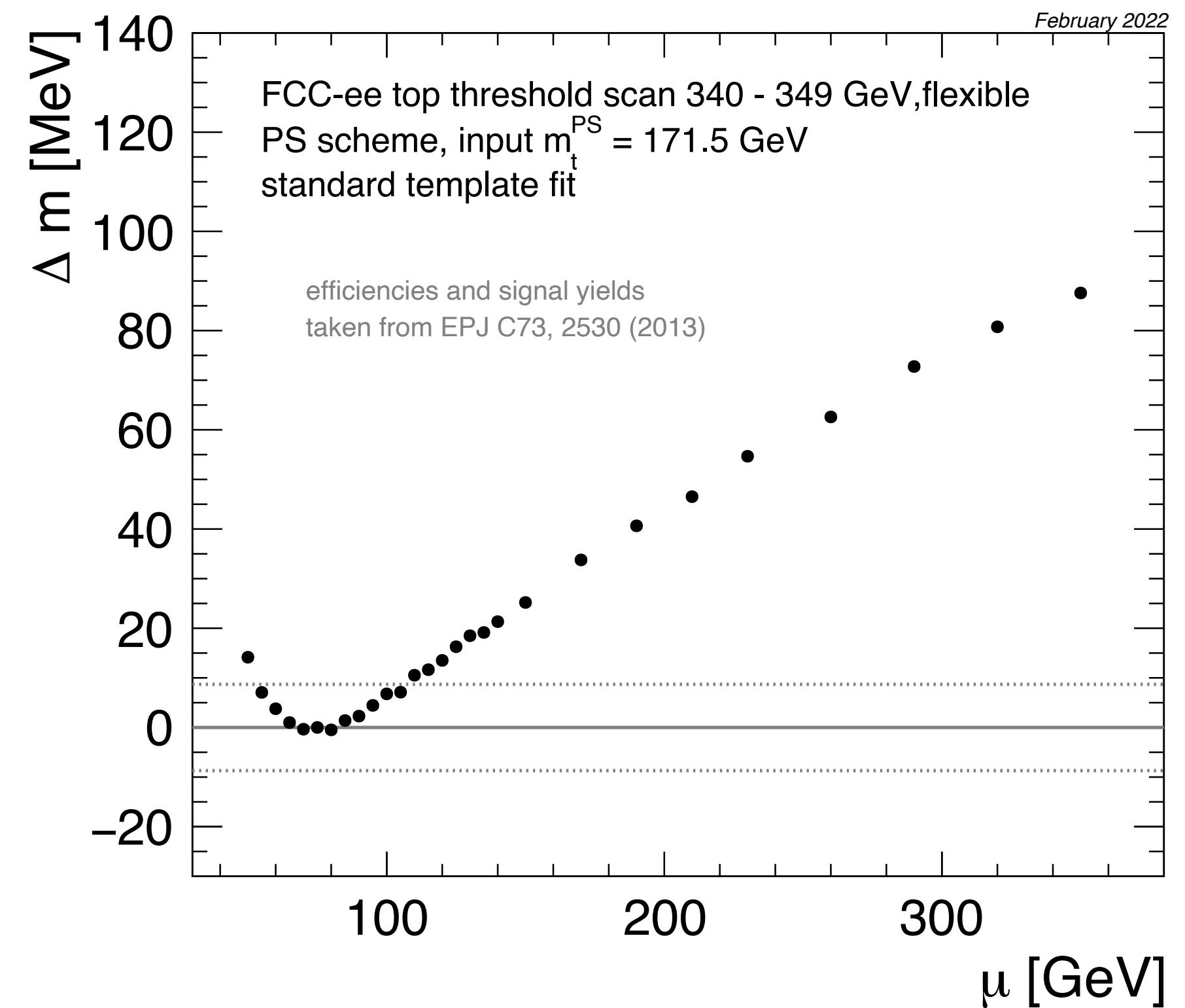
$\Delta m \sim 2.6 \text{ MeV per } 10^{-4} \text{ in } \alpha_s$

$\Delta m \sim 1.6 \text{ MeV per } 1\% \text{ in } y_t: \sim 5 \text{ MeV for } 3.4\%$   
from HL-LHC

# Uncertainties - Scale

A few more details

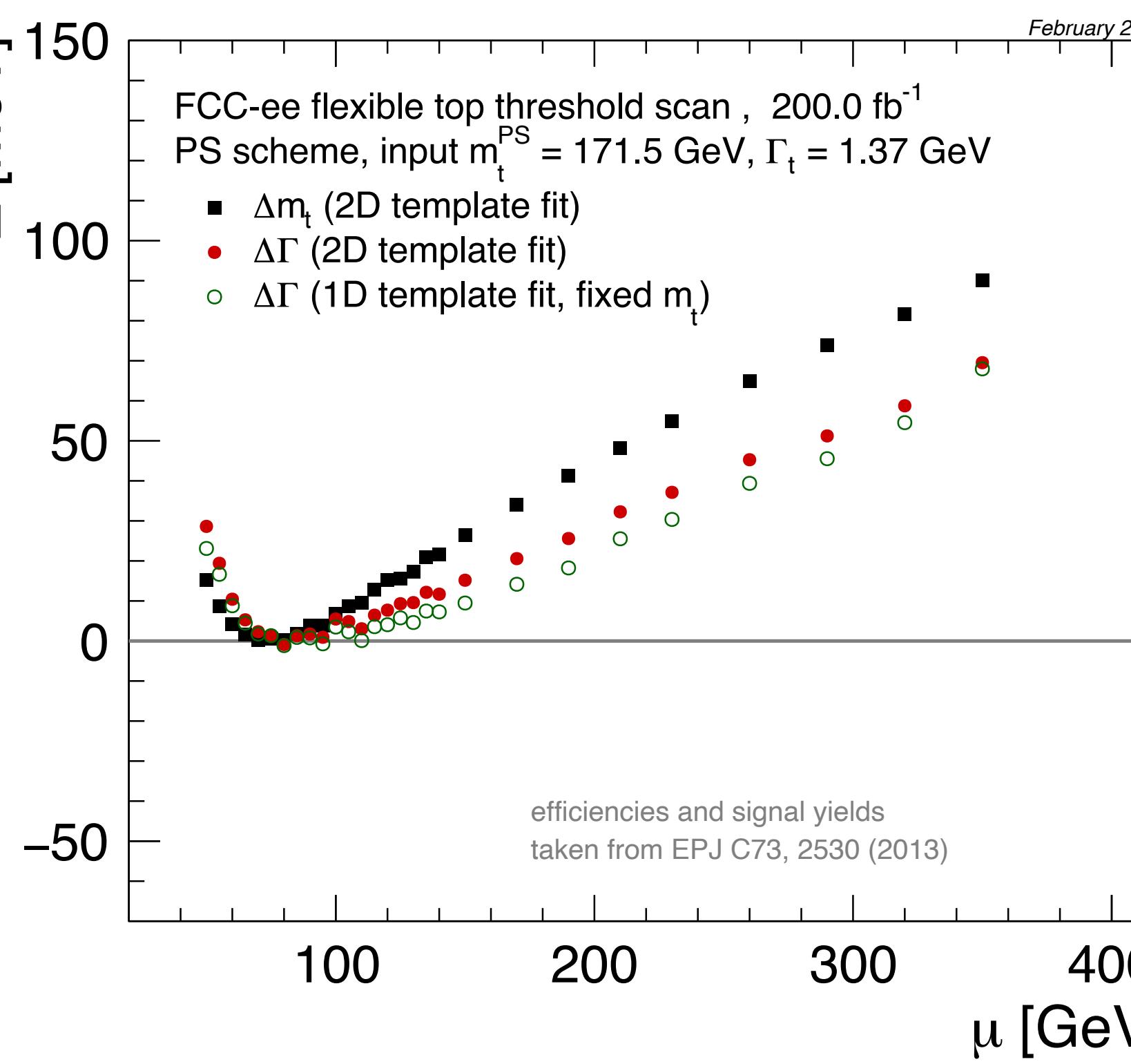
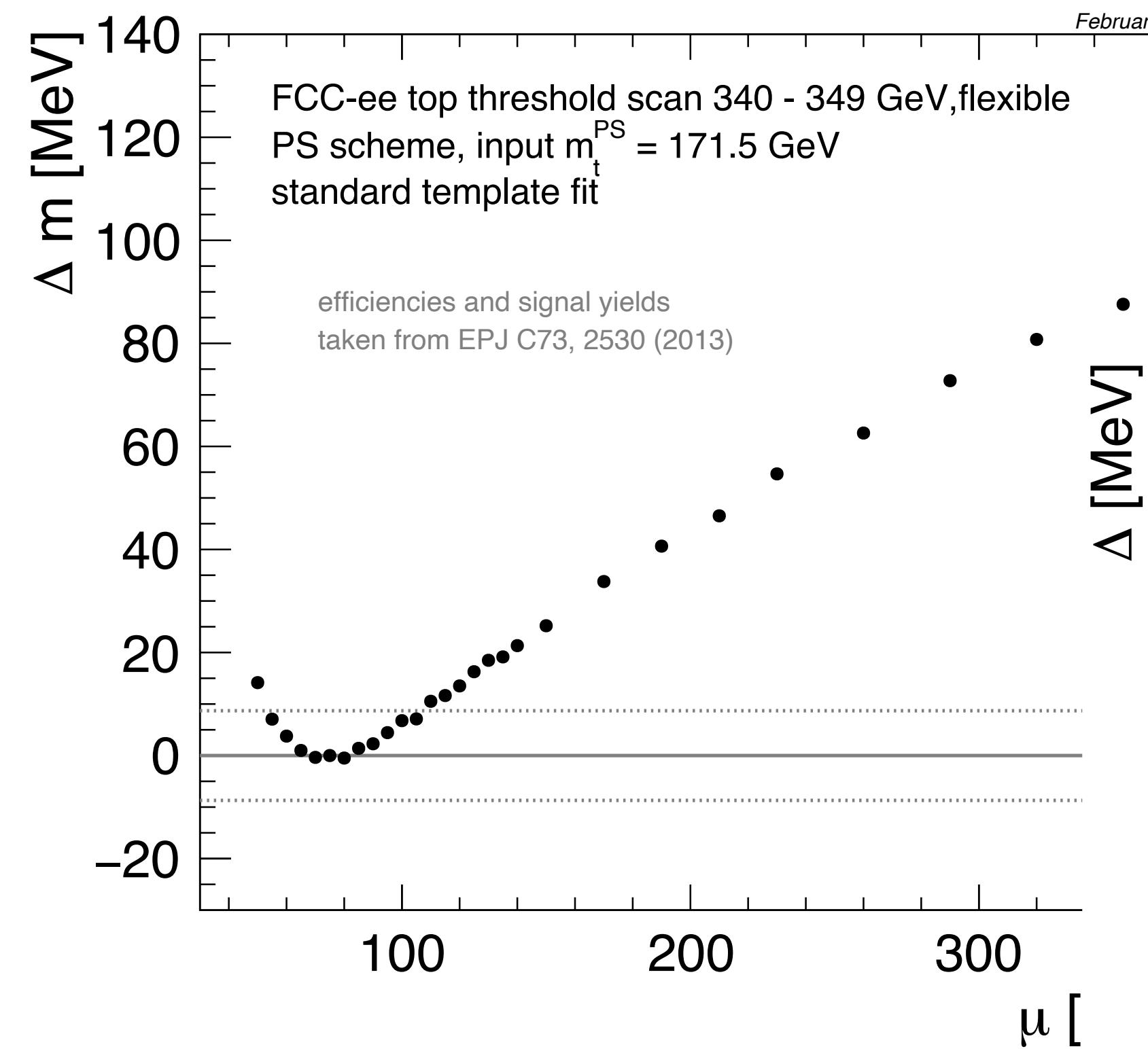
- Impact of QCD scale uncertainties on mass, width, Yukawa extraction



# Uncertainties - Scale

A few more details

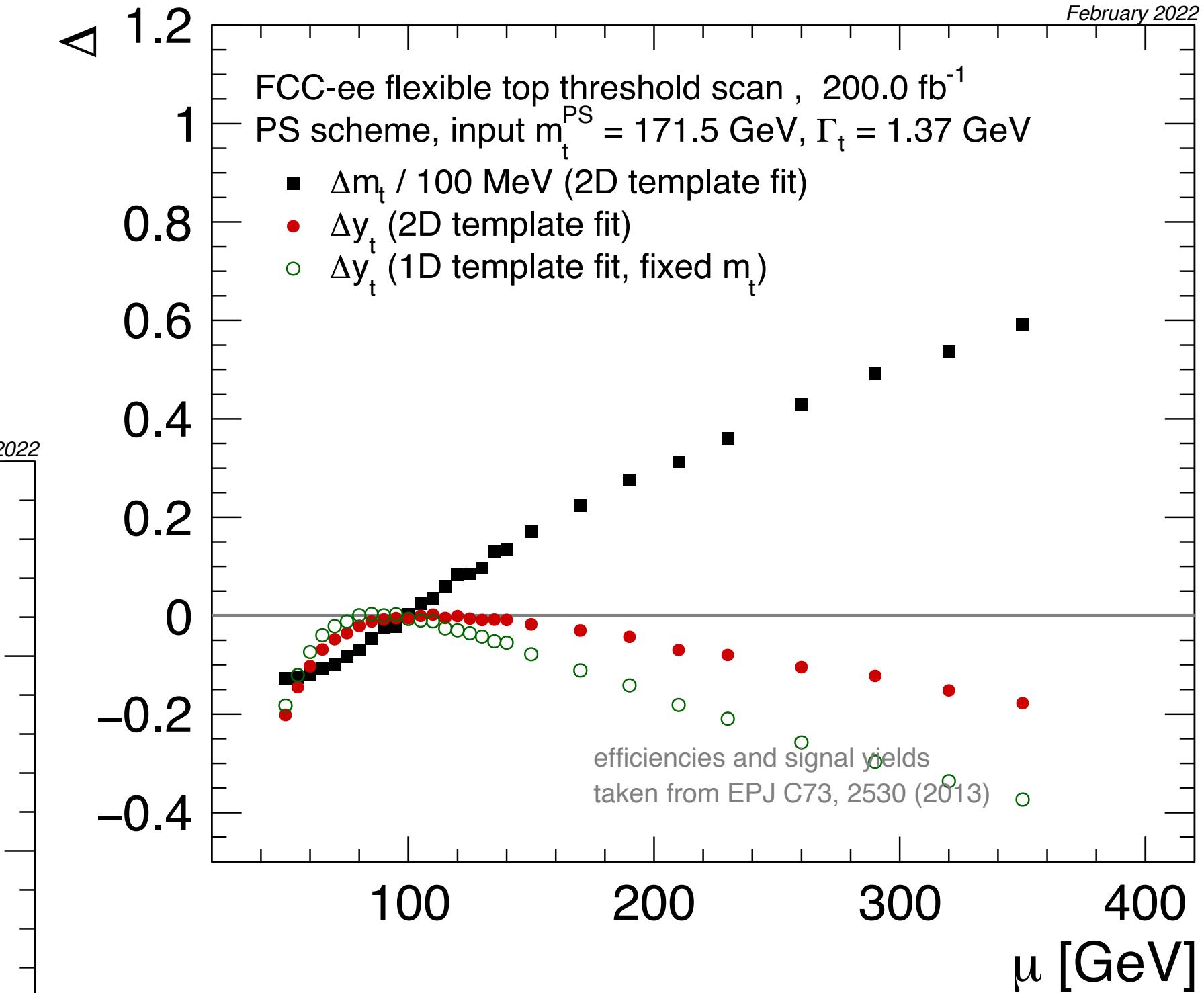
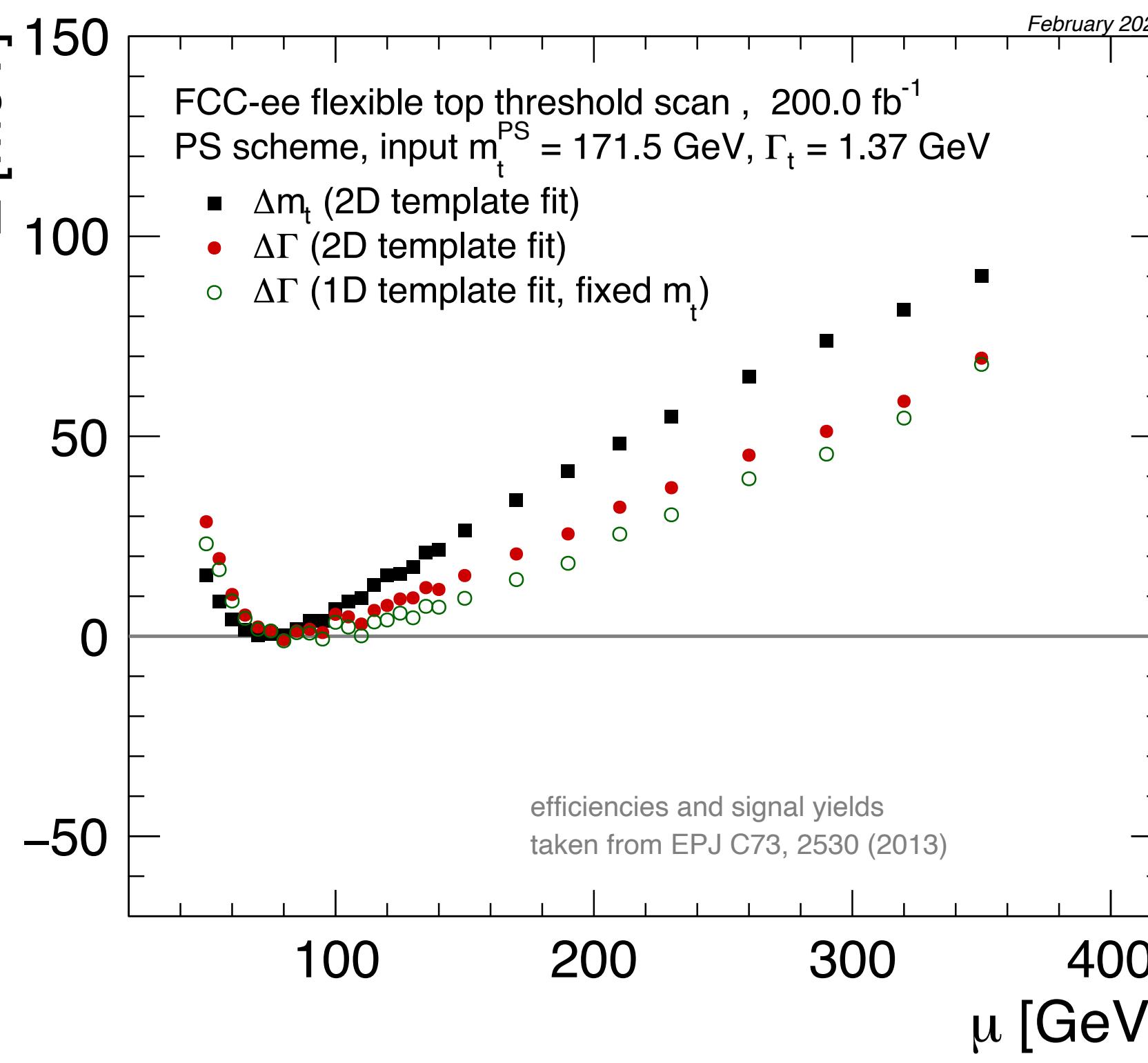
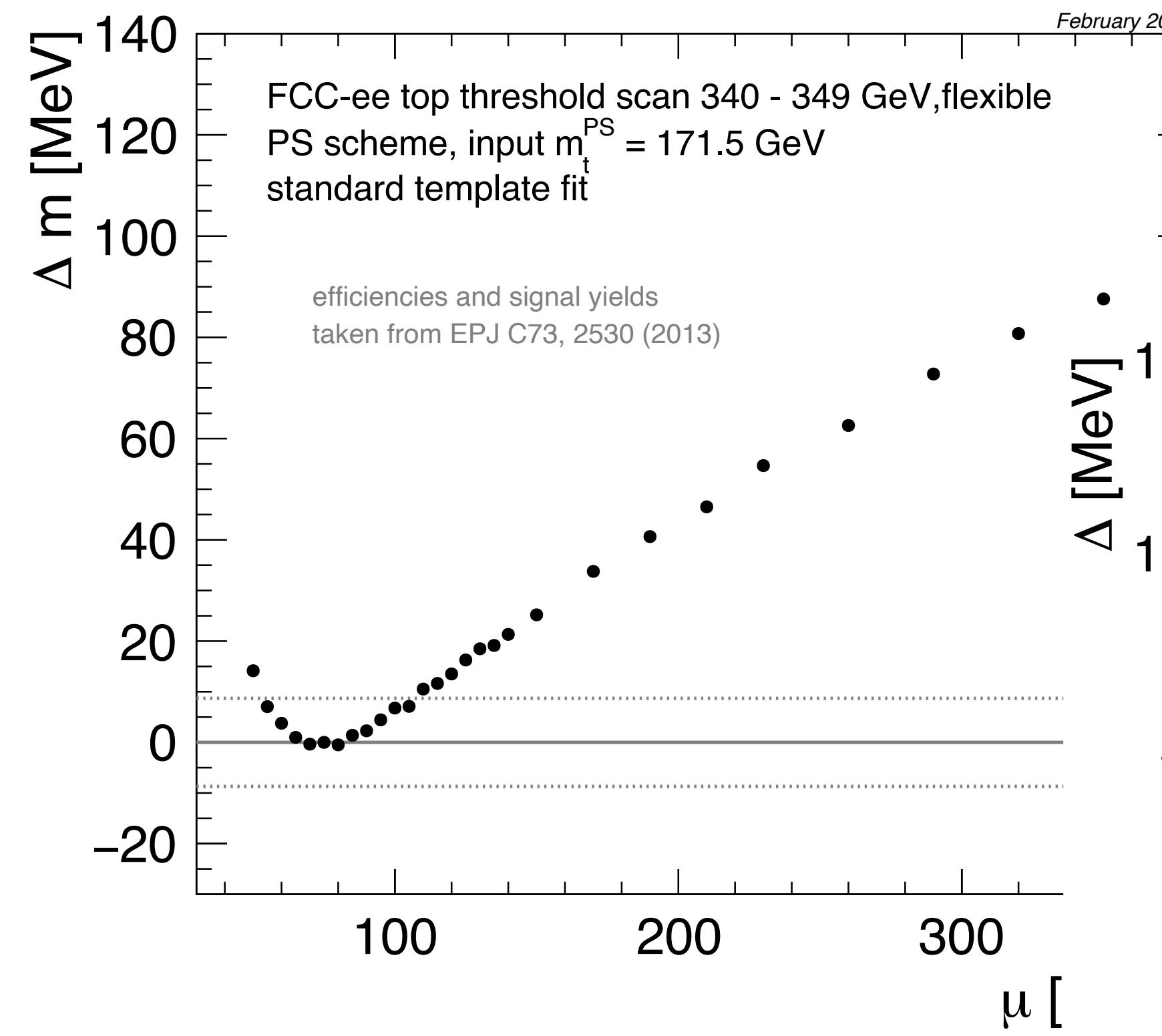
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# Uncertainties - Scale

A few more details

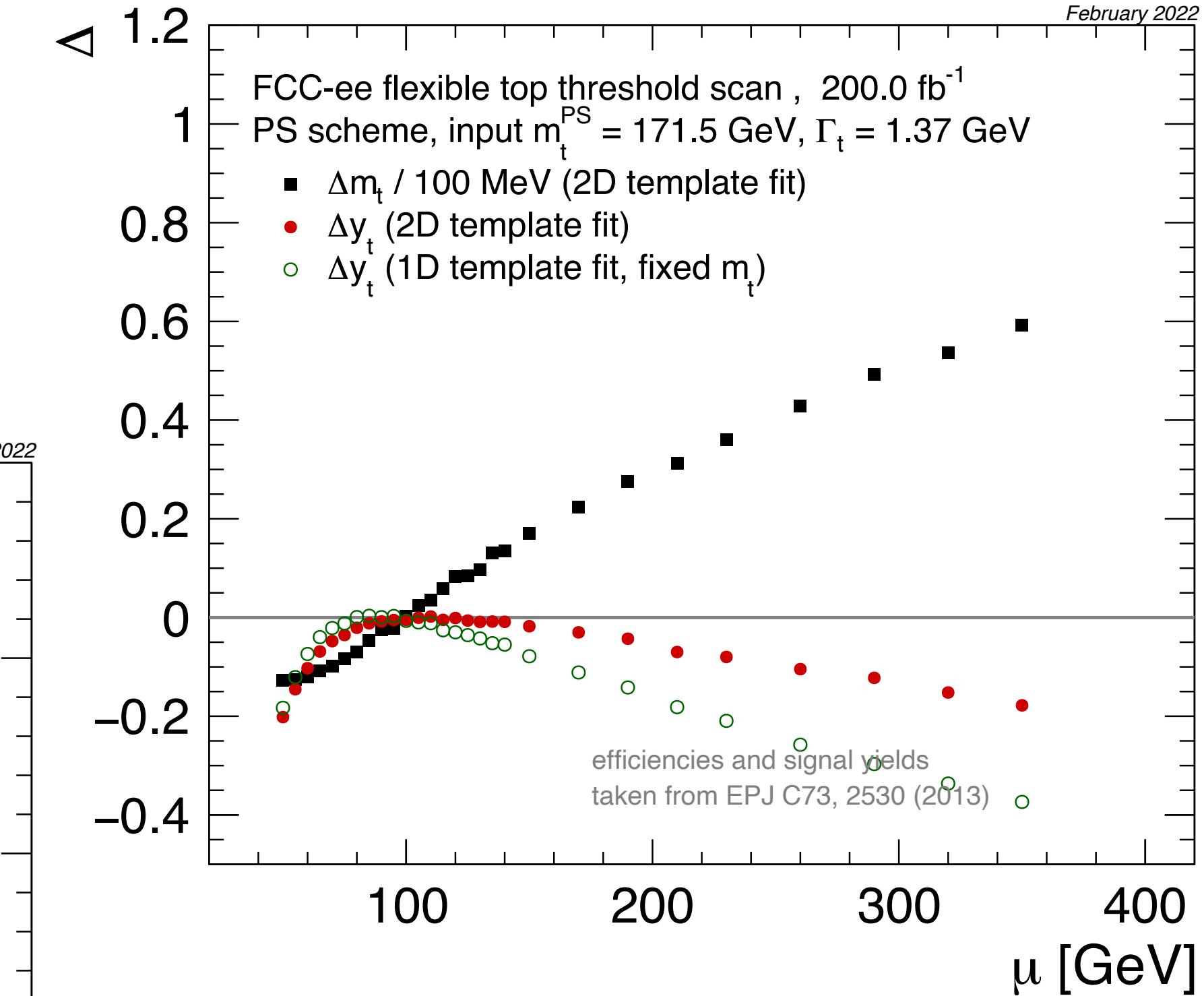
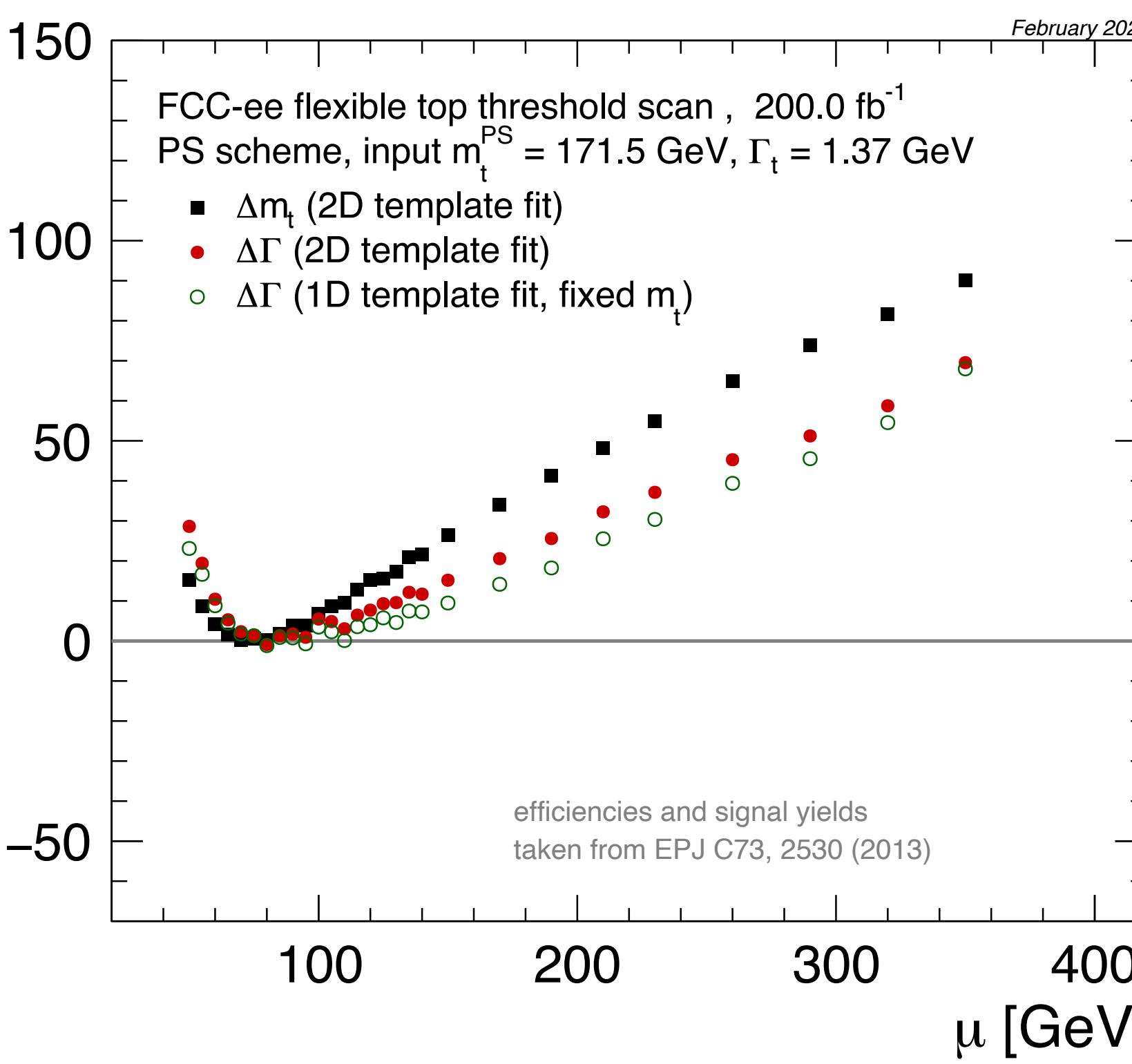
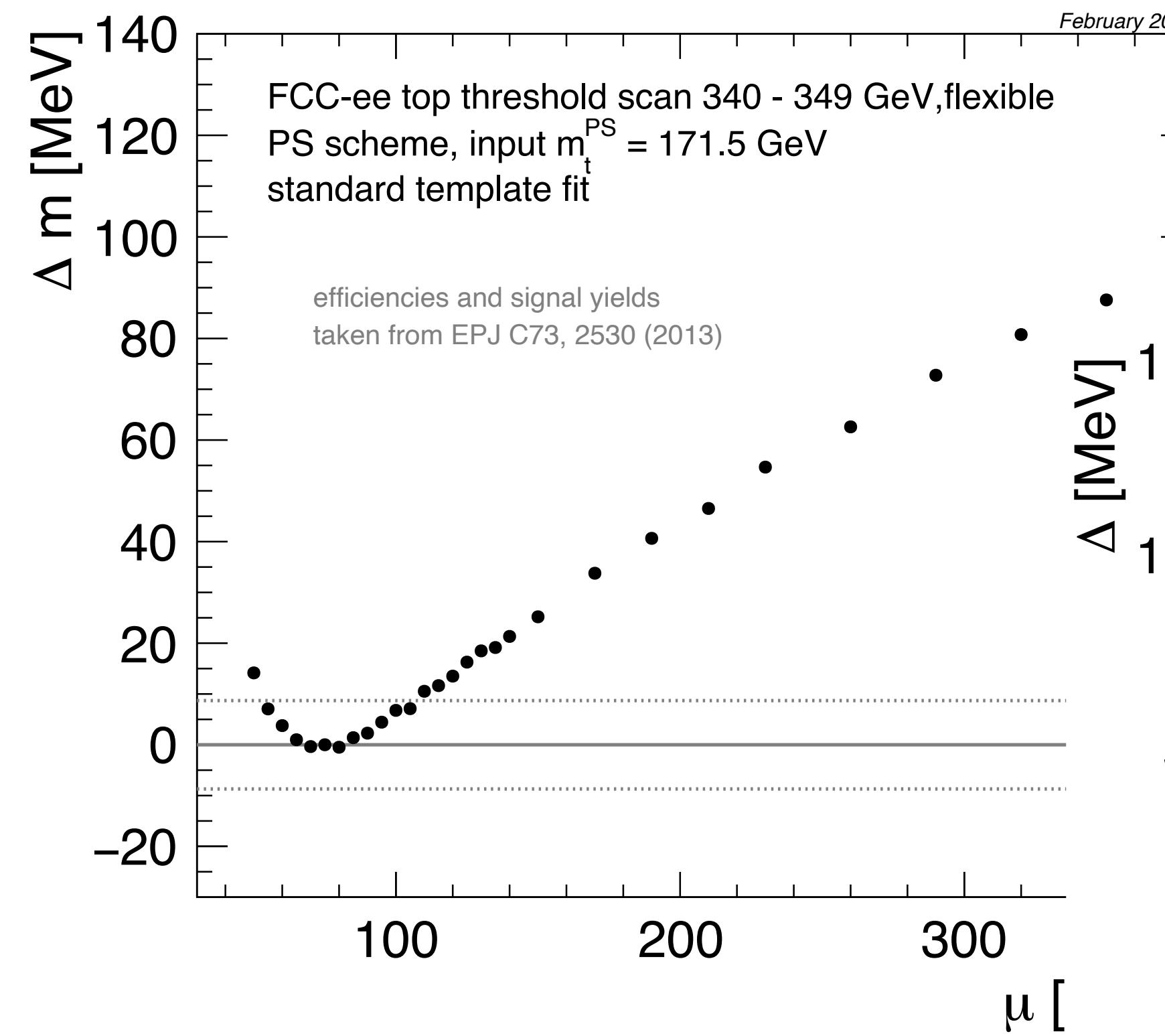
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# Uncertainties - Scale

A few more details

- Impact of QCD scale uncertainties on mass, width, Yukawa extraction



The leading systematic:  
Improvements directly propagate  
to total precision

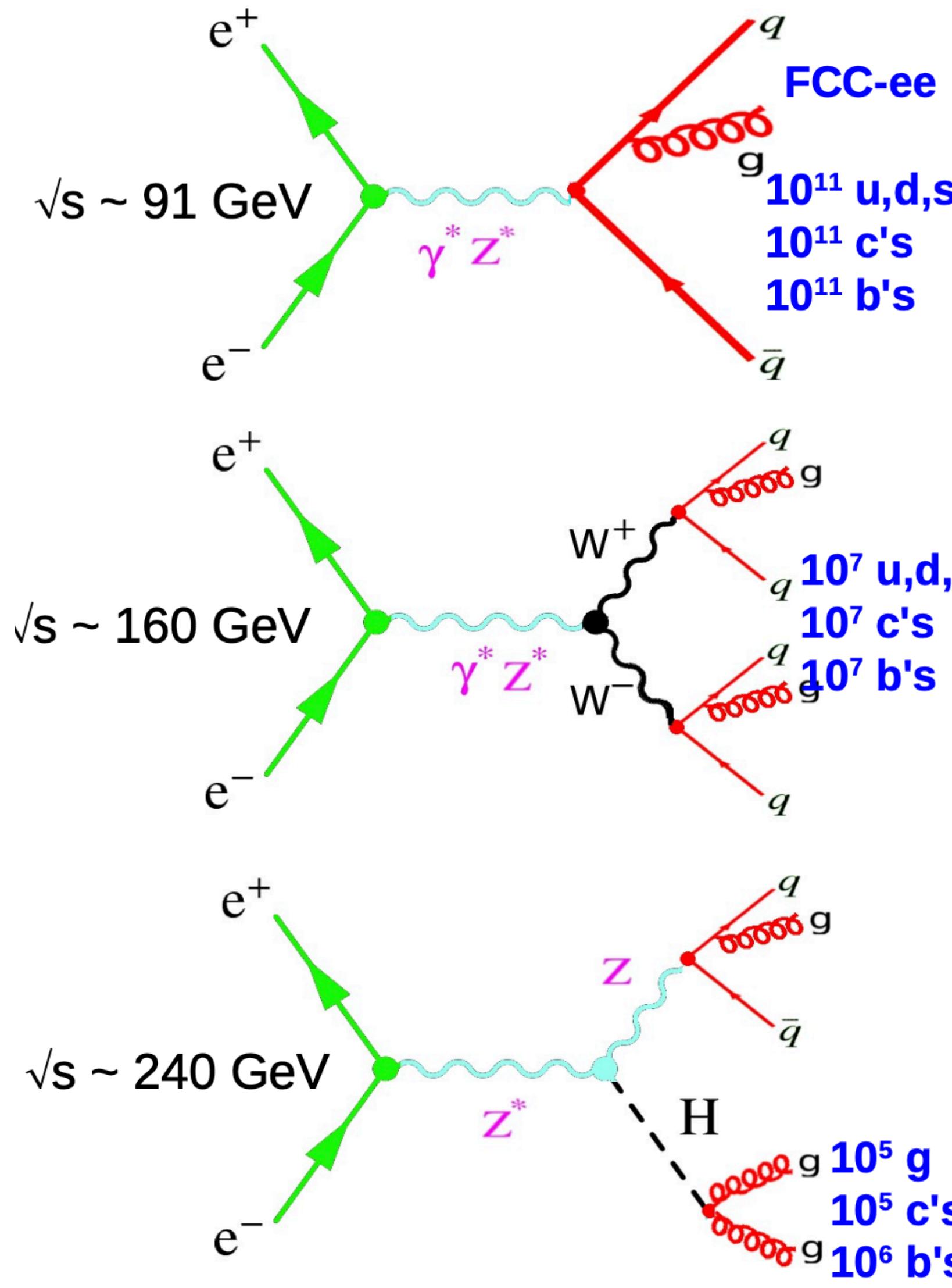
# Part II: QCD

*Based on studies in the context of FCC-ee*

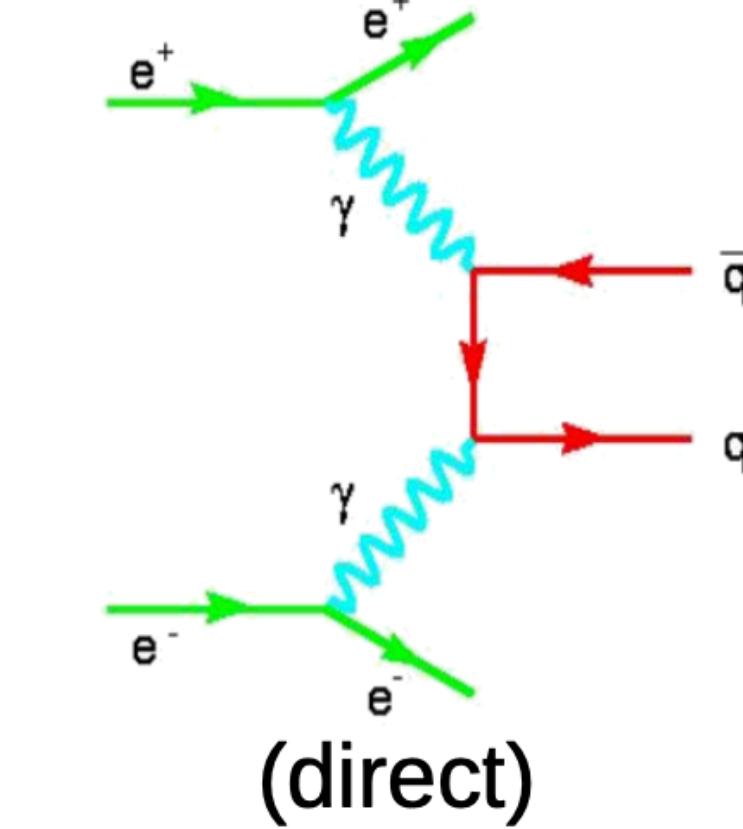
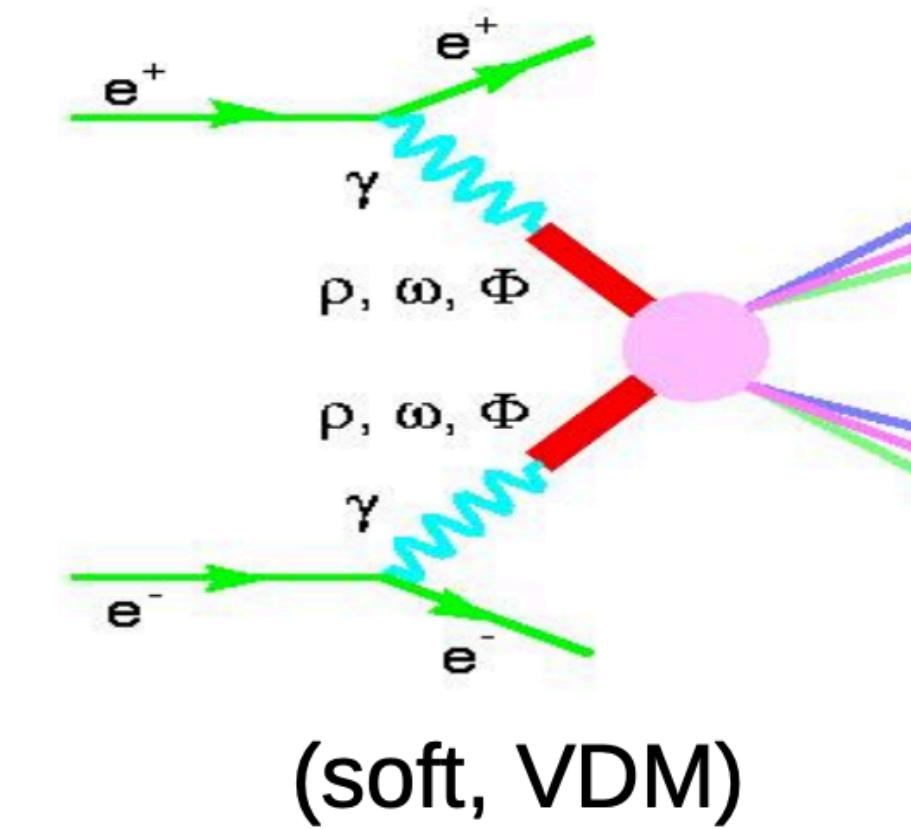
Material taken from Francesco Giuli , LFC 2022

# QCD Precision Measurement at $e^+e^-$ Colliders

With a Circular Collider Perspective

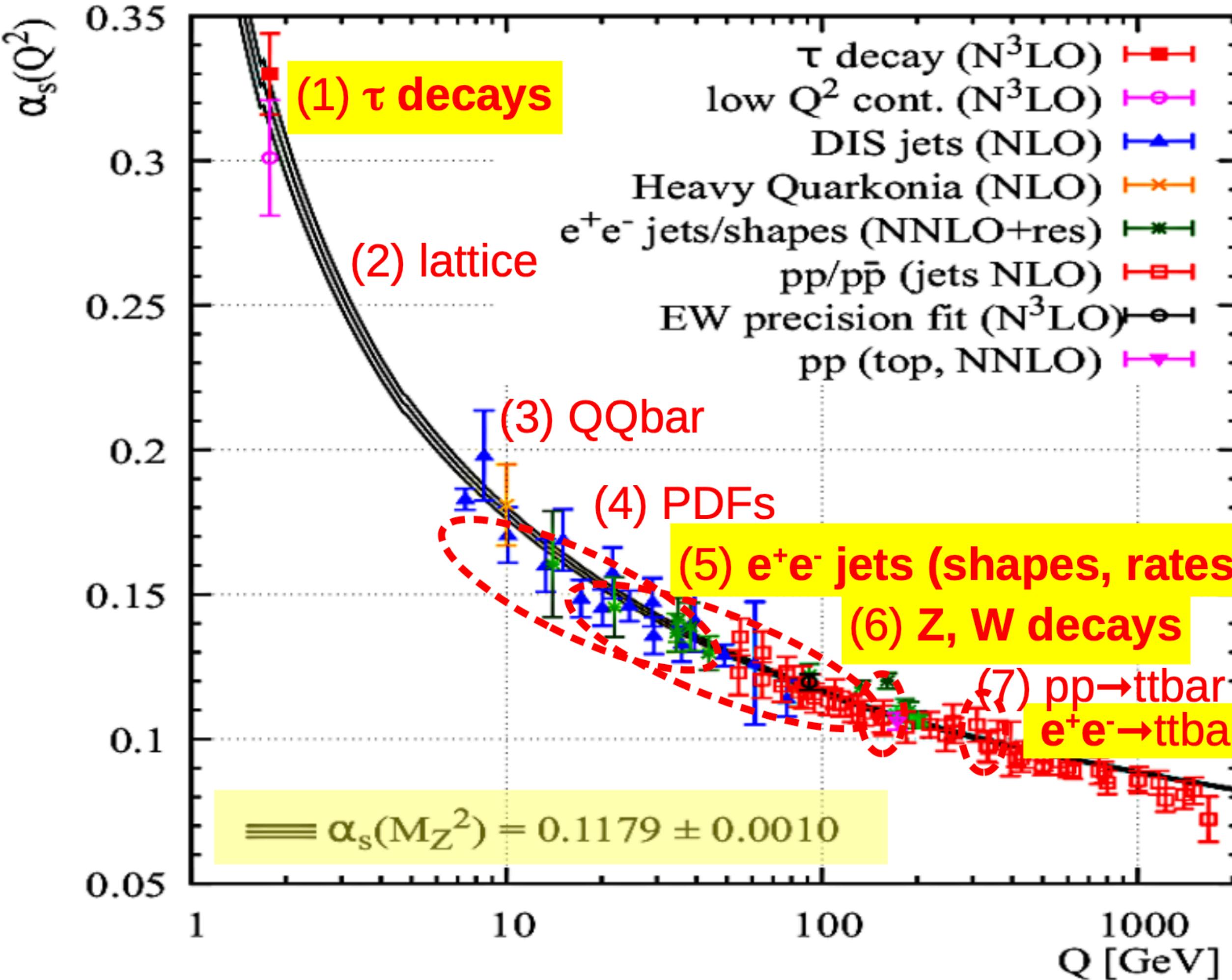


- Clean environment in  $e^+e^-$  collisions, fully controlled initial state
- High statistics QCD samples at all energy stages
- In addition: access to QCD (and other) processes in  $\gamma\gamma$  collisions



# The Strong Coupling Constant

Precise  $a_s$  measurements

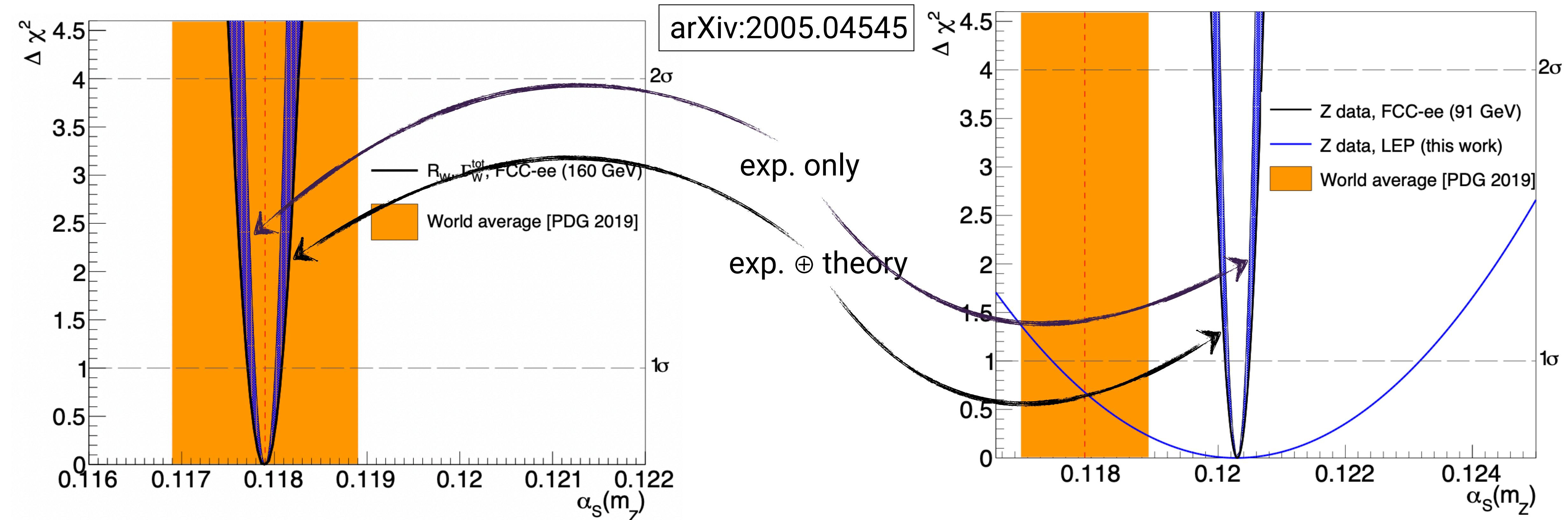


current WA: ~ 0.8% uncertainty

- $e^+e^-$  data contributes in several ways. Significant potential for improvement in particular for:
    - $\tau$  decays: extreme statistics ( $> 10^{11} \tau$  pairs) expect < 1% uncertainty
    - event shapes at lower energies, jet rates at high  $\sqrt{s}$ ; expect < 1% uncertainty
    - hadronic decays of gauge bosons: Combined fits, expect  $\sim 0.1\%$  for  $Z$ ,  $0.2\%$  for  $W$  experimental uncertainty.
- To fully profit, requires reduction of theory uncertainty by computing missing terms (x10 on  $W$ , x4 on  $Z$ )

# The Strong Coupling Constant

Precise  $a_s$  measurements from gauge boson decays

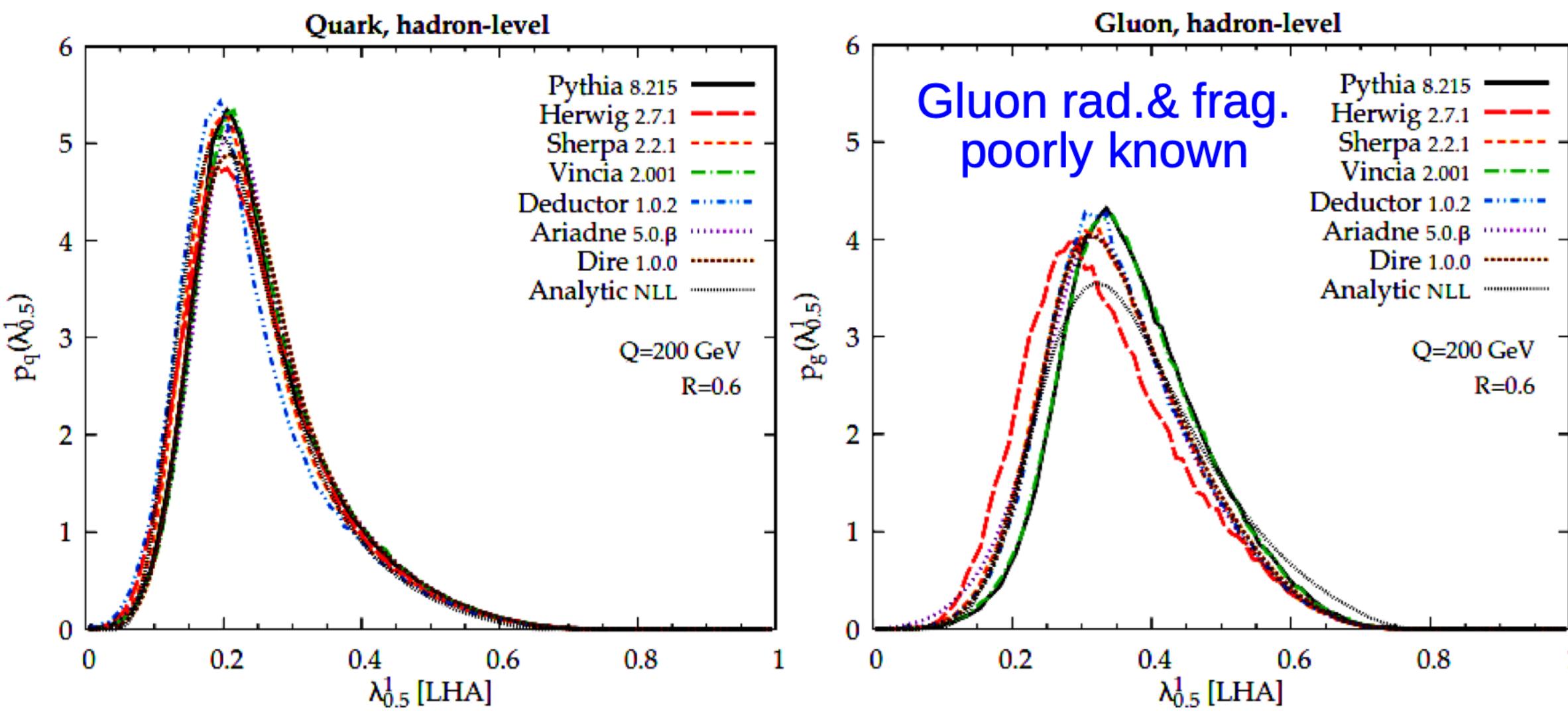


- The highest accuracy: Z (and W) decays:  
~0.2% with theoretical uncertainties, potential to 0.1% ( $1.2 \times 10^{-4}$ ) with improved theory  
(NB: similar potential from lattice)

# Gluon Jets, Colour Reconnection, ...

Examples for non-perturbative effects

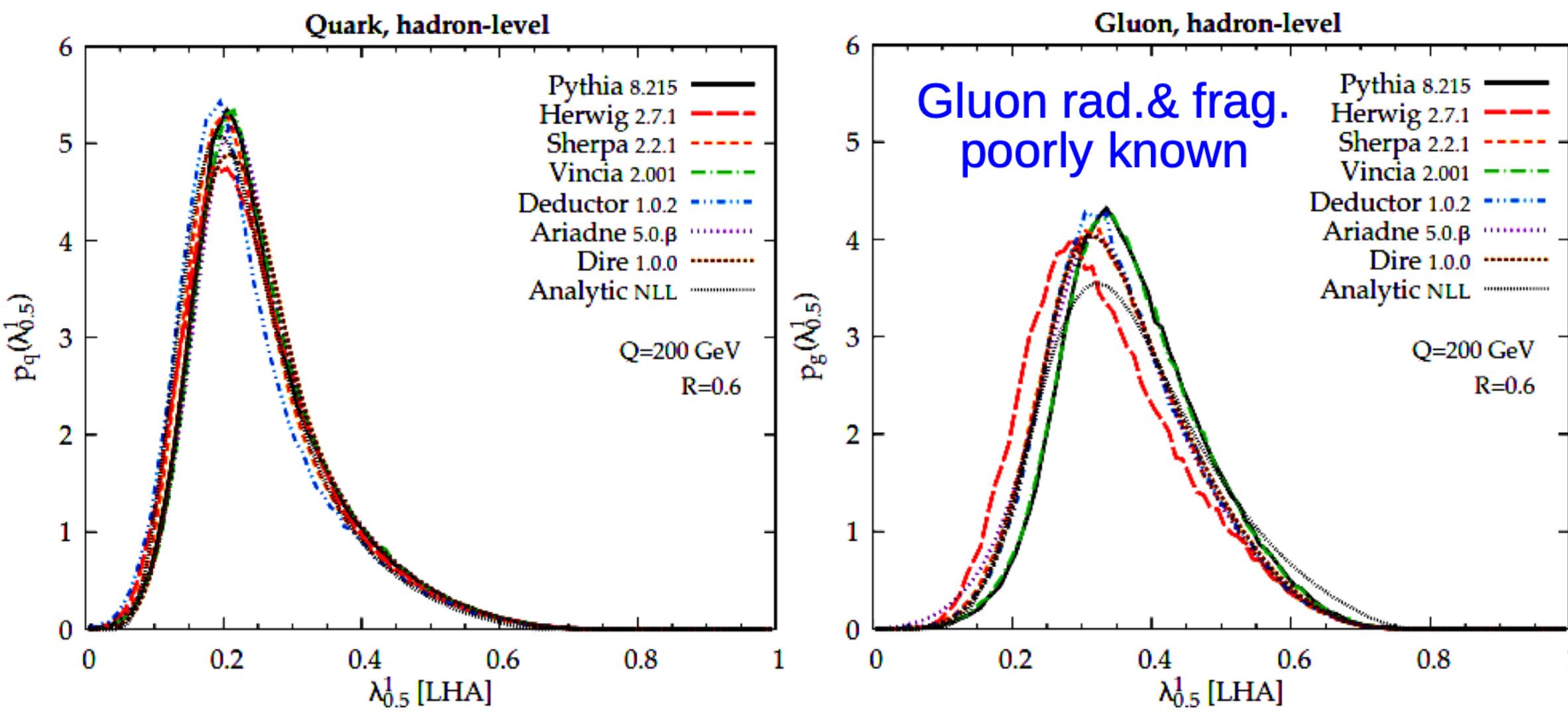
- Understanding gluon jets: Currently sizable uncertainties



# Gluon Jets, Colour Reconnection, ...

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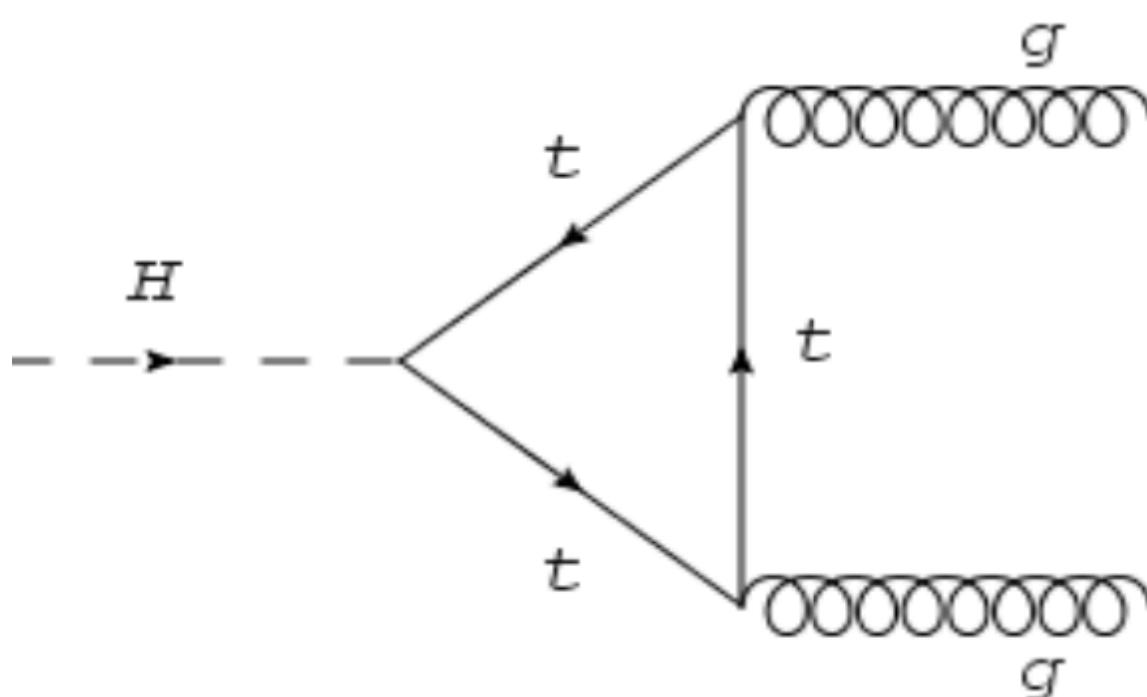
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$e^+e^- \rightarrow Z \rightarrow uu$

$e^+e^- \rightarrow H \rightarrow gg$

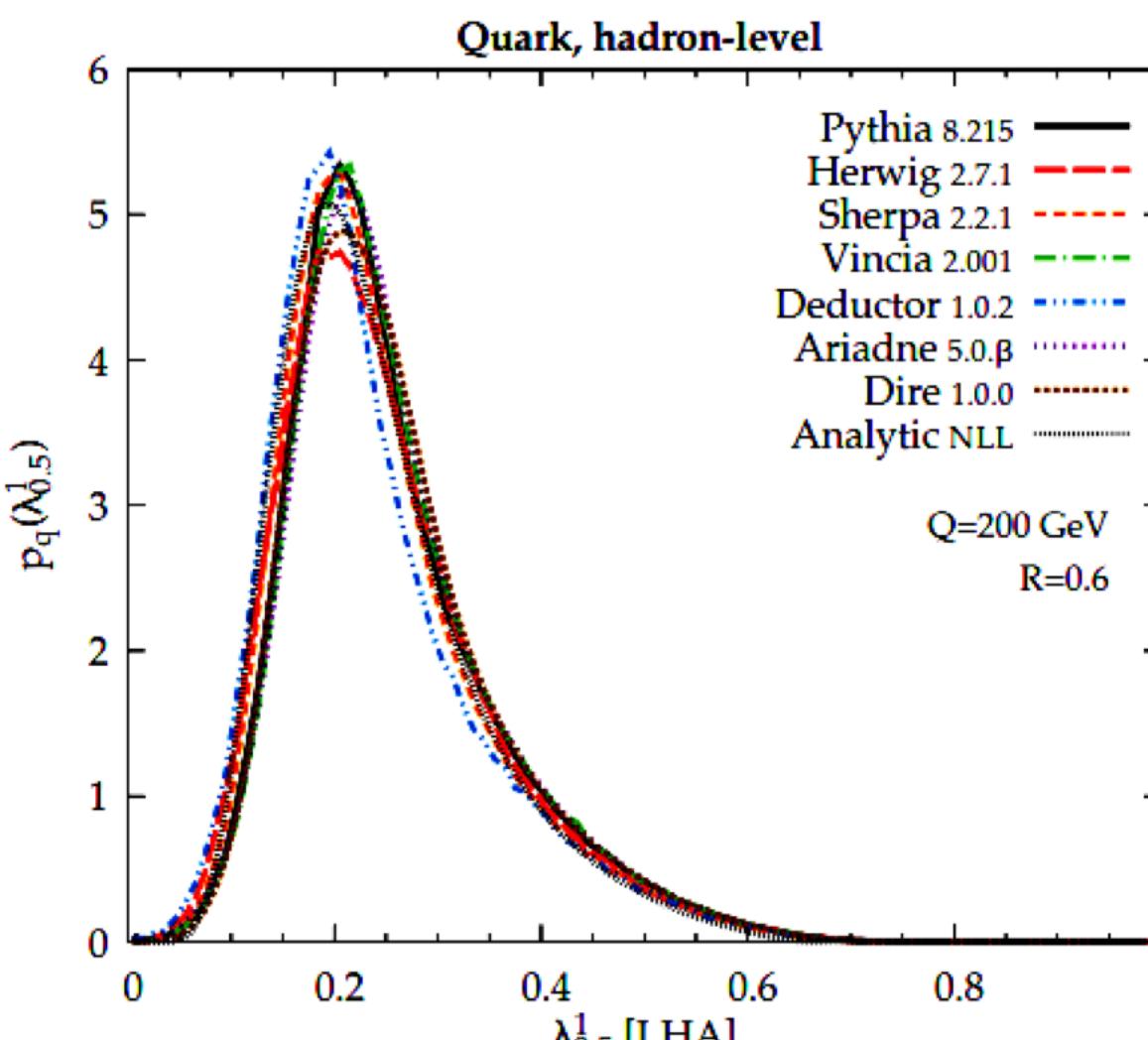
- A clean sample of gluon jets to study gluon jet properties with high precision:  $ZH$ ,  $H \rightarrow gg$



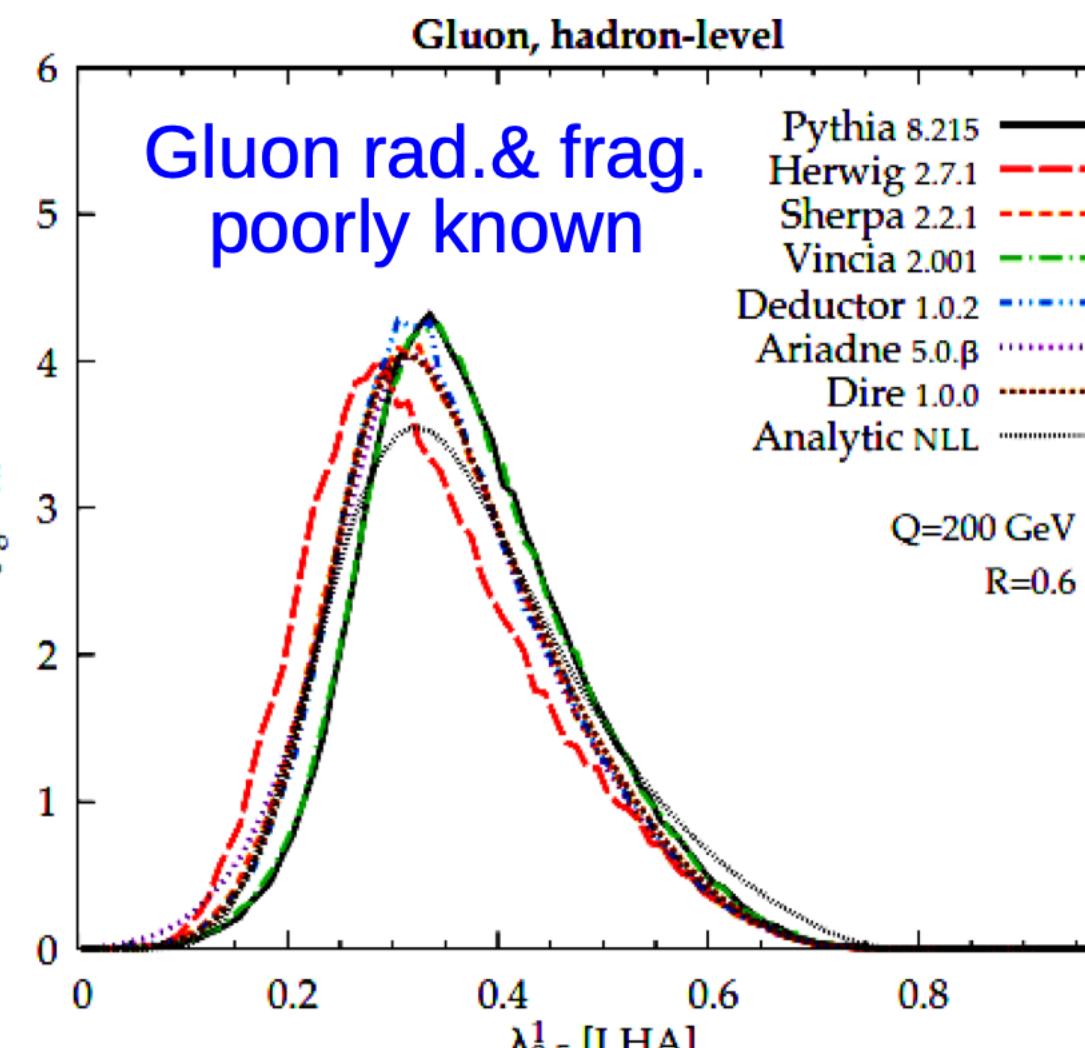
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Examples for non-perturbative effects

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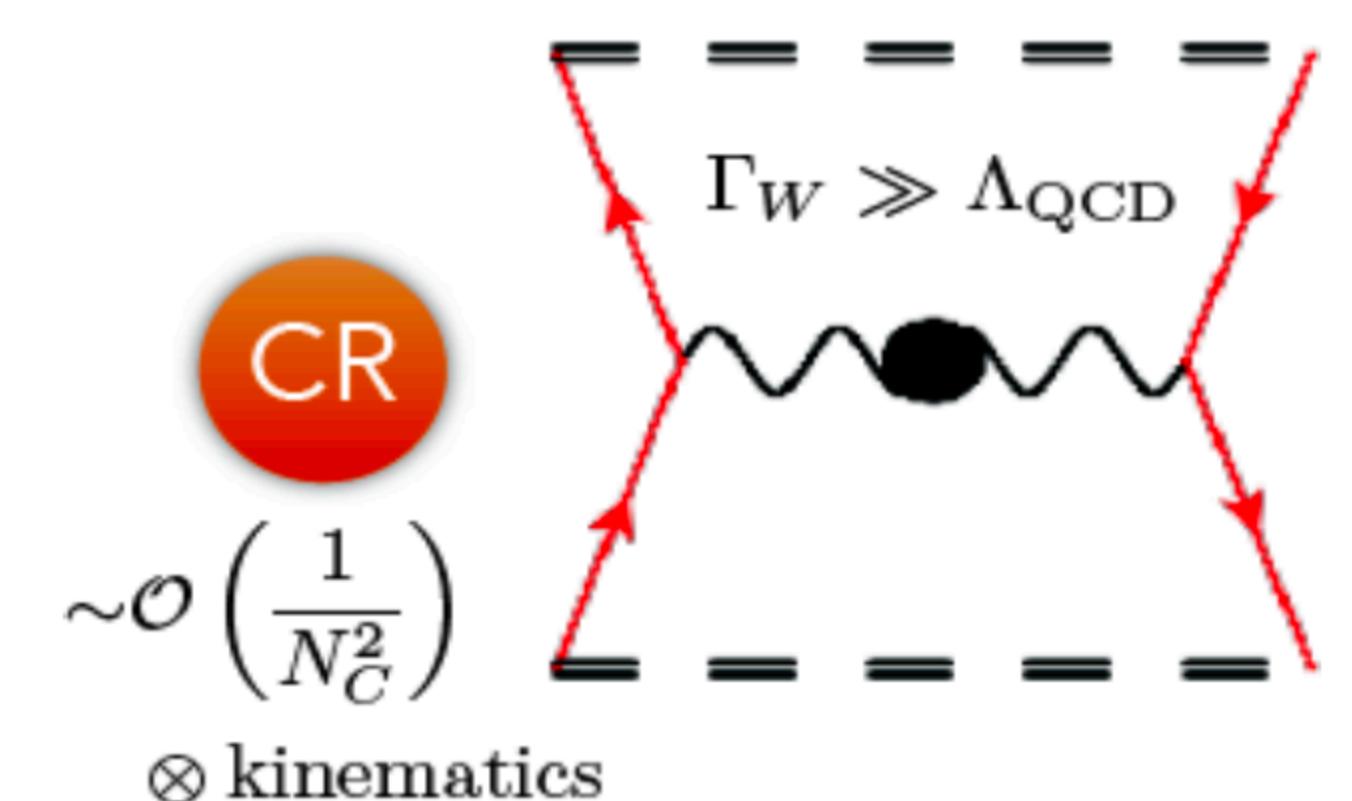
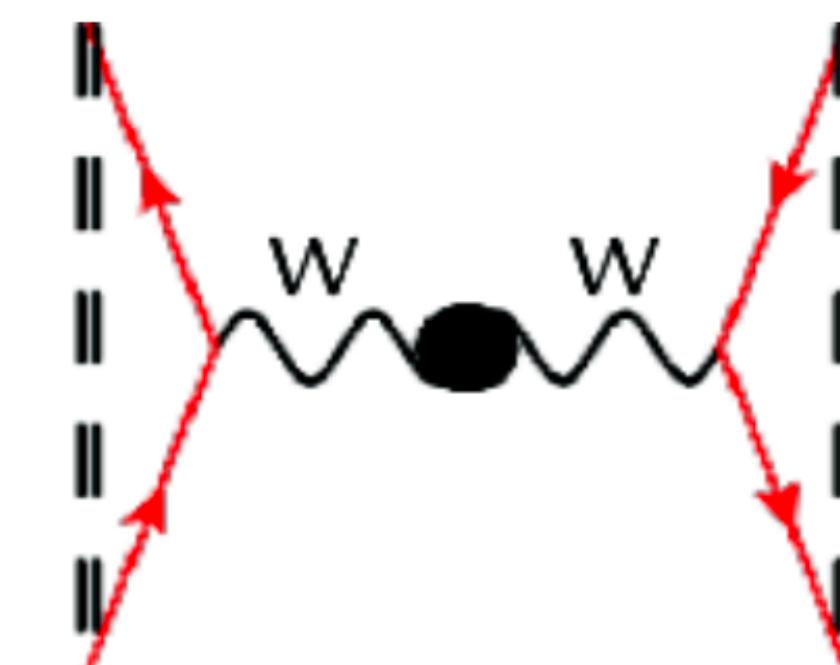
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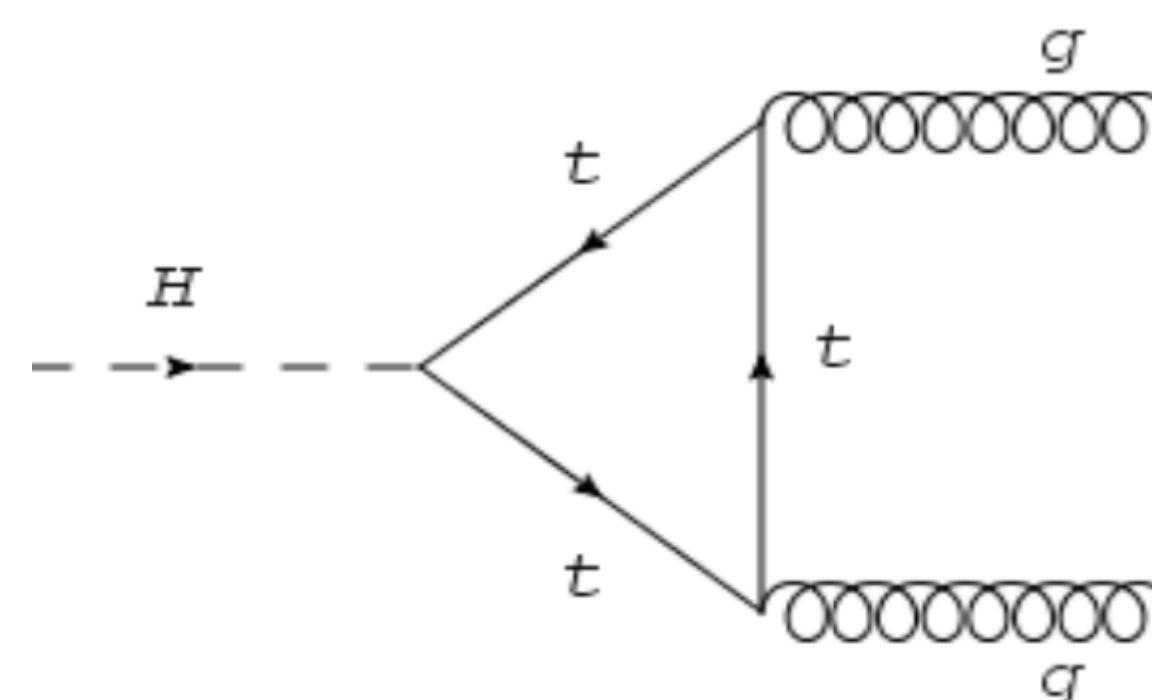
$e^+e^- \rightarrow H \rightarrow gg$

LC  
 $\mathcal{O}(1)$

"leading colour"



- A clean sample of gluon jets to study gluon jet properties with high precision: ZH, H->gg



- Exploit the large W sample to constrain CR experimentally: Compare W mass measured in leptonic and hadronic final states.

# Bottom Line

- $e^+e^-$  Colliders from 91 to 350 GeV are precision tools for top quark physics and QCD measurements  
Ultimate precision on top quark mass and other properties, on the strong coupling constant [and much more!]
- A challenge for theory: Understanding parameters on a level comparable to expected experimental precision. Theory is a / the leading systematic for many measurements - for the top quark mass it is the leading uncertainty overall  
⇒ Advances in theory directly translate into improvements of overall precision.