

ErrorFlow: Jet Error Estimation for Kinematic Fitting in Particle Flow Detectors at Future Higgs Factories

First ECFA Workshop on e^+e^- Higgs/EW/Top Factories

Jenny List ¹, Yasser Radkhorrani^{1,2}

¹DESY, Hamburg

²Universität Hamburg, Hamburg

October 06, 2022



CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

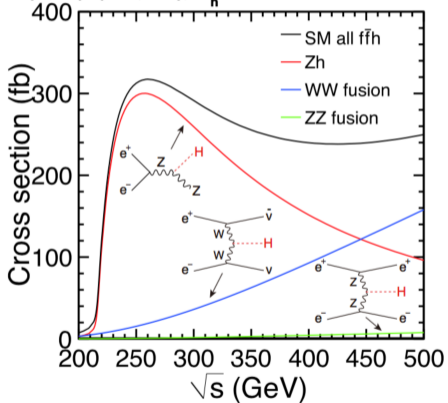
HELMHOLTZ
RESEARCH FOR GRAND CHALLENGES



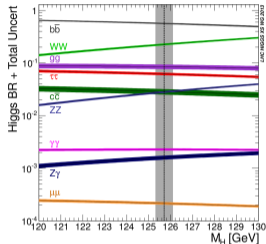
Higgs production mechanisms and decay modes at e^+e^- colliders

- ▶ Higgs strahlung is dominant Higgs production mechanism at 250 GeV

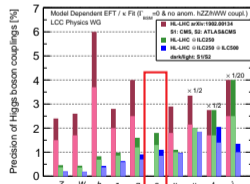
$P(e^-, e^+) = (-0.8, 0.3)$, $M_h = 125$ GeV



- ▶ Most frequent Higgs decay mode: $H \rightarrow b\bar{b}$



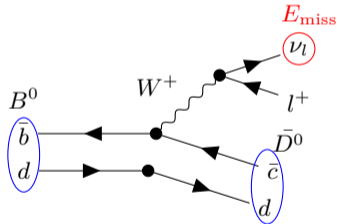
- ▶ Extremely challenging in Hadron colliders: $H \rightarrow c\bar{c}$



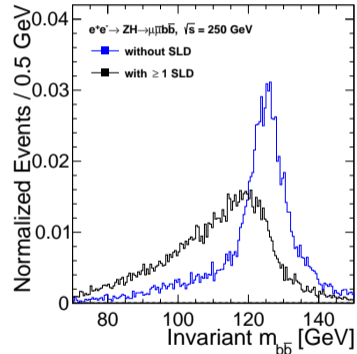
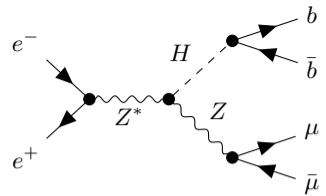
Semi-leptonic b / c decays

- ▶ Number of B-/C-hadron semi-leptonic decays (SLD) in $e^+e^- \rightarrow ZH \rightarrow \mu\bar{\mu}b\bar{b}$ events

		nBSLD		
		0	1	2
nCSDL	0	34%	24%	4%
	1	18%	12%	2%
	2	3%	2%	0%



- ▶ Mis-reconstruction of $b\bar{b}$ invariant mass due to **missing neutrino energy** from semi-leptonic decays
- ▶ Can the **missing momentum** be retrieved from event and decay kinematics in a highly granular detector?



Concept of ν -correction in a semi-leptonic decay

- ▶ Find heavy-quark jets: Identify b or c jet \rightarrow flavour tag
- ▶ Find semi-leptonic decay(s): Identify lepton in jet if present \rightarrow possible using detector's high granularity

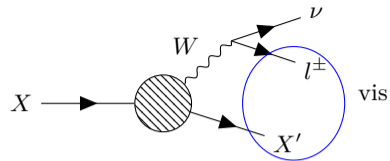
See talk by L. Reichenbach

- ▶ Estimate neutrino energy from decay kinematics:
 - ▶ Assign B^0 or D^0 meson mass to mother hadron.
 - ▶ Reconstruct flight direction of mother hadron from position of primary and secondary vertex.
 - ▶ Calculate neutrino momentum: up to 2-fold ambiguity.
- ▶ As proof-of-principle: CHEAT from MC truth

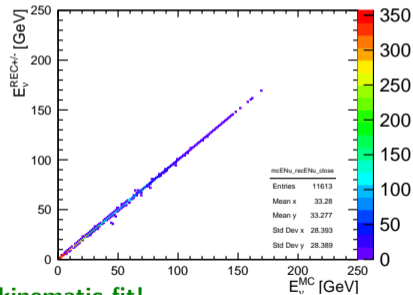
arXiv:2105.08480

The neutrino momentum can be determined up to a two-fold ambiguity

Can we use overall event kinematics to decide between solutions? \Rightarrow kinematic fit!

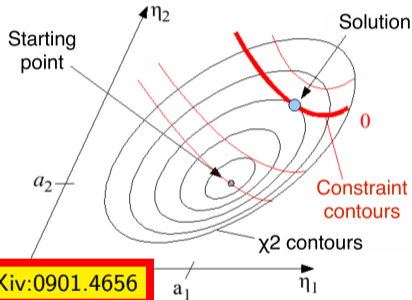


- ▶ Closure test: fully cheated information ($e^+e^- \rightarrow b\bar{b}$ at $\sqrt{s} = 500$ GeV)



Kinematic fit

- ▶ Kinematic fit: adjustment of measured quantities under certain kinematic constraints:
 - ▶ Energy and momentum conservation
 - ▶ Invariant masses of particles



- ▶ Minimize χ^2 :

$$\chi^2(\mathbf{a}, \boldsymbol{\xi}, \mathbf{f}) = (\boldsymbol{\eta} - \mathbf{a})^T \mathbf{V}^{-1} (\boldsymbol{\eta} - \mathbf{a}) - 2\boldsymbol{\lambda}^T \mathbf{f}(\mathbf{a}, \boldsymbol{\xi})$$

$\boldsymbol{\eta}$: vector of measured kinematic variables (x)

\mathbf{a} : vector of fitted quantities

$\boldsymbol{\xi}$: vector of unmeasured kinematic variables

\mathbf{V} : **covariance matrix**

$\boldsymbol{\lambda}$: Lagrange multipliers

$\mathbf{f}(\mathbf{a}, \boldsymbol{\xi})$: vector of constraints

Exploit well-known initial state in e^+e^- colliders

⇒ **need error parametrization, in particular for jets**

Jet specific error parameterisation, ErrorFlow

Parametrize sources of uncertainties (assumed uncorrelated) in jet parameters measurements (ErrorFlow):

$$\sigma_{\text{jet}} = \sigma_{\text{Det}} \oplus \sigma_{\text{Conf}} \oplus \sigma_{\nu} \oplus \sigma_{\text{Clus}} \oplus \sigma_{\text{Had}} \oplus \sigma_{\text{Overlay}}$$

- ▶ σ_{Det} : Detector resolution using track and cluster parameters
- ▶ σ_{Conf} : Particle confusion in Particle Flow Algorithm
Estimated based on jet energy and neutral hadron / photon energy fractions
- ▶ σ_{ν} : Semi-leptonic decays: error propagation from neutrino correction
- ▶ σ_{Clus} : Misassignment of particles in the jet clustering, has not been included yet
- ▶ σ_{Had} : Mismodeling of QCD effects in parton shower and hadronization, has not been included yet
- ▶ σ_{Overlay} : Uncertainties due to imperfect $\gamma\gamma \rightarrow$ low p_T hadrons, has not been included yet

DESY-THESIS-2017-045



Pandora treatment with PFOs

- ▶ Charged PFOs:
 - ▶ four-momentum: calculated using track parameters
 - ▶ uncertainties: propagated from track fit covariance matrix
- ▶ Neutral PFOs:
 - ▶ Cluster energy is assigned to PFO energy (assumed massless); $E_{\text{PFO}} = |\vec{p}_{\text{PFO}}| = E_{\text{clu}}$
 - ▶ photons: OK ($m_\gamma = 0$)
 - ▶ neutral hadrons: identified as neutron $\Rightarrow m_n$ is set for PFO \Rightarrow **inconsistent 4-momentum!**

CovMat of Neutral PFO: calculated using inconsistent 4-momentum

- ▶ energy error: from calorimeter intrinsic energy resolution
- ▶ angular uncertainties: from cluster shape (least correlation with cluster energy)

inconsistent four-momentum of neutral hadrons \Rightarrow wrong error propagation

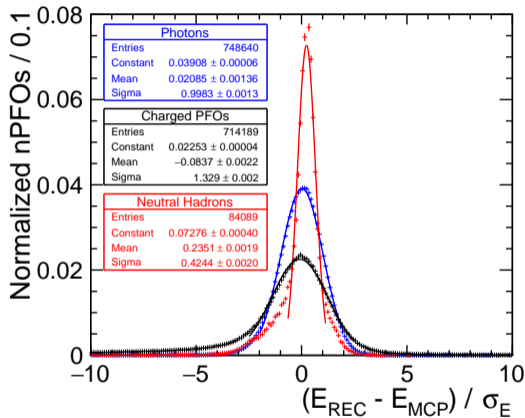


PFO-level detector resolution, σ_{Det}

Energy

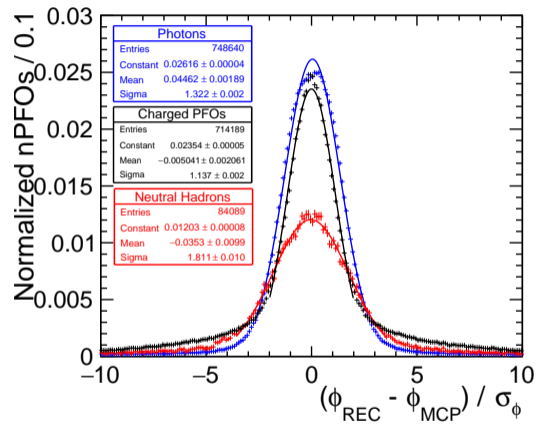
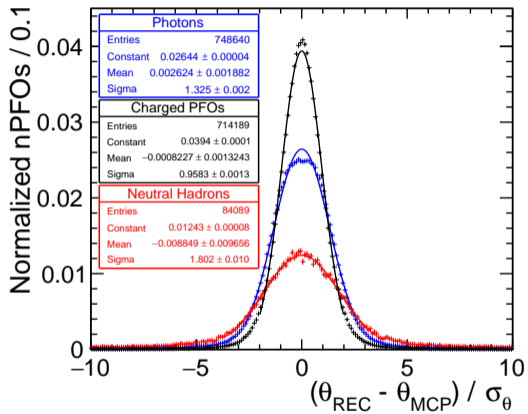
Error estimation in PFO level:

- ▶ **Charged PFOs:** uncertainties propagated from track fit covariance matrix
 - ▶ uncertainties 30% too small
- ▶ **Photons:** energy error is perfectly modeled. (sigma ~ 1)
- ▶ **Neutral Hadrons:** energy error from calorimeter intrinsic energy resolution.



PFO-level detector resolution, σ_{Det} for Neutral PFOs

θ and ϕ : quantities with as little correlation to PFO/cluster energy as possible.



underestimated σ_{θ} and σ_{ϕ} by factor $\sim 30\%$ (for photons) and $\sim 80\%$ (for neutral hadrons)



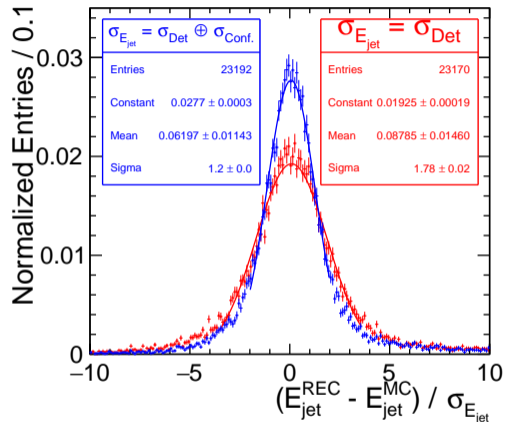
Uncertainties in jet-level

Propagation of errors from PFOs to jets:

- ▶ Transform the covariance matrix of each PFO (E, x, y, z for clusters, track parameters for charged) to (E, p_x, p_y, p_z)
- ▶ Add up scaled ($S^2 \times$) covariance matrices of all PFOs: $S = 1.3$ (photons), $S = 1.8$ (neutral hadrons)
- ▶ Add confusion term to covariance matrix
 - ▶ calculate $\sigma_{E_{\text{jet}}}^{\text{Conf.}}$ using jet energy composition
 - ▶ propagate to all CovMat elements by forcing $\sigma_{\theta_{\text{jet}}}^{\text{Conf.}} = \sigma_{\phi_{\text{jet}}}^{\text{Conf.}} = 0$
- ▶ Transform to ($E_{\text{jet}}, \theta_{\text{jet}}, \phi_{\text{jet}}, m_{\text{jet}}, \sigma_{E_{\text{jet}}}, \sigma_{\theta_{\text{jet}}}, \sigma_{\phi_{\text{jet}}}$) in kinematic fit

10.1016/j.nima.2009.09.009

Confusion term improves the estimate of the jet energy uncertainty, but not quite enough \Rightarrow need adjustment
 \Rightarrow **use scaling factor 1.2 (for $\sigma_{E_{\text{jet}}}$) in kinematic fit**



Application of kinematic fit to $e^+e^- \rightarrow ZH \rightarrow \mu\bar{\mu}b\bar{b}$ events

Parameters of jets and leptons are varied within their uncertainties to satisfy 5 constraints:
Conservation of momentum (hard constraints):

- ▶ p_x : e^+e^- crossing angle: 14 mrad
 $\Sigma p_x = \sqrt{s} \times \sin 0.007 \approx 1.75$ GeV
- ▶ p_y : $\Sigma p_y = 0$
- ▶ p_z : $\Sigma p_z = 0$

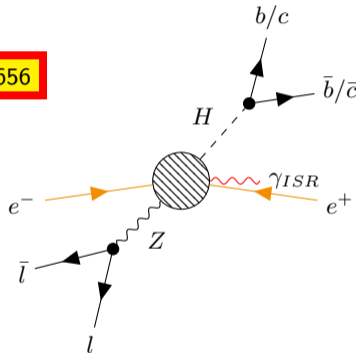
Conservation of total energy (hard constraint):

- ▶ $E_{lab} = 2\sqrt{(\frac{\sqrt{s}}{2})^2 + (\Sigma p_x)^2}$

Constrain di-muon mass to agree with m_Z within its natural width
(soft constraint):

- ▶ $m_Z = 91.2$ GeV , $\sigma_{m_Z} = \frac{2.4952}{2}$

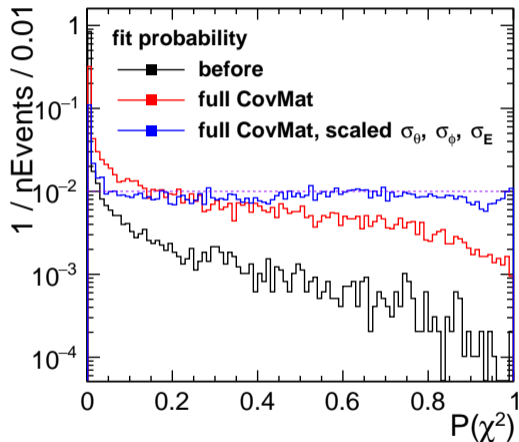
arXiv:0901.4656



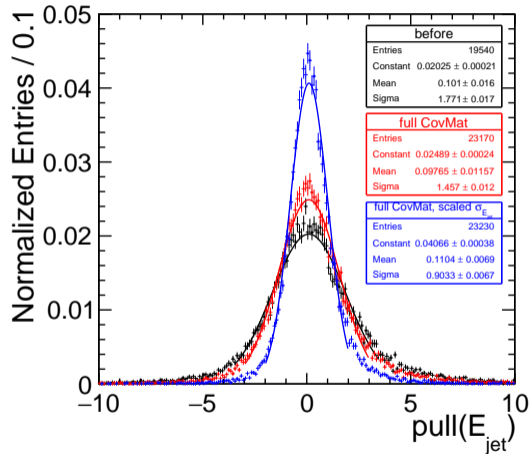
Kinematic fit performance in $e^+e^- \rightarrow ZH \rightarrow \mu\bar{\mu}b\bar{b}$ at $\sqrt{s} = 250$ GeV

without semi-leptonic decays

► fit probability



► pull distribution

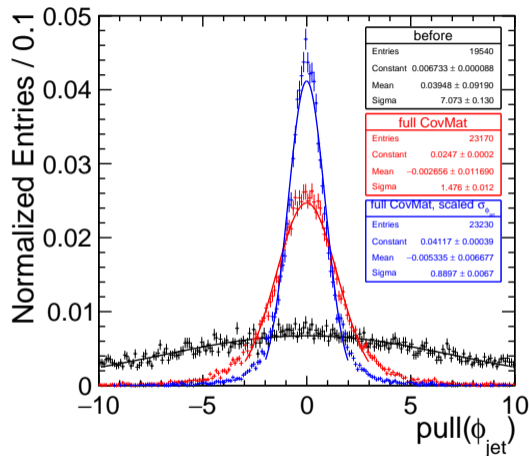
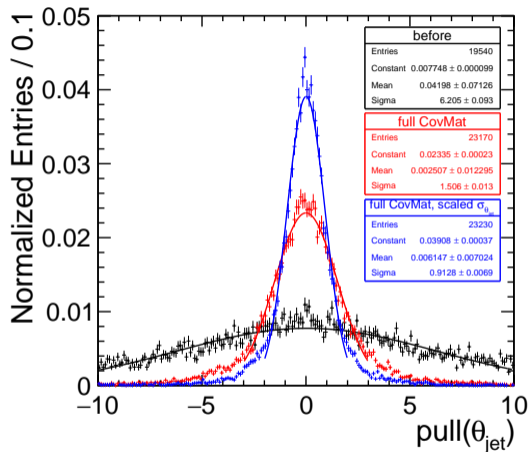


Improved kinematic fit performance with full CovMat of jets + scaled jet energy uncertainty



Kinematic fit performance in $e^+e^- \rightarrow ZH \rightarrow \mu\bar{\mu}b\bar{b}$ at $\sqrt{s} = 250$ GeV (cntd.)

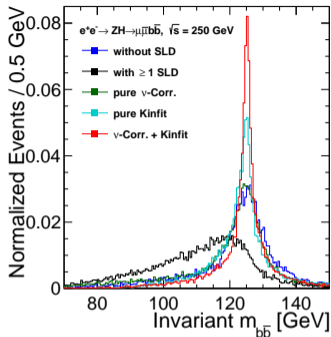
without semi-leptonic decays



Improved kinematic fit performance with full CovMat of jets + scaled jet angular uncertainties

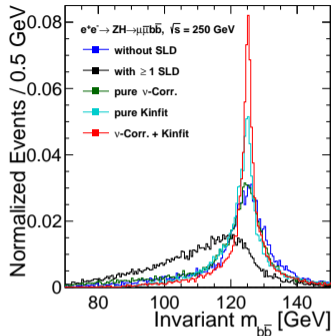


Higgs mass reconstruction with kinematic fit in presence of SLDs



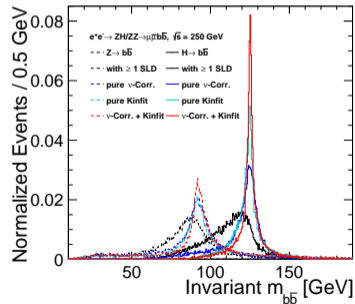
- ▶ ν -correction alone (pre-fit \vec{p}_ν) recovers Higgs mass
- ▶ striking improvement from new jet error parametrisation fed to kinematic fit even without ν -correction
- ▶ significant further improvement by combined kinematic fit and ν -correction (fully cheated), especially for the Higgs peak
- ▶ less powerful ν -correction \Rightarrow performance is expected between Cyan and Red

Higgs mass reconstruction with kinematic fit in presence of SLDs



- ▶ ν -correction alone (pre-fit \vec{p}_ν) recovers Higgs mass
- ▶ striking improvement from new jet error parametrisation fed to kinematic fit even without ν -correction
- ▶ significant further improvement by combined kinematic fit and ν -correction (fully cheated), especially for the Higgs peak
- ▶ less powerful ν -correction \Rightarrow performance is expected between Cyan and Red

- ▶ $Z \rightarrow b\bar{b}/H \rightarrow b\bar{b}$ well separated by combined kinematic fit and ν -correction (fully cheated)
- ▶ potentially large improvement eg for Higgs self-coupling prospects



Conclusions

- ▶ Heavy flavour jets are essential for Higgs physics
- ▶ Correction of semi-leptonic decays of heavy flavour jets is important for Higgs mass reconstruction
- ▶ Kinematic fit exploits well-known initial state in e^+e^- colliders and requires excellent understanding of jet measurement
- ▶ High granular Particle Flow detectors provide full detail for estimating jet measurement uncertainties



Remembering and honoring Mahsa Amini

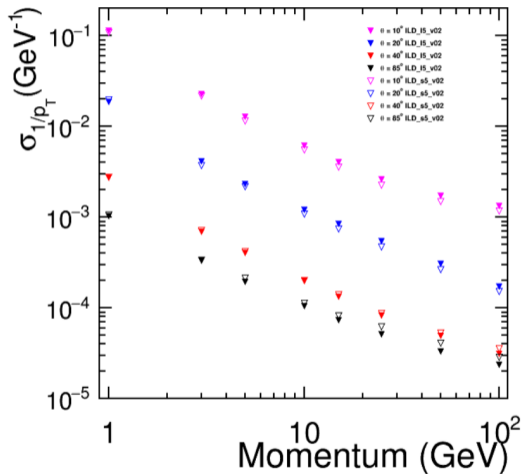


22 years old Iranian woman, arrested by morality police, fell into a coma and died in hospital due to police brutality. Crime? not wearing the hijab in accordance with government standards!

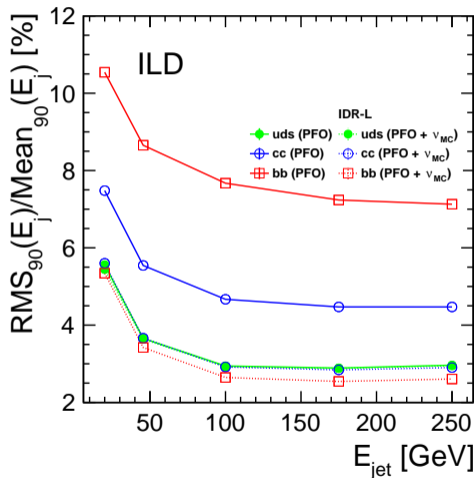
BACKUP



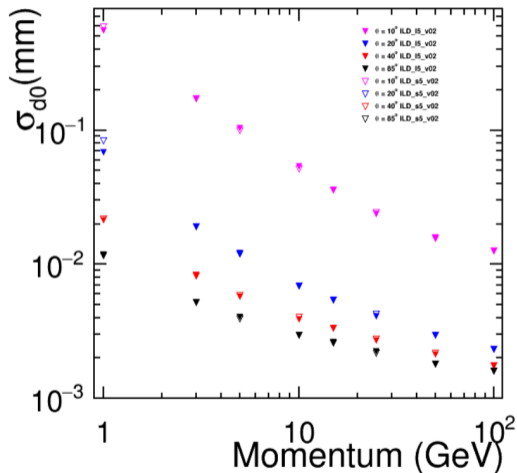
► Momentum Resolution



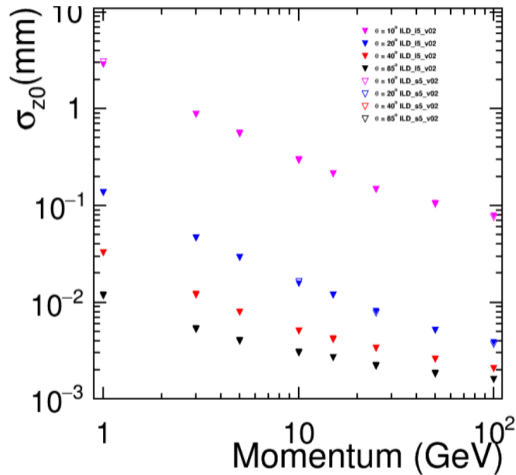
► Jet Energy Resolution ($E_{PFO} + E_\nu^{MC}$)



► Impact Parameter Resolution, d_0



► Impact Parameter Resolution, z_0

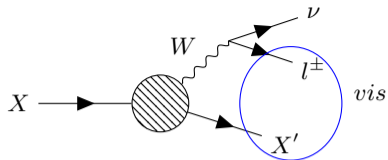


Concept of ν -correction in a semi-leptonic decay

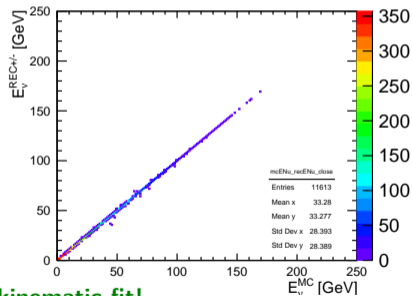
- ▶ Find heavy-quark jets: Identify b or c jet \rightarrow flavour tag
- ▶ Find semi-leptonic decay(s): Identify lepton in jet if present \rightarrow possible using detector's high granularity
- ▶ Estimate neutrino energy from decay kinematics:
 - ▶ Assign B^0 or D^0 meson mass to mother hadron.
 - ▶ Reconstruct flight direction of mother hadron from position of primary and secondary vertex.
 - ▶ Calculate neutrino momentum: up to 3-fold ambiguity.
- ▶ As proof-of-principle: CHEAT from MC truth
 - ▶ Lepton ID
 - ▶ Flavour tag
 - ▶ Mother hadron mass
 - ▶ **Associate of reconstructed particles to secondary vertex**
 - ▶ Momenta of visible decay products

The neutrino momentum can be determined up to a two-fold ambiguity

Can we use overall event kinematics to decide between solutions? \Rightarrow **kinematic fit!**



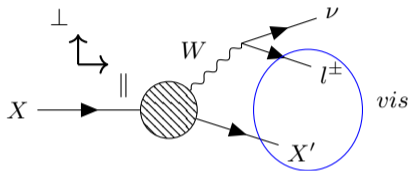
- ▶ Closure test: fully cheated information ($e^+e^- \rightarrow b\bar{b}$ at $\sqrt{s} = 500$ GeV)



correcting neutrino energy

4-vector based approach

- ▶ (E, \vec{p}) -based approach



$$\vec{p}_{\nu, \perp} = -\vec{p}_{vis, \perp}$$

$$\vec{p}_{\nu, \parallel} = \frac{1}{2D}(-A \pm \sqrt{A^2 - BD})\hat{n}$$

$$A = p_{vis, \parallel} (2p_{vis, \perp}^2 + m_{vis}^2 - m_X^2)$$

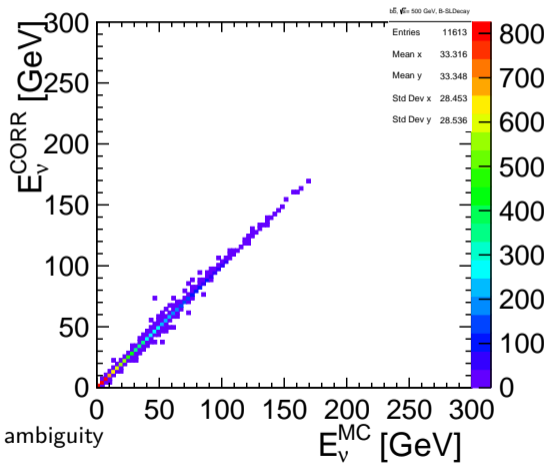
$$B = 4p_{vis, \perp}^2 E_{vis}^2 - (2p_{vis', \perp}^2 + m_{vis}^2 - m_X^2)^2$$

$$D = E_{vis}^2 - p_{vis, \parallel}^2$$

$$\hat{n} = \frac{p_{vis, \parallel}}{|p_{vis, \parallel}|}$$

The neutrino momentum can be determined up to a two-fold ambiguity

- ▶ closure test: apply correction with fully cheated information and compare with true neutrino energy

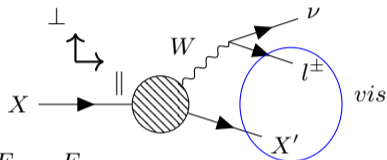


Correcting neutrino energy

Rapidity based approach

Rapidity under Lorentz-transformations \sim velocity under Galileo-transformations: $\omega = \omega_X + \omega'$; $\omega = \frac{1}{2} \ln \frac{E+p'_{\parallel}}{E-p'_{\parallel}}$

ω : rapdity in lab frame , ω' : rapdity in rest frame of X , ω_X : rapdity of X in lab frame



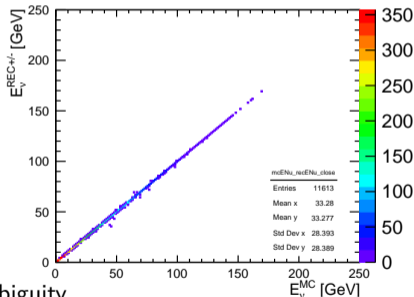
- Closure test: fully cheated information ($e^+e^- \rightarrow b\bar{b}$ at $\sqrt{s} = 500$ GeV)

$$E_{\nu} = E_X - E_{vis}$$

$$E_X = \frac{E_{vis} E'_{vis} - p_{vis\parallel} p'_{vis\parallel}}{m_{vis}^2 + p_{vis\perp}^2} m_X$$

$$E'_{vis} = \frac{m_X^2 + m_{vis}^2}{2m_X}$$

$$p'_{vis\parallel} = \pm \sqrt{\left(\frac{m_X^2 - m_{vis}^2}{2m_X}\right)^2 - p_{vis\perp}^2}$$



The neutrino momentum can be determined up to a two-fold ambiguity

Can we use overall event kinematics to decide between solutions? \Rightarrow kinematic fit!

Pandora treatment with Neutral Hadrons

What Pandora does:

- ▶ Cluster energy is assigned to PFO(massless) energy; $E_{\text{PFO}} = |\vec{p}_{\text{PFO}}| = E_{\text{clu}}$
- ▶ Neutral Hadrons are identified as neutron $\Rightarrow m_n$ is set for PFO \Rightarrow **inconsistent 4-momentum!**
- ▶ CovMat of Neutral PFO is calculated (using inconsistent 4-momentum): $C(\vec{p}, E) = J^T C(\vec{x}_{\text{clu}}, E_{\text{clu}}) J$

$$J = \begin{pmatrix} \frac{\partial p_x}{\partial x_c} & \frac{\partial p_y}{\partial x_c} & \frac{\partial p_z}{\partial x_c} & \frac{\partial E}{\partial x_c} \\ \frac{\partial p_x}{\partial p_y} & \frac{\partial p_y}{\partial p_y} & \frac{\partial p_z}{\partial p_y} & \frac{\partial E}{\partial p_y} \\ \frac{\partial p_x}{\partial y_c} & \frac{\partial p_y}{\partial y_c} & \frac{\partial p_z}{\partial y_c} & \frac{\partial E}{\partial y_c} \\ \frac{\partial p_x}{\partial z_c} & \frac{\partial p_y}{\partial z_c} & \frac{\partial p_z}{\partial z_c} & \frac{\partial E}{\partial z_c} \\ \frac{\partial p_x}{\partial E_c} & \frac{\partial p_y}{\partial E_c} & \frac{\partial p_z}{\partial E_c} & \frac{\partial E}{\partial E_c} \end{pmatrix}$$

Suggestion: Take consistent 4-momentum of massive neutral hadrons for CovMat calculations. CovMat(\vec{p}, E) of Neutral PFOs depend on the mass assumption.

CovMat of Neutral PFOs

- ▶ Current CovMat calculation (MarlinReco/Analysis/AddClusterProperties)

$$E_{PFO} = |\vec{p}_{PFO}| = E_{clu} , p_x = E_{clu} \frac{x}{r} , p_y = E_{clu} \frac{y}{r} , p_z = E_{clu} \frac{z}{r}$$

- ▶ Alternative CovMat calculation (taking consistent 4-momentum of neutral hadrons)

$$E_{PFO} = \sqrt{|\vec{p}_{PFO}|^2 + m_{PFO}^2} = \sqrt{E_{clu}^2 + m_n^2}$$

$$J = \begin{pmatrix} E_{clu} \frac{r^2 - x^2}{r^3} & -E_{clu} \frac{xy}{r^3} & -E_{clu} \frac{xz}{r^3} & 0 \\ -E_{clu} \frac{xy}{r^3} & E_{clu} \frac{r^2 - y^2}{r^3} & -E_{clu} \frac{yz}{r^3} & 0 \\ -E_{clu} \frac{xz}{r^3} & -E_{clu} \frac{yz}{r^3} & E_{clu} \frac{r^2 - z^2}{r^3} & 0 \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} & 1 \end{pmatrix} \rightarrow J = \begin{pmatrix} E_{clu} \frac{r^2 - x^2}{r^3} & -E_{clu} \frac{xy}{r^3} & -E_{clu} \frac{xz}{r^3} & 0 \\ -E_{clu} \frac{xy}{r^3} & E_{clu} \frac{r^2 - y^2}{r^3} & -E_{clu} \frac{yz}{r^3} & 0 \\ -E_{clu} \frac{xz}{r^3} & -E_{clu} \frac{yz}{r^3} & E_{clu} \frac{r^2 - z^2}{r^3} & 0 \\ \frac{E}{E_{clu}} \cdot \frac{x}{r} & \frac{E}{E_{clu}} \cdot \frac{y}{r} & \frac{E}{E_{clu}} \cdot \frac{z}{r} & 1 \end{pmatrix}$$

using error propagation, PFO angular uncertainties are calculated directly from cluster position error:

$$\sigma_\theta^2 = \left(\frac{\partial\theta}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial\theta}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial\theta}{\partial z}\right)^2 \sigma_z^2 + \frac{\partial\theta}{\partial x} \frac{\partial\theta}{\partial y} \sigma_{xy} + \frac{\partial\theta}{\partial x} \frac{\partial\theta}{\partial z} \sigma_{xz} + \frac{\partial\theta}{\partial y} \frac{\partial\theta}{\partial z} \sigma_{yz}$$

$$\sigma_\phi^2 = \left(\frac{\partial\phi}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 \sigma_y^2 + \frac{\partial\phi}{\partial x} \frac{\partial\phi}{\partial y} \sigma_{xy}$$

MUST: angular and energy uncertainties remain unchanged!

Event selection

Select $e^+e^- \rightarrow ZH \rightarrow \mu\bar{\mu}b\bar{b}$ events at $\sqrt{s} = 250$ GeV with (exactly) 2-leptons + 2-jets final state:

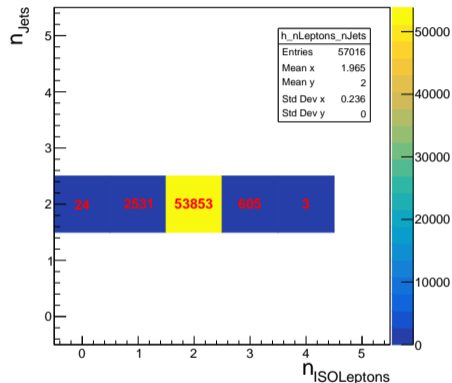
► IsolatedLeptonTagging

Training for the IDR 500 GeV samples is used,

1. Lepton ID: μ^\pm
Deposited energy in subdetectors
2. Vertex: primary or secondary
Significance of impact parameters (d_0, z_0)
3. Isolated: not belong to jets

► FastJetProcessor

- Exclusive k_t (Durham) algorithm (no overlay)
- Find smallest of (d_{ij}, d_{iB})
$$d_{ij} = 2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})$$
$$i, j: \text{particles}, B: \text{Beam}$$
- $d_{ij} < d_{iB}$: combine i & j as pseudojet(p): $p_i + p_j$
- $d_{iB} < d_{ij}$: remove particle i from list
- Repeat iteration until d_{ij} or $d_{iB} > d_{cut}$ (threshold)

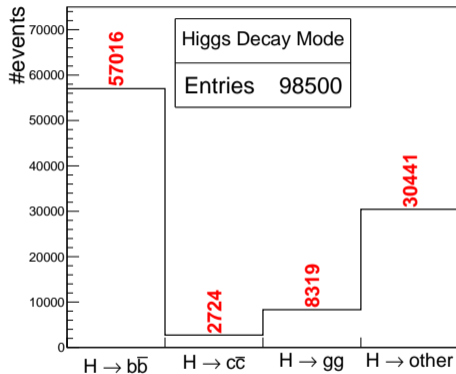


IsolatedLeptonTagging has not been trained for new software at 250 GeV yet!



event selection

separate Higgs decay modes: $H \rightarrow b\bar{b}$, cheat from MCTruth



$\frac{2}{3}$ of $b\bar{b}$ jets contain at-least one semi-leptonic decay \Rightarrow Frequent $H \rightarrow b\bar{b}$ needs neutrino correction.

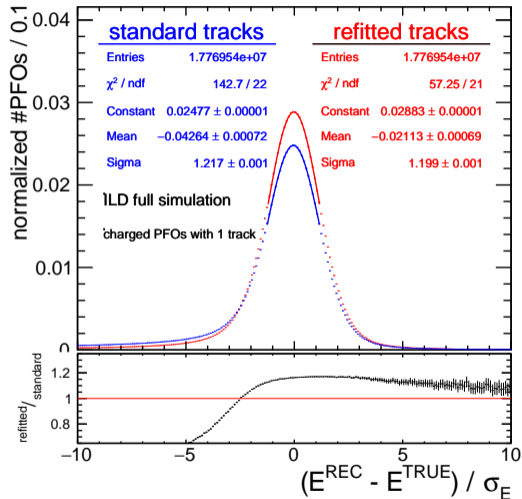
PFO-level detector resolution, σ_{Det} for charged PFOs

Energy

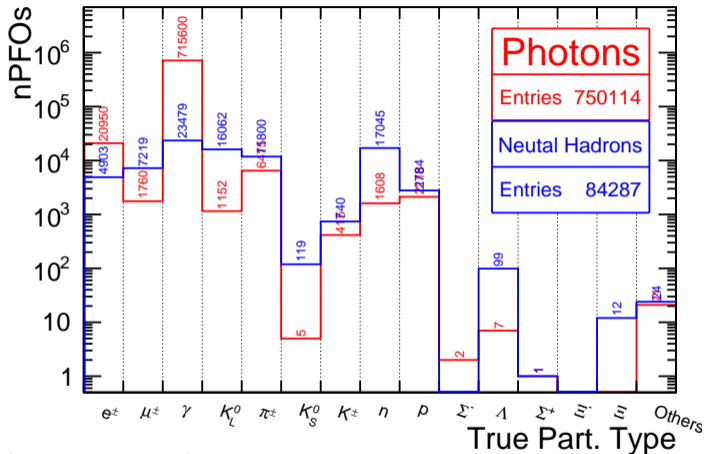
- ▶ **Standard tracks:** all tracks fitted with m_{π^\pm} in standard reconstruction of charged PFOs.
- ▶ **refitting tracks:**
 - ▶ refit K^\pm and p^\pm tracks with true mass of particle
 - ▶ calculated E_{PFO} from momentum using true mass
 - ▶ cheated particle ID, FTB

See talk by B. Dudar

- ▶ slight improvement on energy uncertainty by track refitting with true mass
- ▶ narrower distribution
- ▶ further improvement by calculating energy from momentum using true mass for K^\pm and p^\pm
- ▶ migrating PFOs to core of distribution



Neutral PFO identification by Pandora



Majority of identified photons are true photons.

No explicit decision for mass of identified neutral hadrons due to their multiplicity.

Pandora treatment with Neutral Hadrons

What Pandora does:

- ▶ Cluster energy is assigned to PFO(massless) energy
- ▶ Neutral Hadrons are identified as neutron
- ▶ neutron mass is set for PFO \Rightarrow **inconsistent 4-momentum!**
- ▶ CovMat of Neutral PFO is calculated (using inconsistent 4-momentum):

$$\text{CovMat}(\vec{p}, E) = J^T \text{CovMat}(\vec{x}_{clu}, E_{clu}) J$$

$$J = \begin{pmatrix} \frac{\partial p_x}{\partial x_c} & \frac{\partial p_y}{\partial x_c} & \frac{\partial p_z}{\partial x_c} & \frac{\partial E}{\partial x_c} \\ \frac{\partial p_x}{\partial y_c} & \frac{\partial p_y}{\partial y_c} & \frac{\partial p_z}{\partial y_c} & \frac{\partial E}{\partial y_c} \\ \frac{\partial p_x}{\partial z_c} & \frac{\partial p_y}{\partial z_c} & \frac{\partial p_z}{\partial z_c} & \frac{\partial E}{\partial z_c} \\ \frac{\partial p_x}{\partial E_c} & \frac{\partial p_y}{\partial E_c} & \frac{\partial p_z}{\partial E_c} & \frac{\partial E}{\partial E_c} \end{pmatrix}$$

CovMat(\vec{p}, E) of Neutral PFOs depend on the mass assumption.

Suggestion: Take consistent 4-momentum of massive neutral hadrons for CovMat calculations.

CovMat of Neutral PFOs

- ▶ Current CovMat calculation (MarlinReco/Analysis/AddClusterProperties)

$$E_{PFO} = |\vec{p}_{PFO}| = E_{clu} , p_x = E_{clu} \frac{x}{r} , p_y = E_{clu} \frac{y}{r} , p_z = E_{clu} \frac{z}{r}$$

- ▶ Alternative CovMat calculation (taking consistent 4-momentum of neutral hadrons)

$$E_{PFO} = \sqrt{|\vec{p}_{PFO}|^2 + m_{PFO}^2} = \sqrt{E_{clu}^2 + m_n^2}$$

$$J = \begin{pmatrix} E_{clu} \frac{r^2 - x^2}{r^3} & -E_{clu} \frac{xy}{r^3} & -E_{clu} \frac{xz}{r^3} & 0 \\ -E_{clu} \frac{xy}{r^3} & E_{clu} \frac{r^2 - y^2}{r^3} & -E_{clu} \frac{yz}{r^3} & 0 \\ -E_{clu} \frac{xz}{r^3} & -E_{clu} \frac{yz}{r^3} & E_{clu} \frac{r^2 - z^2}{r^3} & 0 \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} & 1 \end{pmatrix} \rightarrow J = \begin{pmatrix} E_{clu} \frac{r^2 - x^2}{r^3} & -E_{clu} \frac{xy}{r^3} & -E_{clu} \frac{xz}{r^3} & 0 \\ -E_{clu} \frac{xy}{r^3} & E_{clu} \frac{r^2 - y^2}{r^3} & -E_{clu} \frac{yz}{r^3} & 0 \\ -E_{clu} \frac{xz}{r^3} & -E_{clu} \frac{yz}{r^3} & E_{clu} \frac{r^2 - z^2}{r^3} & 0 \\ \frac{E}{E_{clu}} \cdot \frac{x}{r} & \frac{E}{E_{clu}} \cdot \frac{y}{r} & \frac{E}{E_{clu}} \cdot \frac{z}{r} & 1 \end{pmatrix}$$

using error propagation, PFO angular uncertainties are calculated directly from cluster position error:

$$\sigma_\theta^2 = \left(\frac{\partial\theta}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial\theta}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial\theta}{\partial z}\right)^2 \sigma_z^2 + \frac{\partial\theta}{\partial x} \frac{\partial\theta}{\partial y} \sigma_{xy} + \frac{\partial\theta}{\partial x} \frac{\partial\theta}{\partial z} \sigma_{xz} + \frac{\partial\theta}{\partial y} \frac{\partial\theta}{\partial z} \sigma_{yz}$$

$$\sigma_\phi^2 = \left(\frac{\partial\phi}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 \sigma_y^2 + \frac{\partial\phi}{\partial x} \frac{\partial\phi}{\partial y} \sigma_{xy}$$

MUST: angular and energy uncertainties remain unchanged!

CovMat of Jets

► AddClusterProperties/FourMomentumCovMat: CovMat(cluster/track) → CovMat(\vec{p}, E)

► Current CovMat calculation (inconsistent 4-momentum of neutral hadrons):

$$E_{PFO} = |\vec{p}_{PFO}| = E_{clu}, \quad p_x = E_{clu} \frac{x}{r}, \quad p_y = E_{clu} \frac{y}{r}, \quad p_z = E_{clu} \frac{z}{r}, \quad m_{PFO} = m_n$$

► Alternative CovMat calculation (taking consistent 4-momentum of neutral hadrons)

$$E_{PFO} = \sqrt{|\vec{p}_{PFO}|^2 + m_{PFO}^2} = \sqrt{E_{clu}^2 + m_n^2}$$

$J_{(wrong)} \rightarrow J_{(right)}$

$$\begin{pmatrix} E_{clu} \frac{r^2 - x^2}{r^3} & -E_{clu} \frac{xy}{r^3} & -E_{clu} \frac{xz}{r^3} & 0 \\ -E_{clu} \frac{xy}{r^3} & E_{clu} \frac{r^2 - y^2}{r^3} & -E_{clu} \frac{yz}{r^3} & 0 \\ -E_{clu} \frac{xz}{r^3} & -E_{clu} \frac{yz}{r^3} & E_{clu} \frac{r^2 - z^2}{r^3} & 0 \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} & 1 \end{pmatrix}_{wrong} \rightarrow \begin{pmatrix} E_{clu} \frac{r^2 - x^2}{r^3} & -E_{clu} \frac{xy}{r^3} & -E_{clu} \frac{xz}{r^3} & 0 \\ -E_{clu} \frac{xy}{r^3} & E_{clu} \frac{r^2 - y^2}{r^3} & -E_{clu} \frac{yz}{r^3} & 0 \\ -E_{clu} \frac{xz}{r^3} & -E_{clu} \frac{yz}{r^3} & E_{clu} \frac{r^2 - z^2}{r^3} & 0 \\ \frac{E}{E_{clu}} \cdot \frac{x}{r} & \frac{E}{E_{clu}} \cdot \frac{y}{r} & \frac{E}{E_{clu}} \cdot \frac{z}{r} & 1 \end{pmatrix}_{right}$$

► ErrorFlow:

$$\text{CovMat}(\vec{p}_{jet}, E_{jet}) = \sum_{PFO} \text{CovMat}(\vec{p}, E) \quad : \quad \sigma_{E_{jet}}^2 = \sigma_{conf}^2 + \sum_{PFO} \sigma_{E_{PFO}}^2$$

► MarlinKinfittersProcessors:

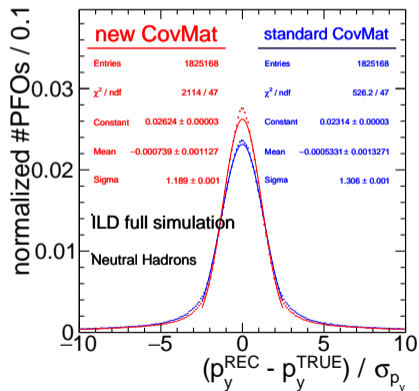
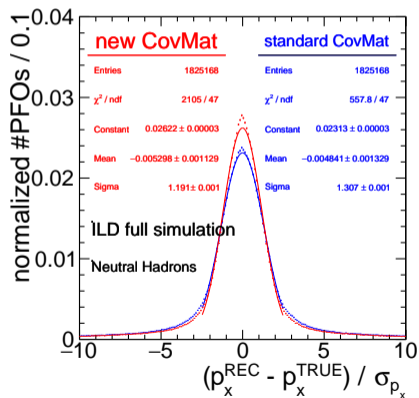
$$\text{CovMat}(\vec{p}_{jet}, E_{jet}) \rightarrow (\sigma_{\theta_{jet}}, \sigma_{\phi_{jet}}, \sigma_{E_{jet}})$$



Pandora treatment with Neutral Hadrons

- ▶ Cluster energy is assigned to PFO(massless) energy; $E_{\text{PFO}} = |\vec{p}_{\text{PFO}}| = E_{\text{clu}}$
- ▶ Neutral Hadrons are identified as neutron $\Rightarrow m_n$ is set for PFO \Rightarrow **inconsistent 4-momentum!**
- ▶ CovMat of Neutral PFO is calculated (using inconsistent 4-momentum): $C(\vec{p}, E) = J^T C(\vec{x}_{\text{clu}}, E_{\text{clu}}) J$

Suggestion: Take consistent 4-momentum of massive neutral hadrons for CovMat calculations.



Covariance of measured quantities

$$\sigma_\phi^2 = \left(\frac{\partial\phi}{\partial p_x}\right)^2 \sigma_{p_x}^2 + \left(\frac{\partial\phi}{\partial p_y}\right)^2 \sigma_{p_y}^2 + 2\frac{\partial\phi}{\partial p_x}\frac{\partial\phi}{\partial p_y}\sigma_{p_x p_y}$$

underestimated σ_{p_x} and σ_{p_y} by $\sim 20\%$, but $\sigma_{p_x p_y}$?

$$\text{Var}\left(\frac{p_x^{\text{REC}} - p_x^{\text{TRUE}}}{\sigma_{p_x}} - \frac{p_y^{\text{REC}} - p_y^{\text{TRUE}}}{\sigma_{p_y}}\right) = \text{Var}\left(\frac{p_x^{\text{REC}} - p_x^{\text{TRUE}}}{\sigma_{p_x}}\right) + \text{Var}\left(\frac{p_y^{\text{REC}} - p_y^{\text{TRUE}}}{\sigma_{p_y}}\right) - 2\text{Cov}\left(\frac{p_x^{\text{REC}} - p_x^{\text{TRUE}}}{\sigma_{p_x}}, \frac{p_y^{\text{REC}} - p_y^{\text{TRUE}}}{\sigma_{p_y}}\right)$$

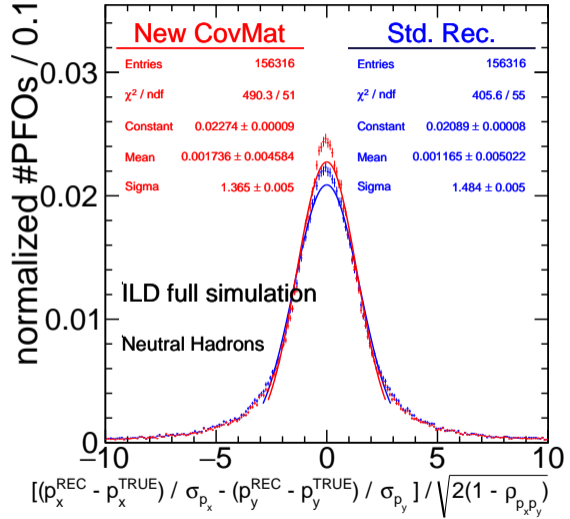
perfect Gaussian $\frac{p_{x/y}^{\text{REC}} - p_{x/y}^{\text{TRUE}}}{\sigma_{p_{x/y}}} \Rightarrow \text{Mean} = 0, \text{Sigma} = 1$

$$\rho_{p_x p_y} = \text{Cor}(p_x, p_y) = \frac{\sigma_{p_x p_y}}{\sigma_{p_x} \sigma_{p_y}}$$

$$\text{Var}\left(\frac{p_x^{\text{REC}} - p_x^{\text{TRUE}}}{\sigma_{p_x}} - \frac{p_y^{\text{REC}} - p_y^{\text{TRUE}}}{\sigma_{p_y}}\right) = 1 + 1 - 2\frac{\sigma_{p_x p_y}}{\sigma_{p_x} \sigma_{p_y}} = 2(1 - \rho_{p_x p_y})$$

For evaluating off-diagonal elements:

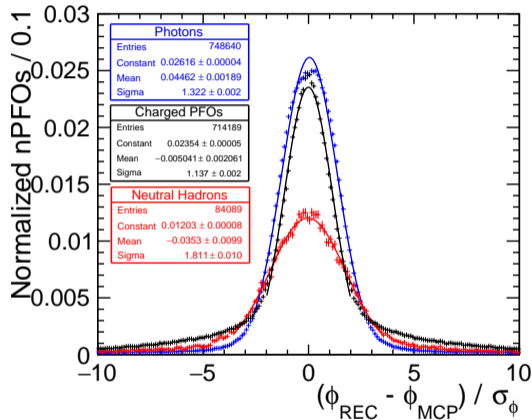
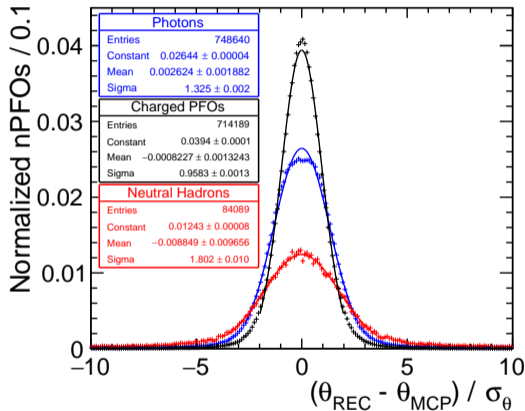
$$\frac{\frac{p_x^{\text{REC}} - p_x^{\text{TRUE}}}{\sigma_{p_x}} - \frac{p_y^{\text{REC}} - p_y^{\text{TRUE}}}{\sigma_{p_y}}}{\sqrt{2(1 - \rho_{p_x p_y})}}$$



ErrorFlow: Jet Error Parametrisation from Particle Flow Objects (PFO)

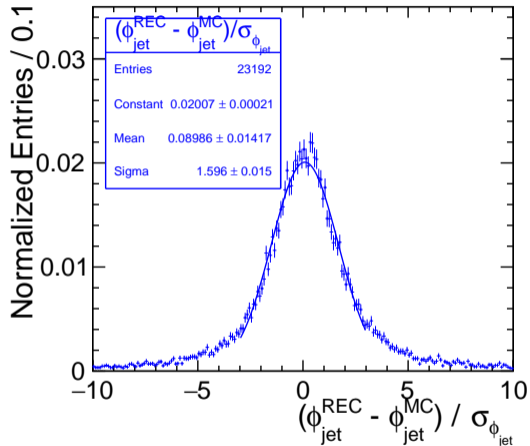
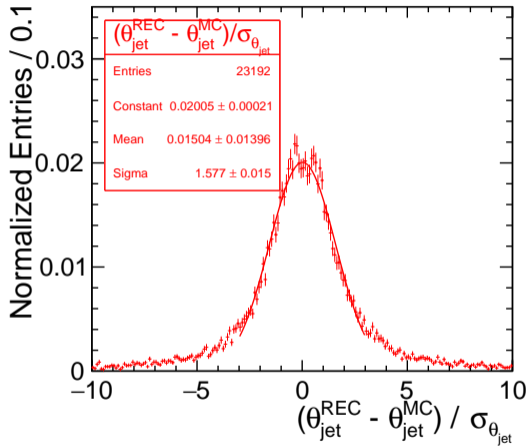
Angles

The angular uncertainties obtained directly from track parameters / cluster position errors



⇒ Scale σ_{θ} and σ_{ϕ} by factor ~ 1.3 (for photons) and ~ 1.8 (for neutral hadrons)

Uncertainties in jet-level: θ & ϕ



Jet angular uncertainties need scaling factor ~ 1.6

Neutrino correction hypothesis

- ▶ Assign semi-leptonic decays to jets

- ▶ Add neutrino momentum to 4-momentum of assigned jet:

Test three hypothesis for neutrino energy in each semi-leptonic decay: E_ν^+ , E_ν^- , 0

3^{nSLD} combination of E_ν 's for adding to jet 4-momentum:

Number of semileptonic decays in a jet: $nSLD = nSLDB + nSLDC$

Example:

If an event contains two jets: jet-1 contains 2 semi-leptonic decays and jet-2 contains 1 semi-leptonic decay, **27**(= $3^2 \times 3^1$) combinations of E_ν 's are available for neutrino correction in the event:

- ▶ jet-1:

comb.	1	2	3	4	5	6	7	8	9
$\vec{p}_{\nu,1}$	-	+	0	-	+	0	-	+	0
$\vec{p}_{\nu,2}$	-	-	-	+	+	+	0	0	0

- ▶ jet-2:

comb.	1	2	3
$\vec{p}_{\nu,3}$	-	+	0

$\vec{p}_{\nu,1} + \vec{p}_{\nu,2}$ is added to 4-momentum of jet-1 and $\vec{p}_{\nu,3}$ is added to 4-momentum of jet-2.

$\vec{p}_{\nu,1} + \vec{p}_{\nu,2} + \vec{p}_{\nu,3} = 0$ allows fitter to neglect neutrino correction

Combination with highest fit probability is chosen as best neutrino correction.



Simple neutrino correction for Higgs mass reconstruction

- ▶ Neutrino energy correction:

Estimating neutrino energy as a fraction of corresponding lepton energy:

$$E_{jet}^{corr} = E_{jet} + E_{\nu} = E_{jet} + \left(\frac{1}{x} - 1\right)E_{lep}$$

- ▶ Uncertainty on jet energy parametrised as:

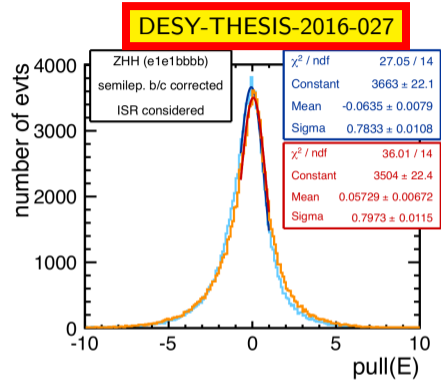
$$\sigma_{E_{jet}}^{corr} = \frac{100\%}{\sqrt{E_{jet}}} \oplus \sigma_{\nu}$$

$$\sigma_{\nu}^2 = \left(\frac{\sigma_{\langle x \rangle}}{\langle x \rangle^2}\right)^2 E_{lep}^2 + \left(\frac{1}{x} - 1\right)\Delta E_{lep}^2$$

- ▶ Fixed uncertainties on angles:

$$\Delta\theta_{jet} = \Delta\phi_{jet} = 100 \text{ mrad}$$

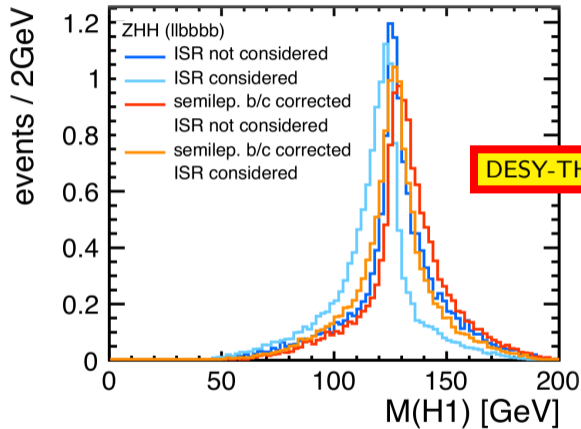
Simple correction to jet energy improves jet energy pull distribution as a measure of fit performance.



Blue: before neutrino energy correction

Orange: After neutrino energy correction

Simple neutrino correction for Higgs mass reconstruction



- Bias and assymetry in m_H is removed by correcting jet energy and adding ISR

Error flow and application in kinematic fit

DESY-THESIS-2017-045

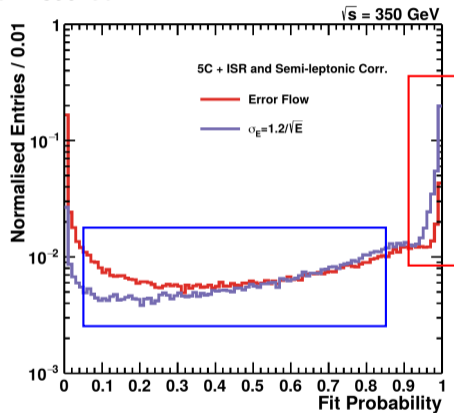
Jet specific energy resolution for $e^+e^- \rightarrow ZH \rightarrow q\bar{q}b\bar{b}$ process at $\sqrt{s} = 350$ GeV

- ▶ Full 4×4 CovMatrix on 4-momentum of jets $\sigma(\vec{p}, E)$:
 - ▶ σ_{Det} : computed using subdetector momentum/energy resolution
 - ▶ σ_{Conf} : computed using jet energy and particle content (charged, neutral and photon)
 - ▶ $\sigma_\nu = 0.73 \cdot E_l$
 - ▶ $\sigma_{Had}, \sigma_{Clus}$ are not accounted for error flow procedure yet.

▶ Fixed (and wide) angular resolution: $\sigma_\theta = \sigma_\phi = 100$ mrad

Kinematic fit: vary jet quantities (E, θ, ϕ) within uncertainties $(\sigma_E, \sigma_\theta, \sigma_\phi)$

Improved fit probability by applying Error Flow on jet energy



⇒ Further improvements by error parametrization and handling sl-decays

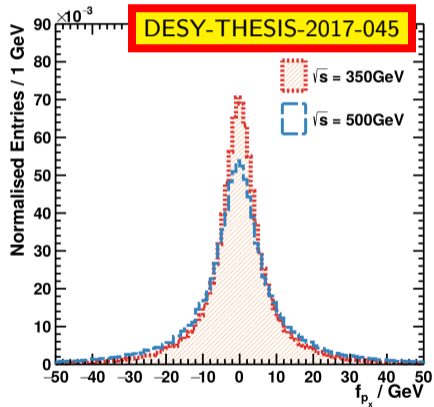


fit constraints

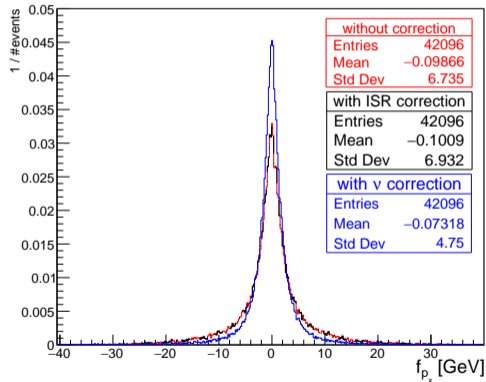
momentum conservation: p_x

ISR is initialized to satisfy momentum conservation on x direction

► by error flow on jet energy



► by error flow on CovMatrix (new)



angular resolution for individual jets: improved constraint on momentum conservation

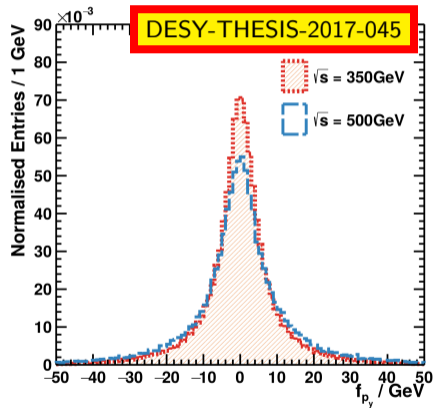


fit constraints

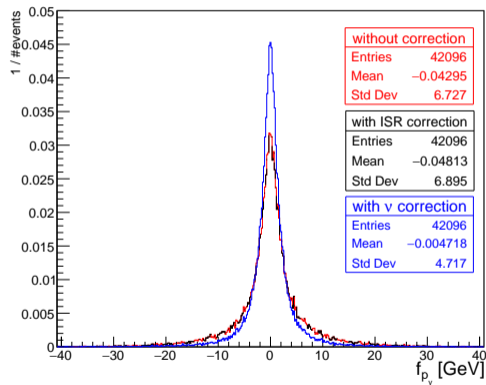
momentum conservation: p_y

ISR is initialized to satisfy momentum conservation on z direction

► by error flow on jet energy



► by error flow on CovMatrix (new)



angular resolution for individual jets: improved constraint on momentum conservation

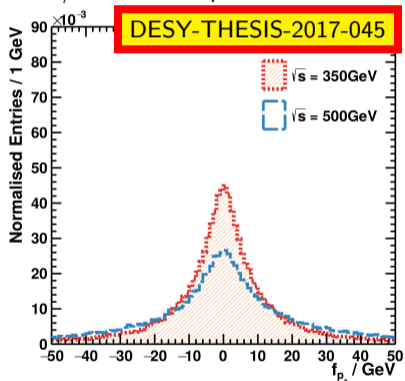


Fit constraints

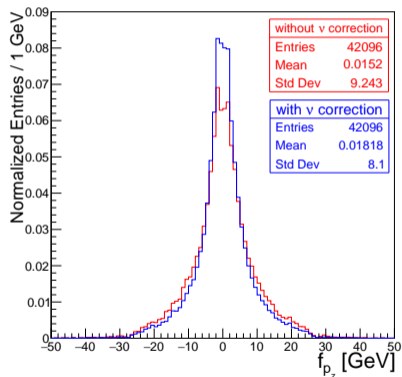
Momentum conservation: p_z

Adding 4-momentum of neutrino improves jet fit object initialization

- ▶ DBD 350/500 GeV samples



- ▶ MC-2020 250 GeV prod. samples



Proper neutrino correction for jets: improved constraint on momentum

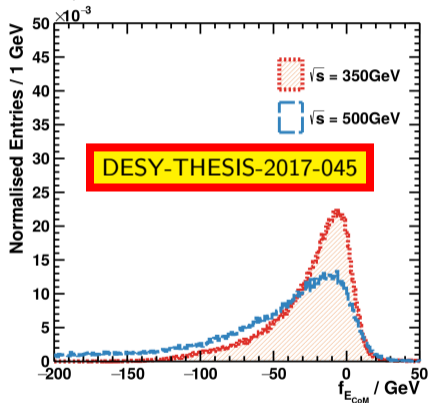


fit constraints

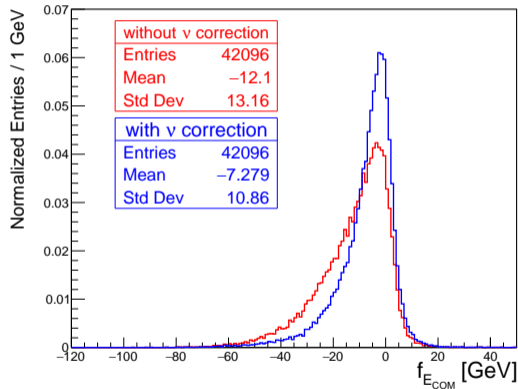
energy conservation: E

Neutrino correction (best pre-fit \vec{p}_ν for succesful fits) improves start values \Rightarrow better fit object initialization

- ▶ DBD 350/500 GeV samples



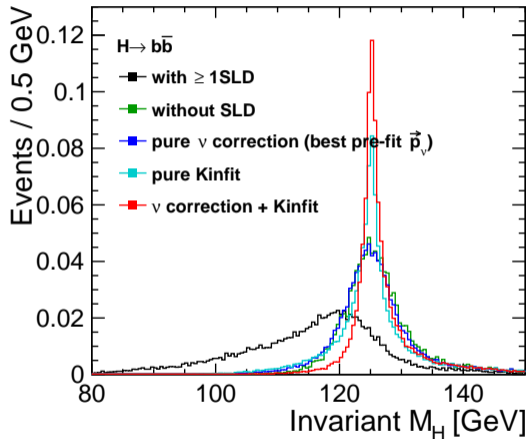
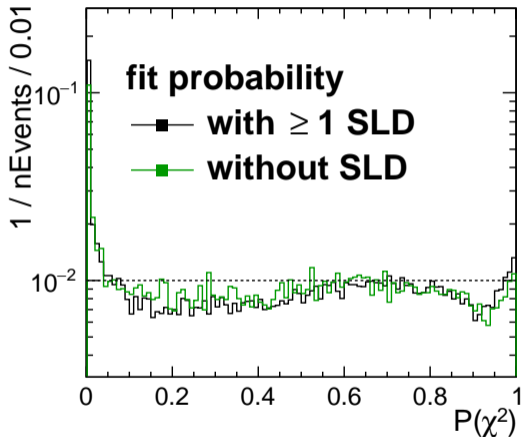
- ▶ MC-2020 250 GeV prod. samples



By neutrino correction, initial value of constraint function closer to target \Rightarrow fit should work better!

Higgs mass in presence of SLDs

ν -correction and kinematic fit on $H \rightarrow b\bar{b}$



► Applying **kinematic fit and ν -correction** gives huge improvement on Higgs mass reconstruction

