Precise predictions for the trilinear Higgs self-coupling in the Standard Model and beyond

predicting κ_λ in *any* model at the one-loop order

Henning Bahl, Johannes Braathen, **Martin Gabelmann**, Georg Weiglein

ECFA, 06.10.2022

The SM scalar sector

> V_{SM} fixed at tree-level by $m_h \approx 125\,\mathrm{GeV}$ and $v = (\sqrt{2}G_F)^{-1/2} \approx 246\,\mathrm{GeV}$:

$$V_{\mathrm{SM}}(h) = \frac{m_h^2}{2}h^2 + 3\frac{m_h^2}{v}h^3 + 3\frac{m_h^2}{v^2}h^4$$
$$= \frac{m_h^2}{2}h^2 + \lambda_{hhh}h^3 + \lambda_{hhhh}h^4$$

> However: λ_{hhh} (and λ_{hhhh}) experimentally unknown.

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$$= \frac{m_h^2}{2}h^2 + \lambda_{hhh}h^3 + \lambda_{hhhh}h^4$$

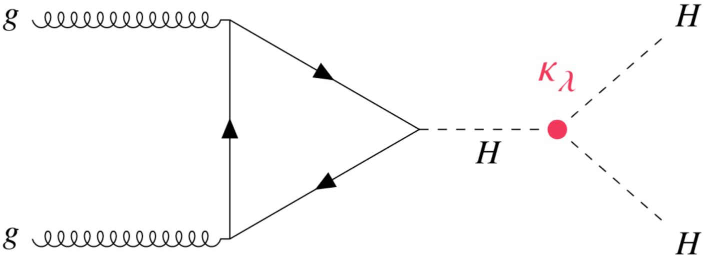
- However: λ_{hhh} (and λ_{hhhh}) experimentally unknown.
- BSM case: deformation of the scalar potential possible!

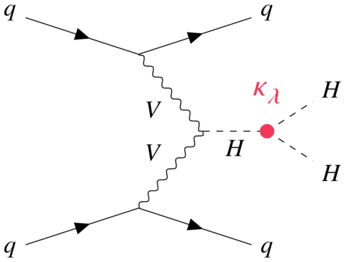
$$V_{\mathrm{BSM}}(h,\dots) = \frac{m_h^2}{2}h^2 + 3\frac{m_h^2}{v}\kappa_\lambda^{\mathrm{BSM}}h^3 + 3\frac{m_h^2}{v^2}\kappa_{2\lambda}h^4 + \dots$$

  $\kappa_\lambda^{\mathrm{BSM}}$: describes deviation from SM: $\kappa_\lambda^{\mathrm{BSM}} = \frac{\lambda_{hhh}^{\mathrm{BSM}}}{\lambda_{hhh}^{\mathrm{SM}}}$ in model "BSM".

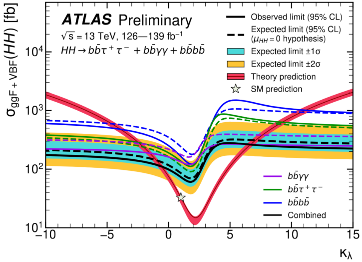
- $|\kappa_\lambda| \gg 1$ possible even though other (known) couplings behave very SM-like
- Example: "BSM"=THDM (talk by Johannes Braathen)
  $\rightarrow$ **higher-order corrections important**

Higgs pair production (in the alignment limit)





$pp \to hh$: $-0.4 < \kappa_\lambda < 6.3$ [ATLAS-CONF-2022-050]



When to apply the κ_λ-constraint to BSM models?

- only κ_λ is *significantly* modified by BSM physics
- all other couplings SM-like

$\to$ in many models given by experimental constraints

Higher-order corrections to $\lambda_{hhh}^{\text{BSM}}$

A few studies of the trilinear Higgs self-couplings (i.e. κ_λ) already exists:

- SM [Kanemura et al. '04]
- SM + singlet [Kanemura et al. '16]
- THDM [Kanemura et al. '04][Basler et al. '17][Braathen et al. '19]
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- MSSM [Brucherseifer et al. '13]
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of which many can have sizeable deviations from $\kappa_\lambda = 1$ and hence also from $\sigma_{hh}^{\text{SM}}(\kappa_\lambda = 1)$ [Abouabid et al. '21]

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However, many more models to explore:

- THDM-variants
- singlet-variants
- extended gauge sectors
- extended fermion sectors
- non-minimal SUSY, e.g. seesaw extensions, dirac gauginos, SplitSUSY, ...
- (+ combinations)
- ...

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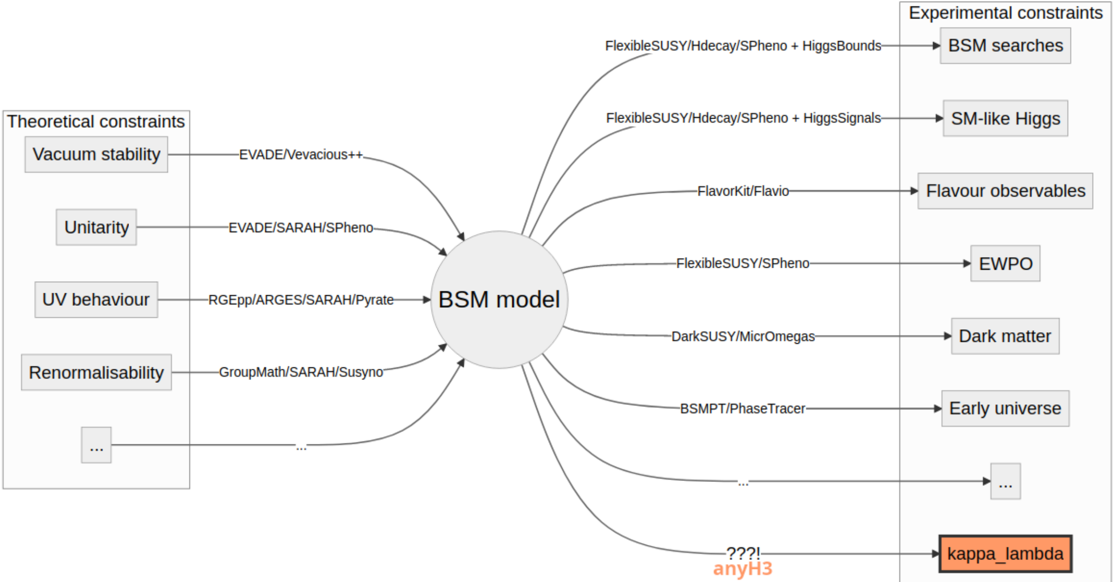
$\rightarrow$ **framework to calculate** $\lambda_{hhh}^{\text{BSM}}$ **for large class of "BSM's"**

$\lambda_{hhh}^{\text{BSM}}$ in the landscape of generic BSM tool-boxes



Ingredients

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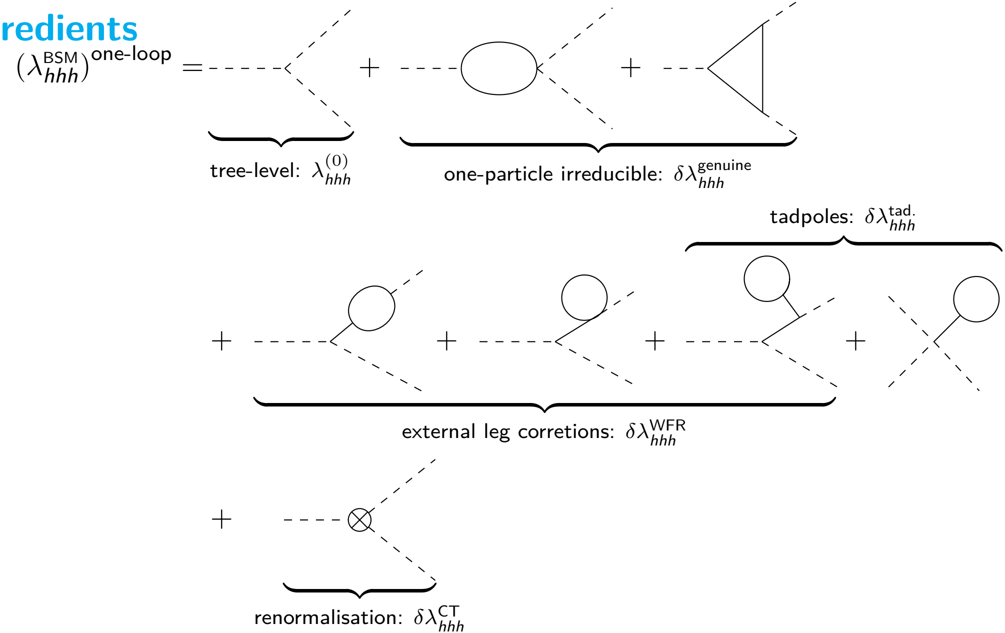
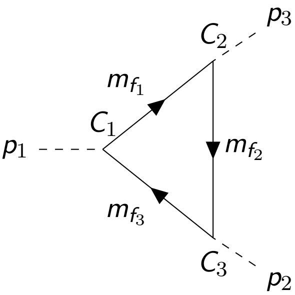


Example ingredient: generic fermion triangle

Idea: compute *generic* diagrams i.e. assume most generic



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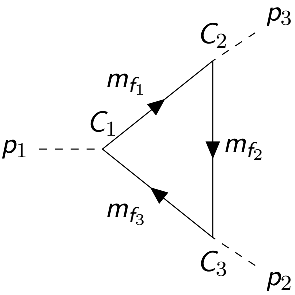

- couplings $C_i = P_L C_i^L + P_R C_i^R$, $P_{R/L} = \frac{1 \pm \gamma_5}{2}$
- as well as loop-masses m_{f_i} and
- external momenta p_i, $i = 1, 2, 3$.

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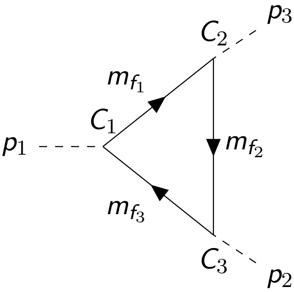

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- external momenta p_i, $i = 1, 2, 3$.

$$= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + 2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3}))$$

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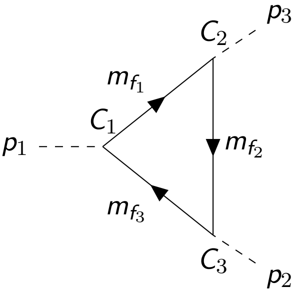

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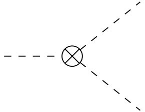
- insert concrete BSM model
- evaluate with the help of `COLLIER` [Denner et al. '16]
- public code `anyH3` [Bahl, Braathen, MG, Weiglein tbp]

Another ingredient: Renormalisation of λ_{hhh}

$\delta\lambda_{hhh}^{\text{CT}} =$  $= \; ?$

- one-loop $\rightarrow$ renormalisation of all parameters entering $\lambda_{hhh}^{(0),\text{BSM}}$
- reminder: $\lambda_{hhh}^{(0),\,\text{SM}} = 3\frac{m_h^2}{v^2}$
- $\rightarrow$ generalization (reminder: quasi-alignment!):

$$\lambda_{hhh}^{(0),\text{BSM}} \equiv \lambda_{hhh}^{(0),\text{BSM}}(\underbrace{v^{\text{SM}}, m_h^{\text{SM}}}_{\text{SM Higgs sector}}, m_{H_i}^{\text{BSM}}, v_i^{\text{BSM}}, \ldots)$$

$m_{H_i}^{\text{BSM}}$ → BSM masses; v_i^{BSM} → vevs; $\ldots$ → couplings etc. (that can't be expressed in terms of masses)

- user's choice:
  - SM sector: fully OS **or** $\overline{\text{MS}}/\overline{\text{DR}}$
  - BSM masses: OS **or** $\overline{\text{MS}}/\overline{\text{DR}}$
  - additional couplings/vevs: most likely $\overline{\text{MS}}$ **but also custom ren. conditions possible!**

$$\delta\lambda_{hhh}^{\text{CT}} = \sum_x \frac{\partial\lambda_{hhh}^{(0),\text{BSM}}}{\partial x}\delta x, \;\; x = (m_h^{SM})^{\text{OS}/\overline{\text{MS}}}, (v^{\text{SM}})^{\text{OS}/\overline{\text{MS}}}, (m_{H_i}^{\text{BSM}})^{\text{OS}/\overline{\text{MS}}}, (\ldots)^{\overline{\text{MS}}/\text{custom}}$$

(Default) Renormalization choice of $(v^{\text{SM}})^{\text{OS}}$ and $(m_i^2)^{\text{OS}}$

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  $\rightarrow$ ability to define *custom* renormalisation conditions

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- attention *(ii)*: scalar mixing may also require further CTs/tree-level relations

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All bosonic one- & two-point functions and their derivatives for general QFTs are required for flexible OS renormalisation.

Feature list (so far) of `anyH3`

- import/convert arbitrary `UFO` models
- definition of renormalisation schemes

```
# schemes.yml
renormalization_schemes:
  OS:
      mass_counterterms:
        h1: OS
        h2: OS
      VEV_counterterm: OS
  MS:
      mass_counterterms:
        h1: MS
        h2: MS
      VEV_counterterm: MS
```

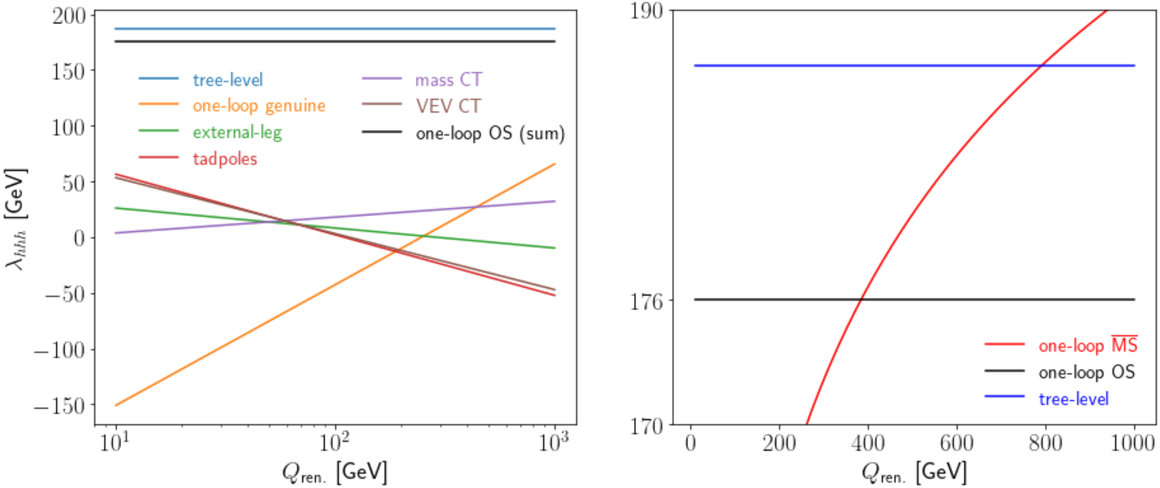
- optional: full p^2 dependence
- numerical / analytical / $\LaTeX$ outputs
- restrict to certain topologies
- restrict to certain particles in the loop
- python-library with command-line- and Mathematica-interface

```
from anyBSM import anyH3
myfancymodel = anyH3(
    'path/to/UFO/model',
    scheme = 'OS')
result = myfancymodel.lambdahhh()
```

- ...

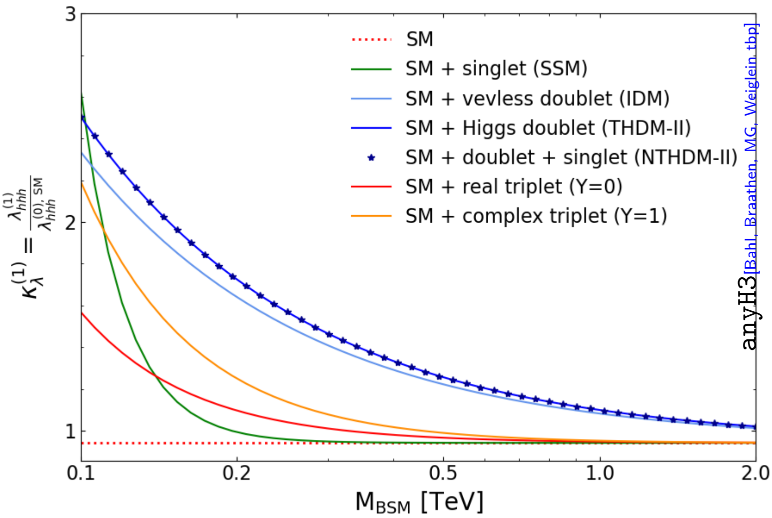
Results

Simplest model to consider for cross-check: SM



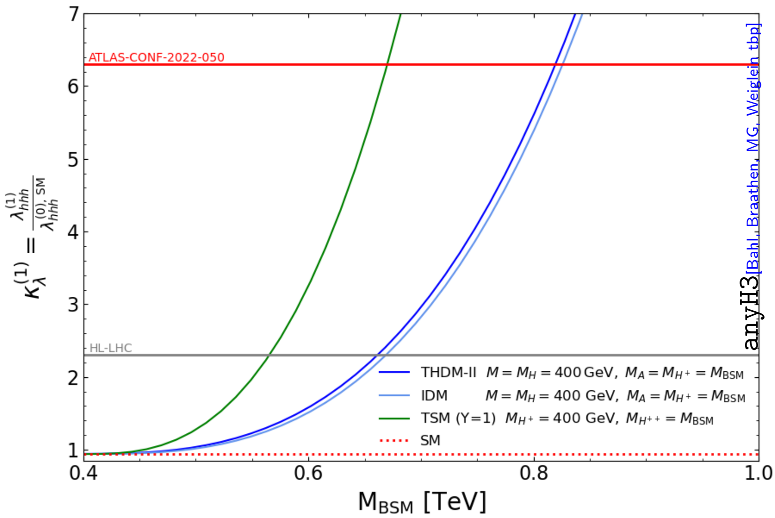
Leading two-loop $\mathcal{O}(\alpha_t(\alpha_t + \alpha_s))^{\text{OS}}$: $\mathcal{O}(+1.4\%)$ [Senaha '18] [Braathen et al. '19]

Decoupling in the alignment limit



- alignment means $\kappa_\lambda^{\text{tree-level}} = 1$
- recover SM result for $M_{BSM} \to \infty$
- many models built-in and cross-checked
- easy to implement new models (UFO)

Non-decoupling in the alignment limit



- mass spliting within the same multiplet
- induces large couplings for $M_{BSM} \to \infty$
- corrections large-enough to exclude otherwise unconstrained parameter space
- (see also talk by Johannes Braathen before)

Summary

- developed computer code anyH3 (anyBSM) for λ_{hhh} in arbitrary ren. QFTs
  - at the full one-loop order
  - with arbitrary choice of renormalization schemes
- uses UFO input (generate with SARAH, FeynRules or use a custom one)
- analytical results; fast numerical results; SM: $\mathcal{O}(0.2\,\mathrm{s})$, MSSM: $\mathcal{O}(0.5\,\mathrm{s})$
- many models already implemented:
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Future todos:

- publish
- more models / cross-checks
- go beyond one-loop
- non-SM self-couplings (e.g. $\kappa_{\lambda_{Hhh}}$)
- κ_t and κ_{tt}
-

Backup

Tadpole contributions to λ_{hhh}

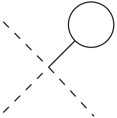
> In the SM: once λ_{hhh} is expressed in terms of *physical* input parameters, its result is independent of the treatment of the tadpoles:

$$\delta^{(1)}\lambda_{hhh} \supset -\frac{3}{v^2}\delta^{(1)}t_{\text{finite}}$$

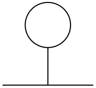
> However: `UFO` models do (often) **not** contain the explicit dependence on the tree-level/one-loop/one-loop-CT tadpoles.

> Thus: we choose the Fleischer-Jegerlehner treatment $t^{\text{tree-level}} = 0$ and renormalize $\delta^{(1)}t^{\text{CT}}|_{\text{finite}} = 0$ in the $\overline{\text{MS}}$ scheme per default (can also turn-off automatic tadpoles and implement own scheme).

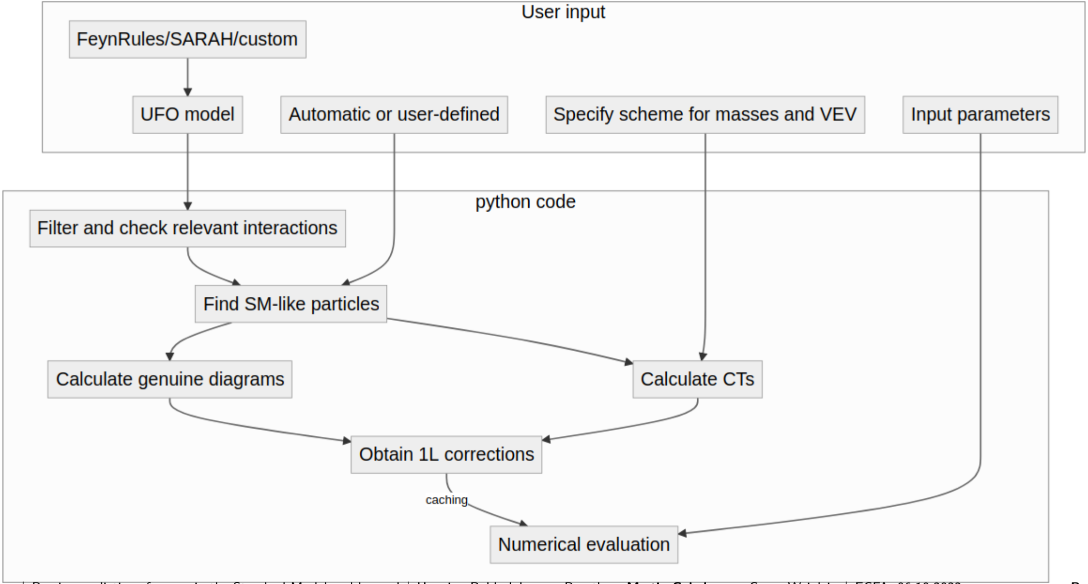
→ only need to take into account tadpole contributions to all two- and three-point functions:



and



Workflow



User input
FeynRules/SARAH/custom
UFO model
Automatic or user-defined
Specify scheme for masses and VEV
Input parameters
python code
Filter and check relevant interactions
Find SM-like particles
Calculate genuine diagrams
Calculate CTs
Obtain 1L corrections
caching
Numerical evaluation

W Mass

- start with HO corrections to muon decay: $M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha_{\text{em}}}{\sqrt{2}G_F}\left[1+\Delta r\right]$
- and solve for: $M_W^2=M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha_{\text{em}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r)}\right]$
- with: $\Delta r^{(1)}=2\delta^{(1)}e+\frac{\Pi_W^{(1),T}(0)-\delta^{(1)}M_W^2}{M_W^2}-\frac{\delta^{(1)}\sin^2\theta_W}{\sin^2\theta_W}+\delta_{\text{vertex+box}}$
- and: $\frac{\delta^{(1)}\sin^2\theta_W}{\sin^2\theta_W}=\frac{\cos^2\theta_W}{\sin^2\theta_W}\left(\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2}-\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2}\right)$

It's all there but:

- $\delta^{\text{SM}}_{\text{vertex+box}}=-\frac{2\,\text{sign}(\sin\theta_W)}{\cos\theta_W\sin\theta_W M_Z^2}\Pi_{Z\gamma}(p^2=0)+\frac{\alpha_{QED}}{4\pi\sin^2\theta_W}\left(6+\frac{7-4\sin^2\theta_W}{2\sin^2\theta_W}\right)\log(\cos^2\theta_W)$
- $\delta^{\text{BSM}}_{\text{vertex+box}}=$ **needs to be implemented**

However:

- in many models $\Delta r\supset\frac{\delta\sin^2\theta_W}{\sin\theta_W}\approx\delta\rho$ is the dominant effect!